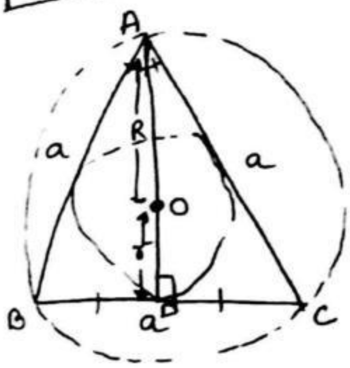
CLASS 58

2D-MENSURATION

Equilateral Δ



$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

$$\text{Altitude} = \frac{\sqrt{3}}{2}a$$

$$r = \frac{a}{2\sqrt{3}}$$

$$R = \frac{a}{\sqrt{3}}$$

O — incentre, circumcentre, orthocentre, centroid.

AD — median, ⊥ bisector, Altitude, Angle bisector.

$$\frac{\text{Area (circumcircle)}}{\text{Area (incircle)}} = \frac{4}{1}$$

$$\frac{\text{Radius (circumcircle)}}{\text{Radius (incircle)}} = \frac{2}{1}$$

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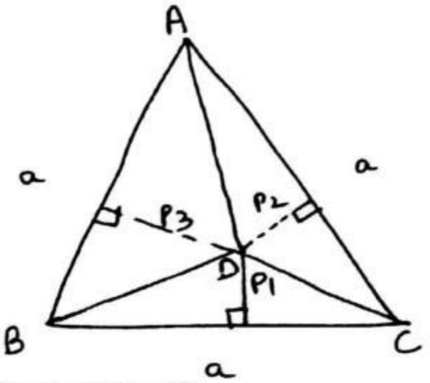


$$Ar(BDC) = \frac{1}{2} \times a \times P_1$$

$$Ar(ADC) = \frac{1}{2} \times a \times P_2$$

$$Ar(ABD) = \frac{1}{2} \times a \times P_3$$

$$\frac{1}{2} \times a \times P_1 + \frac{1}{2} \times a \times P_2 + \frac{1}{2} \times a \times P_3 = \frac{\sqrt{3}}{4}$$

$$\boxed{\frac{\sqrt{3}}{2}a = P_1 + P_2 + P_3}$$

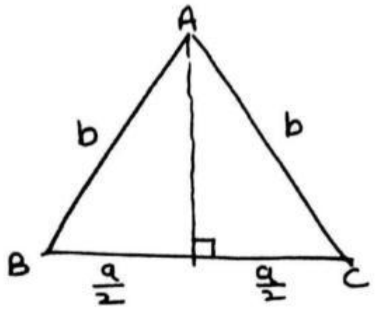
$$\boxed{a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)}$$

1. Find the area of the equilateral Δ in w/c three altitude of length $\sqrt{3}$, $2\sqrt{3}$, $5\sqrt{3}$ are drawn from a ~~side~~ point inside the Δ.

$$a = \frac{2}{\sqrt{3}}\left(\sqrt{3} + 2\sqrt{3} + 5\sqrt{3}\right) = \frac{2}{\cancel{\sqrt{3}}} \times 8\cancel{\sqrt{3}} = 16$$

$$\text{Area} = \frac{\sqrt{3}}{\cancel{4}} \times \cancel{16}^{\,4} \times 16 = 64\sqrt{3} \quad \underline{\text{Ans}}$$

Isosceles Triangle



$$\text{Altitude} = \frac{1}{2}\sqrt{4b^2 - a^2}$$

$$\text{Area} = \frac{a}{4}\sqrt{4b^2 - a^2}$$

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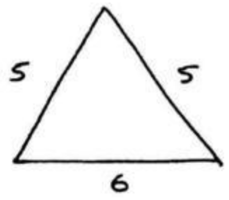
(2) find the area of a Δ whose sides are 5, 5 & 6 cm



$$\text{Area} = \frac{6}{4}\sqrt{100-36}$$

$$= \frac{6}{4} \times 8^2 = 12\ cm^2$$

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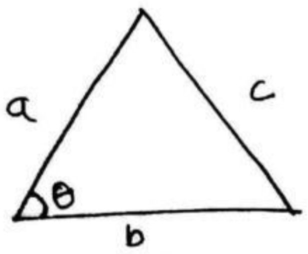
(3) Find the area of Δ whose sides are 5, 6 & 7 cm.

$$s = \frac{5+6+7}{2} = 9$$

$$\text{Area} = \sqrt{9\times4\times3\times2} = \sqrt{216} = \sqrt{36\times6} = 6\sqrt{6}\ cm^2$$

Scalene Δ



$$s = \frac{a+b+c}{2} \quad \Big| \quad \text{Area} = \frac{1}{2}\times a\times b\times \sin\theta$$

$$\text{Area } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$r = \frac{A}{s}, \quad R = \frac{abc}{4A}$$

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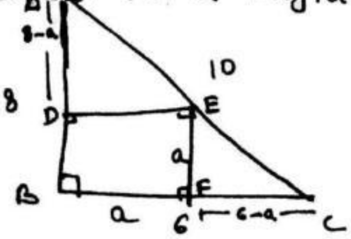
(4) find the area of a square (maximum size) w/c can inscribed in a right angle Δ of side 6, 8, 10 cm



$\frac{1}{2}\times6\times8$

$=24$

$$\underbrace{\frac{1}{2}\times a\times(8-a)}_{\Delta ADE} + \underbrace{\frac{1}{2}(6-a)a}_{\Delta EFC} + \underbrace{a^2}_{\square BDEF} = 24$$

$$7a - \cancel{a^2} + \cancel{a^2} = 24$$

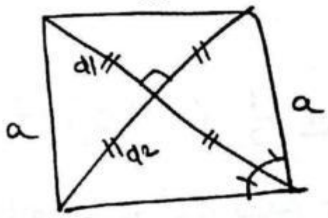
$$\boxed{a = \frac{24}{7}} \qquad \text{Area} = \frac{576}{49}$$

side of the maximum size square inscribed in a right angle Δ =

$$\boxed{a = \frac{P \times B}{P + B}}$$

$\rightarrow \frac{8 \times 6}{8+6} = \frac{\cancel{48}^{24}}{\cancel{14}_{7}} = \frac{24}{7}$

Square

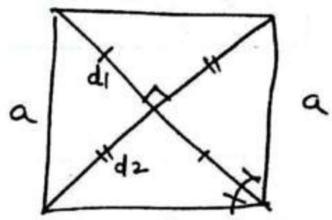


Area $= a^2$

$P = 4a$

$d_1 = d_2$

Rhombus



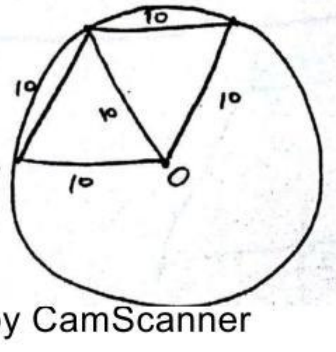
Area $= \frac{1}{2} \times d_1 \times d_2$

$P = 4a$

$d_1 \neq d_2$

Area $= a^2 \sin\theta$

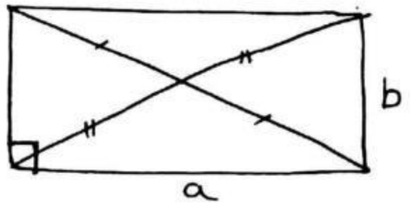
(5) find the area of a rhombus whose 3 vertex lie on the circumferance of a circle and one vertex lie on the centre of circle of radius 10 cm.



$2 \times \left(\frac{\sqrt{3}}{\cancel{4}_{2}} \times \cancel{10}^{5} \times 10\right)$

$= 50\sqrt{3}.$

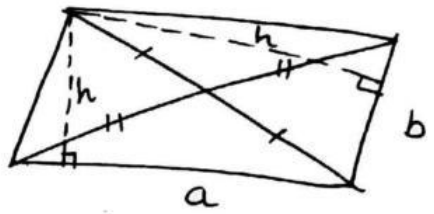
Rectangle



Area $= a \times b$

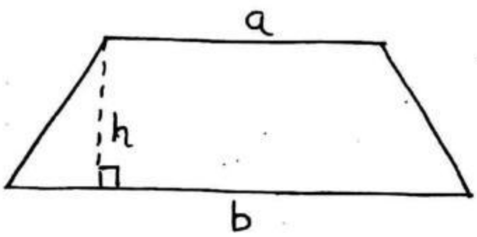
$P = 2(a+b)$

Parallelogram



Area $= a \times h = b \times h$

Trapezium



Area $= \frac{1}{2}(a+b) \times h$

= Avg of 2 parallel lines $\times h$

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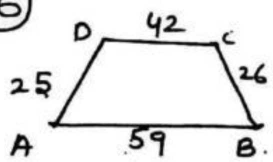
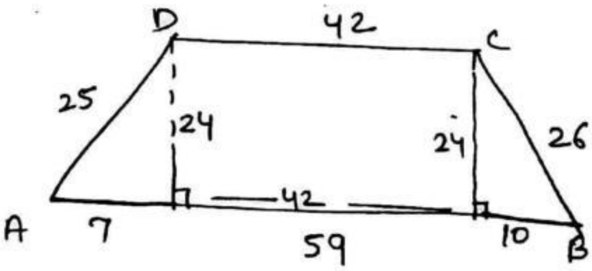
⑥



AB ∥ CD

find Area.



⑰ (7, 24, 25 / 10, 24, 26)

$\frac{1}{\cancel{2}} \times (42+59) \times \cancel{24}\,12$

$101 \times 12 = 1212$

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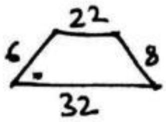
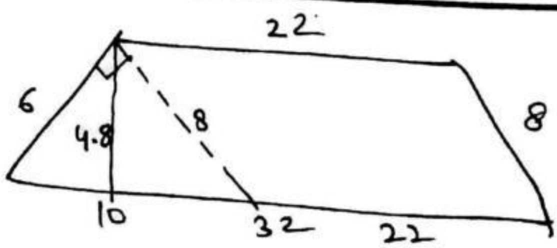
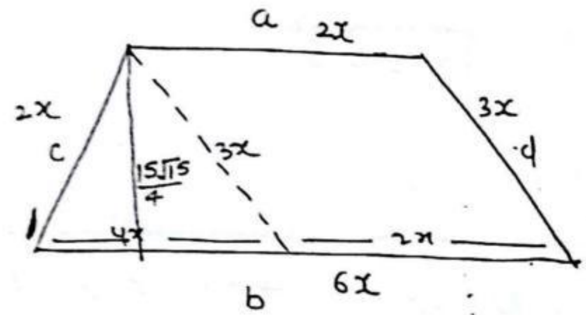
⑦





$\frac{1}{2} \times (22+32) \times 4.8 = 129.6$

8) The ratio of length of two parallel lines is 1:3 of a traperium while non-parallel sides ratio is 2:3 if the ratio of length of larger parallel line to the larger non-parallel line is 2:1. & height is $\frac{15\sqrt{15}}{4}$ cm find the area of the traperium.



$a : b \quad c : d$

$1_{\times 2} : 3_{\times 2} \quad 2 : 3$

$2 : 1$

$2x : 6x : 2x : 3x$

A

$S = \frac{2x+3x+4x}{2} = \frac{9}{2}x$

$Area = \sqrt{\frac{9}{2}x \times \frac{5}{2}x \times \frac{3}{2}x \times \frac{1}{2}x}$

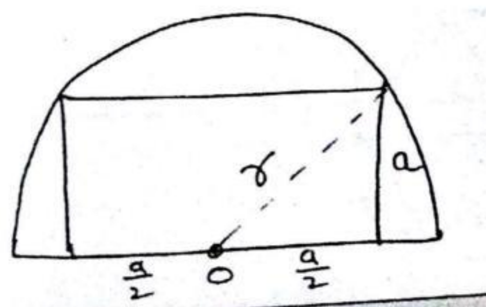
$\frac{3x^2\sqrt{15}}{4}$

$\therefore \quad \frac{1}{2} \times 4x \times \frac{15\sqrt{15}}{4} = \frac{3x^2\sqrt{15}}{4}$ (equating area of Δ)

$\boxed{x = 10}$

Area of Traperium $= \frac{1}{2}(20+60) \times \frac{15\sqrt{15}}{4} = 150\sqrt{15}$

9) find the side of a maximum size square w/c can be inscribed in a semi-circle of radius r. cm.

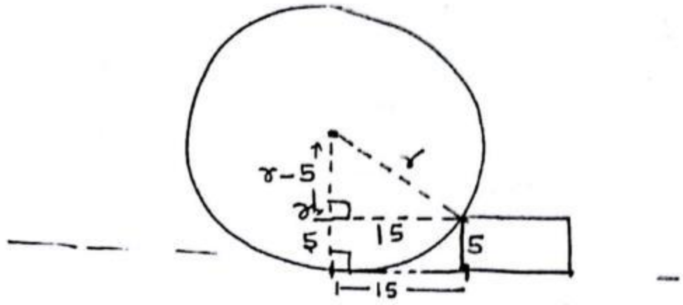


$r^2 = a^2 + \frac{a^2}{4}$

$r = \frac{\sqrt{5}}{2}a$

$a = \frac{2r}{\sqrt{5}}$ Ans

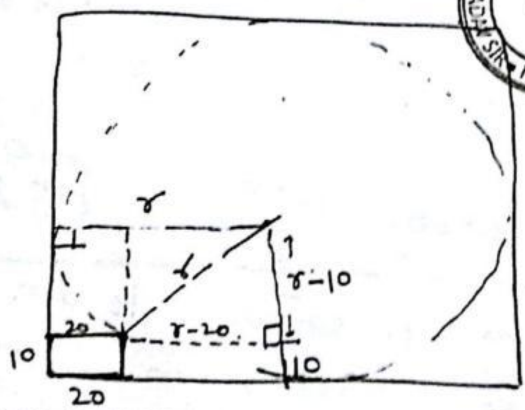
(10) A brick of 5 cm is placed against a wheel. The horizontal distance of the face of the brick stopping the wheel from the point where the wheel touches the ground is 15 cm. find the radius of wheel.



$$r^2 = (r-5)^2 + 15^2$$

$$\boxed{r = 25}$$

(11) find the radius of maximum size circle that can be inscribed in a square. If a rectangle of length 20 cm and breadth 10 cm is constructed in the corner of the square b/w the space of square & circle. The three vertices of ▭ lies on the ~~circumference~~ square and one vertex lies on the circumference of circle.



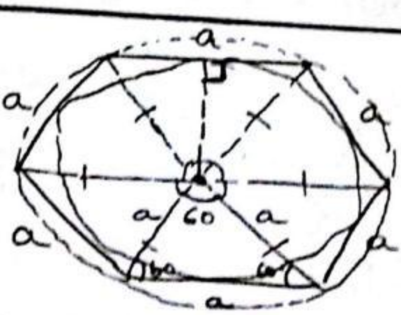
$$r^2 = (r-10)^2 + (r-20)^2$$

$$\boxed{r = 50}$$

$\boxed{\text{Hexagon}}$

$$\text{Area} = 6 \times \frac{\sqrt{3}}{4} a^2$$

$$= \frac{3\sqrt{3}}{2} a^2$$



Radius of circumcircle (R) = a (Side of Hexagon)

Radius of incircle (r) = $\frac{\sqrt{3}}{2}a$

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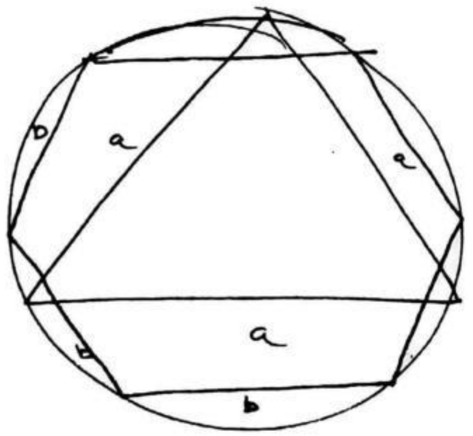
(12) find the ratio of length of an equilateral Δ and a regular hexagon w/c are on the circumference of the circle.



$r = \frac{a}{\sqrt{3}}$ (from Δ)

$r = b$ (from Hexagon)

$\frac{a}{\sqrt{3}} = b$ (Equate both r)

$\boxed{\frac{a}{b} = \frac{\sqrt{3}}{1}}$ Ans.

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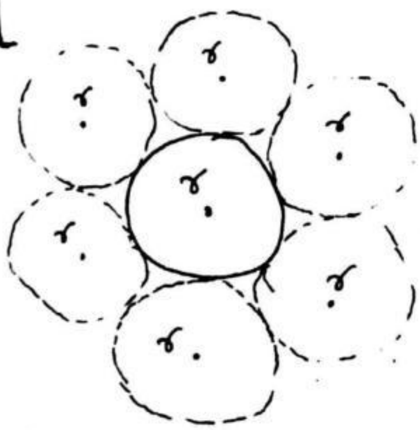
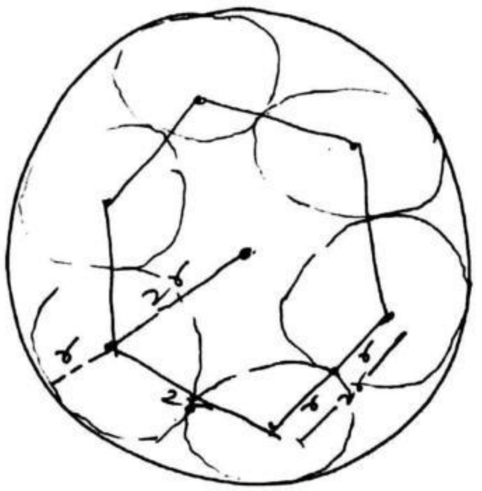


Around a circle of radius r, only 6 circles can be drawn w/c touches the main circle and two other circle of the same radius r.

13



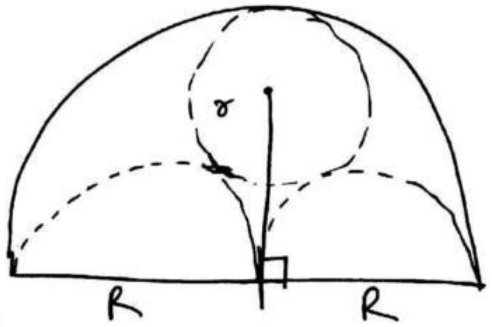
$R = 10$

$r = ?$

$3r = 10$

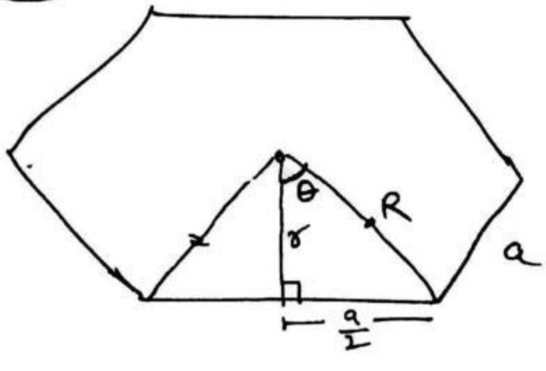
$r = \frac{10}{3}$

\#



$\boxed{r = \frac{R}{3}}$

\#

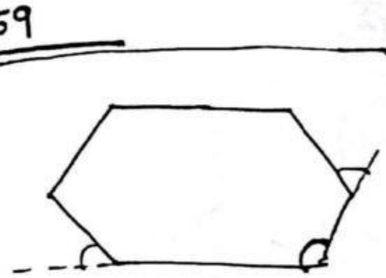


circum-circle radius of any polygon $= \frac{a}{2} \operatorname{cosec} \frac{180^\circ}{n}$

incircle radius of any polygon $= \frac{a}{2} \cot \frac{180^\circ}{n}$

Area of any polygon of n sides $= \frac{na^2}{4} \cot \frac{180^\circ}{n}$

CLASS 59



* Sum of all internal angles $= (n-2)\times 180^\circ$

* Each interior angle $= \dfrac{(n-2)\times 180^\circ}{n}$

* Sum of all exterior angles $= 360^\circ$

* Each exterior angle $= \dfrac{360^\circ}{n}$

* No. of diagonal $= \dfrac{n(n-3)}{2}$

(14) find the no. of sides of a polygon in w/c the no. of diagonals are 27.

$$\frac{n(n-3)}{2} = 27$$

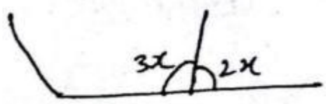
$$n(n-3) = 54$$

$\downarrow \quad \downarrow$

$9 \quad 6$

$\boxed{n=9}$

(15) find the no. of sides in a regular polygon in w/c the ratio of each external angle to each internal angle is 2 : 3



$$5x = 180^\circ$$

$$2x = 72^\circ$$

$$\frac{\cancel{360}^{\,5}}{n} = \cancel{72^\circ}$$

$\boxed{n=5}$

(16) find the length of the side of a regular octagon w/c is formed by cutting the corner of a square of side 10 cm

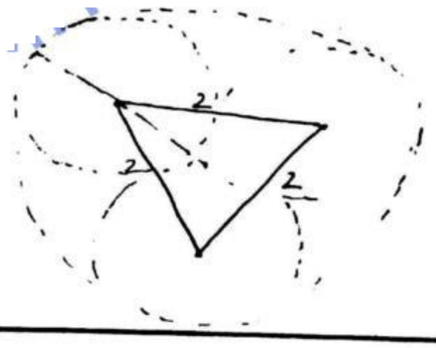
* Area of any octagon = $2a^2(1+\sqrt{2})$

* side of square = side of octagon $(\sqrt{2}+1)$

$10 =$ side of octagon $(\sqrt{2}+1)$

side of octagon $= \dfrac{10}{(\sqrt{2}+1)}$

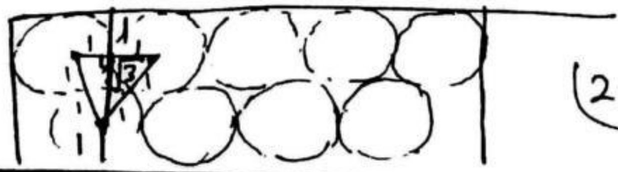
(17) Three circle of radius 1 cm touch one another externally. find the area of the circle circumscribing the three circles.



$R = \dfrac{a}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

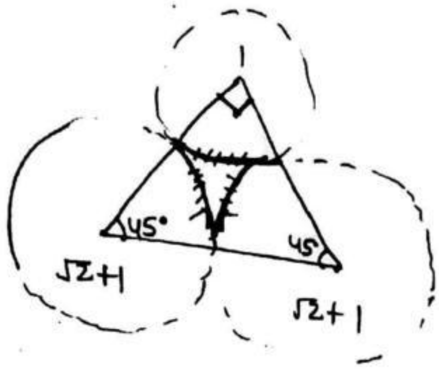
$\dfrac{2}{\sqrt{3}}+1 = \dfrac{2+\sqrt{3}}{\sqrt{3}}$

(18) The length of a rectangular sheet is 10 cm. What would be its minimum breadth so that 9 circular sheets of radius 1 cm can be cut out from it.



$(2+\sqrt{3})$

(19) find the length of common arc of three circle of radius 1 cm, $(\sqrt{2}+1)$ cm, $(\sqrt{2}+1)$ cm touches one another externally.



$$\boxed{Arc = \frac{\theta}{180}\pi r}$$

सिर्फ दो case में arc निकाल सकते हैं या तो Δ equilateral हो या फिर right angle isosceles हो ।

$$\frac{90^\circ}{180}\pi(1) + \frac{45^\circ}{180}\pi(\sqrt{2}+1) \times 2$$

$$\frac{\pi}{2} + \frac{\pi}{2}(\sqrt{2}+1)$$

$$\frac{\pi}{2}[1+\sqrt{2}+1] = \frac{\pi}{2}[2+\sqrt{2}]$$

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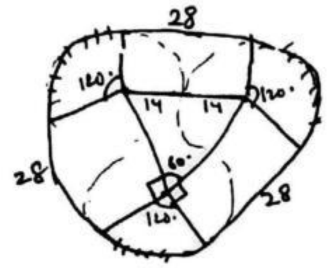
(20) find the length of minimum rubber band w/c can tide three circle of radius 14 cm



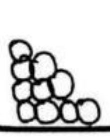
$$\frac{120}{180} \times \pi \times 14 \times 3$$

$$= 88$$

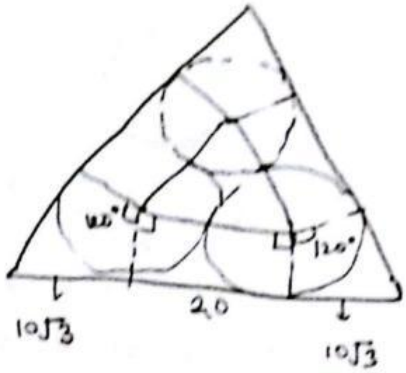
length of Rubber band = 84 + 88
= 172 cm.

\#  ⇒ min. length of Rubber band = $3D + 2\pi r$

\#  ⇒ min. length of Rubber band = $6D + 2\pi r$

\#  ⇒ min length of Rubber band = $9D + 2\pi r$.

(21)



radius = 10 cm

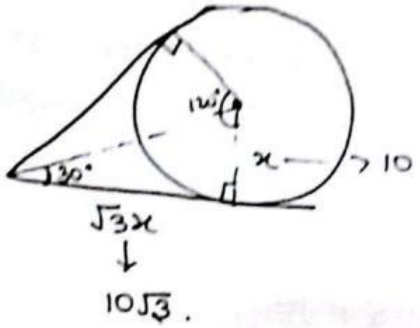
find perimeter of Δ.

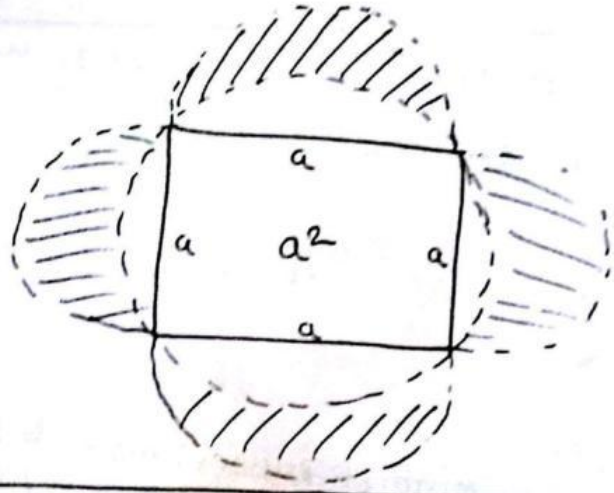
Perimeter =

$3(20 + 10\sqrt{3} + 10\sqrt{3})$

$60 + 60\sqrt{3}$

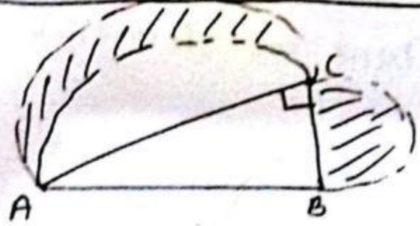
$60(1 + \sqrt{3})$





$\boxed{\text{Area of shaded portion} = \text{Area of base figure} = a^2}$

(22)



Area Δ ABC = 50.

find area of shaded portion.

Area of shaded portion = 50.