SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)#	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)#	–	–	–	**2(2)**
4.	Determinants	1(1)	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)*	1(3)*	–	**3(9)**
7.	Integrals	1(1)*	1(2)*	1(3)	–	**3(6)**
8.	Application of Integrals	1(1)	1(2)	1(3)	–	**3(6)**
9.	Differential Equations	1(1)	1(2)	1(3)	–	**3(6)**
10.	Vector Algebra	3(3)#	1(2)	–	–	**4(5)**
11.	Three Dimensional Geometry	2(2)	1(2)	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2)# + 1(4)	1(2)*	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is a prime number}\}$, find $P(E)$.

OR

If A and B are two events such that $P(A|B) = p$, $P(A) = p$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{5}{9}$, then find the value of p.

2. If A and B are the points (– 3, 4, – 8) and (5, – 6, 4) respectively, then find the ratio in which yz-plane divides $\overrightarrow{AB}$.

3. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix?

OR

If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then find the matrix PQ.

4. Find the distance of the point (2, 3, 4) from the plane $\vec{r}\cdot(3\hat{i}-6\hat{j}+2\hat{k})+11=0$.

5. Evaluate : $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

OR

Evaluate : $\int \frac{\cot x}{\sqrt[3]{\sin x}}\,dx$

6. Prove that the area bounded by the parabola $y^2 = 4ax$ and the line $x = a$ and $x = 4a$ is $\frac{56a^2}{3}$ sq. units.

7. If the position vector $\vec{a}$ of a point (12, n) is such that $|\vec{a}|=13$, then find the value of n .

OR

Find the projection of vector $7\hat{i}+\hat{j}-4\hat{k}$ on the vector $2\hat{i}+6\hat{j}+3\hat{k}$.

8. Find the value of λ for which the vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel.

9. The number of bijective functions from the set A to itself, if A contains 108 elements is $n!$. Find the value of n.

OR

If the set A contains 5 elements and the set B contains 6 elements, then find the number of one-one and onto mappings from A to B.

10. Find the order of the differential equation whose general solution is given by $y = (A + B)\cos(x + C) + De^x$.

11. Construct a 2 × 3 matrix whose elements a_{ij} are given by $a_{ij} = 2i - 3j$.

12. The random variable X has the following probability distribution :

X	0	1	2	3	4	5
$P(X)$	0.1	k	0.1	$3k$	0.3	k

Find the value of k.

13. Find the range of the function $f(x)=\frac{|x-1|}{(x-1)}$.

14. For what value of x, matrix $A = \begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is a singular matrix?

15. Let R be a relation on the set N be defined by $\{(x, y) : x, y \in N, 2x + y = 41\}$. Show that R is neither reflexive nor symmetric.

16. Write the direction cosines of a line parallel to the line $\frac{3-x}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. In a school, a football game is to be organised between students of class 11th and 12th. For which, a team from each class is chosen, say T_1 be the team of class 11th and T_2 be the team class 12th. These teams have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probability of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2},\frac{1}{6}$ and $\frac{1}{3}$ respectively.

Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game.
Let X and Y denote the total points scored by team T_1 and T_2, respectively, after two games.
Based on the above information answer the following :

(i) $P(T_2$ winning a match against $T_1)$ is equal to
(a) 1/2 (b) 1/6 (c) 1/3 (d) none of these

(ii) $P(T_2$ drawing a match against $T_1)$ is equal to
(a) 1/2 (b) 1/3 (c) 1/6 (d) 2/3

(iii) $P(X > Y)$ is equal to
(a) 1/4 (b) 5/12 (c) 1/2 (d) 7/12

(iv) $P(X = Y)$ is equal to
(a) 11/36 (b) 1/3 (c) 13/36 (d) 1/2

(v) $P(X + Y = 12)$ is equal to
(a) 0 (b) 5/12 (c) 13/36 (d) 7/12

18. Mr. Vinay is the owner of apartment complex with 50 units. When he set rent at ₹ 8000/month, all apartments are rented. If he increases rent by ₹ 250/month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹ 500/month.
Based on the above information answer the following :

(i) If P is the rent price per apartment and N is the number of rented apartment, then profit is given by
(a) NP (b) $(N - 500)P$ (c) $N(P - 500)$ (d) none of these

(ii) If x represent the number of apartments which are not rented, then the profit expressed as a function of x is
(a) $(50 - x)(30 + x)$ (b) $(50 + x)(30 - x)$
(c) $250(50 - x)(30 + x)$ (d) $250(50 + x)(30 - x)$

(iii) If $P = 8500$, then $N =$
(a) 50 (b) 48 (c) 49 (d) 47

(iv) If $P = 8250$, then the profit is
(a) ₹ 379750 (b) ₹ 4,00,000 (c) ₹ 4,05,000 (d) ₹ 4,50,000

(v) The rent that maximizes the total amount of profit is
(a) ₹ 5000 (b) ₹ 10500 (c) ₹ 14800 (d) ₹ 14500

PART - B

Section - III

19. Find the value of 'a' if the function $f(x)$ defined by $f(x)=\begin{cases} 2x-1, & x<2 \\ a, & x=2 \\ x+1, & x>2 \end{cases}$ is continuous at $x = 2$.

20. Evaluate : $\int x^2(ax+b)^{-2}dx$

OR

Evaluate : $\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx$

21. Find the area bounded by the line $y = x$, x-axis and lines $x = -1$ to $x = 2$.

22. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, find A^{-1}.

23. A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it. What is the probability of the parts that make it through the inspection machine and get shipped?

OR

If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find $P(B) - P(A)$.

24. Simplify : $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$

25. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point $P(1, 3, 3)$.

26. Find the solution of differential equation $y^3 - \frac{dy}{dx} = x^2 \frac{dy}{dx}$.

27. If $\vec{a} = -3\hat{i} + n\hat{j} + 4\hat{k}$ and $\vec{b} = -2\hat{i} + 4\hat{j} + p\hat{k}$ are collinear, then find the value of n and p.

28. Find the equation of the tangent to the curve $y = 3x^2 - x + 1$ at $P(1, 2)$.

OR

Show, that the function $f(x) = x^9 + 4x^7 + 11$ is increasing on R.

Section - IV

29. Show that $f(x) = [x]$ is not differentiable at $x = 1$.

30. Find the area bounded by the X-axis, part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates at $x = 2$ and $x = 4$.

31. Find the equation of normal to the curve $16x^2 + 9y^2 = 144$ at $(2, y_1)$ where $y_1 > 0$.

OR

Find the interval on which the function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is increasing.

32. Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$.

33. Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation.

34. Show that the exponential function a^x is (where $a > 0$) continuous at every point.

OR

Check whether the function $f(x) = \begin{cases} \frac{(3^{\sin x} - 1)^2}{x \cdot \log(1+x)}, & \text{for } x \neq 0 \\ 2\log 3, & \text{for } x = 0 \end{cases}$ is continuous or not.

35. Evaluate : $\int \frac{dx}{1 - \cos x - \sin x}$

Section-V

36. An amount of ₹5000, is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment by matrix method.

OR

Express the following matrix as the sum of a symmetric matrix and a skew-symmetric matrix and verify your result : $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

37. Find the image of the point (1, – 2, 1) in the line $\dfrac{x-2}{3} = \dfrac{y+1}{-1} = \dfrac{z+3}{2}$.

OR

The lines $\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(m\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ are coplanar. Find the value of m.

38. Find the maximum value of $Z = 4x + 6y$ subject to constraints $3x + 2y \le 12$, $x + y \ge 4$, $x \ge 0$, $y \ge 0$.

OR

Find the number of points at which he objective function $Z = 4x + 3y$ can be maximized subject to the constraints $3x + 4y \le 24$, $8x + 6y \le 48$, $x \le 5$, $y \le 6$, $x \ge 0$, $y \ge 0$.

SOLUTIONS

1. $P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$
$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

OR

We have, $P(A) = p, P(B) = \dfrac{1}{3}$ and $P(A \cup B) = \dfrac{5}{9}$

Now, $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = p \Rightarrow P(A \cap B) = \dfrac{p}{3}$

Since, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{5}{9} = p + \frac{1}{3} - \frac{p}{3} \Rightarrow \frac{5-3}{9} = \frac{2p}{3} \Rightarrow p = \frac{2}{9} \times \frac{3}{2} = \frac{1}{3}$$

2. Let $\vec{a} = -3\hat{i} + 4\hat{j} - 8\hat{k}$, $\vec{b} = 5\hat{i} - 6\hat{j} + 4\hat{k}$

Let $C(\vec{c})$ be the point in yz-plane which divides $\overrightarrow{AB}$ in the ratio $r : 1$.

Then, $0 = \dfrac{5r-3}{r+1}$ $\quad (\because$ In yz-plane, $x = 0)$

$\Rightarrow \quad 5r - 3 = 0 \Rightarrow r = \dfrac{3}{5}$

$\therefore$ Required ratio is 3 : 5.

3. Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. Now, A is an identity matrix then, $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad \cos\alpha = 1$ and $\sin\alpha = 0 \Rightarrow \alpha = 0^\circ$.

OR

Since, $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$$\therefore \quad PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix} \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

4. Here, $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

The distance of the point $(2\hat{i} + 3\hat{j} + 4\hat{k})$ is

$$\left| \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) + 11}{\sqrt{9 + 36 + 4}} \right| = \left| \frac{6 - 18 + 8 + 11}{7} \right|$$

$= 1$ unit

5. Let $I = \displaystyle\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

Put $x = t^6 \Rightarrow dx = 6t^5\, dt$

$$\Rightarrow I = \int \frac{6t^5}{t^3 + t^2} dt = 6\int \frac{t^3}{t+1} dt = 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt$$

$= 2t^3 - 3t^2 + 6t - 6\log(t + 1) + C$

$= 2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x} + 1) + C.$

OR

Let $I = \displaystyle\int \frac{\cot x}{\sqrt[3]{\sin x}} dx = \int \frac{\cos x}{\sin^{1/3} x \cdot \sin x} dx$

$= \displaystyle\int \frac{\cos x}{\sin^{4/3} x} dx = \int \sin^{-4/3} x \cdot \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

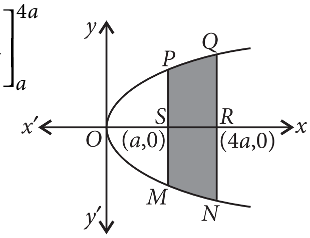
$$\Rightarrow \quad I = \int t^{-4/3}\, dt = \frac{t^{-1/3}}{-1/3} + C = \frac{-3}{\sqrt[3]{\sin x}} + C$$

6. Required area = 2 × area of region $PSRQP$

$$= 2\int_a^{4a} \sqrt{4ax}\, dx = 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a}$$

$$= \frac{8}{3}\sqrt{a}\left(8a^{3/2} - a^{3/2}\right)$$

$$= \frac{56a^2}{3} \text{ sq. units}$$



7. The position vector of the point $(12, n)$ is $12\hat{i} + n\hat{j}$.

$\therefore \quad \vec{a} = 12\hat{i} + n\hat{j} \Rightarrow |\vec{a}| = \sqrt{12^2 + n^2} = 13$ (Given)

$\Rightarrow \quad 12^2 + n^2 = 169 \Rightarrow n^2 = 25 \Rightarrow n = \pm 5$

OR

Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{(7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = \frac{14 + 6 - 12}{7} = \frac{8}{7}$$

8. $\vec{a} = 3\hat{i} - 6\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$

Since, $\vec{a}$ and $\vec{b}$ are parallel $\quad \therefore \quad \vec{a} \times \vec{b} = \vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 1 \\ 2 & -4 & \lambda \end{vmatrix} = \vec{0}$$

$\Rightarrow \quad (-6\lambda + 4)\hat{i} - (3\lambda - 2)\hat{j} + (-12 + 12)\hat{k} = \vec{0}$

$\Rightarrow \quad (-6\lambda + 4)\hat{i} + (2 - 3\lambda)\hat{j} = 0\hat{i} + 0\hat{j}$

Comparing coefficients of $\hat{i}$ and $\hat{j}$, we get

$-6\lambda + 4 = 0$ and $2 - 3\lambda = 0 \Rightarrow \lambda = 2/3$

9. Since number of one-one onto functions from a set A having n elements to itself is $n!$.

$\therefore$ Required value of n is 108.

OR

As A contains 5 elements.

$\therefore$ For any one-one onto mapping $f: A \to B, f(A)$ also contains 5 elements but B contains 6 elements.

$\therefore$ $f(A) \neq B$.

So, no one-one mapping from A to B can be onto.

10. Given $y = (A + B) \cos (x + C) + De^x$

or $y = k \cos (x + C) + De^x$

Now order of a differential equation is same as the number of arbitrary unknowns present in the solution. Hence order of differential equation is 3.

11. Here $a_{ij} = 2i - 3j$, $i = 1, 2$ and $j = 1, 2, 3$

$\therefore$ $a_{11} = 2\cdot1 - 3\cdot1 = -1$, $a_{12} = 2\cdot1 - 3\cdot2 = -4$,

$a_{13} = 2\cdot1 - 3\cdot3 = -7$, $a_{21} = 2\cdot2 - 3\cdot1 = 1$,

$a_{22} = 2\cdot2 - 3\cdot2 = -2$, $a_{23} = 2\cdot2 - 3\cdot3 = -5$

Hence, the required matrix is $\begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \end{bmatrix}$.

12. $\because \sum_{x=0}^{5} P(X = x) = 1$

$\Rightarrow P(X = 0) + P(X = 1) + ... + P(X = 5) = 1$

$\Rightarrow 0.1 + k + 0.1 + 3k + 0.3 + k = 1 \Rightarrow 0.5 + 5k = 1$

$\Rightarrow 5k = 1 - 0.5 = 0.5 \Rightarrow k = 0.1$

13. We have, $|x-1| = \begin{cases} x-1, & x \geq 1 \\ 1-x, & x < 1 \end{cases}$

$\therefore$ $f(x) = \dfrac{|x-1|}{(x-1)} = \begin{cases} 1, & x \geq 1 \\ -1, & x < 1 \end{cases}$

$\therefore$ Range $(f) = \{-1, 1\}$

14. Matrix A is singular, when $|A| = 0$

$$\begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = 0$$

$\Rightarrow 6 - x - 12 + 4x = 0 \Rightarrow 3x = 6 \Rightarrow x = 2$

15. $R = \{(x, y) : x, y \in N, 2x + y = 41\}$

Reflexive : $(1, 1) \notin R$ as $2 \cdot 1 + 1 = 3 \neq 41$. So, R is not reflexive.

Symmetric : $(1, 39) \in R$ but $(39, 1) \notin R$. So R is not symmetric.

16. We have, $\dfrac{x-3}{-3} = \dfrac{y+2}{-2} = \dfrac{z+2}{6}$

$\Rightarrow$ Direction ratios are $-3, -2, 6$.

$\therefore$ Direction cosines are $\dfrac{-3}{7}, \dfrac{-2}{7}, \dfrac{6}{7}$.

These are direction cosines of a line parallel to given line.

17. (i) (c) : Clearly, $P(\text{T}_2$ winning a match against $\text{T}_1)$

$= P(\text{T}_1 \text{ losing}) = \dfrac{1}{3}$

(ii) (c) : Clearly, $P(\text{T}_2$ drawing a match against $\text{T}_1)$

$= P(\text{T}_1 \text{ drawing}) = \dfrac{1}{6}$

(iii) (b) : According to given information, we have the following possibilities for the value of X and Y.

X	6	4	3	2	1	0
Y	0	1	3	2	4	6

Now, $P(X > Y) = P(X = 6, Y = 0) + P(X = 4, Y = 1)$

$= P(\text{T}_1 \text{ win}) P(\text{T}_1 \text{ win}) + P(\text{T}_1 \text{ win}) P(\text{match draw})$
$+ P(\text{match draw}) P(\text{T}_1 \text{ win})$

$= \dfrac{1}{2}\cdot\dfrac{1}{2} + \dfrac{1}{2}\cdot\dfrac{1}{6} + \dfrac{1}{6}\cdot\dfrac{1}{2} = \dfrac{3+1+1}{12} = \dfrac{5}{12}$

(iv) (c) : $P(X = Y) = P(X = 3, Y = 3) + P(X = 2, Y = 2)$

$= P(\text{T}_1 \text{ win}) P(\text{T}_2 \text{ win}) + P(\text{T}_2 \text{ win}) P(\text{T}_1 \text{ win})$
$+ P(\text{match draw}) P(\text{match draw})$

$= \dfrac{1}{2}\cdot\dfrac{1}{3} + \dfrac{1}{3}\cdot\dfrac{1}{2} + \dfrac{1}{6}\cdot\dfrac{1}{6} = \dfrac{1}{6} + \dfrac{1}{6} + \dfrac{1}{36} = \dfrac{13}{36}$

(v) (a) : From the given information, it is clear that maximum sum of X and Y can be 6, therefore $P(X + Y = 12) = 0$

18. (i) (c) : If P is the rent price per apartment and N is the number of rented apartment, the profit is given by $P(N) = NP - 500\,N = N(P - 500)$ [$\because$ ₹ 500/month is the maintenance charges for each occupied unit]

(ii) (c) : Now, if x be the number of non-rented apartments, then $N = 50 - x$ and $P = 8000 + 250\,x$

Thus, $P = N(P - 500) = (50 - x)\,(8000 + 250\,x - 500)$

$= (50 - x)\,(7500 + 250\,x) = 250(50 - x)\,(30 + x)$

(iii) (b) : Clearly, if $P = 8500$, then

$8500 = 8000 + 250\,x \Rightarrow x = 2 \Rightarrow N = 48$

(iv) (a) : Also, if $P = 8250$, then

$8250 = 8000 + 250\,x \Rightarrow x = 1$ and so profit

$P(1) = 250(50 - 1)\,(30 + 1) =$ ₹ 379750

(v) (b) : We have, $P(x) = 250(50 - x)\,(30 + x)$

Now, $P'(x) = 250[50 - x - (30 + x)] = 250[20 - 2x]$

For maxima/minima, put $P'(x) = 0$

$\Rightarrow 20 - 2x = 0 \Rightarrow x = 10$

Thus, price per apartment is, $P = 8000 + 2500 = 10500$

Hence, the rent that maximizes the profit is ₹ 10500.

19. For f to be continuous at $x = 2$, we must have

$\lim_{x \to 2^-} f(x) = f(2) = \lim_{x \to 2^+} f(x)$ ...(i)

Now, $f(2) = a$ ...(ii)

$\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} [2(2-h) - 1] = 3$

$\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0}[(2+h)+1] = 3$

$\therefore$ From (i) and (ii), we get $a = 3$

20. Let $I = \int \frac{x^2}{(ax+b)^2} dx$

Put $ax + b = t \Rightarrow dx = \frac{1}{a} dt$

$$\therefore \quad I = \frac{1}{a^3}\int \frac{(t-b)^2}{t^2} dt = \frac{1}{a^3}\int\left(1+\frac{b^2}{t^2}-\frac{2b}{t}\right)dt$$

$$= \frac{1}{a^3}\left(t - \frac{b^2}{t} - 2b\log t\right) + C$$

$$= \frac{1}{a^3}\left(ax+b-\frac{b^2}{ax+b}-2b\log(ax+b)\right)+C$$

OR

Let $I = \int \frac{\sin 2x}{a^2\sin^2 x + b^2\cos^2 x} dx$

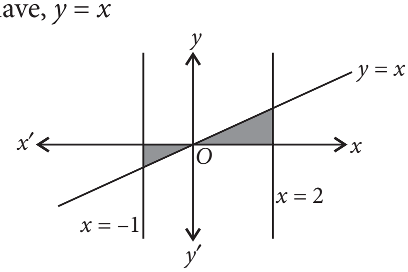
Let $a^2 \sin^2 x + b^2 \cos^2 x = t$

$\Rightarrow (a^2 - b^2)\sin 2x \, dx = dt$

$$\therefore \quad I = \frac{1}{(a^2-b^2)}\int \frac{1}{t} dt = \frac{1}{(a^2-b^2)}\log|t| + C$$

$$\Rightarrow I = \frac{1}{(a^2-b^2)}\log\left|a^2\sin^2 x + b^2\cos^2 x\right| + C$$

21. We have, $y = x$



$\therefore$ Required area = area of shaded region

$$= \left|\int_{-1}^{0} x\,dx\right| + \left|\int_{0}^{2} x\,dx\right| = \left|\frac{x^2}{2}\right|_{-1}^{0} + \left|\frac{x^2}{2}\right|_{0}^{2}$$

$$= \left|-\frac{1}{2}\right| + |2| = 2 + \frac{1}{2} = \frac{5}{2} \text{ sq. units}$$

22. $|A| = 6 + 1 = 7 \neq 0, \therefore A^{-1}$ exists.

$$(\text{adj } A) = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

23. Let G, SD, OD be the events that a randomly chosen part is good, slightly defective, obviously defective respectively.

Then, $P(G) = 0.90$, $P(SD) = 0.02$, and $P(OD) = 0.08$

Required probability = $P(G \mid OD^c)$

$$= \frac{P(G \cap OD^c)}{P(OD^c)} = \frac{P(G)}{1-P(OD)} = \frac{0.90}{1-0.08} = \frac{90}{92} = 0.978$$

OR

Since A and B are independent events, therefore, A^c and B are independent and also A and B^c are independent.

$\therefore \quad P(\overline{A} \cap B) = P(A^c \cap B) = P(A^c)\,P(B) = (1 - P(A))\,P(B)$

and $\quad P(A \cap \overline{B}) = P(A \cap B^c) = P(A)\,P(B^c)$

$= P(A)\,(1 - P(B)) \qquad (\because \overline{A} = A^c \text{ and } \overline{B} = B^c)$

$\Rightarrow (1 - P(A))\,P(B) = P(\overline{A} \cap B) = \frac{2}{15}$ ...(i)

and $P(A)\,(1 - P(B)) = P(A \cap \overline{B}) = \frac{1}{6}$ ...(ii)

Subtracting (ii) from (i), we obtain

$P(B) - P(A) = \frac{2}{15} - \frac{1}{6} \Rightarrow P(B) - P(A) = \frac{4-5}{30}$

$\Rightarrow P(B) - P(A) = -\frac{1}{30}$

24. Here, $\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1$

Put $\frac{3}{5} = \cos\theta$ and $\frac{4}{5} = \sin\theta \Rightarrow \tan\theta = \frac{4}{3}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore \cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$

$= \cos^{-1}(\cos\theta \cos x + \sin\theta \sin x)$

$= \cos^{-1}\{\cos(x-\theta)\} = x - \theta = x - \tan^{-1}\left(\frac{4}{3}\right)$

25. Any point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda(\text{say})$ is of the form $Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ ...(i)

Now, distance PQ, where P is (1, 3, 3), is 5.

So, $(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2 = 5^2$

$\Rightarrow 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 = 25$

$\Rightarrow 17\lambda^2 - 34\lambda = 0 \Rightarrow 17\lambda(\lambda - 2) = 0$

$\Rightarrow \lambda = 0$ or $\lambda = 2$

Putting values of λ in (i), the required points are

(– 2, – 1, 3) and (4, 3, 7).

26. We have, $y^3 - \frac{dy}{dx} = x^2\frac{dy}{dx} \Rightarrow y^3 = (1+x^2)\frac{dy}{dx}$

$\Rightarrow \int \frac{dx}{1+x^2} = \int \frac{dy}{y^3} + c \Rightarrow \tan^{-1} x = \frac{-1}{2y^2} + c$

27. We have, $\vec{a}$ and $\vec{b}$ are collinear.

$\therefore \vec{a} = \lambda\vec{b} \Rightarrow -3\hat{i} + n\hat{j} + 4\hat{k} = \lambda(-2\hat{i} + 4\hat{j} + p\hat{k})$

$\Rightarrow \lambda = \frac{3}{2}$

Also, $n = 4\lambda \Rightarrow n = 4 \times \frac{3}{2} = 6$

And, $\lambda p = 4 \Rightarrow p = 4 \times \frac{2}{3} = \frac{8}{3}$

28. $y = 3x^2 - x + 1$ is the given curve.

Differentiating w.r.t. x, we have

$$\frac{dy}{dx} = 6x - 1 \quad \therefore \quad \left(\frac{dy}{dx}\right)_{x=1} = 6(1) - 1 = 5$$

$\Rightarrow$ The equation of tangent is

$(y - 2) = 5(x - 1) \Rightarrow 5x - y - 3 = 0$

OR

Here, $f(x) = x^9 + 4x^7 + 11$

$\therefore \; f'(x) = 9x^8 + 28x^6 = x^6(9x^2 + 28) > 0$ for all $x \in R$

Thus, $f(x)$ is increasing on R.

29. We have, $f(x) = [x]$

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{[1+h]-[1]}{h} = 0 \qquad (\because [1 + h] = 1 \text{ and } [1] = 1)$$

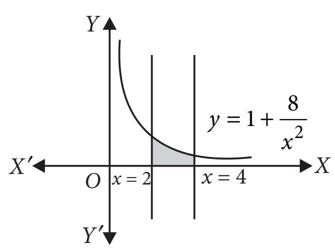
and $Lf'(1) = \lim_{h\to 0} \dfrac{f(1-h) - f(1)}{-h}$

$$= \lim_{h\to 0} \frac{[1-h]-[1]}{-h} = \infty \qquad (\because [1 - h] = 0 \text{ and } [1] = 1).$$

Thus $Rf'(1) \neq Lf'(1)$.

Hence, $f(x) = [x]$ is not differentiable at $x = 1$.

30. Required area $= \displaystyle\int_2^4 \left(1 + \frac{8}{x^2}\right) dx$



$$= \left[x + 8 \times \frac{x^{-1}}{-1}\right]_2^4 = \left[x - \frac{8}{x}\right]_2^4$$

$= (4 - 2) - (2 - 4) = 4$ sq. units.

31. When $x = 2$ we have $y = \sqrt{\dfrac{144 - 16(2)^2}{9}} = \dfrac{4\sqrt{5}}{3}$

So the point of contact is $\left(2, \dfrac{4\sqrt{5}}{3}\right)$

Now $y = \dfrac{\sqrt{144 - 16x^2}}{3} \Rightarrow \dfrac{dy}{dx} = \dfrac{-32x}{6\sqrt{144 - 16x^2}}$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(2, \frac{4\sqrt{5}}{3}\right)} = \frac{-32\,(2)}{6\sqrt{144 - 64}} = \frac{-32}{3 \times 4\sqrt{5}} = \frac{-8}{3\sqrt{5}}$$

$\Rightarrow$ Slope of the normal is $\dfrac{3\sqrt{5}}{8}$.

$\therefore$ Equation of the normal at $\left(2, \dfrac{4\sqrt{5}}{3}\right)$ is

$$\frac{y - \frac{4\sqrt{5}}{3}}{x - 2} = \frac{3\sqrt{5}}{8} \Rightarrow 3\sqrt{5}x - 6\sqrt{5} = 8y - \frac{32\sqrt{5}}{3}$$

$$\Rightarrow 9\sqrt{5}x - 18\sqrt{5} = 24y - 32\sqrt{5}$$

$$\Rightarrow 9\sqrt{5}\,x - 24y + 14\sqrt{5} = 0$$

OR

We have, $f(x) = \dfrac{3}{10}x^4 - \dfrac{4}{5}x^3 - 3x^2 + \dfrac{36}{5}x + 11$

$$\therefore \; f'(x) = \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5}$$

$f'(x) > 0$ as $f(x)$ is increasing

$$\therefore \; \frac{6}{5}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5} > 0$$

$\Rightarrow 6x^3 - 12x^2 - 30x + 36 > 0$

$\Rightarrow 6(x^3 - 2x^2 - 5x + 6) > 0$

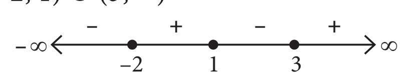
$\Rightarrow 6(x - 1)(x - 3)(x + 2) > 0$

The possible intervals are $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$

For $f(x)$ to be increasing, $f'(x) > 0$.

$\Rightarrow (x - 1)(x - 3)(x + 2) > 0$

$\Rightarrow x \in (-2, 1) \cup (3, \infty)$



So, $f(x)$ is increasing on $x \in (-2, 1) \cup (3, \infty)$.

32. The given D.E. is $\dfrac{dy}{dx} + y\cot x = 2\cos x$ ...(i)

This is a linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$, where $P = \cot x$; $Q = 2\cos x$

Now, I.F. $= e^{\int P\,dx} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$

$\therefore$ The solution of equation (i) is given by

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx + c$$

$$\Rightarrow y\sin x = \int 2\cos x \cdot \sin x\,dx + c$$

$$= \int \sin 2x\,dx + c = -\frac{1}{2}\cos 2x + c$$

$$\Rightarrow y = -\frac{1}{2}\cos 2x \operatorname{cosec} x + c\operatorname{cosec} x$$

This is the required solution of the given differential equation.

33. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(a, b) : |a - b|$ is divisible by $2\}$

(i) Reflexive : For any $a \in A$

$|a - a| = 0$, which is divisible by 2.

Thus, $(a, a) \in R$. So, R is reflexive.

(ii) Symmetric : For any $a, b \in A$

Let $(a, b) \in R$

$\Rightarrow$ $|a - b|$ is divisible by 2

$\Rightarrow$ $|b - a|$ is divisible by 2

$\Rightarrow$ $(b, a) \in R$

$\therefore$ $(a, b) \in R \Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

(iii) Transitive : For any $a, b, c \in A$

Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $|a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2.

$\Rightarrow$ $a - b = \pm 2k_1$ and $b - c = \pm 2k_2$; $k_1, k_2 \in N$

$\Rightarrow$ $a - b + b - c = \pm 2(k_1 + k_2)$

$\Rightarrow$ $a - c = \pm 2k_3$, $k_3 \in N$

$\Rightarrow$ $|a - c|$ is divisible by 2

$\Rightarrow$ $(a, c) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

34. Let $f(x) = a^x$ ...(i)

We have,

$$\lim_{x\to 0} a^x = \lim_{x\to 0}(a^x - 1 + 1) = \lim_{x\to 0}\left(\frac{a^x - 1}{x} \times x + 1\right)$$

$$= \lim_{x\to 0}\frac{a^x - 1}{x} \times \lim_{x\to 0} x + \lim_{x\to 0}(1)$$

$$= \log a \times 0 + 1 = 1$$

$$\therefore \lim_{x\to 0} a^x = 1$$

Let c be an arbitrary real number.

Then $\lim_{x\to c^-} f(x) = \lim_{h\to 0} f(c - h)$

$$= \lim_{h\to 0} a^{c-h} = \lim_{h\to 0} a^c \cdot a^{-h} = a^c \lim_{h\to 0}\frac{1}{a^h}$$

$= a^c \times 1 = a^c = f(c)$ [By (i)]

Likewise $\lim_{x\to c^+} f(x) = a^c$

$$\therefore \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

$\therefore$ f is continuous at $x = c$, where c is an arbitrary real number.

$\therefore$ $f(x) = a^x$ is continuous at every point.

OR

Given, $f(0) = 2\log 3$ ...(i)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{(3^{\sin x} - 1)^2}{x\log(1 + x)}$$

$$= \lim_{x\to 0}\frac{\left(\dfrac{3^{\sin x} - 1}{\sin x}\right)^2 \cdot \left(\dfrac{\sin x}{x}\right)^2}{(1/x)\log(1 + x)}$$

$$= \lim_{x\to 0}\frac{\left(\dfrac{3^{\sin x} - 1}{\sin x}\right)^2 \cdot \left(\dfrac{\sin x}{x}\right)^2}{\dfrac{\log(1 + x)}{x}}$$

$$= \frac{(\log 3)^2 . (1)^2}{1} = (\log 3)^2 \quad \text{...(ii)}$$

From (i) and (ii), $\lim_{x\to 0} f(x) \neq f(0)$

$\therefore$ f is discontinuous at $x = 0$.

35. Let $I = \int \frac{dx}{1 - \cos x - \sin x}$

Since, $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ and $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$\therefore \quad I = \int \frac{dx}{1 - \left(\dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - \dfrac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} = \int \frac{\frac{1}{2}\sec^2 \frac{x}{2} dx}{\tan^2 \frac{x}{2} - \tan \frac{x}{2}}$$

Put $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2 \frac{x}{2} dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2 - t} = \int\left[\frac{1}{t - 1} - \frac{1}{t}\right]dt$$

$$= \log(t - 1) - \log t + C = \log\left|\frac{t - 1}{t}\right| + C$$

$$= \log\left|\frac{\tan \frac{x}{2} - 1}{\tan \frac{x}{2}}\right| + C = \log\left|1 - \cot \frac{x}{2}\right| + C$$

36. Let x, y and z be the investments at the rate of interest of 6%, 7% and 8% per annum respectively. Then, $x + y + z = 5000$.

Income from investment of ₹ x, ₹ y and ₹ z is ₹ $\frac{6x}{100}$, ₹ $\frac{7y}{100}$ and ₹ $\frac{8z}{100}$ respectively.

$\therefore$ Total annual income = ₹ $\left(\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100}\right)$

$\Rightarrow$ $\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$

$\Rightarrow$ $6x + 7y + 8z = 35800$

Also, by given condition $\frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100}$

$\Rightarrow$ $6x + 7y - 8z = 7000$

So, we obtain the following system of linear equations :

$x + y + z = 5000$

$6x + 7y + 8z = 35800$

$6x + 7y - 8z = 7000$

The system of equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix} \text{ or, } AX = B$$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix} = -16 \neq 0.$

So, A^{-1} exist and the solution of the given system of equations is given by $X = A^{-1} B$.

$$\therefore \text{ adj } A = \begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix}' = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{|A|}(\text{adj } A) = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

Hence, the solution is given by

$$X = A^{-1} B = \frac{-1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -560000 + 537000 + 7000 \\ 480000 - 501200 - 14000 \\ 0 - 35800 + 7000 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$$

$\Rightarrow x = 1000, y = 2200, z = 1800$

Hence, three investments are of ₹ 1000, ₹ 2200 and ₹ 1800 respectively.

OR

$$\text{Let } A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A + A') = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

which is clearly a symmetric matrix and

$$A - A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

which is clearly a skew symmetric matrix.

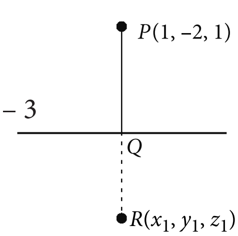
$$\text{Since } A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\therefore A = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

Thus A has been expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

37. Let $P(1, -2, 1)$ be the given point and let Q be the foot of the perpendicular drawn from P on of the given line

$$\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2} = \lambda \text{ (say)}$$



$\Rightarrow x = 3\lambda + 2, y = -\lambda - 1, z = 2\lambda - 3$

Let the coordinates of Q be

$(3\lambda + 2, -\lambda - 1, 2\lambda - 3)$ ...(i)

So, direction ratios of PQ be

$(3\lambda + 2 - 1, -\lambda - 1 + 2, 2\lambda - 3 - 1)$

i.e., $(3\lambda + 1, -\lambda + 1, 2\lambda - 4)$

Since PQ is perpendicular to given line.

$\therefore 3(3\lambda + 1) - 1(-\lambda + 1) + 2(2\lambda - 4) = 0$

$\Rightarrow 9\lambda + 3 + \lambda - 1 + 4\lambda - 8 = 0 \Rightarrow \lambda = 3/7$

Putting $\lambda = 3/7$ in (i), we get the coordinates of Q as

$$\left(\frac{23}{7}, -\frac{10}{7}, -\frac{15}{7}\right)$$

Let $R(x_1, y_1, z_1)$ be the image of $P(1, -2, 1)$ and as Q is the mid point of PR.

$$\therefore \quad \frac{x_1+1}{2} = \frac{23}{7}, \frac{y_1-2}{2} = -\frac{10}{7}, \frac{z_1+1}{2} = -\frac{15}{7}$$

$$\Rightarrow \quad x_1 = \frac{39}{7}, y_1 = -\frac{6}{7}, z_1 = -\frac{37}{7}$$

Hence, image of $P(1, -2, 1)$ in given line is $\left(\frac{39}{7}, -\frac{6}{7}, -\frac{37}{7}\right)$.

OR

Here, $\vec{a}_1 = 2\hat{j} - 3\hat{k}, \ \vec{b}_1 = m\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}, \ \vec{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ m & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i}(8-9) - \hat{j}(4m-6) + \hat{k}(3m-4)$$

$$= -\hat{i} - (4m-6)\hat{j} + (3m-4)\hat{k}$$

Since, given lines are coplanar

$\therefore \ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

$\Rightarrow \ (2\hat{i} + 4\hat{j} + 6\hat{k}) \cdot (-\hat{i} - (4m-6)\hat{j} + (3m-4)\hat{k}) = 0$

$\Rightarrow \ (2)(-1) - 4(4m - 6) + 6(3m - 4) = 0$

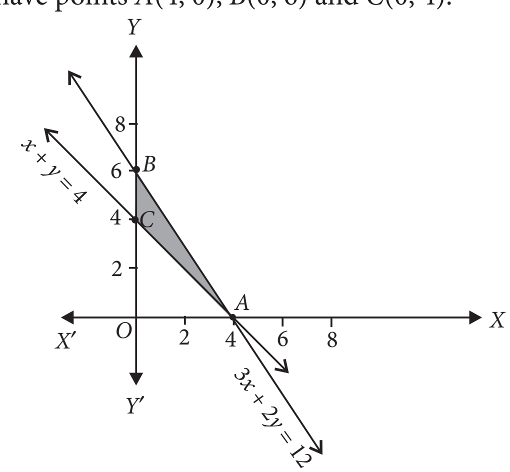
$\Rightarrow \ -2 - 16m + 24 + 18m - 24 = 0 \Rightarrow 2m = 2 \Rightarrow m = 1$

38. Converting inequations into equations and drawing the corresponding lines.

$3x + 2y = 12, \ x + y = 4 \ $ *i.e.*, $\frac{x}{4} + \frac{y}{6} = 1, \ \frac{x}{4} + \frac{y}{4} = 1$

As $x \geq 0, y \geq 0$ solution lies in first quadrant.

We have points $A(4, 0)$, $B(0, 6)$ and $C(0, 4)$.



Now, $Z = 4x + 6y$

$Z(A) = 4(4) + 6(0) = 16$

$Z(B) = 4(0) + 6(6) = 36$

$Z(C) = 4(0) + 6(4) = 24$

$\therefore \ Z$ has maximum value 36 at $B(0, 6)$.

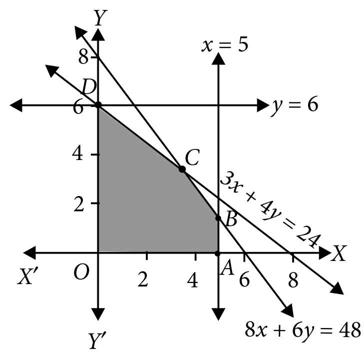
OR

Converting inequations into equations and draw the corresponding lines

$3x + 4y = 24, \ \ 8x + 6y = 48, x = 5, y = 6$

i.e., $\frac{x}{8} + \frac{y}{6} = 1, \ \ \frac{x}{6} + \frac{y}{8} = 1, \ \ x = 5, \ y = 6$

As $x, y \geq 0$, the solution lies in the first quadrant.



B is the point of intersection of the lines

$8x + 6y = 48$ and $x = 5$ *i.e.*, $B = \left(5, \frac{4}{3}\right)$

C is the point of intersection of the lines $3x + 4y = 24$ and $8x + 6y = 48$ *i.e.* $C = \left(\frac{24}{7}, \frac{24}{7}\right)$

We have points $O(0, 0)$ $A(5, 0)$, $B\left(5, \frac{4}{3}\right)$, $C\left(\frac{24}{7}, \frac{24}{7}\right)$ and $D(0, 6)$.

Now, $Z = 4x + 3y$

$\therefore \ Z(O) = 4(0) + 3(0) = 0$

$Z(A) = 4(5) + 3(0) = 20$

$Z(B) = 4(5) + 3\left(\frac{4}{3}\right) = 24$

$Z(C) = 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = 24$

$Z(D) = 4(0) + 3(6) = 18$

Z has maximum value at points B and C. Since both the points lie on the same line $8x + 6y = 48$.

$\therefore$ Each point of the line $8x + 6y = 48$ will give maximum value of Z. Therefore, objective function can be maximized at infinite number of points.

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