MIND MAP : LEARNING MADE SIMPLE CHAPTER - 14

Mathematical Reasoning

Contrapositive and Converse Statements

- The contrapositive of a statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- The converse of a statement $p \Rightarrow q$ is the statement $q \Rightarrow p$

Eg: If the physical environment changes, then biological environmental changes.

- **Contrapositive :** If the biological environment does not change then the physical environment does not change.
- **Converse:** If the biological environment changes then physical environment changes.

Quantifiers

In this statement, the two important symbols are used.

- The symbol '$\forall$' stand for "all values of";
- The symbol '$\exists$' stand for "there exists"

Eg: For every prime number P, P is an irrational number.

- This means that if S denotes the set of all prime numbers, then for all the members of P of the set S, P is an irrational number.

Compound Statement

Many mathematical statements are obtained by combining one or more statements using some connecting words like "and", "or" etc. Each statement is called a "Compound Statement."

Eg:

'The sky is blue' and 'the grass is green' is a compound statement where connecting word is "and";
the components of Compound Statement are

p : The sky is blue
q : The grass is green

Validity of Statements

The following methods are used to check the validity of statements

(i) Direct Method
(ii) Contrapositive Method
(iii) Method of Contradiction
(iv) Using a Counter example

Implications

- These are statements with word "if then", "only if" and "if and only if".

Eg:

- r: if a number is a multiple of 9, then it is multiple of 3.
- p: a number is a multiple of 9.
- q: a number is a multiple of 3.
- Then, if p then q is the same as following:

→ p implies q is denoted by $p \Rightarrow q$, then symbol $\Rightarrow$ stands for implies,
→ p is a sufficient condition for q. then symbol $\Rightarrow$
→ p only if q
→ q is a necessary condition for p
→ $\sim q$ implies $\sim p$

Sentence

A sentence is called a mathematically acceptable statement if it is either true or false but not both.

Eg:

- The sum of two positive numbers is positive.
- All prime numbers are odd numbers.
- In above statements, first is 'true' and second is 'false'.

Negation

A statement which is formed by changing the true value of a given statement by using the word like 'no', 'not' is called negation of given statement.

- If P is a statement, then negation is denoted by $\sim p$

Eg: New Delhi is a city.

The negation of this statement:

→ It is not the case that New Delhi is a city.
→ It is false that New Delhi is a city.
→ New Delhi is not a city.