Mathematics
Class XII
Sample Paper – 9

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Write the element which is denoted by a_{32} in the given matrix

$$\begin{bmatrix} 1 & 16 & 8 & 9 \\ 7 & 5 & 3 & 2 \\ 4 & 10 & 6 & 11 \end{bmatrix}$$

2. Differentiate sin (cos x) w.r.t. x

3. Is the differential equation given by $x+\left(\frac{d}{dx}\right)=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$, linear or nonlinear. Give reason.

4. Find the angle between following pairs of line

$$\frac{x+4}{3}=\frac{y-1}{5}=\frac{z+3}{4} \text{ and } \frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$$

OR

Find the angle between following pairs of line

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \text{ and } \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

SECTION – B

5. Consider the function f: $R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$, where R_+ is the set of all non-negative real numbers. Show that f is invertible. Also find the inverse of f.

6. If A = diag (1 -1 2) and B = diag (2 3 -1), find A + B, 3A + 4B.

7. Evaluate:

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\,dx.$$

8. Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3}\,dx.$

OR

Evaluate: $\int \frac{x^2}{1+x^3}dx$

9. Form the differential equation $y^2 = m(a^2 - x^2)$ by eliminating parameters m and a

10. Find p, if the points (1, 1, p) and (-3, 0, 1) are equidistant from the plane whose equation is

$\vec{r}.\ 3\hat{i} + 4\hat{j} - 12\hat{k}\ + 13 = 0$

OR

Prove that: $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

11. The probability that a student entering a university graduates is 0.4. Find the probability that out of 3 students of the university:

i. None will graduate

ii. Only one will graduate

12. A bag contains 5 white and 3 black balls and another bag contains 3 white and 4 black balls. A ball is drawn from the first bag and without seeing its colour, is put in the second bag. Find the probability that if now a ball is drawn from the second bag, it is black in colour.

OR

A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

SECTION – C

13.

(i) If f: $N \to Z$ s.t f(x) = x and g: $Z \to Z$ s.t g(x) = |x|.
Show that gof is injective but g is not.

(ii) If f: $N \to N$ s.t f(x) = x + 1 and g: $N \to N$ s.t g(x) = $\begin{cases} x-1 & \text{if } x>1 \\ 1 & \text{if } x=1 \end{cases}$.

Show that g of is subjective but f is not.

OR

Show that the function $f: R \to R$ defined by $f(x)=\frac{2x-1}{3}$, $x \in R$ is one-one and onto function. Also find the inverse of the function f.

14. Solve the equation.

$\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$

15. If x, y, and z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$; show that xyz = -1

16. Differentiate x^{x^x} w.r.t. x

OR

If $y=(\sin x)^{\tan x}+(\cos x)^{\sec x}$, find $\frac{dy}{dx}$

17. Differentiate $x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$ w.r.t. x

18. Find the interval in which the function $y = \frac{4\sin\theta}{2 + \cos\theta} - \theta, 0 \le \theta \le \pi$, is an increasing function of θ.

19. Evaluate:

$$\int \frac{x^2}{x^4 + x^2 - 2} dx$$

20. Evaluate: $\int_0^2 \left(x^2 + e^x\right) dx$ using integral as limit of sums.

21. Solve the initial value problem: $(x + y + 1)^2 dy = dx$, $y(-1) = 0$

OR

Solve the initial value problem: $(x - y)(dy + dx) = dx - dy$, $y(0) = -1$

22. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

23. Find the value of λ so that the lines,

$$\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2} \text{ and } \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$$

are perpendicular to each other.

SECTION – D

24. If $A = \begin{bmatrix} -1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, show that $(A + B)^T = A^T + B^T$

OR

If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$, find AB and BA.

25. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

26. Find the smaller of the two areas in which the circle $x^2 + y^2 = 2a^2$ is divided by the parabola $y^2 = ax$, $a > 0$.

OR

Find the area of the region $\{(x, y): y^2 \geq 6x, 4x^2 + 4y^2 \leq 64\}$

27. Find a point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$, which is at the distance of $3\sqrt{2}$ units, from the point (1, 2, 3).

OR

Find the value of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equations of a line passing through a point (3, 2, – 4) and parallel to line l_1.

28. A nutritionist has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packet of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

29. A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced. 6 balls are drawn in this way, find the probability that

i. All balls were purple
ii. Not more than 2 were pink
iii. An equal number of purple and pink balls were drawn.
iv. Atleast one ball was pink

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SECTION – A

1. a_{42}, means element at 3rd row and 2nd column

So,

a_{32} = 10

2. Differentiating w.r.t. x, we get,

$$\frac{d}{dx}(\sin(\cos x))$$
$$=\cos(\cos x)\frac{d}{dx}(\cos x)$$
$$=-\sin x\cos(\cos x)$$

3. DE:

$$x+\left(\frac{dy}{dx}\right)=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

squaring

$$x^2+2x\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2$$
$$x^2+2x\left(\frac{dy}{dx}\right)=1$$

It is linear, since x is independent variable.

4. Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

substituting we get

$$\theta=\cos^1\left(\frac{8}{5\sqrt{3}}\right)$$

OR

Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

substituting we get

$$\theta = \cos^{1}\left(\frac{10}{9\sqrt{22}}\right)$$

SECTION – B

5. f: $R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$

f is 1 – 1

Let $x_1, x_2, \in R_+$ such that $f(x_1) = f(x_2)$

$$\Rightarrow x_1^2 + 4 = x_2^2 + 4$$
$$\Rightarrow x_1^2 = x_2^2$$
$$\Rightarrow x_1 = x_2 \qquad \because x_1, x_2 \in R_+$$

∴ f is 1 – 1

f is onto: Let $y \in [4, \infty)$

$f(x) = y \Rightarrow x^2 + 4 = y$

$$\Rightarrow x = \sqrt{y - 4}$$

Since $y \in [4, \infty) \Rightarrow x \in R_+$

For $y \in [4, \infty)$ there is a $x \in R_+$ such that $f(x) = y$.

So f is onto.

So, f is bijective function and hence f is invertible.

The inverse of f is defined by

f: $[4, \infty) \to R_+$

$f^{-1}(y) = \sqrt{y - 4}$

6.

We have,

$$A=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&2\end{bmatrix} \text{ and, } B=\begin{bmatrix}2&0&0\\0&3&0\\0&0&-1\end{bmatrix}$$

$$A+B=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&2\end{bmatrix}+\begin{bmatrix}2&0&0\\0&3&0\\0&0&-1\end{bmatrix}=\begin{bmatrix}3&0&0\\0&2&0\\0&0&1\end{bmatrix}=\text{diag}(3\ 2\ 1)$$

and,

$$3A+4B=\begin{bmatrix}3&0&0\\0&-3&0\\0&0&6\end{bmatrix}+\begin{bmatrix}8&0&0\\0&12&0\\0&0&-4\end{bmatrix}=\begin{bmatrix}11&0&0\\0&9&0\\0&0&2\end{bmatrix}=\text{diag}(11\ 9\ 2)$$

7. $I=\int\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

Let $e^x=t \quad e^x dx=dt$

Now integral I becomes,

$$I=\int\frac{dt}{\sqrt{5-4t-t^2}}$$

$$\Rightarrow I=\int\frac{dt}{\sqrt{5+4-4-4t-t^2}}$$

$$\Rightarrow I=\int\frac{dt}{\sqrt{9-\left(4+4t+t^2\right)}}$$

$$\Rightarrow I=\int\frac{dt}{\sqrt{9-(t+2)^2}}$$

$$\Rightarrow I=\int\frac{dt}{\sqrt{3^2-(t+2)^2}}$$

$$\Rightarrow I=\sin^{-1}\frac{(t+2)}{3}+C$$

$$\Rightarrow I=\sin^{-1}\frac{(e^x+2)}{3}+C$$

8. $I=\int\frac{(x-4)e^x}{(x-2)^3}.dx$

$$I=\int e^x\left(\frac{x-2}{(x-2)^3}-\frac{2}{(x-2)^3}\right).dx$$

$$I=\int e^x\left(\frac{1}{(x-2)^2}-\frac{2}{(x-2)^3}\right).dx$$

Thus the given integral is of the form,

$I=\int e^x\left|f(x)+f'(x)\right|dx$ where, $f(x)=\frac{1}{(x-2)^2}$; $f'(x)=\frac{-2}{(x-2)^3}$

$$I=\int\frac{e^x}{(x-2)^2}dx-\int\frac{2e^x}{(x-2)^3}dx$$

$$=\int e^x\left[\frac{1}{(x-2)^2}+\frac{d}{dx}\left(\frac{1}{(x-2)^2}\right)\right]dx$$

$$\text{So}, I=\frac{e^x}{(x-2)^2}+C$$

OR

$$\int\frac{x^2}{1+x^3}dx$$

Let $1 + x^3 = t$

$\Rightarrow 0 + 3x^2dx = dt$

$\Rightarrow x^2dx=\frac{dt}{3}$

$$\therefore\int\left(\frac{x^2}{1+x^3}\right)dx=\int\frac{\frac{dt}{3}}{t}$$

$$=\frac{1}{3}\int\frac{dt}{t}$$

$$=\frac{1}{3}\log|t|+c$$

$$=\frac{1}{3}\log\left|1+x^3\right|+c$$

9. We have to differentiate it w.r.t. x two times

Differentiating

$$2y\frac{dy}{dx}=m(-2x)$$

$$y\frac{dy}{dx}=-mx..............(1)$$

differentiating again

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-m$$

from(1)

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{y}{x}\frac{dy}{dx}$$

which is the required differential equation

10.

$$\vec{r}.\ 3\hat{i}+4\hat{j}-12\hat{k}\ +13=0$$

$$\Rightarrow 3x+4y-12z+13=0$$

Distance of point (1, 1, p) from the plane , is given by

$$\frac{|3\times1+4\times1-12\times p+13|}{\sqrt{3^2+4^2+\ -12\ ^2}}=\frac{|20-12p|}{\sqrt{3^2+4^2+\ -12\ ^2}}$$

Distance of point (-3, 0, 1) from the plane , is given by

$$\frac{|3\times\ -3\ +4\times0-12\times1+13|}{\sqrt{3^2+4^2+\ -12\ ^2}}=\frac{|-8|}{\sqrt{3^2+4^2+(-12)^2}}$$

The two distances are equal

$$\Rightarrow\frac{|20-12p|}{\sqrt{3^2+4^2+\ -12\ ^2}}=\frac{|-8|}{\sqrt{3^2+4^2+\ -12\ ^2}}$$

$$\Rightarrow|20-12p|=|-8|$$

$$\Rightarrow 20-12p=\pm8$$

$$\Rightarrow p=1,\frac{7}{3}$$

OR

$$\text{L.H.S.} = [\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$$
$$= \left(\vec{a}+\vec{b}\right).\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right]$$
$$= \left(\vec{a}+\vec{b}\right).[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}]$$
$$= \left(\vec{a}+\vec{b}\right).[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}]$$
$$= \vec{a}.(\vec{b}\times\vec{c})+\vec{a}.\left(\vec{b}\times\vec{a}\right)+\vec{a}.\left(\vec{c}\times\vec{a}\right)+\vec{b}.(\vec{b}\times\vec{c})+\vec{b}.\left(\vec{b}\times\vec{a}\right)+\vec{b}.\left(\vec{c}\times\vec{a}\right)$$
$$= \vec{a}.(\vec{b}\times\vec{c})+0+0+0+0+\vec{b}.\left(\vec{c}\times\vec{a}\right)$$
$$= 2\vec{a}.(\vec{b}\times\vec{c})$$
$$= 2[\vec{a}\ \vec{b}\ \vec{c}] = \text{R.H.S.}$$

11. Let p be the probability that a student entering a university graduates,

So p = 0.4.

Since p + q = 1, thus, q = 1 - 0.4 = 0.6.

Let X denote the random variable representing the number of students who graduate out of the 3. Probability that r students graduate out of n entering the university is given by

$$P\left(X=r\right) = {}^{n}C_{r}p^{r}q^{n-r}$$
$$= {}^{3}C_{r}\left(0.4\right)^{r}\left(0.6\right)^{n-r} \quad ....(1)$$

(i)

Probability that none will graduate

$$= P\left(X = 0\right)$$
$$= {}^{3}C_{0}\left(0.4\right)^{0}\left(0.6\right)^{3-0}$$
$$= \left(0.6\right)^{3}$$
$$= 0.216$$
$$\therefore \text{Probability that none will graduate} = 0.216$$

(ii)

Probability that one will graduate

$= P(X = 1)$

$= {}^3C_1 (0.4)^1 (0.6)^{3-1}$

$= 3 \times (0.4) \times (0.36)$

$= 0.432$

$\therefore$ Probability that only one will graduate = 0.432

12. Let E_1, E_2, and A be the events.

E_1 = white ball is transferred from 1st bag to 2nd bag.

E_2 = black ball is transferred from 1st bag to 2nd bag.

A = a black ball is drawn.

So, $P(E_1) = \frac{5}{8}$; $P(E_2) = \frac{3}{8}$;

Also, $P(A/E_1)$ = Probability of taking out black ball from bag 2 when white ball is already transferred from 1st bag to 2nd bag.

$\Rightarrow P(A/E_1) = \frac{4}{8}$

$P(A/E_2)$ = Probability of taking out black ball from bag 2 when black ball is transferred from 1st bag to 2nd bag.

$\Rightarrow P(A/E_2) = \frac{5}{8}$

By law of total probability

$$P(E_2 | A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{3}{8}.\frac{5}{8}}{\frac{5}{8}.\frac{4}{8} + \frac{3}{8}.\frac{5}{8}} = \frac{15}{35} = \frac{3}{7}$$

OR

The events A, E_1, E_2, E_3, and E_4 are given by

A = event when doctor visits patients late

E_1 = doctor comes by train

E_2 = doctor comes by bus

E_3 = doctor comes by scooter

E_4 = doctor comes by other means of transport

So, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$, $P(E_4) = \frac{2}{5}$

$P(A/E_1)$ = Probability that the doctor arrives late, given that he is comes by train = $\frac{1}{4}$

Similarly $P(A/E_2) = \frac{1}{3}$, $P(A/E_3) = \frac{1}{12}$, $P(A/E_4) = 0$

Required probability of the doctor arriving late by train by using Baye's theorem,

$$P(E_1/A) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_3)P\left(A/E_3\right) + P(E_4)P\left(A/E_4\right)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0}$$

$$= \frac{3}{40} \times \frac{120}{18} = \frac{1}{2}$$

Hence the required probability is $\frac{1}{2}$.

SECTION – C

13. (i) $f: N \to Z$

$f(x) = x$

and f: $Z \to Z$ s.t.

$g(x) = |x|$

Clearly, g the absolute value function is not one–one.

gof: $N \to Z$

$gof(x) = |x| \ \ \forall \ x \in N$

$g(f(x)) = g(x) = |x|$

Let gof (x_1)=g of $((x_2)$

$$\Rightarrow g(f(x_1)) = g(f(x_2))$$
$$|x_1| = |x_2|$$
$$\Rightarrow x_1 = x_2$$
$$\text{since } x_1, x_2 \in N$$

So gof one-one.

(ii) $f: N \to N$ such that

$f(x) = x + 1$ and

g: $N \to N$ such that $g(x) = \begin{cases} x-1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$

Clearly f is not an onto function since set of natural numbers is infinite

gof: $N \to N$ s.t

gof (x) = g (f(x)) = g(x + 1) = x since x is a natural number so x + 1 > 1

Now consider any natural number $x \in N$, the codomain set of gof,

Every natural number has a predecessor i.e every x in N can be mapped to x – 1 in N. So gof is onto.

OR

For one-one function

Let $x_1 = x_2 \in R$ such that $f(x_1) = f(x_2)$

$$\frac{2x_1 - 1}{3} = \frac{2x_2 - 1}{3}$$
$$\Rightarrow 2x_1 - 1 = 2x_2 - 1$$
$$\Rightarrow x_1 = x_2$$

So, f is a 1–1 function

For onto function

Let $y \in R$ such that $f(x) = y$

$$\frac{2x - 1}{3} = y \Rightarrow 2x - 1 = 3y$$
$$\Rightarrow x = \frac{3y + 1}{2}$$
$$f\left(\frac{3y+1}{2}\right) = \frac{2\left(\frac{3y+1}{2}\right) - 1}{3}$$
$$= \frac{3y + 1 - 1}{3} = y$$

Therefore, the function f(x) is onto.

The function is bijective , therefore invertible.

For inverse function

Since, f(x) is one-one and onto, therefore

$f^{-1}(x)$ exists.

Let $y = f(x)$

$$y = \frac{2x - 1}{3}$$
$$\Rightarrow 3y = 2x - 1$$
$$\Rightarrow x = \frac{3y + 1}{2}$$
$$\therefore f^{-1}(y) = \frac{3y + 1}{2}$$

Hence, $f^{-1}(x) = \dfrac{3x + 1}{2}$

14. Let $\sin^{-1}x = \theta$ or $\sin\theta = x$

So given equation becomes

$$\sin^{-1}(1-x) - 2\theta = \frac{\pi}{2}$$

$$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\theta$$

$$1 - x = \sin\left(\frac{\pi}{2} + 2\theta\right)$$

$$1 - x = \cos 2\theta$$

$$1 - x = 1 - 2\sin^2\theta$$

$$\Rightarrow 1 - x = 1 - 2x^2$$

$$\Rightarrow 2x^2 - x = 0$$

$$\Rightarrow x(2x-1) = 0$$

$$\Rightarrow x = 0, \qquad x = \frac{1}{2}$$

If $x = 0$

L.H.S of given equation $= \sin^{-1}1 - 2\sin^{-1}0 = \frac{\pi}{2} =$ R.H.S.

If $x = \frac{1}{2}$

L.H.S. of given equation $= \sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{2} = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6} \neq$ R.H.S.

$\therefore$ solution is $x = 0$

15.

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$

$$\Rightarrow (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix} = 0$$

$R_3 \to R_3 - R_2$

$$\Rightarrow (1+xyz)(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x+y \\ 0 & 0 & z-y \end{vmatrix} = 0$$

$\Rightarrow (1+xyz)(x-y)(y-z)(z-x) = 0$

Given, x , y , z are different $\Rightarrow 1+xyz=0 \Rightarrow$ $xyz = -1$

16.

$y = x^{x^x}$

then,

$y = e^{\log x^{x^x}}$

$y = e^{x^x \log x}$

differentiating w.r.t. x

$$\frac{dy}{dx} = e^{x^x \log x} \frac{d}{dx}\left(x^x \log x\right)$$

$$\frac{dy}{dx} = y\left(x^x \frac{d}{dx}\log x + \log x \frac{d}{dx} x^x\right)$$

$$\frac{dy}{dx} = y\left(x^x \frac{d}{dx}\log x + \log x \frac{d}{dx} e^{\log x^x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{x^x}{x} + \log x \frac{d}{dx} e^{x \log x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{x^x}{x} + \log x \frac{d}{dx} e^{x \log x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{x^x}{x} + \log x\left(e^{x \log x} \frac{dy}{dx} x \log x\right)\right)$$

$$\frac{dy}{dx} = y\left(\frac{x^x}{x} + \log x\left(e^{x \log x}\left(x \times \frac{1}{x} + \log x\right)\right)\right)$$

$$\frac{dy}{dx} = y\left(\frac{x^x}{x} + \log x\left(x^x\left(1 + \log x\right)\right)\right)$$

$$\frac{dy}{dx} = x^{x^x} x^x \left(\frac{1}{x} + \log x\left(1 + \log x\right)\right)$$

OR

we have,

$$y=(\sin x)^{\tan x}+(\cos x)^{\sec x}$$

$$y=e^{\tan x\,.\log\sin x}+e^{\sec x\,.\log\cos x}$$

differentiating

$$\frac{dy}{dx}=\frac{d}{dx}\left(e^{\tan x\,.\log\sin x}\right)+\frac{d}{dx}\left(e^{\sec x\,.\log\cos x}\right)$$

$$\frac{dy}{dx}=e^{\tan x\,.\log\sin x}\,\frac{d}{dx}(\tan x\,.\log\,\sin x)+e^{\sec x\,.\log\cos x}\,\frac{d}{dx}(\sec x\,.\log\,\cos x)$$

$$\frac{dy}{dx}=(\sin x)^{\tan x}\left(\tan x\,\frac{d}{dx}\log\,\sin x+\log\,\sin x\frac{d}{dx}\tan x\right)+$$

$$(\cos x)^{\sec x}\,\frac{d}{dx}\left(\sec x\,\frac{d}{dx}\log\,\cos x+\log\,\cos x\frac{d}{dx}\sec\,x\right)$$

$$\frac{dy}{dx}=(\sin x)^{\tan x}\left(\sec^2 x.\log\sin x+1\right)+(\cos x)^{\sec x}\left(\sec x.\tan x.\log\cos x-\sec x.\tan x\right)$$

17. Let

$$y=x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$$

$$y=e^{\cot x.\log x}+\frac{2x^2-3}{x^2+x+2}$$

differentiate

$$\frac{dy}{dx}=e^{\cot x.\log x}\times\frac{d}{dx}(\cot x.\log x)+\frac{d}{dx}\left(\frac{2x^2-3}{x^2+x+2}\right)$$

$$\frac{dy}{dx}=x^{\cot x}\times\left(\cot x\times\frac{1}{x}+\log x\times-\operatorname{cosec}^2 x\right)+\frac{(x^2+x+2)4x-(2x^2-3)(2x+1)}{(x^2+x+2)^2}$$

$$\frac{dy}{dx}=x^{\cot x}\times\left(\cot x\times\frac{1}{x}+\log x\times-\operatorname{cosec}^2 x\right)+\frac{2x^2+14x+3}{(x^2+x+2)^2}$$

18.

$$y=\frac{4\sin\theta}{2+\cos\theta}-\theta, 0\le\theta\le\pi,$$

$$\frac{dy}{dx}=\frac{(2+\cos\theta)(4\cos\theta)+4\sin\theta(\sin\theta)}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4(\cos^2\theta+\sin^2\theta)-4-\cos^2\theta-4\cos\theta}{(2+\cos\theta)^2}$$

$$=\frac{4\cos\theta-\cos^2\theta}{(2+\cos\theta)^2}$$

$$=\frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

Since, $(2+\cos\theta)^2>0$ for all θ

$4-\cos\theta>0$ for all θ as $|\cos\theta|\le 1$

$\therefore \frac{dy}{d\theta}>0$ if $\cos\theta>0$

$\therefore$ If $\theta\in\left[0,\frac{\pi}{2}\right]$ $\frac{dy}{d\theta}>0$

$\therefore$ y is increasing in $\left[0,\frac{\pi}{2}\right]$

19.

$$\int \frac{x^2}{x^4+x^2-2}dx$$

$$=\int \frac{x^2}{(x^2-1)(x^2+2)}dx$$

$$=\int \frac{x^2}{(x-1)(x+1)(x^2+2)}dx$$

Using partial fraction,

$$\frac{x^2}{(x-1)(x+1)(x^2+2)}=\frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{Cx+D}{(x^2+2)}$$

$$\frac{x^2}{(x-1)(x+1)(x^2+2)}=\frac{A(x+1)(x^2+2)+B(x-1)(x^2+2)+(Cx+D)(x-1)(x+1)}{(x-1)(x+1)(x^2+2)}$$

Equating the coefficients from both the numerators we get,

$A+B+C=0........(1)$

$A-B+D=1........(2)$

$2A+2B-C=0........(3)$

$2A-2B-D=0........(4)$

Solving the above equations we get,

$$A=\frac{1}{6},B=-\frac{1}{6},C=0,D=\frac{2}{3}$$

Our Integral becomes,

$$\int \frac{x^2}{(x-1)(x+1)(x^2+2)}dx=\int \frac{1}{6(x-1)}-\frac{1}{6(x+1)}+\frac{2}{3(x^2+2)}dx$$

$$=\frac{1}{6}\log(x-1)-\frac{1}{6}\log(x+1)+\frac{2}{3}\times\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$$

$$=\frac{1}{6}\left[\log(x-1)-\log(x+1)+2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]+C$$

20. To evaluate: $\int_0^2 \left(x^2 + e^x\right) dx$

Here $f(x) = x^2 + e^x$, $a = 0, b = 2$

So, $nh = b - a = 2$

Now $f(0) = 0 + e^{\circ} = 1$

$f(0 + h) = f(h) = h^2 + e^h$

$f(0 + 2h) = f(2h) = 2^2h^2 + e^{2h}$

$f(0 + (n - 1)h) = f((n - 1)h) = (n - 1)^2h^2 + e^{(n-1)h}$

Now $\int_0^2 f(x)dx = \lim_{h \to 0}\left[f(0) + f(0+h) + f(0+2h) + \ldots + f(0+(n-1)h)\right]$

$$= \lim_{h \to o} h\left[\left(h^2 + 2^2h^2 + \ldots(n-1)^2h^2\right) + \left(1 + e^h + e^{2h} + \ldots + e^{(n-1)h}\right)\right]$$

$$= \lim_{h \to o} h\left[h^2\left(1^2 + 2^2 + \ldots + (n-1)^2\right) + 1.\left(\frac{\left(e^h\right)^n - 1}{e^h - 1}\right)\right]$$

$$= \lim_{h \to o}\left[h^3 \frac{n(n-1)(2n-1)}{6} + \frac{h\left(e^{nh} - 1\right)}{e^h - 1}\right]$$

$$= \lim_{h \to o}\left[\frac{nh(nh-h)(2nh-h)}{6}\right] + \lim_{h \to o}\left(e^{nh} - 1\right) \times \lim_{h \to o}\frac{h}{e^h - 1}$$

$$= \frac{2(2-0)(4-0)}{6} + \left(e^2 - 1\right) \times 1$$

$$= \frac{8}{3} + e^2 - 1 = \frac{5}{3} + e^2$$

21. The given differential equation is

$$(x+y+1)^2 \frac{dy}{dx} = 1$$

sub $x+y+1=v$

$$\frac{dy}{dx}+1=\frac{dv}{dx}$$

so,

$$\Rightarrow v^2\left(\frac{dv}{dx}-1\right)=1$$

$$\Rightarrow \frac{dv}{dx}=\frac{1}{v^2}+1$$

$$\Rightarrow \frac{dv}{dx}=\frac{1+v^2}{v^2}$$

$$\Rightarrow \int \frac{v^2}{1+v^2}dv=\int dx$$

$$\Rightarrow \int\left(1-\frac{1}{1+v^2}\right)dv=x+c$$

$$\Rightarrow v-\tan^{-1}v=x+c$$

$$\Rightarrow x+y+1-\tan^{-1}(x+y+1)=x+c$$

given that $x=-1$, then $y=0$

we get

$c=1$

so,

$$y=\tan^{-1}(x+y+1)$$

$$\tan y=x+y+1$$

OR

The given differential equation is

$$(x-y)(dx+dy)=dx-dy$$

$$\Rightarrow (x-y-1)dx=-(x-y+1)dy$$

$$\Rightarrow \frac{dy}{dx}=-\frac{x-y+1}{x-y-1}$$

let $x-y=v$

$$\frac{dy}{dx}=1-\frac{dv}{dx}$$

so,

$$1-\frac{dv}{dx}=-\frac{v-1}{v+1}$$

$$\Rightarrow \frac{dv}{dx}=\frac{2v}{v+1}$$

$$\Rightarrow \frac{v+1}{v}\,dv=2\,dx$$

$$\Rightarrow \int\left(1+\frac{1}{v}\right)dv=\int dx$$

$$\Rightarrow v+\log|v|=2x+c$$

$$\Rightarrow x-y+\log|x-y|=2x+c$$

$$\Rightarrow \log|x-y|=x+y+c$$

given that

$x=0$, then $y=-1$

so, substituting we get

$c=1$

$$\log|x-y|=x+y+1$$

$$x-y=\pm e^{x+y+1}$$

22. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

The vector which is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is in the direction of

$\vec{a} \times \vec{b}$. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = 32\hat{i} - \hat{j} - 14\hat{k}$

Since $\vec{d}$ is perpendicular to $\vec{a}$ and $\vec{b}$

$\vec{a} \times \vec{b}$ is also perpendicular to both $\vec{a}$ and $\vec{b}$

$\Rightarrow \vec{d}$ is parallel to $\vec{a} \times \vec{b}$.

$$\Rightarrow \vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$$

$$\Rightarrow \vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$$

Also $\vec{c}.\vec{d} = 15 \Rightarrow 2.32\lambda + (-1)(-\lambda) + 4(-14\lambda) = 15$

$$\Rightarrow \lambda = \frac{5}{3}$$

$$\Rightarrow \vec{d} = \frac{5}{3}\left(32\hat{i} - \hat{j} - 14\hat{k}\right)$$

23.

$$\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$$

and

$$\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$$

Let us rewrite the equations of the given lines as follows:

$$\frac{-(x-1)}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$$

and

$$\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{-(z-6)}{7}$$

That is we have,

$$\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$$

and

$$\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-7}$$

The lines are perpendicular so angle between them is 90°

So, $\cos\theta=0$

Here $(a_1, b_1, c_1)=(-3, 2\lambda, 2)$ and $(a_2, b_2, c_2)=(3\lambda, 1, -7)$

For perpendicular lines

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow -9\lambda+2\lambda-14=0$$

$$\Rightarrow -7\lambda-14=0$$

$$\Rightarrow -7\lambda=14$$

$$\Rightarrow \lambda=\frac{14}{-7}$$

$$\Rightarrow \lambda=-2$$

SECTION – D

24.

$$A=\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$$

$$A^T=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix}, B^T=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 1\\ 3 & 3 & 1\end{bmatrix}$$

$$(A+B)^T=$$

$$\left(\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}\right)^T$$

$$=\begin{bmatrix}1+1 & 1 & 0+3\\ 2+2 & 1+1 & 3+3\\ 1+0 & 2+1 & 1+1\end{bmatrix}^T$$

$$=\begin{bmatrix}2 & 1 & 3\\ 4 & 2 & 6\\ 1 & 3 & 2\end{bmatrix}^T$$

$$=\begin{bmatrix}2 & 4 & 1\\ 1 & 2 & 3\\ 3 & 6 & 2\end{bmatrix}$$

$$A^T+B^T=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix}+\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 1\\ 3 & 3 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2+2 & 1+0\\ -1+2 & 1+1 & 2+1\\ 0+3 & 3+3 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 1\\ 1 & 2 & 3\\ 3 & 6 & 2\end{bmatrix}$$

$$(A+B)^T=A^T+B^T$$

OR

$$A=\begin{bmatrix}1 & 1 & -1\\ 2 & 2 & -3\\ -1 & 0 & 2\end{bmatrix}, B=\begin{bmatrix}4 & -2 & -1\\ -1 & 1 & 1\\ 2 & -1 & 0\end{bmatrix}$$

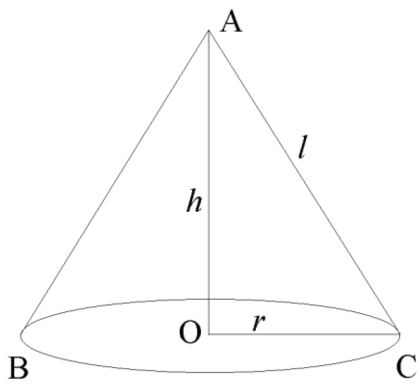
$$AB=\begin{bmatrix}4-1-2 & -2+1+1 & -1+1\\ 8-2-6 & -4+2+3 & -2+2\\ -4+4 & 2-2 & 1\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}4 & -2 & -1\\ -1 & 1 & 1\\ 2 & -1 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & -1\\ 2 & 2 & -3\\ -1 & 0 & 2\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

25.



Here, Volume 'V' of the cone is $V = \frac{1}{3}\pi r^2 h \Rightarrow r^2 = \frac{3V}{\pi h}$ ...(1)

Surface area $S = \pi r l = \pi r\sqrt{h^2 + r^2}$ ...(2)

Where h = height of the cone

r = radius of the cone

l = Slant height of the cone

$S^2 = \pi^2 r^2 \left(h^2 + r^2\right)$ from equation (2)

Let, $S_1 = S^2$

Substituting the value of r^2 from equation (1), we have,

$$S_1 = \frac{3\pi V}{h}\left(h^2 + \frac{3V}{\pi h}\right) = 3\pi V h + \frac{9V^2}{h^2}$$

Differentiating S_1 with respect to h, we get

$$\frac{dS_1}{dh} = 3\pi V + 9V^2\left(\frac{-2}{h^3}\right)$$

$\frac{dS_1}{dh} = 0$ for maxima/minima

$$3\pi V + 9V^2\left(\frac{-2}{h^3}\right) = 0$$

$$\Rightarrow 3\pi V = 9V^2\left(\frac{2}{h^3}\right)$$

$$\Rightarrow h^3 = \frac{6V}{\pi}$$

$$\frac{d^2S_1}{dh^2} = \frac{54V^2}{h^4}$$

$$\frac{d^2S_1}{dh^2} > 0 \text{ at } h^3 = \frac{6V}{\pi^2}$$

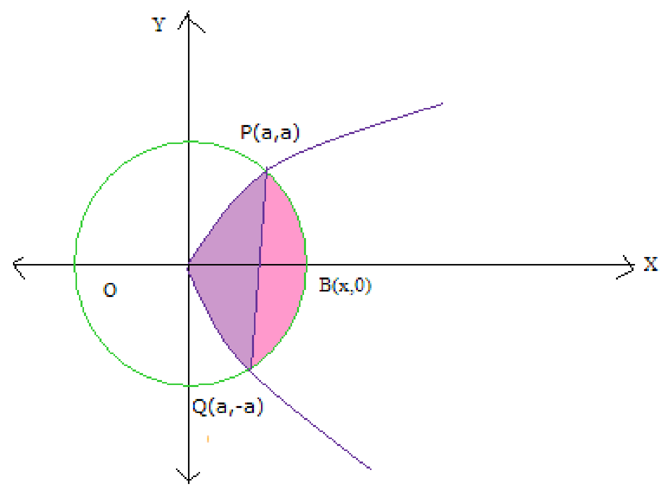
Therefore curved surface area is minimum at $\frac{\pi h^3}{6} = V$

Thus, $\frac{\pi h^3}{6} = \frac{1}{3}\pi r^2 h \;\Rightarrow\; h^2 = 2r^2$

$\Rightarrow\; h = \sqrt{2}r$

Hence for least curved surface the altitude is $\sqrt{2}$ times radius.

26.



The circle is $x^2+y^2=2a^2 \Rightarrow C(O,\sqrt{2}a)$

The parabola is $y^2=ax, a>0 \Rightarrow y^2=4\frac{1}{4}ax, a>0$

Their point of intersection is given by :$x^2+ax=2a^2$

$\Rightarrow x^2+ax-2a^2=0$

$\Rightarrow (x+2a)(x-a)-0$

$\Rightarrow x=a,-2a$

$\Rightarrow x=a$

$\Rightarrow y^2=a^2 \Rightarrow y=\pm a$

$\Rightarrow$ shade region is the smaller of the two areas in which the circle is divided by the parabola

$$A=2\left[\int_0^a \sqrt{ax}dx+\int_a^{\sqrt{2}a}\left(2a^2-x^2\right)^{1/2}dx\right]$$

$$=2\sqrt{a}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^a+2\left[\frac{x}{2}\left(2a^2-x^2\right)^{1/2}+\frac{2a^2}{2}\sin^{-1}\frac{x}{\sqrt{2}a}\right]_a^{\sqrt{2}a}$$

$$=\frac{4}{3}\sqrt{a}\left[a^{3/2}\right]+\left[x\left(2a^2-x^2\right)^{1/2}+2a^2\sin^{-1}\frac{x}{\sqrt{2}a}\right]_a^{\sqrt{2}a}$$

$$= \frac{4}{3}\sqrt{a}\left[a^{3/2}\right] + \left[2a^2 \sin^{-1}\frac{\sqrt{2}a}{\sqrt{2}a} - a\left(2a^2 - a^2\right)^{1/2} - 2a^2 \sin^{-1}\frac{a}{\sqrt{2}a}\right]$$

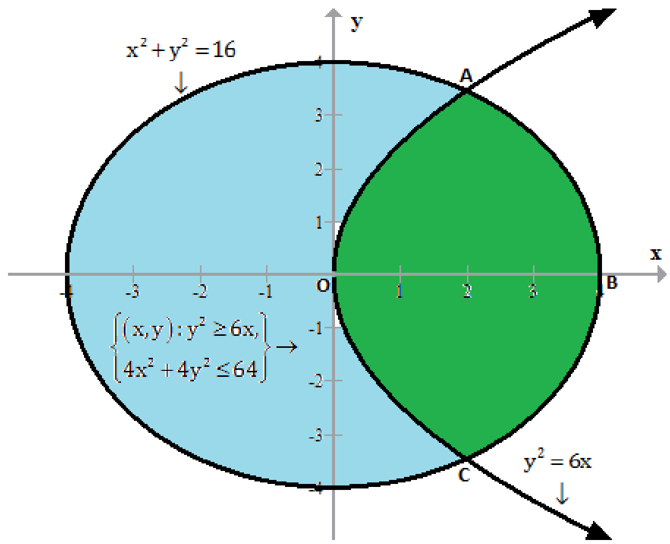
$$= \frac{4}{3}\sqrt{a}\left[a^{3/2}\right] + \left[2a^2 \sin^{-1} 1 - a\left(a^2\right)^{1/2} - 2a^2 \sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= \frac{4}{3}\sqrt{a}\left[a^{3/2}\right] + \left[2a^2\frac{\pi}{2} - a^2 - 2a^2\frac{\pi}{4}\right]$$

$$= \frac{4}{3}\sqrt{a}\left[a^{3/2}\right] + 2a^2\frac{\pi}{4} - a^2$$

$$= \frac{4}{3}\sqrt{a}\left[a^{3/2}\right] + a^2\frac{\pi}{2} - a^2 \text{sq. units}$$

OR



$4x^2 + 4y^2 = 64$

$\Rightarrow x^2 + y^2 = 16$

The points of intersection of the two curves $x^2 + y^2 = 16$ and $y^2 = 6x$

$x^2 + 6x = 16 \Rightarrow x^2 + 6x - 16 = 0 \Rightarrow (x+8)(x-2) = 0 \Rightarrow x = -8, 2$

But x is non negative so $x = 2$

Required area (Blue shaded portion)

= Ar (Circle) − Ar (Green Shaded portion)

$\Rightarrow$ Required area $= \pi(4)^2 - 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$

$$= 16\pi - 2\sqrt{6}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 - 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$$

$$= 16\pi - \frac{4\sqrt{6}}{3}\left(2^{\frac{3}{2}} - 0\right) - 2\left[8\sin^{-1}(1) - \sqrt{16-4} - 8\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= 16\pi - \frac{4\sqrt{6} \times 2\sqrt{2}}{3} - 2\left[8 \times \frac{\pi}{2} - \sqrt{12} - 8 \times \frac{\pi}{6}\right]$$

$$= 16\pi - \frac{16\sqrt{3}}{3} - \frac{16\pi}{3} + 4\sqrt{3} = \frac{32\pi}{3} - \frac{4\sqrt{3}}{3} = \frac{4}{3}\left(8\pi - \sqrt{3}\right) \text{sq.units}$$

27. Let $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}=\lambda$

$x=-2+3\lambda, y=-1+2\lambda, z=3+2\lambda$

Therefore, a point on this line is: {(-2+3 λ), (-1 + 2 λ), (3 + 2 λ)}

The distance of the point {(-2+3 λ), (-1 + 2 λ), (3 + 2 λ)} from point (1, 2, 3) = $3\sqrt{2}$

$\therefore \sqrt{(-2+3\lambda-1)^2+(-1+2\lambda-2)^2+(3+2\lambda-3)^2}=3\sqrt{2}$

$\Rightarrow (-3+3\lambda)^2+(-3+2\lambda)^2+(2\lambda)^2=18$

$\Rightarrow 9+9\lambda^2-18\lambda+9+4\lambda^2-12\lambda+4\lambda^2=18$

$17\lambda^2-30\lambda=0$

$\lambda=0, \lambda=\frac{30}{17}$

When $\lambda=\frac{30}{17}$,

$x=-2+3\lambda=-2+3\left(\frac{30}{17}\right)=-2+\frac{90}{17}=\frac{56}{17}$

$y=-1+2\lambda=-1+2\left(\frac{30}{17}\right)=-1+\frac{60}{17}=\frac{43}{17}$

$z=3+2\lambda=3+2\left(\frac{30}{17}\right)=\frac{51+60}{17}=\frac{111}{17}$

Thus, when $\lambda=\frac{30}{17}$, the point is $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$

and when λ = 0, the point is (-2, -1, 3).

OR

The equation of line L_1 :

$$\frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$$

$$\Rightarrow \frac{x-1}{-3}=\frac{y-2}{\frac{p}{7}}=\frac{z-3}{2} \quad ....(1)$$

The equation of line L_2 :

$$\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$$

$$\Rightarrow \frac{x-1}{\frac{-3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5} \quad ....(2)$$

Since line L_1 and L_2 are perpendicular to each other, we have

$$-3\times\left(\frac{-3p}{7}\right)+\frac{p}{7}\times 1+2\times(-5)=0$$

$$\Rightarrow \frac{9p}{7}+\frac{p}{7}=10$$

$$\Rightarrow 10p=70$$

$$\Rightarrow p=7$$

Thus equations of lines L_1 and L_2 are:

$$\frac{x-1}{-3}=\frac{y-2}{1}=\frac{z-3}{2}$$

$$\frac{x-1}{-3}=\frac{y-5}{1}=\frac{z-6}{-5}$$

Thus the equation of the line passing through the point $(3, 2, -4)$ and parallel to the line L_1 is:

$$\frac{x-3}{-3}=\frac{y-2}{1}=\frac{z+4}{2}$$

28. Let x and y be number of packets of food P and Q respectively.

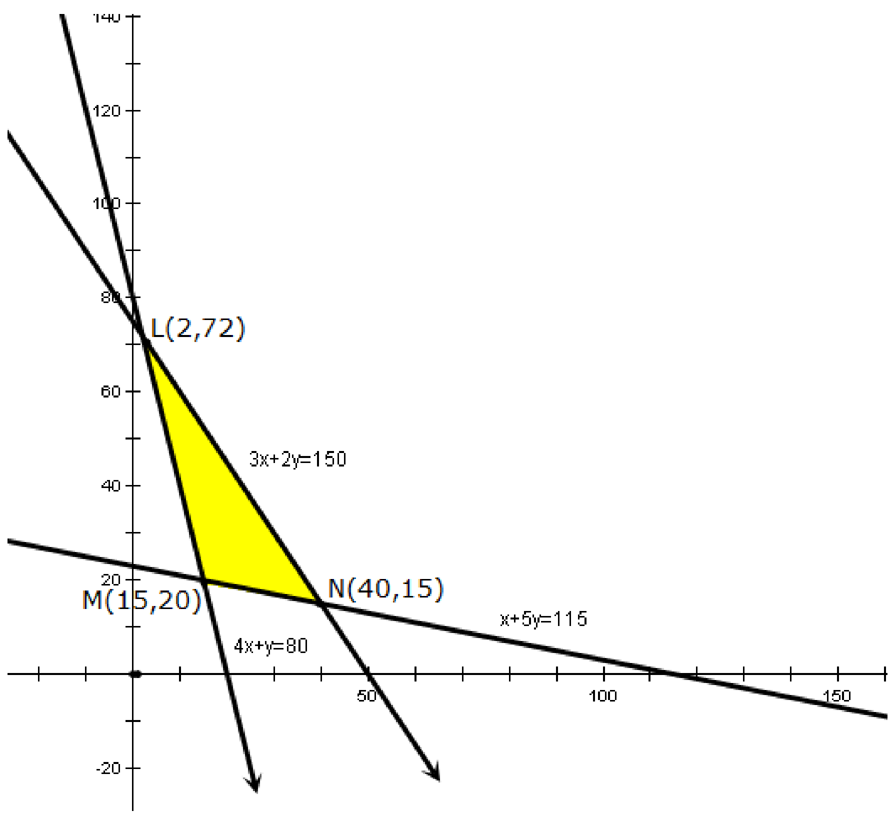
Linear programming problem is

Minimize Z = 6x + 3y (Vitamins A)

s.t.	$12x + 3y \geq 240$	or	$4x + y \geq 80$
	$4x + 20y \geq 460$	or	$x + 5y \geq 115$
	$6x + 4y \leq 300$	or	$3x + 2y \leq 150$

$x \geq 0, y \geq 0,$

Graphically the problem can be represented as



Co-ordinates of corner points L, M, N are (2, 72), (15, 20) and (40, 15), we have

Corner points	z = 6x + 3y
L (2, 72)	228
M (15, 20)	150 → Minimum
N (40, 15)	285

Hence, minimum vitamin A is used at point (15, 20) i.e. when 15 packets of food P and 20 packets of food Q are used.

29. This is a case of Bernoulli's trials.

Let Success: Getting a purple ball on a draw

Failure: Getting a pink ball on a draw

$$p = P(\text{success}) = \frac{10}{25} = \frac{2}{5} \Rightarrow q = \frac{3}{5}$$

$$\text{(i)} P(6\text{success}) = {}^{6}C_{6}p^{6}q^{0} = 1\times\left(\frac{2}{5}\right)^{6}\times 1 = \left(\frac{2}{5}\right)^{6}$$

(ii)P(not more than 2 failures) = P(not less than 4 success) = P(4) + P(5) + P(6)

$$= {}^{6}C_{4}p^{4}q^{2} + {}^{6}C_{5}p^{5}q^{1} + {}^{6}C_{6}p^{6}q^{0} = 15\times\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{2} + 6\times\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right) + 1\times\left(\frac{2}{5}\right)^{6}$$

$$= \left(\frac{2}{5}\right)^{4}\left[15\times\left(\frac{3}{5}\right)^{2} + 6\times\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)^{2}\right]$$

$$= \left(\frac{2}{5}\right)^{4}\left[\frac{135}{25} + \frac{36}{25} + \frac{4}{25}\right]$$

$$= \left(\frac{2}{5}\right)^{4}\left[\frac{175}{25}\right] = 7\times\left(\frac{2}{5}\right)^{4}$$

$$\text{(iii)} P(3\text{success } 3\text{failures}) = P(3) = {}^{6}C_{3}p^{3}q^{3} = 15\left(\frac{2}{5}\right)^{3}\left(\frac{3}{5}\right)^{3} = \frac{648}{3125}$$

(iv)P(atleast 1 failure)=P(at most 5 success)

$$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = 1 - P(6) = 1 - {}^{6}C_{6}p^{6}q^{0}$$

$$= 1 - 1\times\left(\frac{2}{5}\right)^{6}\times 1 = 1 - \left(\frac{2}{5}\right)^{6} = 1 - \frac{64}{15625} = \frac{15561}{15625}$$