12. DIMENSIONAL ANALYSIS

- Velocity potential = $[L^2 T^{-1}]$
  Stream function = $[L^2 T^{-1}]$
  Acceleration = $[LT^{-2}]$
  Vorticity = $[T^{-1}]$
- Total no. of variables influencing the problem is equal to the no. of independent variables plus one, one being the no. of dependent variable.
- **Buckingham π theorem** states that if all the n-variable are described by m fundamental dimensions, they may be grouped into (n - m) dimensionless π terms.
- Selection of 3 repeating variables from the geometry of flow, fluid properties and fluid motion.
- Geometric similarity - similarity of shape
  Kinematic similarity - similarity of motion
  Dynamic similarity - similarity of forces

Number	Equation	Significance
Reynolds No.	$\frac{F_i}{F_v} = \frac{\rho VL}{\mu}$	Flow in closed conduit pipe
Froude No.	$\sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{gL}}$	where a free surface is present, structure eg. weirs spillway, channels, etc. where *gravity* force is predominant.
Eulers No.	$\sqrt{\frac{F_i}{F_p}} = \frac{V}{\sqrt{\frac{p}{\rho}}}$	In cavitation studies.
Mach No.	$\sqrt{\frac{F_i}{F_e}} = \frac{V}{C}$	where fluid compressibility is important.
Weber No.	$\sqrt{\frac{F_i}{F_\sigma}} = \frac{v}{\sqrt{\sigma/\rho L}}$	In capillary studies.

Here, F_i = Inertia force, F_v = Viscous force
F_p = Pressure force, F_e = Elastic force
F_σ = Surface tension force

REYNOLDS MODEL LAW

$$(Re)_m = (Re)_p \qquad \frac{\rho_r V_r L_r}{\mu_r} = 1$$

- Applications of Reynold's Model Law
  - Flow through small sized pipes.
  - Low velocity motion around automobiles and aeroplane.
  - Submarines completely under water.
  - Flow through low speed turbo machines.

FROUDE'S MODEL LAW

$$(F_r)_m = (F_r)_p \qquad \frac{V_r}{\sqrt{L_r g_r}} = 1$$

- Applications of Froude's Model Law
  - Open channels
  - Notches & weirs
  - Spill ways & dams
  - Liquid jets from orifice
  - Ship partially submerged in rough & turbulent sea

DISTORTED MODEL LAW

(i) **Velocity scale ratio:** $V_r = \sqrt{(L_r)_V}$

(ii) **Area scale ratio:** $A_r = (L_r)_H (L_r)_V$

(iii) **Discharge scale ratio:** $Q_r = (L_r)_H (L_r)_V^{3/2}$

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