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Introduction to Three Dimensional Geometry

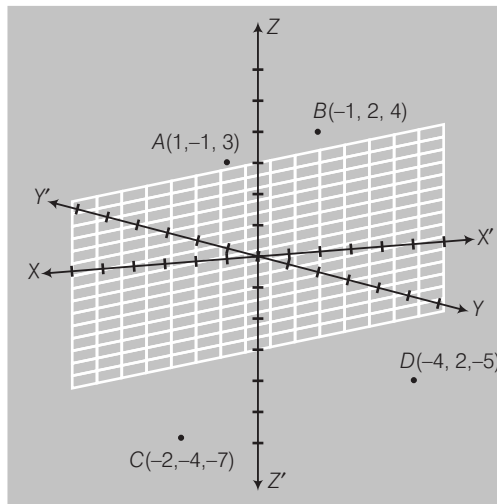
Short Answer Type Questions

Q. 1 Locate the following points

- | | |
|----------------------|--------------------|
| (i) $(1, -1, 3)$ | (ii) $(-1, 2, 4)$ |
| (iii) $(-2, -4, -7)$ | (iv) $(-4, 2, -5)$ |

Sol. Given, coordinates are

- | | |
|-----------------------|---------------------|
| (i) $(1, -1, 3)$ | (ii) $B(-1, 2, 4)$ |
| (iii) $C(-2, -4, -7)$ | (iv) $D(-4, 2, -5)$ |



X-increment = Y-increment = Z-increment = 1

Q. 2 Name the octant in which each of the following points lies.

- | | |
|---------------------|----------------------|
| (i) $(1, 2, 3)$ | (ii) $(4, -2, 3)$ |
| (iii) $(4, -2, -5)$ | (iv) $(4, 2, -5)$ |
| (v) $(-4, 2, 5)$ | (vi) $(-3, -1, 6)$ |
| (vii) $(2, -4, -7)$ | (viii) $(-4, 2, -5)$ |

Sol. (i) Point $(1, 2, 3)$ lies in first quadrant. (ii) $(4, -2, 3)$ in fourth octant.
 (iii) $(4, -2, -5)$ in eighth octant. (iv) $(4, 2, -5)$ in fifth octant.
 (v) $(-4, 2, 5)$ in second octant. (vi) $(-3, -1, 6)$ in third octant.
 (vii) $(2, -4, -7)$ in eighth octant. (viii) $(-4, 2, -5)$ in sixth octant.

Q. 3 If A, B, C be the feet of perpendiculars from a point P on the X, Y and Z -axes respectively, then find the coordinates of A, B and C in each of the following where the point P is

- | | |
|----------------------|--------------------|
| (i) $A(3, 4, 2)$ | (ii) $B(-5, 3, 7)$ |
| (iii) $C(4, -3, -5)$ | |

Sol. The coordinates of A, B and C are the following

- (i) $A(3, 0, 0), B(0, 4, 0), C(0, 0, 2)$
 (ii) $A(-5, 0, 0), B(0, 3, 0), C(0, 0, 7)$
 (iii) $A(4, 0, 0), B(0, -3, 0), C(0, 0, -5)$

Q. 4 If A, B , and C be the feet of perpendiculars from a point P on the XY, YZ and ZX -planes respectively, then find the coordinates of A, B and C in each of the following where the point P is

- | | |
|---------------------|-------------------|
| (i) $(3, 4, 5)$ | (ii) $(-5, 3, 7)$ |
| (iii) $(4, -3, -5)$ | |

Sol. We know that, on XY -plane $z = 0$, on YZ -plane, $x = 0$ and on ZX -plane, $y = 0$. Thus, the coordinate of A, B and C are following

- (i) $A(3, 4, 0), B(0, 4, 5), C(3, 0, 5)$
 (ii) $A(-5, 3, 0), B(0, 3, 7), C(-5, 0, 7)$
 (iii) $A(4, -3, 0), B(0, -3, -5), C(4, 0, -5)$

Q. 5 How far apart are the points $(2, 0, 0)$ and $(-3, 0, 0)$?

Thinking Process

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

Sol. Given points, $A(2, 0, 0)$ and $B(-3, 0, 0)$

$$AB = \sqrt{(2 + 3)^2 + 0^2 + 0^2} = 5$$

Q. 6 Find the distance from the origin to (6, 6, 7).

Sol. Distance from origin to the point (6, 6, 7)

$$\begin{aligned}
 &= \sqrt{(0-6)^2 + (0-6)^2 + (0-7)^2} \quad [\because d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}] \\
 &= \sqrt{36 + 36 + 49} \\
 &= \sqrt{121} = 11
 \end{aligned}$$

Q. 7 Show that, if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1 - x^2 - y^2})$ is at a distance 1 unit from the origin.

Sol. Given that, $x^2 + y^2 = 1$

$\therefore$ Distance of the point $(x, y, \sqrt{1 - x^2 - y^2})$ from origin is given as

$$\begin{aligned}
 d &= \left| \sqrt{x^2 + y^2 + (\sqrt{1 - x^2 - y^2})^2} \right| \\
 &= \left| \sqrt{x^2 + y^2 + 1 - x^2 - y^2} \right| = 1
 \end{aligned}$$

Hence proved.

Q. 8 Show that the point A (1, -1, 3), B (2, -4, 5) and C (5, -13, 11) are collinear.

Thinking Process

If the three points A, B, and C are collinear, then $AB + BC = AC$.

Sol. Given points, A (1, -1, 3), B (2, -4, 5) and C (5, -13, 11).

$$\begin{aligned}
 AB &= \sqrt{(1-2)^2 + (-1-4)^2 + (3-5)^2} \\
 &= \sqrt{1 + 9 + 4} = \sqrt{14} \\
 BC &= \sqrt{(2-5)^2 + (-4+13)^2 + (5-11)^2} \\
 &= \sqrt{9 + 81 + 36} = \sqrt{126} \\
 AC &= \sqrt{(1-5)^2 + (-1+13)^2 + (3-11)^2} \\
 &= \sqrt{16 + 144 + 64} = \sqrt{224}
 \end{aligned}$$

$$\therefore AB + BC = AC$$

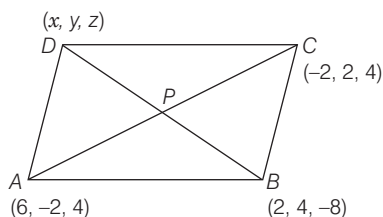
$$\Rightarrow \sqrt{14} + \sqrt{126} = \sqrt{224}$$

$$\Rightarrow \sqrt{14} + 3\sqrt{14} = 4\sqrt{14}$$

So, the points A, B and C are collinear.

Q. 9 Three consecutive vertices of a parallelogram $ABCD$ are $A (6, -2, 4)$, $B (2, 4, -8)$ and $C (-2, 2, 4)$. Find the coordinates of the fourth vertex.

Sol. Let the coordinates of the fourth vertices $D (x, y, z)$.



Mid-points of diagonal AC ,

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}, z = \frac{z_1 + z_2}{2}$$

and
$$x = \frac{6 - 2}{2} = 2, y = \frac{-2 + 2}{2} = 0, z = \frac{4 + 4}{2} = 4$$

Since, the mid-point of AC are $(2, 0, 4)$.

Now, mid-point of BD , $2 = \frac{x + 2}{2} \Rightarrow x = 2$

$\Rightarrow 0 = \frac{y + 4}{2} \Rightarrow y = -4$

$\Rightarrow 4 = \frac{z - 8}{2} \Rightarrow z = 16$

So, the coordinates of fourth vertex D is $(2, -4, 16)$.

Q. 10 Show that the $\triangle ABC$ with vertices $A (0, 4, 1)$, $B (2, 3, -1)$ and $C (4, 5, 0)$ is right angled.

Thinking Process

In a right angled triangle sum of the square of two sides is equal to square of third side.

Sol. Given that, the vertices of the $\triangle ABC$ are $A (0, 4, 1)$, $B (2, 3, -1)$ and $C (4, 5, 0)$.

Now,

$$AB = \sqrt{(0 - 2)^2 + (4 - 3)^2 + (1 - (-1))^2}$$

$$= \sqrt{4 + 1 + 4} = 3$$

$$BC = \sqrt{(2 - 4)^2 + (3 - 5)^2 + (-1 - 0)^2}$$

$$= \sqrt{4 + 4 + 1} = 3$$

$$AC = \sqrt{(0 - 4)^2 + (4 - 5)^2 + (1 - 0)^2}$$

$$= \sqrt{16 + 1 + 1} = \sqrt{18}$$

$\therefore AC^2 = AB^2 + BC^2$

$\Rightarrow 18 = 9 + 9$

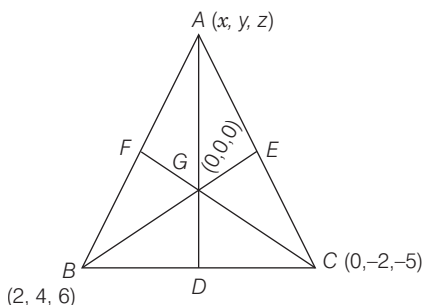
Hence, vertices $\triangle ABC$ is a right angled triangle.

Q. 11 Find the third vertex of triangle whose centroid is origin and two vertices are $(2, 4, 6)$ and $(0, -2, 5)$.

Thinking Process

The vertices of the $\triangle ABC$ are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, then the coordinates of the centroid G are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.

Sol. Let third vertex of $\triangle ABC$ i.e., is $A(x, y, z)$.



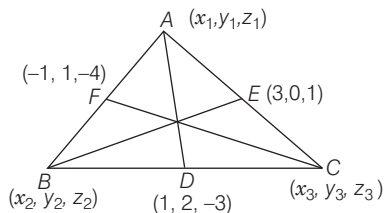
Given that, the coordinate of centroid G are $(0, 0, 0)$.

$$\begin{aligned} \therefore 0 &= \frac{x + 2 + 0}{3} \Rightarrow x = -2 \\ 0 &= \frac{y + 4 - 2}{3} \Rightarrow y = -2 \\ 0 &= \frac{z + 6 - 5}{2} \Rightarrow z = -1 \end{aligned}$$

Hence, the third vertex of triangle is $(-2, -2, -1)$.

Q. 12 Find the centroid of a triangle, the mid-point of whose sides are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$.

Sol. Given that, mid-points of sides are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$.



Let the vertices of the $\triangle ABC$ are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$. Then, mid-point of BC are $(1, 2, -3)$.

$$\begin{aligned} \therefore 1 &= \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 2 & \dots(i) \\ 2 &= \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 4 & \dots(ii) \\ -3 &= \frac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = -6 & \dots(iii) \end{aligned}$$

Similarly for the sides AB and AC ,

$$\Rightarrow -1 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = -2 \quad \dots(\text{iv})$$

$$\Rightarrow 1 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 2 \quad \dots(\text{v})$$

$$\Rightarrow -4 = \frac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = -8 \quad \dots(\text{vi})$$

$$\Rightarrow 3 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 6 \quad \dots(\text{vii})$$

$$\Rightarrow 0 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 0 \quad \dots(\text{viii})$$

$$\Rightarrow 1 = \frac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = 2 \quad \dots(\text{ix})$$

On adding Eqs. (i) and (iv), we get

$$x_1 + 2x_2 + x_3 = 0 \quad \dots(\text{x})$$

On adding Eqs. (ii) and (v), we get

$$y_1 + 2y_2 + y_3 = 6 \quad \dots(\text{xi})$$

On adding Eqs. (iii) and (vi), we get

$$z_1 + 2z_2 + z_3 = -14 \quad \dots(\text{xii})$$

From Eqs. (vii) and (x),

$$2x_2 = -6 \Rightarrow x_2 = -3$$

If $x_2 = -3$, then $x_3 = 5$

If $x_3 = 5$, then $x_1 = 1$, $x_2 = -3$, $x_3 = 5$

From Eqs. (xi) and (viii),

$$2y_2 = 6 \Rightarrow y_2 = 3$$

If $y_2 = 3$, then $y_1 = -1$ If $y_1 = -1$, then $y_3 = 1$, $y_2 = 3$, $y_3 = 1$

From Eqs. (xii) and (ix),

$$2z_2 = -16 \Rightarrow z_2 = -8$$

$$z_2 = -8, \text{ then } z_1 = 0$$

$$z_1 = 0, \text{ then } z_3 = 2$$

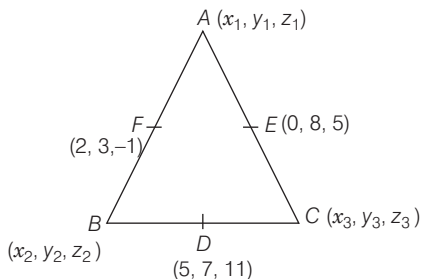
$$z_1 = 0, z_2 = -8, z_3 = 2$$

So, the points are $A(1, -1, 0)$, $B(-3, 3, -8)$ and $C(5, 1, 2)$.

$$\therefore \text{Centroid of the triangle} = G\left(\frac{1-3+5}{3}, \frac{-1+3+1}{3}, \frac{0-8+2}{3}\right) \text{ i.e., } G(1, 1, -2)$$

Q. 13 The mid-points of the sides of a triangle are $(5, 7, 11)$, $(0, 8, 5)$ and $(2, 3, -1)$. Find its vertices.

Sol. Let vertices of the $\triangle ABC$ are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, then the mid-point of BC $(5, 7, 11)$.



$$5 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 10 \quad \dots(i)$$

$$7 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 14 \quad \dots(ii)$$

$$11 = \frac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = 22 \quad \dots(iii)$$

Similarly for the sides AB and AC ,

$$2 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 4 \quad \dots(iv)$$

$$3 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 6 \quad \dots(v)$$

$$-1 = \frac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = -2 \quad \dots(vi)$$

$$0 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 0 \quad \dots(vii)$$

$$8 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 16 \quad \dots(viii)$$

$$5 = \frac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = 10 \quad \dots(ix)$$

From Eqs. (i) and (iv),

$$x_1 + 2x_2 + x_3 = 14 \quad \dots(x)$$

From Eqs. (ii) and (v),

$$y_1 + 2y_2 + y_3 = 20 \quad \dots(xi)$$

From Eqs. (iii) and (vi),

$$z_1 + 2z_2 + z_3 = 20 \quad \dots(xii)$$

From Eqs. (vii) and (x),

$$\begin{aligned} 2x_2 = 14 &\Rightarrow x_2 = 7 \\ x_2 = 7, \text{ then } x_3 &= 10 - 7 = 3 \\ x_3 = 3, \text{ then } x_1 &= -3 \\ x_1 = -3, x_2 = 7, x_3 &= 3 \end{aligned}$$

From Eqs. (viii) and (xi),

$$\begin{aligned} 2y_2 = 4 &\Rightarrow y_2 = 2 \\ y_2 = 2, \text{ then } y_1 &= 4 \\ y_1 = 4, \text{ then } y_3 &= 12 \\ y_1 = 4, y_2 = 2, y_3 &= 12 \end{aligned}$$

From Eqs. (ix) and (xii),

$$\begin{aligned} 2z_2 = 10 &\Rightarrow z_2 = 5 \\ z_2 = 5, \text{ then } z_1 &= -7 \\ z_1 = -7, \text{ then } z_3 &= 17 \\ z_1 = -7, z_2 = 5, z_3 &= 17 \end{aligned}$$

So, the vertices are $A(-3, 4, -7)$, $B(7, 2, 5)$ and $C(3, 12, 17)$.

Q. 14 If the vertices of a parallelogram $ABCD$ are $A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$, then find the fourth vertex D .

Thinking Process

The diagonal of a parallelogram have the same mid-points. Use this property to solve the problem.

Sol. Let the fourth vertex of the parallelogram $ABCD$ is $D(x, y, z)$. Then, the mid-point of AC are

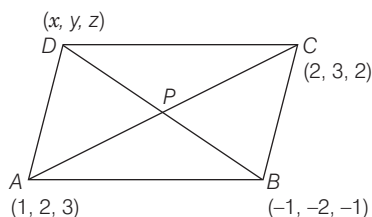
$$P\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) \text{ i.e., } P\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right).$$

Now, mid-point of BD ,

$$\frac{3}{2} = \frac{-1 + x}{2} \Rightarrow x = 4$$

$$\frac{5}{2} = \frac{-2 + y}{2} \Rightarrow y = 7$$

$$\frac{5}{2} = \frac{-1 + z}{2} \Rightarrow z = 6$$



So, the coordinates of fourth vertex is (4, 7, 6).

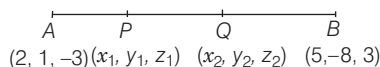
Q. 15 Find the coordinate of the points which trisect the line segment joining the points $A(2, 1, -3)$ and $B(5, -8, 3)$.

Thinking Process

If point P divided line segment joining the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in $m_1:m_2$

internally then the coordinate of P are $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2} \right)$

Sol. Let the $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ trisect line segment AB .



Since, the point P divided line AB in $1:2$ internally, then

$$x_1 = \frac{2 \times 2 + 1 \times 5}{1 + 2} = \frac{9}{3} = 3$$

$$y_1 = \frac{2 \times 1 + 1 \times (-8)}{3} = \frac{-6}{3} = -2$$

$$z_1 = \frac{2 \times (-3) + 1 \times 3}{3} = \frac{-6 + 3}{3} = \frac{-3}{3} = -1$$

Since, the point Q divides the line segment AB in $2:1$, then

$$x_2 = \frac{1 \times 2 + 2 \times 5}{3} = 4,$$

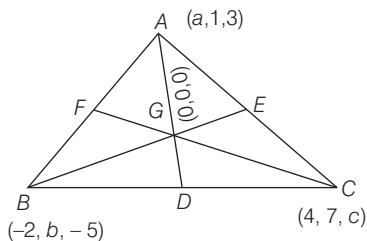
$$y_2 = \frac{1 \times 1 + 2 \times (-8)}{3} = -5$$

$$z_2 = \frac{1 \times (-3) + 2 \times 3}{3} = -1$$

So, the coordinates of P are (3, -2, -1) and the coordinates of Q are (4, -5, 1).

Q. 16 If the origin is the centroid of a $\triangle ABC$ having vertices $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, then find the values of a, b, c .

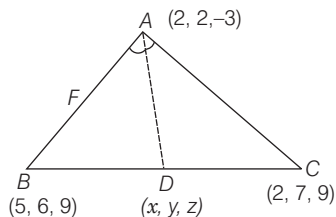
Sol. Given that origin is the centroid of the $\triangle ABC$ i.e., $G(0, 0, 0)$.



$$\begin{aligned} \therefore 0 &= \frac{a - 2 + 4}{3} \Rightarrow a = -2 \\ 0 &= \frac{1 + b + 7}{3} \Rightarrow b = -8 \\ 0 &= \frac{3 - 5 + c}{3} \Rightarrow c = +2 \\ \therefore a &= -2, b = -8 \text{ and } c = 2 \end{aligned}$$

Q. 17 If $A(2, 2, -3)$, $B(5, 6, 9)$, $C(2, 7, 9)$ be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D , then find the coordinates of D .

Sol. Let the coordinates of D are (x, y, z) .



$$\begin{aligned} AB &= \sqrt{9 + 16 + 144} = \sqrt{169} = 13 \\ AC &= \sqrt{0 + 25 + 144} = \sqrt{169} = 13 \\ \Rightarrow \frac{AB}{AC} &= \frac{13}{13} \Rightarrow AB = AC \\ \frac{BD}{DC} &= \frac{1}{1} \Rightarrow BD = DC \end{aligned}$$

Since, D divides the line BC in two equal parts. So, D is the mid-point of BC .

$$\begin{aligned} \therefore x &= \frac{5 + 2}{2} = 7/2 \\ \Rightarrow y &= \frac{6 + 7}{2} = 13/2 \\ \Rightarrow z &= \frac{9 + 9}{2} = 9 \end{aligned}$$

So, the coordinates of D are $\left(\frac{7}{2}, \frac{13}{2}, 9\right)$.

Long Answer Type Questions

Q. 18 Show that the three points $A (2, 3, 4)$, $B (1, 2, -3)$ and $C (-4, 1, -10)$ are collinear and find the ratio in which C divides AB .

Sol. Given points are $A (2, 3, 4)$, $B (-1, 2, -3)$ and $C (-4, 1, -10)$.

$$\begin{aligned} \therefore AB &= \sqrt{(2+1)^2 + (3-2)^2 + (4+3)^2} \\ &= \sqrt{9+1+49} = \sqrt{59} \\ BC &= \sqrt{(-1+4)^2 + (2-1)^2 + (-3+10)^2} \\ &= \sqrt{9+1+49} = \sqrt{59} \\ AC &= \sqrt{(2+4)^2 + (3-1)^2 + (4+10)^2} \\ &= \sqrt{36+4+196} \\ &= \sqrt{236} = 2\sqrt{59} \end{aligned}$$

$$\text{Now, } AB + BC = \sqrt{59} + \sqrt{59} = 2\sqrt{59}$$

$$\therefore AB + BC = AC$$

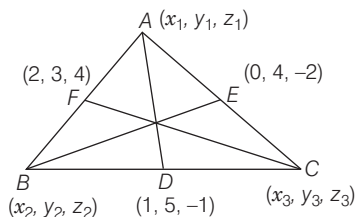
Hence, the points A , B and C are collinear.

$$\text{Now, } AC : BC = 2\sqrt{59} : \sqrt{59} = 2 : 1$$

So, C divides AB in $2 : 1$ externally.

Q. 19 The mid-point of the sides of a triangle are $(1, 5, -1)$, $(0, 4, -2)$ and $(2, 3, 4)$. Find its vertices and also find the centroid of the triangle.

Sol. Let the vertices of $\triangle ABC$ are $A (x_1, y_1, z_1)$, $B (x_2, y_2, z_2)$ and $C (x_3, y_3, z_3)$.



Since, the mid-point of side BC is $D (1, 5, -1)$.

$$\text{Then, } \frac{x_2 + x_3}{2} = 1 \Rightarrow x_2 + x_3 = 2 \quad \dots(i)$$

$$\frac{y_2 + y_3}{2} = 5 \Rightarrow y_2 + y_3 = 10 \quad \dots(ii)$$

$$\frac{z_2 + z_3}{2} = -1 \Rightarrow z_2 + z_3 = -2 \quad \dots(iii)$$

Similarly, the mid-points of AB and AC are $F (2, 3, 4)$ and $E (0, 4, -2)$,

$$\frac{x_1 + x_2}{2} = 2 \Rightarrow x_1 + x_2 = 4 \quad \dots(iv)$$

$$\frac{y_1 + y_2}{2} = 3 \Rightarrow y_1 + y_2 = 6 \quad \dots(v)$$

$$\text{and } \frac{z_1 + z_2}{2} = 4 \Rightarrow z_1 + z_2 = 8 \quad \dots(vi)$$

$$\text{Now, } \frac{x_1 + x_3}{2} = 0 \Rightarrow x_1 + x_3 = 0 \quad \dots(\text{vii})$$

$$\frac{y_1 + y_3}{2} = 4 \Rightarrow y_1 + y_3 = 8 \quad \dots(\text{viii})$$

$$\frac{z_1 + z_3}{2} = -2 \Rightarrow z_1 + z_3 = -4 \quad \dots(\text{ix})$$

From Eqs. (i) and (iv),

$$x_1 + 2x_2 + x_3 = 6 \quad \dots(\text{x})$$

From Eqs. (ii) and (v),

$$y_1 + 2y_2 + y_3 = 16 \quad \dots(\text{xi})$$

From Eqs. (iii) and (vi),

$$z_1 + 2z_2 + z_3 = 6 \quad \dots(\text{xii})$$

From Eqs. (vii) and (x),

$$\begin{aligned} 2x_2 &= 6 \Rightarrow x_2 = 3 \\ x_2 &= 3, \text{ then } x_3 = -1 \\ x_3 &= -1, \\ x_1 &= 1 \Rightarrow x_1 = 1, x_2 = 3, x_3 = -1 \end{aligned}$$

Then,

From Eqs. (viii) and (xi),

$$2y_2 = 8 \Rightarrow y_2 = 4$$

$$y_2 = 4,$$

Then,

$$y_1 = 2$$

$$y_1 = 2,$$

Then,

$$y_3 = 6,$$

$\Rightarrow$

$$y_1 = 2, y_2 = 4, y_3 = 6$$

From Eqs. (ix) and (xii),

$$2z_2 = 10 \Rightarrow z_2 = 5$$

$$z_2 = 5,$$

Then,

$$z_1 = 3$$

$$z_1 = 3,$$

Then,

$$z_3 = -7$$

$\Rightarrow$

$$z_1 = 3, z_2 = 5, z_3 = -7$$

So, the vertices of the triangle A (1, 2, 3), B (3, 4, 5) and C (-1, 6, -7).

Hence, centroid of the triangle $G \left(\frac{1+3-1}{3}, \frac{2+4+6}{3}, \frac{3+5-7}{3} \right)$ i.e., G (1, 4, 1/3).

Q. 20 Prove that the points (0, -1, -7), (2, 1, -9) and (6, 5, -13) are collinear. Find the ratio in which the first point divides the join of the other two.

Thinking Process

First of all find the value of AB, AC and BC using distance formula i.e., $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$, then show that $AB + BC = AC$ for collinearity of the points A, B and C.

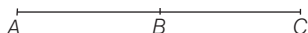
Sol. Given points are A(0, -1, -7), B (2, 1, -9) and C (6, 5, -13)

$$AB = \sqrt{(0-2)^2 + (-1-1)^2 + (-7+9)^2} = \sqrt{4+4+4} = 2\sqrt{3}$$

$$BC = \sqrt{(2-6)^2 + (1-5)^2 + (-9+13)^2} = \sqrt{16+16+16} = 4\sqrt{3}$$

$$AC = \sqrt{(0-6)^2 + (-1-5)^2 + (-7+13)^2} = \sqrt{36+36+36} = 6\sqrt{3}$$

$\therefore AB + BC = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$
 So, $AB + BC = AC$
 Hence, the points A, B and C are collinear.



$$AB : AC = 2\sqrt{3} : 6\sqrt{3} = 1 : 3$$

So, point A divide B and C in $1 : 3$ externally.

Q. 21 What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Sol. The coordinates of the cube which edge is 2 units, are $(2, 0, 0)$, $(2, 2, 0)$, $(0, 2, 0)$, $(0, 2, 2)$, $(0, 0, 2)$, $(2, 0, 2)$, $(0, 0, 0)$ and $(2, 2, 2)$.

Objective Type Questions

Q. 22 The distance of point $P(3, 4, 5)$ from the YZ -plane is

- (a) 3 units (b) 4 units
(c) 5 units (d) 550

Sol. (a) Given, point is $P(3, 4, 5)$.

Distance of P from YZ -plane,

[$\because$ YZ -plane, $x = 0$]

$$d = \sqrt{(0-3)^2 + (4-4)^2 + (5-5)^2} = 3$$

Q. 23 What is the length of foot of perpendicular drawn from the point $P(3, 4, 5)$ on Y -axis?

- (a) $\sqrt{41}$ (b) $\sqrt{34}$
(c) 5 (d) None of these

Sol. (b) We know that, on the Y -axis, $x = 0$ and $z = 0$.

$\therefore$ Point $A(0, 4, 0)$,

$$\begin{aligned}
 PA &= \sqrt{(0-3)^2 + (4-4)^2 + (0-5)^2} \\
 &= \sqrt{9 + 0 + 25} = \sqrt{34}
 \end{aligned}$$

Q. 24 Distance of the point $(3, 4, 5)$ from the origin $(0, 0, 0)$ is

- (a) $\sqrt{50}$ (b) 3
(c) 4 (d) 5

Sol. (a) Given, points $P(3, 4, 5)$ and $O(0, 0, 0)$,

$$\begin{aligned}
 PO &= \sqrt{(0-3)^2 + (0-4)^2 + (0-5)^2} \\
 &= \sqrt{9 + 16 + 25} = \sqrt{50}
 \end{aligned}$$

Q. 25 If the distance between the points $(a, 0, 1)$ and $(0, 1, 2)$ is $\sqrt{27}$, then the value of a is

- (a) 5 (b) ± 5
(c) -5 (d) None of these

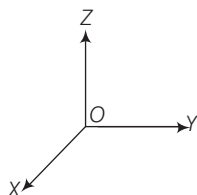
Sol. (b) Given, the points are $A(a, 0, 1)$ and $B(0, 1, 2)$.

$$\begin{aligned}\therefore AB &= \sqrt{(a-0)^2 + (0-1)^2 + (1-2)^2} \\ \Rightarrow \sqrt{27} &= \sqrt{a^2 + 1 + 1} \\ \Rightarrow 27 &= a^2 + 2 \\ \Rightarrow a^2 &= 25 \\ \Rightarrow a &= \pm 5\end{aligned}$$

Q. 26 X-axis is the intersection of two planes

- (a) XY and XZ (b) YZ and ZX
(c) XY and YZ (d) None of these

Sol. (a) We know that, on the XY and XZ-planes, the line of intersection is X-axis.



Q. 27 Equation of Y-axis is considered as

- (a) $x=0, y=0$ (b) $y=0, z=0$
(c) $z=0, x=0$ (d) None of these

Sol. (c) On the Y-axis, $x=0$ and $z=0$.

Q. 28 The point $(-2, -3, -4)$ lies in the

- (a) first octant (b) seventh octant
(c) second octant (d) eighth octant

Sol. (b) The point $(-2, -3, -4)$ lies in seventh octant.

Q. 29 A plane is parallel to YZ-plane, so it is perpendicular to

- (a) X-axis (b) Y-axis
(c) Z-axis (d) None of these

Sol. (a) A plane is parallel to YZ-plane, so it is perpendicular to X-axis.

Q. 30 The locus of a point for which $y = 0$ and $z = 0$, is

- (a) equation of X-axis
- (b) equation of Y-axis
- (c) equation at Z-axis
- (d) None of these

Sol. (a) We know that, equation on the X-axis, $y = 0$, $z = 0$.

So, the locus of the point is equation of X-axis.

Q. 31 The locus of a point for which $x = 0$ is

- (a) XY-plane
- (b) YZ-plane
- (c) ZX-plane
- (d) None of these

Sol. (b) On the YZ-plane, $x = 0$, hence the locus of the point is YZ-plane.

Q. 32 If a parallelopiped is formed by planes drawn through the points $(5, 8, 10)$ and $(3, 6, 8)$ parallel to the coordinate planes, then the length of diagonal of the parallelopiped is

- (a) $2\sqrt{3}$
- (b) $3\sqrt{2}$
- (c) $\sqrt{2}$
- (d) $\sqrt{3}$

Sol. (a) Given points of the parallelopiped are $A(5, 8, 10)$ and $B(3, 6, 8)$.

$$\begin{aligned}\therefore AB &= \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2} \\ &= \sqrt{4 + 4 + 4} = 2\sqrt{3}\end{aligned}$$

Q. 33 L is the foot of the perpendicular drawn from a point $P(3, 4, 5)$ on the XY-plane. The coordinates of point L are

- (a) $(3, 0, 0)$
- (b) $(0, 4, 5)$
- (c) $(3, 0, 5)$
- (d) None of these

Sol. (d) We know that, on the XY-plane $z = 0$.

Hence, the coordinates of the points L are $(3, 4, 0)$.

Q. 34 L is the foot of the perpendicular drawn from a point $(3, 4, 5)$ on X-axis. The coordinates of L are

- (a) $(3, 0, 0)$
- (b) $(0, 4, 0)$
- (c) $(0, 0, 5)$
- (d) None of these

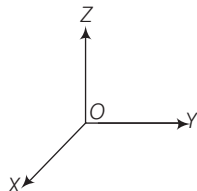
Sol. (a) On the X-axis, $y = 0$ and $z = 0$

Hence, the required coordinates are $(3, 0, 0)$.

Fillers

Q. 35 The three axes OX , OY and OZ determine

Sol. The three axes OX , OY and OZ determine three coordinates planes.



Q. 36 The three planes determine a rectangular parallelopiped which has of rectangular faces.

Sol. Three points

Q. 37 The coordinates of a point are the perpendicular distance from the on the respective axes.

Sol. Given points

Q. 38 The three coordinate planes divide the space into parts.

Sol. Eight parts

Q. 39 If a point P lies in YZ -plane, then the coordinates of a point on YZ -plane is of the form

Sol. We know that, on YZ -plane, $x = 0$. So, the coordinates of the required point is $(0, y, z)$.

Q. 40 The equation of YZ -plane is

Sol. The equation of YZ -plane is $x = 0$.

Q. 41 If the point P lies on Z -axis, then coordinates of P are of the form

Sol. On the Z -axis, $x = 0$ and $y = 0$. So, the required coordinates are $(0, 0, z)$.

Q. 42 The equation of Z -axis, are

Sol. The equation of Z -axis, $x = 0$ and $y = 0$.

Q. 43 A line is parallel to XY -plane if all the points on the line have equal

Sol. z -coordinates.

Q. 44 A line is parallel to X-axis, if all the points on the line have equal

Sol. y and z-coordinates.

Q. 45 $x = a$ represent a plane parallel to

Sol. $x = a$ represent a plane parallel to YZ-plane.

Q. 46 The plane parallel to YZ-plane is perpendicular to

Sol. The plane parallel to YZ-plane is perpendicular to X-axis.

Q. 47 The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 10, 13 and 8 units are

Sol. Given dimensions are $a = 10$, $b = 13$ and $c = 8$.

$$\begin{aligned}\therefore \text{ Required length} &= \sqrt{a^2 + b^2 + c^2} \\ &= \sqrt{100 + 169 + 64} = \sqrt{333}\end{aligned}$$

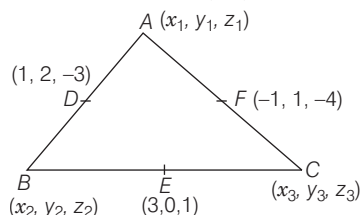
Q. 48 If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then a

Sol. Given points are $(a, 2, 1)$ and $(1, -1, 1)$.

$$\begin{aligned}\therefore \sqrt{(a-1)^2 + (2+1)^2 + (1-1)^2} &= 5 \\ \Rightarrow (a-1)^2 + 9 + 0 &= 25 \\ \Rightarrow a^2 - 2a + 1 + 9 &= 25 \\ \Rightarrow a^2 - 2a - 15 &= 0 \\ \Rightarrow a^2 - 5a + 3a - 15 &= 0 \\ \Rightarrow a(a-5) + 3(a-5) &= 0 \\ \Rightarrow (a-5)(a+3) &= 0 \\ \Rightarrow a-5 = 0 \text{ or } a+3 &= 0 \\ \therefore a &= +5 \text{ or } -3\end{aligned}$$

Q. 49 If the mid-points of the sides of a triangle AB, BC and CA are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$, then the centroid of the $\triangle ABC$ is

Sol. Let the vertices of $\triangle ABC$ is $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$.



Since, D is the mid-point of AB , then

$$\frac{x_1 + x_2}{2} = 1 \Rightarrow x_1 + x_2 = 2 \quad \dots(i)$$

$$\frac{y_1 + y_2}{2} = 2 \Rightarrow y_1 + y_2 = 4 \quad \dots(ii)$$

$$\frac{z_1 + z_2}{2} = -3 \Rightarrow z_1 + z_2 = -6 \quad \dots(iii)$$

Similarly, E and F are the mid-points of sides BC and AC , respectively.

$$\frac{x_2 + x_3}{2} = 3 \Rightarrow x_2 + x_3 = 6 \quad \dots(iv)$$

$$\frac{y_2 + y_3}{2} = 0 \Rightarrow y_2 + y_3 = 0 \quad \dots(v)$$

$$\frac{z_2 + z_3}{2} = 1 \Rightarrow z_2 + z_3 = 2 \quad \dots(vi)$$

$$\frac{x_1 + x_3}{2} = -1 \Rightarrow x_1 + x_3 = -2 \quad \dots(vii)$$

$$\frac{y_1 + y_3}{2} = 1 \Rightarrow y_1 + y_3 = 2 \quad \dots(viii)$$

$$\frac{z_1 + z_3}{2} = -4 \Rightarrow z_1 + z_3 = -8 \quad \dots(ix)$$

From Eqs. (i) and (iv),

$$x_1 + 2x_2 + x_3 = 8 \quad \dots(x)$$

From Eqs. (ii) and (v),

$$y_1 + 2y_2 + y_3 = 4 \quad \dots(xi)$$

From Eqs. (iii) and (vi),

$$z_1 + 2z_2 + z_3 = -4 \quad \dots(xii)$$

From Eqs. (vii) and (x),

$$2x_2 = 10 \Rightarrow x_2 = 5$$

$\Rightarrow$

$$x_2 = 5, \text{ then } x_3 = 1$$

If $x_3 = 1$, then $x_1 = -3$

$\therefore$

$$x_1 = -3, x_2 = 5, x_3 = 1$$

From Eqs. (viii) and (xi),

$$2y_2 = 2 \Rightarrow y_2 = 1$$

If

$$y_2 = 1, \text{ then } y_3 = -1$$

If

$$y_3 = -1, \text{ then } y_1 = 3$$

$\therefore$

$$y_1 = 3, y_2 = 1, y_3 = -1$$

From Eqs. (ix) and (xii),

$$2z_2 = 4 \Rightarrow z_2 = 2$$

If

$$z_2 = 2, \text{ then } z_3 = 0$$

If

$$z_3 = 0, \text{ then } z_1 = -8$$

$\therefore$

$$z_1 = -8, z_2 = 2, z_3 = 0$$

So, the vertices of $\triangle ABC$ are $A(-3, 3, -8)$, $B(5, 1, 2)$ and $C(1, -1, 0)$.

Hence, coordinates of centroid of $\triangle ABC$, $G\left(\frac{-3+5+1}{3}, \frac{3+1-1}{3}, \frac{-8+2+0}{3}\right)$

i.e.,

$$G(1, 1, -2).$$

Q. 50 Match each item given under the Column I to its correct answer given under Column II.

Column I	Column II
(i) In $-XY$ -plane	(a) 1st octant
(ii) Point $(2, 3, 4)$ lies in the	(b) YZ -plane
(iii) Locus of the points having X coordinate 0 is	(c) z -coordinate is zero
(iv) A line is parallel to X -axis if and only	(d) Z -axis
(v) If $X = 0, y = 0$ taken together will represent the	(e) plane parallel to XY -plane
(vi) $z = c$ represent the plane	(f) if all the points on the line have equal y and z -coordinates
(vii) Planes $X = a, Y = b$ represent the line	(f) from the point on the respective
(viii) Coordinates of a point are the distances from the origin to the feet of perpendiculars	(h) parallel to Z -axis
(ix) A ball is the solid region in the space enclosed by a	(i) disc
(x) Region in the plane enclosed by a circle is known as a	(j) sphere

Sol. (i) In XY -plane, z -coordinates is zero.

(ii) The point $(2, 3, 4)$ lies in 1st octant .

(iii) Locus of the points having x -coordinate is zero is YZ -plane.

(iv) A line is parallel to X -axis if and only if all the points on the line have equal y and z -coordinates.

(v) $x = 0, y = 0$ represent Z -axis.

(vi) $z = c$ represent the plane parallel to XY -plane.

(vii) The planes $x = a, y = b$ represent the line parallel to Z -axis.

(viii) Coordinates of a point are the distances from the origin to the feet of perpendicular from the point on the respective.

(ix) A ball is the solid region in the space enclosed by a sphere.

(x) The region in the plane enclosed by a circle is known as a disc.