31. Capacitors

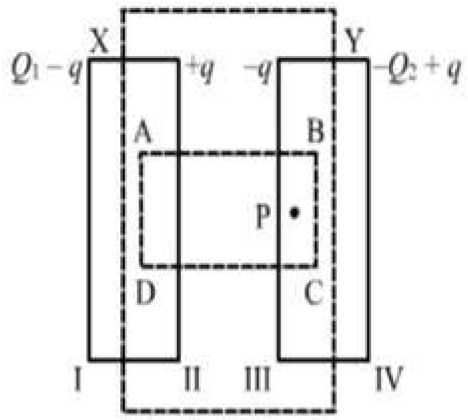
Short Answer

Answer.1

Given:

Charge on positive plate=Q_1

Charge on negative plate=Q_2



Assume a rectangular gaussian surface ABCD as shown in fig.

Let the charge on the capacitor plates be "q" and the area of plates be A. Then,

Charge appearing on face 1=Q_1-q.

Charge appearing on face 2=q.

Similarly,

Charge appearing on face 3= -q.

Since, the total charge enclosed by a closed surface =0)

Charge appearing on face 4=Q_2 +q.

Formula used:

We know that,

I) Electric field inside any conductor=0.

∴ Electric field at point Pinside plate)=0.

$E_1+E_2+E_3+E_4=0$ ...i)

This Electric field is the net effect of fields at point P due to faces I, II, III and IV.

II) Electric field due a thin sheet, $E=\frac{Q}{2A\varepsilon_0}$

Where

E is the electric filed due to thin plate

Q is the total charge enclosed in the gaussian surface

A is the area of the plate

ε_0 is the permittivity of the vacuum

Thus, Electric field at point P due to face I $E_1=\frac{Q_1-q}{2A\epsilon}$

Electric field at point P due to face II $E_2=\frac{q}{2A\epsilon}$

Electric field at point P due to face III $E_3=-\frac{q}{2A\epsilon}$

Electric field at point P due to face IV $E_4=-\frac{Q_2+q}{2A\epsilon}$

negative sign because electric field due to face IV is in leftwards direction).

Putting the values in equation (i) we get,

$$\frac{Q_1-q}{2A\varepsilon}+\frac{q}{2A\varepsilon}+\frac{-q}{2A\varepsilon}+\frac{-Q_2-q}{2A\varepsilon}=0$$

On solving the above equation, we get

$Q_1-q+q-q-Q_2-q=0$

$Q_1-Q_2-2q=0$

$q=\frac{Q_1+Q_2}{2}$

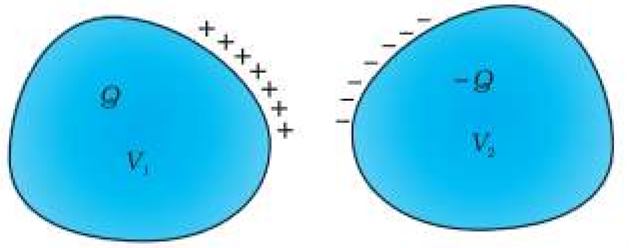
Thus, the charge on the capacitor is $\frac{Q_1+Q_2}{2}$

Answer.2

We know that,

Potential difference V is the work done per unit positive charge in taking a small test charge from conductor 2 to 1 against the field. Consequently, V is also proportional to Q and the ratio Q/V is a constant C known as capacitance of the capacitor.

$$C = \frac{Q}{V}$$



The value of this capacitance depends only on the size, shape and position of conductor and its plates and not on the potential difference applied by the battery or th charge on the plates.

For example: the capacitance in case of an isolated spherical capacitor is given by

$$C = 4\pi\epsilon_0 R$$

Where,

R=radius of the spherical conductor.

∴ Capacitance cannot be said to be dependent on charge Q.

Thus, capacitance of the capacitor is independent of the charge on the capacitor.

Answer3

We know that,

Charge given to any conductor appears entirely on its outer surface evenly.

Therefore, if equal amount of charge Q are given to a hollow and solid spheres, the entire charge Q will appear on their spherical surfaces and since they both have equal radius, capacitance of both spheres are given by

$C=4\pi\epsilon_0 R$

Where

R= radius of the spherical capacitor.

Now, using $Q = CV$

We get $V = \frac{Q}{4\pi\epsilon R}$

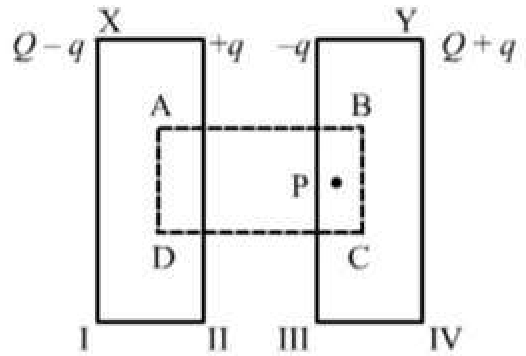
∴ Potential of both the spheres hollow and solid) will be same.

Answer.4

<u>Given:</u>

Two plates of a parallel plate capacitor with equal charge.

Here, both the plates are given same charge +Q.



Consider q charge on face II so that induced charge on face III is -q

Assume a rectangular Gaussian surface ABCD having area, A as shown in the above fig.

Thus, Electric field at point P due to face I $E_1 = \frac{Q-q}{2A\epsilon_0}$

Electric field at point P due to face II $E_2 = \frac{q}{2A\epsilon}$

Electric field at point P due to face III $E_3 = -\frac{q}{2A\epsilon}$

Electric field at point P due to face IV $E_4 = -\frac{Q+q}{2A\epsilon_0}$

negative sign because electric field due to face IV is in leftwards direction).

Since , point P lies inside the conductor thee total electric field at P must be zero

∴ $E_1+E_2+E_3+E_4=0$

$\Rightarrow \frac{Q-q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} - \frac{Q+q}{2A\epsilon_0} = 0$

∴ q=0

Therefore zero charge appears on face II and III and Q charge appears on face I and IV

Potential difference b/w the plates is given by

$$V = \frac{Q}{C}$$

∴ V=0 both the plates are at same potential since both are given equal charges)

1)Potential difference between the plates=0.

2)Charges on outer faces of plates=+Q.

3)Charges on inner faces of plates=0.

Answer.5

Charge on the capacitor is given by product of capacitance and potential difference across capacitor plates

Charge on the capacitor, $\boldsymbol{Q = C \times V}$

Where,

C is the capacitance of the capacitor

V is the potential on the capacitor

Therefore, without knowing the potential difference and only capacitance we cannot find out the maximum charge capacitor can contain.

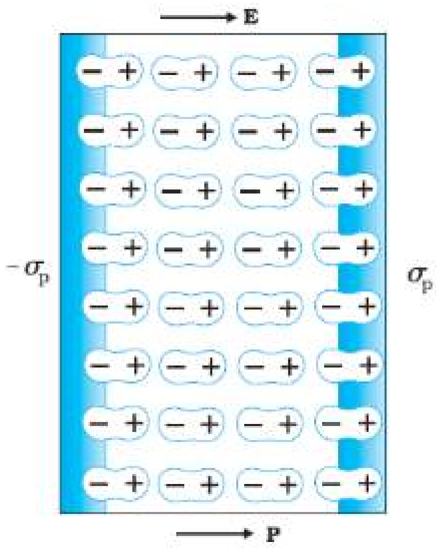
∴ The following information is insufficient.

Answer.6

When a polar or non polar material is placed in an external electric field , the electron charge distribution inside the material is slightly shifted opposite to the electric field and this induces a dipole moment in any volume of the material.

The polarization vector $\vec{P}$ is defined as this dipole moment per unit volume.

When a dielectric rectangular slab is placed in an external electric field the dipoles get aligned along the field and the right and left surfaces of slab gets positive and negative charges as shown in fig. known as induced charge.



The more the dipoles are aligned with the external field , the more the dipole moment and thus more is the polarization.

Because of these induced charges an extra electric field is produced inside the material opposite to the direction of external field and the net electric field is given by

$$E = \frac{E_0}{K}$$

Where ,

K is the constant for a given dielectric known as dielectric constant of the dielectric >1)

E_0 is the field in vacuum.

On increasing temperature, the random motion of molecules or dipoles increases due to thermal agitation and the dipoles get less aligned with the electric field and thus dipole moment decreases.

Since polarization is given by dipole moment per unit volume, it also decreases

And since ,dielectric constant is described by the polarization of the material

Thus, on increasing temperature, dielectric constant decreases.

Answer.7

When a dielectric slab is gradually inserted between the plates of an isolated parallel-plate capacitor, the energy of the system come out to be a linear function of xdisplacement of the slab inside capacitor measured from the center of the plate).

since x decreases, the energy of the system decreases.

We know that

$$F = -\frac{\partial U}{\partial x}$$

Where

$\frac{\partial U}{\partial x}$ is the rate of change of potential energy function with x

-ve sign indicates that force is in negative direction when energy increases with respect to x)

Therefore, Force on the slab exerted by the electric field is constant and positive.

Objective I

Answer.1

Given: a capacitor of capacitance C charged to a potential V

Gauss's law:

Electric flux ϕ) through a closed surface S is given by

$$\phi = \frac{Q}{\epsilon_0}$$

Where,

Q is the charge enclosed by S

ε_0 is the permittivity of the free space

According to the gauss law

Electric flux, $\varphi = \frac{Q}{\varepsilon_o}$

Where

Q is the total charge enclosed in the gaussian surface

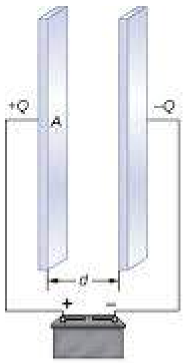
ε_0 is the absolute permittivity of the vacuum

Since, the two plates of capacitor contains equal and opposite charges

∴ Total charge enclosed by the surface ⇒ Q-Q=0

Putting the values of total charge in gauss law, we get

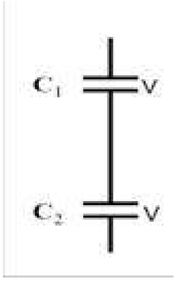
$$\varphi = \frac{Q}{\varepsilon_o} = \frac{0}{\varepsilon_o} = 0$$



∴ the electric flux through the closed surface enclosing the capacitor=0.

Answer.2

Two capacitance each having capacitance C and breakdown voltage V joined in series.



The general formula for effective capacitance of a series combination of n capacitors is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \ldots \frac{.1}{C_n}$$

The equivalent capacitance of two capacitors in series is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Where

C_1 is the capacitance of the first capacitor

C_2 is the capacitance of the second capacitor

On Solving for C, we get

$$C = \frac{C_1 \times C_2)}{C_1 + C_2}$$

Since, C_1=C_2=C

Putting the values of C_1 and C_2, we get

$$C = \frac{C \times C}{2C} = \frac{C^2}{2C} = \frac{C}{2}$$

Also, Capacitors in series have same amount of charge

∴ Q_1=Q_2=Q

Therefore, potential difference across both the capacitors are also equal to V

So, the voltage across the system is the sum of voltage across each capacitor.

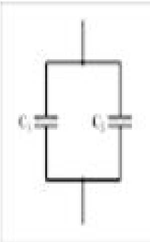
Therefore, **the breakdown voltage of the combination =V+V⇒ 2V**

Thus, the capacitance and breakdown voltage of the combination is C/2 and 2V respectively

Answer.3

Given:

Two capacitors of capacitance C each and breakdown voltage V connected in parallel



Explanation:

The general formula for effective capacitance of a series combination of n capacitors is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \ldots \frac{.1}{C_n}$$

The equivalent capacitance of two capacitors connected in parallel are given by

$$C_{eq} = C_1 + C_2$$

Where

C_1 is the capacitance of the capacitor C_1

C_2 is the capacitance of the capacitor C_2

Here C_1=C_2=C

Putting the values in the above formula, we get

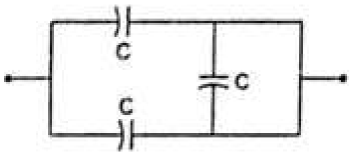
∴ C_{eq}=C+C=2C

Since, potential difference across capacitors in parallel are equal

Therefore voltage across the system is equal to the voltage across a single capacitor.

Therefore, breakdown voltage of the combination =V

Answer.4



Since the both ends of the capacitor on the right is connected at same point.

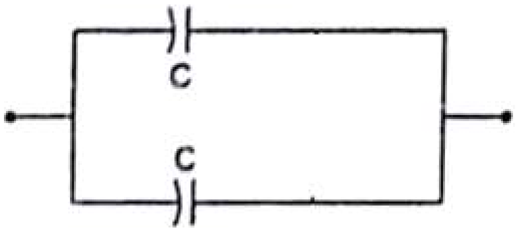
Both the plates of the capacitor are at same potential and potential difference across capacitor becomes 0

∴ V=0

And the capacitor C on the right now becomes useless and

Thus, capacitor is replaced by a short circuit.

The circuit now becomes



Which involve two equal capacitors of capacitance C connected in parallel.

The general formula for effective capacitance Ceq for parallel combination of n capacitors is given by

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \ldots C_n$$

Thus, the equivalent capacitance of the two capacitor in parallel combination is

$C_{eq}=C_1+C_2$

Since $C_1=C_2=C$

Putting the value in the above formula, we get

$C_{eq}=C+C=2C$

Therefore, equivalent capacitance of the combination is C+C=2C.

Thus, the equivalent capacitance of the combination is 2C

Answer.5

We know that,

Force between the plates of the capacitor is given by

$$F = \frac{Q^2}{2A\epsilon_0}$$

Where,

Q=charge on the capacitor

A=area of plates

Derivation:

Suppose charge Q and -Q are provided on plates of capacitor of area A.

Since, F=QE

Where

Q is the test charge on the point charge

E is the electric field intensity.

Field due to charge Q on one plate is

$$E = \frac{Q}{2A\epsilon_0}$$

Force on the plate with charge -Q will be

$$F = \frac{Q(-Q)}{2A\epsilon_0} = -\frac{Q^2}{2A\varepsilon_o}$$

Considering magnitude, each plate applies a force of $\frac{Q^2}{2A\epsilon_0}$

Each.

On increasing a dielectric slab between the plates of the capacitor, the charge on the plates remains constant as the plates are isolated) .

Since, area of plates does not change, force between the plates remain constant.

Answer.6

We know that,

The energy stored per unit volumeenergy density) in an electric field E is given by

$$U = \frac{1}{2}\epsilon_0 E^2$$

Where,

E=magnitude of electric field intensity

ϵ_0=absolute permittivity of vacuum

As we know that,

And the electric field due to a point charge Q at a distance r is given by

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Therefore, energy density $U \propto E^2$by formula)

And E$\propto \frac{1}{r^2}$

∴

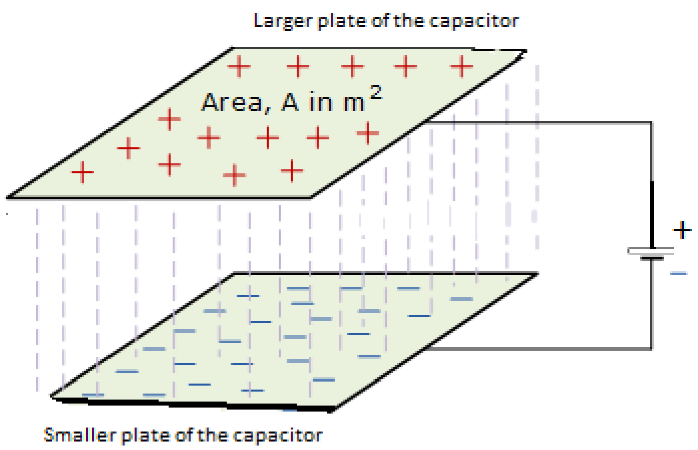
$\Rightarrow U \propto \frac{1}{r^4}$

Thus , the energy density in the electric field created by a point charge falls of with distance from a point charge as $\frac{1}{r^4}$

Answer.7

Given:

A parallel plate capacitor with plates of unequal area and charges on larger and smaller plates are Q_+ and Q_- respectively

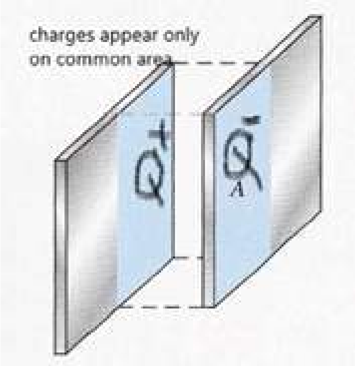


Explanation:

When you have two plates of unequal areas facing each other , the electric field is present only in their common area ignoring fringe effects.)

Therefore, charges acquire only on the facing common areas of the plates of the capacitor. So, if the plates have unequal area it doesn't matter as only the common facing area of both the plates acquire charges

Therefore, we are left with a capacitor with plates area A where A is the common area



Answer.8

<u>Given:</u> a parallel plate capacitor with a thin metal plate P inserted in between such that it touches the two plates.

Explanation:

When two plates of a capacitor are connected by a conductor) redistribution of charge takes place and both plates acquire same potential.

Thin metal plate P is a conductor and when connecting it to both plates of capacitor, charges gets neutralized and both the plates acquire same potential.

∴ potential difference = 0

We know that,

$$C = \frac{Q}{V}$$

Where,

V is the potential difference across capacitor

Q=charge on the capacitor

$\Rightarrow C = \frac{Q}{0} = \infty$

∴ Capacitance of the capacitor becomes infinite and it can hold any amount of charge.

Thus, a thin metal plate p is inserted between the plates of a parallel plate capacitor of capacitance C in such a way that its edge touch the two plates. The capacitance now becomes ∞.

Answer.9

We know that,

Voltage dropor potential difference) across capacitor is given by

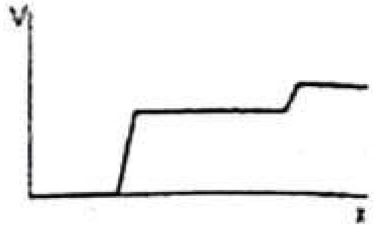
$$V = \frac{Q}{C}$$

Where,

Q=charge on the capacitor

C=capacitance of the capacitor

By looking at the graph,



We can see that first increment in voltage is greater than the second increment. Therefore,

we can conclude that voltage drop across capacitor C_1 is greater than the voltage drop across capacitor C_2

on moving left to right C_1 comes first)

Since charges on the capacitors in series are same ,

$\therefore Q_1=Q_2$

The potential drop across the capacitor C_1 is more than Capacitor C_2.

$V_1>V_2$

Putting the values of V, we get

$$\frac{Q}{C_1} > \frac{Q}{C_2}$$

On solving, we get

$$\frac{1}{C_1} > \frac{1}{C_2}$$

Or

$C_2>C_1$

Thus, the capacitance of the capacitor C_1 is less than C_2

Answer.10

Dielectric constant of a substance is the factor>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of the capacitor

$$\therefore K = \frac{C}{C_0}$$

Where,

C=capacitance in presence of dielectric

C_0=capacitance in presence of vacuumK=1)

The electric field between the plates of a capacitor when the space between the plates is filled with a dielectric of dielectric constant K is given by

$$\boldsymbol{E = \frac{Q}{A\epsilon_0 K}}$$

Where,

Q=charge on the capacitor

A=area of metal plates

K=dielectric constant

ϵ_0=permittivity of vacuum

When dipped in oil tank value of K>1

When oil is removed there is air between the plates with K~1

∴ the value of K decreases when oil is pumped out

By the formula,

$$E \propto \frac{1}{K}$$

So as K decrease from greater than 1 to 1 , the electric field increases.

Therefore, after pumping out oil, the electric field between the plates increases.

Answer.11

Given: two metal spheres of capacitances C_1 and C_2 carrying some charges.

Explanation:

Two metal spheres carrying different charges have different electric fields on their surfaces and have different potential. When they are put in contact , due to potential difference, charge transfer takes place between them such that they acquire same potential .

∴ When two conductors are placed in contact with each other they acquire same potential.

Potential difference V across capacitor is given by the formula

$$V = \frac{Q}{C}$$

Where

Q=charge on the capacitor

C=capacitance of the capacitor

Therefore, after putting them in contact and separating them, if the final charges are given by Q_1 and Q_2 then

$$V_1 = V_2$$

$$\Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

Answer.12

Given:

Three capacitors of capacitances 6μF each

i)The minimum capacitance can be obtained by connecting all three capacitors in series.

In series combination, charges on the two plates are same on each capacitor.

The general formula for effective capacitance of a series combination of n capacitors is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \ldots \frac{.1}{C_n}$$

The equivalent capacitance in this case is given by

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$\Rightarrow C_{eq} = \frac{C}{3}$$

$$\boldsymbol{C_{eq} = \frac{6}{3} = 2\mu F}$$

ii) The maximum capacitance can be obtained by connecting all three capacitors in parallel.

In this case, the same potential difference is applied across all capacitors.

The general formula for effective capacitance Ceq for parallel combination of n capacitors is given by

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \ldots C_n$$

The equivalent capacitance in this case is given by

$$C_{eq} = C + C + C$$

$\Rightarrow C_{eq}=3C$

$C_{eq}=3\times 6=18\mu F$

Therefore, the maximum and minimum capacitance that can be obtained is 18μF and 2μF respectively.

Objective II

Answer.1

We know that,

Capacitance of a capacitor only depends on shape, size and geometrical placing.

For example:

Capacitance of a parallel plate capacitor is given by

$$C = \frac{kA\epsilon_0}{d}$$

Where,

A=area of cross-section of plates

K=dielectric constant

ϵ_0=absolute permittivity of medium

d=distance between the plates

∴ It does not depend on charges on the plates

So, The capacitor does depends on the shape and size of the plates and separation between the plates.

Answer.2

Since, the capacitor is isolated, it has no connections to any battery. Therefore, it is not possible to exchange charge due to absence of any external voltage source.

On inserting a dielectric slab of dielectric constant K, capacitance will change to KC.

Since, Charge remains constant and capacitance changes, voltage will also change according to the formula $V = \frac{Q}{C}$

Energy stored in the capacitor is given by

$$E = \frac{Q^2}{2C}$$

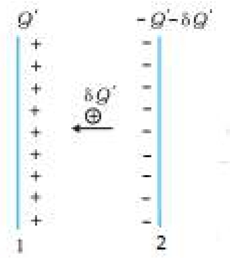
Where

Q is the charge on the capacitor

C is the capacitance of the capacitor

Derivation:

Initially consider two uncharged conductors 1 and 2 . We are transferring charge from conductor 2 to 1 such that at the end 1 gets charge Q and 2 gets charge -Q



Consider an intermediate stage where conductors 1 and 2 have charges Q' and -Q' respectively. At this stage potential difference V' between conductors is given by Q'/C where C is the capacitance of the system.

For transferring a small charge dQ' from 2 to 1 work done is given by

$$dW' = \frac{Q'}{C} dQ'$$

The total energy stored in the capacitor is summation of all these works done in transferring charge from 0 to Q.

$$W = \int_0^Q \frac{Q'}{C} dQ' = \frac{Q^2}{2C}$$

Therefore total energy stored in capacitor is given by

$$E = \frac{Q^2}{2C}$$

Where,

Q=charge on the capacitor

C=capacitance of the capacitor

Which also changes due to change in capacitance.

Therefore on inserting dielectric slab between the plates of an isolated charge capacitor the charge on the capacitor does not change.

Answer.3

When a dielectric slab of dielectric constant K is introduced between the plates of the capacitor, the net electric field in the dielectric becomes

$$E = \frac{E_0}{K}$$

Where,

E_0 is the electric field when there is vacuum between the plates.

K is the dielectric constant of the dielectric

The net electric field is due to charges +Q, -Q and due to induced charges +Q',-Q'in opposite direction).

So, the net electric field becomes

$$\frac{Q}{A\epsilon_0} - \frac{Q'}{A\varepsilon} = \frac{Q}{KA\epsilon_0}$$

$$\Rightarrow \boldsymbol{Q'} = \boldsymbol{Q}\left(\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{K}}\right)$$

Where,

Q'=induced charge due to dielectric

Q=charge on the capacitor in vacuum

K=dielectric constant

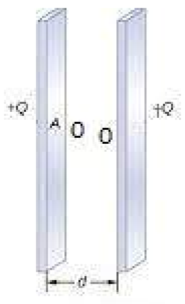
Since dielectric constant K>1

Q’<Q

Therefore, on inserting a dielectric slab between plates of capacitor the induced charge Q’ is less than Q.

Answer.4

When battery is not connected, the outer surfaces will have charge +q and inner faces of the plates will have zero charge each.



When a battery is connected to the plates of the capacitor the charges on the plate redistribute in such a way that the potential difference between the plates becomes equal to the emf of the battery.

So, the inner surfaces will have equal and opposite charges according to Q=CV

Where C is the capacitance and V is the emf of the battery. Thus we can say that the battery supplies equal and opposite charges CV) to two plates.

And the charges on the outer surfaces remain same as on connecting the battery only charges are transferred and total charge remains constant so to have zero field inside plate the outer face charges have to be same.

Therefore when a parallel plate capacitor with each plate having charge q is connected to a battery then the facing surfaces have equal and opposite charge and the outer surface will have equal charge.

Answer.5

When we increase the separation between the plates of a charged parallel capacitor the value of Capacitance decreases by the formula

$$C = \frac{A\epsilon_0}{d}$$

Where,

d=separation between the plates

A=area of plates

ϵ_0=absolute permittivity of vacuum

Charge on the capacitor remains unchanged because no charge transfer takes place.

∴ Potential difference across the capacitor changes by the formula

$$V = \frac{Q}{C}$$

Where,

Q= charge on the capacitor

C=capacitance

Similarly Energy across the capacitor given by

$$E = \frac{1}{2}CV^2$$

Where,

C=capacitance

V=potential difference across capacitor

So, as V changes energy stored also changes

Therefore, on increasing separation between the plates of capacitor, potential difference and energy of capacitor changes whereas charge and energy density remains the same.

Answer.6

Let $E_0=V_0/d$ be the electric field between the plates when there is no electric and the potential difference is V_0. If the dielectric of dielectric constant K is now inserted , the electric field in the dielectric will be $E = \frac{E_0}{K}$

The potential difference will then be

$$V = \frac{E_0}{K}t + E_0(d - t)$$

Where,

t=thickness of dielectric slab

d=separation between the plates of capacitor

Here, since metal plate is of negligible thickness , t=0

And $V=E_0d=V_0$

Thus the potential remains same c) is incorrect) and the **charge Q_0 on plates also remains same.**

Therefore

$$C = \frac{Q_0}{V} = C_0$$

∴ capacitance remains same.b) is incorrect)

Since charge on the capacitor remains same, no extra charge is supplied by the batterya) is incorrect)

We know that when dielectric is introduced between the plates of capacitor this polarized dielectric is equivalent to two charged surfaces with induced surface charges Q’ and -Q’

Here, the dielectric is the metal plate and therefore equal and opposite charges appear on the two faces of metal plate.

Answer.7

Option b) is correct because when a dielectric slab W is inserted in the capacitor in the presence of a battery the capacitance increases by a factor of Kdielectric constant)

$$C = \frac{k\varepsilon_o A}{D}$$

Where

K is the dielectric constant

ε_0 is the permittivity of the free space

A is the area of the plate,

d is the distance between the plates of the capacitor,

As the capacitance increases with the insertion of the dielectric, the charge appearing on the capacitor increases

Since,

Q=CV

Where,

Q is the charge on the capacitor

C is the capacitance of the capacitor

V is the potential difference supplied by the battery

Whereas capacitance does not change in case of inserting slab after removing the battery.

Optionc) is correct as

In process WXY after inserting a dielectric slab in the capacitor, the capacitance becomes

$C' = KC_0$

Where C_0 is the capacitance in a vacuum and K is the dielectric constant. Now if the capacitor is connected to the battery of emf ϵ, then potential difference across the capacitor is given by ϵ, and the stored electrical energy is given by

$$E = \frac{1}{2}CV^2 = \frac{1}{2}KC_0\epsilon^2$$

Whereas in process XYW the energy is given by

$$E = \frac{1}{2}C\epsilon^2$$

Therefore, The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.

Option→d) is correct because in both cases Electric field in the capacitor reduces to $E = \frac{E_0}{K}$

Where

E_0=electric field in c=vacuum

K=dielectric constant

note that it does not matter whether the battery is connected afterwards or before in 4th part)

The electric field in the capacitor after the action XW is the same as that after WX.

Exercises

Answer.1

Given:

$n = 1.0 \times 10^{12}$

V= 10 V

Formula used:

1. Charge can be given by the formula

$$Q = ne$$

Where Q→ Charge on the capacitor

n → number of the electrons

e → electric charge of an electron $= 1.6 \times 10^{-19}\ C$

2. The capacitance of a parallel plate Capacitor is given by

$$C = \frac{Q}{V}$$

Where Q → charge on the capacitor

V → Voltage or potential difference

Putting the values in the formula 1, we get

$$=> Q = (1 \times 10^{12}) \times (1.6 \times 10^{-19}) = 1.6 \times 10^{-7}\ C$$

Substitute Q and C in Formula 2), we get

$$\Rightarrow C = \frac{(1.6 \times 10^{-7})}{10}\ C = 1.6 \times 10^{-8} F$$

Thus, the capacitance of the parallel plate capacitor is $C = 1.6 \times 10^{-8}\ F$

So, if 1.0×10^{12} electrons are transferred between two conductors the capacitance of the parallel plate capacitor is 1.6×10^{-8} F when a potential difference is 10V

Answer.2

Parallel plate capacitor: When two conducting plates are connected in parallel and separated by some distance then parallel plate capacitor will be formed.

Given:

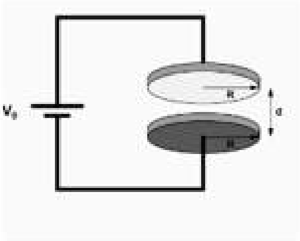


Fig.Circular disc parallel plate capacitor

Radius r= 5cm

Separation between the plates is $d = 1mm = 1 \times 10^{-3} m$

Formula used:

Capacitance is given by the formula

$$C = \frac{\varepsilon_0 A}{d}$$

Where,

C is the capacitance of the capacitor

ε_0 is the permittivity of free space $= 8.85 \times 10^{-12} F/m$

A is the area of the circle $= \Pi r^2 = 3.14 \times (5 \times 10^{-2})^2 m^2$

Because capacitor plates are made of circular discs)

d is the separation between the plates

Putting the values in the above relation, we get

$$C = \frac{8.85 \times 10^{-12} \times 3.15 \times (5 \times 10^{-2})^2}{1 \times 10^{-3}} = 6.95 \times 10^{-5}\ \mu F$$

Capacitance is of a circular disc parallel plate capacitor

$$C = 6.95 \times 10^{-5} \mu F$$

Answer.3

Given:

C = 1 F d=1 mm

Formula used :

Capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}$$

Where

C is the capacitance of the capacitor

D is the separation between the capacitor plates

A is the area of a circular plate capacitor

ε_0 is the permittivity of the free space=8.85×10^{-12} F/m

A = area of the circle = πr^2.Because capacitor plates are circular discs.

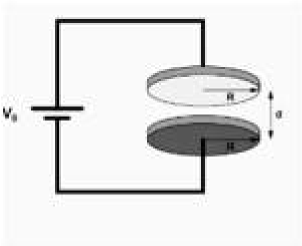


Fig. Circular disc parallel plate capacitor

$$C = \frac{\varepsilon_0 \pi r^2}{d}$$

$$r = \sqrt{\frac{C \times d}{\pi \varepsilon_0}}$$

$$= \sqrt{\frac{1 \times (1 \times 10^{-3})}{3.14 \times 8.85 \times 10^{-12}}}$$

$$= \sqrt{35.98 \times 10^6}\,\text{m}$$

$= \sqrt{36 \times 10^6} = 6 \times 10^3$ m=6000 m=6 km

For the construction of 1F capacitor with 1mm separation, we need to take the radius r=6 Km.

Answer.4

Given

Area, A=25 cm^2 =25×10^{-4} m^2

Voltage, V=6V

The separation between the plates is d= 1mm=1×10^{-3}

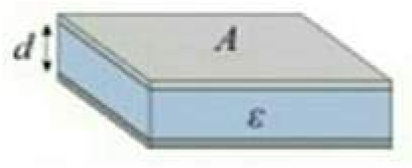


Fig. parallel plate capacitor

Formula used:

When a capacitor is connected to a capacitor, the charge can be calculated

$Q = C \times V$

Where

Q is the charge of the capacitor

C is the capacitance of the capacitor

V is the Voltage or potential difference across the plates of the capacitor.

Capacitance C can be calculated by the formula

$$C = \frac{\varepsilon_0 A}{d}$$

Where

C is the capacitance of the capacitor

D is the separation between the capacitor plates

A is the area of a circular plate capacitor

ε_0 is the permittivity of the free space, $\varepsilon_0 = 8.85 \times 10^{-12} F/m$

$$C = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1 \times 10^{-3}}$$

$C = 2.21 \times 10^{-11}\ F$

When the capacitor is connected to a 6V battery, Charge flow through the battery is the same as the charge that can be withstand with the capacitor.

Substitute the value of C in 1)

$Q = 2.21 \times 10^{-11} \times 6$

$= 1.33 \times 10^{-10} C$

Work is done by the battery

$W = QV$

Where

W is the work done by the battery

Q is the charge on the plates of the capacitor

V is the voltage across the plates of the capacitor

$W = 1.33 \times 10^{-10} \times 6$

$= 7.98 \times 10^{-10} J$

$= 8 \times 10^{-10} J$

Charge flows through the battery is $1.33 \times 10^{-10} C$ and work done by the battery is $=8\times10^{-10}$ J

Answer.5

a) Given Area $A = 25cm^2 = 25 \times 10^{-4}$

Voltage V=12V

Separation between the plates d=2mm

Formula used :

Charge of the capacitor can be calculated as

$Q = C \times V$

The capacitance of the parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

Where,

C is the capacitance of the parallel plate capacitor

D is the separation between the capacitor plates

A is the area of a circular plate capacitor

ε_0 is the permittivity of the free space, $\varepsilon_0=8.85\times10^{-12}$ F/m

$$C = \frac{8.85x10^{-12} \times 25 \times 10^{-4}}{2 \times 10^{-3}}$$

$$C = 11.06 \times 10^{-12}\ F$$

$$Q_1 = C \times V$$

$$= 11.06 \times 10^{-12} \times 12$$

$$= 1.33 \times 10^{-10} C$$

b) d is decreased to 1.0mm. We have to calculate the extra charge given by the battery to the positive plate.

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85x10^{-12} \times 25 \times 10^{-4}}{1 \times 10^{-3}}$$

$$= 22.12 \times 10^{-12} F$$

$$Q_2 = 22.12 \times 10^{-12} \times 12$$

$$= 2.652 \times 10^{-10} C$$

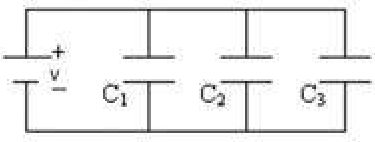
Extra charge is $=Q2 - Q1 = 2.652 - 1.33) \times 10^{-10} = 1.33 \times 10^{-10}\ C$

Charge on the capacitor when d = 2mm is = $1.33 \times 10^{-10} C$

When d is decreased to 1.00 mm the extra charge given by the battery is = $1.33 \times 10^{-10}\ C$

Answer.6

Redraw the circuit given



Given

Capacitance of C_1 capacitor =2μF

Capacitance of C_2 Capacitor =4μf

Capacitance of C_3 capacitor =6μF

Voltage V=12V

Formula used

All three capacitors are in parallel. When capacitors are in parallel, we will add them.

Equivalent Capacitance

$$C_{eqv} = C_1 + C_2 + C_3$$

Charge of a capacitor can be calculated by the for formula

$$Q = C_{eq} \times V$$

Where

Q is the charge of the capacitor

Ceq is the equivalent Capacitance of the capacitor

V is the voltage or potential difference across the plates of the capacitor

Charge on capacitor C_1 is

$Q_1 = C_1 \times V = 2 \times 12 = 24$ Coulombs

Charge on capacitor C_2 is

$$Q_2 = C_2 \times V = 4 \times 12 = 48\ C$$

Charge on capacitor C_3 is

$$Q_3 = C_3 \times V = 6 \times 12 = 72\ C$$

Charge on capacitors 2μF, 4μF and 6μF are 24C, 48C, 72C respectively.

Answer.7

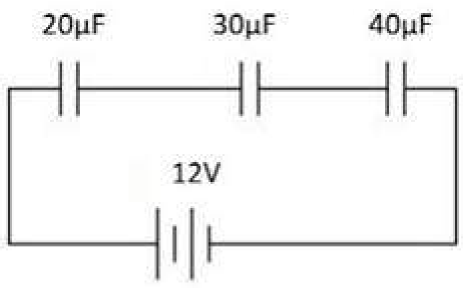
Given

Capacitance of the C_1 capacitor = 20µF

Capacitance of the C2 capacitor = 30µF

Capacitance of the C3 capacitor = 40µF

All the Capacitors are connected in series.



Formula used :

Equalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1C_2C_3}$$

$$C_{eq} = \frac{C1C2C3}{C1C2 + C2C3 + C3C1}$$

$$C_{eq} = \frac{20 \times 30 \times 40}{(20 \times 30) + (30 \times 40) + (40 \times 20)}$$

$$C_{eq} = 9.23\mu F$$

Capacitors are connected in series, so the charge on each of them is the same .

Formula used:

Charge can be calculated as

$$Q = C_{eq} \times V$$

Where

Q is the charge on the capacitor

C_{eq} is the equivalent Capacitance

V is the voltage

$Q = 9.23 \times 12$

$Q = 110.76\mu C$

Work done by the battery is

$W = Q \times V$

Where

W is the work done by the battery

Q is the charge on the capacitor

V is the voltage across the end of the capacitor

$W = 110.76 \times 12 \times 10^{-6}$

$= 1.33 \times 10^{-3} J$

Charge on capacitors 20μF, 30μF and 40μF are 110.76μC.

Work is done by the battery W $= 1.33 \times 10^{-3}$J

Answer.8

In the given fig. Capacitors B and C are in parallel.

The formula for parallel combination of capacitor is

$C_{eq}=C_1+C_2= C_A + C_B= 4 + 4 = 8$ μF

The formula for series combination of capacitors is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

From the figure, the 8 μF is connected in series with C_{eqv}.

Thus, we get $\frac{1}{C_{eq'}} = \frac{1}{C_A} + \frac{1}{C_{eqv}}$

Putting the values in the above equation, we get

$$\frac{1}{C_{eqv}} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$C_{eq} = 4\ \mu C$$

Capacitors B and C are in parallel. C_C

System of B,C and A has the same capacitor values. So they exhibit the same potential difference between them.

Hence Voltage across A is =6V

The voltage across B and C is = 6V

The charge can be calculated as

$$Q = CV$$

Where

Q is the charge of the capacitor

C is the Capacitance of the capacitor

V is the voltage across the circuit

Charge flows through A is Q_A = 8×6= 48μC

Charge flows through B is Q_B= 4×6 =24μC

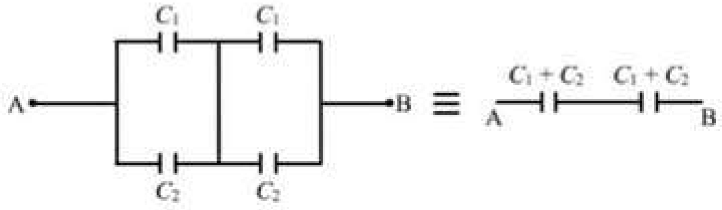
Charge flows through C is Q_C= 4×6 = 24μC

Charge appearing on the capacitors A, B and C is 48μC, 24μC and 24μC respectively.

Answer.9

Let's name the points indicated in fig as A and B.

In a) C_1 and C_2 are in parallel. If we redraw the diagram that will look like



$$\frac{1}{C_{eq}} = \frac{1}{C_1 + C_2} + \frac{1}{C_1 + C_2}$$

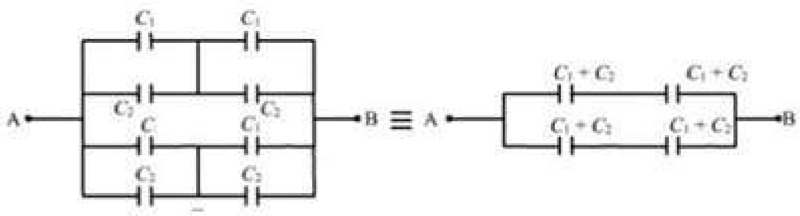
$$\frac{1}{C_{eq}} = \frac{2}{C_1 + C_2}$$

$$C_{eq} = \frac{C_1 + C_2}{2}$$

$$C_{eq} = \frac{4 + 6}{2} = \frac{10}{2}$$

$$= 5\mu F$$

In b) also C_1 and C_2 are in parallel. Redraw the diagram.



$$\frac{1}{C_a} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_1 + C_2}\right)$$

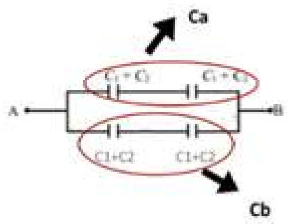
$$= \left(\frac{1}{4 + 6} + \frac{1}{4 + 6}\right) = \frac{2}{10} = \frac{1}{5}$$

$$\frac{1}{C_a} = \frac{1}{5}$$

$$C_a = 5\mu F$$

Also $C_b = 5\mu F$

$C_{eq}=C_a+C_b=5+5=10\mu F$



Equalent capacitance in figa) is 5μF.

Equalent capacitance in figb) is 10μF.

Answer.10

Given V= 10V

C_1=5μF

C_2=6μF

Formula used :

Charge supplied by the battery is

$Q = CV$

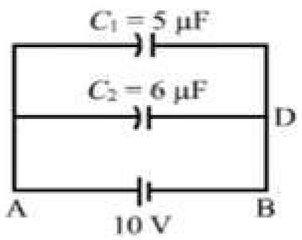
Where

C is the capacitance of the capacitor

V is the voltage across the potential difference

Q is the Charge on the capacitor

First, we have to calculate the capacitance C .To do this short circuit the voltage source and open circuit the current source. We don't have any current sources over here. So short circuit the Voltage source. Then two capacitors will come to parallel. In parallel connection of the capacitor we add the capacitor values.



Equalent Capacitance is

$C_{eq} = C_1 + C_2$

$= 5 + 6$

$Ceq = 11\mu F$

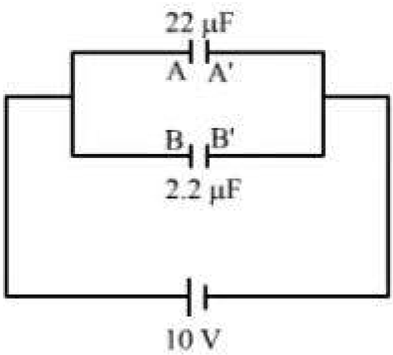
Charge Q can be calculated as

$Q = 11 \times 10 = 110\mu C$

Charge supplied by the battery is 110μC.

Answer.11

Lets take inner cylinders as A and B. and outer cylinders as A^1 and B^1. Inner cylinders A and B are connected through a wire. Outer cylinders kept in contact. If we draw the diagram, it will be look like as fig.



Given

capacitance C= 2.2μF

Voltage=10V

Capacitors are kept in parallel. So the potential difference across them is the same.

Formula used:

The magnitude of the charge on each capacitor is

$$Q = CV$$

Where

Q is the charge of the capacitor

C is the Capacitance of the capacitor

V is the voltage across the capacitor

$$Q = 2.2 \times 10 = 22\mu C$$

Inner cylinders of the capacitor are connected to the positive terminal of the battery. So the charge on each of them is +22μC

Net charge on the inner cylinders is = 22μC+22μC= +44μC

Note: If it is asked for a charge on outer cylinders of the capacitor. A net charge will be equal to -44μC because they are connected to the negative terminal of the battery).

Answer.12

The radius of conducting sphere 1 =R_1

Radius conducting sphere 2 =R_2

First, we need to calculate the capacitance of isolated charged sphere.

Formula used :

The capacitance of a sphere is given by the formula

$$C = \frac{4\pi\varepsilon_o}{\frac{1}{a} - \frac{1}{b}}$$

Where

C is the capacitance of the capacitor

ε_0 is the permittivity of the free space is $\varepsilon_0 = 8.85 \times \frac{10^{-12}F}{m}$

Taking limits as aR and b∞, Capacitance of charged sphere is found by imagining the concentric sphere with an infinite radius having some -Q charge).

$$C = \frac{4\pi\varepsilon_o}{\frac{1}{R} - 0} = 4\pi\varepsilon_0 R$$

The capacitance of isolated charge sphere is

$$C = 4\pi\varepsilon_0 R$$

The capacitance of isolated charge sphere 1 is $C_1 = 4\pi\varepsilon_o R_1$

The capacitance of isolated charge sphere 2 is $C_2 = 4\pi\varepsilon_o R_2$

If the two spheres are connected by a metal wire, then the charge will flow one sphere to another up to their potential becomes the same.

The potential will be the same only when they are connected in parallel. So two spheres are connected by a metal wire in parallel.

$$C_{eq} = C_1 + C_2$$

$$= 4\pi\varepsilon_0 R_1 + 4\pi\varepsilon_0 R_2$$

$$= 4\pi\varepsilon_0 R_1 + R_2)$$

The capacitance of individual spheres of radius R_1 and R_2 is $C_1 = 4\pi\varepsilon_0 R_1$ and $C_2 = 4\pi\varepsilon_0 R_2$ respectively.

Combinational capacitance when charged spheres are connected by a wire is $4\pi\varepsilon_0 R_1 + R_2)$.

Answer.13

Given

Capacitance C= 2μF

The equalent capacitance of the first row is calculated as

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$C = \frac{2}{3}\mu F$$

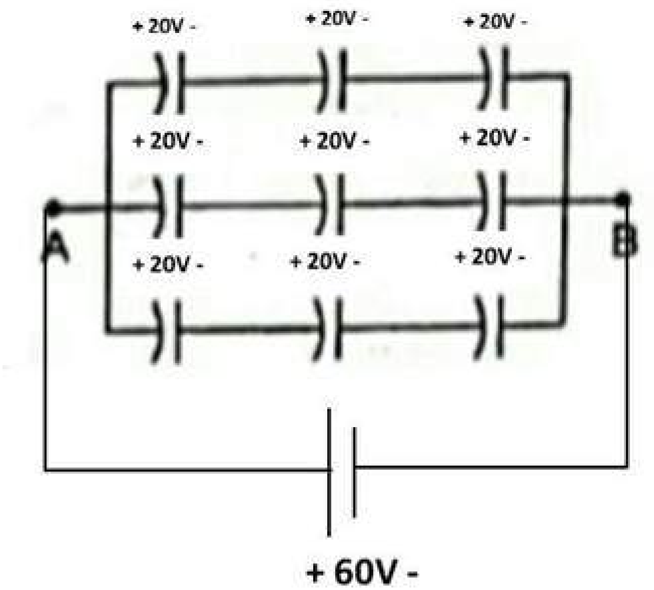
$\frac{2}{3}$ μF is the capacitance of only one row)

$$C_{eq} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2\mu F$$

The capacitance of each row is the same, and it is equal to 2μF.

All the three rows are arranged in parallel. So, Voltage or potential difference across each row is the same and is equal to 60V.

So, Voltage across each capacitor is =20V.



The equalent capacitance between A and B is 2μF.

Potential difference across each capacitor is 20V.

Answer.14

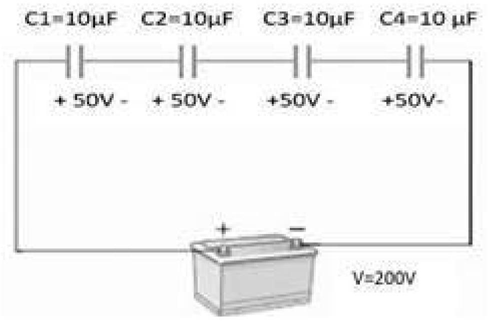
Requirement : We have to construct a 10μF capacitor, and it has to connect across a 200V battery.

Capacitors of 10μF are available, but the voltage rating is 50V only. By using these capacitors with this voltage rating, we have to meet our requirement.

Let's assume some X capacitors are placed in series. 200V battery connected across the.

So $X \times 50 = 200$

$$X = \frac{200}{50} = 4\ Capacitors$$



We have to construct 4 capacitors in a series so that we get the potential difference of 200V

If we calculate the capacitance of the parallel combination of four 10μF capacitors

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

Putting the value of the capacitor in the above formula, we get

$$\frac{1}{C} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$$

Thus, the capacitance of the combination is C=2.5μF

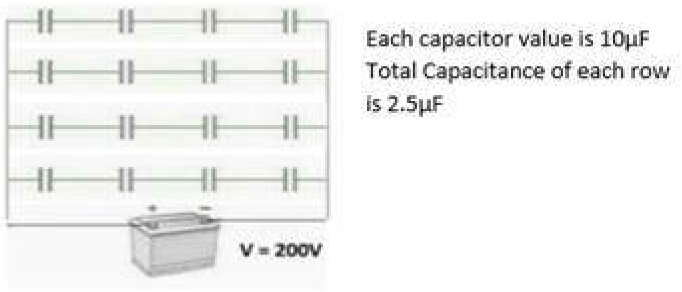
With this arrangement, we get the required potential difference value, but we are not getting the capacitor value 10μF instead of this we get only 2.5μF. So we have to add some columns. Let us take Y as columns,

$$\Rightarrow 2.5 \times Y = 10\mu F$$

$$Y = \frac{10}{2.5} = 4 \text{ columns}$$

So we have to add 4 columns as the same row.

That circuit will look like



$$Ceq = 2.5 + 2.5 + 2.5 + 2.5 = \ 10\mu F$$

Finally, the above fig will be the design for our requirements; each capacitor value is $10\mu F$ with voltage rating 50V.

Answer.15

From the fig. the capacitors 4μF and 8μF are in series.

$$C_1 = \frac{4 \times 8}{4 + 8} = \frac{8}{3}\mu F$$

Capacitors 3μF and 6μF are in series

$$C_2 = \frac{3 \times 6}{3 + 6} = 2\mu F$$

C_1 and C_2 are in parallel combination

Two rows are in parallel. So the voltage across each row is the same, and that is equal to 50V.

The charge on the branch ACB is

Charge on the branch ADB is

$$Q_{ADB} = C_2 \times V = 2 \times 50 = 100\mu C$$

Voltage, $V = \frac{Q}{C}$

Where

Q is the charge on the capacitor

C is the capacitance of the capacitor

The voltage at 4µF is $=\frac{400}{3\times4}=\frac{100}{3}V$

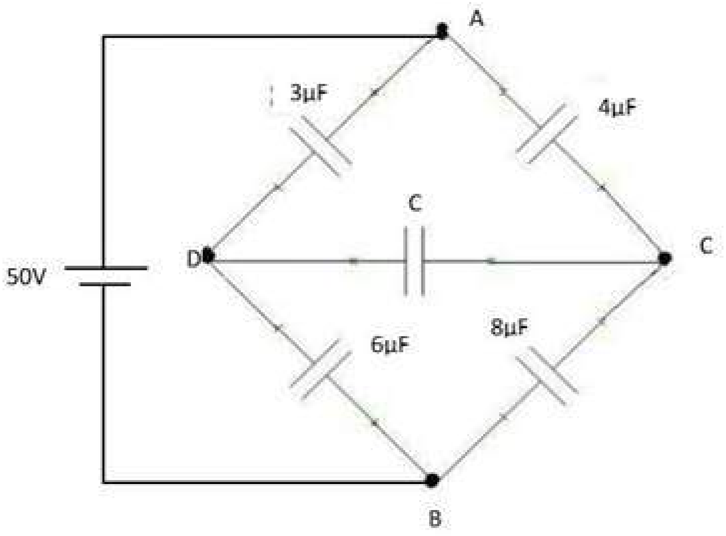
The voltage at 8µF is $=\frac{400}{3\times8}=\frac{50}{3}V$

Voltage at node C is $=\frac{100}{3}-\frac{50}{3}=\frac{50}{3}$V

The voltage at 3µF is $=\frac{100}{3}V$

The voltage at 6µF is $=\frac{100}{6}=\frac{50}{3}V$

The voltage at node D $=\frac{100}{3}-\frac{50}{3}=\frac{50}{3}V$



b) if a capacitor is connected between node C and D. if we redraw the circuit, it will look like. Here bridge is balanced at the condition

$$\frac{C_{AD}}{C_{BD}}=\frac{C_{AC}}{C_{BC}}$$

$$\frac{3}{6}=\frac{4}{8}$$

$$\frac{1}{2}=\frac{1}{2}$$

the charge on the capacitor will be zero.

Because the bridge is balanced so the potential difference between C and D will be zero. So no charge flow will occur.

The voltage at node C and node D is same and is equal to $\frac{50}{3}$V.

If a capacitor is connected between node C and D, the charge flow will be zero.

Answer.16

C_1 and C_2 are in series Equivalent capacitance, $\frac{1}{C_a} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C_a = \frac{C_1 \times C_2}{C_1 + C_2}$$

$$Cb = \frac{C_1 \times C_2}{C_1 + C_2}$$

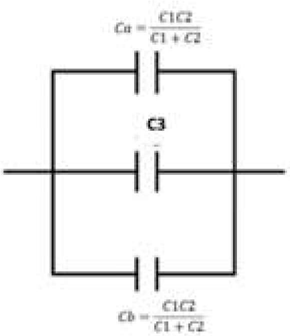
The capacitance C_a, C_b and C_3 are connected in parallel combination across each other

$$Ceq = C_a + C_3 + C_b$$

Putting the values in the above equation, we get

$$= \frac{C_1 \times C_2}{C_1 + C_2} + C_3 + \frac{C_1 \times C_2}{C_1 + C_2}$$

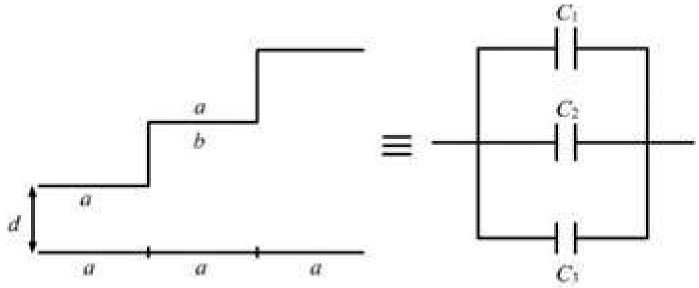
$$C_{eq} = 2\frac{C_1 \times C_2}{C_1 + C_2} + C_3$$



Equalent capacitance between a and b is $C_{eq} = 2\frac{C_1 \times C_2}{C_1 + C_2} + C_3$

Answer.17

It looks like this capacitor is made up of 3 capacitors with different d separation between the plates) and arranged in parallel. Redraw the fig. it looks like



Area of the flat plate is = A

Width of the second plate is the same for all the three capacitors is =a

For C_1 area is A_1 = A/3

C_2 area is A_2 = A/3

C_3 area is A_3 = A/3

Height of the second plate of three capacitors is same and is =a

d_1= d

d_2= d+b)

d_3=d +b +b)=d+2b)

Formula used:

Capacitance can be calculated by the

$$C = \frac{\varepsilon_0 A}{d}$$

$$C_1 = \frac{\varepsilon_0 A_1}{d1} = \frac{\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 A_2}{d2} = \frac{\varepsilon_0 A}{3d + b)}$$

$$C_3 = \frac{\varepsilon_0 A_3}{d_3} = \frac{\varepsilon_0 A}{3d + 2b)}$$

Capacitors are in parallel.

$$C_{eq} = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3(d+b)} + \frac{\varepsilon_0 A}{3(d+2b)}$$

$$C_{eq} = \frac{3d^2 + 6bd + 2b^2}{d(d+b)(d+2b)}$$

The capacitance of the assembly of the capacitors is

$$C_{eq} = \frac{3d^2 + 6bd + 2b^2}{d(d+b)(d+2b)}$$

Answer.18

Given

Length, l = 10cm

Radius, R_1 = 2mm

Radius, R_2 = 4mm

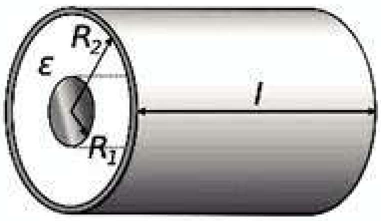


Fig. Cylindrical capacitor

Formula used :

a) The capacitance of the cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_0 l}{ln\left(\frac{R_2}{R_1}\right)}$$

Where

l→ length of the cylinder

R_2→ radius of outer cylinder

R_1→ radius of inner cylinder permittivity of the free space

ε_0→ permittivity of the free space = $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

Putting the value in the above formula, we get

$$C = \frac{2 \times 3.14 \times 8.85 \times 10}{ln\left(\frac{4}{2}\right)} = 8\ pF$$

b) Another cylindrical capacitor of same but different radius R_1=4mm and R_2= 8mm

If we compare the radii in a) with b), they give the same ratio

$$\frac{R_2}{R_1} => \frac{4}{2} = \frac{8}{4}$$

$$2 = 2$$

So capacitance is also same as a) is

$$C = \frac{2\pi\varepsilon_0 l}{ln\left(\frac{R_2}{R_1}\right)} = \frac{2 \times 3.14 \times 8.85 \times 10^{-12} \times 10}{ln\left(\frac{8}{4}\right)} = 8pF$$

Capacitance of cylindrical capacitor for both a) and b) is same and is =8pF

Answer.19

Given

for charged capacitor C_1 =100μF

V=24V

Formula used :

Charge is given by the formula

Q=CV

Where

Q is the charge of the capacitor

C is the capacitance of the capacitor

V is the voltage across the capacitor

Putting the values in the above formula, we get

$$Q = 100 \times 10^{-6} \times 24 \ = 2.4mC$$

This capacitor is connected to an uncharged capacitor of C_2=20μF

When a charged capacitor is connected to an uncharged capacitor, then the total charge will be equal to

$$Q_1 + Q_2 = \ 2.4 + 0 = 2.4mC$$

The potential difference across both capacitors will be the same.

$$V = \frac{Q}{C}$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{Q_1}{100} = \frac{Q_2}{20}$$

$$Q_1 = 5Q_2$$

Substitute in eq1)

$$Q_1 + \frac{Q_1}{5} = 2.4mC$$

$$6 \times \ Q_1 = \ 5 \times 2.4 = 12mC$$

$$Q_1 = \frac{12}{6} = 2mC$$

New potential difference is = $\frac{Q_1}{C_1} = \frac{2mC}{100\mu F} = 20V$

If 100 μF capacitor which is charged to 24V is connected to an uncharged capacitor of 20 μF then potential difference across it is 20V.

Answer.20

Given :

Capacitance C= 5.0 μF

Voltage V= 50V

Formula used :

1) If switch S is closed, it will be a short circuit. Current flow always chooses a low resistance path. No current will flow through capacitor at switch S., So we don't need to consider it. Finally, we will left with two capacitor which are in parallel. We add the capacitance when the capacitors are in parallel.

$C_{eqv}=C_1+C_2$

Where

C_1, C_2 are the capacitance of the capacitors

2) Charge supplied by the battery

$Q = C \times V$

Where

C is the capacitance of the capacitor

V is the voltage across the capacitance

Q is the charge on the capacitor

Putting the values in the formula 1, we get

C_{eqv}= 5+5=10μF

Putting the values in the formula 1, we get

$Q \ = \ 10 \times 50 = 500\mu C$

Charge supplied by the battery Q=500μC.

But when the switch has not connected the charge $Q=C_{eq}\times V$

The capacitors are connected in series connection, we get

$$C_{eq} = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = \frac{10}{3}\mu F$$

$$\Rightarrow\ Q^1 = \frac{10}{3} \times 50 = \frac{500}{3}\mu C$$

After switch S is closed the initial charge Q^1 stored in the capacitor will discharge.

$$\Rightarrow\ Q^1 = -\frac{500}{3}\mu C$$

Note: Q^1 will be negative because the capacitor is discharging.)

So the net charge flows from A to B is $-ve$

Total $3.3 \times 10^{-4}C$ Charge will flow through A and B when switch S is closed.

Answer.21

43 mV

Given,

Mass of the particle, m=10 mg

The charge of the particle, q= -0.01 μC = -0.01 $\times 10^{-6}$ C

The capacitance of each pair of the parallel capacitor plates, C=0.04 μF = 0.04$\times 10^{-6}$ F

Area of each capacitor plates, A= 100 cm^2=100$\times 10^{-4}$ m^2

V is the potential difference required for the particle to be in equilibrium=?

Formula used

For the particle of mass 'm' to stay in equilibrium in the given set up, the weight of the particle W) should be opposed by the electric force F), acting on the same charged particle. The electric force is exerted by the electric field in between the capacitor plates. As the weight is acting downward, the electrical force should act upward for the equilibrium.

So,

$W = F$

Or

$mg = qE\ eqn.1)$

Where,

g= Acceleration due to gravity= $9.81 m/s^2$

q= charge of the particle= -0.01 μC= -0.01×10^{-6} C;

m=10 mg=10×10^{-4}kg;

E= Magnitude of Electric field in between the capacitor plates;

But from Gauss's law, we have,

$$E = \frac{Q}{\varepsilon_0 A}\ eqn.2)$$

Where,

Q= Charge on the capacitor plates same on both capacitors for series arrangement)

ε_0= Permittivity of free space= $8.85\times 10^{-12} Fm^{-1}$

A= Area of the plate in the parallel plate capacitor=$100\times10^{-4}\ m^2$

We know that, for capacitors connected in series across the voltage V, the effective capacitance, C_{eff} will be

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Or,

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2}$$

Here C_1=C_2= C = 0.04 μF

Hence,

$$C_{eff} = \frac{0.04\ \mu F \times 0.04\ \mu F}{0.04\ \mu F + 0.04\ \mu F}$$

Or,

$$C_{eff} = 0.02 \mu F = 0.02 \times 10^{-6} F$$

With this, we can calculate the value of charge stored Q) in the given capacitor arrangement as,

$$Q = C_{eff} \times V \; eqn.3)$$

Where, V is the potential difference required to produce enough electric field to oppose the weight of the particle.

Putting eqn.3 in eqn.2, we get,

$$E = \frac{C_{eff} \times V}{\varepsilon_0 A} \; eqn.4)$$

Now, substituting eeqn.4 in eqn.1, we get,

$$mg = q\frac{C_{eff} \times V}{\varepsilon_0 A}$$

Or,

$$V = \frac{m \times g \times \varepsilon_0 \times A}{q \times C_{eff}}$$

Substituting the known values, we get

$$V = \frac{10 \times 10^{-4} kg \times 9.81 \mathrm{m}/s^2 \times 8.85 \times 10^{-12} \ \mathrm{F}m^{-1} \times 100 \times 10^{-4} \ m^2}{0.01 \times 10^{-6} C \times 0.02 \times 10^{-6} F}$$

Or,

V=0.434V=43.4 mV

Hence, to keep the particle of mass 10mg, the potential difference in the set up should be 43 mV.

Answer.22

Given,

d_1,d_2 are the separations between capacitor plates in the upper and lower capacitors respectively.

a is the length of each plate

Area of each plates$=$ a^2

S_y is the distance that the electron must travel in order to avoid collision in Y-direction$=$ $d_1/2$

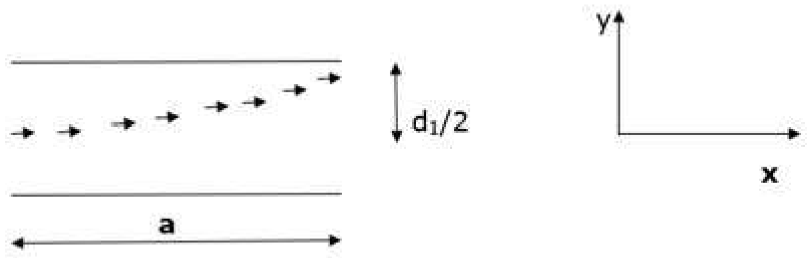
S_x is the distance that the electron must travel in order to avoid collision in X-direction$=$ a

V is the potential difference between the given series arrangement of capacitors.

e is the charge of electron released in between the plates

Formula used

In order to avoid a collision with plates, the electron should have an initial velocity, v. Hence, with 'v' velocity, the electron should travel a distance of '$d_1/2$' in Y-direction and 'a' in X-direction



Since the electric field is acting only in Y-direction, the electron will travel with constant velocity, v, in X-direction.

Hene the external force, neglecting gravitational and other forces, acting on the electron is the force due to the electric fieldqE). Hence, according to Newton's second law of motion, we can write,

$$ma_y = qE$$

$$a_y = \frac{eE}{m}\ eqn.1)$$

Where,

m$=$ mass of electron;

$a_y$$=$ acceleration of electron in Y-direction;

q=e=charge of electron;

E= Magnitude of Electric field acting between the plates of capacitor.

We know that the distance that must be traveled in X-direction=a

So, by the equations of motion, this can be represented as,

$$a = v \times t$$

Or,

$$t = \frac{a}{v} \; eqn.2)$$

Where,

t= time taken to travel 'a' distance

Acceleration in X-direction is Zero)

And the distance that must be traveled in Y-direction=$d_1/2$

Hence, by the equation of motion, assuming no initial velocity in Y-direction as the electron is projected horizontally.

$$d_1/_2 = \frac{1}{2} \times a_y \times t^2 \; eqn.3)$$

From eqn.1 and eqn.2, eqn. 3 can be modified as,

$$d_1/_2 = \frac{1}{2} \times \frac{eE}{m} \times \left(\frac{a}{v}\right)^2 \; eqn.4)$$

Now, let C_1 and C_2 be the capacitance of the upper and lower capacitors. Hence the effective capacitance, C_{eff} of the series arrangement is,

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Or,

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2}$$

Where,

$C_1 = \frac{\varepsilon_0 \times a^2}{d_1}$ and $C_2 = \frac{\varepsilon_0 \times a^2}{d_2}$

ε_0= Permittivity of free space, in between the capacitor plates.

Hence,

$$C_{eff} = \frac{\frac{\varepsilon_0 \times a^2}{d_1} \times \frac{\varepsilon_0 \times a^2}{d_2}}{\frac{\varepsilon_0 \times a^2}{d_1} + \frac{\varepsilon_0 \times a^2}{d_2}}$$

Or,

$$C_{eff} = \frac{\varepsilon_0 \times a^2}{(d_1 + d_2)}$$

Now, the magnitude of electric field, E, in the upper capacitor is given by,

$$E = \frac{V_1}{d_1} \ eqn.5)$$

Where, $V_1 =$ Potential difference in the upper capacitor and is equal to,

$$V_1 = \frac{Q}{C_1} \ eqn.6)$$

Where,

Q= charge in each capacitor $=$ total charge in the arrangement, since it is a series arrangement

Hence, Q can be calculated as,

$$Q = C_{eff} \times V$$

Where V $=$ total potential difference

Or,

$$Q = \frac{\varepsilon_0 \times a^2}{(d_1 + d_2)} \times V$$

Substituting the above equation and the value of C1 in eqn.6, we get,

$$V_1 = \frac{\frac{\varepsilon_0 \times a^2}{(d_1 + d_2)} \times V}{\frac{\varepsilon_0 \times a^2}{d_1}}$$

Or,

$$V_1 = \frac{d_1}{d_1 + d_2} \times V$$

Substituting the above expression in eqn.5, we get,

$$E = \frac{\frac{d_1}{d_1 + d_2} \times V}{d_1}$$

Or,

$$E = \frac{V}{d_1 + d_2}$$

Substituting the above expression in eqn.4, we get

$$d_1/_2 = \frac{1}{2} \times \frac{e\frac{V}{d_1 + d_2}}{m} \times \left(\frac{a}{v}\right)^2$$

By re-arranging,

$$v = \left[\frac{Vea^2}{md_1(d_1 + d_2)}\right]^{1/2}$$

The above expression is the least value of horizontal initial velocity needed for the electron to cross the capacitor plates without collision.

Answer.23

Given,

The distance in between the capacitor plates= 2cm

Formula used

Let t be the time, in seconds, with which proton and electron reach negative and positive charged plates respectively.

Let m_p, m_e be the mass and q_p, q_e be the charge of proton and electron respectively.

a_p, a_e be the acceleration of proton and electron respectively, in direction of Electric field, E Let's say Y-direction)

Let x= vertical distance traveled by proton to reach the negatively charged plate, in cm. Hence x is the distance is where we should place the electron-proton pair initially.

Hence, the distance traveled by electron= 2-x) cm

The proton and electron are accelerated to the oppositely charged plates, and the expression for the respective acceleration can be written from Newton's second law of motion.

So,

$$Mass\ of\ particleM) \times accelerationA) = External\ force\ due\ to\ electric\ fieldF_e)$$

The external electric field acting on the proton= The external electric field acting on the electron= E

Hence, for proton of mass m_p , the expression for second law of motion can be written as,

$$m_p \times a_p = q_p \times E$$

Here the term 'qE' represents the external force acting on the charged particle with a charge q in an electric field of magnitude E.

Similarly the expression for electron is,

$$m_e \times a_e = q_e \times E$$

From the above equations, the accelerations can be written as,

$$Acceleration\ of\ proton = a_p = \frac{q_p \times E}{m_p}$$

And

$$Acceleration\ of\ electron = a_e = \frac{q_e \times E}{m_e}$$

Hence, the distance travelled by proton in a time t seconds, x, by equations of motion

$$x \times 10^{-2} m = \frac{1}{2} \times a_p \times t^2 \ \because the\ initial\ velocit = 0)$$

Or,

$$x \times 10^{-2} m = \frac{1}{2} \times \frac{q_p \times E}{m_p} \times t^2\ eqn. 1)$$

Similarly for electron, the distance traveled,

$$(2 - x) \times 10^{-2} m = \frac{1}{2} \times a_e \times t^2 \ \because the\ initial\ velocit = 0)$$

Or,

$$(2-x)\times 10^{-2}m = \frac{1}{2}\times\frac{q_e\times E}{m_e}\times t^2 \; eqn.2)$$

Now, to find x, the distance traveled by proton, we divide eqn.1 by eqn.2, that is,

$$\frac{x}{2-x} = \frac{\frac{1}{2}\times\frac{q_p\times E}{m_p}\times t^2}{\frac{1}{2}\times\frac{q_e\times E}{m_e}\times t^2}$$

Or,

$$\frac{x}{2-x} = \frac{q_p\times m_e}{q_e\times m_p}$$

But we know, charge of proton, $q_p =$ charge of electron, q_e

Hence the above expression will reduce to,

$$\frac{x}{2-x} = \frac{m_e}{m_p} \; eqn.3)$$

Now, mass of electron, m_e=9.1×10^{-31} kg

And mass of proton, m_p= 1.67×10^{-27} kg

Substitution the above values in eqn.3, we get,

$$\frac{x}{2-x} = \frac{9.1\times 10^{-31}\text{ kg}}{1.67\times 10^{-27}\text{ kg}}$$

Or,

$$\frac{x}{2-x} = 5.449\times 10^{-4}$$

By rearranging the above expression we get,

$$x = 5.449\times 10^{-4}\; 2-x)$$

Or,

$$x = 10.898\times 10^{-4} - \; 5.449\times 10^{-4}x$$

Or,

$$1.000549x = 10.898\times 10^{-4}$$

Or,

$$x = \frac{10.898\times 10^{-4}}{1.000549} = 1.08\times 10^{-3}cm\; Ans.)$$

Hence the pair should be released at a distance of 1.08×10^{-3} cm from the negative plate.

Answer.24

Given,

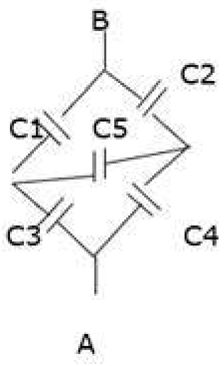
In the figure, part a), b), and c) are same.

Capacitances are 1μF,3μF,2μF,6μF and 5μF

Formula used

Each parts of the figure represents a bridge circuit. A bridge circuit is the one in which, two electrical paths are branched in parallel between the same potential difference, but are bridged by a third path, from intermediate points.

Inorder to check the balancing of the bridge circuits, the following conditions must be satisfied,



For a balanced bridge with capacitance arranged as shown in figure,

$$\frac{C_1}{C_3} = \frac{C_2}{C_4}$$

If this condition is satisfied the current through the C_5 capacitor will be zero. Hence, C_5 will be ineffective.

In the given figures, we have to check this condition before calculating the effective capacitance.

By comparing the above figure and the question figures, we can write,

C1=3 μF, C2=6 μF, C3=1 μF, C4=2 μF, C5=5 μF

And,

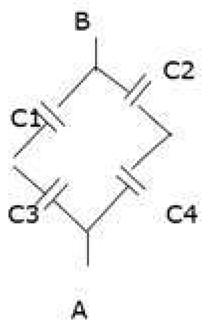
$$\frac{C_1}{C_3} = \frac{C_2}{C_4}$$

Or,

$$\frac{3\mu F}{1\mu F} = \frac{6\mu F}{2\mu F}$$

So, the balancing condition is satisfied, and hence, the 5 μF capacitor will be ineffective.

Thus the setup will reduce to the below form.



This is a simple capacitor combination, with two series connections connected in parallel. This can be solved in parts

Effective capacitance with C_1 and C_3 are,

$$\frac{1}{C_{eff1}} = \frac{1}{C_1} + \frac{1}{C_3}$$

Or,

$$C_{eff1} = \frac{C_1 \times C_3}{C_1 + C_3}$$

Substituting the values of C_1 and C_3

$$C_{eff1} = \frac{1\mu F \times 3\mu F}{1\mu F + 3\mu F} = \frac{3}{4}\mu F$$

Similarly on the other branch,

$$C_{eff2} = \frac{C_2 \times C_4}{C_2 + C_4}$$

Or,

$$C_{eff2} = \frac{6\mu F \times 2\mu F}{6\mu F + 2\mu F} = \frac{12}{8}\mu F$$

The above two series arrangements are arranged in parallel to each other across a potential difference. Hence their equivalent capacitance, C_{eq}, can be found by,

$$C_{eq} = C_{eff1} + C_{eff2}$$

Or,

$$C_{eq} = \frac{3}{4}\mu F + \frac{12}{8}\mu F = \frac{9}{4}\mu F = 2.25\mu F$$

Hence, the equivalent capacitance in each of the arrangement will be 2.25μF.

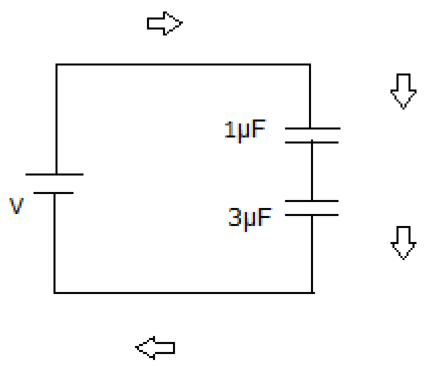
Thus, for the case A), B) and C) the equivalent capacitance of the circuit remains constant.

Answer.25

<u>Explanation:</u>

The potential difference $V_a - V_b$can be found out using Kirchoff's loop rule. Kirchoffs loop rule states that, in any closed loops, the algebraic sum of voltage is equal to zero.

To solve a problem, follow some simple procedure as explained below with an example figure.



The two capacitors 1 μF and 3 μF are connected in series with the battery of V voltage.

We consider the loop and travel through it in any direction, clockwise or anti-clockwise. In the figure we choose to go in clockwise direction as shown.

Note: In the case of a DC source inside the loop, a change from –ve to +ve will be assigned as a positive potential.

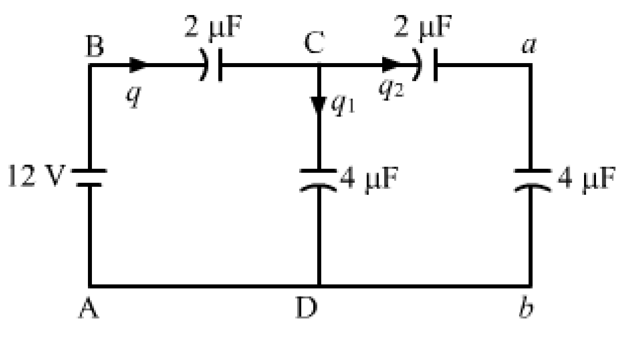
Assume the total charge in the loop is q. So, as per kirchoff's loop rule, the sum of voltages will be,

$$V - \frac{q}{1\mu\text{F}} - \frac{q}{3\mu\text{F}} = 0$$

From this equation, we can find the unknown values depending on the problem.

This same principles are extended to the following problems.

a)



In the figure there are three loops: ABCabDA, ABCDA,CabDC. We goes in clockwise direction in every loops.

And assume, total charge, q is splitted into q_1 and q_2, since they branches in parallel.

So,

$$q = q_1 + q_2$$

Applying kirchoff's rule in CabDC, we get

$$\frac{q_2}{2\mu F}+\frac{q_2}{4\mu F}-\frac{q_1}{4\mu F}=0\ \because q_1\ is\ in\ oposite\ to\ the\ traveling\ direction)$$

Or,

$$3q_2 = q_1\ (eqn.1)$$

Now, apply kirchoff's rule in the loop ABCDA,

So,

$$\frac{q}{2\mu F}+\frac{q_1}{4\mu F}-12V=0$$

Or,

$$\frac{2q}{4\mu F}+\frac{q_1}{4\mu F}=12V$$

But we know, $q=q_1+q_2$

$$2q_2+3q_1=48V$$

So the above expression becomes,

$$2q_2+3q_1=48V\ (eqn.2)$$

Substituting eqn.1 in eqn.2, we get

$$2q_2+9q_2=11q_2=48V$$

Or,

$$q_2=\frac{48V}{11}C$$

Hence, the potential difference $V_a - V_b$ is,

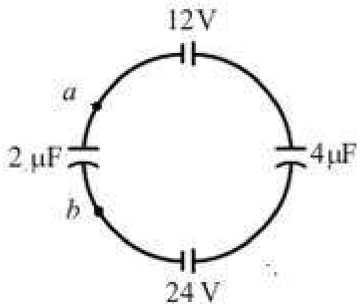
$$V_a-V_b=\frac{q_2}{4\mu F}$$

Or,

$$V_a-V_b=\frac{\left(\frac{48V}{11}C\right)}{4\mu F}=\frac{12}{11}V\ Ans.)$$

Hence the potential difference $V_a - V_b$ is $\left(\frac{12}{11}\right)$V

b) Let's assume there a charge of q amount is in the one loop involved.



By applying Kirchoff's loop rule, by going in clockwise direction, starting from the point a, the sum of potential difference is,

$$12V + \frac{q}{4\mu\text{F}} - 24V + \frac{q}{2\mu\text{F}} = 0$$

Or,

$$\frac{3q}{4\mu\text{F}} = 12V$$

Or,

$$q = \frac{12 \times 4}{3} = 16\mu C$$

Now, we have to find the potential difference across 2 μF capacitor. Since we considering Clockwise as positive direction,

$$V_b - V_a = \frac{q}{2\mu\text{F}}$$

Or,

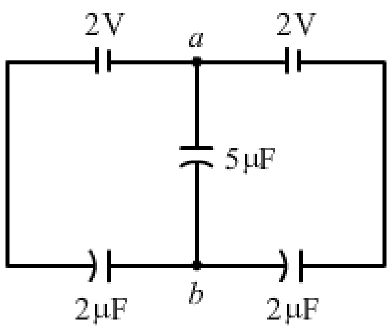
$$V_b - V_a = \frac{16\mu C}{2\mu\text{F}} = 8V$$

Hence

$$V_a - V_b = -8V\ Ans.)$$

Hence $V_a - V_b$ is -8V

c)



From the figure, we can see that, the either side of the terminal a-b are similar or the loops are symmetrical with respect to the terminal a-b. The charge in either of the loop will be same, which can be assumed as q.

Hence the effect on the 5 μF capacitor due to the loop on the left side will be cancelled by the loop of the right side. The charging on the 5 μF due to the left loop will get nullified by the charging by the right side loop. Hence there will be no charge accumulation on the 5 μF capacitor due to either of the battery due to their opposite orientation and symmetry.

Which means, between the terminals a-b,

$$Q = 0C$$

Hence the Potential difference across 5μF,

$$V_a - V_b = \frac{Q}{C}$$

Or,

$$V_a - V_b = \frac{0C}{5\mu F} = 0V\ (Ans.)$$

Hence $V_a - V_b$ is 0V

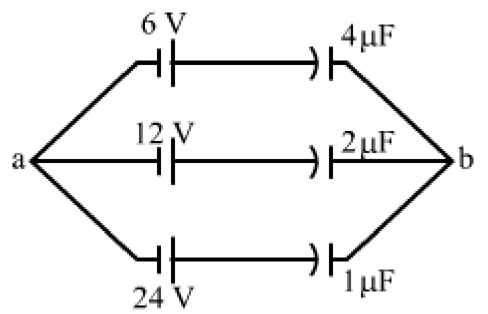
d)

The three branches are connected in parallel across the terminal a-b.

The potential difference $V_a - V_b$ can be found out by,

$$V_a - V_b = \frac{Net\ charge}{Net\ Capacitance}\quad eqn. 1)$$

Where the net charge and net capacitance are the algebraic sum of charges and capacitance ein each branches. So,



In the upper branch, Capacitance is 4μF, and Charge, Q is,

Charge on the capacitor, $Q = CV$

Where

C is the capacitance of the capacitor

V is the potential difference across the end of the capacitor.

Or,

$$Q = 6V \times 4\mu F = 24\mu C$$

In the upper branch, Capacitance is 2μF, and Charge,Q is,

$$Q = CV$$

Or,

$$Q = 12V \times 2\mu F = 24\mu C$$

In the bottom branch, Capacitance is 1μF, and Charge, Q is,

$$Q = CV$$

Or,

$$Q = 24V \times 1\mu F = 24\mu C$$

Hence Net charge between a-b, by adding all the charges, Q_{net}

$$Q_{net} = 24\mu C + 24\mu C + 24\mu C = 72\mu C$$

And Net capacitance, C_{net}

$$C_{net} = 4\mu F + 2\mu F + 1\mu F = 7\mu F \because \text{Since they are connected in parallel)}$$

Hence from eqn.1.

$$V_a - V_b = \frac{Q_{net}}{C_{net}}$$

Or,

$$V_a - V_b = \frac{72\mu C}{7\mu F} = 10.3V\ (Ans.)$$

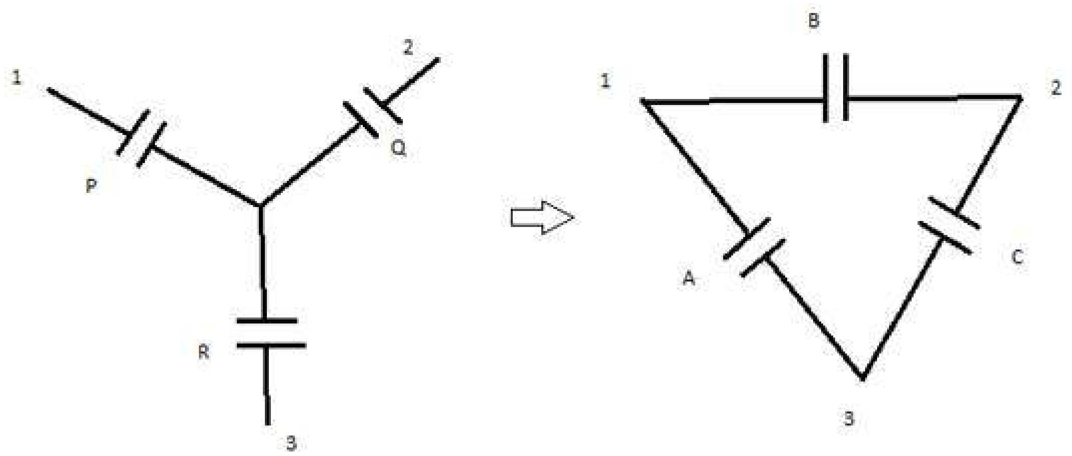
Hence $V_a - V_b$is 10.3V

Answer.26

Figure ‘a’ and ‘b’ can be solved using Y- Delta transformation while figure ‘c’ and ‘d’ can be solved using the concept of Balanced bridge circuit.

Y- Delta or Star-Delta) Transformation:

The Y-Delta transformation technique is used to simplify electrical circuits. It is an extension of Kirchoff’s Loop Rule.



As shown on the figure, the capacitance arranged in between 3 terminals of the first figure can be transformed into the form shown in the second figure. As we converts from the first form to the second one, the capacitance P,Q and R will be replaced by capacitance A, B and C.

The capacitance between terminals 1 and 2 in the second figure corresponding to that of in the first figure, can be written as,

$$B = \frac{P \times Q}{P + Q + R)}$$

Similarly between terminals 2 and 3 will be

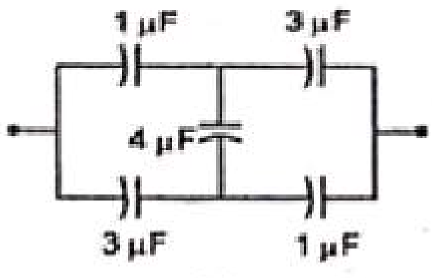
$$C = \frac{Q \times R}{P + Q + R)}$$

Similarly between terminals 3 and 1 will be

$$A = \frac{R \times P}{P + Q + R)}$$

With these values of B, C, and A , the first figure can be transformed into an easier second figure.

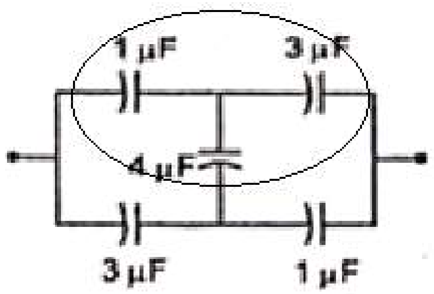
a)



(a)

In the figure 'a' , as the circuit is not balanced $\because \frac{1\mu F}{3\mu F} \neq \frac{3\mu F}{1\mu F}$), this must be changed into a simpler form using Y-Delta transformation.

Below we consider the capacitance in the 'circled portion', and by the transformation equations,



(a)

The capacitance equivalent to 1μF and 3μF is,

$$C_1 = \frac{1\mu F \times 3\mu F}{1\mu F + 3\mu F + 4\mu F)} = \frac{3}{8}\mu F$$

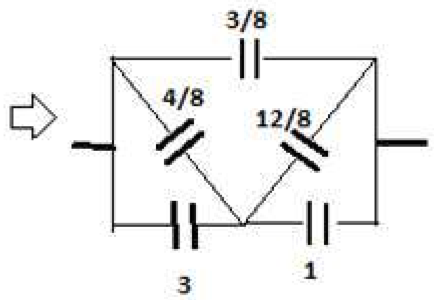
Similarly, corresponding to the capacitance 1μF and 4μF, the equivalent capacitance after transformation is,

$$C_2 = \frac{1\mu F \times 4\mu F}{1\mu F + 3\mu F + 4\mu F)} = \frac{4}{8}\mu F$$

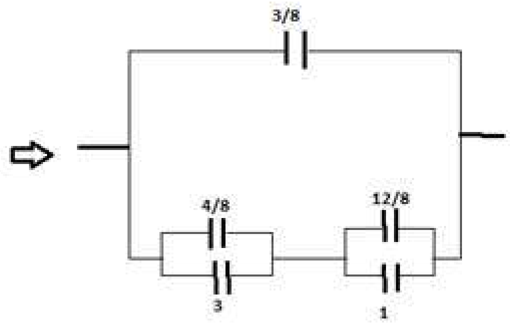
Similarly, corresponding to the capacitance 3μF and 4μF, the equivalent capacitance after transformation is,

$$C_2 = \frac{3\mu F \times 4\mu F}{1\mu F + 3\mu F + 4\mu F)} = \frac{12}{8}\mu F$$

Hence the resultant figure can be drawn as shown,



All the values are in μF)



And it can be further simplified, by re-arranging parallel and series arrangements as shown in figure below.

The above arrangement of capacitances is a simple one, and can be done using the basic equations.

First, consider the two parallel arrangements at the bottom, the equivalent capacitance in the left one is,

$$C_{eq1} = C_1 + C_2$$

Or,

$$C_{eq1} = {}^{4}/_{8}\,\mu F + 3\mu F = \frac{7}{2}\mu F$$

Similarly for the bottom right arrangement,

$$C_{eq1} = C_1 + C_2$$

Or,

$$C_{eq2} = {}^{12}/_{8}\,\mu F + 1\mu F = \frac{5}{2}\mu F$$

Hence the effective capacitance, considering two series capacitance C_{eq1},C_{eq2}) connected in series with the 3/8 μF, is

$$C_{eff} = \left(\frac{\frac{7}{2}\,\mu F \times \frac{5}{2}\,\mu F}{\frac{7}{2}\,\mu F + \frac{5}{2}\,\mu F}\right) + \frac{3}{8}\,\mu F$$

$$= \frac{3}{8}\,\mu F + \frac{35}{24}\,\mu F = \frac{11}{6}\,\mu F$$

Hence, the Effective capacitance between the terminals is 11/6)μF

b)

In figure ‘b’ we have to apply Y-Delta transformation at two portions, as circled in the picture below.

We apply Y- Delta transformation in each circled portion.

In the upper portion,

The capacitance equivalent to 1μF and 3μF is,

$$C_1 = \frac{1\mu F \times 3\mu F}{1\mu F + 3\mu F + 4\mu F)} = \frac{3}{8}\,\mu F$$

Similarly, corresponding to the capacitance 1μF and 4μF, the equivalent capacitance after transformation is,

$$C_2 = \frac{1\mu F \times 4\mu F}{1\mu F + 3\mu F + 4\mu F)} = \frac{4}{8}\,\mu F$$

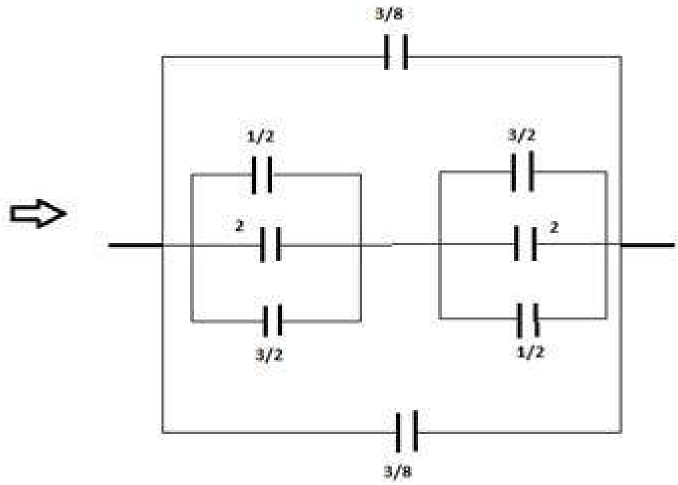
Similarly, corresponding to the capacitance 3μF and 4μF, the equivalent capacitance after transformation is,

$$C_2 = \frac{3\mu F \times 4\mu F}{1\mu F + 3\mu F + 4\mu F)} = \frac{12}{8}\,\mu F$$

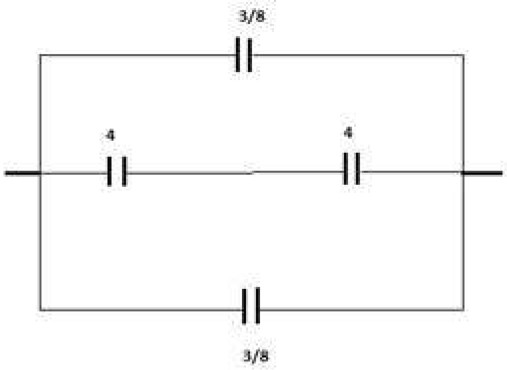
At the lower circled portion,

The same values will come , as the two portions are symmetrical with respect to the central horizontal line. Hence the arrangement becomes,

All the values are in μF)



By simplifying further, it becomes,



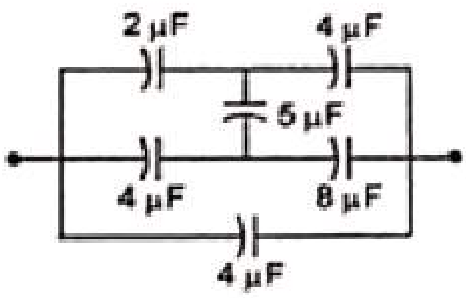
Hence Effective capacitance is,

$$C_{eff} = \left(\frac{4\mu F \times 4\mu F}{4\mu F + 4\mu F}\right) + \frac{3}{8}\mu F + \frac{3}{8}\mu F$$

$$= 2\mu F + \frac{3}{8}\mu F + \frac{3}{8}\mu F = \frac{11}{4}\mu F$$

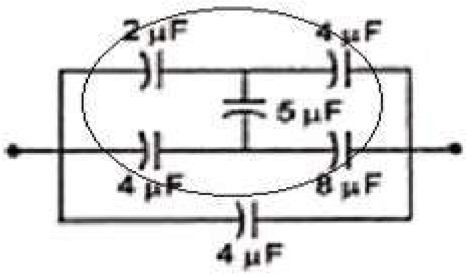
Hence, the Effective capacitance between the terminals is 11/4)μF.

c)



(c)

This problem can be done by either Y-Delta transformation or by the concept of balanced bridge circuits. Here we choose the concept of balanced bridge circuits for simplicity.



(c)

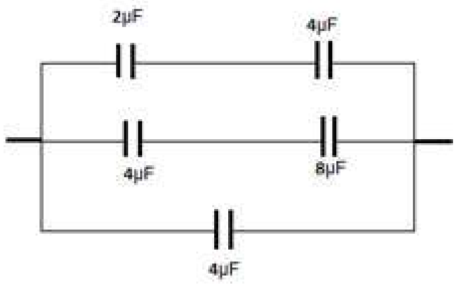
In the below figure, the circled portion is a balance bridge since it obeys balancing condition which is,

$$\frac{C_1}{C_3} = \frac{C_2}{C_4}$$

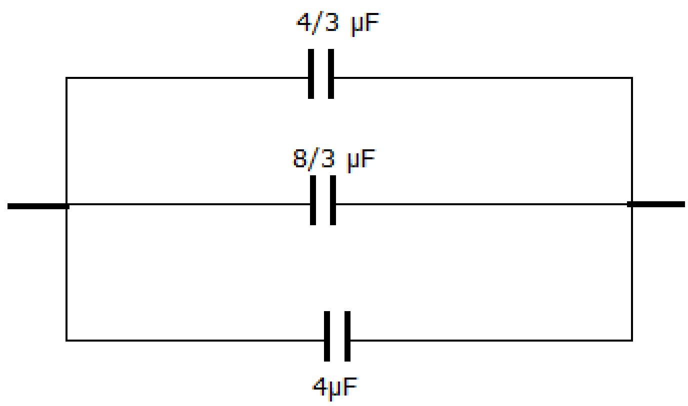
Or,

$$\frac{2\mu F}{4\mu F} = \frac{4\mu F}{8\mu F}$$

And hence the 5μF capacitor will be ineffective as per the principle. Hence the arrangement will be reduced into,



Or, by combining the series capacitance together, it will be reduced into,



This is a simple parallel arrangement, and effective capacitance can be calculated as,

$$C_{eff} = C_1 + C_2 + C_3$$

By substituting the values, we get

$C_{eff} = {}^4/_3 \, \mu F + {}^8/_3 \, \mu F + 4\mu F$

$= 8\mu F$

Hence, the Effective capacitance between the terminals is 8μF.

d)

This problem can be done by the concept of balanced bridge circuits. There are three balanced bridges present in the arrangement.

For which $\frac{C_1}{C_3} = \frac{C_2}{C_4}$

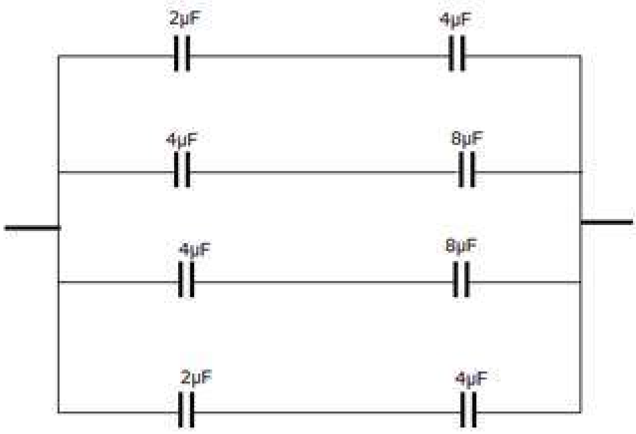
Since,

$$\frac{2\mu F}{4\mu F} = \frac{4\mu F}{8\mu F}$$

and

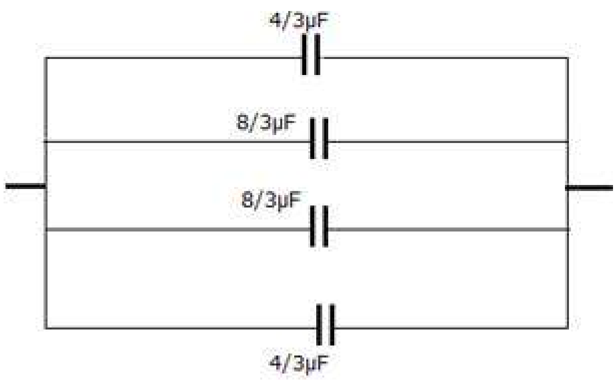
$$\frac{4\mu F}{8\mu F} = \frac{4\mu F}{8\mu F}$$

They are balanced and hence the three 6 μF capacitance will be ineffective.



Hence the resultant arrangement will be,

It is further reduced, by combining series capacitors together, into,



This is a simple parallel arrangement, and effective capacitance can be calculated as,

$$C_{eff} = C_1 + C_2 + C_3 + C_4$$

Or,

$$C_{eff} = \frac{4}{3}\mu F + \frac{8}{3}\mu F + \frac{8}{3}\mu F + \frac{4}{3}\mu F$$

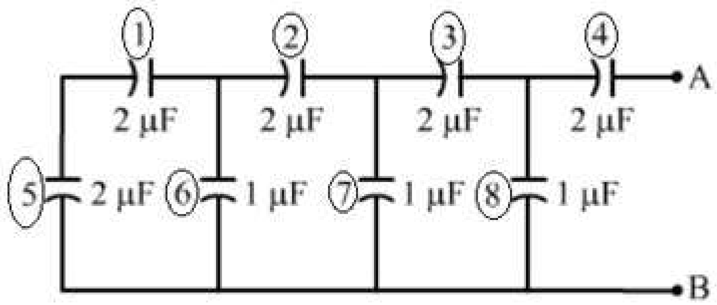
$$= 8\mu F$$

Hence, the Effective capacitance between the terminals is 8μF.

Answer.27

The question figure is a simple arrangement of parallel andseries configurations.

Let us number each capacitor as C_1, C_2,... and C_8 for simplification.



In the figure 5^{th} and 1^{st} capacitors are in series, hence the effective capacitance, C_{51} is

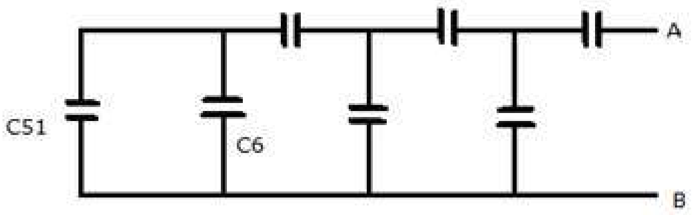
$$C_{51} = \frac{C_5 \times C_1}{C_5 + C_1}$$

Or,

$$C_{51} = \frac{2\mu F \times 2\mu F}{2\mu F + 2\mu F}$$

$$= 1\mu F$$

Now, C_{51} and C_6 are in parallel,



Hence the effective capacitance, C_{61} is,

$$C_{61} = C_{51} + C_6$$

On substituting,

$$C_{61} = 1\mu F + 1\mu F = 2\mu F$$

Now, C_{61} and C_2 are in series, hence the effective capacitance, C_{62} is,

$$C_{62} = \frac{C_{61} \times C_2}{C_{61} + C_2}$$

Or,

$$C_{51} = \frac{2\mu F \times 2\mu F}{2\mu F + 2\mu F}$$

$$= 1\mu F$$

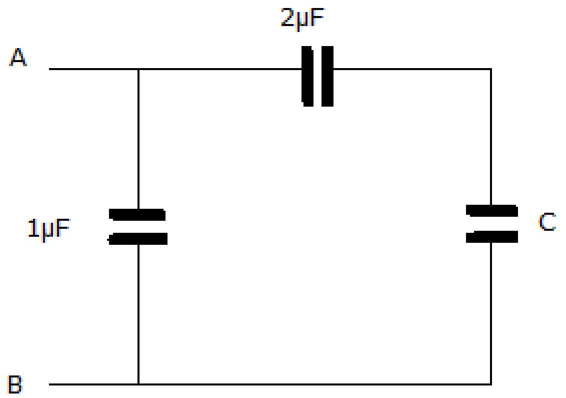
This above pattern repeats for 2 more times. Hence at the end, the effective capacitance, C_{eff} will be 1μF,

So, $C_{eff} = 1\mu F$

The capacitance of the combination is hence 1μF.

Answer.28

This is an infinite series and hence deletion or addition of any repetitive portions of the arrangement does not affect the overall effect. So, let’s convert this into a simpler figure for calculation.



In the above figure, 'C' represents the effective capacitance of the infinite ladder. For the calculations, we have added a 1μF and a 2μF as shown since they both constitute the repetitive portion of the question figure. Hence C and 2μF are in series and they instead is parallel to 1μF.

Now, we calculate the value of C as,

$$C_{eqv} = \frac{2\mu F \times C}{2\mu F + C} + 1\mu F$$

Which is equals to C itself,

So, $C\mu F = \frac{2\mu F \times C\mu F}{2\mu F + C\mu F} + 1\mu F$

Or,

$$C - 1 = \frac{2 \times C}{2 + C}$$

Or,

$$(C - 1) \times (2 + C) = 2C$$

Or,

$$C^2 - C - 2 = 0$$

Or,

$$(C - 2) \times (C + 1) = 0$$

Or,

$$C = 2\mu F \; or \; C = -1\mu F$$

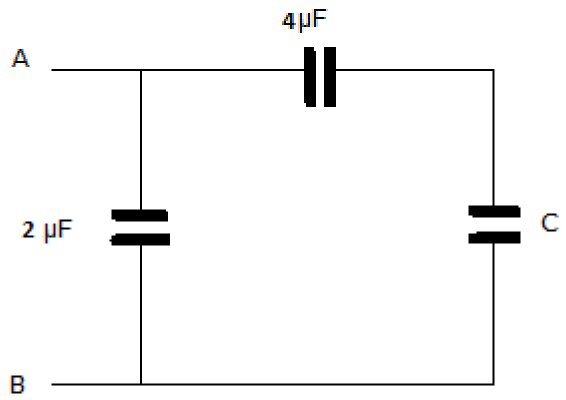
Since capacitance value cannot be negative, we neglect C=-1μF. Hence the equivalent capacitance of the infinite ladder is 2μF.

Answer.29

Given,

The capacitance C should be equal to the equivalent capacitance.

Since the arrangement is an infinite series, addition or deletion of the repetiting components which is the 2 μF, 4 μF capacitor combinations) would not make any effect on the overall capacitance.



Hence, for simplification, we represent it as shown below,

In the figure , C in μF) represents the capacitance that gives the same value for equivalent capacitance to the infinite ladder even after it is terminated at the end. So that C and 4 μF are in series, and these are parallel to 2μF.

In this case, the effective capacitance C_{eff}

$$C_{eff} = \frac{4\mu F \times C}{4\mu F + C} + 2\mu F$$

Which is equals to C itself, since C should not alter the effective capacitance.

So,$C\mu F = \frac{4\mu F \times C\mu F}{4\mu F + C\mu F} + 2\mu F$

Or,

$$C - 2 = \frac{4 \times C}{4 + C}$$

Or,

$$(C-2)\times(4+C)=4C$$

Or,

$$C^2-2C-8=0$$

Or,

$$(C-4)\times(C+2)=0$$

$$C=4\mu F \; or \; C=-2\mu F$$

Since capacitance value cannot be negative, we neglect C=-2μF. Hence the equivalent capacitance of the infinite ladder is 4μF.

So, the value of capacitance that should be assigned with the terminating capacitor is 4 μF.

Answer.30

Given,

Charge on plate 1, $Q_1 = +2.0 \times 10^{-8} C$

Charge on plate 2, $Q_2 = -1.0 \times 10^{-8} C$

Capacitance of the capacitor, C= 1.2×10^{-3} μF= 1.2×10^{-9} F

Formula used

We know that for a capacitor with net charge, Q and capacitance, C, the Potential difference deceloped in between the plates, V is,

$$V=\frac{|Q|}{C} \; eqn.1)$$

The charges on the inner plates of the capacitor with plates having charges Q_1 and Q_2 is,

$$Q=\pm\left(\frac{Q_1-Q_2}{2}\right)$$

Note: Charges on the outer plates of the capacitor with plates having charges Q1 and Q2 is,

$$Q = \pm\left(\frac{Q_1 + Q_2}{2}\right)$$

In the given example, the plates has individual charges Q_1 and Q_2. But when they placed as a capacitor, their charges re-arrange and equal and opposite charges will be distributed in each plates. The total net charge, Q_{net} on the inner sides of each plates will be

$$Q_{net} = \frac{Q_1 \sim Q_2}{2}$$

Substituting the values,

$$Q_{net} = \frac{(+2.0 \times 10^{-8}C) \sim (-1.0 \times 10^{-8}C)}{2}$$

$$= \pm 1.5 \times 10^{-8}C$$

Hence the inner side of each plates will have a charge of $\pm 1.5 \times 10^{-8}C$.

Hence from eqn.1, Potential difference, V is

$$V = \frac{|Q|}{C}$$

Or,

$$V = \frac{+1.5 \times 10^{-8}C}{1.2 \times 10^{-9}\,F} = 12.5V\ (Ans.)$$

Hence the potential difference developed between the plates is 12.5V

Answer.31

<u>Explanation:</u>

Given,

Charge on plate 1, Q_1 = 20 μC

Charge on plate 2, Q_2 = 0C Since no charge is given to the other plate and the setup is isolated)

Capacitance of the capacitor, C= 10 μF

<u>Formula used</u>

We know that for a capacitor with net charge, Q and capacitance, C, the Potential difference deceloped in between the plates, V is,

$$V = \frac{|Q|}{C}\ eqn.1)$$

The charges on the inner plates of the capacitor with plates having charges Q_1 and Q_2 is,

$$Q = \pm\left(\frac{Q_1 - Q_2}{2}\right)$$

Note: Charges on the outer plates of the capacitor with plates having charges Q1 and Q2 is,

$$Q = \pm\left(\frac{Q_1 + Q_2}{2}\right)$$

In the given question, the charges on the inner plates , according to above formulas,

$$Q = \pm\left(\frac{20\ \mu C - 0C}{2}\right) = \pm 10\mu C$$

Hence from eqn.1, the potential difference

$$V = \frac{10\mu C}{10\ \mu F} = 1V\ (Ans.)$$

Hence the potential difference developed in between the plates is 1V.

32. Question

A charge of 1 μC is given to one plate of a parallel-plate capacitor of capacitance 0.1 μF and a charge of 2 μC is given to the other plate. Find the potential difference developed between the plates.

Answer

Explanation:

Given,

Charge on plate 1, Q_1 = 1 μ*C*

Charge on plate 2, Q_2 = 2 μ*C*

Capacitance of the capacitor, C= 0.1 μF

Formula used

We know that for a capacitor with net charge, Q and capacitance, C, the Potential difference deceloped in between the plates, V is,

$$V = \frac{|Q|}{C}\ eqn.1)$$

The charges on the inner plates of the capacitor with plates having charges Q_1 and Q_2 is,

$$Q = \pm\left(\frac{Q_1 - Q_2}{2}\right)$$

So, the charge, Q by substituting the given values, is

$$Q = \pm\left(\frac{1\mu C - 2\mu C}{2}\right) = \pm 0.5\mu C$$

Hence from eqn.1, the potential difference

$$V = \frac{0.5\mu C}{0.1\ \mu F} = 5V\ (Ans.)$$

Hence the potential difference developed in between the plates is 5V.

Answer.33

Given,

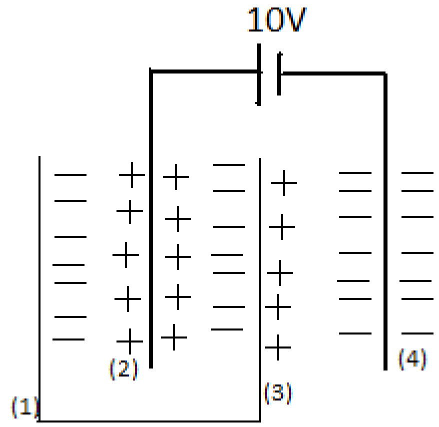
The area of the capacitor plates, A $=96/\epsilon_0) \times 10^{-12}$ Fm

The distance in between each pairs of plates, d$=$ 4mm$=4\times10^{-3}$ m

The emf of the connected battery, V$=$ 10V

Formula used

The schematic representation of distribution of charges when connected to the DC battery is shown in the figure. The plate 2) connected to the positive terminal will be positively charged and the one 4) connected to the negative terminal will be negatively charged.



The oposite charges will be induced in plates 1) and 3), whe the battery is connected as shown.

So, there will be three capacitors that are formed namely, 1-2, 2-3 and 3-4. And they are connected in series arrangement.

Let's assume that each capacitors has a charge Q, and since they are connected in series, the total charge will also be Q.

Hence the charge, Q

$$Q = C_{eq} \times V \; eqn.1)$$

Where,

V= Potential difference= 10V

C_{eq}= Equivalent capacitance of the arrangement.

Now, for series arrangement, we know

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

And C_1, C_2 and C_3 are the capacitance of capacitors formed by plates 1-2, 2-3 and 3-4 respectively.

Since area and the separation of all the plates are same,

$$C_1 = C_2 = C_3 = C$$

And we know,

Capacitance of the capacitor, $C = \frac{\varepsilon_0 A}{d}$

Where

ε_0 is the permittivity of the free space

A is the area of the plates of the capacitor d is the separation between the plates of the capacitor

Substituting the given values in the above equation, we get

$$C = \frac{\varepsilon_0 \times 96/\varepsilon_0) \times 10^{-12}\text{Fm}}{4 \times 10^{-3}\ \text{m}}$$

$$C = 24 \times 10^{-9}\text{F}$$

Hence, Equivalent capacitance is,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Or,

$$\frac{1}{C_{eq}} = \frac{3}{C} \because C_1 = C_2 = C_3 = C)$$

or,

$$\frac{1}{C_{eq}} = \frac{3}{24 \times 10^{-9}\text{F}}$$

Or,

$$C_{eq} = \frac{24 \times 10^{-9}\text{F}}{3} = 8 \times 10^{-9}\text{F}$$

Hence, from eqn.1, the charge on each pairs will be,

$$Q = 8 \times 10^{-9}\text{F} \times 10 = 8 \times 10^{-8}C$$

This is the charge on each side of the plates constituting a capacitor. But the plates connected to the battery has either positive charge or negative charge on both sides, as shown in figure. So, the total charge accumulated in the plates connected to the battery will be two times the above value.

Hence, charge on the plates connected to battery will be 2×Q,

$$Charge = 2 \times Q$$

Or,

$$Charge = 2 \times 8 \times 10^{-8}C = 16 \times 10^{-8}C = 0.16\mu\text{C}$$

Hence the charge on the specific plates will be ±0.16μC , since one plate is positively charged and the other is negatively charged.

Answer.34

Given,

The charge given to the middle plate Q) is 1.0 μC

The capacitance between the plates, C is 50 nF=50× 10^{-3} μF

Formula used

For a capacitor with net charge, Q and capacitance, C, the Potential difference deceloped in between the plates, V is,

$$V = \frac{|Q|}{C} \; eqn.1)$$

The charges on the inner plates of the capacitor with plates having charges Q_1 and Q_2 is,

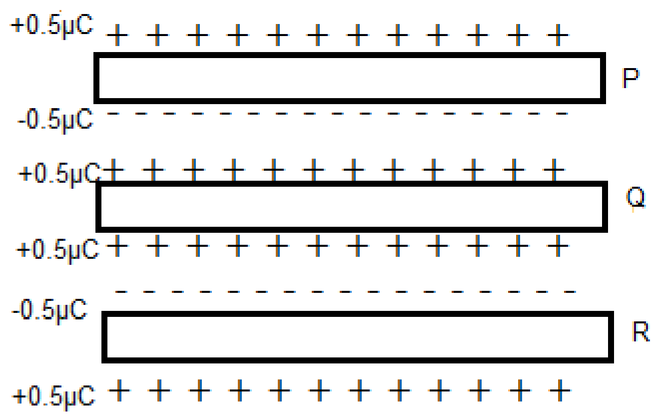
$$Q = \pm\left(\frac{Q_1 - Q_2}{2}\right) \; eqn.2)$$

Charges on the outer plates of the capacitor with plates having charges Q1 and Q2 is,

$$Q = \pm\left(\frac{Q_1 + Q_2}{2}\right) \; eqn.3)$$

a)

The charge given to the plate Q will be distributed equally on the either sides of plates as shown in figure. Hence the upper and lower sides of plate Q will be charged to +0.5 μC.



Here, we get two capacitors namingly as P-Q and Q-R.

In capacitor P-Q, the upper plate is neither connected to any battery nor given any charges. So the total charge on the plate is $0C$. But when it is made into a capacitor plate, a charge is induced in it from the plate Q. The amount of the charge can be calculated from the eqn.2,

Which is,

$$Q = \pm\left(\frac{Q_1 - Q_2}{2}\right)$$

Where Q_1 is the charge on one plate Q= 1.0 μC

And Q_2 is the charge on plate P = $0C$

Hence by substituting in the above equation, we get,

$$Q = \pm\left(\frac{1.0\ \mu C - 0C}{2}\right) = \pm 0.5\mu C$$

Hence the inner surfaces get a charge of ±0.5μC on each plates. Since the plate Q is positively charged, Plate P will get -0.5μC charge.

But we know that the net charge on plate P is zero. Hence to nutralise the inner surface charge, the outer surface will get a charge of +0.5μC

b) From the above calculation, we found that the inner surfaces of the capacitor P-Q has a charge of ±0.5μC.

The potential difference between the plates can be found by the eqn.1, as

$$V = \frac{0.5\mu\text{C}}{50 \times 10^{-3}\mu\text{F}} = 10V\ Ans.)$$

Hence the potential difference between the upper and middle plates of the arrangements is 10V.

Answer.35

Given,

Charge given to the upper plate, plate P, is 1.0 μC.

Formula used

For a capacitor with net charge, Q and capacitance, C, the Potential difference deceloped in between the plates, V is,

$$V = \frac{|Q|}{C} \; eqn.1)$$

The charges on the inner plates of the capacitor with plates having charges Q_1 and Q_2 is,

$$Q = \pm\left(\frac{Q_1 - Q_2}{2}\right) \; eqn.2)$$

a)

By giving a charge of 1.0 μC to plate P, it will get distributed on either side of the plate as +0.5 μC. The other plates get induced with this charge as shown in figure.

To find the charge on the plate Q, eqn.2 shall be used. So we get,

$$Q = \pm\left(\frac{Q_1 - Q_2}{2}\right)$$

Where Q_1 is the charge on one plate P= 1.0 μC

And Q_2 is the charge on plate Q = 0C

Hence by substituting in the above equation, we get,

$$Q = \pm\left(\frac{1.0\ \mu C - 0C}{2}\right) = \pm 0.5\mu C$$

So the upper face of plate Q will get a charge of -0.5 μC and this will induce a charge of +0.5 μC on the bottom side of plate Q.

Hence the potential difference in capacitor P-Q, by eqn.1 is

$$V = \frac{|Q|}{C}$$

Where Q= 0.5 μC

And C= 50×10^{-3}μF

Hence,

$$V = \frac{0.5\mu C}{50 \times 10^{-3}\mu F} = 10V\ (Ans.)$$

So the potential difference in between the upper and middle plates is 10V

b)

From the above condition, the upper face of plate Q will get a charge of -0.5 μC and this will induce a charge of +0.5 μC on the bottom side of plate Q.

We know that, the capacitor Q-R is made of the bottom surface of plate Q and the upper side of plate R. As the bottom surface of plate Q already has a charge of +0.5 μC, it will induce -0.5 μC charge on the upper face of plate R As shown in figure).

Hence the potential difference in between the lower and middle plates can be calculated from the eqn.1, as

$$V = \frac{|Q|}{\mathrm{C}}$$

Where Q is the charge in each plates=±0.5 μC

And C= 50×10^{-3}μF

Hence,

$$V = \frac{|\pm 0.5\mu\mathrm{C}|}{50 \times 10^{-3}\mu\mathrm{F}} = 10V\ (Ans.)$$

So the potential difference in between the middle and lower plates is 10V

Answer.36

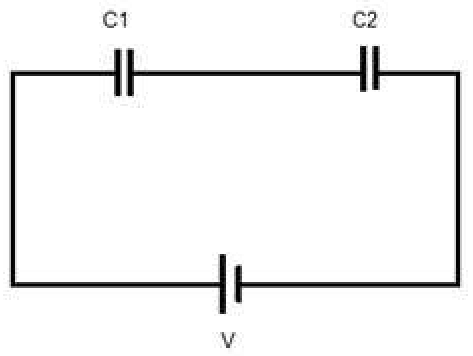
Given,

Capacitances of the two capacitors, are 20.0 pF and 50.0 pF.

The voltage of the battery, V is 6V

Formula used

Let us represent the arrangement as



In a series arrangement the the charge on both the capacitance are same equal to total charge),can be found out by the equation,

$$Q = C_{eff} \times V \; eqn. 1)$$

Where Q and V represents the Charge and Potential difference respectively.

in series arrangement with Capacitance C_1 and C_2, C_{eff} can be found out as,

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Or,

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2} \; eqn. 2)$$

And thus the potential difference on each capacitance, V_1 and V_2 can be calculated by the below relations,

$$V_1 = \frac{Q}{C_1} \; eqn. 3)$$

And,

$$V_2 = \frac{Q}{C_2} \; eqn. 4)$$

Now,

The energy stored in a capacitor,E in Jules) can be found out by the relation,

$$E = \frac{1}{2} \times C \times V^2 \; eqn. 5)$$

Where

C is the capacitance of the capacitor in Farad

V is the potential difference across the capacitor.

a) First we calculate the ewuivalent capacitance by eqn.2.

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2}$$

Where C_1=20 pF and C_2=50pF

So,$C_{eff} = \frac{50\text{pF} \times 20\text{pF}}{50\text{pF} + 20\text{pF}} = 14.286pF$

Hence, the total charge, Q from eqn.1 is

$Q = 14.286pF \times 6V = 85.714pC$

To find potential difference on each capacitor, we use eqn.3 and eqn.4. So the potential difference on 50pF capacitor is,

$$V_1 = \frac{Q}{C_1}$$

Or,

$$V_1 = \frac{85.714pC}{20pF} = 4.29V\,(Ans.)$$

Similarly, on 20pF capacitor, V_2 is

$$V_2 = \frac{Q}{C_2}$$

Or,

$$V_2 = \frac{85.714pC}{50pF} = 1.7V\,(Ans.)$$

Hence the potential differences across 50pF and 20pF capacitors are 1.714V and 4.29V respectively.

b) Energy stored in each capacitors can be calculat4ed by eqn.5

Hence for, 20pF capacitance across 4.29V potential difference, energy stored is,

$$E = \frac{1}{2} \times 20\text{pF} \times (4.29V)^2 = 184.04pJ\ Ans.)$$

Similarly for, 50pF capacitance across 1.71V potential difference, energy stored is,

$$E = \frac{1}{2} \times 50\text{pF} \times (1.71V)^2 = 73.469pJ \approx 73.5pJ\ Ans.)$$

Hence Energy stored in each capacitors are 73.5pJ and 184.04pJ for 50pF and 20pF capacitors respectively.

Answer.37

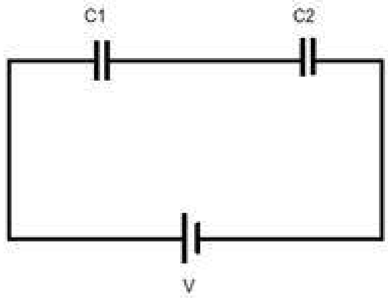
Given,

Capacitance are 4.0 μF C_1) and 6.0 μF C_2)

The voltage of battery, V is 20V

Formula used

Let us represent the arrangement as



In a series arrangement the the charge on both the capacitance are same, and can be found out by the equation,

$$Q = C_{eff} \times V \; eqn. 1)$$

Where Q and V represents the Charge and Potential difference respectively.

in series arrangement with Capacitance C_1 and C_2, C_{eff} can be found out as,

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Or,

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2} \; eqn. 2)$$

The energy stored in the capacitor,E in Jules) can be found out by the relation,

$$E = \frac{1}{2} \times C \times V^2 \;\; eqn. 3)$$

Where C is the capacitance of the capacitor in Farad and V is the potential difference across the capacitor.

We have to find the equivalent capacitance by eqn.2

So,

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2}$$

Or,

$$C_{eff} = \frac{4.0\ \mu\text{F} \times 6.0\ \mu\text{F}}{4.0\ \mu\text{F} + 6.0\ \mu\text{F}} = 2.4\ \mu\text{F}$$

Now the energy supplied by the battery is equivalent to the energy stored in the equivalent capacitor with capacitance C_{eff}. Hence by eqn.3, we get

$$E = \frac{1}{2} \times C_{eff} \times V^2$$

Or,

$$E = \frac{1}{2} \times 2.4\ \mu F \times (20V)^2 = 480\mu J$$

The supplied energy will be twice of the stored energy, since half of the supplied energy will be dissipated by the resistance of the circuit.

Hence the supplied energy will be

$$E_S = 2 \times E = 2 \times 480\mu J = 960\mu J\ )$$

Hence an amount of 960 μJ will be supplied by the battery.

Answer.38

Given, capacitance of a, b, c, d capacitors are 10 μF each.

The voltage of the DC battery is 100V

Formula used

Energy stored in a capacitor can be calculated from the relation,

$$E = \frac{1}{2} \times C \times V^2 \ \ eqn. 1)$$

Or,

$$E = \frac{1}{2} \times \frac{Q^2}{C} eqn. 2)$$

Where C represents the capacitance, V is the potential difference across the capacitor and Q is the charge in the capacitor.

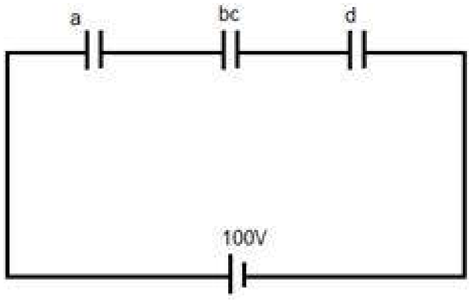
And, effective capacitance of capacitors C_1 and C_2 arranged in series is

$$C_{eff} = \frac{C_1 \times C_2}{C_1 + C_2} \ eqn. 3)$$

And those connected in parallel is

$C_{eff} = C_1 + C_2 + C_3 \; eqn.4)$ Considering three capacitors)

The capacitors b and c are in parallel. For simplification, we reduce it into capacitor bc as shown,



and the capacitance of bc is, from eqn.4

$$C_{bc} = C_b + C_c$$

By substituting the values,

$$C_{bc} = 10\mu\text{F} + 10\mu\text{F} = 20\mu\text{F}$$

Now the whole arrangement is a series connection and charges in each capacitor will be the same.

To find out effective capacitance of this arrangement, we find equivalent capacitance, C_{ad} between a and d initially, by eqn.3

$$C_{ad} = \frac{C_a \times C_d}{C_a + C_d}$$

By substituting the values,

$$C_{ad} = \frac{10\mu\text{F} \times 10\mu\text{F}}{10\mu\text{F} + 10\mu\text{F}} = 5\mu\text{F}$$

Now the total capacitance considering $C_{ad}$and C_{bc} in series, using eqn.3,

$$C_{eff} = \frac{C_{ad} \times C_{bc}}{C_{ad} + C_{bc}}$$

Or,

$$C_{eff} = \frac{5\mu\text{F} \times 20\mu\text{F}}{5\mu\text{F} + 20\mu\text{F}} = \; 4\mu\text{F}$$

The capacitors a ,d and the parallel arrangement will have same charge,Q in it, which can be calculated as,

$$Q = C_{eff}V$$

Where,

C_{eff}= Capacitance, V= Potential difference=$100V$

By substitution, we get , Q as

$Q = 4\mu F \times 100V = 400\mu C$

The energy stored in a and d are same due to the same capacitance value and the same charge accumulation. So energy stored in a and d are, from eqn.2

$$E = \frac{1}{2} \times \frac{Q^2}{C}$$

Or,

$$E = \frac{1}{2} \times \frac{(400\mu C)^2}{10\mu F} = 8 \times 10^{-3} J = 8mJ\ (Ans.)$$

In the parallel arrangement, the charge, Q=400µC will be splitted in half as the two branches are symmetrical. So each capacitors b and c will have Q=200µC amount of charge.

Hence, by the energy relation, eqn.2 , the energy in each capacitors b and c, will be,

$$E = \frac{1}{2} \times \frac{Q^2}{C}$$

Or,

$$E = \frac{1}{2} \times \frac{(200\mu C)^2}{10\mu F} = 2 \times 10^{-3} J = 2mJ\ (Ans.)$$

Hence 8mJ will be stored in the capacitors a and d , while 2mJ will be stored in b and c.

Answer.39

Given,

The stored energy in the first capacitor is 4.0J

Formula used

Energy stored in a capacitor of capacitance C across a potential difference V is,

Energy stored in the capacitor, $E = \frac{1}{2} \times C \times V^2\ eqn.1)$

Whenever an uncharged capacitor is connected with a charged capacitor, the charge will redistribute according to the capacitance of both of the capacitors.

In the given case, both the capacitors are identical and hence the charge will distribute equally in both.

We assume that the charge in the first capacitor is initially as q. After the charge distribution, the charge on both capacitors will be q/2.

Since the capacitance are equal and there is no electric field placed in between, according to the eqn.1 the energy stored in both the capacitors are same.

So, by conservation of energy, the total 4J will be distributed to both of the capacitors. So each capacitor will store energy of amount 2J.

Answer.40

Given,

Potential difference, V is $12V$

Capacitance of initially charged capacitor, C_1 is 2 μF

Capacitance of initially uncharged capacitor, C_2 is 4 μF

<u>Formula used</u>,

Energy stored in a capacitor of capacitance C and charge Q is,

$$E = \frac{1}{2} \times \frac{Q^2}{C} eqn.1)$$

Energy stored in a capacitor of capacitance C across a potential difference V is,

$$E = \frac{1}{2} \times C \times V^2 \ eqn.2)$$

a)

Initial charge on C_1capacitor, Q_1 is

$$Q_1 = C_1 \times V$$

Or,

$$Q_1 = 2\ \mu F \times 12V = 24\mu C$$

Now, let's assume that after connecting the second capacitor C_2, the charge on C_1 and C_2 as q_1 and q_2 respectively.

So,$Q_1 = q_1 + q_2 = 24\mu C\ eqn. 2)$

We know that for a parallel arrangement of capacitors across a single battery, the potential differences are the same. So

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

Or, by substituting the values for C_1 and C_2, we can re-write it as,

$$\frac{q_1}{2\mu F} = \frac{q_2}{4\mu F}$$

Or,

$$2q_1 = q_2\ eqn. 3)$$

Substituting eqn.3 in eqn.2, we get

$$3q_1 = 24\mu C$$

Or,

$$q_1 = 8\mu C$$

And

$$q_2 = 16\mu C$$

b) Energy stored on each capacitor, by eqn.1 is

On C_1

$$E_1 = \frac{1}{2} \times \frac{{q_1}^2}{C_1}$$

Or,

$$E_1 = \frac{1}{2} \times \frac{8\mu C^2}{2\mu F} = 16\mu J$$

Similarly,

On C_2

$$E_2 = \frac{1}{2} \times \frac{{q_2}^2}{C_2}$$

Or,

$$E_2 = \frac{1}{2} \times \frac{16\mu C^2}{4\mu F} = 32\mu J$$

Hence the energy stored is 16μJ and 32μJ on 2μF and 4μF capacitors respectively.

c) For heat dissipation, we have to find the initial energy stored.

Hence from eqn.1, the initial energy with 2μF capacitor only in the circuit, E_b is

$$E_b = \frac{1}{2} \times C \times V^2$$

Where V=12V

So after substitution,

$$E_b = \frac{1}{2} \times 2\mu F \times 12V^2 = 144\mu J$$

Hence heat produced is the difference between the initial energy and the algebraic sum of the energy stored after connection.

So,

$$Heat = E_b - (E_1 + E_2)$$

Or,

$$Heat = 144\mu J - (16\mu J + 32\mu J) = 96\mu J$$

The heat produced/dissipated during the charging is 96 μJ

Answer.41

Given,

R is the radius of the sphere and Q is a point charge

<u>Formula used</u>

We know that for a sphere or a point charge, the capacitance can be found out by the equation,

$$C = 4\pi\varepsilon_0 \times R$$

Now, to find energy stored, we have the relation,

$$E = \frac{1}{2} \times \frac{Q^2}{C}$$

Here the point charge has Q amount of charge and capacitance C is as given above. So by substitution,

$$E = \frac{1}{2} \times \frac{Q^2}{4\pi\varepsilon_0 \times R} = \frac{Q^2}{8\pi\varepsilon_0 \times R} \; Ans.)$$

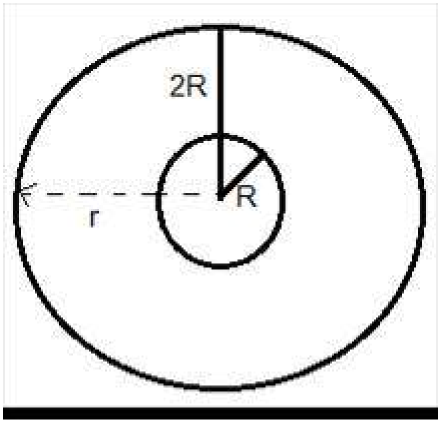
Hence the expression for energy stored on a sphere around a point charge placed at the origin is $Q^2/8\pi\varepsilon_0 \times R)$ J

Answer.42

$U_E = \pi\varepsilon_0 R V^2$ J

Explanation:

The given condition is represented in the figure. The outer sphere has a radius 2R while the metal sphere has a radius R.



Now potential difference, V of the sphere is given by,

$$V = \frac{Q}{C} \; eqn. 1)$$

Where Q and C represents Charge and Capacitance of sphere

For sphere of radius R, C is

$$C = 4\pi\varepsilon_0 \times R$$

Substituting this in eqn.1, we get,

$$V = \frac{Q}{4\pi\varepsilon_0 \times R} \; eqn. 2)$$

a)

Energy density at a distance r from the centre is,

$$U_E = \frac{1}{2}\varepsilon_o E^2$$

Or

$$U_E = \frac{1}{2}\varepsilon_o \left(\frac{q}{4\pi\varepsilon_o r^2}\right)^2 = \frac{q^2}{32\,\pi^2\varepsilon_o r^4}$$

Consider a spherical element at a distance r from the centre, with a thickness dr, such that R>r>2R.

Now the volume of the spherical element is,

$$dV = 4\pi r^2 dr$$

So, energy stored will be

$$dU_E = u_E \times dv = \frac{q^2}{32\,\pi^2\varepsilon_o r^4} \times 4\pi r^2 dr$$

Or

$$dU_E = \frac{q^2}{8\pi\varepsilon_o}\left(\frac{dr}{r^2}\right)$$

For finding the electrostatic energy on a surface at 2R, we have to integrate the expression for dU_E in between R and 2R. So,

$$U_E = \int_R^{2R} \frac{q^2}{8\pi\varepsilon_o}\left(\frac{dr}{r^2}\right)$$

Or,

$$U_E = \frac{q^2}{8\pi\varepsilon_o}\int_R^{2R}\left(\frac{dr}{r^2}\right) = \frac{q^2}{8\pi\varepsilon_o}\left[-\frac{1}{r}\right]_R^{2R} = \frac{q^2}{16\,\pi\varepsilon_o R}$$

But from eqn.2,

$q = 4\pi\varepsilon_0 \times R \times V$

Hence, U_E becomes,

$$U_E = \frac{q^2}{16\pi\varepsilon_o R} = \frac{(4\pi\varepsilon_o RV)^2}{16\pi\varepsilon_o R}$$

Or,

Electrical energy at a distance 2R is

$$U_E = \pi\varepsilon_0 RV^2 \; (Ans.)$$

b)

To find the electrostatic stored energy outside the radius 2R, we integrate the above expression for differential of stored energy from 2R to infinity.

So,

$$U_E = \int_{2R}^{\infty} \frac{q^2}{8\pi\varepsilon_o} \left(\frac{dr}{r^2}\right)$$

Or,

$$U_E = \frac{q^2}{8\pi\varepsilon_o} \int_{2R}^{\infty} \left(\frac{dr}{r^2}\right) = \frac{q^2}{8\pi\varepsilon_o} \left[-\frac{1}{r}\right]_{2R}^{\infty} = \frac{q^2}{16\,\pi\varepsilon_o R}$$

By substituting q as $4\pi\varepsilon_0 \times R \times V$ in the above expression, we get,

$$U_E = \frac{q^2}{16\,\pi\varepsilon_o R} = \frac{(4\pi\varepsilon_o RV)^2}{16\pi\varepsilon_o R}$$

Or it will reduce to,

$$U_E = \pi\varepsilon_0 RV^2 \; (Ans.)$$

This is same as that of inside the sphere of radius 2R.

Thus electrostatic field energy stored outside the sphere of radius 2R equals that stored within it.

Answer.43

Given,

Surface charge density,σ=1.0×10^{-4} C m^{-2}

Edge length of the cube, e=1.0 cm=0.01m

Permittivity of free space, ε_0= 8.85×10^{-12} F/m

Formula used

For a conducting plate infinite length), the electric field, E is,

$$E = \frac{\sigma}{\varepsilon_0}\ (eqn. 1)$$

And the electrostatic energy density or the energy per volume is,

$$u = \frac{1}{2}\varepsilon_0 E^2\ (eqn. 2)$$

Substituting eqn.1 in eqn.2 will result in,

$$u = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2$$

Now the energy stored in volume V is

$$U = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 \times V\ eqn. 3)$$

In the problem, we have to find the force inside a cube of edge e length.

So, we replace V with e^3 in eqn.3

So,

$$U = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 \times e^3$$

Now, substituting the known values in the above equation, it becomes,

$$U = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0} \times e^3$$

Or,

$$U = \frac{1}{2}\frac{(1.0\times10^{-4}\ C\,m^{-2})^2}{8.85\times\frac{10^{-12}F}{m}} \times (0.01m)^3$$

$$= 5.6\times10^{-4} J\ Ans.)$$

Answer.44

Given –

Plate area 20 cm

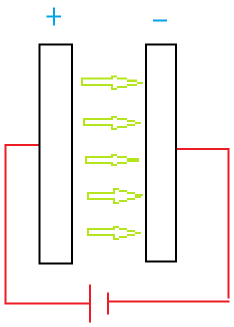
Separation between the plates 1.00 mm = 0.001m

Battery Voltage = 12.0 V

We know capacitance, C

$$C = \frac{\epsilon_0 A}{d} \quad 1)$$

^{2} = 0.002m^2



Where,

A= Plate Area

d= separation between the plates,

ϵ_0 = Permittivity of free space

= 8.854×10^{-12}

From1),

Capacitance when distance d = 0.001m, C

$$C_1 = \frac{\epsilon_0 \; A_1}{d_1}$$

$m^{-3}\ kg^{-1}\ s^4\ A^2$

Substituting values,

$C_1 = \frac{\epsilon_0 \times 0.002}{0.001} = 2\,\epsilon_0$ 2)

When The plates are pulled apart to increase the separation to –

2.0 mm = 0.002m, then capacitance C_2 becomes,

$$C_2 = \frac{\epsilon_0 \; A_2}{d_2}$$

Substituting values

$C_2 = \frac{\epsilon_0 \times 0.002}{0.002} = \epsilon_0$ 3)

From 1) and 2)

$C_2 = 2C_1$ 4)

We know capacitance in terms of voltage is given by –

Q = CV 5)

Where

Q= charge stored on the capacitor

C= capacitance of the capacitor

V = applied voltage

Given applied v = 12V

From 2) and 3) and 5)

$Q_1 = 2 \times \epsilon_0 \times v$

$= 2 \times \epsilon_0 \times 12$

$= 24 \times \epsilon_0$ 6)

And

$Q_2 = \epsilon_0 \times v$

$= 12\,\epsilon_0$ 7)

a) The charge flown through the circuit during the process –

$Q_{flown} = Q_1 - Q_2$

From 6) and 7)

$Q_{flown} = 24 \times \epsilon_0) - 12\ \epsilon_0\ )$

$= 12\ \epsilon_0$

$= 12 \times 8.854 \times 10^{-12}$

$= 1.06 \times 10^{-10}$ C 8)

b)Energy absorbed by the battery during the process-

We know Energy E is given by -

E=QV 9)

Where

Q = charged present on the surface

V= Applied voltage

From 8),

$Q_{flown} = 1.06 \times 10^{-10}$ c

Applied voltage V = 12V

From 9),

Energy absorbed,

$\mathrm{E} = 12 \times 1.06 \times 10^{-10}$

$= 2.7 \times 10^{-10}$ J

c)Stored energy in the electric field before and after the process

We know that stored energy in the electric field,

$$\mathrm{E} = \frac{1}{2}\,\mathrm{C}\,\mathrm{V}^2$$

Before process, the energy stored -

$$\mathrm{E_{before}} = \frac{1}{2}\,\mathrm{C_1}\,\mathrm{V}^2$$

From 2),

$$\mathrm{E_{before}} = \frac{1}{2} \times 2\ \epsilon_0 \times 12 \times 12$$

$= 12.7 \times 10 - 10\,J$ 10)

From 3), After process, the energy stored will become

$$E_{after} = \frac{1}{2} \times \epsilon_0 \times 12 \times 12$$

$= 6.35 \times 10^{-10} J$ 11)

d) The work done by the person pulling the plates apart.

We know, work done, W

$W = f \times d$ 12)

Where, f = force

d = displacement

We, know in parallel plate capacitor, the force between the plates is given by.

$f = Q \times \frac{\sigma}{2\epsilon_0}$ 13)

Where, Q = charge enclosed,

σ = surface charge density,

σ, surface charge density is given by ,

$$\sigma = \frac{Charge Q)}{Area}$$

From 12) and 13)

Work done, $w = \frac{Q \times Q}{2\epsilon_0 \times Area} \times d$

Given, Plate area 20 cm^2 = 0.002m^2

Separation between the plates changed to, 2.00 mm = 0.002 m by

pulling apart.

Substituting values, from 7), $Q_2 = 12\ \epsilon_0$

$$w = \frac{12 \times \epsilon_0 \times 12 \times \epsilon_0}{2 \times \epsilon_0 \times 0.002} \times 0.002$$

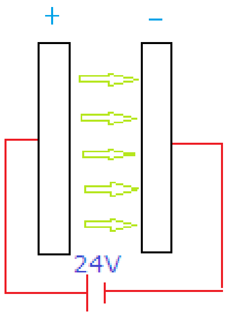
$$= \frac{12 \times \epsilon_0 \times 12 \times \epsilon_0}{2 \times \epsilon_0 \times 0.002} \times 0.002$$

$$= \frac{12 \times \epsilon_0 \times 6 \times 0.001}{0.002}$$

$$= 6.375 \times 10^{-10} J$$

Answer.45

Given



Capacitance =100 μF

Initial battery voltage used = 24V

Second voltage used = 12V

a) Charges on the capacitor before and after the reconnection.

Before reconnection, the battery used is 24V, hence

$C = 100\mu f$

$V = 24V$

We know charge present on a capacitor is given by

q = CV 1)

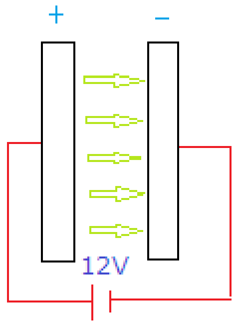
Where q = charge

C=capacitance

V=voltage

Substituting in 1)

$q = CV$



$= 100 \times 24$

$= 2400\mu c$

Similarly, after connection of 12V battery –

When C = 100μf

V = 12V

Using equation 1)

$q = CV$

$= 100 \times 12$

$= 1200\mu c$

b) Charge flown through the 12V battery.

C = 100, V = 12V

From 1),

$q = CV$

Substituting the values,

$q = CV$

$= 100 \times 12$

$= 1200\ \mu C$

c) Work is done by the battery, and its magnitude is as follows

We know

$$V = \frac{W}{q}$$

Where V = applied voltage across the capacitor

W = work done and

q = charge on the surface of the parallel plate capacitor

Which gives,

$W = v \times q$

$= 12 \times 1200$

$= 14400J$

$= 14.4MJ$ is the amount of work done on the battery.

d) Decrease in electrostatic field energy

Electrostatic field energy stored is given by –

$U = \frac{1}{2} C \times v^2$

where , c = capacitance

v= voltage across capacitor

Initially, electrostatic field energy stored is given by -

$U_i = \frac{1}{2} C \times v_1^2$

Final Electrostatic field energy

$U_f = \frac{1}{2} C \times v_2^2$

Decrease in Electrostatic field energy

$= U_i - U_f$

$= \frac{1}{2} C \times v_1^2 - \frac{1}{2} C \times v_2^2$

$= \frac{1}{2} C(v_1^2 - v_2^2)$

Substituting the values

Capacitance =100 μF

Initial battery voltage used = 24V

Second voltage used = 12V

$= \frac{1}{2} \times 100\,(24^2 - 12^2)$

$= 21600J$

e) Heat developed during the flow of charge after reconnection

After reconnection

C = 100μc, V = 12v

We know the energy stored, E in capacitor is given by

$$E = \frac{1}{2}\ CV^2$$

Where c is the capacitance and v is the applied voltage

Substituting values –

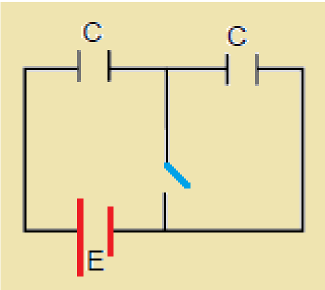
$$E = \frac{1}{2}\ 100\ \times 12^2$$

$= 7200J$

This is the amount of energy developed as heat when the charge flows through the capacitor.

Answer.46

Given circuit as shown below -



a) Charge flown through the battery when the switch S is closed. Since the switch was open for a long time, hence the charge flown must be due to the both.

When the switch is closed, the capacitor is in series, the equivalent capacitance is given by

$$c_{eqv} = \frac{c}{2}$$

Now, we know the relation between capacitance, charge q and voltage v given by ,

$$q = c \times v$$

$$\Rightarrow q = c_{eqv} \times v$$

$$= \frac{c}{2} \times v$$

b) Work done by the battery

We know, work done is given by

$$W = q \times v$$

Where q is charge stored and v is the applied voltage

We have, $q = c_{eqv} \times v$ and $c_{eqv} = \frac{c}{2}$

Substituting the values, we get,

$$w = \frac{c}{2} \times v \times v$$

$$= \frac{c}{2} \times v^2$$

c) Change in energy stored in the capacitors

Energy stored in capacitor is given by –

$$E = \frac{1}{2} C \times v^2$$

Initially, the energy stored in the capacitor is given by

$$\text{Ei} = \frac{1}{2} C \times v^2 \; 1)$$

After closing the switch, the capacitance changes to

$$c_{eqv} = \frac{c}{2}$$

Energy stored after closing the switch is given by -

$\text{Ef} = \frac{1}{2} \times \frac{c}{2} \times v^2$

$= \frac{C \times v^2}{4}$ 2)

From 1) and 2)

Change in energy stored in the capacitors

$= E_i - E_f)$

$= \frac{C \times v^2}{2} - \frac{C \times v^2}{4})$

$= \frac{C \times v^2}{4}$ 3)

d) Heat developed in the system

The net change in the stored energy is wasted as heat developed in the system,

Hence, heat developed in the systems is given as-

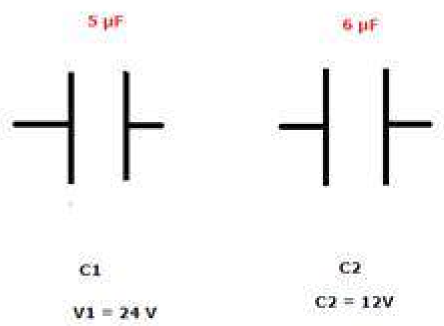
H=ΔE

Where, H is the heat developed and ΔE is the change in the stored energy in the capacitor

From 3)

$\Rightarrow H = \frac{C \times v^2}{4}$

Answer.47



Given:

C_1=5 μF

V_1=24 V

To calculate the charge present on the capacitor, we use the formula

$$q = c \times v$$

where,

c = capacitance of the capacitor and

v = voltage across the capacitor

For first capacitor, the stored charge q_1 is given by

$$q_1 = C_1 V_1$$

$$= 5 \times 24$$

$$= 120\ \mu C$$

Similarly for second capacitor, the stored charge q_2 is given by-

$$q_2 = C_2 V_2$$

Given, C_2=6 μF and V_2=12

$$q_2 = 6 \times 12$$

$$= 72\ \mu C$$

a) Energy stored in each capacitor-

Energy stored in a capacitor is given by

$$E = \frac{1}{2} \times c \times V^2$$

Where, v = applied voltage

C =capacitance

For capacitor $C_{1,}$ energy stored is given by

$$E_1 = \frac{1}{2} \times c_1 \times V^2$$

$$= \frac{1}{2} \times 5\,\mu \times 24^2$$

$$= 1440\,J$$

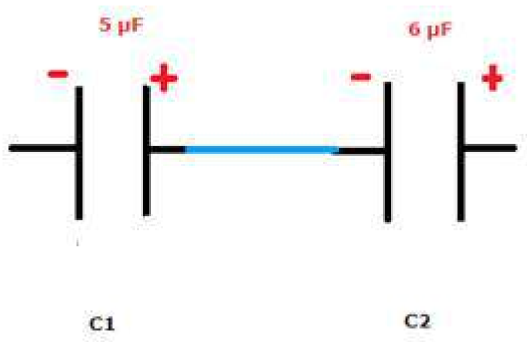
Similarly, for capacitor $C_{2,}$ energy stored is given by

$$E_2 = \frac{1}{2} \times c_2 \times V^2$$

$$= \frac{1}{2} \times 6\,\mu \times 12^2$$

$$= 432\,J$$

b) New charges on the capacitors when the positive plate of the first capacitor is now connected to the negative plate of the second nd vice versa



The capacitors are connected as shown on the right hand side.

The positive of first capacitor is connected to the negative of the second capacitor.

So charge flows from positive of first capacitor to the negative of the second capacitor.

Then, the net charge for connected capacitors becomes

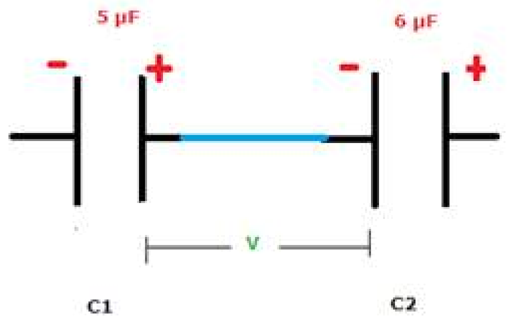
$$Q_{net} = q_1 - q_2$$

$$= 120\,C - 72\,C$$

$$= 48\,C$$

Now, let V be the common potential of the two capacitors

Since, charge is conserved, we know that electric charge can neither be created nor be destroyed, hence net charge is always conserved.



From the conservation of charge before and after connecting, we get, common voltage V

$$V = \frac{Q_{net}}{C_1 + C_2)}$$

$$= \frac{48}{5+6}$$

$= 4.36$ V

We know,

$q1 = C \times V$

where v = applied voltage and C is the capacitance

Using above relation, the new charges becomes-

$q1' = C1 \times V$

$= 5 \times 4.36$

$= 21.8\ \mu C$

$q2' = C2 \times V$

$= 6 \times 4.36$

$= 26.2\ \mu C$

c) Loss of electrostatic energy during the process

Energy stored in a capacitor is given by

$E = \frac{1}{2} \times c \times v^2$ 1)

Where, v = applied voltage

C =capacitance

For capacitor C_1, energy stored is given by

$$E_1 = \frac{1}{2} \times c_1 \times V^2$$

$= \frac{1}{2} \times 5\ \mu \times 4.36^{\ 2}$

Similarly, for capacitor $C_{2\ ,}$ energy stored is given by

$$E_2 = \frac{1}{2} \times c_2 \times V^2$$

$$= \frac{1}{2} \times 6\ \mu \times 4.36^{\ 2}$$

$\Delta E = E_1 + E2$

$= \left(\frac{1}{2} \times 5\ \mu \times 4.36^{\ 2}\right) + \frac{1}{2} \times 6\ \mu \times 4.36^{\ 2})$

$= 104.5 \times 10^{-6}\ J$ 2)

But given

for c_1, actual V_1 = 24V

and $c_{2,}$ actualV_2 = 12V

Using 1)

$$E_1 = \frac{1}{2} \times 5\ \mu \times 24^{\ 2} = 1.44 \times 10^{-3}\ \text{J}$$

And

$$E_2 = \frac{1}{2} \times 6\ \mu \times 12^{\ 2} = 0.433 \times 10^{-4}\ \text{J}$$

Now, change in energy, $\Delta E' = E_1 + E2$

$= 1.44 \times 10^{-3} + 0.433 \times 10^{-4}$

$= 1.873 \times 10^{-3}\ J$ 3)

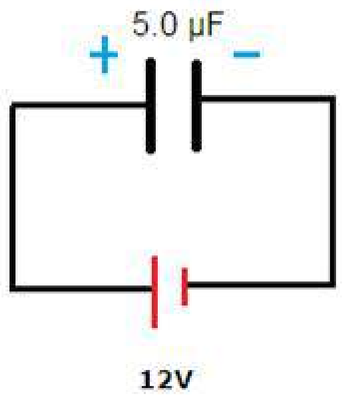
From 2) and3)

Loss of electrostatic energy = $1.873 \times 10^{-3} - 104.5 \times 10^{-6}$

$= 1.77 \times 10^{-3}\ J$

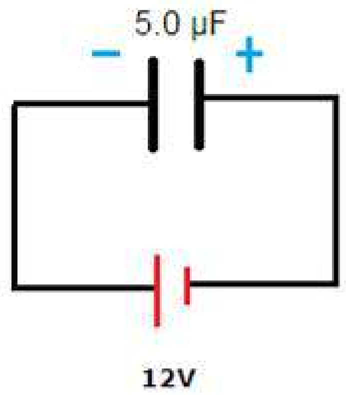
d) This energy, which is lost as electrostatic energy gets converted and dissipated from the capacitor in the from of heat energy.

Answer.48



Given

Initially 5.0 μF capacitor is charged to 12V as shown in fig.

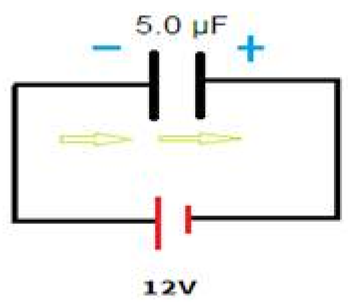


Next, the positive plate of this capacitor is now connected to the negative terminal of a 12V battery as shown in fig.

When the capacitor is connected to the battery of 12V with first plate to positive and second plate to negative, a positive charge Q = CV appears on one plate where, C is the capacitance and v is the voltage applied, and –Q charge appears on the other.

When the polarity is reversed, a charge –Q appears on the first plate and +Q on the second plate.

A total charge of 2Q accumulates on the negative plate.



Therefore, 2Q charge passes through the battery from the negative to the positive terminal.

The battery does a work-

$W = Q \times v$

Where,

Q = charge and v= applied voltage

Since, a total charge of 2Q accumulates on the negative plate

$\Rightarrow W = 2 \times Q \times v$

We know, Q = C×v

Where c= capacitance and v= applied voltage

$\Rightarrow W = 2 \times C \times v^2$

The energy stored in the capacitor is the same in the two cases

and the work done by battery dissipates as heat in the connecting wires.

Hence, the heat produced is -

$\Rightarrow W = 2 \times C \times v^2$

Given , C = 5.0 μF and voltage v = 12V

$\Rightarrow W = 2 \times 5 \times 10^{-6} \times 12)2$

$= 144 \times 10^{-5} J$

$= 1.44 mJ$

Answer.49

Given,

dielectric constant of slab = 4.0

Area of slab = 20 cm × 20 cm $= 400\, cm^2 = 0.04\, m^2$

Separation between slab, the thickness of the slab= 1.0 mm $= 0.001 m$

We know capacitance is given by

$$c = \frac{\epsilon_0\ Ak}{d}$$

Where

A= area of cross section

K = dielectric constant

d= separation between the slab and

ϵ_0 = permittivity of free space = 8.85×10^{-12}

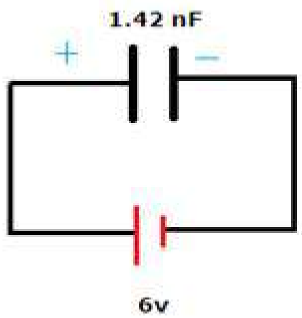
Hence, capacitance is given by-

$$c = \frac{8.85 \times 10^{-12}\ \times 0.04 \times 4}{0.001}$$

$= 1.42\ nF$

Answer.50

Given-



Capacitance of the capacitor, C = 1.42 nFDielectric constant, k = 4Voltage of the battery connected, V = 6 V

a)The charge supplied by the battery is given by-

We know from definition of capacitance, charge q on capacitor is given by -

q=CV

Where C= capacitance and V = applied voltage.

$\Rightarrow q\ = 1.42 \times 10^{-9} \times 6$

$= 8.52 \times 10^{-9}\ C$

$= 8.5\ nC$ 1)

b) The charge induced on the dielectric –

We know, the induced polarization charge on a dielectric material is given by-

$$q_p = q1 - \frac{1}{k})$$

Where,

q_p = polarized charge

q = charge on the capacitance

k = dielectric constant

$$q_p = 8.52 \times 10^{-9}\ \times 1 - \frac{1}{4})$$

$= 6.39 \times 10^{-9}$

$= 6.4\ nC$ 2)

c)The net charge appearing on one of the coated plates –

The net charge appearing will be the charge on the plat minus the charge on dielectric material

From 1) and 2) $Qnet\ = \ 8.5 - 6.4$

= 2.1 nC

Answer.51

Given-

Area of the plate= 100 cm^2 = 0.01m

Separation between the plates = 0.500 cm = 5 × 10^{-3} m

Thickness of the metal, *t* = 4 × 10^{-3} m

We know capacitance is given by

$$c = \frac{\epsilon_0 A}{d - t + \frac{t}{k})}$$

Where

A= area of cross section

K = dielectric constant

d= separation between the slab and

ϵ_0 = permittivity of free space = 8.85×10^{-12}

t = Thickness of the metal

Since, it's a metal, for metals k = infinite

Hence, capacitance is given by-

$$c = \frac{\epsilon_0 Ak}{d - t)}$$

$$= \frac{(8.85 \times 10^{-12}) \times 0.01}{5 \times 10^{-3} - 4 \times 10^{-3})}$$

$=88\ pF$

Here capacitance is a constant value, hence the capacitance

is independent of the position of the metal.

At any position, the net separation is $d - t$).

Answer.52

Given-

Initially, the charge on the capacitor = 50 μC

Now, let the dielectric constant of the material inserted in the gap be k.

When this dielectric material is inserted, 100 μC of extra charge flows through the battery

Therefore , the net charge on the capacitor becomes

50 + 100 = 150 μC

Now, we know capacitance of a material is given by –

$$c = \frac{q}{v}$$

Where q is charge on the capacitance and v is the applied voltage

Also

$$c = \frac{\in_0 A}{d}$$

Where A is the plate area and $\in_0$ is the permittivity of the free space.

Initially, without dielectric material inserted, capacitance is given by

$$C1 = \frac{q1}{V}$$

$\frac{= \in_0 A}{d}$ 1)

Similarly, with the dielectric material place, capacitance is given by

$$C_2 = \frac{q_2}{V}$$

$\frac{= \in_0 k A}{d}$ 2)

On dividing *1*) by *2*), we get

$$\frac{C_1}{C_2} = \frac{q_1}{q_2}$$

$$\Rightarrow \frac{150}{50} = k$$

$$\Rightarrow k = 3$$

Thus, the dielectric constant of the given material is 3.

Answer.53

Given-

Capacitance C=5 μF =5×10^{-6} F

Voltage, V=6v

Separation between plates, d=2 mm=2×10^{-3} m

a)The charge on the positive plate is calculated using

q = c×v

where, c is the capacitance

and v = voltage applied

Thus,

q=5 μF×6 V

=30 μC

b) The electric field between the plates of the capacitor is given by

$$E_f = \frac{v}{d}$$

Where, v is the applied voltage and d is the distance between the capacitor plates

$$\Rightarrow E_f = \frac{6}{2\times10-3}$$

=3×10^3 V/m

c)Given-

Distance between the plates of the capacitor, d =2×10^{-3} mDielectric constant of the dielectric material inserted, k = 5Thickness of the dielectric material inserted, t = 1×10^{-3} m

capacitance of the capacitor= 5 μF

Now,To calculate area of the plates of the capacitor,

$$c = \frac{\epsilon_0 \text{ Ak}}{d)}$$

Where,

A = area

k = dielectric constant of the material placed

d= separation between the plates

substituting the values,

$$\Rightarrow 5 \times 10^{-6} = \frac{8.85 \times 10^{-12} \times A}{2 \times 10^{-3}}$$

$$\Rightarrow 10^4 = 8.85 \times A$$

$$\Rightarrow A = \frac{10^4}{8.85} = 1129.433\ m^2$$

When the dielectric placed in it, the capacitance becomes

$$c_1 = \frac{\epsilon_0\ \text{A}}{d - t + \frac{t}{k})}$$

t=thickness of the material

substituting the values,

$$\Rightarrow C_1 = 8.85 \times 10^{-12} \times \frac{10^4}{8.852\ 2 \times 10^{-3} - 10^{-3} + \frac{10^{-3}}{5})}$$

$$\Rightarrow C1 = \frac{10^{-12} \times 10^4 \times 5}{6 \times 10^{-3}}$$

$= 8.33\ \mu F$ 1)

d)The charge stored in the capacitor initially is -

C=5×10^{-6} F

Also, V=6 V

Now, we know

$$Q = CV$$

where v is the applied voltage and c is the capacitance

$$\Rightarrow Q\ = 3 \times 10^{-5}\ F$$

$$\Rightarrow Q = 30\ \mu C$$

Now, when the dielectric slab is inserted ,charge on the capacitor, from 1)

$$C_1 = 8.3 \times 10^{-6}\ F$$

Charge, Q’

$$Q' = C_1 V$$

$= 8.3 \times 6 \times 10^{-6}$

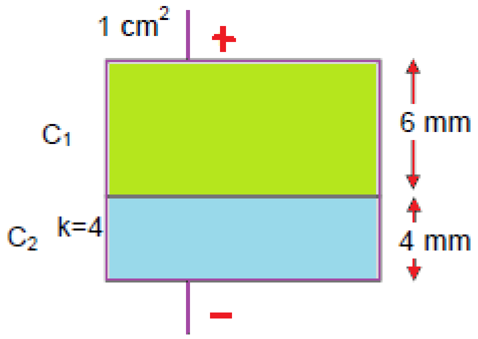
$\Rightarrow Q' = 50\ \mu C$

Now, charge flow is given by,

$Q' - Q = 20\ \mu C$

Answer.54

Given:



Area of the plate, A is 100 cm^2

Separation of the plate, d is 1 cm

Dielectric constant of the glass plate is 6

Thickness of the glass plate is 6.0 mm

Dielectric constant of an ebonite plate is 4.0

The given system of the capacitor will connected as shown in the fig.

The capacitors behave as two capacitors connected in series.

Let the capacitances be C_1 and C_2.

Now,

capacitance c

$$c = \frac{\epsilon_0\ Ak}{d)}$$

Where, A = area

k = dielectric constant of the material placed

d=separation between the plates

and ϵ_0 is permittivity of free space whose value is 8.85×10^{-12}

Now, first capacitor C_1

$c_1 = \frac{\epsilon_0 \mathrm{Ak_1}}{d)}$ 1)

and

$c_2 = \frac{\epsilon_0 \mathrm{Ak_2}}{d)}$ 2)

Hence, the net capacitance for a series connected capacitor is given by-

$C_{net} = \frac{C_1 \times C_2}{C_1 + C_2}$ 3)

from 1),2), and 3)

$$= \frac{\epsilon_0\ Ak1d1 \times \epsilon\ 0Ak2d2}{\epsilon\ 0Ak1d1 + \epsilon\ 0Ak2d2}$$

$$= \frac{\epsilon_0\ A \times d_1 \times d_2 k_1 + k_2)}{k_1 d_1 + k_2 d_2}$$

$$= \frac{8.85 \times 10^{-12}) \times 10^{-2}) \times 24}{6 \times 4 \times 10^{-3} + 4 \times 6 \times 10^{-3})}$$

$$= 4.425 \times 10^{-11} C$$

$$= 44.25\ pF$$

Answer.55

Given,

Area, A = 400cm^2 = 400 × 10^{-4}m^2

distance between plates d = 1cm = 1× 10^{-3}m

voltage V = 100V

Thickness t = 0.5 = 5 × 10^{-4}m

dielectric constant, k = 5

The capacitance of the capacitor without the dielectric slab is given by $C_1 = \frac{\epsilon 0 A}{d}$

Where,

A= Plate Area

d= separation between the plates,

ϵ_0 = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

$$\Rightarrow C = \frac{8.85 \times 10^{-12}) \times 400 \times 10^{-4})}{1.0 \times 10^{-3})}$$

$$\Rightarrow C = 3.54 \times 10^{-10}\ F$$

When the dielectric slab is inserted, the capacitance becomes

$$c' = \frac{\epsilon_0\ A}{d - t + \frac{t}{k})}$$

where, t is the thickness of the slab

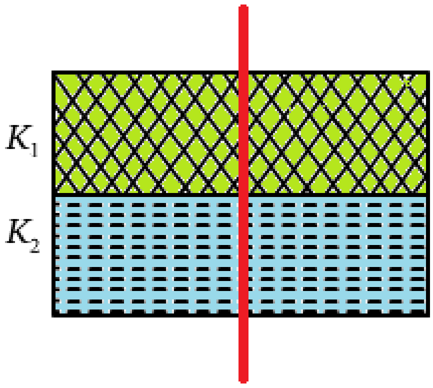
$$C' = \frac{8.85 \times 10^{-12}) \times 400 \times 10^{-4})}{1 \times 10^{-3} - 0.5 \times 10^{-3} + \frac{0.5 \times 10^{-3}}{5}}$$

$$\Rightarrow C' = \frac{5 \times 8.85 \times 10^{-12}) \times 400 \times 10^{-4})}{6 \times 0.5 \times 10^{-3})}$$

$$\Rightarrow C' = 5.9 \times 10^{-10}\ F$$

Answer.56

a)The capacitors are as shown in the fig



Here, the two parts of the capacitor

are in series

Capacitances C_1 and C_2 with dielectric constants as K_1 and K_2

$$C_1 = \frac{\epsilon_0 \times A \times k}{d}$$

Where,

A= Plate Area

d= separation between the plates,

ϵ_0 = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

k = dielectric strengthof the material

Here,

$C_1 = \frac{K_1 \epsilon_0 A}{d_1}$ and $C_2 = \frac{K_2 \epsilon_0 A}{d_2}$

Since, the distance between the plates is divided into two parts,

hence, separation between the plates becomes = $\frac{d}{2}$

$\Rightarrow C_1 = \frac{K1 \epsilon 0 A}{\frac{d}{2}}$ and

$$C_2 = \frac{K_2 \epsilon_0 A}{\frac{d}{2}}$$

Because they are in series, the equivalent capacitance is

calculated as:

$$C_{eqv} = \frac{C_1 \times C_2}{C_1 + C_2}$$

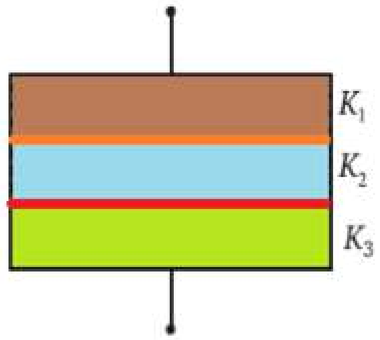
Substituting the values,

$$C_{eqv} = \frac{2K_1\ \epsilon_0\ \ A\,d \times 2K_2\ \epsilon_0\ \ Ad}{2K_1\,\epsilon_0\ \ Ad + 2K2\ \epsilon_0\ \ Ad}$$

$$= \frac{2K_1\,K_2\ \epsilon_0\ \ Ad}{K_1 + K_2}$$

Here, the capacitor has three parts. These can be taken in series.

b)



Now, in this case, there are three capacitors connected as shown in fig.

These capacitors are connected in series.

capacitance c is given by –

$$c = \frac{\in 0\,A\ \times k}{d}$$

Where,

A= Plate Area

d= separation between the plates,

ϵ_0 = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

k = dielectric strengthof the material

Capacitors are as follows –

Since, the entire distance is separated into three parts,

$$C_1 = \frac{K_1\ \epsilon_0\ \ A}{\frac{d}{3}}$$

$$= \frac{3\,K_1\,\epsilon_0\,A}{d}$$

Similarly, the other two capacitors

$$C_2 = \frac{3\,K_2\,\epsilon_0\,A}{d}$$

$$C_3 = \frac{3\,K_3\,\epsilon\,0\,A}{d}$$

These three capacitors are connected in series

Thus, the net capacitance is calculated as-

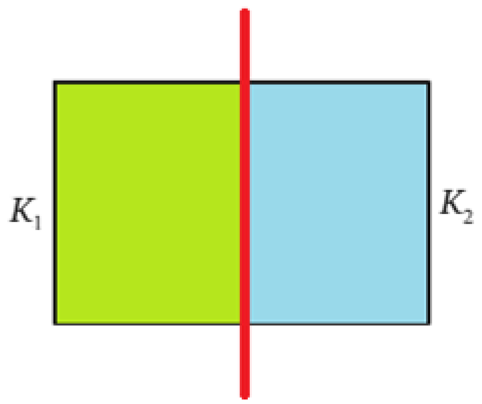
$$\frac{1}{C_{net}} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1 \times C_2 \times C_2}$$

$$\Rightarrow C_{net}$$

$$= \frac{3K_1\,\epsilon_0\,Ad \times 3K_2\,\epsilon_0\,Ad \times 3K_3\,\epsilon_0\,Ad}{3K_1\,\epsilon_0\,Ad \times 3K_2\,\epsilon_0\,Ad + 3K_2\,\epsilon_0\,Ad \times 3K_3\,\epsilon_0\,Ad + 3K_3\,\epsilon_0\,Ad \times 3K_1\,\epsilon_0\,Ad}$$

$$= \frac{3\,\epsilon_0\,K_1K_2K_3\,d}{K_1K_2 + K_2K_3 + K_3K_1)}$$

c) Here, the capacitors are connected as shown in fig.



We know, capacitance c is given by-

$$c = \frac{\epsilon\,0\,A \times k}{d}$$

Where,

A= Plate Area

d= separation between the plates,

ϵ_0 = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

k = dielectric strengthof the material

Capacitors C_1 andC_2 is given by-

$$C_1 = \frac{K_1\ \epsilon_0 \times A}{\frac{d}{2}}$$

$$= \frac{K_1\ \epsilon_0\ \ A\ \times 2}{d}$$

$$C_2 = \frac{K_2 \times \epsilon \times\ A \times 2}{d}$$

These two capacitors are connected in parallel, net capacitance

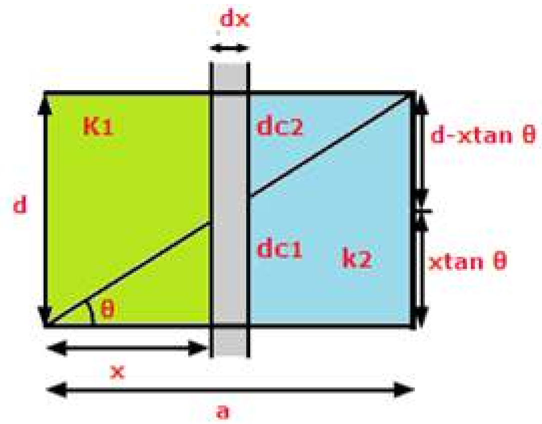
becomes

$$C_{net} = C_1 + C_2$$

$$= \frac{\epsilon_0 \times\ A \times 2K_1 + K_2)}{d}$$

Answer.57

These two capacitors are connected in series.



To find out the capacitance, let us consider a small capacitor of

differential width *dx* at a distance *x* from

the left end of the capacitor.

The two capacitive elements of dielectric constants K_1 and K_2 are with plate

separations as -

$x \tan \theta)$ and $d - x \tan \theta)$ in series,

respectively as seen from fig.

Also, differential plate areas of the capacitors are *adx*.

We know, capacitance c is given by-

$$c = \frac{\in 0 A \times k}{d}$$

Where,

A= Plate Area

d= separation between the plates,

$\in_0$ = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

k = dielectric strengthof the material

Then, looking into the fig, the capacitances of the capacitive elements of the elemental capacitors are given by –

$dC_1 = \frac{\in_0 K_2 (adx)}{x \tan \theta}$ and

$$dC_2 = \frac{\in_0 K_1 adx)}{d - x \tan \theta}$$

We know that equivalent capacitance of capacitors connected in

series is given by the expression –

$$\frac{1}{dC_{eqv}} = \frac{1}{dC_1} + \frac{1}{dC_1}$$

$$= \frac{x \tan \theta}{\in_0 K_2 adx)} + \frac{d - x \tan \theta}{\in_0 K_1 adx)}$$

$$\Rightarrow dC_{eqv} = \frac{\in_0 K_1 K_2 (adx)}{K_1 x \tan \theta + K_2 d - x \tan \theta)}$$

Now, integrating both sides to get the actual capacitance,

$$C = \int_0^a adC_{eqv}$$

$$= \int_0^a \frac{\epsilon_0 K_1 K_2 \times a\, dx}{K_1 x \tan\theta + K_2 d - x \tan\theta)C}$$

$$= \epsilon_0 K_1 K_2 a \int_0^a \frac{dx}{K_2 d + x \tan\theta K_1 - K_2)}$$

$$\Rightarrow C = \frac{\epsilon_0 K_1 K_2 \times a}{\tan\theta K_1 - K_2)} \left(\log_e [K2d + x \tan\theta K1 - K2)]_0^a\right)$$

$$\Rightarrow C = \frac{\epsilon_0 K_1 K_2 \times a}{\tan\theta K_1 - K_2)} \log_e [K_2 d + x \tan\theta (K_1 - K_2) - \log_e K_2 d]$$

Looking back into the fig.

$$\tan\theta = \frac{d}{a}$$

Substituting in the expression for capacitance C,

Now,

$$\Rightarrow C = \frac{\epsilon_0 K_1 K_2 a}{K_1 - K_2) \times \frac{d}{a}} \, loge[K_2 d + a \times daK_1 - K_2)] - logeK_2 d$$

$$\Rightarrow C = \frac{\epsilon_0 K_1 K_2 a}{(K1 - K2)\frac{d}{a}} [loge\, K_1 d - logeK_2 d]$$

$$\Rightarrow C = \frac{\epsilon_0 K_1 K_2 a^2}{dK_1 - K_2)} \log_e \frac{K_1}{K_2}$$

Answer.58

Initially the switch is closed and the capacitors are fully charged.

When the switch is closed, both capacitors are in parallel as shown in fig, Hence the total energy stored by the capacitor when switch is closed is –

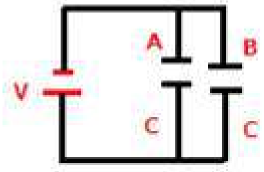
$$E_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$$

$$= CV^2$$

where C is the capacitance and V is the applied voltage.

When the switch is opened and dielectric is induced, the capacitance is

$$c' = k \times c$$



Given dielectric constant as 3

$$c' = 3 \times c$$

The total energy stored by the capacitor when switch is closed is –

For capacitor at AB

$$E_{AB} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}3C \times V^2$$

And E_{BC} is –

$$E_{BC} = \frac{1}{2} \times \frac{C}{3}V^2$$

$$= \frac{1}{6}C \times V^2$$

Total energyE_f)

$$E_{AB} + E_{BC}$$

$$= \frac{1}{2}3C \times V^2 + \frac{1}{6}C \times V^2$$

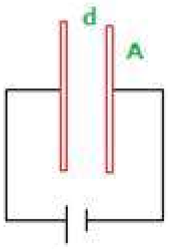
$$= \frac{10C\,V^2}{6}$$

Now, the ratio of the initial total energy stored in the capacitors to the final total energy stored –

$$\frac{E_i}{Ef} = \frac{CV^2}{\frac{10C\,V^2}{6}}$$

=3/5

Answer.59



Before inserting slab-

We know, capacitance c is given by-

$$c = \frac{\in 0\, A}{d}$$

Where,

A= Plate Area

d= separation between the plates,

$\in_0$ = Permittivity of free space = $8.854 \times 10^{-12}\ m^{-3}\ kg^{-1}\ s^4\ A^2$

Energy stored by the capacitor

$$E_i = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times \frac{\in 0\, A}{d} \times V^2$$

where C is the capacitance and V is the applied voltage.

After inserting slab capacitance c is given by-

$$c = \frac{\in 0\, A}{d} \times k$$

Where,

A= Plate Area

d= separation between the plates,

ϵ_0 = Permittivity of free space = 8.854×10^{-12} m^{-3} kg^{-1} s^4 A^2

k = dielectric strengthof the material

Also, the final voltage becomes $\frac{v}{k}$

Energy stored by the capacitor–

$$E_f = \frac{1}{2} \times \frac{\epsilon 0 A}{d} \times k \times \frac{V^2}{k^2}$$

where ,

C is the capacitance and V is the applied voltage, k is the dielectric constant of the material

The work done on the system in the process of inserting the slab

= change in energy

$= E_f - E_i$

$$= \frac{1}{2} \times \frac{\epsilon_0 \ A}{d} \times V^2 - \frac{1}{2} \times \frac{\epsilon_0 \ A}{d} \times k \times \frac{V^2}{k^2}$$

$$= \frac{1}{2} \times \frac{\epsilon_0 \ A}{d} \times V^2 \ \frac{1}{k} - \ 1)$$

Answer.60

Given –

Capacitance, C = 100 μF

Potential difference, V = 50V.

a) We know the magnitude of the charge on each plate is given by

Charge stored on the capacitor, q = c × v

where c is the capacitance and v is the potential difference

⇒ q = 100 μF×50

=5 mC

b)Now, the charging battery is disconnected and a dielectric of dielectric constant 2.5 is inserted.

The new potential difference between the plates will be –

$$\text{Potential difference} = \frac{Initial\ potential}{dielectric}$$

$$= \frac{5.0}{2.5}$$

$$= 20V$$

c)

Now, the charge on the capacitance can be calculated as:

Charge, q= Capacitance, C × Potential difference, V

Putting the value in the above formula, we get

Q= 20 × 100 × 10^{-6} =2 mC

d)The charge induced at a surface of the dielectric slab –

$$q_i = q1 - \frac{1}{k})$$

Where, q_i is the induced charge, q is the initial charge and k is the dielectric constant of the material inserted.

$$\Rightarrow q_i = 5mC\ 1 - \frac{1}{2.5})$$

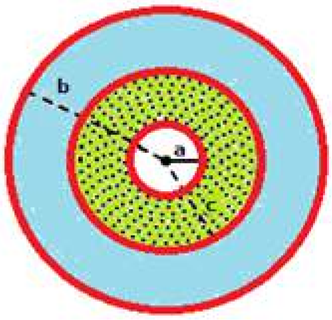
$$= 3mc$$

Answer.61

Lets re-draw the diagram-

We know, capacitance for a spherical capacitance c is given by-

$$C = \frac{4\,\pi\,k\,\in}{\frac{1}{r_i} - \frac{1}{r_o})}$$



Where,

C: Capacitance

r_i: inner radius

r_o: outer radius

k: relative permittivity

∈: permittivity of space

Capacitance between c and a-

$$c_{ac} = \frac{4\,\pi\,k\,\epsilon_0 \times\ ac))}{c - a))} \times k$$

Similarly, between b and c

$$c_{bc} = \frac{4\,\pi\,k\,\epsilon_0 \times bc)}{b - c)}$$

From fig, we can see that the two capacitors are connected in series, hence the net capacitance is given by-

$$\frac{1}{C_{net}} = \frac{1}{c_{ac}} + \frac{1}{c_{bc}}$$

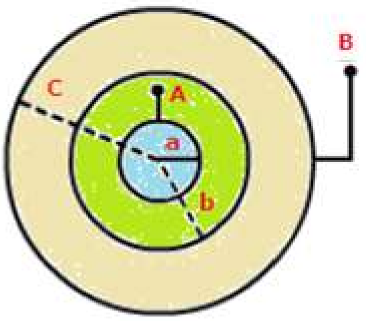
$$= \frac{1}{\frac{4\,\pi\, k\, \epsilon_0 \times\ ac))}{c - a)} \times k} + \frac{1}{\frac{4\,\pi\, k\, \epsilon_0 \times bc)}{b - c)}}$$

$$= \frac{b(c - a) + k \times ab - c)}{k\ \times 4\,\pi\, k\, \epsilon_0 \times\ abc}$$

$$\Rightarrow C_{net} = \frac{k\ \times 4\,\pi\, k\, \epsilon_0 \times\ abc}{b(c - a) + k \times ab - c)}$$

Answer.62

These three metallic hollow spheres form two spherical capacitors, which are connected in series.



Solving them individually, for 1) and 2)

For a spherical capacitor formed by two spheres of radii $r_o > r_i$ is given by

$$C = \frac{4\,\pi\, k\, \in}{\frac{1}{ri} - \frac{1}{ro})}$$

Where,

C: Capacitance

r_i: inner radius

r_o: outer radius

k: relative permittivity or dielectric constant

∈: permittivity of space

Similarly,

$$C_{AB} = \frac{4\pi \in_0 \times BA)}{B - A)}$$

And

$$C_{BC} = \frac{4\pi \in_0 \times CB)}{C - B)}$$

Type equation here.

Now, the capacitors are connected in series, net capacitance for series connected capacitors is given by –

$$\frac{1}{C_{net}} = \frac{1}{C_{AB}} + \frac{1}{C_{BC}}$$

$$= \frac{1}{\frac{4\pi \in_0 \times CB)}{C - B)}} + \frac{1}{\frac{4\pi \in_0 \times BA)}{B - A)}}$$

$$= \frac{\frac{(4\pi \in_0)^2 AB^2 C}{C - A) \times B - A)}}{4\pi \in_0 \frac{AB(C - B) + BC B - A)}{C - B) \times B - A)}}$$

$$= 4\pi \in_0 \frac{ac}{c - a)}$$

Answer.63

The two capacitors are connected in series, hence the net capacitance is given by

$$\frac{1}{C_{net}} = \frac{1}{C_{AB}} + \frac{1}{C_{BC}}$$

For a spherical capacitor formed by two spheres of radii $r_o > r_i$ is given by

$$C = \frac{4 \pi k \in}{\frac{1}{ri} - \frac{1}{ro})}$$

Where,

C: Capacitance

r_i: inner radius

r_o: outer radius

k: relative permittivity or dielectric constant

∈: permittivity of space

Similarly,

$$C_{AB} = \frac{4\pi \in_0 \times BA)}{B \ - \ A)} \times k$$

And

$$C_{BC} = \frac{4\pi \in_0 \times CB)}{C \ - \ B)}$$

$$\frac{1}{C_{net}} = \frac{1}{c_{AB}} + \frac{1}{c_{BC}}$$

$$= \frac{1}{\frac{4\pi \in_0 \times BA)}{B \ - \ A)} \times k} + \frac{1}{\frac{4\pi \in_0 \times CB)}{C \ - \ B)}}$$

$$= \frac{4\pi \in_0 \text{k ABC}}{kA(C \ - \ B) \ + C \, B \ - \ A)}$$

Answer.64

Given,

Q = 12μc = $12 \times 10^{-6} \ c$

V = 1200V

dielectric strength, b = 3 x 10^{6}V/m

Plate Area can be calculated as follows –

The separation between the plates of the capacitor is given by-

$d = \frac{V}{b}$

where v is the applied voltage and b is the dielectric strength

$\frac{1200}{3 \times 10^6}$

$= 4 \times 10^{-4}$ m

Now, the capacitance of the capacitor is given by

$C = \frac{Q}{V}$

where Q is the charge stored and V is the voltage applied.

$= \frac{12 \times 10^{-6}}{1200} 1200$

$= 10^{-8} F$

We know, capacitance c is given by-

$c = \frac{\in 0 A}{d}$

Where,

A= Plate Area

d= separation between the plates,

$\in_0$ = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

Thus, the area of the plates is given by –

$c = \frac{\in 0 A}{d}$

$10^{-8} = \frac{8.854 \times 10^{-12} A}{4 \times 10^{-4}}$

⇒ A= $10^{-8} \times 10^{-8} \frac{10^{-8} \times 4 \times 10^{-4}}{8.854 \times 10^{-12}}$ m^2

=0.45 m^2

Answer.65

Given -

Area of the plates of the capacitor, A = 100 cm^2 = 10^{-2} m^2

Separation between the plates, d = 1 cm = 10^{-2} m

Emf of battery, V = 24 V

We know, capacitance c is given by-

$$c = \frac{\in 0\ A}{d}$$

Where,

A= Plate Area

d= separation between the plates,

$\in_0$ = Permittivity of free space = 8.854 × 10^{-12} m^{-3} kg^{-1} s^4 A^2

Therefore,

Capacitance,

$$c = \frac{\in 0\ A}{d} =$$

$$= \frac{(8.85 \times 10^{-12}) \times (10^{-2})}{10^{-2}}$$

$$= 8.85 \times 10^{-12} F$$

Energy stored by the capacitor

$$\mathrm{E} = \frac{1}{2}\mathrm{CV}^2$$

$$= \frac{1}{2} \times 8.85 \times 10^{-12}) \times (24)^2$$

$$= 2548.8 \times 10^{-12} J$$

Now, force of attraction between the plates,

$$F = Ed$$

where

E = energy stored and d is the separation between the plates

⇒

$f = 2548.8 \times 10^{-12} \times 10^{-2}$

$= 2548.8 \times 10^{-10}\,N$

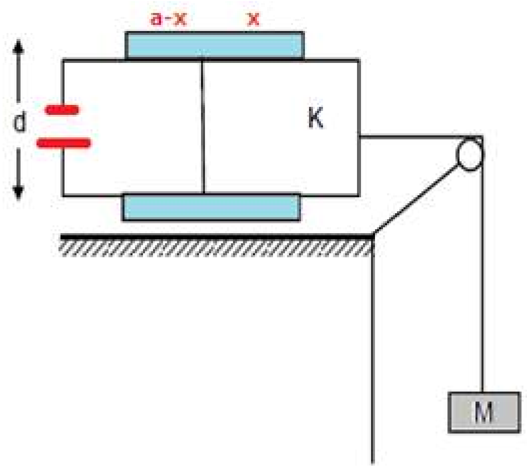
Answer.66

Let the battery connected to the capacitor be of potential V.

Let the length of the part of the slab inside the capacitor be x.

b – Width of plates

The capacitor can be considered to be two capacitors which are connected in parallel.



The capacitances of the two capacitors in parallel is given by –

$$C_1 = \frac{k \in_0 bx}{d}$$

$$C_2 = \frac{\in_0 ba - x)}{d}$$

C_1 is the part of the capacitor having the dielectric inserted in it and C_2 is the capacitance of the part of the capacitor without dielectric.

As, C_1 and C_2 are in parallel therefore, the net capacitance is given by

$$C_{net} = C_1 + C_2$$

$$= \frac{k \in_0 bx}{d} + \frac{\in_0 ba - x)}{d}$$

$$= \frac{k \in_0 b + \in_0 ba - x)}{d}$$

This dielectric slab is attracted by the electric field of the capacitor and applies a force

The direction of force is in left direction.

Let assume that electric force of magnitude F pulls the slab toward left direction.

Let there be an differential displacement dx towards the left direction by the force F.

The work done by the force

$$= F.dx$$

Let V_1 and V_2 be the potential of the battery connected to the left capacitor and that of the battery connected with the right capacitor

With increase in the displacement of slab, the capacitance will increase, hence the energy stored in the capacitor will also increase.

In order to maintain constant voltage, the battery will supply extra charge, and gets damage .

Therefore the battery will do work.

Now,

Work done by the battery

= Energy change of capacitor + work done by the force F on the capacitor

$$dW_B = dU + dW_F \quad 1)$$

Let's take the differential charge dq is supplied by the battery, and the change in the capacitor be dC

We know that energy in capacitor dW_B

$$dW_B = (dq).V$$

we know q = cv

$$\Rightarrow dW_B = dC).V^2 \quad 2)$$

And force F is given by,

$$dU = \frac{1}{2}dC).V^2$$

$$\Rightarrow dC).V^2 = \frac{1}{2}dC).V^2 + F.dx$$

From 1) and 2)

$$\Rightarrow \frac{1}{2}dC).V^2 = F.dx$$

$$\Rightarrow F = \frac{1}{2}\frac{dC}{dx}V_1^2$$

$$\Rightarrow F = \frac{1}{2}\frac{d}{dx}\{\frac{\epsilon_0 b}{d}[l_1 + x(k-1)]\,V_1^2\}$$

$$\Rightarrow F = \frac{\epsilon_0 bV_1^2 k - 1)}{2d}$$

In order to keep the dielectric slab in equilibrium, the electrostatic force acting on it must be balanced by the weight of the block attached.

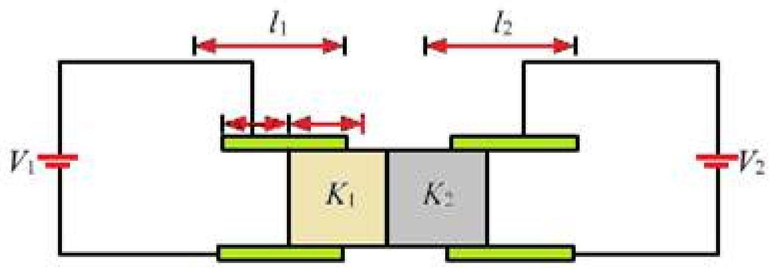
Therefore,

$$\frac{\epsilon_0 bV_1^2(K_1 - 1)}{2d} = Mg$$

$$\Rightarrow M = \frac{\epsilon_0 bV_1^2 K_1 - 1)}{2d \times g}$$

Answer.67

Let $V_{1,}$ V_2 be the potential of the battery connected to the left capacitor and that of the battery connected to the right capacitor



Considering the left capacitor -

Let the length of the part of the slab inside the capacitor be x.

The left capacitor can be considered to be two capacitors in parallel.

Let the battery connected to the capacitor be of potential V.

Let the length of the part of the slab inside the capacitor be x.

b – Width of plates

The capacitances of the two capacitors in parallel is given by –

$$C_1 = \frac{k \in_0 bx}{d}$$

$$C_2 = \frac{\in_0 ba - x)}{d}$$

C_1 is the part of the capacitor having the dielectric inserted in it and C_2 is the capacitance of the part of the capacitor without dielectric.

As, C_1 and C_2 are in parallel therefore, the net capacitance is given by

$$C_{net} = C_1 + C_2$$

$$= \frac{k_1 \in_0 bx}{d} + \frac{\in_0 bl_1 - x)}{d}$$

$$= \frac{k_1 \in_0 b + \in_0 bl_1 - x)}{d}$$

$$= \frac{\in_0 b}{d} [l_1 + x(K_1 - 1)]$$

Therefore, the potential energy stored in the left capacitor will be

$$U = \frac{1}{2} C V_1^2$$

$$\Rightarrow U = \frac{\in_0 b V_1^2}{2d} [l_1 + x(K1 - 1)] \text{ 1)}$$

This dielectric slab is attracted by the electric field of the capacitor and applies a force.

Let assume that electric force of magnitude F pulls the slab toward left direction.

Let there be an differential displacement dx towards the left direction by the force F.

The work done by the force

$$= F.dx$$

Let V_1 and V_2 be the potential of the battery connected to the left capacitor and that of the battery connected with the right capacitor

With increase in the displacement of slab, the capacitance will increase, hence the energy stored in the capacitor will also increase.

Let us consider a small displacement dx of the slab towards the inward direction.

In order to maintain constant voltage, the battery will supply extra charge, and gets damage .

Therefore the battery will do work.

Now,

Work done by the battery

= Energy change of capacitor + work done by the force F on the capacitor

$$dW_B = dU + dW_F \quad 1)$$

Let's take the differential charge dq is supplied by the battery, and the change in the capacitor be dC

We know that energy in capacitor dW_B

$$dW_B = (dq).V$$

we know q = cv

$$\Rightarrow dW_B = dC).V^2 \quad 2)$$

And force F is given by,

$$dU = \frac{1}{2}dC).V^2$$

$$\Rightarrow dC).V^2 = \frac{1}{2}dC).V^2 + F.dx$$

From 1) and 2)

$$\Rightarrow \frac{1}{2}dC).V^2 = F.dx$$

$$\Rightarrow F = \frac{1}{2}\frac{dC}{dx}V_1^2$$

$$\Rightarrow F = \frac{1}{2}\frac{d}{dx}\{\frac{\epsilon_0 \; b}{d}[l_1 + x(k_1 - 1)]\; V_1^2\}$$

$$\Rightarrow F = \frac{\epsilon_0 \; bV_1^2 k_1 - 1)}{2d}$$

Solving for voltages V_1 and V_2 -

$$\Rightarrow V_1^2 = \frac{F \times 2d}{\epsilon_0 \; bK_1 - 1)}$$

$$V1 = \sqrt{\frac{F \times 2d}{\epsilon_0 \; bK_1 - 1)}}$$

Similarly, for the right side the voltage of the battery is given by-

$$V_2 = \sqrt{\frac{F \times 2d}{\epsilon_0 \; bK_2 - 1)}}$$

Now, the ratio of the voltages is given by-

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{F \times 2d}{\epsilon_0 \; bK_1 - 1)}}}{\sqrt{\frac{F \times 2d}{\epsilon_0 \; bK_2 - 1)}}}$$

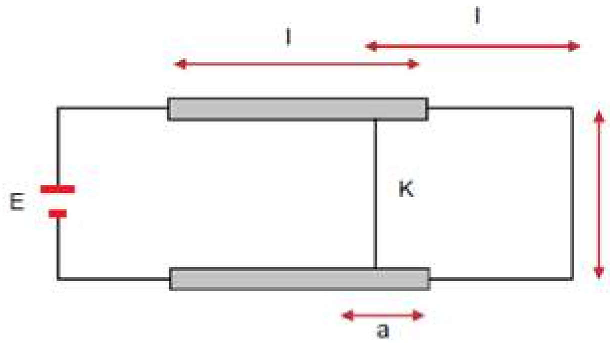
$$\Rightarrow \frac{V_1}{V_2} = \frac{K_1 - 1)}{K_2 - 1)}$$

Thus, the ratio of the emfs of the left battery to the right battery is given by -

$$\frac{V_1}{V_2} = \frac{K_1 - 1)}{K_2 - 1)}$$

Answer.68

Given



area of the plates of the capacitors = *A*.

a = length of the dielecric slab is inside the capacitor.

Therefore, the area of the plate covered with dielectric is =

$$\frac{A}{l}a$$

The capacitance of the portion with dielectric is given by

$$C_1 = \frac{K \in_0 \ A \ \times a}{ld}$$

The capacitance of the portion without dielectric is given by

$$C_2 = \frac{K \in_0 \ A\, l - a)}{ld}$$

The two parts can be considered to be in parallel.

Therefore, the net capacitance is given by-

$$C = C_1 + C_2$$

$$\Rightarrow C = \frac{K \in_0 \ A \ \times a}{ld} + \frac{K \in_0 \ A\, l - a)}{ld}$$

$$\Rightarrow C = \frac{\in 0A}{ld}[l + a(K - 1)]$$

Let us consider a small displacement *da* of the slab towards the inward direction.

With increase in the displacement of slab, the capacitance will increase, hence the energy stored in the capacitor will also increase.

In order to maintain constant voltage, the battery will supply extra charge, and gets damage .

Therefore the battery will do work.

Now,

Work done by the battery

= Energy change of capacitor + work done by the force F on the capacitor

$dW_B = dU + dW_F$ 1)

Let's take the differential charge dq is supplied by the battery, and the change in the capacitor be dC

We know that energy in capacitor dW_B

$dW_B = (dq).V$

we know q = cv

$\Rightarrow dW_B = dC).V^2$ 2)

And force F is given by,

$dU = \frac{1}{2}dC).V^2$

$\Rightarrow dC).V^2 = \frac{1}{2}dC).V^2 + F.da$

From 1) and 2)

$\Rightarrow \frac{1}{2}dC).V^2 = F.da$

$\Rightarrow F = \frac{1}{2}\frac{dC}{da}V^2$

$\Rightarrow F = \frac{1}{2}\frac{d}{da}\{\frac{\epsilon_0 A}{ld}1 + a(k-1))] V_1^2\}$

$\Rightarrow F = \frac{1}{2}\frac{\epsilon_0 Ak-1)}{l \times d}$ 1)

We know

$Force, F = m \times a$ 2)

where m is the mass of the object

a is the acceleration

From 1) and2)-

the acceleration of the dielectric a_0 is given by =

$$\frac{1}{2}\frac{\epsilon_0 \text{ Ak} - 1)}{\text{l} \times \text{d} \times \text{m}}$$

where, m is the mass.

As, the force is in inward direction, it tends to make the dielectric to completely fill the space inside the capacitors.

As, the dielectric tends to completely fills the space inside the capacitor, at this instant its velocity is not zero.

After that the dielectric slab tends to move outside the capacitor.

As the slab tends to move out, the direction of force reverses.

Hence, the dielectric slab will maintain periodic motion.

Now, the time required for moving a distance *l*-a) can be-

$$(l - a) = \frac{1}{2}a_0 t^2$$

$$t \;=\sqrt{\frac{2l - a)}{a_0 t}}$$

$$= \sqrt{2l - a) \times \frac{2ldm}{\epsilon_0 \; AV^2 K - 1)}}$$

$$= \sqrt{\frac{4ml-a)ld}{\epsilon_0 AV^2 K-1)}}$$

For completing cycle, the time taken will be four times the time taken for covering distance *l*-a)

hence,

T=4t

$$=4\times\sqrt{\frac{4ml-a)ld}{\epsilon_0 AV^2 K-1)}}$$

$$=8\sqrt{\frac{ml-a)ld}{\epsilon_0 AV^2 K-1)}}$$