

Trigonometric Functions

Short Answer Type Questions

Q. 1 Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$.

Thinking Process

Here, use the formulae i.e., $\sec^2 A - \tan^2 A = 1$ and $a^2 - b^2 = (a+b)(a-b)$ to solve the above problem.

Sol.

$$\begin{aligned}
 \text{LHS} &= \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{(\tan A - \sec A + 1)} \quad [\because \sec^2 A - \tan^2 A = 1] \\
 &= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{(1 - \sec A + \tan A)} \\
 &= \frac{(\sec A + \tan A)(1 - \sec A + \tan A)}{1 - \sec A + \tan A} \\
 &= \sec A + \tan A = \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
 &= \frac{1 + \sin A}{\cos A} = \text{RHS}
 \end{aligned}$$

Hence proved.

Q. 2 If $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$, then prove that $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is also equal to y .

Sol. Given that, $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$

$$\begin{aligned}
 \text{Now, } \frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha} &= \frac{(1 - \cos\alpha + \sin\alpha)}{(1 + \sin\alpha)} \cdot \frac{(1 + \cos\alpha + \sin\alpha)}{(1 + \cos\alpha + \sin\alpha)} \\
 &= \frac{\{(1 + \sin\alpha) - \cos\alpha\}}{(1 + \sin\alpha)} \cdot \frac{\{(1 + \sin\alpha) + \cos\alpha\}}{(1 + \cos\alpha + \sin\alpha)} \\
 &= \frac{(1 + \sin\alpha)^2 - \cos^2\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)} \\
 &= \frac{(1 + \sin^2\alpha + 2\sin\alpha) - \cos^2\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1 + \sin^2 \alpha + 2 \sin \alpha - 1 + \sin^2 \alpha}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)} \\
 &= \frac{2 \sin^2 \alpha + 2 \sin \alpha}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)} \\
 &= \frac{2 \sin \alpha(1 + \sin \alpha)}{(1 + \sin \alpha)(1 + \sin \alpha + \cos \alpha)} \\
 &= \frac{2 \sin \alpha}{1 + \sin \alpha + \cos \alpha} = y
 \end{aligned}$$

Hence proved.

Q. 3 If $m \sin \theta = n \sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha) \cot \alpha = \frac{m + n}{m - n}$.

Sol. Given that, $m \sin \theta = n \sin(\theta + 2\alpha)$
 $\therefore \frac{\sin(\theta + 2\alpha)}{\sin \theta} = \frac{m}{n}$

Using componendo and dividendo, we get

$$\begin{aligned}
 &\frac{\sin(\theta + 2\alpha) + \sin \theta}{\sin(\theta + 2\alpha) - \sin \theta} = \frac{m + n}{m - n} \\
 \Rightarrow &\frac{2 \sin\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \cos\left(\frac{\theta + 2\alpha - \theta}{2}\right)}{2 \cos\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \sin\left(\frac{\theta + 2\alpha - \theta}{2}\right)} = \frac{m + n}{m - n} \\
 &[\because \sin x + \sin y = 2 \sin \frac{x + y}{2} \cdot \cos \frac{x - y}{2} \text{ and } \sin x - \sin y = 2 \cos \frac{x + y}{2} \cdot \sin \frac{x - y}{2}] \\
 \Rightarrow &\frac{\sin(\theta + \alpha) \cdot \cos \alpha}{\cos(\theta + \alpha) \cdot \sin \alpha} = \frac{m + n}{m - n} \\
 \Rightarrow &\tan(\theta + \alpha) \cdot \cot \alpha = \frac{m + n}{m - n}
 \end{aligned}$$

Hence proved.

Q. 4 If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, then find that value of $\tan 2\alpha$.

Sol. Given that, $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

$$\therefore \sin(\alpha + \beta) = \frac{3}{5}$$

$$\text{and } \cos(\alpha - \beta) = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \pm \frac{12}{13}$$

$$\therefore \cos(\alpha - \beta) = \frac{12}{13}$$

$$\text{Now, } \tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \quad \left[\text{since, } \alpha \text{ lies between } 0 \text{ and } \frac{\pi}{4} \right]$$

$$= \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

and $\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$

$\therefore \tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} \quad \left[\because \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y} \right]$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{16-5}{16}} = \frac{14 \times 16}{12 \times 11} = \frac{56}{33}$$

Q. 5 If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Thinking Process

First of all rationalise the given expression and used the formula, i.e., $\cos 2x = \cos^2 x - \sin^2 x$.

Sol. Given that, $\tan x = \frac{b}{a}$

$$\therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{\sqrt{(a+b)^2} + \sqrt{(a-b)^2}}{\sqrt{(a-b)(a+b)}}$$

$$= \frac{(a+b) + (a-b)}{\sqrt{a^2 - b^2}} = \frac{2a}{\sqrt{a^2 - b^2}} = \frac{2a}{a\sqrt{1 - \left(\frac{b}{a}\right)^2}} \quad \left[\because \frac{b}{a} = \tan x \right]$$

$$= \frac{2}{\sqrt{1 - \tan^2 x}} = \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}} \quad [\because \cos 2x = \cos^2 x - \sin^2 x]$$

$$= \frac{2 \cos x}{\sqrt{\cos 2x}}$$

Q. 6 Prove that $\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 7\theta \sin 8\theta$.

Sol. LHS = $\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2}$

$$= \frac{1}{2} \left[2 \cos \theta \cdot \cos \frac{\theta}{2} - 2 \cos 3\theta \cdot \cos \frac{9\theta}{2} \right]$$

$$= \frac{1}{2} \left[\cos \left(\theta + \frac{\theta}{2} \right) + \cos \left(\theta - \frac{\theta}{2} \right) - \cos \left(3\theta + \frac{9\theta}{2} \right) - \cos \left(3\theta - \frac{9\theta}{2} \right) \right]$$

$$= \frac{1}{2} \left(\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} - \cos \frac{15\theta}{2} - \cos \frac{3\theta}{2} \right)$$

$$= \frac{1}{2} \left[\cos \frac{\theta}{2} - \cos \frac{15\theta}{2} \right]$$

$$= -\frac{1}{2} \left[2 \sin \left(\frac{\theta + 15\theta}{2} \right) \cdot \sin \left(\frac{\theta - 15\theta}{2} \right) \right] \quad \left[\because \cos x - \cos y = -2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2} \right]$$

$$= + (\sin 8\theta \cdot \sin 7\theta) = \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

Q. 7 If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then show that $a^2 + b^2 = m^2 + n^2$.

Sol. Given that, $a \cos \theta + b \sin \theta = m$... (i)
and $a \sin \theta - b \cos \theta = n$... (ii)
On squaring and adding of Eqs. (i) and (ii), we get
$$m^2 + n^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$$
$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cdot \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cdot \cos \theta$$
$$\Rightarrow m^2 + n^2 = a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$
$$\Rightarrow m^2 + n^2 = a^2 + b^2$$

Hence proved.

Q. 8 Find the value of $\tan 22^\circ 30'$.

Sol. Let $\theta = 45^\circ$
We know that, $\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} \Rightarrow \tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$
 $\therefore \tan 22^\circ 30' = \frac{\sin 45^\circ}{1 + \cos 45^\circ}$ [$\because \theta = 45^\circ$]
$$= \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2} + 1}$$

Q. 9 Prove that $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$.

Thinking Process

Here, apply the formula i.e., $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$

Sol. LHS = $\sin 4A$
 $= 2 \sin 2A \cdot \cos 2A$
 $= 2 (2 \sin A \cdot \cos A) (\cos^2 A - \sin^2 A)$
 $= 4 \sin A \cdot \cos^3 A - 4 \cos A \sin^3 A$ [$\because \cos 2A = \cos^2 A - \sin^2 A$
and $\sin 2A = 2 \sin A \cdot \cos A$]
 $\therefore$ LHS = RHS **Hence proved.**

Q. 10 If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, then prove that $m^2 - n^2 = 4 \sin \theta \tan \theta$.

Sol. Given that, $\tan \theta + \sin \theta = m$... (i)
and $\tan \theta - \sin \theta = n$... (ii)
Now, $m + n = \tan \theta + \sin \theta + \tan \theta - \sin \theta$
 $m + n = 2 \tan \theta$... (iii)
Also, $m - n = \tan \theta + \sin \theta - \tan \theta + \sin \theta$
 $m - n = 2 \sin \theta$... (iv)
From Eqs. (iii) and (iv),
$$(m + n)(m - n) = 4 \sin \theta \cdot \tan \theta$$
$$m^2 - n^2 = 4 \sin \theta \cdot \tan \theta$$

Hence proved.

Q. 11 If $\tan(A + B) = p$ and $\tan(A - B) = q$, then show that $\tan 2A = \frac{p+q}{1-pq}$.

Sol. Given that $\tan(A + B) = p$... (i)
 and $\tan(A - B) = q$... (ii)
 $\therefore \tan 2A = \tan(A + B + A - B)$
 $= \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B)\tan(A - B)} \quad \left[\because \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \right]$
 $= \frac{p + q}{1 - pq}$ [from Eqs. (i) and (ii)]

Q. 12 If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$.

Sol. Given that, $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$
 $\Rightarrow (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$
 $\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2\cos \alpha \cos \beta - \sin^2 \alpha - \sin^2 \beta - 2\sin \alpha \sin \beta = 0$
 $\Rightarrow \cos^2 \alpha - \sin^2 \alpha + \cos^2 \beta - \sin^2 \beta = 2(\sin \alpha \sin \beta - \cos \alpha \cos \beta)$
 $\Rightarrow \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$ Hence proved.

Q. 13 If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

Sol. Given that, $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$
 Using componendo and dividendo,
 $\Rightarrow \frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$
 $\Rightarrow \frac{2\sin\left(\frac{x+y+x-y}{2}\right) \cdot \cos\left(\frac{x+y-x-y}{2}\right)}{2\cos\left(\frac{x+y+x-y}{2}\right) \cdot \sin\left(\frac{x+y-x-y}{2}\right)} = \frac{2a}{2b}$
 $\left[\because \sin x + \sin y = 2\sin\frac{x+y}{2} \cdot \cos\frac{x-y}{2} \text{ and } \sin x - \sin y = 2\cos\frac{x+y}{2} \cdot \sin\frac{x-y}{2} \right]$
 $\Rightarrow \frac{\sin x \cdot \cos y}{\cos x \cdot \sin y} = \frac{a}{b}$
 $\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$

Q. 14 If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then show that $\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$.

Sol. Given that, $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$
 $\Rightarrow \tan \theta = \frac{\cos \alpha (\tan \alpha - 1)}{\cos \alpha (\tan \alpha + 1)}$
 $\Rightarrow \tan \theta = \frac{\tan \alpha - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{4} \cdot \tan \alpha} \quad \left[\because \tan \frac{\pi}{4} = 1 \right]$

$$\begin{aligned}
 \Rightarrow \quad \tan \theta &= \tan \left(\alpha - \frac{\pi}{4} \right) \\
 \Rightarrow \quad \theta &= \alpha - \frac{\pi}{4} \Rightarrow \alpha = \theta + \frac{\pi}{4} \\
 \therefore \quad \sin \alpha + \cos \alpha &= \sin \left(\theta + \frac{\pi}{4} \right) + \cos \left(\theta + \frac{\pi}{4} \right) \\
 &= \sin \theta \cdot \cos \frac{\pi}{4} + \cos \theta \cdot \sin \frac{\pi}{4} + \cos \theta \cdot \cos \frac{\pi}{4} - \sin \theta \cdot \sin \frac{\pi}{4} \\
 &= \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta \left[\because \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right] \\
 &= \frac{2}{\sqrt{2}} \cdot \cos \theta = \sqrt{2} \cos \theta
 \end{aligned}$$

Q. 15 If $\sin \theta + \cos \theta = 1$, then find the general value of θ .

Thinking Process

If $\sin \theta = \sin \alpha$, then $\theta = n\pi + (-1)^n \cdot \alpha$, gives general solution of the given equation.

Sol. Given that, $\sin \theta + \cos \theta = 1$

On squaring both sides, we get

$$\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cdot \cos \theta = 1$$

$$\Rightarrow \quad 1 + 2\sin \theta \cdot \cos \theta = 1 \quad [\because \sin 2x = 2\sin x \cos x]$$

$$\Rightarrow \quad \sin 2\theta = 0 \Rightarrow 2\theta = n\pi + (-1)^n \cdot 0$$

$$\therefore \quad \theta = \frac{n\pi}{2}$$

Alternate Method

$$\sin \theta + \cos \theta = 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} \cdot \sin \theta + \frac{1}{\sqrt{2}} \cdot \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin \theta \cdot \cos \frac{\pi}{4} + \cos \theta \cdot \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \left[\because \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4} \right]$$

$$\Rightarrow \quad \sin \left(\theta + \frac{\pi}{4} \right) = \sin \frac{\pi}{4} \quad [\because \sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y]$$

$$\Rightarrow \quad \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$$

$$\therefore \quad \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$$

Q. 16 Find the most general value of θ satisfying the equation $\tan \theta = -1$ and

$$\cos \theta = \frac{1}{\sqrt{2}}.$$

Sol. The given equations are

$$\tan \theta = -1 \quad \dots (i)$$

$$\text{and} \quad \cos \theta = \frac{1}{\sqrt{2}} \quad \dots (ii)$$

$$\text{From Eq. (i),} \quad \tan \theta = -\tan \frac{\pi}{4}$$

$$\Rightarrow \quad \tan \theta = \tan \left(2\pi - \frac{\pi}{4} \right) \Rightarrow \tan \theta = \tan \frac{7\pi}{4}$$

$$\therefore \quad \theta = \frac{7\pi}{4}$$

From Eq. (ii), $\cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta = \cos \frac{\pi}{4}$

$\Rightarrow \cos \theta = \cos \left(2\pi - \frac{\pi}{4} \right) \Rightarrow \cos \theta = \cos \frac{7\pi}{4}$

$\therefore \theta = \frac{7\pi}{4}$

Hence, the most general value of θ i.e., $\theta = 2n\pi + \frac{7\pi}{4}$.

Q. 17 If $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$, then find the general value of θ .

Sol. Given that, $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$

$\Rightarrow \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{2}{\sin \theta}$

$\Rightarrow \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cdot \cos \theta} = \frac{2}{\sin \theta}$

$\Rightarrow \frac{1}{\cos \theta} = 2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$

$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \cos \theta = \cos \frac{\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$

Q. 18 If $2\sin^2 \theta = 3\cos \theta$, where $0 \leq \theta \leq 2\pi$, then find the value of θ .

Sol. Given that, $2\sin^2 \theta = 3\cos \theta$

$\Rightarrow 2 - 2\cos^2 \theta = 3\cos \theta$

$\Rightarrow 2\cos^2 \theta + 3\cos \theta - 2 = 0$

$\Rightarrow 2\cos^2 \theta + 4\cos \theta - \cos \theta - 2 = 0$

$\Rightarrow 2\cos \theta (\cos \theta + 2) - 1(\cos \theta + 2) = 0$

$\Rightarrow (\cos \theta + 2)(2\cos \theta - 1) = 0$

$\Rightarrow \cos \theta = -2$ not possible $[\because -1 \leq \cos \theta \leq 1]$

$\Rightarrow 2\cos \theta = 1$

$\Rightarrow \cos \theta = \frac{1}{2}$

$\Rightarrow \cos \theta = \cos \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

Also, $\cos \theta = \cos \left(2\pi - \frac{\pi}{3} \right)$

$\Rightarrow \cos \theta = \cos \frac{5\pi}{6}$

$\therefore \theta = \frac{5\pi}{6}$

So, the values of θ are $\frac{\pi}{3}$ and $\frac{5\pi}{6}$.

Q. 19 If $\sec x \cos 5x + 1 = 0$, where $0 < x \leq \frac{\pi}{2}$, then find the value of x .

Sol. Given that,

$$\begin{aligned} \sec x \cos 5x + 1 &= 0 \\ \frac{\cos 5x}{\cos x} + 1 &= 0 \Rightarrow \cos 5x + \cos x = 0 \\ \Rightarrow 2 \cos \left(\frac{5x + x}{2} \right) \cdot \cos \left(\frac{5x - x}{2} \right) &= 0 \quad \left[\because \cos x + \cos y = 2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right] \\ \Rightarrow 2 \cos 3x \cdot \cos 2x &= 0 \\ \Rightarrow \cos 3x &= 0 \text{ or } \cos 2x = 0 \\ \Rightarrow \cos 3x &= \cos \frac{\pi}{2} \text{ or } \cos 2x = \cos \frac{\pi}{2} \\ \therefore 3x &= \frac{\pi}{2} \Rightarrow 2x = \frac{\pi}{2} \\ \text{and } x &= \frac{\pi}{6} \Rightarrow x = \frac{\pi}{4} \\ \text{Hence, the solutions are } \frac{\pi}{2}, \frac{\pi}{4} \text{ and } \frac{\pi}{6}. \end{aligned}$$

Long Answer Type Questions

Q. 20 If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos(\alpha + \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.

Thinking Process

Express $\cos(\alpha - \beta) = \cos(\theta + \alpha) - (\theta + \beta)$.

Sol. Given that, $\sin(\theta + \alpha) = a$

...(i)

and $\sin(\theta + \beta) = b$

...(ii)

$$\therefore \cos(\theta + \alpha) = \sqrt{1 - a^2} \text{ and } \cos(\theta + \beta) = \sqrt{1 - b^2}$$

$$\therefore \cos(\alpha - \beta) = \cos\{\theta + \alpha - (\theta + \beta)\}$$

$$= \cos(\theta + \beta) \cos(\theta + \alpha) + \sin(\theta + \alpha) \sin(\theta + \beta)$$

$$= \sqrt{1 - a^2} \sqrt{1 - b^2} + a \cdot b = ab + \sqrt{(1 - a^2)(1 - b^2)}$$

$$= ab + \sqrt{1 - a^2 - b^2 + a^2 b^2}$$

$$\text{and } \cos(\alpha - \beta) = ab + \sqrt{1 - a^2 - b^2 + a^2 b^2}$$

$$= \cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta)$$

$$= 2 \cos^2(\alpha - \beta) - 1 - 4ab \cos(\alpha - \beta)$$

$$= 2 \cos(\alpha - \beta) (\cos \alpha - \beta - 2ab) - 1$$

$$= 2(ab + \sqrt{1 - a^2 - b^2 + a^2 b^2})(ab + \sqrt{1 - a^2 - b^2 + a^2 b^2} - 2ab) - 1$$

$$= 2[(\sqrt{1 - a^2 - b^2 + a^2 b^2} + ab)(\sqrt{1 - a^2 - b^2 + a^2 b^2} - ab)] - 1$$

$$= 2[1 - a^2 - b^2 + a^2 b^2 - a^2 b^2] - 1$$

$$= 2 - 2a^2 - 2b^2 - 1$$

$$= 1 - 2a^2 - 2b^2$$

Hence proved.

Q. 21 If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then prove that $\tan \theta = \frac{1-m}{1+m} \cot \phi$.

Sol. Given that,

$$\Rightarrow \frac{\cos(\theta + \phi)}{\cos(\theta - \phi)} = \frac{m \cos(\theta - \phi)}{1}$$

Using componendo and dividendo rule,

$$\begin{aligned} \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{\cos(\theta - \phi) + \cos(\theta + \phi)} &= \frac{1-m}{1+m} \\ \Rightarrow \frac{-2 \sin\left(\frac{\theta - \phi + \theta + \phi}{2}\right) \cdot \sin\left(\frac{\theta - \phi - \theta - \phi}{2}\right)}{2 \cos\left(\frac{\theta - \phi + \theta + \phi}{2}\right) \cdot \cos\left(\frac{\theta - \phi - \theta - \phi}{2}\right)} &= \frac{1-m}{1+m} \\ \Rightarrow \frac{\sin \theta \cdot \sin \phi}{\cos \theta \cdot \cos \phi} &= \frac{1-m}{1+m} \quad \left[\because \sin(-\theta) = -\sin \theta \right. \\ &\quad \left. \text{and } \cos(-\theta) = \cos \theta \right] \\ \Rightarrow \tan \theta \cdot \tan \phi &= \frac{1-m}{1+m} \\ \Rightarrow \tan \theta &= \left(\frac{1-m}{1+m} \right) \cot \phi \end{aligned}$$

Q. 22 Find the value of the expression

$$3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4(3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6(5\pi - \alpha) \right].$$

Sol. Given expression,

$$\begin{aligned} &3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4(3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6(5\pi - \alpha) \right] \\ &= 3 [\cos^4 \alpha + \sin^4(\pi + \alpha)] - 2 [\cos^6 \alpha + \sin^6(\pi - \alpha)] \\ &= 3 [\cos^4 \alpha + \sin^4 \alpha] - 2 [\cos^6 \alpha + \sin^6 \alpha] = 3 - 2 = 1 \end{aligned}$$

Q. 23 If $a \cos 2\theta + b \sin 2\theta = c$ has α and β as its roots, then prove that

$$\tan \alpha + \tan \beta = \frac{2b}{a+c}.$$

Sol. Given that, $a \cos 2\theta + b \sin 2\theta = c$

$$\begin{aligned} \Rightarrow a \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + b \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) &= c \quad \left[\because \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \text{ and } \cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right] \\ \Rightarrow a(1 - \tan^2 \theta) + 2b \tan \theta &= c(1 + \tan^2 \theta) \\ \Rightarrow a - a \tan^2 \theta + 2b \tan \theta &= c + c \tan^2 \theta \\ \Rightarrow (a + c) \tan^2 \theta - 2b \tan \theta + c - a &= 0 \end{aligned}$$

Since, this equation has $\tan \alpha$ and $\tan \beta$ as its roots.

$$\therefore \tan \alpha + \tan \beta = \frac{-(-2b)}{a+c} = \frac{2b}{a+c}$$

Q. 24 If $x = \sec \phi - \tan \phi$ and $y = \operatorname{cosec} \phi + \cot \phi$, then show that $xy + x - y + 1 = 0$.

Sol. Given that, $x = \sec \phi - \tan \phi$... (i)
 and $y = \operatorname{cosec} \phi + \cot \phi$... (ii)
 Now, $1 \cdot xy = (\sec \phi - \tan \phi)(\operatorname{cosec} \phi + \cot \phi)$
 $\Rightarrow xy = \sec \phi \cdot \operatorname{cosec} \phi - \operatorname{cosec} \phi \cdot \tan \phi + \sec \phi \cdot \cot \phi - \tan \phi \cdot \cot \phi$
 $\Rightarrow xy = \sec \phi \cdot \operatorname{cosec} \phi - \frac{1}{\cos \phi} + \frac{1}{\sin \phi} - 1$
 $\Rightarrow 1 + xy = \sec \phi \operatorname{cosec} \phi - \sec \phi + \operatorname{cosec} \phi$... (iii)
 From Eqs. (i) and (ii), we get
 $x - y = \sec \phi - \tan \phi - \operatorname{cosec} \phi - \cot \phi$
 $\Rightarrow x - y = \sec \phi - \operatorname{cosec} \phi - \frac{\sin \phi}{\cos \phi} - \frac{\cos \phi}{\sin \phi}$
 $\Rightarrow x - y = \sec \phi - \operatorname{cosec} \phi - \left(\frac{\sin^2 \phi + \cos^2 \phi}{\sin \phi \cdot \cos \phi} \right)$
 $\Rightarrow x - y = \sec \phi - \operatorname{cosec} \phi - \frac{1}{\sin \phi \cdot \cos \phi}$
 $\Rightarrow x - y = \sec \phi - \operatorname{cosec} \phi - \operatorname{cosec} \phi \cdot \sec \phi$
 $\Rightarrow x - y = -(\sec \phi \cdot \operatorname{cosec} \phi - \sec \phi + \operatorname{cosec} \phi)$
 $\Rightarrow x - y = -(xy + 1)$ [from Eq. (iii)]
 $\Rightarrow xy + x - y + 1 = 0$ **Hence proved.**

Q. 25 If θ lies in the first quadrant and $\cos \theta = \frac{8}{17}$, then find the value of $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.

Sol. Given that, $\cos \theta = \frac{8}{17} \Rightarrow \sin \theta = \sqrt{1 - \frac{64}{289}}$
 $\Rightarrow \sin \theta = \sqrt{\frac{289 - 64}{289}} \Rightarrow \sin \theta = \pm \frac{15}{17}$
 $\Rightarrow \sin \theta = \frac{15}{17}$ [since, θ lies in first quadrant]

Now, $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$
 $= \cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(90^\circ + 30^\circ - \theta)$
 $= \cos(30^\circ + \theta) + \cos(45^\circ - \theta) - \sin(30^\circ - \theta)$
 $= \cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta + \cos 45^\circ \cos \theta + \sin 45^\circ \sin \theta$
 $\quad \quad \quad - \sin 30^\circ \cos \theta + \cos 30^\circ \sin \theta$
 $= \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta - \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$
 $= \left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2} \right) \cos \theta + \left(\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{\sqrt{3}}{2} \right) \sin \theta$
 $= \left(\frac{\sqrt{6} + 2 - \sqrt{2}}{2\sqrt{2}} \right) \cos \theta + \left(\frac{2 - \sqrt{2} + \sqrt{6}}{2\sqrt{2}} \right) \sin \theta$
 $= \left(\frac{\sqrt{6} + 2 - \sqrt{2}}{2\sqrt{2}} \right) \frac{8}{17} + \left(\frac{2 - \sqrt{2} + \sqrt{6}}{2\sqrt{2}} \right) \frac{15}{17}$

$$\begin{aligned}
 &= \frac{1}{17(2\sqrt{2})}(8\sqrt{6} + 16 - 8\sqrt{2} + 30 - 15\sqrt{2} + 15\sqrt{6}) \\
 &= \frac{1}{17(2\sqrt{2})}(23\sqrt{6} - 23\sqrt{2} + 46) \\
 &= \frac{23\sqrt{6}}{17(2\sqrt{2})} - \frac{23\sqrt{2}}{17(2\sqrt{2})} + \frac{46}{17(2\sqrt{2})} \\
 &= \frac{23\sqrt{3}}{17(2)} - \frac{23}{17(2)} + \frac{23}{17\sqrt{2}} \\
 &= \frac{23}{17} \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right)
 \end{aligned}$$

Q. 26 Find the value of $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$.

Sol. Given expression, $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$\begin{aligned}
 &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \left(\pi - \frac{3\pi}{8} \right) + \cos^4 \left(\pi - \frac{\pi}{8} \right) \\
 &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8} \\
 &= 2 \left[\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} \right] = 2 \left[\cos^4 \frac{\pi}{8} + \cos^4 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right] \\
 &= 2 \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right] \\
 &= 2 \left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\
 &= 2 \left[1 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] = 2 - \left(2 \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \right)^2 \\
 &= 2 - \left(\sin \frac{2\pi}{8} \right)^2 = 2 - \left(\frac{1}{\sqrt{2}} \right)^2 \\
 &= 2 - \frac{1}{2} = \frac{3}{2}
 \end{aligned}$$

Q. 27 Find the general solution of the equation $5\cos^2 \theta + 7\sin^2 \theta - 6 = 0$.

Sol. Given equation, $5\cos^2 \theta + 7\sin^2 \theta - 6 = 0$

$$\begin{aligned}
 \Rightarrow & 5\cos^2 \theta + 7(1 - \cos^2 \theta) - 6 = 0 \\
 \Rightarrow & 5\cos^2 \theta + 7 - 7\cos^2 \theta - 6 = 0 \\
 \Rightarrow & 5\cos^2 \theta + 7 - 7\cos^2 \theta - 6 = 0 \Rightarrow -2\cos^2 \theta + 1 = 0 \\
 \Rightarrow & 2\cos^2 \theta - 1 = 0 \quad \left[\begin{array}{l} \because \cos^2 \theta = \cos^2 \alpha \\ \therefore \theta = n\pi \pm \alpha \end{array} \right] \\
 \Rightarrow & \cos^2 \theta = \frac{1}{2} \\
 \Rightarrow & \cos^2 \theta = \cos^2 \frac{\pi}{4} \\
 \therefore & \theta = n\pi \pm \frac{\pi}{4}
 \end{aligned}$$

Q. 28 Find the general of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

Sol. Given equation, $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$$\begin{aligned} \Rightarrow & 2\sin\left(\frac{x+3x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right) - 3\sin 2x \\ & = 2\cos\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right) - 3\cos 2x \\ \Rightarrow & 2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cdot \cos x - 3\cos 2x \\ \Rightarrow & \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3) \\ \Rightarrow & \frac{\sin 2x}{\cos 2x} = 1 \\ \Rightarrow & \tan 2x = 1 \\ \Rightarrow & \tan 2x = \tan \frac{\pi}{4} \\ \Rightarrow & 2x = n\pi + \frac{\pi}{4} \\ \therefore & x = \frac{n\pi}{2} + \frac{\pi}{8} \end{aligned}$$

Q. 29 Find the general solution of the equation

$$(\sqrt{3} - 1)\cos \theta + (\sqrt{3} + 1)\sin \theta = 2$$

Sol. Given equation is,

$$\begin{aligned} & (\sqrt{3} - 1)\cos \theta + (\sqrt{3} + 1)\sin \theta = 2 \quad \dots(i) \\ \text{Put } & \sqrt{3} - 1 = r\sin \alpha \quad \text{and} \quad \sqrt{3} + 1 = r\cos \alpha \\ \therefore & r^2 = (\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2 \\ \Rightarrow & = 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} \\ \Rightarrow & r^2 = 8 \\ \therefore & r = 2\sqrt{2} \\ \text{now, } & \tan \alpha = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \cdot \tan \frac{\pi}{4}} \\ \Rightarrow & \tan \alpha = \tan \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \\ \Rightarrow & \tan \alpha = \tan \frac{\pi}{12} \\ \therefore & \alpha = \frac{\pi}{12} \\ \text{From Eq. (i), } & r\sin \alpha \cos \theta + r\cos \alpha \sin \theta = 2 \\ & r[\sin(\theta + \alpha)] = 2 \\ \Rightarrow & \sin(\theta + \alpha) = \frac{2}{2\sqrt{2}} \\ \Rightarrow & \sin(\theta + \alpha) = \frac{1}{\sqrt{2}} \\ \Rightarrow & \sin(\theta + \alpha) = \sin \frac{\pi}{4} \quad \theta + \alpha = n\pi + (-1)^n \frac{\pi}{4} \\ & \theta = n\pi + (-1)^n \cdot \frac{\pi}{4} - \frac{\pi}{12} \end{aligned}$$

Alternate Method

$$\begin{aligned}
 & (\sqrt{3} - 1)\cos \theta + (\sqrt{3} + 1)\sin \theta = 2 \quad \dots(i) \\
 \text{Put } & \sqrt{3} - 1 = r\cos \alpha \text{ and } \sqrt{3} + 1 = r\sin \alpha \\
 \therefore & r = 2\sqrt{2} \\
 \text{Now, } & \tan \alpha = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \\
 \Rightarrow & \tan \alpha = \frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{\pi}{4} \cdot \tan \frac{\pi}{6}} \\
 \Rightarrow & \tan \alpha = \tan \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \Rightarrow \tan \alpha = \tan \frac{5\pi}{12} \\
 \therefore & \alpha = \frac{5\pi}{12} \\
 \text{From Eq. (i), } & r\cos \alpha \cos \theta + r\sin \alpha \sin \theta = 2 \\
 & r[\cos(\theta - \alpha)] = 2 \\
 \Rightarrow & \cos(\theta - \alpha) = \frac{2}{2\sqrt{2}} \\
 \Rightarrow & \cos(\theta - \alpha) = \frac{1}{\sqrt{2}} \\
 \Rightarrow & \cos(\theta - \alpha) = \cos \frac{\pi}{4} \\
 \Rightarrow & \theta - \alpha = 2n\pi \pm \frac{\pi}{4} \\
 \therefore & \theta = 2n\pi \pm \frac{\pi}{4} + \frac{5\pi}{12}
 \end{aligned}$$

Objective Type Questions**Q. 30** If $\sin \theta + \operatorname{cosec} \theta = 2$, then $\sin^2 \theta + \operatorname{cosec}^2 \theta$ is equal to

- (a) 1 (b) 4 (c) 2 (d) None of these

Sol. (c) Given that, $\sin \theta + \operatorname{cosec} \theta = 2$

$$\begin{aligned}
 \Rightarrow & \sin^2 \theta + \operatorname{cosec}^2 \theta + 2\sin \theta \cdot \operatorname{cosec} \theta = 4 \\
 \Rightarrow & \sin^2 \theta + \operatorname{cosec}^2 \theta = 4 - 2 \\
 \Rightarrow & \sin^2 \theta + \operatorname{cosec}^2 \theta = 2
 \end{aligned}$$

Q. 31 If $f(x) = \cos^2 x + \sec^2 x$, then

- (a) $f(x) < 1$ (b) $f(x) = 1$ (c) $2 < f(x) < 1$ (d) $f(x) \geq 2$

Sol. (d) Given that, $f(x) = \cos^2 x + \sec^2 x$ We know that, $AM \geq GM$

$$\begin{aligned}
 & \frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x \cdot \sec^2 x} \\
 \Rightarrow & \cos^2 x + \sec^2 x \geq 2 \quad [\because \cos x \cdot \sec x = 1] \\
 \Rightarrow & f(x) \geq 2
 \end{aligned}$$

Q. 32 If $\tan \theta = \frac{1}{2}$ and $\tan \phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

(a) $\frac{\pi}{6}$

(b) π

(c) 0

(d) $\frac{\pi}{4}$

Sol. (d) Given that,

$$\tan \theta = \frac{1}{2} \text{ and } \tan \phi = \frac{1}{3}$$

Now,

$$\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \cdot \tan \phi}$$

$$\Rightarrow \tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} \Rightarrow \tan(\theta + \phi) = \frac{\frac{3+2}{6}}{\frac{6-1}{6}} = \frac{5}{5} = 1$$

$$\Rightarrow \tan(\theta + \phi) = \tan \frac{\pi}{4}$$

$$\therefore \theta + \phi = \frac{\pi}{4}$$

Q. 33 Which of the following is not correct?

(a) $\sin \theta = -\frac{1}{5}$

(b) $\cos \theta = 1$

(c) $\sec \theta = \frac{1}{2}$

(d) $\tan \theta = 20$

Sol. (c) We know that, the range of $\sec \theta$ is $R - (-1, 1)$.

Hence, $\sec \theta$ cannot be equal to $\frac{1}{2}$.

Q. 34 The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ is

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) Not defined

Sol. (b) Given expression, $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$

$$= \tan 1^\circ \tan 2^\circ \dots \tan 45^\circ \cdot \tan(90^\circ - 44^\circ) \tan(90^\circ - 43^\circ) \dots \tan(90^\circ - 1^\circ)$$

$$= \tan 1^\circ \cdot \cot 1^\circ \cdot \tan 2^\circ \cdot \cot 2^\circ \dots \tan 89^\circ \cdot \cot 89^\circ$$

$$= 1 \cdot 1 \dots 1 \cdot 1 = 1$$

Q. 35 The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is

(a) 1

(b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$

(d) 2

Sol. (c) Given expression, $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$

Let

$$\theta = 15^\circ$$

We know that,

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$\therefore$

$$\cos 30^\circ = \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$$

$\Rightarrow$

$$\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \frac{\sqrt{3}}{2}$$

$$\left[\because \cos 30^\circ = \frac{\sqrt{3}}{2} \right]$$

Q. 36 The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$ is

- (a) $\frac{1}{\sqrt{2}}$ (b) 0 (c) 1 (d) -1

Sol. (b) Given expression, $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$
 $= \cos 1^\circ \cos 2^\circ \dots \cos 90^\circ \dots \cos 179^\circ$ [$\because \cos 90^\circ = 0$]
 $= 0$

Q. 37 If $\tan \theta = 3$ and θ lies in third quadrant, then the value of $\sin \theta$ is

- (a) $\frac{1}{\sqrt{10}}$ (b) $-\frac{1}{\sqrt{10}}$ (c) $-\frac{3}{\sqrt{10}}$ (d) $\frac{3}{\sqrt{10}}$

Sol. (c) Given that, $\tan \theta = 3$
 $\Rightarrow \sec^2 \theta = 1 + \tan^2 \theta$
 $\Rightarrow \sec \theta = \sqrt{1 + 9} = \pm \sqrt{10}$
 $\Rightarrow \sec \theta = -\sqrt{10}$
 $\Rightarrow \cos \theta = -\frac{1}{\sqrt{10}}$
 $\Rightarrow \sin \theta = \pm \sqrt{1 - \frac{1}{10}} = \pm \sqrt{\frac{9}{10}} = \pm \frac{3}{\sqrt{10}}$ [since, θ lies in third quadrant]
 $\therefore \sin \theta = -\frac{3}{\sqrt{10}}$

Q. 38 The value of $\tan 75^\circ - \cot 75^\circ$ is

- (a) $2\sqrt{3}$ (b) $2 + \sqrt{3}$ (c) $2 - \sqrt{3}$ (d) 1

Sol. (a) Given expression, $\tan 75^\circ - \cot 75^\circ$
 $= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ}$
 $= \frac{\sin^2 75^\circ - \cos^2 75^\circ}{\sin 75^\circ \cdot \cos 75^\circ}$
 $= \frac{-2\cos 150^\circ}{\sin 150^\circ}$
 $= \frac{-2\cos(90^\circ + 60^\circ)}{\sin(90^\circ + 60^\circ)}$
 $= \frac{+2\sin 60^\circ}{\cos 60^\circ}$
 $= \frac{2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}$

Q. 39 Which of the following is correct?

- (a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ < \sin 1$
 (c) $\sin 1^\circ = \sin 1$ (d) $\sin 1^\circ = \frac{\pi}{18^\circ} \sin 1$

Sol. (b) We know that, if θ is increasing, then $\sin \theta$ is also increasing.
 $\therefore \sin 1^\circ < \sin 1$ [$\because 1 \text{ rad} = 57^\circ 30'$]

Q. 40 If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Sol. (d) Given that, $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$

$$\begin{aligned} \text{Now,} \quad \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} \\ \Rightarrow \tan(\alpha + \beta) &= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)} \\ \Rightarrow \tan(\alpha + \beta) &= \frac{m(2m+1) + m+1}{(m+1)(2m+1) - m} \\ \Rightarrow \tan(\alpha + \beta) &= \frac{2m^2 + m + m + 1}{2m^2 + 2m + m + 1 - m} \\ \Rightarrow \tan(\alpha + \beta) &= \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} \Rightarrow \tan(\alpha + \beta) = 1 \\ \therefore \alpha + \beta &= \frac{\pi}{4} \end{aligned}$$

Q. 41 The minimum value of $3\cos x + 4\sin x + 8$ is

- (a) 5 (b) 9 (c) 7 (d) 3

Thinking Process

For the expression $A\cos \theta + B\sin \theta$, then the minimum value is $-\sqrt{A^2 + B^2}$.

Sol. (d) Given expression, $3\cos x + 4\sin x + 8$

$$\begin{aligned} \text{Let} \quad y &= 3\cos x + 4\sin x + 8 \\ \Rightarrow y - 8 &= 3\cos x + 4\sin x \\ \therefore \text{Minimum value of } y - 8 &= -\sqrt{9 + 16} \\ \Rightarrow y - 8 &= -5 \Rightarrow y = -5 + 8 \\ \therefore y &= 3 \end{aligned}$$

Hence, the minimum value of $3\cos x + 4\sin x + 8$ is 3.

Q. 42 The value of $\tan 3A - \tan 2A - \tan A$ is

- (a) $\tan 3A \tan 2A \tan A$
(b) $-\tan 3A \tan 2A \tan A$
(c) $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$
(d) None of the above

Sol. (a) Let

$$\begin{aligned} 3A &= A + 2A \\ \tan 3A &= \tan(A + 2A) \\ \Rightarrow \tan 3A &= \frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A} \\ \Rightarrow \tan A + \tan 2A &= \tan 3A - \tan 3A \cdot \tan 2A \cdot \tan A \\ \Rightarrow \tan 3A - \tan 2A - \tan A &= \tan 3A \cdot \tan 2A \cdot \tan A \end{aligned}$$

Q. 43 The value of $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is

- (a) $2 \cos \theta$ (b) $2 \sin \theta$ (c) 1 (d) 0

Thinking Process

Use formula i.e., $\sin(A + B) = \sin A \cos B + \cos A \sin B$

and $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Sol. (d) Given expression,

$$\begin{aligned} \sin(45^\circ + \theta) - \cos(45^\circ - \theta) \\ &= \sin 45^\circ \cdot \cos \theta + \cos 45^\circ \cdot \sin \theta - \cos 45^\circ \cdot \cos \theta - \sin 45^\circ \cdot \sin \theta \\ &= \frac{1}{\sqrt{2}} \cdot \cos \theta + \frac{1}{\sqrt{2}} \cdot \sin \theta - \frac{1}{\sqrt{2}} \cdot \cos \theta - \frac{1}{\sqrt{2}} \sin \theta \\ &= 0 \end{aligned}$$

Q. 44 The value of $\cot\left(\frac{\pi}{4} + \theta\right) \cot\left(\frac{\pi}{4} - \theta\right)$ is

- (a) -1 (b) 0 (c) 1 (d) Not defined

Thinking Process

Use formula i.e., $\cot(A + B) = \left(\frac{\cot A \cot B - 1}{\cot A + \cot B}\right)$ and $\cot(A - B) = \left(\frac{\cot A \cot B + 1}{\cot A - \cot B}\right)$.

Sol. (c) Given expression,

$$\begin{aligned} &\cot\left(\frac{\pi}{4} + \theta\right) - \cot\left(\frac{\pi}{4} - \theta\right) \\ &= \left(\frac{\cot \frac{\pi}{4} \cot \theta - 1}{\cot \frac{\pi}{4} + \cot \theta}\right) \cdot \left(\frac{\cot \frac{\pi}{4} \cot \theta + 1}{\cot \theta - \cot \frac{\pi}{4}}\right) \\ &= \left(\frac{\cot \theta - 1}{\cot \theta + 1}\right) \cdot \left(\frac{\cot \theta + 1}{\cot \theta - 1}\right) \\ &= 1 \end{aligned}$$

Q. 45 $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to

- (a) $\sin 2(\theta + \phi)$ (b) $\cos 2(\theta + \phi)$ (c) $\sin 2(\theta - \phi)$ (d) $\cos 2(\theta - \phi)$

Sol. (b) Given expression, $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$$\begin{aligned} &= \cos 2\theta \cdot \cos 2\phi + \sin(\theta - \phi + \theta + \phi) \cdot \sin(\theta - \phi - \theta - \phi) \\ &= \cos 2\theta \cdot \cos 2\phi - \sin 2\theta \cdot \sin 2\phi \\ &= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi) \end{aligned}$$

Q. 46 The value of $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$ is

- (a) $\frac{1}{2}$ (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{1}{8}$

Thinking Process

Use the formula $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$ and

$\cos A - \cos B = -2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2}$ to solve this problem.

Sol. (c) Given expression, $\cos 12^\circ + \cos 84^\circ + \cos 150^\circ + \cos 132^\circ$

$$\begin{aligned}
 &= \cos 12^\circ + \cos 150^\circ + \cos 84^\circ + \cos 132^\circ \\
 &= 2\cos\left(\frac{12^\circ + 150^\circ}{2}\right) \cdot \cos\left(\frac{12^\circ - 150^\circ}{2}\right) + 2\cos\left(\frac{84^\circ + 132^\circ}{2}\right) \cdot \cos\left(\frac{84^\circ - 132^\circ}{2}\right) \\
 &= 2\cos 84^\circ \cos 72^\circ + 2\cos 108^\circ \cos 24^\circ \\
 &= 2\cos 84^\circ \cos(90^\circ - 18^\circ) + 2\cos(90^\circ + 18^\circ) \cdot \cos 24^\circ \\
 &= 2\cos 84^\circ \sin 18^\circ - 2\sin 18^\circ \cos 24^\circ \\
 &= 2\sin 18^\circ (\cos 84^\circ - \cos 24^\circ) \\
 &= 2\sin 18^\circ \cdot 2\sin\left(\frac{84^\circ + 24^\circ}{2}\right) \cdot \sin\left(\frac{84^\circ - 24^\circ}{2}\right) \\
 &= -4\sin 18^\circ \cdot \sin 54^\circ \sin 30^\circ \\
 &= -4\left(\frac{\sqrt{5}-1}{4}\right) \cdot \cos 36^\circ \cdot \frac{1}{2} \\
 &= -(\sqrt{5}-1) \left(\frac{\sqrt{5}+1}{4}\right) \cdot \frac{1}{2} = -\left(\frac{5-1}{8}\right) = \frac{-4}{8} = \frac{-1}{2}
 \end{aligned}$$

Q. 47 If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to

- (a) 1 (b) 2 (c) 3 (d) 4

Sol. (c) Given that, $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

Now, $\tan(2A + B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B} \quad \dots(i)$

Also, $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$

From Eq. (i), $\tan(2A + B) = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \cdot \frac{1}{3}} = \frac{\frac{4+1}{3}}{\frac{9-4}{9}} = \frac{5}{5} = 1$

Q. 48 The value of $\sin \frac{\pi}{10} \sin \frac{13\pi}{10}$ is

- (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) 1

Sol. (c) Given expression, $\sin \frac{\pi}{10} \sin \frac{13\pi}{10} = \sin \frac{\pi}{10} \sin \left(\pi + \frac{3\pi}{10}\right)$

$$= -\sin \frac{\pi}{10} \sin \frac{3\pi}{10} = -\sin 18^\circ \cdot \sin 54^\circ$$

$$= -\sin 18^\circ \cdot \cos 36^\circ$$

$$= -\left(\frac{\sqrt{5}-1}{4}\right) \left(\frac{\sqrt{5}+1}{4}\right)$$

[since, put this value here]

$$= -\left(\frac{5-1}{16}\right) = -\frac{1}{4}$$

Q. 49 The value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is

- (a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 2

Thinking Process

Here, use the formula i.e., $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$ also $\sin(-\theta) = -\sin \theta$

Sol. (b) Given expression, $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$

$$\begin{aligned} &= 2 \cos\left(\frac{50^\circ + 70^\circ}{2}\right) \cdot \sin\left(\frac{50^\circ - 70^\circ}{2}\right) + \sin 10^\circ \\ &= -2 \cos 60^\circ \sin 10^\circ + \sin 10^\circ \\ &= -2 \cdot \frac{1}{2} \sin 10^\circ + \sin 10^\circ = 0 \end{aligned}$$

Q. 50 If $\sin \theta + \cos \theta = 1$, then the value of $\sin 2\theta$ is

- (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) -1

Sol. (c) Given that, $\sin \theta + \cos \theta = 1$

On squaring both sides, we get

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cdot \cos \theta = 1$$

$$\Rightarrow 1 + \sin 2\theta = 1$$

$$\therefore \sin 2\theta = 0$$

Q. 51 If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan \alpha)(1 + \tan \beta)$ is

- (a) 1 (b) 2 (c) -2 (d) Not defined

Thinking Process

Formula i.e., $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$ to solve this problem.

Sol. (b) Given that, $\alpha + \beta = \frac{\pi}{4}$

$$\text{Now, } (1 + \tan \alpha)(1 + \tan \beta) = 1 + \tan \alpha + \tan \beta + \tan \alpha \tan \beta \quad \dots(i)$$

$$\text{We know that, } \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\Rightarrow 1 = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\Rightarrow \tan \alpha + \tan \beta = 1 - \tan \alpha \tan \beta$$

From Eq. (i),

$$\begin{aligned} (1 + \tan \alpha)(1 + \tan \beta) &= 1 + 1 - \tan \alpha \cdot \tan \beta + \tan \alpha \cdot \tan \beta \\ &= 2 \end{aligned}$$

Q. 52 If $\sin \theta = \frac{-4}{5}$ and θ lies in third quadrant, then the value of $\cos \frac{\theta}{2}$ is

(a) $\frac{1}{5}$

(b) $-\frac{1}{\sqrt{10}}$

(c) $-\frac{1}{\sqrt{5}}$

(d) $\frac{1}{\sqrt{10}}$

Thinking Process

Use $\cos \theta = \sqrt{1 - \sin^2 \theta}$ and $\cos \theta = 2 \cos^2 \frac{\theta}{2} - 1$.

Sol. (c) Given that,

$$\sin \theta = \frac{-4}{5}$$

$$\cos \theta = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \pm \frac{3}{5}$$

$$\cos \theta = \frac{-3}{5} \quad [\text{since, } \theta \text{ lies in third quadrant}]$$

$$\Rightarrow 2 \cos^2 \frac{\theta}{2} - 1 = \frac{-3}{5}$$

$$\Rightarrow 2 \cos^2 \frac{\theta}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow 2 \cos^2 \frac{\theta}{2} = \frac{2}{5}$$

$$\therefore \cos \frac{\theta}{2} = \pm \frac{1}{\sqrt{5}}$$

$$\Rightarrow \cos \frac{\theta}{2} = -\frac{1}{\sqrt{5}} \quad [\text{since, } \theta \text{ lies in third quadrant}]$$

Q. 53 The number of solutions of equation $\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0

(b) 1

(c) 2

(d) 3

Sol. (c) Given equation,

$$\tan x + \sec x = 2 \cos x$$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \cos x$$

$$\Rightarrow 1 + \sin x = 2 \cos^2 x$$

$$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$$

$$\Rightarrow 1 + \sin x = 2 - 2 \sin^2 x$$

$$\Rightarrow 2 \sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow 2 \sin^2 x + 2 \sin x - \sin x - 1 = 0$$

$$\Rightarrow 2 \sin x (\sin x + 1) - 1 (\sin x + 1) = 0$$

$$\Rightarrow (\sin x + 1)(2 \sin x - 1) = 0$$

$$\Rightarrow \sin x + 1 = 0 \text{ or } (2 \sin x - 1) = 0$$

$$\Rightarrow \sin x = -1, \sin x = \frac{1}{2}$$

$$\therefore x = \frac{3\pi}{2}, x = \frac{\pi}{6}$$

Hence, only two solutions possible.

Q. 54 The value of $\sin \frac{\pi}{18} + \sin \frac{\pi}{9} + \sin \frac{2\pi}{9} + \sin \frac{5\pi}{18}$ is

(a) $\sin \frac{7\pi}{18} + \sin \frac{4\pi}{9}$

(b) 1

(c) $\cos \frac{\pi}{6} + \cos \frac{3\pi}{7}$

(d) $\cos \frac{\pi}{9} + \sin \frac{\pi}{9}$

Thinking Process

Here, apply the formulae i.e., $\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$.

Sol. (a) Given expression, $\sin \frac{\pi}{18} + \sin \frac{\pi}{9} + \sin \frac{2\pi}{9} + \sin \frac{5\pi}{18}$

$$= \sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ$$

$$= \sin 50^\circ + \sin 10^\circ + \sin 40^\circ + \sin 20^\circ$$

$$= \sin 130^\circ + \sin 10^\circ + \sin 140^\circ + \sin 20^\circ$$

$$= 2 \sin 70^\circ \cos 60^\circ + 2 \sin 80^\circ \cos 60^\circ \quad \left[\because \sin x + \sin y = 2 \sin \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right]$$

$$= 2 \cdot \frac{1}{2} \sin 70^\circ + 2 \cdot \frac{1}{2} \sin 80^\circ \quad \left[\because \cos 60^\circ = \frac{1}{2} \right]$$

$$= \sin 70^\circ + \sin 80^\circ = \sin \frac{7\pi}{18} + \sin \frac{4\pi}{9}$$

Q. 55 If A lies in the second quadrant and $3 \tan A + 4 = 0$, then the value of $2 \cot A - 5 \cos A + \sin A$ is

(a) $\frac{-53}{10}$

(b) $\frac{23}{10}$

(c) $\frac{37}{10}$

(d) $\frac{7}{10}$

Thinking Process

Use the formulae i.e., $\sec A = \sqrt{1 + \tan^2 A}$ and $\sin A = \sqrt{1 - \cos^2 A}$, $\sec A = \frac{1}{\cos A}$ and

$$\tan A = \frac{1}{\cot A}.$$

Sol. (b) Given equation ,

$$3 \tan A + 4 = 0$$

$$\Rightarrow 3 \tan A = -4$$

$$\Rightarrow \tan A = \frac{-4}{3}$$

$$\Rightarrow \cot A = \frac{-3}{4}$$

$$\Rightarrow \sec A = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \pm \frac{5}{3}$$

$$\Rightarrow \sec A = \frac{-5}{3} \quad [\text{since, } A \text{ lies in second quadrant}]$$

$$\cos A = \frac{-3}{5}$$

$$\sin A = \sqrt{1 - \frac{9}{25}} = \frac{\sqrt{25-9}}{25} = \pm \frac{4}{5}$$

$$\sin A = \frac{4}{5} \quad [\text{since, } A \text{ lies in second quadrant}]$$

$$\begin{aligned}
 \therefore 2 \cot A - 5 \cos A + \sin A &= 2 \left(\frac{-3}{4} \right) - 5 \left(\frac{-3}{5} \right) + \frac{4}{5} \\
 &= \frac{-6}{4} + 3 + \frac{4}{5} \\
 &= \frac{-30 + 60 + 16}{20} = \frac{46}{20} \\
 &= \frac{23}{10}
 \end{aligned}$$

Q. 56 The value of $\cos^2 48^\circ - \sin^2 12^\circ$ is

- (a) $\frac{\sqrt{5} + 1}{8}$ (b) $\frac{\sqrt{5} - 1}{8}$ (c) $\frac{\sqrt{5} + 1}{5}$ (d) $\frac{\sqrt{5} + 1}{2\sqrt{2}}$

Sol. (a) Given expression, $\cos^2 48^\circ - \sin^2 12^\circ$

$$\begin{aligned}
 &= \cos(48^\circ + 12^\circ) \cdot \cos(48^\circ - 12^\circ) \\
 &= \cos 60^\circ \cdot \cos 36^\circ \\
 &= \frac{1}{2} \cdot \frac{\sqrt{5} + 1}{4} \\
 &= \frac{\sqrt{5} + 1}{8}
 \end{aligned}$$

Q. 57 If $\tan \alpha = \frac{1}{7}$ and $\tan \beta = \frac{1}{3}$, then $\cos 2\alpha$ is equal to

- (a) $\sin 2\beta$ (b) $\sin 4\beta$ (c) $\sin 3\beta$ (d) $\cos 2\beta$

Thinking Process

$$\text{Use } \cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} \text{ and } \sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

Sol. (b) Given that, $\tan \alpha = \frac{1}{7}$ and $\tan \beta = \frac{1}{3}$

$$\begin{aligned}
 \cos 2\alpha &= \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{\frac{48}{49}}{\frac{50}{49}} \\
 &= \frac{48}{50} = \frac{24}{25}
 \end{aligned}$$

$$\Rightarrow \cos 2\alpha = \frac{24}{25} \quad \dots(i)$$

$$\text{We know that, } \sin 4\beta = \frac{2 \tan 2\beta}{1 + \tan^2 2\beta} \quad \dots(ii)$$

$$\begin{aligned}
 \text{and } \tan 2\beta &= \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}} \\
 &= \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2 \times 9}{3 \times 8} = \frac{3}{4}
 \end{aligned}$$

From Eq. (ii),

$$\sin 4\beta = \frac{2 \times \frac{3}{4}}{1 + \frac{9}{16}} = \frac{\frac{6}{4}}{\frac{25}{16}} = \frac{6 \times 16}{4 \times 25}$$

$$\Rightarrow \sin 4\beta = \frac{24}{25}$$

$$\Rightarrow \sin 4\beta = \cos 2\alpha$$

$$\therefore \cos 2\alpha = \sin 4\beta \quad [\text{from Eq. (i)}]$$

Q. 58 If $\tan \theta = \frac{a}{b}$, then $b \cos 2\theta + a \sin 2\theta$ is equal to

- (a) a (b) b (c) $\frac{a}{b}$ (d) None of these

Sol. (b) Given that, $\tan \theta = \frac{a}{b}$

$$\begin{aligned} \therefore b \cos 2\theta + a \sin 2\theta &= b \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + a \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \\ &= b \left(\frac{1 - \frac{a^2}{b^2}}{1 + \frac{a^2}{b^2}} \right) + a \left(\frac{\frac{2a}{b}}{1 + \frac{a^2}{b^2}} \right) \\ &= b \left(\frac{b^2 - a^2}{b^2 + a^2} \right) + \frac{2a^2 b}{a^2 + b^2} \\ &= \frac{b}{a^2 + b^2} [b^2 - a^2 + 2a^2] = \frac{(a^2 + b^2)b}{(a^2 + b^2)} \\ &= b \end{aligned}$$

Q. 59 If for real values of x , $\cos \theta = x + \frac{1}{x}$, then

- (a) θ is an acute angle (b) θ is right angle
(c) θ is an obtuse angle (d) No value of θ is possible

Thinking Process

The quadratic equation $ax^2 + bx + c = 0$ has real roots, then $b^2 - 4ac \geq 0$, use this condition to solve the above problem.

Sol. (d) Here, $\cos \theta = x + \frac{1}{x}$

$$\Rightarrow \cos \theta = \frac{x^2 + 1}{x}$$

$$x^2 - x \cos \theta + 1 = 0$$

$$\text{For real value of } x, (-\cos \theta)^2 - 4 \times 1 \times 1 \geq 0$$

$$\cos^2 \theta \geq 4$$

$$\cos \theta = \pm 2$$

which is not possible.

$$[\because -1 \leq \cos \theta \leq 1]$$

Fillers

Q. 60 The value of $\frac{\sin 50^\circ}{\sin 130^\circ}$ is

Sol. Here,
$$\frac{\sin 50^\circ}{\sin 130^\circ} = \frac{\sin(180^\circ - 130^\circ)}{\sin 130^\circ}$$
$$= \frac{\sin 130^\circ}{\sin 130^\circ} = 1$$

Q. 61 If $k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, then the numerical value of k is

Sol. Here,
$$k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$$
$$= \sin 10^\circ \sin 50^\circ \sin 70^\circ$$
$$= \sin 10^\circ \cos 40^\circ \cdot \cos 20^\circ$$
$$= \frac{1}{2} \sin 10^\circ [2 \cos 40^\circ \cdot \cos 20^\circ]$$
$$= \frac{1}{2} \sin 10^\circ [\cos 60^\circ + \cos 20^\circ] \quad [\because 2 \cos x \cdot \cos y = \cos(x+y) + \cos(x-y)]$$
$$= \frac{1}{2} \sin 10^\circ \cdot \frac{1}{2} + \frac{1}{2} \sin 10^\circ \cos 20^\circ$$
$$= \frac{1}{4} \sin 10^\circ + \frac{1}{4} [\sin 30^\circ - \sin 10^\circ]$$
$$= \frac{1}{8}$$

Q. 62 If $\tan A = \frac{1 - \cos \theta}{\sin B}$, then $\tan 2A = \dots\dots\dots$.

💡 Thinking Process

Use $\cos \theta = 1 - 2 \sin^2 \frac{\theta}{2}$ and $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$.

Sol. Given that,
$$\tan A = \frac{1 - \cos B}{\sin B}$$
$$= \frac{1 - 1 + 2 \sin^2 \frac{B}{2}}{2 \sin \frac{B}{2} \cdot \cos \frac{B}{2}} = \tan \frac{B}{2}$$

Now,
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\Rightarrow \tan 2A = \frac{2 \cdot \tan \frac{B}{2}}{1 - \tan^2 \frac{B}{2}}$$

$$\Rightarrow \tan 2A = \tan B$$

Q. 63 If $\sin x + \cos x = a$, then

(i) $\sin^6 x + \cos^6 x = \dots\dots\dots$

(ii) $|\sin x - \cos x| = \dots\dots\dots$

Sol. Given that, $\sin x + \cos x = a$

On squaring both sides, we get

$$\begin{aligned} & (\sin x + \cos x)^2 = (a)^2 \\ \Rightarrow & \sin^2 x + \cos^2 x + 2\sin x \cos x = a^2 \\ \Rightarrow & \sin x \cdot \cos x = \frac{1}{2}(a^2 - 1) \end{aligned}$$

$$\begin{aligned} \text{(i) } \sin^6 x + \cos^6 x &= (\sin^2 x)^3 + (\cos^2 x)^3 \\ &= (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) \\ &= \sin^4 x + \cos^4 x - \frac{1}{4}(a^2 - 1)^2 \\ &= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \frac{1}{4}(a^2 - 1)^2 \\ &= 1 - 2 \cdot \frac{1}{4}(a^2 - 1)^2 - \frac{1}{4}(a^2 - 1)^2 = \frac{1}{4}[4 - 3(a^2 - 1)^2] \end{aligned}$$

$$\begin{aligned} \text{(ii) } |\sin x - \cos x| &= \sqrt{(\sin x - \cos x)^2} \\ &= \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x} \\ &= \sqrt{1 - 2 \cdot \frac{1}{4}(a^2 - 1)} = \sqrt{1 - a^2 + 1} = \sqrt{2 - a^2} \end{aligned}$$

Q. 64 In right angled $\triangle ABC$ with $\angle C = 90^\circ$ the equation whose roots are $\tan A$ and $\tan B$ is $\dots\dots\dots$

Sol. In right angled $\triangle ABC$, $\angle C = 90^\circ$

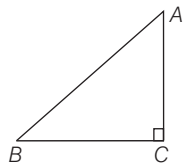
$$\therefore \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \frac{1}{0} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan A \tan B = 1 \quad \dots\text{(i)}$$

$$\begin{aligned} \tan A + \tan B &= \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} \\ &= \frac{\sin A}{\cos A} + \frac{\sin(90^\circ - A)}{\cos(90^\circ - A)} \\ &= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \\ &= \frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A} \\ &= \frac{1}{\sin A \cdot \cos A} = \frac{2}{2 \cdot \sin A \cdot \cos A} \\ &= \frac{2}{\sin 2A} \end{aligned}$$

$$[\because \angle C = 90^\circ, \angle B = 90^\circ - A]$$



$$[\because \sin 2x = 2\sin x \cos x]$$

So, the required equation is $x^2 - \left(\frac{2}{\sin A}\right)x + 1 = 0$.

Q. 65 $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = \dots\dots\dots$

Thinking Process

Use formulae i.e., $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$ and $a^2 + b^2 = (a + b)^2 - 2ab$.

Sol. Given expression, $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$$= 3[\sin^2 x + \cos^2 x - 2\sin x \cos x]^2 + 6[\sin^2 x + \cos^2 x + 2 \cdot \sin x \cdot \cos x]$$

$$+ 4[(\sin^2 x)^3 + (\cos^2 x)^3]$$

$$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4[(\sin^2 + \cos^2 x)(\sin^4 x - \sin x \cos^2 x + \cos^4 x)]$$

$$= 3(1 + \sin^2 2x - 2\sin 2x) + 6 + 6\sin 2x + 4[(\sin^2 x + \cos^2 x)^2 3\sin x \cos^2 x]$$

$$= 3 + 3\sin^2 2x - 6\sin 2x + 6 + 6\sin 2x$$

$$= 4 - 3\sin^2 2x = 13$$

Q. 66 Given $x > 0$, the value of $f(x) = -3\cos\sqrt{3 + x + x^2}$ lie in the interval
.....

Sol. Given function, $f(x) = -3\cos\sqrt{3 + x + x^2}$

We know that, $-1 \leq \cos x \leq 1$

$$\Rightarrow -3 \leq 3\cos x \leq 3$$

$$\Rightarrow 3 \geq -3\cos x \geq -3$$

$$\Rightarrow -3 \leq -3\cos x \leq 3$$

So, the value of $f(x)$ lies in $[-3, 3]$.

Q. 67 The maximum distance of a point on the graph of the function $y = \sqrt{3}\sin x + \cos x$ from X-axis is

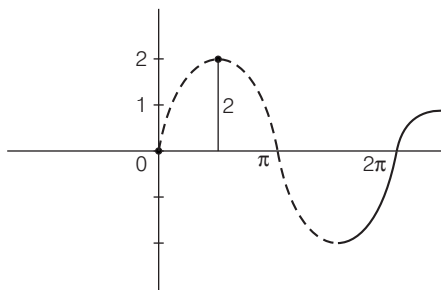
Sol. Given that, $y = \sqrt{3}\sin x + \cos x$

$$y = 2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right]$$

$$= 2\left[\sin x \cdot \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6}\right]$$

$$= 2\sin(x + \pi/6)$$

Graph of $y = 2\sin x$



Hence, the maximum distance is 2 units.

True/False

Q. 68 In each of the questions 68 to 75, state whether the statements is True or False? Also, give justification.

Thinking Process

$$\text{If } \tan A = \frac{1 - \cos B}{\sin B}, \text{ then } \tan 2A = \tan B$$

Sol. True

$$\begin{aligned} \text{Given that, } \tan A &= \frac{1 - \cos B}{\sin B} = \frac{1 - 1 + 2\sin^2 \frac{B}{2}}{2\sin \frac{B}{2} \cdot \cos \frac{B}{2}} = \tan \frac{B}{2} \\ \text{Now, } \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \tan \frac{B}{2}}{1 - \tan^2 \frac{B}{2}} = \tan B \end{aligned}$$

Q. 69 The equality $\sin A + \sin 2A + \sin 3A = 3$ holds for some real value of A .

Sol. False

Given that, $\sin A + \sin 2A + \sin 3A = 3$

It is possible only if $\sin A, \sin 2A, \sin 3A$ each has a value one because maximum value of $\sin A$ is a certain angle is 1. Which is not possible because angle are different.

Q. 70 $\sin 10^\circ$ is greater than $\cos 10^\circ$.

Sol. False

$$\begin{aligned} \sin 10^\circ &= \sin(90^\circ - 80^\circ) \\ \sin 10^\circ &= \cos 80^\circ \\ \cos 80^\circ &< \cos 10^\circ \\ \therefore \sin 10^\circ &< \cos 10^\circ \end{aligned}$$

Q. 71 $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = \frac{1}{16}$

Sol. True

$$\begin{aligned} \text{LHS} &= \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} \\ &= \cos 24^\circ \cos 48^\circ \cos 96^\circ \cos 192^\circ \\ &= \frac{1}{16 \sin 24^\circ} [(2 \sin 24^\circ \cos 24^\circ) (2 \cos 48^\circ) (2 \cos 96^\circ) (2 \cos 192^\circ)] \\ &= \frac{1}{16 \sin 24^\circ} [2 \sin 48^\circ \cos 48^\circ (2 \cos 96^\circ) (2 \cos 192^\circ)] \\ &= \frac{1}{16 \sin 24^\circ} [(2 \sin 96^\circ \cos 96^\circ) (2 \cos 192^\circ)] \\ &= \frac{1}{16 \sin 24^\circ} [2 \sin 192^\circ \cos 192^\circ] \\ &= \frac{1}{16 \sin 24^\circ} \sin 384^\circ = \frac{\sin(360^\circ + 24^\circ)}{16 \sin 24^\circ} \\ &= \frac{1}{16} = \text{RHS} \end{aligned}$$

Hence proved.

Q. 72 One value of θ which satisfies the equation $\sin^4 \theta - 2\sin^2 \theta - 1$ lies between 0 and 2π .

Sol. *False*

Given equation, $\sin^4 \theta - 2\sin^2 \theta - 1 = 0$

$$\Rightarrow \sin^2 \theta = \frac{2 \pm \sqrt{4+4}}{2}$$

$$\Rightarrow \sin^2 \theta = \frac{2 \pm 2\sqrt{2}}{2}$$

$$\Rightarrow \sin^2 \theta = (1 + \sqrt{2}) \text{ or } (1 - \sqrt{2}) \Rightarrow -1 \leq \sin \theta \leq 1$$

$$\Rightarrow \sin^2 \theta \leq 1$$

$$\therefore \sin^2 \theta = \sqrt{2} + 1 \text{ or } (1 - \sqrt{2})$$

which is not possible.

Q. 73 If $\operatorname{cosec} x = 1 + \cot x$, then $x = 2n\pi, 2n\pi + \frac{\pi}{2}$

Sol. *True*

Given that,

$$\operatorname{cosec} x = 1 + \cot x$$

$$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x} \Rightarrow \frac{1}{\sin x} = \frac{\sin x + \cos x}{\sin x}$$

$$\Rightarrow \sin x + \cos x = 1$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cdot \sin x + \frac{1}{\sqrt{2}} \cdot \cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin \frac{\pi}{4} \sin x + \cos x \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \left(x - \frac{\pi}{4} \right) = \cos \frac{\pi}{4}$$

$$\therefore x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\text{For positive sign, } x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4} = 2n\pi + \frac{\pi}{2}$$

$$\text{For negative sign, } x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{4} = 2n\pi$$

Q. 74 If $\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3}$, then $\theta = \frac{n\pi}{3} + \frac{\pi}{9}$.

Sol. *True*

$$\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3}$$

$$\Rightarrow \tan \theta + \tan 2\theta = \sqrt{3} - \sqrt{3} \tan \theta \tan 2\theta$$

$$\Rightarrow \tan \theta + \tan 2\theta = \sqrt{3} (1 - \tan \theta \tan 2\theta)$$

$$\Rightarrow \frac{\tan \theta + \tan 2\theta}{1 - \tan \theta \tan 2\theta} = \sqrt{3}$$

$$\Rightarrow \tan(\theta + 2\theta) = \tan \frac{\pi}{3} \Rightarrow \tan 3\theta = \tan \frac{\pi}{3}$$

$$\therefore 3\theta = n\pi + \frac{\pi}{3}$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{9}$$

Q. 75 If $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$, then $\cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$.

Thinking Process

Use the formulae i.e., $\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$ and $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Sol. True

We have,

$$\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$$

$$\Rightarrow \tan(\pi \cos \theta) = \tan\left[\frac{\pi}{2} - (\pi \sin \theta)\right]$$

$$\Rightarrow \pi \cos \theta = \frac{\pi}{2} - \pi \sin \theta$$

$$\Rightarrow \pi(\sin \theta + \cos \theta) = \frac{\pi}{2}$$

$$\Rightarrow \sin \theta + \cos \theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cdot \sin \theta + \frac{1}{\sqrt{2}} \cdot \cos \theta = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \sin \theta \cdot \sin \frac{\pi}{4} + \cos \theta \cdot \cos \frac{\pi}{4} = \frac{1}{2\sqrt{2}}$$

$$\therefore \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$

Q. 76 In the following match each item given under the Column I to its correct answer given under the Column II.

Column I	Column II
(i) $\sin(x + y) \sin(x - y)$	(a) $\cos^2 x - \sin^2 y$
(ii) $\cos(x + y) \cos(x - y)$	(b) $1 - \tan \theta / 1 + \tan \theta$
(iii) $\cot\left(\frac{\pi}{4} + \theta\right)$	(c) $1 + \tan \theta / 1 - \tan \theta$
(iv) $\tan\left(\frac{\pi}{4} + \theta\right)$	(d) $\sin^2 x - \sin^2 y$

Sol.

$$(i) \sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$$

$$(ii) \cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$$

$$(iii) \cot\left(\frac{\pi}{4} + \theta\right) = \frac{\cot \frac{\pi}{4} \cot \theta - 1}{\cot \frac{\pi}{4} + \cot \theta}$$

$$= \frac{-1 + \cot \theta}{1 + \cot \theta} = \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$(iv) \tan\left(\frac{\pi}{4} + \theta\right) = \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} = \frac{1 + \tan \theta}{1 - \tan \theta}$$

Hence, the correct matches are (i) $\rightarrow$ (d), (ii) $\rightarrow$ (a), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (c).