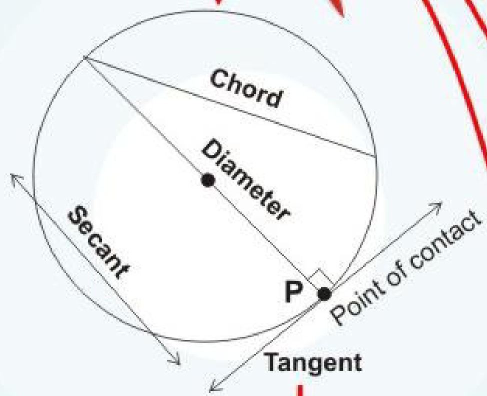
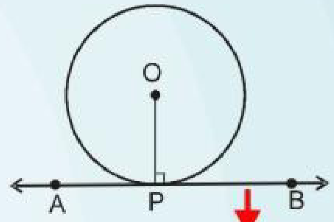
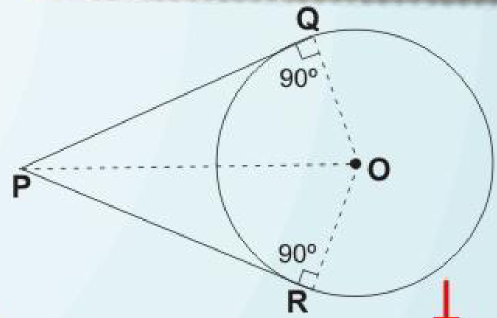
CIRCLES



Special case of secant with only one point of contact.



A tangent to a circle is perpendicular to the radius through the point of contact. i.e. $OP \perp AB$



Length of two tangent drawn from external point are equal

In ΔOQP & ΔORP

$\angle OQP = \angle ORP$ (Each 90°)

$OP = OP$, (Common

$OQ = OR$ (radius)

$\therefore \Delta OQP \cong \Delta ORP \Rightarrow PQ = PR$

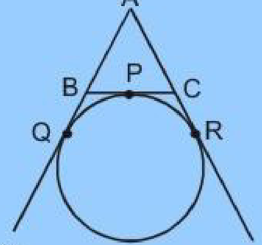
Results :

(i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. $\angle POQ = \angle POR$

(ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle OPQ = \angle OPR$

Applications

A circle touches the side BC of a ΔABC at P and touches AB and AC when produced at Q and R respectively then $AQ = \frac{1}{2}$(Perimeter of ΔABC).



$AQ = AR$....(i)

$BQ = BP$....(ii)

$CP = CR$....(iii)

[Tangents drawn from an external point to a circle are equal]

Now, perimeter of ΔABC

$= AB + BC + CA$

$= AB + BP + PC + CA$

$= (AB + BQ) + (CR + CA)$ [From (ii) and (iii)]

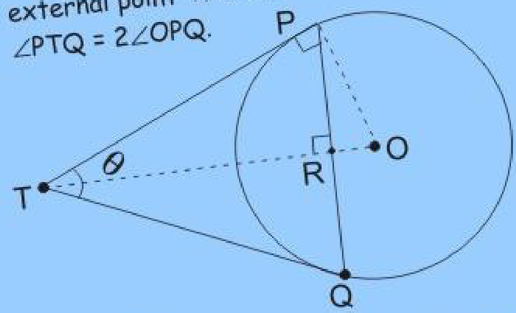
$= AQ + AR = AQ + AQ$ [From(i)]

$AQ = \frac{1}{2}$(Perimeter of ΔABC).

Special Note :

1. If quadrilateral ABCD is circumscribing a circle, then $AB + CD = AD + BC$.
2. If all the sides of a parallelogram touches a circle, then the parallelogram is a rhombus.

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



In ΔPTQ

$TP = TQ$, hence $\angle TQP = \angle QPT = 1/2\ (180 - \theta)$

Since $\angle OPT = 90°$, $\angle OPQ = \angle OPT - \angle TPQ$

$\angle OPQ = 90° - \frac{1}{2}(180° - \theta)$

$\Rightarrow \angle PTQ = 2\angle OPQ$.