

# ICSE 2024 EXAMINATION

## MATHEMATICS

### SAMPLE PAPER - 5

Time Allowed : 2½ hours

Max. Marks : 80

#### General Instructions :

*Attempt all questions from Section A and any four questions from Section B.  
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.  
Omission of essential working will result in loss of marks.  
The intended marks for questions or parts of questions are given in brackets [ ]  
Mathematical tables are provided.*

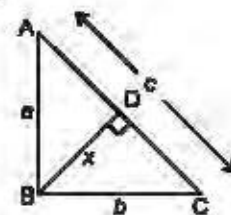
#### SECTION - A (40 Marks)

(Attempt all questions from this Section)

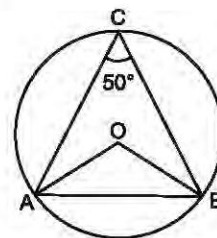
Question 1 : Choose the correct answers to the questions from the given options:

[15]

- (i) A dealer in Rohtak (Haryana) sold a table for ₹16,000 to a consumer in Sonapat (Haryana). If the GST rate is 18%, then IGST is :  
(a) ₹1440 (b) ₹2880 (c) ₹3000 (d) nil
- (ii) The roots of  $x^2 - 5x + 1 = 0$  are :  
(a) real and unequal (b) real and equal (c) imaginary (d) none of these
- (iii) On dividing  $x^2 - 4x + m$  by  $(x - 2)$ , the remainder is  $-1$ . The value of  $m$  is :  
(a) 1 (b) 2 (c)  $-2$  (d) 3
- (iv) An identity matrix is always :  
(a) a square matrix (b) rectangular matrix (c) a row matrix (d) a null matrix
- (v) The sum of  $1 + 3 + 7 + \dots + 199$  is :  
(a) 10000 (b) 9000 (c) 8000 (d) 8500
- (vi) Which of the following points is invariant with respect to the line  $y = -2x$ ?  
(a) (3, 2) (b) (3,  $-2$ ) (c) (2, 3) (d) ( $-2$ , 3)
- (vii) In the figure, the product  $ab$  is equal to :  
(a)  $c + x$   
(b)  $cx$   
(c)  $bc$   
(d)  $b + c$
- (viii) A right circular cylinder of radius  $r$  and height  $h$  ( $h > 2r$ ) just encloses a sphere of diameter:  
(a)  $r$   
(b)  $2r$   
(c)  $h$   
(d)  $2h$
- (ix) If Diha invests ₹15,500 on ₹100 shares at a premium of ₹25, then the number of shares she buys is:  
(a) 124 (b) 155 (c) 160 (d) 180
- (x) What is the probability of not picking a face card when you draw a card at random from a deck of playing cards?  
(a)  $\frac{3}{13}$  (b)  $\frac{10}{13}$  (c) 1 (d)  $\frac{2}{13}$



- (xi) 12th term of the GP 4, 8, 16, 32, ... is :  
 (a) 8000 (b) 8050 (c) 8120 (d) 8192
- (xii) The y-axis divides the line-segment joining the points  $(-4, 5)$  and  $(3, -7)$  in the ratio :  
 (a) 2 : 7 (b) 3 : 7 (c) 4 : 3 (d) 3 : 4
- (xiii) In the figure, if  $\angle ACB = 50^\circ$ , then  $\angle OAB$  is :  
 (a)  $40^\circ$   
 (b)  $50^\circ$   
 (c)  $60^\circ$   
 (d)  $70^\circ$
- (xiv) A replica of a cone is made. If their surface areas are in the ratio 4 : 25, then the ratio of their radii is:  
 (a) 4 : 25 (b) 8 : 125 (c) 2 : 5 (d) 1 : 5
- (xv) While computing mean of grouped data, we assumed that the frequencies are:  
 (a) evenly distributed over all the classes (b) centred at the class marks of the classes  
 (c) centred at the upper limits of the classes (d) centred at the lower limits of the classes



**Question 2 :**

- (i) Shalini has a cumulative time deposit account of ₹ 340 per month at 6% p.a. If she gets ₹ 7157 at the time of maturity, find the total time for which the account was held. [4]
- (ii) A man bought 1000 shares, each of face value ₹5 at ₹7 per share. At the end of the year, the company from which he bought the shares declared a dividend of 8%. Calculate [4]  
 (a) the amount of money invested by the man (b) the percentage return on his outlay
- (iii) Prove that  $\frac{1 + \cos A}{1 - \cos A} = (\operatorname{cosec} A + \cot A)^2$ . [4]

**Question 3 :**

- (i) The  $n$ th term of a sequence is  $(4^n + 7n)$ . Find the sum of first  $n$  terms of this sequence. [4]
- (ii) A  $(2, 7)$  and B  $(-3, 5)$  are two given points. Find [4]  
 (a) the gradient of AB, (b) the equation of AB.
- (iii) Use graph paper for this question. [5]  
 (Take 2 cm = 1 unit along both  $x$  and  $y$  axis)  
 Plot the points O  $(0, 0)$ , A  $(-4, 4)$ , B  $(-3, 0)$  and C  $(0, -3)$   
 (a) Reflect points A and B on the  $y$  axis and name them A' and B' respectively. Write down their coordinates.  
 (b) Name the figure OACB'A'  
 (c) State the line of symmetry of this figure.

**SECTION - B (40 Marks)**

**(Attempt any four questions from this Section)**

**Question 4 :**

- (i) Three friends X, Y and Z live in Ghaziabad (U.P.) X sells medicines worth ₹50,000 to Y, Y sells the same medicines to Z at a profit of ₹6000. If the rate of GST is 12%, find :  
 (a) SGST paid by Y (b) total CGST (c) the amount paid by Z for the medicines. [3]
- (ii) The scale of a model ship was 1 : 300. [3]  
 (a) If the length of the model is 250 cm, find the actual length in m.  
 (b) If the deck area of the model is  $1 \text{ m}^2$ , find the deck area of the ship  
 (c) If the volume of the ship is  $10,80,00,000 \text{ m}^3$ , find the volume of the model.
- (iii) A mathematics aptitude test of 50 students was recorded as follows: [4]

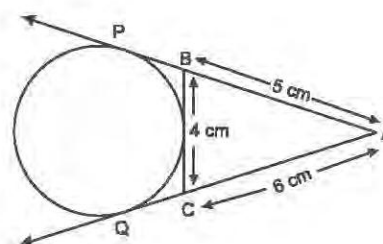
Marks	50–60	60–70	70–80	80–90	90–100
Number of students	4	8	14	19	5

Draw a histogram for the above data and locate the mode.

**Question 5 :**

- (i) Given  $A = \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ . Find the  $2 \times 2$  matrix X such that  $A + X = 2B - C$ . [3]

- (ii) In the figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then find the length of AP.



[3]

- (iii) If  $(2x + 1)$  is a factor of  $(3k + 2)x^3 + (k - 1)$ , find the value of  $k$ .

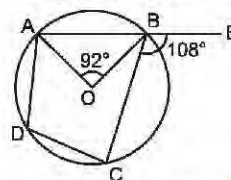
[4]

**Question 6 :**

- (i) Find the slope of the line passing through the point (2, 4) and (-2, -3). [3]  
 (ii) Draw two intersecting lines to include an angle of  $30^\circ$ . Use ruler and compasses to locate points which are equidistant from these lines and also 2 cm away from their point of intersection. How many such points exist? [3]  
 (iii) Which term of the AP 5, 12, 19, 26, 33, ... will be 35 more than its 12th term? [4]

**Question 7 :**

- (i) A card is drawn at random from a well shuffled pack of playing cards. Find the probability that the card drawn is :  
 (a) a king or a jack (b) a non ace (c) a red card [3]  
 (ii) From a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm has been removed as shown. Find the total surface area of the remaining solid. [3]  
 (iii) In the given figure, O is the centre of the circle. If  $\angle EBC = 108^\circ$  and  $\angle AOB = 92^\circ$ , calculate the value of  $\angle BDC$ . [4]

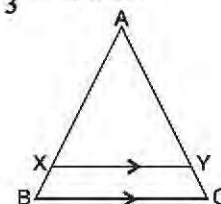


**Question 8 :**

- (i) Solve the following inequation and graph the solution set on the number line.  $3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2; x \in \mathbb{R}$  [3]  
 (ii) Find the mean of the following frequency distribution: [3]

Class	0-100	100-200	200-300	300-400	400-500
Frequency	6	9	15	12	8

- (iii) In the figure,  $XY \parallel BC$ . If AX = 3 cm, XB = 1.5 cm and BC = 6 cm, find XY.



**Question 9 :**

- (i) The distance by road between two towns A and B is 216 km and by rail it is 200 km. A car travels at a speed of  $x$  km/h and the train travels at a speed which is 16 km/h faster than the car. Calculate:  
 (a) The time taken by the car to reach town B from A, in terms of  $x$ .  
 (b) The time taken by the train to reach town B from A in terms of  $x$ .  
 (c) If the train takes 2 hours less than the car to reach town B, obtain an equation in  $x$  and solve it. Hence, find the speed of the train. [4]  
 (ii) The marks obtained by 200 students in an examination are given below : [6]

Marks	1-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	5	10	11	20	27	38	40	29	14	6

Draw an ogive for the above distribution. From the ogive, determine (i) the median (ii) the lower quartile

**Question 10 :**

- (i) Using the properties of proportion, solve for  $x$ :  $\frac{\sqrt{3x} + \sqrt{2x-1}}{\sqrt{3x} - \sqrt{2x-1}} = 5$  [3]  
 (ii) Draw a circle of radius 3.2 cm. Draw two tangents to it inclined at an angle of  $45^\circ$  to each other. [3]  
 (iii) At the foot of a mountain, the elevation of its summit is  $45^\circ$ . After ascending 500 m, towards the mountain up an incline of  $30^\circ$ , the elevation changes to  $60^\circ$ . Find the height of the mountain. [4]

## ANSWERS

1. (i) (d) (ii) (a) (iii) (d) (iv) (a) (v) (a) (vi) (b) (vii) (b) (viii) (b) (ix) (a)  
 (x) (b) (xi) (d) (xii) (c) (xiii) (a) (xiv) (c) (xv) (b)
2. (i) 20 months (ii) (a) ₹7000 (b) 5.7% 3. (i)  $\frac{1}{6} [8(4^n - 1) + 21n(n + 1)]$   
 (ii) (a)  $\frac{2}{5}$  (b)  $2x - 5y = -31$  (iii)  $(-2, 7), (3, 5)$
4. (i) (a) ₹360 (b) ₹3360 (c) ₹6720 (ii) (a) 750 m (b) 90,000 m<sup>2</sup> (c) 4 m<sup>3</sup> (iii) 82.5
5. (i)  $\begin{bmatrix} 1 & 1 \\ 4 & -5 \end{bmatrix}$  (ii) 7.5 cm (iii) 2 6. (i)  $\frac{7}{4}$  (ii) 4 (iii) 17th
7. (i) (a)  $\frac{2}{13}$  (b)  $\frac{12}{13}$  (c)  $\frac{1}{2}$  (ii) 350.09 cm<sup>2</sup> (iii) 62° 8. (i)  $\{4.8 \leq x \leq 6, x \in \mathbb{R}\}$  (ii) 264 (iii) 4 cm
9. (i) (a)  $\frac{216}{x}$  km (b)  $\frac{200}{x + 16}$  (c) 53.76 km/h 10. (i)  $x = \frac{2}{3}$  (iii) 683 m

# SOLUTION

Time Allowed : 2½ hours

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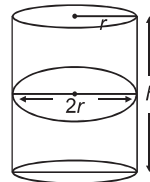
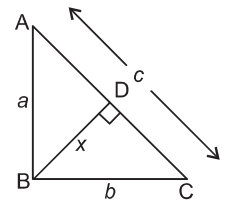
## SECTION - A (40 Marks)

(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options:

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(a) (3, 2) (b) (3,  $-2$ ) (c) (2, 3) (d) ( $-2$ , 3)
- (vii) In the figure, the product  $ab$  is equal to :  
(a)  $c + x$   
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- (ix) If Disha invests ₹15,500 on ₹100 shares at a premium of ₹25, then the number of shares she buys is:  
(a) 124 (b) 155 (c) 160 (d) 180



(x) What is the probability of not picking a face card when you draw a card at random from a deck of playing cards?

- (a)  $\frac{3}{13}$  (b)  $\frac{10}{13}$  (c) 1 (d)  $\frac{2}{13}$

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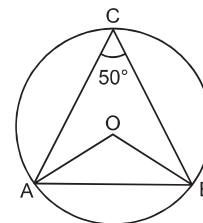
- (a) 8000 (b) 8050 (c) 8120 (d) 8192

(xii) The y-axis divides the line-segment joining the points  $(-4, 5)$  and  $(3, -7)$  in the ratio :

- (a) 2 : 7 (b) 3 : 7 (c) 4 : 3 (d) 3 : 4

(xiii) In the figure, if  $\angle ACB = 50^\circ$ , then  $\angle OAB$  is :

- (a)  $40^\circ$   
(b)  $50^\circ$   
(c)  $60^\circ$   
(d)  $70^\circ$



(xiv) A replica of a cone is made. If their surface areas are in the ratio 4 : 25, then the ratio of their radii is:

- (a) 4 : 25 (b) 8 : 125 (c) 2 : 5 (d) 1 : 5

(xv) Assertion (A) : For a data, if mean = 20 and mode = 22, then the value of median will be 20.7.

Reason (R) : The empirical relationship between mean, mode and median is given by : mean = 3 median – 2 mode

- (a) A is true, R is false (b) A is false, R is true (c) both A and R are true (d) both A and R are false

**Solution :**

(i) (d) IGST = 0

(ii) (a)  $D = (-5)^2 - 4 \times 1 \times 1 = 25 - 4 = 21 > 0$   
So, roots are real and unequal.

(iii) (d)  $2^2 - 4 \times 2 + m = -1 \Rightarrow m = 3$

(iv) (a)

(v) (a)  $199 = 1 + (n - 1) \times 2 \Rightarrow n = 100$

So,  $S_{100} = \frac{100}{2} (1 + 199) = 10000$

(vi) (b) Image of  $(3, -2)$  when reflected in the line  $y = -2$  is  $(3, 2 + 2 \times (-2))$ , i.e.,  $(3, -2)$

(vii) (b)  $\triangle ABC = \triangle ADB$

$$\Rightarrow \frac{BC}{BD} = \frac{AC}{AB} \Rightarrow \frac{b}{x} = \frac{c}{a} \Rightarrow ab = cx.$$

(viii) (b)

(ix) (a) Number of shares bought =  $\frac{15500}{125} = 124$

(x) (b) Required probability =  $\frac{40}{52} = \frac{10}{13}$

(xi) (d)  $T_{12} = 4 \times (-2)^{11} = 8192$ .

(xii) (c) Let the required ratio be m : 1

$$\text{Then } 0 = \frac{3m - 4}{m + 1} \Rightarrow m = \frac{4}{3}$$

So, required ratio = 4 : 3.

(xiii) (a)  $\angle AOB = 2\angle ACB = 100^\circ$

$$\angle OAB = \angle OBA = \frac{180^\circ - \angle AOB}{2} = 40^\circ.$$

(xiv) (c)  $\frac{S_1}{S_2} = \frac{r_1^2}{r_2^2}$

$$\Rightarrow \frac{4}{25} = \frac{r_1^2}{r_2^2} \Rightarrow \frac{r_1}{r_2} = \frac{2}{5}.$$

(xv) (a) We have, mode = 3 median – 2 mean

$$\Rightarrow 22 = 3 \text{ median} - 2 \times 20 \Rightarrow 3 \text{ median} = 22 + 40 \Rightarrow \text{Median} = \frac{62}{3} = 20.7.$$

**Question 2 :**

- (i) Shalini has a cumulative time deposit account of ₹340 per month at 6% p.a. If she gets ₹7157 at the time of maturity, find the total time for which the account was held. [4]
- (ii) A man bought 1000 shares, each of face value ₹5 at ₹7 per share. At the end of the year, the company from which he bought the shares declared a dividend of 8%. Calculate [4]
- (a) the amount of money invested by the man (b) the percentage return on his outlay
- (iii) Prove that  $\frac{1 + \cos A}{1 - \cos A} = (\operatorname{cosec} A + \cot A)^2$ . [4]

**Solution :**

$$\begin{aligned} \text{(i) Maturity value} &= Pn + \frac{Pn(n+1)}{2} \times \frac{r}{12 \times 100} \\ \Rightarrow 7157 &= 340n + \frac{340n(n+1)}{2} \times \frac{6}{12 \times 100} \\ \Rightarrow 7157 &= 340n + \frac{17n^2 + 17n}{20} \Rightarrow 7157 \times 20 = 6800n + 17n^2 + 17n \\ \Rightarrow 143140 &= 6817n + 17n^2 \Rightarrow 17n^2 + 6817n - 143140 = 0 \\ \Rightarrow n^2 + 401n - 8420 &= 0 \\ \Rightarrow n &= \frac{-401 \pm \sqrt{(401)^2 - 4 \times 1 \times (-8420)}}{2} = \frac{-401 \pm \sqrt{160801 + 33680}}{2} \\ &= \frac{-401 \pm \sqrt{194481}}{2} = \frac{-401 \pm 441}{2} \\ \Rightarrow n &= \frac{-401 + 441}{2} = \frac{40}{2} = 20 \text{ or } \frac{-401 - 441}{2} = -421 \\ \Rightarrow n &= 20 \text{ months. [Rejecting } n = -421] \text{ Ans.} \end{aligned}$$

- (ii) (a) Face value of 1000 shares = ₹5 × 1000 = ₹5000  
Market value of 1000 shares = ₹7 × 1000 = ₹7000 Ans.

- (b) Dividend earned = 8% of ₹5000 = ₹400.

$$\text{Percentage return} = \frac{400 \times 100}{7000} \% = 5.7\% \text{ Ans.}$$

$$\begin{aligned} \text{(iii) LHS} &= \frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A} \\ &= \frac{(1 + \cos A)^2}{1 - \cos^2 A} = \frac{(1 + \cos A)^2}{\sin^2 A} = \left( \frac{1 + \cos A}{\sin A} \right)^2 \\ &= \left[ \frac{1}{\sin A} + \frac{\cos A}{\sin A} \right]^2 = (\operatorname{cosec} A + \cot A)^2 = \text{RHS Proved.} \end{aligned}$$

**Question 3 :**

- (i) The  $n$ th term of a sequence is  $(4^n + 7n)$ . Find the sum of first  $n$  terms of this sequence. [4]
- (ii) A (2, 7) and B (-3, 5) are two given points. Find [4]
- (a) the gradient of AB, (b) the equation of AB.
- (iii) Use graph paper for this question. [5]
- (Take 2 cm = 1 unit along both  $x$  and  $y$  axis)
- Plot the points O (0, 0), A(-4, 4), B(-3, 0) and C(0, -3)



- (a) Reflect points A and B on the  $y$  axis and name them A' and B' respectively. Write down their coordinates.  
 (b) Name the figure OACB'A'  
 (c) State the line of symmetry of this figure.

**Solution :**

(i)  $T_n = 4^n + 7n$

$$T_1 = 4^1 + 7 \times 1$$

$$T_2 = 4^2 + 7 \times 2$$

$$T_3 = 4^3 + 7 \times 3$$

.....

$$T_n = 4^n + 7 \times n$$

$$S_n = T_1 + T_2 + T_3 + \dots + T_n$$

$$= (4^1 + 4^2 + \dots + 4^n) + 7(1 + 2 + \dots + n)$$

$$= \frac{4(4^n - 1)}{4 - 1} + 7 \times \frac{n(n + 1)}{2}$$

$$= \frac{8}{6} (4^n - 1) + \frac{21}{6} n(n + 1)$$

$$= \frac{1}{6} [8(4^n - 1) + 21n(n + 1)] \quad \text{Ans.}$$

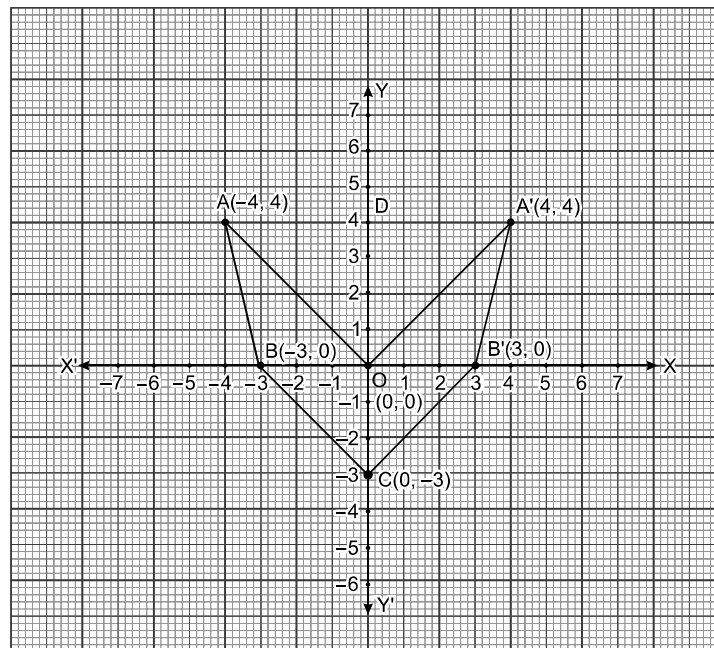
(ii) (a) Gradient of AB =  $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 7}{-3 - 2} = \frac{-2}{-5} = \frac{2}{5} \quad \text{Ans.}$

(b) Equation of line AB =  $(y - y_1) = m(x - x_1)$

$$\Rightarrow (y - 7) = \frac{2}{5}(x - 2) \Rightarrow 5y - 35 = 2x - 4 \Rightarrow 2x - 5y = -31.$$

(iii) (a) Coordinates of A' and B' are (4, 4) and (3, 0) respectively. **Ans.**

(b) OACB'A' is a hexagon (arrowhead)



(c)  $y$ -axis the line of symmetry of the given figure. **Ans.**



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**SECTION - B (40 Marks)**  
**(Attempt *any four* questions from this Section)**

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**Question 4 :**

- (i) Three friends X, Y and Z live in Ghaziabad (U.P.) X sells medicines worth ₹50,000 to Y, Y sells the same medicines to Z at a profit of ₹6000. If the rate of GST is 12%, find :

(a) SGST paid by Y                      (b) total CGST                      (c) the amount paid by Z for the medicines. [3]

- (ii) The scale of a model ship was 1 : 300. [3]

- (a) If the length of the model is 250 cm, find the actual length in m.  
 (b) If the deck area of the model is 1 m<sup>2</sup>, find the deck area of the ship  
 (c) If the volume of the ship is 10,80,00,000 m<sup>3</sup>, find the volume of the model.

- (iii) A mathematics aptitude test of 50 students was recorded as follows: [4]

Marks	50–60	60–70	70–80	80–90	90–100
Number of students	4	8	14	19	5

Draw a histogram for the above data and locate the mode.

**Solution :**

- (i) GST rate = 12%

So, SGST rate = CGST rate = 6%

- (a) SGST paid by Y = 6% of ₹6000 = ₹360 **Ans.**

- (b) Total CGST  
 = 6% of ₹(50,000 + 6,000)  
 = ₹3360 **Ans.**

- (c) Amount paid by Z  
 = ₹56,000 + 12% of (₹56,000)  
 = ₹62,720 **Ans.**

- (ii) Here scale factor,  $k = \frac{1}{300}$

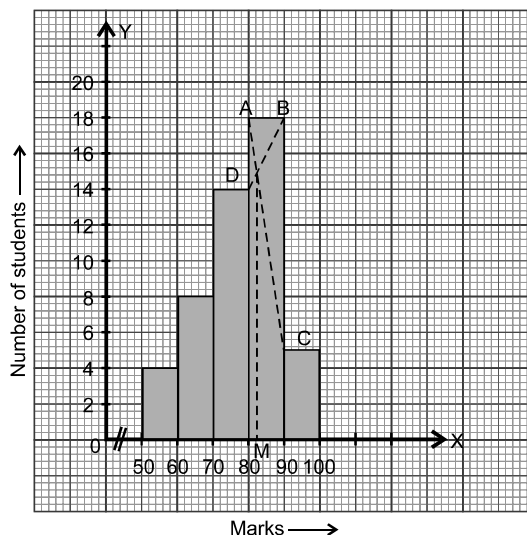
- (a) Actual length =  $\frac{1}{k} \times$  length on the map  
 =  $300 \times 250 \text{ cm} = \frac{300 \times 250}{100} \text{ m} = 750 \text{ m}$  **Ans.**

- (b) Deck area of the ship =  $\frac{1}{k^2} \times$  deck area of the model  
 =  $(300)^2 \times 1 \text{ m}^2 = 90,000 \text{ m}^2$  **Ans.**

- (c) Volume of the model =  $k^3 \times$  volume of the ship  
 =  $\frac{1}{(300)^3} \times 10,80,00,000 \text{ m}^3 = 4 \text{ m}^3$  **Ans.**

- (iii) The histogram has been shown below :

Mark the upper corners of highest rectangle and the corners of the adjoining rectangle as A, B, C, D. Join AC and BD to intersect, at point P. Draw PM  $\perp$  x-axis, abscissa of M is 82.5. So, Mode = 82.5 **Ans.**

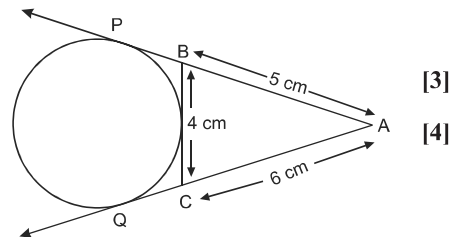


**Question 5 :**

- (i) Given  $A = \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ . Find the  $2 \times 2$  matrix X such that  $A + X = 2B - C$ . [3]

- (ii) In the figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then find the length of AP. [3]

- (iii) If  $(2x + 1)$  is a factor of  $(3k + 2)x^3 + (k - 1)$ , find the value of k. [4]



**Solution :**

- (i)  $A = \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$$2B - C = 2 \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 4 & -1 \end{bmatrix}$$

$$A + X = 2B - C$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix} + X = \begin{bmatrix} 2 & -2 \\ 4 & -1 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & -2 \\ 4 & -1 \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & -5 \end{bmatrix} \text{ Ans.}$$

- (ii) We know that tangents drawn from an external point are equal. So,

$$BP = BD \quad \dots(1)$$

$$CQ = CD \quad \dots(2)$$

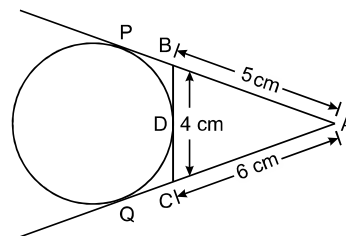
$$AP = AQ \quad \dots(3)$$

$$AP = AB + BP$$

$$\Rightarrow AP = AB + BD \quad \dots(4)$$

$$AQ = AC + CQ$$

$$\Rightarrow AQ = AC + CD \quad \dots(5)$$



Adding (4) and (5), we have

$$AP + AQ = AB + BD + AC + CD$$

$$AP + AP = AB + BC + AC$$

$$\Rightarrow 2AP = 11 + BC \Rightarrow 2AP = 11 + 4 \Rightarrow 2AP = 15 \Rightarrow AP = 7.5 \text{ Ans.}$$

$$(iii) f(x) = (3k + 2)x^3 + (k - 1)$$

$$f\left(-\frac{1}{2}\right) = 0 \Rightarrow (3k + 2)\left(-\frac{1}{2}\right)^3 + (k - 1) = 0 \Rightarrow (3k + 2)\left(-\frac{1}{8}\right) + (k - 1) = 0$$

$$\Rightarrow \frac{-3k - 2}{8} + \frac{k - 1}{1} = 0 \Rightarrow \frac{-3k - 2 + 8k - 8}{8} = 0$$

$$\Rightarrow 5k - 10 = 0 \Rightarrow k = 2. \text{ Ans.}$$

**Question 6 :**

(i) Find the slope of the line passing through the point (2, 4) and (-2, -3). [3]

(ii) Draw two intersecting lines to include an angle of  $30^\circ$ . Use ruler and compasses to locate points which are equidistant from these lines and also 2 cm away from their point of intersection. How many such points exist? [3]

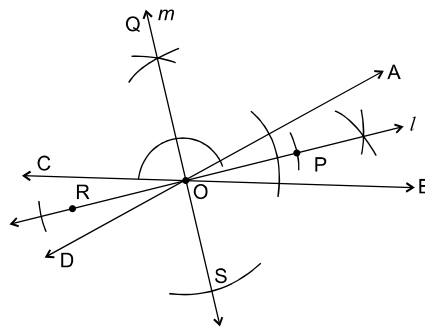
(iii) Which term of the AP 5, 12, 19, 26, 33, ... will be 35 more than its 12th term? [4]

**Solution :**

$$(i) \text{ Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 4}{-2 - 2} = \frac{-7}{-4} = \frac{7}{4} \text{ Ans.}$$

(ii) Here,  $\angle AOB = 30^\circ$

P, Q, R, S are the four points which satisfy the given conditions.



$$(iii) a_{12} = a + (12 - 1) \times d = 5 + (12 - 1) \times 7 = 5 + 11 \times 7$$

$$\therefore a_{12} = 82$$

$$a_n = a_{12} + 35$$

$$a + (n - 1) \times d = 82 + 35 \Rightarrow 5 + (n - 1) \times 7 = 117 \Rightarrow (n - 1) \times 7 = 112$$

$$\Rightarrow n - 1 = \frac{112}{7} \Rightarrow n - 1 = 16 \Rightarrow n = 17. \text{ Ans.}$$

**Question 7 :**

(i) The distance by road between two towns A and B is 216 km and by rail it is 200 km. A car travels at a speed of  $x$  km/h and the train travels at a speed which is 16 km/h faster than the car. Calculate: [5]

(a) The time taken by the car to reach town B from A, in terms of  $x$ .

(b) The time taken by the train to reach town B from A in terms of  $x$ .

(c) If the train takes 2 hours less than the car to reach town B, obtain an equation in  $x$  and solve it. Hence, find the speed of the train.

(ii) The marks obtained by 200 students in an examination are given below : [5]

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	5	10	11	20	27	38	40	29	14	6

Draw an ogive for the above distribution. From the ogive, determine (a) the median (b) the lower quartile

**Solution :**

(i) (a) Speed =  $\frac{\text{distance}}{\text{Time}}$

Time taken by car from A to B =  $\frac{216}{x}$  hrs **Ans.**

(b) Time taken by train from A to B =  $\frac{200}{x+16}$  hrs **Ans.**

(c)  $\frac{216}{x} - \frac{200}{x+16} = 2 \Rightarrow \frac{216(x+16) - 200x}{x(x+16)} = 2 \Rightarrow \frac{216x + 3456 - 200x}{x^2 + 16x} = 2$

$\Rightarrow \frac{16x + 3456}{2} = x^2 + 16x \Rightarrow 8x + 1728 = x^2 + 16x \Rightarrow x^2 + 16x - 8x - 1728 = 0 \Rightarrow x^2 + 8x - 1728 = 0$

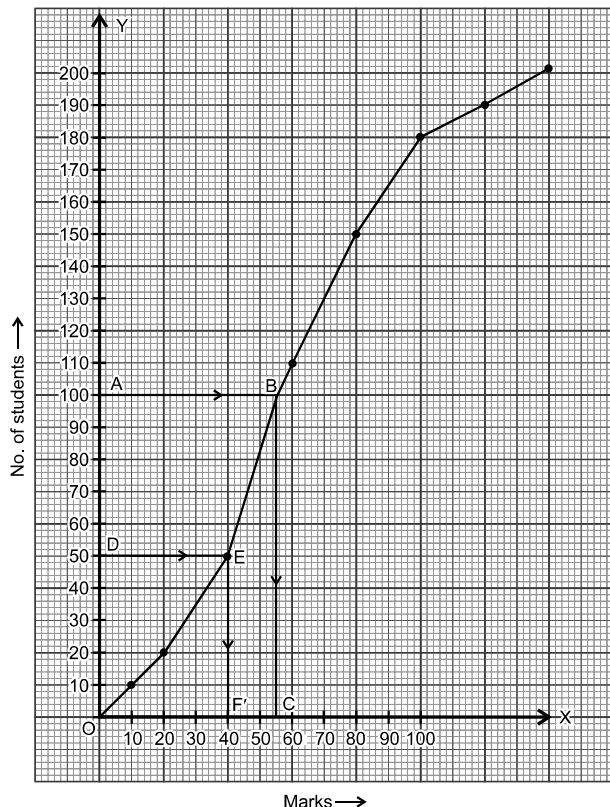
$\Rightarrow x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-1728)}}{2 \times 1} = \frac{-8 \pm \sqrt{64 + 6912}}{2} = \frac{-8 \pm \sqrt{6976}}{2} = \frac{-8 \pm 83.52}{2} = \frac{75.52}{2} = 37.76$

$\therefore$  Speed of train =  $(x + 16) = (37.76 + 16) = 53.76$  km/hr **Ans.**

(ii)

Marks	No. of Students	c.f
0 – 10	5	5
10 – 20	10	15
20 – 30	11	26
30 – 40	20	46
40 – 50	27	73
50 – 60	38	111
60 – 70	40	151
70 – 80	29	180
80 – 90	14	194
90 – 100	6	200

On a graph paper, plot the points (0, 0), (10, 5), (20, 15), (30, 26), (40, 46), (50, 73), (60, 111), (70, 151), (80, 180), (90, 194), (100, 200). Join these points by a free hand curve to get the ogive.



(a) Here,  $N = 200$ .

Therefore,  $\frac{N}{2} = \frac{200}{2} = 100$

A represents 100 on y-axis.  $AB \parallel x$ -axis.

$BC \perp x$ -axis, meeting the  $x$ -axis at C.

The abscissa of C is 57

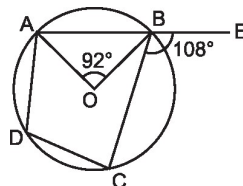
$\therefore$  Median = 57. **Ans.**

(b)  $\frac{N}{4} = \frac{200}{4} = 50$ .

From the graph lower quartile  $Q_1 = 40$  **Ans.**

**Question 8 :**

- (i) A card is drawn at random from a well shuffled pack of playing cards. Find the probability that the card drawn is :  
 (a) a king or a jack      (b) a non ace      (c) a red card [3]
- (ii) From a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm has been removed. Find the total surface area of the remaining solid. [3]
- (iii) In the given figure, O is the centre of the circle. If  $\angle EBC = 108^\circ$  and  $\angle AOB = 92^\circ$ , calculate the value of  $\angle BDC$ . [4]



**Solution :**

- (i)  $P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$
- (a)  $P(\text{a king or jack}) = \frac{8}{52} = \frac{2}{13}$ . **Ans.**
- (b)  $P(\text{a non ace}) = \frac{48}{52} = \frac{12}{13}$ . **Ans.**
- (c)  $P(\text{a red card}) = \frac{26}{52} = \frac{1}{2}$ . **Ans.**

- (ii)  $\triangle OAB \sim \triangle OCD$  [AA similarity]

$$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow AB = \frac{4 \times 6}{12} \Rightarrow AB = 2 \text{ cm. Ans.}$$

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{16 + 4} = 2\sqrt{5}$$

$$OD = \sqrt{OC^2 + CD^2} = \sqrt{144 + 36} = 6\sqrt{5}$$

Total surface area of the remaining solid

= Total surface area of the bigger cone – curved surface area of the smaller cone + base area of smaller cone.

$$= \pi RL + \pi R^2 - \pi rl + \pi r^2 = \pi R(L + R) - \pi r(l - r)$$

$$= \left[ \frac{22}{7} \times 6(6\sqrt{5} + 6) - \frac{22}{7} \times 2(2\sqrt{5} - 2) \right] \text{ cm}^2$$

$$= \left( \frac{22}{7} \times 36 \times 3.24 - \frac{22}{7} \times 4 \times 1.24 \right) \text{ cm}^2 = 350.99 \text{ cm}^2. \text{ Ans.}$$

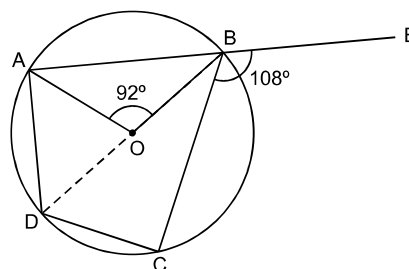
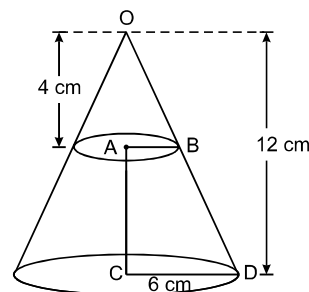
- (iii)  $\angle ADC = \angle CBE = 108^\circ$

$$\angle ADB = \frac{1}{2} \angle AOB$$

$$= \frac{1}{2} \times 92^\circ = 46^\circ$$

$$\angle BDC = \angle ADC - \angle ADB$$

$$= 108^\circ - 46^\circ = 62^\circ \text{ Ans.}$$

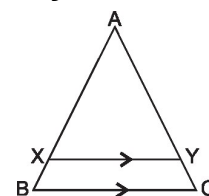


**Question 9 :**

- (i) Solve the following inequation and graph the solution set on the number line.  $3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2; x \in \mathbb{R}$  [3]
- (ii) Find the mean of the following frequency distribution: [3]

Class	0–100	100–200	200–300	300–400	400–500
Frequency	6	9	15	12	8

- (iii) In the figure,  $XY \parallel BC$ . If  $AX = 3$  cm,  $XB = 1.5$  cm and  $BC = 6$  cm, find  $XY$ . [4]

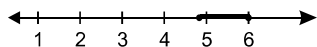


**Solution :**

$$(i) \quad 3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2, x \in \mathbb{R} \Rightarrow 3 \geq \frac{3x-12+2x}{6} \geq 2, x \in \mathbb{R} \Rightarrow 18 \geq 5x - 12 \geq 12, x \in \mathbb{R}$$

$$\Rightarrow 30 \geq 5x \geq 24, x \in \mathbb{R} \Rightarrow 6 \geq x \geq 4.8, x \in \mathbb{R}$$

Solution set =  $\{6 \geq x \geq 4.8, x \in \mathbb{R}\}$



(ii)

Class	Frequency ( $f$ )	$x$	$f \times x$
0 – 100	6	50	300
100 – 200	9	150	1350
200 – 300	15	250	3750
300 – 400	12	350	4200
400 – 500	8	450	3600
Total	50		13200

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{13200}{50} = 264. \quad \text{Ans.}$$

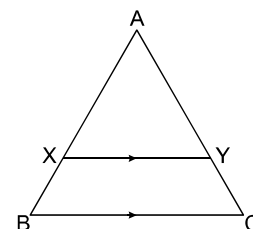
- (iii)  $XY \parallel BC \Rightarrow \angle AXY = \angle ABC$  [Corresponding  $\angle$ s]  
 $\angle AYX = \angle ACB$  [Corresponding  $\angle$ s]  
 $\Delta AXY \sim \Delta ABC$  [AA similarity]

$$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC} \quad [\text{By BPT}]$$

$$\Rightarrow \frac{AX}{AX + XB} = \frac{XY}{BC}$$

$$\Rightarrow \frac{3}{3 + 1.5} = \frac{XY}{6} \Rightarrow 4.5 XY = 18 \Rightarrow XY = \frac{18}{4.5} = 4 \text{ cm}$$

$\therefore XY = 4 \text{ cm.} \quad \text{Ans.}$



**Question 10 :**

(i) Using the properties of proportion, solve for  $x$ :  $\frac{\sqrt{3x} + \sqrt{2x-1}}{\sqrt{3x} - \sqrt{2x-1}} = 5$  [3]

(ii) Draw a circle of radius 3.2 cm. Draw two tangents to it inclined at an angle of  $45^\circ$  to each other. [3]

(iii) At the foot of a mountain, the elevation of its summit is  $45^\circ$ . After ascending 500 m, towards the mountain up an incline of  $30^\circ$ , the elevation changes to  $60^\circ$ . Find the height of the mountain. [4]

**Solution :**

(i) We have,  $\frac{\sqrt{3x} + \sqrt{2x-1}}{\sqrt{3x} - \sqrt{2x-1}} = 5$

Using componendo and dividendo, we get

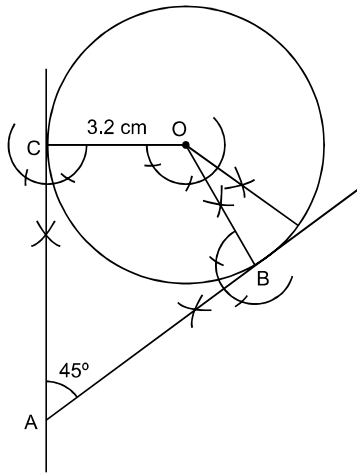
$$\frac{\sqrt{3x} + \sqrt{2x-1} + \sqrt{3x} - \sqrt{2x-1}}{\sqrt{3x} + \sqrt{2x-1} - \sqrt{3x} + \sqrt{2x-1}} = \frac{5+1}{5-1}$$

$$\Rightarrow \frac{2\sqrt{3x}}{2\sqrt{2x-1}} = \frac{6}{4}$$

On squaring both sides, we have,  $\frac{3x}{2x-1} = \frac{9}{4} \Rightarrow 18x - 9 = 12x$

$$\Rightarrow 18x - 12x = 9 \Rightarrow 6x = 9 \Rightarrow x = \frac{9}{6} \Rightarrow x = \frac{3}{2} \quad \text{Ans.}$$

(ii)



(iii) AB is the mountain of height  $h$ .

In  $\triangle OCE$ ,

$$\sin 30^\circ = \frac{CE}{OC} \Rightarrow \frac{1}{2} = \frac{CE}{500} \Rightarrow CE = 250$$

$$\cos 30^\circ = \frac{OE}{OC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OE}{500} \Rightarrow OE = 250\sqrt{3}$$

$$\text{In } \triangle AOB, \tan 45^\circ = \frac{AB}{OA}$$

$$\Rightarrow 1 = \frac{AB}{OA} \Rightarrow OA = AB = h$$

$$CD = EA = OA - OE$$

$$\Rightarrow CD = h - 250\sqrt{3}$$

$$\Rightarrow BD = AB - AD \Rightarrow BD = h - 250$$

$$\tan 60^\circ = \frac{BD}{CD}$$

$$\Rightarrow \sqrt{3} = \frac{h - 250}{h - 250\sqrt{3}} \Rightarrow \sqrt{3}h - 750 = h - 250$$

$$\Rightarrow h = \frac{500}{0.732} \approx 683 \text{ m} \quad \text{Ans.}$$

