LINEAR EQUATION IN TWO VARIABLES

1
CHAPTER

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➢ LINEAR EQUATIONS IN ONE VARIABLE

A statement of equality of two algebraic expressions, which involve one or more unknown quantities is known as an equation.

A linear equation is an equation which involves linear polynomials.

A value of the variable which makes the two sides of the equation equal is called the solution of the equation.

Same quantity can be added/subtracted to/from both the sides of an equation without changing the equality.

Both the sides of an equation can be multiplied/divided by the same non-zero number without changing the equality.

➢ GENERAL FORM OF LINEAR EQUATION IN TWO VARIABLES

$ax + by + c = 0$, $a \neq 0$, $b \neq 0$ or any one from a & b can zero.

❖ EXAMPLES ❖

Ex.1 Express the following linear equations in general form and identify coefficients of x, y and constant term.

Sol.

S.No.	Equation	General form	Coeff. of x, y, constant
(1)	$3x - 2y = 5$	$3x - 2y - 5 = 0$	3, –2, –5
(2)	$\frac{3}{7}x - 2 + y = 0$	$\frac{3}{7}x + y - 2 = 0$	$\frac{3}{7}$, 1, –2
(3)	$5y = 2x + 7$	$2x - 5y + 7 = 0$	2, –5, 7
(4)	$18y - 72x = 8$	$72x - 18y + 8 = 0$	72, –18, 8
(5)	$3.\bar{7}x - y - \frac{1}{7} = 0$	$3.\bar{7}x - y - \frac{1}{7} = 0$	$3.\bar{7}, -1, -\frac{1}{7}$
(6)	$y = 5$	$0x + y - 5 = 0$	0, 1, –5
(7)	$\frac{x}{7} = 5$	$\frac{x}{7} + 0.y - 5 = 0$	$\frac{1}{7}$, 0, –5
(8)	$2x + 3 = 0$	$2x + 0y + 3 = 0$	2, 0, 3

Ex.2 Make linear equation by the following statements :

(1) The cost of 2kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically.

Sol. Let cost of per kg apples & grapes are x & y respectively then by I[st] condition :

$$2x + y = 160 \quad ......(i)$$

& by II[nd] condition : $4x + 2y = 300$ …..(ii)

(2) The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically.

Sol. Let cost of a bat and a ball are ₹ x & ₹ y respectively. According to questions

$3x + 6y = 3900$ …..(i)

& $x + 3y = 1300$ …..(ii)

(3) 10 students of class IX took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys.

Sol. Let no. of boys and girls are x & y then according to question

$x + y = 10$ ……(i)

& $y = x + 4$ ……(ii)

(4) Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m.

Sol. Let length & breadth are x m and y m.

$\therefore$ according to question $\frac{1}{2}$ perimeter = 36

$$\frac{1}{2}\,[2(\lambda + b)] = 36$$

$\Rightarrow$ $x + y = 36$ …..(i)

also length = 4 + breadth

$x = 4 + y$ …..(ii)

(5) The difference between two numbers is 26 and one number is three times the other.

Sol. Let the numbers are x and y & x > y

$\therefore$ $x - y = 26$ ……(i)

and $x = 3y$ ……(ii)

(6) The larger of two supplementary angles exceeds the smaller by 18 degrees.

Sol. Let the two supplementary angles are x and y & x > y

Then $x + y = 180°$ ……(i)

and $x = y + 18°$ …...(ii)

(7) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$.

Sol. Let fraction is $\frac{x}{y}$

Now according to question $\frac{x+2}{y+2} = \frac{9}{11}$

$\Rightarrow 11x + 22 = 9y + 18$

$\Rightarrow 11x - 9y = -4$ …..(i)

and $\frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6x + 18 = 5y + 15$

$\Rightarrow 6x - 5y = -3$ ….(ii)

(8) Five years hence, the age of Sachin will be three times that of his son. Five years ago, Sachin's age was seven times that of his son.

Sol. Let present ages of Sachin & his son are x years and y years.

Five years hence,

age of Sachin = (x + 5) years & his son's age = (y + 5) years

according to question $(x + 5) = 3(y + 5)$

$\Rightarrow x + 5 = 3y + 15$

$\Rightarrow x - 3y = 10$ ……(i)

and 5 years ago age of both were (x – 5) years and (y – 5) years respectively

according to question $(x - 5) = 7(y - 5)$

$\Rightarrow x - 5 = 7y - 35$

$\Rightarrow x - 7y = -30$ ….(ii)

➤ SOLUTION OF LINEAR EQUATION

Method : Put the value of x (or y) = 0, ±1, ±2, ±3,……, we get values of y (or x). By this we can find many solutions of given equation.

❖ EXAMPLES ❖

Ex.3 Find five solutions of

(i) $2x + 3y = 6$ (ii) $3x - 2y = 12$

(iii) $7x + y = 15$

Sol. (i) $2x = 6 - 3y$

$\Rightarrow x = \frac{6-3y}{2}$

Now put $y = 0$, $x = \frac{6-0}{2} = 3$

for $y = 1$, $x = \frac{6-3(1)}{2} = \frac{3}{2}$

for $y = 2$, $x = \frac{6-3(2)}{2} = 0$

for $y = 3$, $x = \frac{6-3(3)}{2} = -\frac{3}{2}$

for $y = 4$, $x = \frac{6-3(4)}{2} = -3$

$\therefore$

x	3	3/2	0	−3/2	−3
y	0	1	2	3	4

(ii) $3x - 12 = 2y \Rightarrow y = \frac{3x-12}{2}$

Put value of $x = 0, 1, 2, 3, -1$

we get $y = -6, -\frac{9}{2}, -3, -\frac{3}{2}, -8$

x	0	1	2	3	−1
y	−6	−9/2	−3	−3/2	−8

(iii) $y = 15 - 7x$

Put $x = 0, 1, 2, 3, 4$ we get $y = 15, 8, 1, -6, -13$

$\therefore$

x	0	1	2	3	4
y	15	8	1	−6	−13

Ex.4 Find two solutions of

(i) $3x - 7y = 21$ (ii) $8x - 5y = 16$

Sol. (i) $3x - 7y = 21$

Put $x = 0$, $3(0) - 7y = 21$

$y = \frac{21}{-7} = -3$

$\therefore x = 0, y = -3$

and put $y = 0 \Rightarrow 3x - 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore x = 7, y = 0$

$\therefore$

x	0	7
y	−3	0

(ii) $8x - 5y = 16$

Put $x = 0 \Rightarrow 8(0) - 5y = 16$

$\Rightarrow -5y = 16 \Rightarrow y = \frac{16}{-5} = -3.2$

$\therefore x = 0, y = -3.2$

and put $y = 0 \Rightarrow 8x - 5(0) = 16$

$\Rightarrow 8x = 16 \Rightarrow x = \frac{16}{8} = 2$

$\therefore x = 2 ; y = 0$

$\therefore$

x	0	2
y	−3.2	0

Ex.5 Find five solutions of

(i) $3x = 5$

(ii) $7y = 10$

Sol. (i) The equation is only in one variable. So we have to convert into 2 variable $3x + 0.y = 5$

put $y = 0, 1, 2, 3, 4$ $x = \frac{5}{3}, \frac{5}{3}, \frac{5}{3}, \frac{5}{3}, \frac{5}{3}$

x	5/3	5/3	5/3	5/3	5/3
y	0	1	2	3	4

(ii) $7y = 10$

$\Rightarrow 0.x + 7y = 10$

put $x = 0, 1, 2, 3, 4,$

we get $y = \frac{10}{7}, \frac{10}{7}, \frac{10}{7}, \frac{10}{7}, \frac{10}{7}$

x	0	1	2	3	4
y	10/7	10/7	10/7	10/7	10/7

Note :

Ordered pair : If value of x & y are represent in form (x, y) then this form is called ordered pair form : Eg. $x = 5$, $y = \frac{7}{3}$

then ordered pair form = $\left(5, \frac{7}{3}\right)$. First part is called abscissa (x part) and second part is ordinate (y part).

Ex.6 Check the following value of x & y are solution of equation $9x - 8y = 72$ or not

(i) (0, 9) (ii) (0, – 9) (iii) (– 8, 0)

(iv) (+8, 0) (v) (1, 1) (vi) $\left(\frac{1}{3}, \frac{1}{2}\right)$

Sol. Given equation $9x - 8y = 72$

(i) LHS at point x = 0, y = 9

$= 9(0) - 8(9) = -72 \neq \text{RHS} \therefore$ No

(ii) LHS at x = 0, y = –9

$= 9(0) - 8(-9)$

$= +72 = \text{RHS} \quad \therefore$ Yes

(iii) LHS = $9(-8) - 8(0)$ (at x = –8, y = 0)

$= -72 \neq \text{RHS} \quad \therefore$ No

(iv) LHS = $9(-8) - 8(0)$ (at x = 8, y = 0)

$= 72 = \text{RHS} \quad \therefore$ Yes

(v) LHS = $9(1) - 8(1)$ (at x = 1, y = 1)

$= 9 - 8$

$= 1 \neq \text{RHS} \quad \therefore$ No

(vi) LHS = $9\left(\frac{1}{3}\right) - 8\left(\frac{1}{2}\right) \quad \left(\text{at } x = \frac{1}{3}, y = \frac{1}{2}\right)$

$= 3 - 4$

$= -1 \neq \text{RHS} \quad \therefore$ No

Ex.7 Find the value of k in equation $2x + ky = 6$ if (–2, 2) is a solution.

Sol. Θ (–2, 2) is a solution of $2x + ky = 5$

$\therefore \; 2(-2) + k(2) = 6$

$-4 + 2k = 6 \Rightarrow 2k = 6 + 4$

$k = \frac{10}{2} = 5$ **Ans.**

Ex.8 Find value of p if (4, –4) is a solution of $x - py = 8$.

Sol. $x - py = 8$

$4 - p(-4) = 8$

$4p = 8 - 4$

$4p = 4$

$p = 1$ **Ans.**

Ex.9 Find the value of a if (a, –3a) is a solution of $14x + 3y = 35$.

Sol. Put x = a and y = –3a in given equation

$14(a) + 3(-3a) = 35$

$14a - 9a = 35$

$5a = 35$

$a = 7$ **Ans.**

➤ GRAPH OF LINEAR EQUATION ax + by + c = 0 IN TWO VARIABLES, WHERE a ≠ 0, b ≠ 0

(i) **Step I :**

Obtain the linear equation, let the equation be $ax + by + c = 0$.

(ii) **Step II :**

Express y in terms of x to obtain

$y = -\left(\frac{ax + c}{b}\right)$

(iii) **Step III :**

Give any two values to x and calculate the corresponding values of y from the expression in step II to obtain two solutions, say (α_1, β_1) and (α_2, β_2). If possible take values of x as integers in such a manner that the corresponding values of y are also integers.

(iv) **Step IV :**

Plot points (α_1, β_1) and (α_2, β_2) on a graph paper.

(v) **Step V :**

Join the points marked in step IV to obtain a line. The line obtained is the graph of the equation $ax + by + c = 0$.

❖ EXAMPLES ❖

Ex.10 Draw the graph of the equation $y - x = 2$.

Sol. We have,

$y - x = 2$

$\Rightarrow \; y = x + 2$

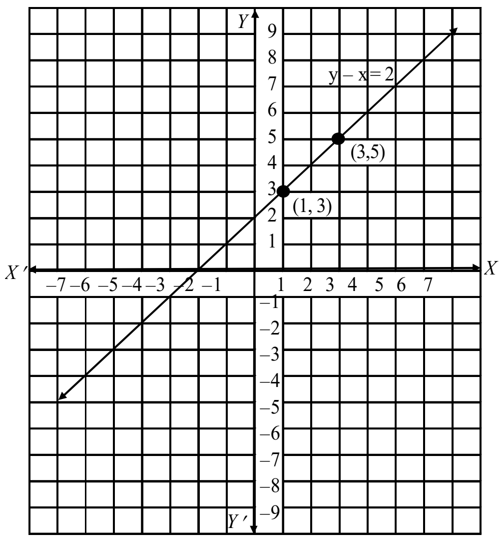
When x = 1, we have : y = 1 + 2 = 3

When x = 3, we have : y = 3 + 2 = 5

Thus, we have the following table exhibiting the abscissa and ordinates of points on the line represented by the given equation.

x	1	3
y	3	5

Plotting the points (1, 3) and (3, 5) on the graph paper and drawing a line joining them, we obtain the graph of the line represented by the given equation as shown in Fig.



Ex.11 Draw a graph of the line x – 2y = 3. From the graph, find the coordinates of the point when (i) x = – 5

(ii) y = 0.

Sol. We have x – 2y = 3

$\Rightarrow$ y = $\frac{x-3}{2}$

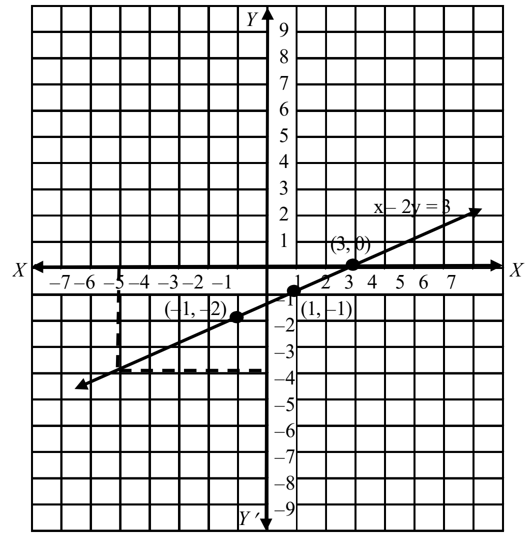
When x = 1, we have : y = $\frac{1-3}{2}$ = –1

When x = –1, we have : y = $\frac{-1-3}{2}$ = –2

Thus, we have the following table :

x	1	–1
y	–1	–2

Plotting points (1, –1) & (–1, –2) on graph paper & joining them, we get straight line as shown in fig. This line is required graph of equation x – 2y = 3.



To find the coordinates of the point when x = –5, we draw a line parallel to y-axis and passing through (–5, 0). This line meets the graph of x – 2y = 3 at a point from which we draw a line parallel to x-axis which crosses y-axis at y = –4. So, the coordinates of the required point are (–5, –4).

Since y = 0 on x-axis. So, the required point is the point where the line meets x-axis. From the graph the coordinates of such point are (3, 0).

Hence, required points are (–5, –4) and (3, 0).

Ex.12 Draw the graph of

(i) x – 7y = – 42

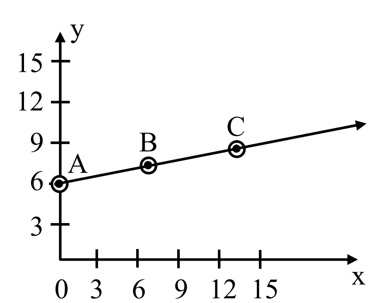
(ii) x – 3y = 6

(iii) x – y + 1 = 0

(iv) 3x + 2y = 12

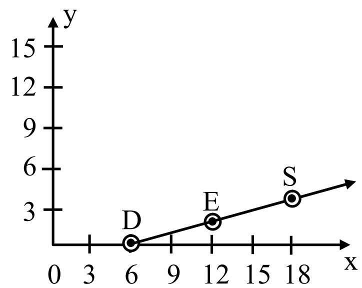
Sol. (i) x – 7y = – 42

x	0	7	14
$y = \frac{x+42}{7}$	6	7	8
Points	A	B	C



(ii) $x - 3y = 6$

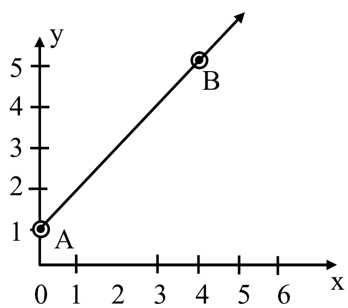
x	6	12	18
$y = \frac{x-6}{3}$	0	2	4
Points	D	E	F



(iii) $x - y + 1 = 0$

In tabular form

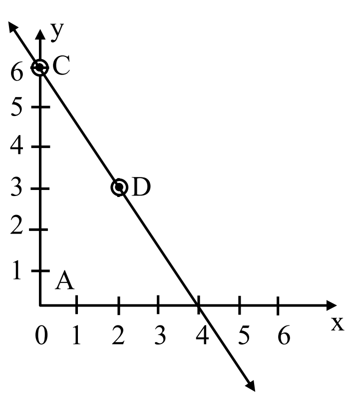
x	0	4
$y = x + 1$	1	5
Points	A	B



(iv) $3x + 2y = 12$

In tabular form

x	0	2
$y = \frac{12-3x}{2}$	6	3
Points	C	D

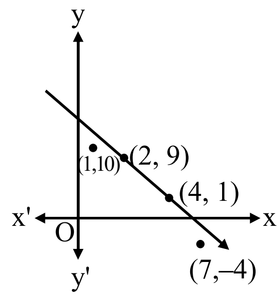


Note :

(i) The graph of any linear equation is a line and every solution of equations lies on the graph of that equation.

(ii) If a point (a, b) is not on the line then this point is not a solution of given equation.

Eg.



∵ (2, 9) and (4, 1) are on the line

∴ these two points are solution of given equation

But (1, 10) and (7, –4) are not on the line so these two are not solutions.

Ex.13 If $\left(\frac{9}{2}, 6\right)$ is lies on graph of $4x + ky = 12$ then find value of k.

Sol. ∵ $x = \frac{9}{2}$ and $y = 6$ are on the line

∴ put these value in given equation

$$4\left(\frac{9}{2}\right) + k(6) = 12$$

$$18 + 6k = 12$$

$$6k = 12 - 18$$

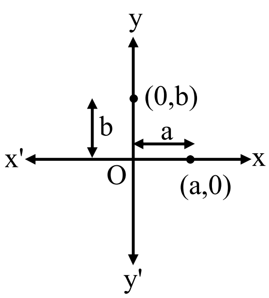
$$6k = -6$$

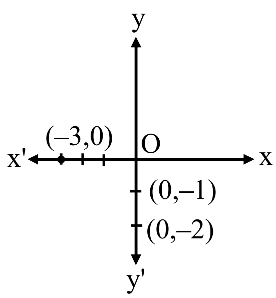
$$k = -1$$ **Ans.**

Note :

(1) Equation of x-axis is $y = 0$ and any point in ordered pair form which is on the x axis is $(\pm a, 0)$.

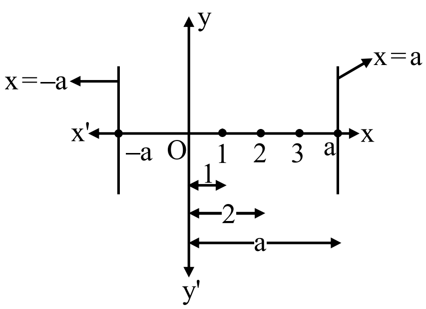
(2) Equation of y axis is $x = 0$ and any point on y axis is $(0, \pm b)$



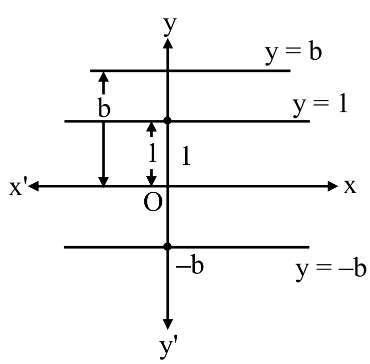


(3) Graph of line $x = \pm a$ is parallel to y axis

(4) Graph of line $y = \pm b$ is parallel to x axis



Graph of $x = -a$ and $x = +a$

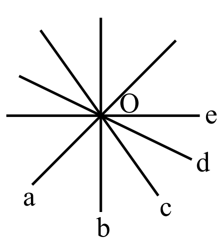


Graph of $y = 1$, $y = b$, $y = -b$

◈ **Concurrent lines :**

Three or more lines are called concurrent if all lines passes through a common point. These all lines a, b, c, d, e are passes through O.

∴ These are concurrent lines



Note :

From a point there are infinite lines can pass, so we can find (or make) infinite equations of lines which passes through a given point.

Ex.14 Find five equations of lines which passes through (3, –5).

Sol. $x + y = -2$, $x - y = 8$,

$2x + y = 1$, $2x - y = 11$,

$2x + 3y + 9 = 0$

➤ EQUATIONS OF LINES PARALLEL TO THE X-AXIS AND Y-AXIS

We can represent graph of these equations in two types of geometrically

(A) in one variable or on number line

(B) in two variable or on the Cartesian plane

In one variable, the solution is represent by a point. While in two variable, the solution is represent by a line parallel to x or y axis.

❖ EXAMPLES ❖

Ex.15 Give the geometric representation of $x = 5$ as an equation in

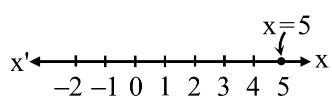
(i) one variable

(ii) two variable

(iii) also find the common solution of $x = 5$ & $x = 0$

Sol. (i) $x = 5$

it is in only one variable so representation on number line



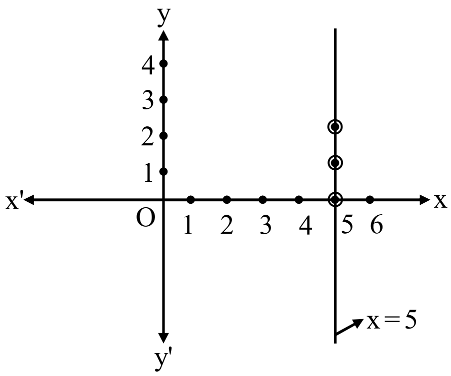
(ii) In two variables (or on Cartesian plane)

first we have to represent equation in two variables $x + 0.y = 5$ …..(i)

now we have to find two or three solutions of equations (i)

x	5	5	5
y	0	1	2

Then mark these points on graph with proper scale & join them



Scale : on both axis 10 lines or
1 big box = 1 cm

(iii) Θ x = 5 is line parallel to y axis and x = 0 is y axis.

∴ both are parallel

∴ no common solution

Ex.16 Give geometric representation of 5x + 7 = 0 as an equation

(i) in one variable (or on a number line)

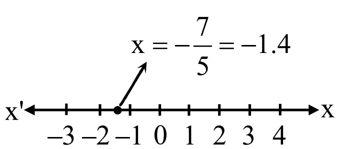
(ii) in two variable (or on Cartesian plane)

Sol. (i) $5x + 7 = 0$

$\Rightarrow \; 5x = -7$

$\Rightarrow \; x = -\frac{7}{5}$

$= -1.4$

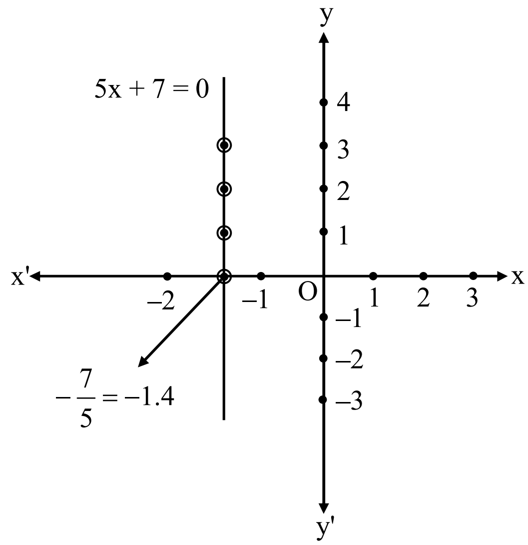


(ii) $5x + 0.y = -7$

x	− 7 / 5	− 7 / 5	− 7 / 5	− 7 / 5
y	0	1	2	3

Scale : on both axis 10 lines or 1 box

= 1 cm



Note :

If constant term 'c' is zero in equation ax + by + c = 0 then line will pass through origin (always)