

Chapter : 11. ARITHMETIC PROGRESSION

Exercise : 11A

Question: 1 A

Show that each of

Solution:

$$\text{Here, } T_2 - T_1 = 15 - 9 = 6$$

$$T_3 - T_2 = 21 - 15 = 6$$

$$T_4 - T_3 = 27 - 21 = 6$$

Since the difference between each consecutive term is same, $\therefore$ the progression is an AP.

So, first term = 9

$$\text{Common difference} = 15 - 9 = 6$$

$$\text{Next term} = T_5 = T_4 + d = 27 + 6 = 33$$

Question: 1 B

Show that each of

Solution:

$$\text{Here, } T_2 - T_1 = 6 - 11 = -5$$

$$T_3 - T_2 = 1 - 6 = -5$$

$$T_4 - T_3 = -4 - 1 = -5$$

Since the difference between each consecutive term is same, $\therefore$ the progression is an AP.

So, first term = 11

$$\text{Common difference} = 6 - 11 = -5$$

$$\text{Next term} = T_5 = T_4 + d = -4 + (-5) = -9$$

Question: 1 C

Show that each of

Solution:

$$\text{Here, } T_2 - T_1 = (-5/6) - (-1) = 1/6$$

$$T_3 - T_2 = (-2/3) - (-5/6) = 1/6$$

$$T_4 - T_3 = (-1/2) - (-2/3) = 1/6$$

Since the difference between each consecutive term is same, $\therefore$ the progression is an AP.

So, first term = -1

$$\text{Common difference} = (-5/6) - (-1) = 1/6$$

$$\text{Next term} = T_5 = T_4 + d$$

$$= (-1/2) + (1/6)$$

$$= (-2/6)$$

$$= (-1/3)$$

Question: 1 D

Show that each of

Solution:

$$\text{Here, } T_2 - T_1 = \sqrt{8} - \sqrt{2}$$

$$= 2\sqrt{2} - \sqrt{2}$$

$$= \sqrt{2}$$

$$T_3 - T_2 = \sqrt{18} - \sqrt{8}$$

$$= 3\sqrt{2} - 2\sqrt{2}$$

$$= \sqrt{2}$$

$$T_4 - T_3 = \sqrt{32} - \sqrt{18}$$

$$= 4\sqrt{2} - 3\sqrt{2}$$

$$= \sqrt{2}$$

Since the difference between each consecutive term is same, $\therefore$ the progression is an AP.

So, first term $= \sqrt{2}$

$$\text{Common difference} = \sqrt{8} - \sqrt{2} = \sqrt{2}$$

$$\text{Next term} = T_5 = T_4 + d$$

$$= \sqrt{32} + \sqrt{2}$$

$$= 4\sqrt{2} + \sqrt{2}$$

$$= 5\sqrt{2}$$

$$= \sqrt{50}$$

Question: 1 E

Show that each of

Solution:

$$\text{Here, } T_2 - T_1 = \sqrt{45} - \sqrt{20}$$

$$= 3\sqrt{5} - 2\sqrt{5}$$

$$= \sqrt{5}$$

$$T_3 - T_2 = \sqrt{80} - \sqrt{45}$$

$$= 4\sqrt{5} - 3\sqrt{5}$$

$$= \sqrt{5}$$

$$T_4 - T_3 = \sqrt{125} - \sqrt{80}$$

$$= 5\sqrt{5} - 4\sqrt{5}$$

$$= \sqrt{5}$$

Since the difference between each consecutive term is same, $\therefore$ the progression is an AP.

So, first term $= \sqrt{20}$

$$\text{Common difference} = \sqrt{45} - \sqrt{20} = \sqrt{5}$$

$$\text{Next term} = T_5 = T_4 + d$$

$$= \sqrt{125} + \sqrt{5}$$

$$= 5\sqrt{5} + \sqrt{5}$$

$$= 6\sqrt{5}$$

$$= \sqrt{180}$$

Question: 2 A

Find:

<

Solution:

Here, First term = $a = 9$

Common difference = $d = 13 - 9 = 4$

To find = 20th term, $\therefore n = 20$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = 9 + (20 - 1) \times 4$$

$$\Rightarrow a_n = 9 + 19 \times 4 = 9 + 76 = 85$$

$\therefore$ 20th term of the given AP is 85.

Question: 2 B

Find:

<

Solution:

Here, First term = $a = 20$

Common difference = $d = 17 - 20 = -3$

To find = 35th term, $\therefore n = 35$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = 20 + (35 - 1) \times (-3)$$

$$\Rightarrow a_n = 20 + 34 \times (-3) = 20 - 102 = -82$$

$\therefore$ 35th term of the given AP is -82.

Question: 2 C

Find:

<

Solution:

The given AP can be rewritten as $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, \dots$

Here, First term = $a = \sqrt{2}$

Common difference = $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

To find = 18th term, $\therefore n = 18$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = \sqrt{2} + (18 - 1) \times 2\sqrt{2}$$

$$\Rightarrow a_n = \sqrt{2} + 17 \times 2\sqrt{2} = \sqrt{2} + 34\sqrt{2} = 35\sqrt{2}$$

$\therefore$ 18th term of the given AP is $35\sqrt{2}$.

Question: 2 D

Find:

<

Solution:

Here, First term = $a = 3/4$

Common difference = $d = 5/4 - 3/4 = 2/4$

To find = 9th term, $\therefore n = 9$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = (3/4) + (9 - 1) \times (2/4)$$

$$\Rightarrow a_n = 3/4 + 8 \times (2/4) = 3/4 + 16/4 = 19/4$$

$\therefore$ 9th term of the given AP is $19/4$.

Question: 2 E

Find:

<

Solution:

Here, First term = $a = -40$

Common difference = $d = -15 - (-40) = 25$

To find = 15th term, $\therefore n = 15$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = -40 + (15 - 1) \times (25)$$

$$\Rightarrow a_n = -40 + 14 \times (25) = -40 + 350 = 310$$

$\therefore$ 15th term of the given AP is 310.

Question: 3

Find the 37th term

Solution:

The given AP can be rewritten as $6, 31/4, 19/2, 45/4, \dots$

Here, First term = $a = 6$

Common difference = $d = (31/4) - 6 = 7/4$

To find = 37th term, $\therefore n = 37$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = 6 + (37 - 1) \times (7/4)$$

$$\Rightarrow a_n = 6 + 36 \times (7/4) = 6 + 63 = 69$$

$\therefore$ 37th term of the given AP is 69.

Question: 4

Find the 25th term

Solution:

Here, First term = $a = 5$

Common difference = $d = 9/2 - 5 = - (1/2)$

To find = 25th term, $\therefore n = 25$

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = 5 + (25 - 1) \times (-1/2)$$

$$\Rightarrow a_n = 5 + 24 \times (-1/2) = 5 - 12 = - 7$$

$\therefore$ 25th term of the given AP is - 7.

Question: 5 A

Find the n^{th} term

Solution:

Here, First term = $a = 5$

Common difference = $d = 11 - 5 = 6$

To find = n^{th} term

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = 5 + (n - 1) \times 6$$

$$\Rightarrow a_n = 5 + 6n - 6 = 6n - 1$$

$\therefore n^{\text{th}}$ term of the given AP is $(6n - 1)$.

Question: 5 B

Find the n^{th} term

Solution:

Here, First term = $a = 16$

Common difference = $d = 9 - 16 = - 7$

To find = n^{th} term

Using the formula for finding n^{th} term of an A.P.,

$$a_n = a + (n - 1) \times d$$

$$\therefore a_n = 16 + (n - 1) \times (-7)$$

$$\Rightarrow a_n = 16 - 7n + 7 = 23 - 7n$$

$\therefore n^{\text{th}}$ term of the given AP is $(23 - 7n)$.

Question: 6

If the n^{th} term o

Solution:

n^{th} term of the AP is $(4n - 10)$.

For $n = 1$, we have $a_1 = 4 - 10 = - 6$

For $n = 2$, we have $a_2 = 8 - 10 = -2$

For $n = 3$, we have $a_3 = 12 - 10 = 2$

For $n = 4$, we have $a_4 = 16 - 10 = 6$, and so on.

$\therefore a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = 4 = \text{constant}$.

$\therefore$ the given progression is an AP.

Hence, (i) Its first term = $a = -6$

(ii) common difference = 4

(iii) To find : 16th term

$$\therefore a_{16} = a + (16 - 1)d$$

$$\Rightarrow a_{16} = -6 + 15 \times 4 = 54$$

$\therefore$ 16th term of the given AP is 54.

Question: 7

How many terms are

Solution:

In the given AP, the first term = $a = 6$

Common difference = $d = 10 - 6 = 4$

Last term = 174

To find: No. of terms in the AP.

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 174 = 6 + (n - 1) \times 4$$

$$\Rightarrow 174 - 6 = 4n - 4$$

$$\Rightarrow 168 = 4n - 4$$

$$\Rightarrow 168 + 4 = 4n$$

$$\Rightarrow 4n = 172$$

$$\Rightarrow n = 172/4$$

$$\Rightarrow n = 43$$

$\therefore$ Number of terms = 43.

Question: 8

How many terms are

Solution:

In the given AP, the first term = $a = 41$

Common difference = $d = 38 - 41 = -3$

Last term = 8

To find: No. of terms in the AP.

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 8 = 41 + (n - 1) \times (-3)$$

$$\Rightarrow 8 - 41 = -3n + 3$$

$$\Rightarrow -33 = -3n + 3$$

$$\Rightarrow -33 - 3 = -3n$$

$$\Rightarrow -3n = -36$$

$$\Rightarrow n = 36/3$$

$$\Rightarrow n = 12$$

$\therefore$ Number of terms = 12.

Question: 9

How many terms are

Solution:

In the given AP, the first term = $a = 18$

Common difference = $d = (31/2) - 18 = (-5/2)$

Last term = -47

To find: No. of terms in the AP.

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore -47 = 18 + (n - 1) \times (-5/2)$$

$$\Rightarrow -47 - 18 = (n - 1) \times (-5/2)$$

$$\Rightarrow -65 = (n - 1) \times (-5/2)$$

$$\Rightarrow -65 \times (-2/5) = n - 1$$

$$\Rightarrow n - 1 = 26$$

$$\Rightarrow n = 26 + 1$$

$$\Rightarrow n = 27$$

$\therefore$ Number of terms = 27.

Question: 10

Which term of the

Solution:

In the given AP, the first term = $a = 3$

Common difference = $d = 8 - 3 = 5$

To find: place of the term 88.

So, let $a_n = 88$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 88 = 3 + (n - 1) \times 5$$

$$\Rightarrow 88 - 3 = 5n - 5$$

$$\Rightarrow 85 = 5n - 5$$

$$\Rightarrow 85 + 5 = 5n$$

$$\Rightarrow 5n = 90$$

$$\Rightarrow n = 90/5$$

$$\Rightarrow n = 18$$

$\therefore$ 18th term of the AP is 88.

Question: 11

Which term of the

Solution:

In the given AP, the first term = $a = 72$

Common difference = $d = 68 - 72 = -4$

To find: place of the term 0.

So, let $a_n = 0$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 0 = 72 + (n - 1) \times (-4)$$

$$\Rightarrow 0 - 72 = -4n + 4$$

$$\Rightarrow -72 - 4 = -4n$$

$$\Rightarrow -76 = -4n$$

$$\Rightarrow n = 76/4$$

$$\Rightarrow n = 19$$

$\therefore$ 19th term of the AP is 0.

Question: 12

Which term of the

Solution:

In the given AP, the first term = $a = 5/6$

Common difference = $d = 1 - 5/6 = 1/6$

To find: place of the term 3.

So, let $a_n = 3$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 3 = (5/6) + (n - 1) \times (1/6)$$

$$\Rightarrow 3 - (5/6) = (n - 1) \times (1/6)$$

$$\Rightarrow 13/6 = (n - 1) \times (1/6)$$

$$\Rightarrow 13 = n - 1$$

$$\Rightarrow n = 13 + 1$$

$$\Rightarrow n = 14$$

$\therefore$ 14th term of the AP is 3.

Question: 13

Which term of the

Solution:

In the given AP, the first term = $a = 21$

$$\text{Common difference} = d = 18 - 21 = -3$$

To find: place of the term - 81.

$$\text{So, let } a_n = -81$$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore -81 = 21 + (n - 1) \times (-3)$$

$$\Rightarrow -81 - 21 = -3n + 3$$

$$\Rightarrow -102 = -3n + 3$$

$$\Rightarrow -102 - 3 = -3n$$

$$\Rightarrow -3n = -105$$

$$\Rightarrow n = 105/3$$

$$\Rightarrow n = 35$$

$\therefore$ 35th term of the AP is - 81.

Question: 14

Which term of the

Solution:

In the given AP, the first term = $a = 3$

$$\text{Common difference} = d = 8 - 3 = 5$$

To find: place of the term which is 55 more than its 20th term.

So, we first find its 20th term.

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore a_{20} = 3 + (20 - 1) \times 5$$

$$\Rightarrow a_{20} = 3 + 19 \times 5$$

$$\Rightarrow a_{20} = 3 + 95$$

$$\Rightarrow a_{20} = 98$$

$\therefore$ 20th term of the AP is 98.

Now, 55 more than 20th term of the AP is $55 + 98 = 153$.

So, to find: place of the term 153.

$$\text{So, let } a_n = 153$$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 153 = 3 + (n - 1) \times 5$$

$$\Rightarrow 153 - 3 = 5n - 5$$

$$\Rightarrow 150 = 5n - 5$$

$$\Rightarrow 150 + 5 = 5n$$

$$\Rightarrow 5n = 155$$

$$\Rightarrow n = 155/5 = 31$$

$\therefore 31^{\text{st}}$ term of the AP is the term which is 55 more than 20^{th} term.

Question: 15

Which term of the

Solution:

In the given AP, the first term = $a = 5$

Common difference = $d = 15 - 5 = 10$

To find: place of the term which is 130 more than its 31^{st} term.

So, we first find its 31^{st} term.

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore a_{31} = 5 + (31 - 1) \times 10$$

$$\Rightarrow a_{31} = 5 + 30 \times 10$$

$$\Rightarrow a_{31} = 5 + 300$$

$$\Rightarrow a_{31} = 305.$$

$\therefore 31^{\text{st}}$ term of the AP is 305.

Now, 130 more than 31^{st} term of the AP is $130 + 305 = 435$.

So, to find: place of the term 435.

So, let $a_n = 435$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 435 = 5 + (n - 1) \times 10$$

$$\Rightarrow 435 - 5 = 10n - 10$$

$$\Rightarrow 430 = 10n - 10$$

$$\Rightarrow 430 + 10 = 10n$$

$$\Rightarrow 10n = 440$$

$$\Rightarrow n = 440/10 = 44$$

$\therefore 44^{\text{th}}$ term of the AP is the term which is 130 more than 31^{st} term.

Question: 16

If the 10th term

Solution:

Given: 10^{th} term of the AP is 52.

17^{th} term is 20 more than the 13^{th} term.

Let the first term be a and the common difference be d .

Since,

$$a_n = a + (n - 1) \times d$$

therefore for 10^{th} term, we have,

$$52 = a + (10 - 1) \times d$$

$$\Rightarrow 52 = a + 9d \dots\dots\dots (1)$$

Now, 17th term is 20 more than the 13th term.

$$\therefore a_{17} = 20 + a_{13}$$

$$\Rightarrow a + (17 - 1)d = 20 + a + (13 - 1)d$$

$$\Rightarrow 16d = 20 + 12d$$

$$\Rightarrow 4d = 20$$

$$\Rightarrow d = 5$$

$\therefore$ from equation (1), we have,

$$52 = a + 9d$$

$$\Rightarrow 52 = a + 9 \times 5$$

$$\Rightarrow 52 = a + 45$$

$$\Rightarrow a = 52 - 45$$

$$\Rightarrow a = 7$$

$\therefore$ AP is $a, a + d, a + 2d, a + 3d, \dots$

$\therefore$ AP is 7, 12, 17, 22....

Question: 17

Find the middle t

Solution:

First term of the AP = 6

Common difference = $d = 13 - 6 = 7$

Last term = 216

Since

$$a_n = a + (n - 1) \times d$$

$$\therefore 216 = 6 + (n - 1) \times 7$$

$$\Rightarrow 216 - 6 = 7n - 7$$

$$\Rightarrow 210 = 7n - 7$$

$$\Rightarrow 210 + 7 = 7n$$

$$\Rightarrow 7n = 217$$

$$\Rightarrow n = 217/7 = 31$$

$\therefore$ Middle term is $(31 + 1)/2 = 16^{\text{th}}$

So, $a_{16} = a + (16 - 1) \times d$

$$\therefore a_{16} = 6 + 15 \times 7$$

$$\Rightarrow a_{16} = 6 + 105 = 111$$

$\therefore$ Middle term of the AP is 111.

Question: 18

Find the middle t

Solution:

First term of the AP = 10

Common difference = $d = 7 - 10 = -3$

Last term = - 62

Since

$$a_n = a + (n - 1) \times d$$

$$\therefore - 62 = 10 + (n - 1) \times (-3)$$

$$\Rightarrow - 62 - 10 = - 3n + 3$$

$$\Rightarrow - 72 = - 3n + 3$$

$$\Rightarrow - 72 - 3 = - 3n$$

$$\Rightarrow 3n = 75$$

$$\Rightarrow n = 75/3 = 25$$

$$\therefore \text{Middle term is } (25 + 1)/2 = 13^{\text{th}}$$

$$\text{So, } a_{13} = a + (13 - 1) \times d$$

$$\therefore a_{13} = 10 + 12 \times (-3)$$

$$\Rightarrow a_{13} = 10 - 36 = - 26$$

$\therefore$ Middle term of the AP is - 26.

Question: 19

Find the sum of t

Solution:

First term of the AP = - (4/3)

Common difference = d = - 1 - (-4/3) = - 1 + (4/3) = 1/3

Last term = 13/3

Since

$$a_n = a + (n - 1) \times d$$

$$\therefore 13/3 = (-4/3) + (n - 1) \times (1/3)$$

$$\Rightarrow (13/3) + (4/3) = (n - 1) \times (1/3)$$

$$\Rightarrow 17/3 = (n - 1) \times (1/3)$$

$$\Rightarrow 17 = n - 1$$

$$\Rightarrow n = 17 + 1$$

$$\Rightarrow n = 18$$

$\therefore$ Two middle most terms of the AP are 18/2 and (18/2) + 1 terms, i.e. 9th and 10th terms.

$$\text{So, } a_9 = a + (9 - 1) \times d$$

$$\therefore a_9 = (-4/3) + [8 \times (1/3)]$$

$$\Rightarrow a_9 = (-4/3) + (8/3) = 4/3$$

$$\text{Also, } a_{10} = a_9 + d$$

$$= (4/3) + (1/3)$$

$$= 5/3$$

$$\text{Now, } a_{10} + a_9 = (4/3) + (5/3)$$

$$= 9/3$$

$$= 3$$

∴ Sum of two middle most terms of the AP is 3.

Question: 20

Find the 8th term

Solution:

Here, First term = $a = 7$

Common difference = $d = 10 - 7 = 3$

Last term = $l = 184$

To find: 8th term from end.

So, nth term from end is given by:

$$a_n = l - (n - 1)d$$

∴ 8th term from end is:

$$a_8 = 184 - (8 - 1) \times 3$$

$$= 184 - 21$$

$$= 163$$

Question: 21

Find the 6th term

Solution:

Here, First term = $a = 17$

Common difference = $d = 14 - 17 = -3$

Last term = $l = -40$

To find: 6th term from end.

So, nth term from end is given by:

$$a_n = l - (n - 1)d$$

∴ 6th term from end is:

$$a_6 = -40 - (6 - 1) \times (-3)$$

$$= -40 + 15$$

$$= -25$$

Question: 22

Is 184 a term of

Solution:

Here, First term = $a = 3$

Common difference = $d = 7 - 3 = 4$

Now, to check: 184 is a term of the AP or not.

Since, nth term of an AP is given by:

$$a_n = a + (n - 1)d$$

If 184 is a term of the AP, then it must satisfy this equation.

So, let $a_n = 184$

$$\therefore 184 = 3 + (n - 1) \times 4$$

$$\Rightarrow 184 - 3 = 4n - 4$$

$$\Rightarrow 181 = 4n - 4$$

$$\Rightarrow 181 + 4 = 4n$$

$$\Rightarrow 4n = 185$$

$$\Rightarrow n = 185/4 = 46.25$$

But this is not possible because n is the number of terms which can't be a fraction.

Therefore, 184 is not a term of the given AP.

Question: 23

Is - 150 a term o

Solution:

Here, First term = $a = 11$

Common difference = $d = 8 - 11 = -3$

Now, to check: - 150 is a term of the AP or not.

Since, n^{th} term of an AP is given by:

$$a_n = a + (n - 1)d$$

If - 150 is a term of the AP, then it must satisfy this equation.

So, let $a_n = -150$

$$\therefore -150 = 11 + (n - 1) \times (-3)$$

$$\Rightarrow -150 - 11 = -3n + 3$$

$$\Rightarrow -161 = -3n + 3$$

$$\Rightarrow -161 - 3 = -3n$$

$$\Rightarrow 3n = 164$$

$$\Rightarrow n = 164/3 = 54.66$$

But this is not possible because n is the number of terms which can't be a fraction.

Therefore, - 150 is not a term of the given AP.

Question: 24

Which term of the

Solution:

Here, First term = $a = 121$

Common difference = $d = 117 - 121 = -4$

Let n^{th} term of the AP be its first negative term.

$$\therefore a_n < 0$$

Since, n^{th} term of an AP is given by:

$$a_n = a + (n - 1)d$$

$$\therefore a + (n - 1)d < 0$$

$$\Rightarrow 121 + (n - 1) \times (-4) < 0$$

$$\Rightarrow -4n + 125 < 0$$

$$\Rightarrow -4n < -125$$

$$\Rightarrow 4n > 125$$

$$\Rightarrow n > 31.25$$

Since n is an integer, therefore n must be 32.

$\therefore$ 32nd term will be the first negative term of the AP.

Question: 25

Which term of the

Solution:

Here, First term = $a = 20$

Common difference = $d = (77/4) - 20 = (-3/4)$

Let n^{th} term of the AP be its first negative term.

$$\therefore a_n < 0$$

Since, n^{th} term of an AP is given by:

$$a_n = a + (n - 1)d$$

$$\therefore a + (n - 1)d < 0$$

$$\Rightarrow 20 + (n - 1) \times (-3/4) < 0$$

$$\Rightarrow 80 + (n - 1) \times (-3) < 0 \text{ (multiplying both sides by 4)}$$

$$\Rightarrow 80 - 3n + 3 < 0$$

$$\Rightarrow -3n < -83$$

$$\Rightarrow 3n > 83$$

$$\Rightarrow n > 27.66$$

Since n is an integer, therefore n must be 28.

$\therefore$ 28th term will be the first negative term of the AP.

Question: 26

The 7th term of a

Solution:

Let a be the first term and d be the common difference.

Given: $a_7 = -4$

$$a_{13} = -16$$

Now, Consider $a_7 = -4$

$$\Rightarrow a + (7 - 1)d = -4$$

$$\Rightarrow a + 6d = -4 \text{(1)}$$

Consider $a_{13} = -16$

$$\Rightarrow a + (13 - 1)d = -16$$

$$\Rightarrow a + 12d = -16 \text{(2)}$$

Now, subtracting equation (1) from (2), we get,

$$6d = -12$$

$$\Rightarrow d = -2$$

$\therefore$ from equation (1), we get,

$$a = -4 - 6d$$

$$\Rightarrow a = -4 - 6 \times (-2)$$

$$\Rightarrow a = -4 + 12$$

$$\Rightarrow a = 8$$

Thus the AP is $a, a + d, a + 2d, a + 3d, a + 4d, \dots$

Therefore the AP is 8, 6, 4, 2, 0,

Question: 27

The 4th term of a

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_4 = 0$$

$$\text{To prove: } a_{25} = 3 \times a_{11}$$

$$\text{Now, Consider } a_4 = 0$$

$$\Rightarrow a + (4 - 1)d = 0$$

$$\Rightarrow a + 3d = 0$$

$$\Rightarrow a = -3d \dots\dots\dots (1)$$

$$\text{Consider } a_{25} = a + (25 - 1)d$$

$$\Rightarrow a_{25} = -3d + 24d \text{ (from equation (1))}$$

$$\Rightarrow a_{25} = 21d \dots\dots\dots (2)$$

$$\text{Now, consider } a_{11} = a + (11 - 1)d$$

$$\Rightarrow a_{11} = -3d + 10d \text{ (from equation (1))}$$

$$\Rightarrow a_{11} = 7d \dots\dots\dots (3)$$

From equation (2) and (3), we get,

$$a_{25} = 3 \times a_{11}$$

Hence, proved.

Question: 28

The 8th term of a

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_8 = 0$$

$$\text{To prove: } a_{38} = 3 \times a_{18}$$

$$\text{Now, Consider } a_8 = 0$$

$$\Rightarrow a + (8 - 1)d = 0$$

$$\Rightarrow a + 7d = 0$$

$$\Rightarrow a = -7d \dots\dots\dots (1)$$

$$\text{Consider } a_{38} = a + (38 - 1)d$$

$$\Rightarrow a_{38} = -7d + 37d \text{ (from equation (1))}$$

$$\Rightarrow a_{38} = 30d \dots\dots\dots (2)$$

Now, consider $a_{18} = a + (18 - 1)d$

$$\Rightarrow a_{18} = -7d + 17d \text{ (from equation (1))}$$

$$\Rightarrow a_{18} = 10d \dots\dots\dots(3)$$

From equation (2) and (3), we get,

$$a_{38} = 3 \times a_{18}$$

Hence, proved.

Question: 29

The 4th term of a

Solution:

Let a be the first term and d be the common difference.

Given: $a_4 = 11$

$$a_5 + a_7 = 34$$

To find: common difference = d

Now, Consider $a_4 = 11$

$$\Rightarrow a + (4 - 1)d = 11$$

$$\Rightarrow a + 3d = 11 \dots\dots\dots(1)$$

Consider $a_5 + a_7 = 34$

$$\Rightarrow a + (5 - 1)d + a + (7 - 1)d = 34$$

$$\Rightarrow 2a + 10d = 34$$

$$\Rightarrow a + 5d = 17 \dots\dots\dots(2)$$

Subtracting equation (1) from equation (2), we get,

$$2d = 6$$

$$\Rightarrow d = 3$$

$$\therefore \text{Common difference} = d = 3$$

Question: 30

The 9th term of a

Solution:

Let a be the first term and d be the common difference.

Given: $a_9 = -32$

$$a_{11} + a_{13} = -94$$

To find: common difference = d

Now, Consider $a_9 = -32$

$$\Rightarrow a + (9 - 1)d = -32$$

$$\Rightarrow a + 8d = -32 \dots\dots\dots(1)$$

Consider $a_{11} + a_{13} = -94$

$$\Rightarrow a + (11 - 1)d + a + (13 - 1)d = -94$$

$$\Rightarrow 2a + 22d = -94$$

$$\Rightarrow a + 11d = -47 \dots\dots\dots(2)$$

Subtracting equation (1) from equation (2), we get,

$$3d = -15$$

$$\Rightarrow d = -5$$

$$\therefore \text{Common difference} = d = -5$$

Question: 31

Determine the n th

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_7 = -1$$

$$a_{16} = 17$$

$$\text{Now, Consider } a_7 = -1$$

$$\Rightarrow a + (7 - 1)d = -1$$

$$\Rightarrow a + 6d = -1 \dots\dots\dots(1)$$

$$\text{Consider } a_{16} = 17$$

$$\Rightarrow a + (16 - 1)d = 17$$

$$\Rightarrow a + 15d = 17 \dots\dots\dots(2)$$

Now, subtracting equation (1) from (2), we get,

$$9d = 18$$

$$\Rightarrow d = 2$$

$\therefore$ from equation (1), we get,

$$a = -1 - 6d$$

$$\Rightarrow a = -1 - 6 \times (2)$$

$$\Rightarrow a = -1 - 12$$

$$\Rightarrow a = -13$$

Now, the n^{th} term of the AP is given by:

$$a_n = a + (n - 1)d$$

$$\therefore a_n = -13 + (n - 1) \times 2$$

$$\Rightarrow a_n = 2n - 15$$

$$\therefore n^{\text{th}} \text{ term of the AP is } (2n - 15)$$

Question: 32

If 4 times the 4th

Solution:

$$\text{Given: } 4 \times a_4 = 18 \times a_{18}$$

$$\text{To find : } a_{22}$$

$$\text{Consider } 4 \times a_4 = 18 \times a_{18}$$

$$\Rightarrow 4[a + (4 - 1)d] = 18[a + (18 - 1)d]$$

$$\Rightarrow 4a + 12d = 18a + 306d$$

$$\Rightarrow -14a = 294d$$

$$\Rightarrow a = -21d \dots\dots\dots(1)$$

$$\text{Now, } a_{22} = a + (22 - 1)d$$

$$\Rightarrow a_{22} = a + 21d$$

$$\Rightarrow a_{22} = -21d + 21d \text{ (from equation 1)}$$

$$\Rightarrow a_{22} = 0$$

$$\therefore a_{22} = 0$$

Question: 33

If 10 times the 1

Solution:

$$\text{Given: } 10 \times a_{10} = 15 \times a_{15}$$

$$\text{To show : } a_{25} = 0$$

$$\text{Consider } 10 \times a_{10} = 15 \times a_{15}$$

$$\Rightarrow 10 [a + (10 - 1)d] = 15 [a + (15 - 1)d]$$

$$\Rightarrow 10a + 90d = 15a + 210d$$

$$\Rightarrow -5a = 120d$$

$$\Rightarrow a = -24d \dots\dots\dots(1)$$

$$\text{Now, } a_{25} = a + (25 - 1)d$$

$$\Rightarrow a_{25} = a + 24d$$

$$\Rightarrow a_{25} = -24d + 24d \text{ (from equation 1)}$$

$$\Rightarrow a_{25} = 0$$

Hence, proved.

Question: 34

Find the common d

Solution:

Let a be the first term and d be the common difference of the AP.

$$\text{Given: } a = 5$$

$$\text{Sum of first four terms} = 1/2(\text{sum of next four terms})$$

$$\Rightarrow a + (a + d) + (a + 2d) + (a + 3d) = 1/2 ((a + 4d) + (a + 5d) + (a + 6d) + (a + 7d))$$

$$\Rightarrow 4a + 6d = 1/2(4a + 22d)$$

$$\Rightarrow 4a + 6d = 2a + 11d$$

$$\Rightarrow 2a = 5d$$

$$\Rightarrow d = 2a/5$$

As a = 5, therefore,

$$d = 10/5 = 2$$

Thus, Common difference = d = 2

Question: 35

The sum of the 2n

Solution:

Let a be the first term and d be the common difference of the AP.

$$\text{Given: } a_2 + a_7 = 30$$

$$\text{Also, } a_{15} = 2a_8 - 1$$

$$\text{Consider } a_2 + a_7 = 30$$

$$\Rightarrow (a + d) + (a + 6d) = 30$$

$$\Rightarrow 2a + 7d = 30 \dots\dots\dots (1)$$

$$\text{Consider } a_{15} = 2a_8 - 1$$

$$\Rightarrow a + 14d = 2(a + 7d) - 1$$

$$\Rightarrow a + 14d = 2a + 14d - 1$$

$$\Rightarrow a = 1$$

$$\therefore \text{First term} = a = 1$$

Thus, from equation (1), we get,

$$7d = 30 - 2a$$

$$\Rightarrow 7d = 30 - 2$$

$$\Rightarrow 7d = 28$$

$$\Rightarrow d = 4$$

Thus, the AP is $a, a + d, a + 2d, a + 3d, \dots$

Therefore, the AP is 1, 5, 9, 13, 17,...

Question: 36

For what value of

Solution:

Let a_1 and d_1 be the first term and common difference of the AP 63, 65, 67, 69,....

Let a_2 and d_2 be the first term and common difference of the AP 3, 10, 17,....

$$\therefore a_1 = 63, d_1 = 2$$

$$a_2 = 3, d_2 = 7$$

Let a_n be the n^{th} term of the first AP and b_n be the n^{th} term of the second AP.

$$\text{So, } a_n = a_1 + (n - 1)d_1$$

$$\Rightarrow a_n = 63 + (n - 1)2$$

$$\Rightarrow a_n = 61 + 2n$$

$$\text{and, } b_n = a_2 + (n - 1)d_2$$

$$\Rightarrow b_n = 3 + (n - 1)7$$

$$\Rightarrow b_n = -4 + 7n$$

Since for n^{th} terms of both the AP's to be same, $a_n = b_n$

$$\Rightarrow 61 + 2n = -4 + 7n$$

$$\Rightarrow 61 + 4 = 7n - 2n$$

$$\Rightarrow 65 = 5n$$

$$\Rightarrow n = 13$$

Therefore, 13th term of both the AP's will be same.

Question: 37

The 17th term of

Solution:

Let a and d be the first term and common difference of the AP Given: $a_{17} = 2 \times a_8 + 5$

$$a_{11} = 43$$

To find: n^{th} term = a_n

$$\text{Consider } a_{11} = 43$$

$$\Rightarrow a + (11 - 1)d = 43$$

$$\Rightarrow a + 10d = 43 \dots\dots\dots (1)$$

$$\text{Consider } a_{17} = 2 \times a_8 + 5$$

$$\Rightarrow a + (17 - 1)d = 2[a + (8 - 1)d] + 5$$

$$\Rightarrow a + 16d = 2a + 14d + 5$$

$$\Rightarrow -a + 2d = 5 \dots\dots\dots (2)$$

Adding equation (1) and equation (2), we get

$$12d = 48$$

$$\Rightarrow d = 4$$

$\therefore$ from equation (1), we get,

$$a = 43 - 10d$$

$$= 43 - 40$$

$$= 3$$

Now, n^{th} term is given by:

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_n = 3 + (n - 1)4$$

$$\Rightarrow a_n = 4n - 1$$

Therefore, n^{th} term is given by $(4n - 1)$.

Question: 38

The 24th term of

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_{24} = 2(a_{10})$$

$$\text{To prove: } a_{72} = 4 \times a_{15}$$

$$\text{Now, Consider } a_{24} = 2a_{10}$$

$$\Rightarrow a + 23d = 2[a + 9d]$$

$$\Rightarrow a + 23d = 2a + 18d$$

$$\Rightarrow a = 5d \dots\dots\dots (1)$$

$$\text{Consider } a_{72} = a + (72 - 1)d$$

$$\Rightarrow a_{72} = 5d + 71d \text{ (from equation (1))}$$

$$\Rightarrow a_{72} = 76d \dots\dots\dots (2)$$

Now, consider $a_{15} = a + (15 - 1)d$

$$\Rightarrow a_{15} = 5d + 14d \text{ (from equation (1))}$$

$$\Rightarrow a_{18} = 19d \dots\dots\dots (3)$$

From equation (2) and (3), we get,

$$a_{72} = 4 \times a_{18}$$

Hence, proved.

Question: 39

The 19th term of

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_9 = 19$$

$$a_{19} = 3 a_6$$

$$\text{Now, Consider } a_9 = 19$$

$$\Rightarrow a + (9 - 1)d = 19$$

$$\Rightarrow a + 8d = 19 \dots\dots\dots (1)$$

$$\text{Consider } a_{19} = 3 a_6$$

$$\Rightarrow a + 18d = 3(a + 5d)$$

$$\Rightarrow a + 18d = 3a + 15d$$

$$\Rightarrow 2a - 3d = 0 \dots\dots\dots (2)$$

Now, subtracting twice of equation (1) from (2), we get,

$$- 19d = - 38$$

$$\Rightarrow d = 2$$

$\therefore$ from equation (1), we get,

$$a = 19 - 8d$$

$$\Rightarrow a = 19 - 8 \times 2$$

$$\Rightarrow a = 19 - 16$$

$$\Rightarrow a = 3$$

Thus the AP is $a, a + d, a + 2d, a + 3d, a + 4d, \dots$

Therefore the AP is 3, 5, 7, 9....

Question: 40

If the p th term o

Solution:

Let a be the first term and d be common difference.

$$\text{Given: } a_p = q$$

$$a_q = p$$

$$\text{To show: } a_{(p+q)} = 0$$

We know, n th term of an AP is $a_n = a + (n - 1)d$ where, a is first term and d is common

difference Consider $a_p = q$

$$\Rightarrow a + (p - 1)d = q \quad (1)$$

Consider $a_q = p$

$$\Rightarrow a + (q - 1)d = p \quad (2)$$

Now, subtracting equation (2) from equation (1), we get

$$(p - q)d = (q - p)$$

$$\Rightarrow d = -1$$

$\therefore$ From equation (1), we get,

$$a - p + 1 = q$$

$$\Rightarrow p + q = a + 1 \dots\dots\dots(3)$$

Consider $a_{(p+q)} = a + (p + q - 1)d$

$$= a + (p + q - 1)(-1)$$

$$= a + (a + 1 - 1)(-1)$$

(putting the value of $p + q$ from equation 3)

$$= a + (-a)$$

$$= 0$$

$$\therefore a_{(p+q)} = 0$$

Hence, proved.

Question: 41

The first and last

Solution:

Let d be the common difference of the AP.

First term = a

Last term = $l = 1$

n^{th} term from beginning of an AP is given by:

$$a_n = a + (n - 1)d \dots\dots\dots(1)$$

n^{th} term from the end of an AP is given by:

$$T_n = l - (n - 1)d$$

$$= 1 - (n - 1)d \dots\dots\dots(2)$$

Sum of the n^{th} term from the beginning and end is given by:

$$a_n + T_n = a + (n - 1)d + 1 - (n - 1)d$$

$$= a + 1$$

Hence, proved.

Question: 42

How many two - digit

Solution:

The two digit numbers divisible by 6 are 12, 18, 24, 30,...96.

This forms an AP with first term $a = 12$

and common difference = $d = 6$

Last term is 96.

Now, number of terms in this AP are given as:

$$96 = a + (n - 1)d$$

$$\Rightarrow 96 = 12 + (n - 1)6$$

$$\Rightarrow 96 - 12 = 6n - 6$$

$$\Rightarrow 84 + 6 = 6n$$

$$\Rightarrow 90 = 6n$$

$$\Rightarrow n = 15$$

There are 15 two - digit numbers that are divisible by 6.

Question: 43

How many two - di

Solution:

The two digit numbers divisible by 3 are 12, 15, 18, 21, ..., 99.

This forms an AP with first term $a = 12$

and common difference = $d = 3$

Last term is 99.

Now, number of terms in this AP are given as:

$$99 = a + (n - 1)d$$

$$\Rightarrow 99 = 12 + (n - 1)3$$

$$\Rightarrow 99 - 12 = 3n - 3$$

$$\Rightarrow 87 + 3 = 3n$$

$$\Rightarrow 90 = 3n$$

$$\Rightarrow n = 30$$

There are 30 two - digit numbers that are divisible by 3.

Question: 44

How many three -

Solution:

The three digit numbers divisible by 9 are 108, 117, 126, ..., 999.

This forms an AP with first term $a = 108$

and common difference = $d = 9$

Last term is 999.

Now, number of terms in this AP are given as:

$$999 = a + (n - 1)d$$

$$\Rightarrow 999 = 108 + (n - 1)9$$

$$\Rightarrow 999 - 108 = 9n - 9$$

$$\Rightarrow 891 + 9 = 9n$$

$$\Rightarrow 900 = 9n$$

$$\Rightarrow n = 100$$

There are 100 three - digit numbers that are divisible by 9.

Question: 45

How many numbers

Solution:

The numbers between 101 and 999 that are divisible by both 2 and 5 are 110, 120, 130,..., 990.

This forms an AP with first term $a = 110$

and common difference $= d = 10$

Last term is 990.

Now, number of terms in this AP are given as:

$$990 = a + (n - 1)d$$

$$\Rightarrow 990 = 110 + (n - 1)10$$

$$\Rightarrow 990 - 110 = 10n - 10$$

$$\Rightarrow 880 + 10 = 10n$$

$$\Rightarrow 890 = 10n$$

$$\Rightarrow n = 89$$

There are 89 numbers between 101 and 999 that are divisible by both 2 and 5.

Question: 46

In a flower bed,

Solution:

The no of rose plants in each row can be arranged in the form of an AP as 43, 41, 39, ..., 11.

Here, First term $= a = 43$

Common difference $= d = 41 - 43 = -2$

No of terms in the AP = No of rows in the flower bed.

$$\therefore 11 = a + (n - 1)d$$

$$\Rightarrow 11 = 43 + (n - 1)(-2)$$

$$\Rightarrow 11 - 43 = -2n + 2$$

$$\Rightarrow 11 - 43 - 2 = -2n$$

$$\Rightarrow 2n = 34$$

$$\Rightarrow n = 17$$

$\therefore$ No of rows in the flower bed = 17

Question: 47

A sum of Rs. 2800

Solution:

Let the first prize be Rs. x . Thus each succeeding prize is Rs. 200 less than the preceding prize.

$\therefore$ Second prize is Rs. $(x - 200)$

Third prize is Rs. $(x - 400)$

Fourth prize is Rs. $(x - 600)$

This forms an AP as $x, x - 200, x - 400, x - 600$.

Since, Total sum of prize amount = 2800.

$$\therefore x + (x - 200) + (x - 400) + (x - 600) = 2800$$

$$\Rightarrow 4x - 1200 = 2800$$

$$\Rightarrow 4x = 2800 + 1200$$

$$\Rightarrow 4x = 4000$$

$$\Rightarrow x = 1000$$

Thus, the first, second, third and fourth prizes are as Rs. 1000, Rs. 800, Rs. 600, Rs. 400.

Exercise : 11B

Question: 1

Determine k so th

Solution:

If three terms are in AP, the difference between the terms should be equal, i.e. if a, b and c are in AP then, $b - a = c - b$. Since, the terms are in an AP, therefore

$$(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$$

$$\Rightarrow k - 4 = -3k + 8$$

$$\Rightarrow 4k = 12$$

$$\Rightarrow k = 3$$

$$\therefore k = 3$$

Question: 2

Find the value of

Solution:

Given: The numbers $(5x + 2)$, $(4x - 1)$ and $(x + 2)$ are in AP. **To find:** The value of x. **Solution:** Let $a_1 = (5x + 2)$, $a_2 = (4x - 1)$, $a_3 = (x + 2)$. Since, the terms are in an AP, therefore common difference is same. $\Rightarrow a_2 - a_1 = a_3 - a_2 \Rightarrow (4x - 1) - (5x + 2) = (x + 2) - (4x - 1)$

$$\Rightarrow 4x - 1 - 5x - 2 = x + 2 - 4x + 1$$

$$\Rightarrow -x - 3 = -3x + 3$$

$$\Rightarrow -x + 3x = 3 + 3$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$\therefore x = 3$$

Question: 3

If $(3y - 1)$, $(3y$

Solution:

Since, the terms are in an AP, therefore

$$(3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$$

$$\Rightarrow 6 = 2y - 4$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = 5$$

$$\therefore y = 5$$

Question: 4

Find the value of

Solution:

Given: $(x + 2)$, $2x$, $(2x + 3)$ are three consecutive terms of an AP. **To find:** the value of x **Solution:** Let $a_1 = x + 2$

$$a_2 = 2x$$

$$a_3 = 2x + 3$$

As, a_1 , a_2 and a_3 are in AP, common difference will be equal

$$\Rightarrow a_2 - a_1 = a_3 - a_2$$

$$\Rightarrow (2x) - (x + 2) = (2x + 3) - (2x) \Rightarrow 2x - x - 2 = 2x + 3 - 2x$$

$$\Rightarrow x - 2 = 3$$

$$\Rightarrow x = 5$$

Question: 5

Show that $(a - b)$

Solution:

$$\text{Consider } (a^2 + b^2) - (a - b)^2$$

$$= (a^2 + b^2) - (a^2 + b^2 - 2ab)$$

$$= 2ab$$

$$\text{Consider } (a + b)^2 - (a^2 + b^2)$$

$$= (a^2 + b^2 + 2ab) - (a^2 + b^2)$$

$$= 2ab$$

Since, the difference between consecutive terms is constant, therefore the terms are in AP.

Question: 6

Find three number

Solution:

Let the numbers be $(a - d)$, a , $(a + d)$.

Now, sum of the numbers = 15

$$\therefore (a - d) + a + (a + d) = 15$$

$$\Rightarrow 3a = 15$$

$$\Rightarrow a = 5$$

Now, product of the numbers = 80

$$\Rightarrow (a - d) \times a \times (a + d) = 80$$

$$\Rightarrow a^3 - ad^2 = 80$$

Put the value of a , we get,

$$125 - 5d^2 = 80$$

$$\Rightarrow 5d^2 = 125 - 80 = 45$$

$$d^2 = 9$$

$$d = \pm 3$$

$\therefore$ If $d = 3$, then the numbers are 2, 5, 8.

If $d = -3$, then the numbers are 8, 5, 2.

Question: 7

The sum of three

Solution:

Let the numbers be $(a - d)$, a , $(a + d)$.

Now, sum of the numbers = 15

$$\therefore (a - d) + a + (a + d) = 3$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

Now, product of the numbers = - 35

$$\Rightarrow (a - d) \times a \times (a + d) = - 35$$

$$\Rightarrow a^3 - ad^2 = - 35$$

Put the value of a , we get,

$$1 - d^2 = - 35$$

$$\Rightarrow d^2 = 35 + 1 = 36$$

$$d^2 = 36$$

$$d = \pm 6$$

$\therefore$ If $d = 6$, then the numbers are - 5, 1, 7.

If $d = - 6$, then the numbers are 7, 1, - 5.

Question: 8

Divide 24 in thre

Solution:

Let 24 be divided in numbers which are in AP as $(a - d)$, a , $(a + d)$.

Now, sum of the numbers = 24

$$\therefore (a - d) + a + (a + d) = 24$$

$$\Rightarrow 3a = 24$$

$$\Rightarrow a = 8$$

Now, product of the numbers = 440

$$\Rightarrow (a - d) \times a \times (a + d) = 440$$

$$\Rightarrow a^3 - ad^2 = 440$$

Put the value of a , we get,

$$512 - 8d^2 = 440$$

$$\Rightarrow 8d^2 = 512 - 440 = 72$$

$$d^2 = 9$$

$$d = \pm 3$$

$\therefore$ If $d = 3$, then the numbers are 5, 8, 11.

If $d = - 3$, then the numbers are 11, 8, 5.

Question: 9

The sum of three

Solution:

Let the numbers be $(a - d)$, a , $(a + d)$.

Now, sum of the numbers = 21

$$\therefore (a - d) + a + (a + d) = 21$$

$$\Rightarrow 3a = 21$$

$$\Rightarrow a = 7$$

Now, sum of the squares of the terms = 165

$$\Rightarrow (a - d)^2 + a^2 + (a + d)^2 = 165$$

$$\Rightarrow a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 165$$

$$\Rightarrow 3a^2 + 2d^2 + a = 165$$

Put the value of $a = 7$, we get,

$$3(49) + 2d^2 = 165$$

$$\Rightarrow 2d^2 = 165 - 147$$

$$\Rightarrow 2d^2 = 18$$

$$\Rightarrow d^2 = 9$$

$$\Rightarrow d = \pm 3$$

$\therefore$ If $d = 3$, then the numbers are 4, 7, 10.

If $d = -3$, then the numbers are 10, 7, 4.

Question: 10

The angles of a q

Solution:

Let these angles be x° , $(x + 10)^\circ$, $(x + 20)^\circ$ and $(x + 30)^\circ$.

Since, Sum of all angles of a quadrilateral = 360° .

$$\Rightarrow x^\circ + (x + 10)^\circ + (x + 20)^\circ + (x + 30)^\circ = 360^\circ$$

$$\Rightarrow 4x + 60^\circ = 360^\circ$$

$$\Rightarrow 4x = 300^\circ$$

$$\Rightarrow x = 75^\circ$$

$\therefore$ the angles will be 75° , 85° , 95° , 105° .

Question: 11

Find four numbers

Solution:

Let the numbers be $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$.

Now, sum of the numbers = 28

$$\therefore (a - 3d) + (a - d) + (a + d) + (a + 3d) = 28$$

$$\Rightarrow 4a = 28$$

$$\Rightarrow a = 7$$

Now, sum of the squares of the terms = 216

$$\Rightarrow (a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 216$$

$$\Rightarrow a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + d^2 + 2ad + a^2 + 9d^2 + 6ad = 216$$

$$\Rightarrow 4a^2 + 20d^2 = 216$$

Put the value of $a = 7$, we get,

$$4(49) + 20d^2 = 216$$

$$\Rightarrow 20d^2 = 216 - 196$$

$$\Rightarrow 20d^2 = 20$$

$$\Rightarrow d^2 = 1$$

$$\Rightarrow d = \pm 1$$

$\therefore$ If $d = 1$, then the numbers are 4, 6, 8, 10.

If $d = -1$, then the numbers are 10, 8, 6, 4.

Question: 12

Divide 32 into fo

Solution:

Let 32 be divided into parts as $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$.

$$\text{Now } (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$$

$$\Rightarrow 4a = 32$$

$$\Rightarrow a = 8$$

Now, we are given that product of the first and the fourth terms is to the product of the second and the third terms as 7 : 15.

$$\text{i.e. } [(a - 3d) \times (a + 3d)] : [(a - d) \times (a + d)] = 7 : 15$$

$$\Rightarrow \frac{(a - 3d) \times (a + 3d)}{(a - d) \times (a + d)} = \frac{7}{15}$$

$$\Rightarrow 15[(a - 3d) \times (a + 3d)] = 7[(a - d) \times (a + d)]$$

$$\Rightarrow 15[a^2 - 9d^2] = 7[a^2 - d^2]$$

$$\Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$$

$$\Rightarrow 8a^2 - 128d^2 = 0$$

$$\Rightarrow 8a^2 = 128d^2$$

Putting the value of a , we get,

$$512 = 128 d^2$$

$$\Rightarrow d^2 = 4$$

$$\Rightarrow d = \pm 2$$

$\therefore$ If $d = 2$, then the numbers are 2, 6, 10, 14.

If $d = -2$, then the numbers are 14, 10, 6, 2.

Question: 13

The sum of first

Solution:

Let the numbers be $(a - d)$, a , $(a + d)$.

$$\text{Now, sum of the numbers} = 48$$

$$\therefore (a - d) + a + (a + d) = 48$$

$$\Rightarrow 3a = 48$$

$$\Rightarrow a = 16$$

Now, we are given that,

Product of first and second terms exceeds 4 times the third term by 12.

$$\Rightarrow (a - d) \times a = 4(a + d) + 12$$

$$\Rightarrow a^2 - ad = 4a + 4d + 12$$

On putting the value of a in the above equation, we get,

$$256 - 16d = 64 + 4d + 12$$

$$\Rightarrow 20d = 180$$

$$\Rightarrow d = 9$$

$\therefore$ The numbers are a - d, a, a + d

i.e. the numbers are 7, 16, 25.

Exercise : 11C

Question: 1

The first three t

Solution:

Since, the terms are in an AP, therefore

$$(3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$$

$$\Rightarrow 6 = 2y - 4$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = 5$$

$$\therefore y = 5$$

Question: 2

If k, (2k - 1) an

Solution:

Since, the terms are in an AP, therefore

$$(2k - 1) - k = (2k + 1) - (2k - 1)$$

$$\Rightarrow k - 1 = 2$$

$$\Rightarrow k = 3$$

$$\therefore k = 3$$

Question: 3

If 18, a, (b - 3)

Solution:

Since, the terms are in an AP, therefore

$$a - 18 = (b - 3) - a$$

$$\Rightarrow 2a - b = -3 + 18$$

$$\Rightarrow 2a - b = 15$$

$$\therefore 2a - b = 15$$

Question: 4

If the numbers a,

Solution:

Since, the terms are in an AP, therefore

$$9 - a = b - 9 = 25 - b$$

$$\text{Consider } b - 9 = 25 - b$$

$$\Rightarrow 2b = 34$$

$$\Rightarrow b = 17$$

Now, consider the first equality,

$$9 - a = b - 9$$

$$\Rightarrow a = 18 - b$$

$$\Rightarrow a = 18 - 17$$

$$\Rightarrow a = 1$$

$$\therefore a = 1, b = 17$$

Question: 5

If the numbers (2

Solution:

Since, the terms are in an AP, therefore

$$(3n + 2) - (2n - 1) = (6n - 1) - (3n + 2)$$

$$\Rightarrow n + 3 = 3n - 3$$

$$\Rightarrow 2n = 6$$

$$\Rightarrow n = 3$$

$\therefore n = 3$, and hence the numbers are 5, 11, 17.

Question: 6

How many three -

Solution:

The three digit numbers divisible by 7 are 105, 112, 119,, 994.

This forms an AP with first term $a = 105$

and common difference $= d = 7$

Last term is 994.

Now, number of terms in this AP are given as:

$$994 = a + (n - 1)d$$

$$\Rightarrow 994 = 105 + (n - 1)7$$

$$\Rightarrow 994 - 105 = 7n - 7$$

$$\Rightarrow 889 + 7 = 7n$$

$$\Rightarrow 896 = 7n$$

$$\Rightarrow n = 128$$

Therefore 994 is the 128th term in the AP.

$\therefore$ There are 128 three - digit natural numbers that are divisible by 7.

Question: 7

How many three -

Solution:

The three digit natural numbers divisible by 9 are 108, 117, 126,, 999.

This forms an AP with first term $a = 108$

and common difference $= d = 9$

Last term is 999.

Now, number of terms in this AP are given as:

$$999 = a + (n - 1)d$$

$$\Rightarrow 999 = 108 + (n - 1)9$$

$$\Rightarrow 999 - 108 = 9n - 9$$

$$\Rightarrow 891 + 9 = 9n$$

$$\Rightarrow 900 = 9n$$

$$\Rightarrow n = 100$$

Therefore 999 is the 100th term in the AP.

$\therefore$ There are 100 three - digit natural numbers that are divisible by 9.

Question: 8

If the sum of fir

Solution:

Let S_n denotes the sum of first n terms of an AP.

$$\text{Sum of first } m \text{ terms} = S_m = 2m^2 + 3m$$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

We need to find the 2nd term, so put $n = 2$, we get

$$a_2 = S_2 - S_1$$

$$= (2(2)^2 + 3(2)) - (2(1)^2 + 3(1))$$

$$= 14 - 5$$

$$= 9$$

$\therefore$ the second term of the AP is 9.

Question: 9

What is the sum o

Solution:

Here, first term $= a$

Common difference $= 3a - a = 2a$

Now, Sum of first n terms of an AP is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

where a is the first term and d is the common difference.

$\therefore$ Sum of first n terms of given AP is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)2a]$$

$$= \frac{n}{2} [2a + 2an - 2a]$$

$$= \frac{n}{2} [2an]$$

$$= n^2a$$

Question: 10

What is the 5th term

Solution:

Here, First term = $a = 2$

Common difference = $d = 7 - 2 = 5$

Last term = $l = 47$

To find: 5th term from end.

So, n^{th} term from end is given by:

$$a_n = l - (n - 1)d$$

$\therefore$ 5th term from end is:

$$a_5 = 47 - (5 - 1) \times 5$$

$$= 47 - 20$$

$$= 27$$

$\therefore$ 5th term from the end is 27.

Question: 11

If a_n

Solution:

Here, First term = $a = 2$

Common difference = $d = 7 - 2 = 5$

To find: $a_{30} - a_{20}$

So, n^{th} term is given by:

$$a_n = a + (n - 1)d$$

$\therefore$ 30th term is:

$$a_{30} = 2 + (30 - 1) \times 5$$

$$= 2 + 145$$

$$= 147$$

Now, 20th term is:

$$a_{20} = 2 + (20 - 1) \times 5$$

$$= 2 + 95$$

$$= 97$$

Now, $(a_{30} - a_{20}) = 147 - 97$

$$= 50$$

$\therefore (a_{30} - a_{20}) = 50$

Question: 12

The n^{th}

Solution:

$$n^{\text{th}} \text{ term of an AP} = a_n = 3n + 5$$

Common difference (= d) of an AP is the difference between a term and its preceding term.

$$\therefore d = a_n - a_{n-1}$$

$$= (3n + 5) - (3(n - 1) + 5)$$

$$= 3n + 5 - 3n + 3 - 5$$

$$= 3$$

$$\therefore \text{Common difference} = 3$$

Question: 13

The n^{th} term of a

Solution:

$$n^{\text{th}} \text{ term of an AP} = a_n = 7 - 4n$$

Common difference (= d) of an AP is the difference between a term and its preceding term.

$$\therefore d = a_n - a_{n-1}$$

$$= (7 - 4n) - (7 - 4(n - 1))$$

$$= 7 - 4n - 7 + 4n - 4$$

$$= -4$$

$$\therefore \text{Common difference} = -4.$$

Question: 14

Write the next te

Solution:

$$\text{Here, first term} = \sqrt{8}$$

$$\text{Common difference} = \sqrt{18} - \sqrt{8} = \sqrt{2}$$

$$\text{Next term} = T_4 = T_3 + d$$

$$= \sqrt{32} + \sqrt{2}$$

$$= 4\sqrt{2} + \sqrt{2}$$

$$= 5\sqrt{2}$$

$$= \sqrt{50}$$

Question: 15

Write the next te

Solution:

$$\text{Here, first term} = \sqrt{2}$$

$$\text{Common difference} = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

$$\text{Next term} = T_4 = T_3 + d$$

$$= \sqrt{18} + \sqrt{2}$$

$$= 3\sqrt{2} + \sqrt{2}$$

$$= 4\sqrt{2}$$

$$= \sqrt{32}$$

Question: 16

Which term of the

Solution:

Here first term = 21

Common difference = $18 - 21 = -3$

Let a_n be the term which is zero.

$$\therefore a_n = 0$$

$$\Rightarrow a + (n - 1)d = 0$$

$$\Rightarrow 21 + (n - 1)(-3) = 0$$

$$\Rightarrow 21 - 3n + 3 = 0$$

$$\Rightarrow 3n = 24$$

$$\Rightarrow n = 8$$

$\therefore 8^{\text{th}}$ term of the given AP will be zero.

Question: 17

Find the sum of f

Solution:

First n natural numbers are 1, 2, 3,..., n .

To find: sum of these n natural numbers.

The natural numbers forms an AP with first term 1 and common difference 1.

Now, Sum of first n terms of an AP is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

where a is the first term and d is the common difference.

$\therefore$ Sum of first n natural numbers is given by:

$$S_n = \frac{n}{2} [2(1) + (n - 1)(1)]$$

$$= \frac{n}{2} [2 + n - 1]$$

$$= \frac{n}{2} [n + 1]$$

$\therefore$ Sum of first n natural numbers is $n(n + 1)/2$.

Question: 18

Find the sum of f

Solution:

First n even natural numbers are 2, 4, 6,..., $2n$.

To find: sum of these n even natural numbers.

The even natural numbers forms an AP with first term 2 and common difference 2.

Now, Sum of first n terms of an AP is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

where a is the first term and d is the common difference.

∴ Sum of first n natural numbers is given by:

$$S_n = \frac{n}{2} [2(2) + (n - 1)(2)]$$

$$= \frac{n}{2} [4 + 2n - 2]$$

$$= \frac{n}{2} [2n + 2]$$

$$= n(n + 1)$$

∴ Sum of first n even natural numbers is $n(n + 1)$.

Question: 19

The first term of

Solution:

Here, given: first term = p

Common difference = q

To find: a_{10}

$$a_{10} = a + (10 - 1)d$$

$$\Rightarrow a_{10} = p + 9q$$

∴ 10th term of the given AP will be $p + 9q$.

Question: 20

If $4/5$, a, 2 are

Solution:

Since, the terms are in an AP, therefore

$$a - (4/5) = 2 - a$$

$$\Rightarrow 2a = 2 + (4/5)$$

$$\Rightarrow 2a = 14/5$$

$$\Rightarrow a = 14/10$$

$$\Rightarrow a = 7/5$$

$$\therefore a = 7/5$$

Question: 21

If $(2p + 1)$, 13,

Solution:

Since, the terms are in an AP, therefore

$$13 - (2p + 1) = (5p - 3) - (13)$$

$$\Rightarrow 12 - 2p = 5p - 16$$

$$\Rightarrow 7p = 28$$

$$\Rightarrow p = 4$$

$$\therefore p = 4$$

Question: 22

If $(2p - 1)$, 7, 3

Solution:

Since, the terms are in an AP, therefore

$$7 - (2p - 1) = 3p - 7$$

$$\Rightarrow 8 - 2p = 3p - 7$$

$$\Rightarrow 5p = 15$$

$$\Rightarrow p = 3$$

$$\therefore p = 3$$

Question: 23

If the sum of fir

Solution:

Let S_p denotes the sum of first p terms of an AP.

$$\text{Sum of first } p \text{ terms} = S_p = ap^2 + bp$$

Then p^{th} term is given by: $a_p = S_p - S_{p-1}$

$$\therefore a_p = (ap^2 + bp) - [a(p-1)^2 + b(p-1)]$$

$$= (ap^2 + bp) - [a(p^2 + 1 - 2p) + bp - b]$$

$$= ap^2 + bp - ap^2 - a + 2ap - bp + b$$

$$= b - a + 2ap$$

$$\text{Now, common difference} = d = a_p - a_{p-1}$$

$$= b - a + 2ap - [b - a + 2a(p-1)]$$

$$= b - a + 2ap - b + a - 2ap + 2a$$

$$= 2a$$

$$\therefore \text{common difference} = 2a$$

ALITER: Let S_p denotes the sum of first p terms of an AP.

$$\text{Sum of first } p \text{ terms} = S_p = ap^2 + bp$$

$$\text{Put } p = 1, \text{ we get } S_1 = a + b$$

$$\text{Put } p = 2, \text{ we get } S_2 = 4a + 2b$$

$$\text{Now } S_1 = a_1$$

$$a_2 = S_2 - S_1$$

$$\therefore a_2 = 3a + b$$

$$\text{Now, } d = a_2 - a_1$$

$$= 3a + b - (a + b)$$

$$= 2a$$

$$\therefore \text{Common difference} = 2a$$

Question: 24

If the sum of fir

Solution:

Let S_n denotes the sum of first n terms of an AP.

$$\text{Sum of first } n \text{ terms} = S_n = 3n^2 + 5n$$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

$$\begin{aligned}
\therefore a_n &= (3n^2 + 5n) - [3(n-1)^2 + 5(n-1)] \\
&= (3n^2 + 5n) - [3(n^2 + 1 - 2n) + 5n - 5] \\
&= 3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5 \\
&= 2 + 6n
\end{aligned}$$

Now, common difference = $d = a_n - a_{n-1}$

$$\begin{aligned}
&= 2 + 6n - [2 + 6(n-1)] \\
&= 2 + 6n - 2 - 6n + 6 \\
&= 6
\end{aligned}$$

$\therefore$ Common difference = 6

ALITER: Let S_n denotes the sum of first n terms of an AP.

$$\text{Sum of first } n \text{ terms} = S_n = 3n^2 + 5n$$

$$\text{Put } n = 1, \text{ we get } S_1 = 8$$

$$\text{Put } n = 2, \text{ we get } S_2 = 22$$

$$\text{Now } S_1 = a_1$$

$$a_2 = S_2 - S_1$$

$$\therefore a_2 = 22 - 8 = 14$$

$$\text{Now, } d = a_2 - a_1$$

$$= 14 - 8$$

$$= 6$$

$\therefore$ Common difference = 6

Question: 25

Find an AP whose

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_4 = 9$$

$$a_6 + a_{13} = 40$$

$$\text{Now, Consider } a_4 = 9$$

$$\Rightarrow a + (4-1)d = 9$$

$$\Rightarrow a + 3d = 9 \dots\dots\dots(1)$$

$$\text{Consider } a_6 + a_{13} = 40$$

$$\Rightarrow a + (6-1)d + a + (13-1)d = 40$$

$$\Rightarrow 2a + 17d = 40 \dots\dots\dots(2)$$

Subtracting twice of equation (1) from equation (2), we get,

$$11d = 22$$

$$\Rightarrow d = 2$$

$\therefore$ Common difference = $d = 2$

Now from equation (1), we get

$$a = 9 - 3d$$

$$= 9 - 6$$

$$= 3$$

$\therefore$ AP is $a, a + d, a + 2d, a + 3d, \dots$

i.e. AP is 3, 5, 7, 9, 11,

Exercise : 11D

Question: 1 A

Find the sum of e

Solution:

Here, first term = 2

Common difference = $7 - 2 = 5$

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_{19} = \frac{19}{2} [2(2) + (19 - 1)5]$$

$$= (19)(4 + 90)/2$$

$$= (19 \times 94)/2$$

$$= 893$$

Thus, sum of 19 terms of this AP is 893.

Question: 1 B

Find the sum of e

Solution:

Here, first term = 9

Common difference = $7 - 9 = -2$

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_{14} = \frac{14}{2} [2(9) + (14 - 1)(-2)]$$

$$= (7)(18 - 26)$$

$$= (7) \times (-8)$$

$$= -56$$

Thus, sum of 14 terms of this AP is -56.

Question: 1 C

Find the sum of e

Solution:

Here, first term = -37

Common difference = $(-33) - (-37) = 4$

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\begin{aligned}
 \therefore S_{12} &= \frac{12}{2} [2(-37) + (12 - 1)(4)] \\
 &= (6)(-74 + 44) \\
 &= 6 \times (-30) \\
 &= -180
 \end{aligned}$$

Thus, sum of 12 terms of this AP is - 180.

Question: 1 D

Find the sum of e

Solution:

Here, first term = $1/15$

Common difference = $(1/12) - (1/15) = 1/60$

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\begin{aligned}
 \therefore S_{11} &= \frac{11}{2} [2(1/15) + (11 - 1)(1/60)] \\
 &= (11/2) \times [(2/15) + (1/6)] \\
 &= (11/2) \times [(3/10)] \\
 &= 33/20
 \end{aligned}$$

Thus, sum of 11 terms of this AP is $33/20$.

Question: 1 E

Find the sum of e

Solution:

Here, first term = 0.6

Common difference = $1.7 - 0.6 = 1.1$

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\begin{aligned}
 \therefore S_{100} &= \frac{100}{2} [2(0.6) + (100 - 1)(1.1)] \\
 &= (50) \times [1.2 + (99 \times 1.1)] \\
 &= 50 \times [1.2 + 108.9] \\
 &= 50 \times 110.1 \\
 &= 5505
 \end{aligned}$$

Thus, sum of 100 terms of this AP is 5505.

Question: 2 A

Find the sum of e

Solution:

Here, First term = 7

Common difference = $d = (21/2) - 7 = (7/2)$

Last term = $l = 84$

Now, $84 = a + (n - 1)d$

$$\therefore 84 = 7 + (n - 1)(7/2)$$

$$\Rightarrow 84 - 7 = (n - 1)(7/2)$$

$$\Rightarrow 77 = (n - 1)(7/2)$$

$$\Rightarrow 154 = 7n - 7 \text{ (multiplying both sides by 2)}$$

$$\Rightarrow 154 + 7 = 7n$$

$$\Rightarrow 7n = 161$$

$$\Rightarrow n = 23$$

$\therefore$ there are 23 terms in this Arithmetic series.

Now, Sum of these 23 terms is given by

$$\therefore S_{23} = \frac{23}{2} [2(7) + (23 - 1)(7/2)]$$

$$= (23/2) \times [14 + (22)(7/2)]$$

$$= (23/2) \times [14 + 77]$$

$$= (23/2) \times [91]$$

$$= 2093/2$$

$$= 1046.5$$

Thus, sum of 23 terms of this AP is 1046.5.

Question: 2 B

Find the sum of e

Solution:

Here, First term = 34

Common difference = $d = 34 - 32 = -2$

Last term = $l = 10$

Now, $10 = a + (n - 1)d$

$$\therefore 10 = 34 + (n - 1)(-2)$$

$$\Rightarrow 10 - 34 = (n - 1)(-2)$$

$$\Rightarrow -24 = -2n + 2$$

$$\Rightarrow -24 - 2 = -2n$$

$$\Rightarrow -26 = -2n$$

$$\Rightarrow n = 13$$

$$\Rightarrow n = 13$$

$\therefore$ there are 13 terms in this Arithmetic series.

Now, Sum of these 13 terms is given by

$$\therefore S_{13} = \frac{13}{2} [2(34) + (13 - 1)(-2)]$$

$$= (13/2) \times [68 + (12)(-2)]$$

$$= (13/2) \times [68 - 24]$$

$$= (13/2) \times [44]$$

$$= 13 \times 22$$

$$= 286$$

Thus, sum of 23 terms of this AP is 286.

Question: 2 C

Find the sum of e

Solution:

Here, First term = - 5

Common difference = $d = - 8 - (-5) = - 3$

Last term = $l = - 230$

Now, $- 230 = a + (n - 1)d$

$$\therefore - 230 = - 5 + (n - 1)(-3)$$

$$\Rightarrow - 230 + 5 = (n - 1)(-3)$$

$$\Rightarrow - 225 = - 3n + 3$$

$$\Rightarrow - 225 - 3 = - 3n$$

$$\Rightarrow - 228 = - 3n$$

$$\Rightarrow n = 76$$

$\therefore$ there are 76 terms in this Arithmetic series.

Now, Sum of these 76 terms is given by

$$\therefore S_{76} = \frac{76}{2} [2(-5) + (76 - 1)(-3)]$$

$$= 38 \times [- 10 + (75)(-3)]$$

$$= 38 \times [- 10 - 225]$$

$$= 38 \times (-235)$$

$$= - 8930$$

Thus, sum of 23 terms of this AP - 8930.

Question: 3

Find the sum of f

Solution:

Since, nth term is given as $(5 - 6n)$

Put $n = 1$, we get $a_1 = - 1 =$ first term

Put $n = 2$, we get $a_2 = - 7 =$ second term

Now, $d = a_2 - a_1 = - 7 - (-1) = - 6$

Sum of first n terms = $S_n = \frac{n}{2} [2a + (n - 1)d]$; where a is the first term

and d is the common difference.

$$= \frac{n}{2} [- 2 + (n - 1)(-6)]$$

$$= n[- 1 - 3n + 3]$$

$$= n(2 - 3n)$$

$\therefore$ sum of first 20 terms = S_{20}

$$= \frac{20}{2} [2(-1) + (20 - 1)(-6)]$$

$$= 10 \times [- 2 - 114]$$

$$= 10 \times [- 116]$$

$$= - 1160$$

Question: 4

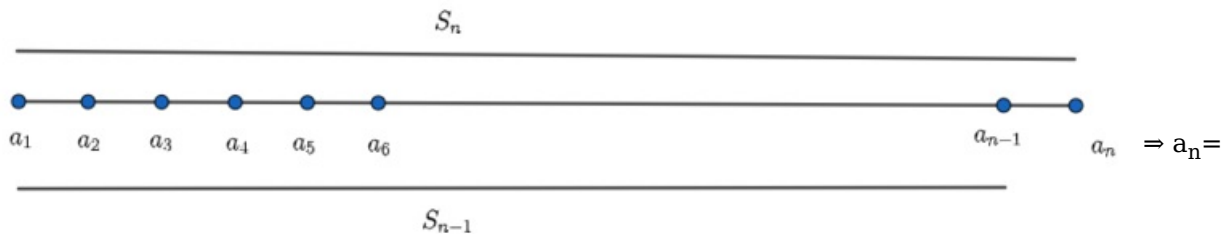
The sum of the fi

Solution:

Given: The sum of the first n terms of an AP is $(3n^2 + 6n)$. **To find:** the n th term and the 15th term of this AP.
Solution: Sum of first n terms = $S_n = 3n^2 + 6n$

Now let a_n be the n^{th} term of the AP.

To find: a_n and a_{15} Since $a_n = S_n - S_{n-1}$



$$(3n^2 + 6n) - (3(n-1)^2 + 6(n-1)) \Rightarrow a_n = (3n^2 + 6n) - (3(n^2 + 1 - 2n) + 6(n-1))$$

$$\Rightarrow a_n = (3n^2 + 6n) - (3n^2 + 3 - 6n + 6n - 6)$$

$$\Rightarrow a_n = 3n^2 + 6n - 3n^2 - 3 + 6n - 6n + 6$$

$$\Rightarrow a_n = 6n + 3$$

$$\text{Now, } a_{15} = 6(15) + 3$$

$$\Rightarrow a_{15} = 93$$

Question: 5

The sum of the fi

Solution:

(i) Let a_n be the n^{th} term of the AP.

To find: a_n

$$\text{Then } a_n = S_n - S_{n-1}$$

$$= (3n^2 - n) - (3(n-1)^2 - (n-1))$$

$$= (3n^2 - n) - (3n^2 + 3 - 6n - n + 1)$$

$$= 6n - 4$$

$$(ii) \text{ Since } a_n = 6n - 4$$

$$\therefore \text{ For first term, } n = 1$$

By putting $n = 1$ in the n^{th} term, we get,

$$a_1 = 6(1) - 4$$

$$= 2$$

$$\therefore a = 2$$

(iii) Put $n = 2$ in the n th term, we get

$$a_2 = 6 \times (2) - 4$$

$$= 12 - 4$$

$$= 8$$

Now common difference = $d = a_2 - a_1$

$$= 8 - 2$$

$$= 6$$

$\therefore$ Common difference = 6

Question: 6

The sum of the fi

Solution:

Let a_n be the n^{th} term of the AP.

To find: a_n and a_{20}

Since, $a_n = S_n - S_{n-1}$

$$= \left(\frac{5n^2}{2} + \frac{3n}{2} \right) - \left(\frac{5(n-1)^2}{2} + \frac{3(n-1)}{2} \right)$$

$$= 1/2 (5n^2 + 3n) - 1/2 [5(n-1)^2 + 3(n-1)]$$

$$= 1/2 (5n^2 + 3n - 5n^2 - 5 + 10n - 3n + 3)$$

$$= 1/2 (10n - 2)$$

$$= 5n - 1$$

Since $a_n = 5n - 1$

$\therefore$ For 20^{th} term, put $n = 20$, we get,

$$a_{20} = 5(20) - 1$$

$$= 100 - 1$$

$$= 99$$

Question: 7

The sum of the fi

Solution:

Let a_n be the n^{th} term of the AP.

To find: a_n and a_{25}

Since, $a_n = S_n - S_{n-1}$

$$= \left(\frac{3n^2}{2} + \frac{5n}{2} \right) - \left(\frac{3(n-1)^2}{2} + \frac{5(n-1)}{2} \right)$$

$$= 1/2 (3n^2 + 5n) - 1/2 [3(n-1)^2 + 5(n-1)]$$

$$= 1/2 (3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5)$$

$$= 1/2 (6n - 2)$$

$$= 3n + 1$$

Since $a_n = 3n + 1$

$\therefore$ For 25^{th} term, put $n = 25$, we get,

$$a_{25} = 3(25) + 1$$

$$= 75 + 1$$

$$= 76$$

Question: 8

How many terms of

Solution:

Here, first term = $a = 21$

Common difference = $d = 18 - 21 = -3$

Let first n terms of the AP sums to zero.

$$\therefore S_n = 0$$

To find: n

$$\text{Now, } S_n = (n/2) \times [2a + (n - 1)d]$$

$$\text{Since, } S_n = 0$$

$$\therefore (n/2) \times [2a + (n - 1)d] = 0$$

$$\Rightarrow (n/2) \times [2(21) + (n - 1)(-3)] = 0$$

$$\Rightarrow (n/2) \times [42 - 3n + 3] = 0$$

$$\Rightarrow (n/2) \times [45 - 3n] = 0$$

$$\Rightarrow [45 - 3n] = 0$$

$$\Rightarrow 45 = 3n$$

$$\Rightarrow n = 15$$

$\therefore$ 15 terms of the given AP sums to zero.

Question: 9

How many terms of

Solution:

Here, first term = $a = 9$

Common difference = $d = 17 - 9 = 8$

Let first n terms of the AP sums to 636.

$$\therefore S_n = 636$$

To find: n

$$\text{Now, } S_n = (n/2) \times [2a + (n - 1)d]$$

$$\text{Since, } S_n = 636$$

$$\therefore (n/2) \times [2a + (n - 1)d] = 636$$

$$\Rightarrow (n/2) \times [2(9) + (n - 1)(8)] = 636$$

$$\Rightarrow (n/2) \times [18 + 8n - 8] = 636$$

$$\Rightarrow (n/2) \times [10 + 8n] = 636$$

$$\Rightarrow n[5 + 4n] = 636$$

$$\Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow (n - 12)(4n + 53) = 0$$

$$\Rightarrow n = 12 \text{ or } n = -53/4$$

But n can't be negative and fraction.

$$\therefore n = 12$$

$\therefore$ 12 terms of the given AP sums to 636.

Question: 10

How many terms of

Solution:

Here, first term = $a = 63$

Common difference = $d = 60 - 63 = -3$

Let first n terms of the AP sums to 693.

$$\therefore S_n = 693$$

To find: n

$$\text{Now, } S_n = (n/2) \times [2a + (n - 1)d]$$

$$\text{Since, } S_n = 693$$

$$\therefore (n/2) \times [2a + (n - 1)d] = 693$$

$$\Rightarrow (n/2) \times [2(63) + (n - 1)(-3)] = 693$$

$$\Rightarrow (n/2) \times [126 - 3n + 3] = 693$$

$$\Rightarrow (n/2) \times [129 - 3n] = 693$$

$$\Rightarrow n[129 - 3n] = 1386$$

$$\Rightarrow 129n - 3n^2 = 1386$$

$$\Rightarrow 3n^2 - 129n + 1386 = 0$$

$$\Rightarrow (n - 22)(n - 21) = 0$$

$$\Rightarrow n = 22 \text{ or } n = 21$$

$$\therefore n = 22 \text{ or } n = 21$$

$$\text{Since, } a_{22} = a + 21d$$

$$= 63 + 21(-3)$$

$$= 0$$

$\therefore$ Both the first 21 terms and 22 terms give the sum 693 because the 22nd term is 0. So, the sum doesn't get affected.

Question: 11

How many terms of

Solution:

Here, first term = $a = 20$

Common difference = $d = 58/3 - 20 = -2/3$

Let first n terms of the AP sums to 300.

$$\therefore S_n = 300$$

To find: n

$$\text{Now, } S_n = (n/2) \times [2a + (n - 1)d]$$

$$\text{Since, } S_n = 300$$

$$\therefore (n/2) \times [2a + (n - 1)d] = 300$$

$$\Rightarrow (n/2) \times [2(20) + (n - 1)(-2/3)] = 300$$

$$\Rightarrow (n/2) \times [40 - (2/3)n + (2/3)] = 300$$

$$\Rightarrow (n/2) \times [(120 - 2n + 2)/3] = 300$$

$$\Rightarrow n[122 - 2n] = 1800$$

$$\Rightarrow 122n - 2n^2 = 1800$$

$$\Rightarrow 2n^2 - 122n + 1800 = 0$$

$$\Rightarrow n^2 - 61n + 900 = 0$$

$$\Rightarrow (n - 36)(n - 25) = 0$$

$$\Rightarrow n = 36 \text{ or } n = 25$$

$$\therefore n = 36 \text{ or } n = 25$$

$$\text{Now, } S_{36} = (36/2)[2a + 35d]$$

$$= 18(40 + 35(-2/3))$$

$$= 18(120 - 70)/3$$

$$= 6(50)$$

$$= 300$$

$$\text{Also, } S_{25} = (25/2)[2a + 24d]$$

$$= (25/2)(40 + 24(-2/3))$$

$$= (25/2)(40 - 16)$$

$$= (24 \times 25)/2$$

$$= 12 \times 25$$

$$= 300$$

$$\text{Now, sum of 11 terms from 26}^{\text{th}} \text{ term to 36}^{\text{th}} \text{ term} = S_{36} - S_{25} = 0$$

$\therefore$ Both the first 25 terms and 36 terms give the sum 300 because the sum of last 11 terms is 0.
So, the sum doesn't get affected.

Question: 12

Find the sum of a

Solution:

Odd numbers from 0 to 50 are 1, 3, 5, ..., 49

Sum of these numbers is $1 + 3 + 5 + \dots + 49$.

This forms an Arithmetic Series with first term $= a = 1$

and Common Difference $= d = 3 - 1 = 2$

There are 25 terms in this Arithmetic Series.

Now, sum of n terms is given as:

$$S_n = (n/2)[2a + (n - 1)d]$$

$$S_{25} = (25/2)[2(1) + (25 - 1)2]$$

$$= (25/2)[2 + 48]$$

$$= (25 \times 50)/2$$

$$= 25 \times 25$$

$$= 625$$

$\therefore$ Sum of odd numbers from 0 to 50 is 625.

Question: 13

Find the sum of a

Solution:

Natural numbers between 200 and 400 which are divisible by 7 are 203, 210, 217, ..., 399.

Sum of these numbers forms an arithmetic series $203 + 210 + 217 + \dots + 399$.

Here, first term = $a = 203$

Common difference = $d = 7$

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow 399 = 203 + (n - 1)7$$

$$\Rightarrow 399 = 7n + 196$$

$$\Rightarrow 7n = 203$$

$$\Rightarrow n = 29$$

$\therefore$ there are 29 terms in the AP.

Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 28 terms of this arithmetic series is given by:

$$\therefore S_{29} = \frac{29}{2} [2(203) + (29 - 1)(7)]$$

$$= (29/2) [406 + 196]$$

$$= (29/2) \times 602$$

$$= 7279$$

Question: 14

Find the sum of f

Solution:

First 40 positive integers divisible by 6 are 6, 12, 18, ..., 240.

Sum of these numbers forms an arithmetic series $6 + 12 + 18 + \dots + 240$.

Here, first term = $a = 6$

Common difference = $d = 6$

Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 40 terms of this arithmetic series is given by:

$$\therefore S_{40} = \frac{40}{2} [2(6) + (40 - 1)(6)]$$

$$= 20 [12 + 234]$$

$$= 20 \times 246$$

$$= 4920$$

Question: 15

Find the sum of t

Solution:

First 15 multiples of 8 are 8, 16, 24, ..., 120.

Sum of these numbers forms an arithmetic series $8 + 16 + 24 + \dots + 120$.

Here, first term = $a = 8$

Common difference = $d = 8$

Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 15 terms of this arithmetic series is given by:

$$\therefore S_{15} = \frac{15}{2} [2(8) + (15 - 1)(8)]$$

$$= (15/2) [16 + 112]$$

$$= (15/2) \times 128$$

$$= 15 \times 64$$

$$= 960$$

Question: 16

Find the sum of a

Solution:

Multiples of 9 lying between 300 and 700 are 306, 315, 324, ..., 693.

Sum of these numbers forms an arithmetic series $306 + 315 + 324 + \dots + 693$.

Here, first term = $a = 306$

Common difference = $d = 9$

We first find the number of terms in the series.

Here, last term = $l = 693$

$$\therefore 693 = a + (n - 1)d$$

$$\Rightarrow 693 = 306 + (n - 1)9$$

$$\Rightarrow 693 - 306 = 9n - 9$$

$$\Rightarrow 387 = 9n - 9$$

$$\Rightarrow 387 + 9 = 9n$$

$$\Rightarrow 9n = 396$$

$$\Rightarrow n = 44$$

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 44 terms of this arithmetic series is given by:

$$\therefore S_{44} = \frac{44}{2} [2(306) + (44 - 1)(9)]$$

$$= 22 \times [612 + 387]$$

$$= 22 \times 999$$

$$= 21978$$

Question: 17

Find the sum of a

Solution:

Three - digit natural numbers which are divisible by 13 are 104, 117, 130, ..., 988.

Sum of these numbers forms an arithmetic series $104 + 117 + 130 + \dots + 988$.

Here, first term = $a = 104$

Common difference = $d = 13$

We first find the number of terms in the series.

Here, last term = $l = 988$

$$\therefore 988 = a + (n - 1)d$$

$$\Rightarrow 988 = 104 + (n - 1)13$$

$$\Rightarrow 988 - 104 = 13n - 13$$

$$\Rightarrow 884 = 13n - 13$$

$$\Rightarrow 884 + 13 = 13n$$

$$\Rightarrow 13n = 897$$

$$\Rightarrow n = 69$$

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 69 terms of this arithmetic series is given by:

$$\therefore S_{69} = \frac{69}{2} [2(104) + (69 - 1)(13)]$$

$$= (69/2) \times [208 + 884]$$

$$= (69/2) \times 1092$$

$$= 69 \times 546$$

$$= 3767$$

Question: 18

Find the sum of f

Solution:

First 100 even natural numbers which are divisible by 5 are 10, 20, 30, ..., 1000

Here, first term = $a = 10$

Common difference = $d = 10$

Number of terms = 100

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 100 terms of this arithmetic series is given by:

$$\therefore S_{100} = \frac{100}{2} [2(10) + (100 - 1)(10)]$$

$$= 50 \times [20 + 990]$$

$$= 50 \times 1010$$

$$= 50500$$

Question: 19

Find the sum of t

Solution:

The given sum can be written as $(1 + 1 + 1 + \dots) - (1/n, 2/n, 3/n, \dots)$

Sum of first series up to n terms = $1 + 1 + 1 + \dots$ up to n terms

$$= n$$

Now, consider the second series:

Here, first term = $a = 1/n$

Common difference = $d = (2/n) - (1/n) = (1/n)$

Now, Sum of n terms of an arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of n terms of second arithmetic series is given by:

$$\therefore S_n = \frac{n}{2} [2(1/n) + (n - 1)(1/n)]$$

$$= \frac{n}{2} [(2/n) + 1 - (1/n)]$$

$$= \frac{n}{2} [(1/n) + 1]$$

$$= \frac{n}{2} \times \frac{n+1}{n} = (n + 1)/2$$

Now, sum of n terms of the complete series = Sum of n terms of first series - Sum of n terms of second series

$$= n - (n + 1)/2$$

$$= (2n - n - 1)/2$$

$$= 1/2 (n - 1)$$

Question: 20

In an AP, it is g

Solution:

Let the first term be a .

Let Common difference be d .

Given: $S_5 + S_7 = 167$

$$S_{10} = 235$$

Now, Sum of n terms of an arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

So, consider

$$S_5 + S_7 = 167$$

$$\Rightarrow (5/2) [2a + (5 - 1)d] + (7/2) [2a + (7 - 1)d] = 167$$

$$\Rightarrow (5/2) [2a + 4d] + (7/2) [2a + 6d] = 167$$

$$\Rightarrow 5 \times [a + 2d] + 7 \times [a + 3d] = 167$$

$$\Rightarrow 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow 12a + 31d = 167 \dots\dots\dots (1)$$

Now, consider $S_{10} = 235$

$$\Rightarrow (10/2) [2a + (10 - 1)d] = 235$$

$$\Rightarrow 5 \times [2a + 9d] = 235$$

$$\Rightarrow 10a + 45d = 235$$

$$\Rightarrow 2a + 9d = 47 \dots\dots\dots (2)$$

Subtracting equation (1) from 6 times of equation (2), we get,

$$\Rightarrow 23d = 115$$

$$\Rightarrow d = 5$$

So, from equation (2), we get,

$$a = 1/2 (47 - 9d)$$

$$\Rightarrow a = 1/2 (47 - 45)$$

$$\Rightarrow a = 1/2 (2)$$

$$\Rightarrow a = 1$$

Therefore the AP is a, a + d, a + 2d, a + 3d,...

i.e. 1, 6, 11, 16,

Question: 21

In an AP, the fir

Solution:

Here, first term = a = 2

Let the Common difference = d

Last term = l = 29

Sum of all terms = $S_n = 155$

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [a + a + (n - 1)d]$$

$$= \frac{n}{2} [a + l]$$

Therefore sum of n terms of this arithmetic series is given by:

$$\therefore S_n = \frac{n}{2} [2 + 29] = 155$$

$$\Rightarrow 31n = 310$$

$$\Rightarrow n = 10$$

$\therefore$ there are 10 terms in the AP.

Thus 29 be the 10th term of the AP.

$$\therefore 29 = a + (10 - 1)d$$

$$\Rightarrow 29 = 2 + 9d$$

$$\Rightarrow 27 = 9d$$

$$\Rightarrow d = 3$$

$\therefore$ common difference = d = 3

Question: 22

In an AP, the fir

Solution:

Here, first term = a = - 4

Let the Common difference = d

Last term = $l = 29$

Sum of all terms = $S_n = 150$

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [a + a + (n - 1)d]$$

$$= \frac{n}{2} [a + l]$$

Therefore sum of n terms of this arithmetic series is given by:

$$\therefore S_n = \frac{n}{2} [-4 + 29] = 150$$

$$\Rightarrow 25n = 300$$

$$\Rightarrow n = 12$$

$\therefore$ there are 12 terms in the AP.

Thus 29 is the 12th term of the AP.

$$\therefore 29 = a + (12 - 1)d$$

$$\Rightarrow 29 = -4 + 11d$$

$$\Rightarrow 29 + 4 = 11d$$

$$\Rightarrow 11d = 33$$

$$\Rightarrow d = 3$$

$\therefore$ Common difference = $d = 3$

Question: 23

The first and the

Solution:

Here, first term = $a = 17$

Common difference = 9

Last term = $l = 350$

To find: number of terms and their sum.

Let there be n terms in the AP.

Since, $l = 350$

$$\therefore 350 = 17 + (n - 1)9$$

$$\Rightarrow 350 - 17 = 9n - 9$$

$$\Rightarrow 333 = 9n - 9$$

$$\Rightarrow 333 + 9 = 9n$$

$$\Rightarrow 9n = 342$$

$$\Rightarrow n = 38$$

Therefore number of terms = 38

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [a + a + (n - 1)d]$$

$$= \frac{n}{2} [a + l]$$

Therefore sum of 38 terms of this arithmetic series is given by:

$$\therefore S_{38} = \frac{38}{2} [17 + 350]$$

$$= 19 \times 367$$

$$= 6973$$

$$\therefore n = 38 \text{ and } S_n = 6973$$

Question: 24

The first and the

Solution:

Here, first term = $a = 5$

Let the Common difference = d

Last term = $l = 45$

Sum of all terms = $S_n = 400$

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [a + a + (n - 1)d]$$

$$= \frac{n}{2} [a + l]$$

Therefore sum of n terms of this arithmetic series is given by:

$$\therefore S_n = \frac{n}{2} [5 + 45] = 400$$

$$\Rightarrow 50n = 800$$

$$\Rightarrow n = 16$$

$\therefore$ there are 16 terms in the AP.

Thus 45 is the 16th term of the AP.

$$\therefore 45 = a + (16 - 1)d$$

$$\Rightarrow 45 = 5 + 15d$$

$$\Rightarrow 40 = 15d$$

$$\Rightarrow 15d = 40$$

$$\Rightarrow d = 8/3$$

$\therefore$ Common difference = $d = 8/3$

Question: 25

In an AP, the fir

Solution:

Here, first term = $a = 22$

Let the Common difference = d

$$n^{\text{th}} \text{ term} = a_n = -11$$

$$\text{Sum of first } n \text{ terms} = S_n = 66$$

Let there be n terms in the AP.

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [a + a + (n - 1)d]$$

$$= \frac{n}{2} [a + a_n]$$

Therefore sum of n terms of this arithmetic series is given by:

$$\therefore S_n = \frac{n}{2} [22 + (-11)] = 66$$

$$\Rightarrow 11n = 132$$

$$\Rightarrow n = 12$$

$\therefore$ there are 12 terms in the AP.

Thus n^{th} is the 12^{th} term of the AP.

$$\therefore -11 = a + (12 - 1)d$$

$$\Rightarrow -11 = 22 + 11d$$

$$\Rightarrow -11 - 22 = 11d$$

$$\Rightarrow 11d = -33$$

$$\Rightarrow d = -3$$

$$\therefore \text{Common difference} = d = -3$$

$$\therefore n = 12, d = -3$$

Question: 26

The 12th term of

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_{12} = -13$$

$$S_4 = 24$$

To find: Sum of first 10 terms.

$$\text{Consider } a_{12} = -13$$

$$\Rightarrow a + 11d = -13 \dots\dots\dots(1)$$

$$\text{Also, } S_4 = 24$$

$$\Rightarrow (4/2) \times [2a + (4 - 1)d] = 24$$

$$\Rightarrow 2 \times [2a + 3d] = 24$$

$$\Rightarrow 2a + 3d = 12 \dots\dots\dots(2)$$

Subtracting equation (2) from twice of equation (1), we get,

$$19d = -38$$

$$\Rightarrow d = -2$$

Now, from equation (1), we get

$$a = -13 - 11d$$

$$\Rightarrow a = -13 - 11(-2)$$

$$\Rightarrow a = -13 + 22$$

$$\Rightarrow a = 9$$

Now, Sum of first n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of first 10 terms of this arithmetic series is given by:

$$\therefore S_{10} = \frac{10}{2} [2(9) + (10 - 1)(-2)]$$

$$= 5 \times [18 - 18]$$

$$= 0$$

$$\therefore S_{10} = 0$$

Question: 27

The sum of the fi

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } S_7 = 182$$

4th and 17th terms are in the ratio 1 : 5.

$$\text{i.e. } [a + 3d] : [(a + 16d)] = 1 : 5$$

$$\Rightarrow \frac{(a + 3d)}{(a + 16d)} = \frac{1}{5}$$

$$\Rightarrow 5(a + 3d) = (a + 16d)$$

$$\Rightarrow 5a + 15d = a + 16d$$

$$\Rightarrow 4a = d$$

$$\text{Now, consider } S_7 = 182$$

$$\Rightarrow (7/2)[2a + (7 - 1)d] = 182$$

$$\Rightarrow (7/2)[2a + 6(4a)] = 182$$

$$\Rightarrow 7 \times [26a] = 182 \times 2$$

$$\Rightarrow 182a = 364$$

$$\Rightarrow a = 2$$

$$\therefore d = 4a$$

$$\Rightarrow d = 8$$

Thus the AP will be $a, a + d, a + 2d, \dots$

i.e. AP is 2, 10, 18, 26,

Question: 28

The sum of the fi

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } S_9 = 81, S_{20} = 400$$

Now, consider $S_9 = 81$

$$\Rightarrow (9/2)[2a + (9 - 1)d] = 81$$

$$\Rightarrow (9/2)[2a + 8d] = 81$$

$$\Rightarrow [2a + 8d] = 18 \dots\dots\dots(1)$$

Now, consider $S_{20} = 400$

$$\Rightarrow (20/2)[2a + (20 - 1)d] = 400$$

$$\Rightarrow 10 \times [2a + 19d] = 400$$

$$\Rightarrow [2a + 19d] = 40 \dots\dots\dots(2)$$

Now, on subtracting equation (2) from equation (1), we get,

$$11d = 22$$

$$\Rightarrow d = 2$$

$\therefore$ from equation (1), we get

$$a = 1/2 (18 - 8d)$$

$$\Rightarrow a = 9 - 4d$$

$$\Rightarrow a = 9 - 8$$

$$\Rightarrow a = 1$$

$$\therefore a = 1, d = 2$$

Question: 29

The sum of the fi

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } S_7 = 49, S_{17} = 289$$

To find: sum of first n terms.

Now, consider $S_7 = 49$

$$\Rightarrow (7/2)[2a + (7 - 1)d] = 49$$

$$\Rightarrow (7/2)[2a + 6d] = 49$$

$$\Rightarrow [a + 3d] = 7 \dots\dots\dots(1)$$

Now, consider $S_{17} = 289$

$$\Rightarrow (17/2)[2a + (17 - 1)d] = 289$$

$$\Rightarrow (17/2) \times [2a + 16d] = 289$$

$$\Rightarrow [a + 8d] = 17 \dots\dots\dots(2)$$

Now, on subtracting equation (2) from equation (1), we get,

$$5d = 10$$

$$\Rightarrow d = 2$$

$\therefore$ from equation (1), we get

$$a = (7 - 3d)$$

$$\Rightarrow a = 7 - 6$$

$$\Rightarrow a = 1$$

$$\therefore a = 1, d = 2$$

Now, Sum of first n terms = $S_n = (n/2)[2a + (n - 1)d]$

$$= (n/2)[2 + (n - 1)2]$$

$$= (n/2)[2n]$$

$$= n^2$$

$$\therefore S_n = n^2$$

Question: 30

Two APs have the

Solution:

Let a_1 and a_2 be the first terms of the two APs

Let d_1 and d_2 be the common difference of the respective APs.

Given: $d_1 = d_2$ and $a_1 = 3$, $a_2 = 8$

To find: Difference between the sums of their first 50 terms.

i.e. to find: $(S_2)_{50} - (S_1)_{50}$

where $(S_1)_{50}$ denotes the sum of first 50 terms of first AP and $(S_2)_{50}$

denotes the sum of first 50 terms of second AP.

Now, consider $(S_1)_{50} = (50/2)[2a_1 + (50 - 1)d_1]$

$$= 25 \times [2(3) + 49 \times d_1]$$

$$= 25[6 + 49d_1]$$

$$= 150 + 1225d_1$$

Now, consider $(S_2)_{50} = (50/2)[2a_2 + (50 - 1)d_2]$

$$= 25 \times [2(8) + 49 \times d_2]$$

$$= 25[16 + 49d_1]$$

$$= 400 + 1225d_2$$

Now, $(S_2)_{50} - (S_1)_{50} = 400 + 1225d_2 - (150 + 1225d_2)$

$$= 400 - 150 (\because d_1 = d_2)$$

$$= 250$$

$$\therefore (S_2)_{50} - (S_1)_{50} = 250$$

Question: 31

The sum of first

Solution:

Let a be the first term and d be the common difference.

Given: Sum of first 10 terms = $S_{10} = -150$

Sum of next 10 terms = -550

$$\text{i.e. } S_{20} - S_{10} = -550$$

Consider $S_{10} = -150$

$$\Rightarrow (10/2)[2a + (10 - 1)d] = -150$$

$$\Rightarrow 5 \times [2a + 9d] = -150$$

$$\Rightarrow [2a + 9d] = -30 \dots\dots\dots(1)$$

Now, consider $S_{20} - S_{10} = -550$

$$\Rightarrow (20/2)[2a + (20 - 1)d] - (10/2)[2a + (10 - 1)d] = -550$$

$$\Rightarrow 10 \times [2a + 19d] - 5[2a + 9d] = -550$$

$$\Rightarrow 10a + 145d = -550 \dots\dots\dots(2)$$

On subtracting equation (2) from 5 times of equation (1), we get,

$$-100d = 400$$

$$\Rightarrow d = -4$$

$$\therefore a = 1/2 (-30 - 9d)$$

$$\Rightarrow a = 1/2 (-30 + 36)$$

$$\Rightarrow a = 3$$

Therefore the AP is 3, -1, -5, -9,....

Question: 32

The 13th term of

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_5 = 16$$

$$a_{13} = 4a_3$$

$$\text{Now, Consider } a_5 = 16$$

$$\Rightarrow a + (5 - 1)d = 16$$

$$\Rightarrow a + 4d = 16 \dots\dots\dots(1)$$

$$\text{Consider } a_{13} = 4a_3$$

$$\Rightarrow a + 12d = 4(a + 2d)$$

$$\Rightarrow a + 12d = 4a + 8d$$

$$\Rightarrow 3a - 4d = 0 \dots\dots\dots(2)$$

Now, adding equation (1) and (2), we get,

$$4a = 16$$

$$\Rightarrow a = 4$$

$\therefore$ from equation (2), we get,

$$4d = 3a$$

$$\Rightarrow 4d = 12$$

$$\Rightarrow d = 3$$

Now, Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 10 terms is given by:

$$S_{10} = \frac{10}{2} [2(4) + (10 - 1)(3)]$$

$$= 5 \times [8 + 27]$$

$$= 5 \times 35$$

$$= 175$$

$$\therefore S_{10}=175$$

Question: 33

The 16th term of

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_{10} = 41$$

$$a_{16} = 5 a_3$$

$$\text{Now, Consider } a_{10} = 41$$

$$\Rightarrow a + (10 - 1)d = 41$$

$$\Rightarrow a + 9d = 41 \dots\dots\dots(1)$$

$$\text{Consider } a_{16} = 5 a_3$$

$$\Rightarrow a + 15d = 5(a + 2d)$$

$$\Rightarrow a + 15d = 5a + 10d$$

$$\Rightarrow 4a - 5d = 0 \dots\dots\dots(2)$$

Now, subtracting equation (2) from 4 times of equation (1), we get,

$$41d = 164$$

$$\Rightarrow d = 4$$

$\therefore$ from equation (2), we get,

$$4a = 5d$$

$$\Rightarrow 4a = 20$$

$$\Rightarrow a = 5$$

Now, Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 15 terms is given by:

$$S_{15} = \frac{15}{2} [2(5) + (15 - 1)(4)]$$

$$= (15/2) \times [10 + 56]$$

$$= 15 \times 33$$

$$= 495$$

$$\therefore S_{15} = 495$$

Question: 34

An AP 5, 12, 19,

Solution:

Here, First term = $a = 5$

Common difference = $d = 12 - 5 = 7$

No. of terms = 50

$\therefore$ last term will be 50th term.

Using the formula for finding n^{th} term of an A.P.,

$$l = a_{50} = a + (50 - 1) \times d$$

$$\therefore l = 5 + (50 - 1) \times 7$$

$$\Rightarrow l = 5 + 343 = 348$$

Now, sum of last 15 terms = sum of first 50 terms - sum of first 35 terms

$$\text{i.e. sum of last 15 terms} = S_{50} - S_{35}$$

Now, Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 50 terms is given by:

$$S_{50} = \frac{50}{2} [2(5) + (50 - 1)(7)]$$

$$= 25 \times [10 + 343]$$

$$= 25 \times 353$$

$$= 8825$$

Now, Sum of first 35 terms is given by:

$$S_{35} = \frac{35}{2} [2(5) + (35 - 1)(7)]$$

$$= (35/2) \times [10 + 238]$$

$$= (35/2) \times 248$$

$$= 35 \times 124$$

$$= 4340$$

$$\text{Now, } S_{50} - S_{35} = 8825 - 4340$$

$$= 4485$$

$$\therefore \text{last term} = 348, \text{ sum of last 15 terms} = 4485$$

Question: 35

An AP 8, 10, 12,

Solution:

Here, First term = $a = 8$

Common difference = $d = 10 - 8 = 2$

No. of terms = 60

$\therefore$ last term will be 60th term.

Using the formula for finding nth term of an A.P.,

$$l = a_{60} = a + (60 - 1) \times d$$

$$\therefore l = 8 + (60 - 1) \times 2$$

$$\Rightarrow l = 8 + 118 = 126$$

Now, sum of last 10 terms = sum of first 60 terms - sum of first 50 terms

$$\text{i.e. sum of last 10 terms} = S_{60} - S_{50}$$

Now, Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 50 terms is given by:

$$S_{50} = \frac{50}{2} [2(8) + (50 - 1)(2)]$$

$$= 25 \times [16 + 98]$$

$$= 25 \times 114$$

$$= 2850$$

Now, Sum of first 60 terms is given by:

$$S_{60} = \frac{60}{2} [2(8) + (60 - 1)(2)]$$

$$= 30 \times [16 + 118]$$

$$= 30 \times 248$$

$$= 4020$$

$$\text{Now, } S_{60} - S_{50} = 4020 - 2850$$

$$= 1170$$

$\therefore$ last term = 126, sum of last 10 terms = 1170

Question: 36

The sum of the 4t

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_4 + a_8 = 24$$

$$\text{and } a_6 + a_{10} = 44$$

To find: S_{10}

$$\text{Now, Consider } a_4 + a_8 = 24$$

$$\Rightarrow a + 3d + a + 7d = 24$$

$$\Rightarrow 2a + 10d = 24 \dots\dots\dots(1)$$

$$\text{Consider } a_6 + a_{10} = 44$$

$$\Rightarrow a + 5d + a + 9d = 44$$

$$\Rightarrow 2a + 14d = 44 \dots\dots\dots(2)$$

Subtracting equation (1) from equation (2), we get,

$$4d = 20$$

$$\Rightarrow d = 5$$

$$\therefore \text{ Common difference} = d = 5$$

Thus from equation (1), we get,

$$a = -13$$

Now, Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 10 terms is given by:

$$S_{10} = \frac{10}{2} [2(-13) + (10 - 1)(5)]$$

$$= 5 \times [-26 + 45]$$

$$= 5 \times 19$$

$$= 95$$

$$\therefore S_{10} = 95$$

Question: 37

The sum of first

Solution:

Let a be the first term and d be the common difference.

Given: Sum of first m terms of an AP is given by:

$$S_m = \frac{m}{2} [2a + (m - 1)d] = 4m^2 - m$$

Now, n^{th} term is given by: $a_n = S_n - S_{n-1}$

$$\therefore a_n = (4n^2 - n) - [4(n - 1)^2 - (n - 1)]$$

$$= (4n^2 - n) - [4(n^2 + 1 - 2n) - n + 1]$$

$$= 4n^2 - n - 4n^2 - 4 + 8n + n - 1$$

$$= 8n - 5 \dots\dots\dots(1)$$

Now, given that $a_n = 107$

$$\Rightarrow 8n - 5 = 107$$

$$\Rightarrow 8n = 112$$

$$\Rightarrow n = 14$$

For 21st term of AP, put $n = 21$ in the value of the nth term in equation (1), we get

$$a_{21} = 8 \times (21) - 5$$

$$\Rightarrow a_{21} = 168 - 5$$

$$= 163$$

$$\therefore a_{21} = 163$$

Question: 38

The sum of first

Solution:

Let a be the first term and d be the common difference.

Given: Sum of first q terms of an AP is given by:

$$S_q = \frac{q}{2} [2a + (q - 1)d] = 63q - 3q^2$$

Now, p^{th} term is given by: $a_p = S_p - S_{p-1}$

$$\therefore a_p = (63p - 3p^2) - [63(p - 1) - 3(p - 1)^2]$$

$$= (63p - 3p^2) - [63p - 63 - 3p^2 - 3 + 6p]$$

$$= 63p - 3p^2 - 63p + 63 + 3p^2 + 3 - 6p$$

$$= 66 - 6p \dots\dots\dots(1)$$

Now, given that $a_p = -60$

$$\Rightarrow 66 - 6p = -60$$

$$\Rightarrow 6p = 126$$

$$\Rightarrow p = 21$$

For 11th term of AP, put $p = 11$ in the value of the p^{th} term in equation (1), we get

$$a_{11} = 66 - 6 \times (11)$$

$$\Rightarrow a_{11} = 66 - 66$$

$$= 0$$

$$\therefore a_{11} = 0$$

Question: 39

Find the number o

Solution:

Here, first term = $a = -12$

Common difference = $d = -9 - (-12) = 3$

Last term is 21.

Now, number of terms in this AP are given as:

$$21 = a + (n - 1)d$$

$$\Rightarrow 21 = -12 + (n - 1)3$$

$$\Rightarrow 21 + 12 = 3n - 3$$

$$\Rightarrow 33 + 3 = 3n$$

$$\Rightarrow 36 = 3n$$

$$\Rightarrow n = 12$$

If 1 is added to each term, then the new AP will be - 11, - 8, - 5,..., 22.

Here, first term = $a = -11$

Common difference = $d = -8 - (-11) = 3$

Last term = $l = 22$.

Number of terms will be the same,

i.e, number of terms = $n = 12$

$\therefore$ Sum of 12 terms of the AP is given by:

$$S_{12} = (12/2) \times [a + l]$$

$$= 6 \times [-11 + 22]$$

$$= 6 \times 11$$

$$= 66$$

$\therefore$ Sum of 12 terms of the new AP will be 66.

Question: 40

Sum of the first

Solution:

Here, first term = $a = 10$

Let the Common difference = d

Sum of first 14 terms = $S_{14} = 1505$

Now, Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_{14} = \frac{14}{2} [2(10) + (14 - 1)d] = 1505$$

$$\Rightarrow 7 \times [20 + 13d] = 1505$$

$$\Rightarrow [20 + 13d] = 215$$

$$\Rightarrow 13d = 195$$

$$\Rightarrow d = 15$$

Now, n^{th} term is given by:

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow a_{25} = 10 + (25 - 1)15$$

$$= 10 + (24 \times 15)$$

$$= 10 + 360$$

$$= 370$$

Question: 41

Find the sum of f

Solution:

Here, second term = $a_2 = 14$

Third term = $a_3 = 18$

$$\therefore \text{Common difference} = a_3 - a_2 = 18 - 14 = 4$$

$$\text{Thus first term} = a = a_2 - d = 14 - 4 = 10$$

Now, Sum of first n terms of an AP is given by

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 51 terms is given by:

$$S_{51} = \frac{51}{2} [2(10) + (51 - 1)(4)]$$

$$= (51/2) \times [20 + 200]$$

$$= (51/2) \times 220$$

$$= (51) \times 110$$

$$= 5610$$

$$\therefore S_{51} = 5610$$

Question: 42

In a school, stud

Solution:

Number of trees planted by one section of class 1st = 2

Now, there are 2 sections, $\therefore$ Number of trees planted by class 1st = 4

Number of trees planted by one section of class 2nd = 4

Now, there are 2 sections, $\therefore$ Number of trees planted by class 2nd = 8

This will follow up to class 12th and we will obtain an AP as

4, 8, 12, ... upto 12 terms.

Now, Total number of trees planted by the students = 4 + 8 + 12 + ... upto 12 terms.

∴ In this Arithmetic series, first term = $a = 4$

Common difference = $d = 4$

Now, $S_{12} = (12/2)[2a + (12 - 1)d]$

$$= 6[2(4) + 11(4)]$$

$$= 6 \times [8 + 44]$$

$$= 6 \times 52$$

$$= 312$$

∴ Total number of trees planted by the students = 312

Values shown in the question are care and awareness about conservation of nature and environment.

Question: 43

In a potato race,

Solution:

To pick the first potato, the competitor has to run 5 m to reach the potato and 5 m to run back to the bucket.

∴ Total distance covered by the competitor to pick first potato = $2 \times (5) = 10$ m

To pick the second potato, the competitor has to run $(5 + 3)$ m to reach the potato and $(5 + 3)$ m to run back to the bucket.

∴ Total distance covered by the competitor to pick second potato = $2 \times (5 + 3) = 16$ m

To pick the third potato, the competitor has to run $(5 + 3 + 3)$ m to reach the potato and $(5 + 3 + 3)$ m to run back to the bucket.

∴ Total distance covered by the competitor to pick third potato = $2 \times (5 + 3 + 3) = 22$ m

This will continue and we will get a sequence of distance as 10, 16, 22,... upto 10 terms (as there are 10 potatoes to pick).

Total distance covered by the competitor to pick all the 10 potatoes = $10 + 16 + 22 + \dots$ upto 10 terms.

This forms an Arithmetic series with first term = $a = 10$

and Common difference = $d = 6$

Number of terms = $n = 10$

Now, $S_{10} = (10/2)[2a + (10 - 1)d]$

$$= 5 \times [2(10) + 9(6)]$$

$$= 5 \times [20 + 54]$$

$$= 5 \times 74$$

$$= 370$$

∴ Total distance covered by the competitor = 370 m

Question: 44

There are 25 tree

Solution:

To water the first tree, the gardener has to cover 10 m to reach the tree and 10 m to go back to the tank.

∴ Total distance covered by the gardener to water first tree = $2 \times (10) = 20$ m

To water the second tree, the gardener has to cover $(10 + 5)$ m to reach the tree and $(10 + 5)$ m to go back to the tank.

∴ Total distance covered by the gardener to water second tree = $2 \times (10 + 5) = 30$ m

To water the third tree, the gardener has to cover $(10 + 5 + 5)$ m to reach the tree and $(10 + 5 + 5)$ m to go back to the tank.

∴ Total distance covered by the gardener to water third tree = $2 \times (10 + 5 + 5) = 40$ m

This will continue and we will get a sequence of distance as 20, 30, 40,... upto 25 terms (as there are 25 trees to be watered).

Total distance covered by the gardener to water all 25 trees = $20 + 30 + 40 + \dots$ upto 25 terms.

This forms an Arithmetic series with first term = $a = 20$

and Common difference = $d = 10$

Number of terms = $n = 25$

Now, $S_{25} = (25/2)[2a + (25 - 1)d]$

$$= (25/2) \times [2(20) + 24(10)]$$

$$= (25/2) \times [40 + 240]$$

$$= (25/2) \times 280$$

$$= 25 \times 140$$

$$= 3500$$

∴ Total distance covered by the gardener = 3500 m

Question: 45

A sum of Rs. 700

Solution:

Let the first prize be Rs. x . Thus each succeeding prize is Rs. 20 less than the preceding prize.

∴ Second prize, third prize, ..., seventh prize be Rs. $(x - 20)$, $(x - 40)$, ..., $(x - 120)$.

This forms an AP as $x, x - 20, \dots, x - 120$.

Here, first term = x

Common difference = $x - 20 - x = -20$

Total number of terms = 7

Since, Total sum of prize amount = 700.

∴ Sum of all the terms = 700

Now, sum of first n terms of an AP is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

∴ Sum of 7 terms of an AP is given by:

$$S_7 = \frac{7}{2} [2a + (7 - 1)d] = 700$$

$$\Rightarrow \frac{7}{2} [2x + (7 - 1)(-20)] = 700$$

$$\Rightarrow 7[2x - 120] = 1400$$

$$\Rightarrow 2x - 120 = 200$$

$$\Rightarrow x - 60 = 100$$

$$\Rightarrow x = 160$$

Thus, the prizes are as Rs. 160, Rs.140, Rs.120, Rs. 100, Rs. 80, Rs. 60, Rs. 40.

Question: 46

A man saved Rs. 3

Solution:

Let the amount of money the man saved in first month = Rs. x

Now, the amount of money he saved in second month = Rs. $(x + 100)$

The amount of money he saved in third month = Rs. $(x + + 100 + 100)$

This will continue for 10 months.

$\therefore$ We get a an AP as $x, x + 100, x + 200, \dots$ up to 10 terms.

Here, first term = x

Common difference = $d = 100$

Number of terms = $n = 10$

Total amount of money saved by the man = $x + (x + 100) + (x + 200) + \dots$ up to 10 terms. = Rs. 33000 (given)

$\therefore$ Sum of 10 terms of the Arithmetic Series = 33000

$$\Rightarrow S_{10} = 33000$$

$$\Rightarrow (10/2) \times [2a + (10 - 1)d] = 33000$$

$$\Rightarrow (10/2) \times [2(x) + 9(100)] = 33000$$

$$\Rightarrow 5 \times [2x + 900] = 33000$$

$$\Rightarrow 2x + 900 = 6600$$

$$\Rightarrow 2x = 6600 - 900$$

$$\Rightarrow 2x = 5700$$

$$\Rightarrow x = 2850$$

$\therefore$ Amount of money saved by the man in first month = Rs. 2850

Question: 47

A man arranges to

Solution:

Let the first installment = Rs. x

Since the instalments form an arithmetic series, therefore let the common difference = d

Now, amount paid in 30 installments = two - third of the amount = $(2/3) \times (36000) = \text{Rs. } 24000$

$\therefore$ Total amount paid by the man in 30 installments = 24000

Let S_n be that amount paid in 30 installments.

$$\therefore S_{30} = 24000$$

$$\Rightarrow (30/2) \times [2x + (30 - 1)d] = 24000$$

$$\Rightarrow 15 \times [2x + 29d] = 24000$$

$$\Rightarrow 2x + 29d = 1600 \dots\dots\dots(1)$$

Now, Total sum of the amount = 36000

$$\therefore S_{40} = 36000$$

$$\Rightarrow (40/2) \times [2x + (40 - 1)d] = 36000$$

$$\Rightarrow 20 \times [2x + 39d] = 36000$$

$$\Rightarrow 2x + 39d = 1800 \dots\dots\dots(2)$$

Subtracting equation (1) from equation (2), we get:

$$10d = 200$$

$$\Rightarrow d = 20$$

$\therefore$ from equation (1), we get

$$x = \frac{1}{2}(1600 - 29d)$$

$$= \frac{1}{2}(1600 - 580)$$

$$= \frac{1}{2}(1020)$$

$$= 510$$

Therefore the amount of first installment = Rs. 510

Question: 48

A contract on con

Solution:

Penalty for delay for first day = Rs. 200

Penalty for delay for second day = Rs. 250

Penalty for delay for third day = Rs. 300

Penalty for each succeeding day is Rs. 50 more than for the preceding day.

$\therefore$ The amount of penalties are in AP with common difference

$$= d = \text{Rs.}50$$

Also, number of days in delay of the work = 30 days

Thus the penalties are 200, 250, 300, ... up to 30 terms

Thus the amount of money paid by the contractor is $200 + 250 + 300 + \dots$ up to 30 terms

Here, first term = $a = 200$

Common difference = $d = 50$

Number of terms = $n = 30$

$$\therefore \text{The sum} = S_{30} = \frac{(30)}{2} \times [2(200) + (30 - 1)(50)]$$

$$= 15 \times [400 + 1450]$$

$$= 15 \times 1850$$

$$= 27750$$

Thus the total amount of money paid by the contractor = Rs. 27750

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

The common differ

Solution:

$$\text{Common difference} = T_2 - T_1 = \frac{1-p}{p} - \frac{1}{p}$$

$$= \frac{1-p-1}{p} = -1$$

Question: 2

The common differ

Solution:

$$\text{Common difference} = T_2 - T_1 = \frac{1-3b}{3} - \frac{1}{3}$$

$$= \frac{1-3b-1}{3} = -b$$

Question: 3

The next term of

Solution:

Here, first term = $\sqrt{7}$

$$\text{Common difference} = \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7} = \sqrt{7}$$

$$\text{Next term} = T_4 = T_3 + d$$

$$= \sqrt{63} + \sqrt{7}$$

$$= 3\sqrt{7} + \sqrt{7}$$

$$= 4\sqrt{7}$$

$$= \sqrt{112}$$

Question: 4

If 4, x_1

Solution:

Here, first term = $a = 4$

Last term = $l = 28$

Number of terms = $n = 5$

$$\therefore l = a + (n - 1)d$$

$$\Rightarrow 28 = 4 + (5 - 1)d$$

$$\Rightarrow 28 - 4 = 4d$$

$$\Rightarrow 4d = 24$$

$$\Rightarrow d = 6$$

Therefore $x_3 = a + 3d$

$$= 4 + 3(6)$$

$$= 4 + 18$$

$$= 22$$

Question: 5

If the n th term o

Solution:

Given: n^{th} term = $2n + 1$

$$\therefore a_n = 2n + 1$$

$$\text{Put } n = 1, a_1 = 3$$

$$\text{Put } n = 2, a_2 = 5$$

$$\text{Put } n = 3, a_3 = 7$$

Now, sum of first three terms = $3 + 5 + 7 = 15$

Question: 6

The sum of first

Solution:

Let S_n denotes the sum of first n terms of an AP.

$$\text{Sum of first } n \text{ terms} = S_n = 3n^2 + 6n$$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

$$\therefore a_n = (3n^2 + 6n) - [3(n-1)^2 + 6(n-1)]$$

$$= (3n^2 + 6n) - [3(n^2 + 1 - 2n) + 6n - 6]$$

$$= 3n^2 + 6n - 3n^2 - 3 + 6n - 6n + 6$$

$$= 3 + 6n$$

Now, common difference = $d = a_n - a_{n-1}$

$$= 3 + 6n - [3 + 6(n-1)]$$

$$= 3 + 6n - 3 - 6n + 6$$

$$= 6$$

$\therefore$ Common difference = 6

Question: 7

The sum of first

Solution:

Let S_n denotes the sum of first n terms of an AP.

$$\text{Sum of first } n \text{ terms} = S_n = 5n - n^2$$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

$$\therefore a_n = (5n - n^2) - [5(n-1) - (n-1)^2]$$

$$= (5n - n^2) - [5n - 5 - (n^2 + 1 - 2n)]$$

$$= 5n - n^2 - 5n + 5 + n^2 + 1 - 2n$$

$$= 6 - 2n$$

Question: 8

The sum of first

Solution:

Let S_n denotes the sum of first n terms of an AP.

$$\text{Sum of first } n \text{ terms} = S_n = 4n^2 + 2n$$

Then n^{th} term is given by: $a_n = S_n - S_{n-1}$

$$\therefore a_n = (4n^2 + 2n) - [4(n-1)^2 + 2(n-1)]$$

$$= (4n^2 + 2n) - [4(n^2 + 1 - 2n) + 2n - 2]$$

$$= 4n^2 + 2n - 4n^2 - 4 + 8n - 2n + 2$$

$$= 8n - 2$$

Question: 9

The 7th term of a

Solution:

Let a be the first term and d be the common difference.

Given: $a_7 = -1$

$$a_{16} = 17$$

Now, Consider $a_7 = -1$

$$\Rightarrow a + 6d = -1 \dots\dots\dots(1)$$

Consider $a_{16} = 17$

$$\Rightarrow a + 15d = 17 \dots\dots\dots(2)$$

Subtract equation (1) from (2), we get,

$$9d = 18$$

$$\Rightarrow d = 2$$

$$\therefore \text{Common difference} = d = 2$$

Now, from equation (1), we get,

$$a = -1 - 6d$$

$$= -1 - 6(2)$$

$$= -13$$

Now, n^{th} term of the AP is given by

$$a_n = a + (n - 1)d$$

$$= -13 + (n - 1)2$$

$$= 13 + 2n - 2$$

$$= 2n - 15$$

Question: 10

The 5th term of a

Solution:

Let a be the first term and d be the common difference.

Given: $a_5 = -3$

Common difference = $d = -4$

Now, Consider $a_5 = -3$

$$\Rightarrow a + 4d = -3$$

$$\Rightarrow a + 4(-4) = -3$$

$$\Rightarrow a - 16 = -3$$

$$\Rightarrow a = 16 - 3$$

$$\Rightarrow a = 13$$

Now, Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of first 10 terms is given by:

$$S_{10} = \frac{10}{2} [2(13) + (10 - 1)(-4)]$$

$$= 5[26 - 36]$$

$$= 5 \times (-10)$$

$$= -50$$

Question: 11

The 5th term of a

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_5 = 20$$

$$a_7 + a_{11} = 64$$

$$\text{Now, Consider } a_5 = 20$$

$$\Rightarrow a + 4d = 20 \dots\dots\dots(1)$$

$$\text{Consider } a_7 + a_{11} = 64$$

$$\Rightarrow a + 6d + a + 10d = 64$$

$$\Rightarrow 2a + 16d = 64 \dots\dots\dots(2)$$

Subtract twice of equation (1) from (2), we get,

$$8d = 24$$

$$\Rightarrow d = 3$$

Question: 12

The 13th term of

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_{13} = 4(a_3)$$

$$a_5 = 16$$

To find: Sum of first ten terms.

$$\text{Now, Consider } a_{13} = 4a_3$$

$$\Rightarrow a + 12d = 4[a + 2d]$$

$$\Rightarrow a + 12d = 4a + 8d$$

$$\Rightarrow 3a = 4d \dots\dots\dots (1)$$

$$\text{Consider } a_5 = a + (5 - 1)d = 16$$

$$\Rightarrow a + 4d = 16$$

$$\Rightarrow a + 3a = 16 \text{ (from equation (1))}$$

$$\Rightarrow 4a = 16$$

$$\Rightarrow a = 4 \dots\dots\dots (2)$$

$$\therefore d = 3$$

Sum of n terms of an arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 10 terms of the arithmetic series is given by:

$$\therefore S_{10} = \frac{10}{2} [2(4) + (10 - 1)(3)]$$

$$= 5 \times [8 + 27]$$

$$= 5 \times 35$$

$$= 175$$

Question: 13

An AP 5, 12, 19,

Solution:

Here, first term = 5

Common difference = $12 - 5 = 7$

Given that there are 50 terms in the AP.

To find: Last term, i.e. 50th term = a_{50}

Since $a_n = a + (n - 1)d$

$$\therefore a_{50} = 5 + (50 - 1)7$$

$$= 5 + (49) \times 7$$

$$= 5 + 343$$

$$= 348$$

Question: 14

The sum of first

Solution:

Sum of first 20 odd natural numbers is $1 + 3 + 5 + 7 + \dots + 39$.

This forms an arithmetic series with first term = $a = 1$

and common difference = $d = 2$

Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 20 terms of this arithmetic series is given by:

$$\therefore S_{20} = \frac{20}{2} [2(1) + (20 - 1)(2)]$$

$$= 10 [2 + 38]$$

$$= 10 \times 40$$

$$= 400$$

Question: 15

The sum of first

Solution:

First 40 positive integers divisible by 6 are 6, 12, 18, ..., 240.

Sum of these numbers forms an arithmetic series $6 + 12 + 18 + \dots + 240$.

Here, first term = $a = 6$

Common difference = $d = 6$

Sum of n terms of this arithmetic series is given by:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Therefore sum of 40 terms of this arithmetic series is given by:

$$\therefore S_{40} = \frac{40}{2} [2(6) + (40 - 1)(6)]$$

$$= 20 [12 + 234]$$

$$= 20 \times 246$$

$$= 4920$$

Question: 16

How many two - di

Solution:

The two digit numbers divisible by 3 are 12, 15, 18, 21, ..., 99.

This forms an AP with first term $a = 12$

and common difference $= d = 3$

Last term is 99.

Now, number of terms in this AP are given as:

$$99 = a + (n - 1)d$$

$$\Rightarrow 99 = 12 + (n - 1)3$$

$$\Rightarrow 99 - 12 = 3n - 3$$

$$\Rightarrow 87 + 3 = 3n$$

$$\Rightarrow 90 = 3n$$

$$\Rightarrow n = 30$$

There are 30 two - digit numbers that are divisible by 3.

Question: 17

How many three -

Solution:

The three digit numbers divisible by 9 are 108, 117, 126, ..., 999.

This forms an AP with first term $a = 108$

and common difference $= d = 9$

Last term is 999.

Now, number of terms in this AP are given as:

$$999 = a + (n - 1)d$$

$$\Rightarrow 999 = 108 + (n - 1)9$$

$$\Rightarrow 999 - 108 = 9n - 9$$

$$\Rightarrow 891 + 9 = 9n$$

$$\Rightarrow 900 = 9n$$

$$\Rightarrow n = 100$$

There are 100 three - digit numbers that are divisible by 9.

Question: 18

What is the commo

Solution:

Let a be the first term and d be the common difference.

$$\text{Given: } a_{18} - a_{14} = 32$$

$$\Rightarrow (a + 17d) - (a + 13d) = 32$$

$$\Rightarrow 17d - 13d = 32$$

$$\Rightarrow 4d = 32$$

$$\Rightarrow d = 8$$

$$\therefore d = \text{common difference} = 8$$

Question: 19

If a_n

Solution:

Here, First term = $a = 3$

$$\text{Common difference} = d = 8 - 3 = 5$$

To find: $a_{30} - a_{20}$

So, n^{th} term is given by:

$$a_n = a + (n - 1)d$$

$\therefore 30^{\text{th}}$ term is:

$$a_{30} = 3 + (30 - 1) \times 5$$

$$= 3 + 145$$

$$= 148$$

Now, 20^{th} term is:

$$a_{20} = 3 + (20 - 1) \times 5$$

$$= 3 + 95$$

$$= 98$$

$$\text{Now, } (a_{30} - a_{20}) = 148 - 98$$

$$= 50$$

$$\therefore (a_{30} - a_{20}) = 50$$

Question: 20

Which term of the

Solution:

In the given AP, the first term = $a = 72$

$$\text{Common difference} = d = 63 - 72 = -9$$

To find: place of the term 0.

$$\text{So, let } a_n = 0$$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 0 = 72 + (n - 1) \times (-9)$$

$$\Rightarrow -72 = -9n + 9$$

$$\Rightarrow -72 - 9 = -9n$$

$$\Rightarrow -9n = -81$$

$$\Rightarrow n = 9$$

$$\therefore 9^{\text{th}} \text{ term of the AP is } -81.$$

Question: 21

Which term of the

Solution:

In the given AP, the first term = $a = 25$

Common difference = $d = 20 - 25 = -5$

To find: place of first negative term.

So, $a_n < 0$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 25 + (n - 1) \times (-5) < 0$$

$$\Rightarrow 25 - 5n + 5 < 0$$

$$\Rightarrow -5n + 30 < 0$$

$$\Rightarrow -5n < -30$$

$$\Rightarrow 5n > 30$$

$$\Rightarrow n > 6$$

$\therefore 7^{\text{th}}$ term of the AP is the first negative term.

Question: 22

Which term of the

Solution:

In the given AP, the first term = $a = 21$

Common difference = $d = 42 - 21 = 21$

To find: place of the term 210.

So, let $a_n = 210$

Since, we know that

$$a_n = a + (n - 1) \times d$$

$$\therefore 210 = 21 + (n - 1) \times (21)$$

$$\Rightarrow 210 = 21 + 21n - 21$$

$$\Rightarrow 210 = 21n$$

$$\Rightarrow n = 10$$

$\therefore 10^{\text{th}}$ term of the AP is 210.

Question: 23

What is 20th term

Solution:

Here, First term = $a = 3$

Common difference = $d = 8 - 3 = 5$

Last term = $l = 253$

To find: 20th term from end.

So, n^{th} term from end is given by:

$$a_n = l - (n - 1)d$$

$\therefore$ 20th term from end is:

$$a_{20} = 253 - (20 - 1) \times 5$$

$$= 253 - 95$$

$$= 158$$

$\therefore$ 20th term from the end is 158.

Question: 24

$$(5 + 13 + 21 + +$$

Solution:

Here, first term = 5

Common difference = $d = 13 - 5 = 8$

Last term = $l = 253$

To find: number of terms in the Arithmetic series

So, nth term is given by:

$$a_n = a + (n - 1)d$$

$$\therefore 181 = 5 + (n - 1) \times 8$$

$$\Rightarrow 181 - 5 = 8n - 8$$

$$\Rightarrow 176 = 8n - 8$$

$$\Rightarrow 176 + 8 = 8n$$

$$\Rightarrow 8n = 184$$

$$\Rightarrow n = 23$$

Thus there are 23 terms in the arithmetic series.

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$\therefore$ Sum of 23 terms is given by:

$$S_{23} = \frac{23}{2} [2(5) + (23 - 1)(8)]$$

$$= (23/2) \times [10 + 176]$$

$$= (23/2) \times 186$$

$$= 23 \times 93$$

$$= 2139$$

Thus, sum of 23 terms of this Arithmetic series is 2139.

Question: 25

The sum of first

Solution:

Here, first term = 10

Common difference = $d = 6 - 10 = -4$

Sum of first n terms of an AP is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\begin{aligned}
 \therefore S_{16} &= \frac{16}{2} [2(10) + (16 - 1)(-4)] \\
 &= 8 \times [20 - 60] \\
 &= 8 \times (-40) \\
 &= -320
 \end{aligned}$$

Thus, sum of 16 terms of this AP is - 320.

Question: 26

How many terms of

Solution:

Here, first term = $a = 3$

Common difference = $d = 7 - 3 = 4$

Let first n terms of the AP sums to 406.

$$\therefore S_n = 406$$

To find: n

$$\text{Now, } S_n = (n/2) \times [2a + (n - 1)d]$$

$$\text{Since, } S_n = 406$$

$$\therefore (n/2) \times [2a + (n - 1)d] = 406$$

$$\Rightarrow (n/2) \times [2(3) + (n - 1)4] = 406$$

$$\Rightarrow (n/2) \times [6 + 4n - 4] = 406$$

$$\Rightarrow (n/2) \times [(2 + 4n)] = 406$$

$$\Rightarrow n[1 + 2n] = 406$$

$$\Rightarrow n + 2n^2 = 406$$

$$\Rightarrow 2n^2 + n - 406 = 0$$

$$\Rightarrow 2n^2 - 28n + 29n - 406 = 0$$

$$\Rightarrow 2(n - 14) + 29(n - 14) = 0$$

$$\Rightarrow (2n + 29)(n - 14) = 0$$

$$\Rightarrow n = 14 \text{ or } n = -29/2$$

$$\therefore n = 14 \text{ (}\because n \text{ can't be a fraction or negative number)}$$

Question: 27

The 2nd term of a

Solution:

$$\text{Given: } a_2 = 13$$

$$a_5 = 25$$

To find: a_{17}

$$\text{Consider } a_2 = 13$$

$$\Rightarrow a + d = 13 \dots\dots\dots(1)$$

$$\text{Consider } a_5 = 25$$

$$\Rightarrow a + 4d = 25 \dots\dots\dots(2)$$

Subtracting equation (1) from equation (2), we get,

$$3d = 12$$

$$\Rightarrow d = 4$$

$\therefore$ Common difference = 4

From equation (1), we get

$$a = 13 - d$$

$$= 13 - 4$$

$$= 9$$

$$\text{Thus } a_{17} = a + 16d$$

$$= 9 + 16(4)$$

$$= 73$$

Question: 28

The 17th term of

Solution:

|

Let a be the first term and d be the common difference.

$$\text{Given: } a_{17} = a_{10} + 21$$

To find: common difference = d

$$\text{Consider } a_{17} = a_{10} + 21$$

$$\Rightarrow a + 16d = a + 9d + 21$$

$$\Rightarrow 16d = 9d + 21$$

$$\Rightarrow 16d - 9d = 21$$

$$\Rightarrow 7d = 21$$

$$\Rightarrow d = 3$$

$\therefore$ Common difference = 3

Question: 29

The 8th term of a

Solution:

$$\text{Given: } a_8 = 17$$

$$a_{14} = 29$$

To find: common difference = d

$$\text{Consider } a_8 = 17$$

$$\Rightarrow a + 7d = 17 \dots\dots\dots(1)$$

$$\text{Consider } a_{14} = 29$$

$$\Rightarrow a + 13d = 29 \dots\dots\dots(2)$$

Subtracting equation (1) from equation (2), we get,

$$6d = 12$$

$$\Rightarrow d = 2$$

$\therefore$ Common difference = 2

Question: 30

The 7th term of a

Solution:

Given: $a_7 = 4$

Common difference = $d = -4$

To find: First term = a

Since, $a_7 = 4$

$$\Rightarrow a + 6d = 4$$

$$\Rightarrow a + 6(-4) = 4$$

$$\Rightarrow a = 4 + 24$$

$$\Rightarrow a = 28$$