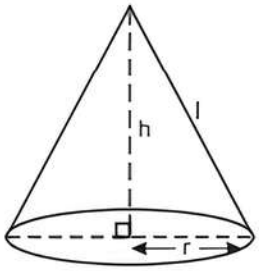
11 Surface Areas and Volumes

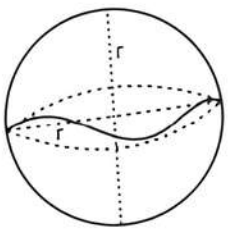
Fastrack Revision

- **Solid Figures:** Three-dimensional figures having definite shape, size and occupying a fixed amount of space in three dimensions. *e.g.,* cuboid, cylinder, cone, sphere etc.
- **Surface Area:** The sum of areas of all the faces of a solid figure. Its unit is unit^2 or square units.
- **Volume:** The space occupied by a solid. Its unit is unit^3 or cubic units.
- **Right Circular Cone:** A solid generated by revolving a line segment that passes through a fixed point and makes a constant angle with a fixed line. It has one curved face and one plane circular face. *e.g.,* funnel, joker's cap, etc. Let 'r' be the radius, 'h' be the vertical height and 'l' be the slant height of a right circular cone.



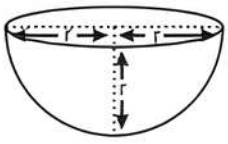
1. Volume $= \frac{1}{3}\pi r^2 h$
2. Curved Surface Area (CSA)
$= \frac{1}{2} \times l \times$ Circumference of base
$= \frac{1}{2} \times l \times 2\pi r = \pi rl$
3. Total Surface Area (TSA) = CSA + Area of the base
$= \pi rl + \pi r^2 = \pi r(l + r)$
4. Slant height $(l) = \sqrt{h^2 + r^2}$

- **Sphere:** A solid figure made up of all points in space which lie at a constant distance called radius from a fixed point called the centre of the sphere. Let 'r' be the radius of sphere.



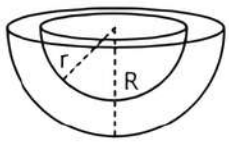
1. Volume $= \frac{4}{3}\pi r^3$
2. Curved Surface Area (CSA)
= Total Surface Area (TSA) $= 4\pi r^2$

- **Hemisphere:** A plane through the centre of a sphere cuts it into two equal parts, each part is called a hemisphere. Let 'r' be the radius of hemisphere.



1. Volume $= \frac{2}{3}\pi r^3$
2. Curved Surface Area (CSA) $= 2\pi r^2$
3. Total Surface Area (TSA) = CSA + Area of the base
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

- **Hemispherical Shell:** Let 'R' be the external radius and 'r' be the internal radius.



1. Area of base $= \pi(R^2 - r^2)$
2. External CSA $= 2\pi R^2$
3. Internal CSA $= 2\pi r^2$
4. Volume of material $= \frac{2}{3}\pi(R^3 - r^3)$
5. Thickness $= R - r$

Knowledge BOOSTER

1. *The total surface area of any object is greater than its lateral surface area.*
2. *When an object of certain volume is recast into a new shape, the volume of new shape formed will always be equal to the volume of the original object.*
3. *The solids having the same curved surface do not necessarily occupy the same volume.*
4. *When an object is dropped into a liquid, the volume of the displaced liquid is equal to the volume of the object that is dipped.*
5. *The volume of the material in a hollow body is equal to the difference between the external volume and internal volume.*
6. *To find the cost of polishing/covering/painting of a solid, we first find its surface area and then multiply by its per unit cost.*
7. *To find the quantity of a substance contained in a solid, we find its volume.*
8. *Volume of water accumulated on a roof after rain = Surface area of roof × Height of Rainfall.*
9. *Volume of water released by a pipe in unit time = Rate of flow × Area of cross-section of pipe.*
10. *If a large solid is transformed into a number of small identical solids, then number of small solids* $= \frac{\text{Volume of large solid}}{\text{Volume of small solid}}$

Practice Exercise

Multiple Choice Questions

Q 1. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is:

a. $2\pi r\,(l + r)$ b. $\pi r\left(l + \frac{r}{4}\right)$

c. $\pi r\,(l + r)$ d. $2\pi rl$

Q 2. The volume of a right circular cone is 924 cm^3. If its height is 18 cm, then the area of its base (in cm^2) is:

a. 154 b. 132

c. 176 d. 198

Q 3. If a right circular cone of height 24 cm has a volume of 1232 cm^3, then the area of its curved surface area is: $\left(\text{take } \pi = \frac{22}{7}\right)$

a. 1254 cm^2 b. 704 cm^2

c. 550 cm^2 d. 154 cm^2

Q 4. The volume of a right circular cone is equal to that of a sphere, whose radius is half the radius of the base of the cone. What is the ratio of the radius of the base to the height of the cone?

a. 1 : 4 b. 1 : 2

c. 4 : 1 d. 2 : 1

Q 5. The curved surface area and the slant height of a right circular cone are 99 cm^2 and 9 cm respectively. Find its diameter (in cm).

a. 3.5 b. 7

c. 14 d. 10.5

Q 6. The ratio of height and the diameter of a right circular cone is 3 : 2 and its volume is 1078 cm^3. Then, its height is: $\left(\text{take } \pi = \frac{22}{7}\right)$

a. 7 cm b. 14 cm

c. 21 cm d. 28 cm

Q 7. The circumference of the circular base of a cone is 50 cm. If the slant height of cone is 10 cm, then its curved surface area will be:

a. 125 cm^2 b. 2500 cm^2

c. 500 cm^2 d. 250 cm^2

Q 8. Find the volume (in cm^3) of a sphere of diameter 42 cm.

a. 17779 b. 36922

c. 38808 d. 13371

Q 9. The total surface area of a hemisphere is 1848 sq. cm, what is its radius? $\left(\text{take } \pi = \frac{22}{7}\right)$

a. 28 cm b. 7 cm

c. 21 cm d. 14 cm

Q 10. Find the curved surface area (in cm^2) of a hemisphere of diameter 28 cm.

a. 1152 b. 1024 c. 956 d. 1232

Q 11. Find the total surface area (in cm^2) of a hemisphere of diameter 42 cm.

a. 4158 b. 5782 c. 6321 d. 7782

Q 12. The volume of a solid hemisphere is 19404 cm^3. Its total surface area is:

a. 4158 cm^2 b. 2858 cm^2

c. 1738 cm^2 d. 2038 cm^2

Q 13. If the radius of a sphere is $2r$, then its volume will be:

a. $\frac{4}{3}\pi r^3$ b. $4\pi r^3$ c. $\frac{8\pi r^3}{3}$ d. $\frac{32}{3}\pi r^3$

Q 14. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is:

a. 1 : 4 b. 1 : 3 c. 2 : 3 d. 2 : 1

Q 15. The cost of electroplating of 1 m2 is ₹ 35. If the radius of sphere is 13 m, then the cost of electroplating of the curved surface area of the sphere will be:

a. ₹ 74400 b. ₹ 74360

c. ₹ 75500 d. ₹ 7600

Q 16. The volume of two spheres are in the ratio of 64 : 27. The ratio of their surface area is:

a. 4 : 3 b. 3 : 4

c. 16 : 9 d. 5 : 9

Assertion & Reason Type Questions

Directions (Q.Nos. 17-20): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct choice as:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 17. Assertion (A): If the ratio of the radii of two right circular cones of the same height is 1:3, then the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone, is $\sqrt{5}:9$.

Reason (R): The ratio of curved surface area of two different cones cannot be in the same ratio of their radii.

Q 18. Assertion (A): If the volumes of two spheres in in the ratio 125 : 8, then their surface area are in the ratio 25 : 4.

Reason (R): If R is the radius of a sphere, then volume and surface area of sphere are $\frac{4}{3}\pi R^3$ cubic units and $4\pi R^2$ sq. units.

Q 19. Assertion (A): A shotput is a metallic sphere of radius 4 cm. If the density of the metal is 10 gm per cm^3, then the mass of the shotput is 2 kg.

Reason (R): Volume of sphere with radius r is $\frac{4}{3}\pi r^3$.

Q 20. Assertion (A): If diameter of a sphere is 20 cm and it is reduced by 10% then the change in its volume will be 26.1%.

Reason (R): Change in volume

$$= \frac{\text{Volume of original sphere} - \text{Volume of new sphere}}{\text{Volume of original sphere}} \times 100$$

Fill in the Blanks Type Questions

Q 21. If the radius of a right circular cone is halved and height is doubled, then volume will remain ..... . (change/unchange)

Q 22. If the heights of two cones are in the ratio 1 : 3 and radii of their bases are in the ratio 3 : 1, then the ratio of their volumes is ..... .

Q 23. If the radius of the sphere is 7 cm, the surface area of the sphere is ..... .

Q 24. The total surface area of a hemisphere of radius 10 cm using value of $\pi = 3.14$ is ..... .

Q 25. If the volume of two hemispheres are in the ratio 8 : 27, then the ratio of their radii is ..... .

True/False Type Questions

Q 26. In a right circular cone, height, radius and slant height do not always be sides of a right triangle.

Q 27. If radius of the cone is doubled and height is halved, the volume will be halved.

Q 28. If the radius of a solid sphere is 24 cm, then 8 spheres of 12 cm radius can be made from it.

Q 29. If the ratio of the volume of two spheres is 27 : 64, then the ratio of their surface areas is 4 : 9.

Solutions

1. (b) Given, radius of cone is $\frac{r}{2}$ and slant height is $2l$

$\therefore$ TSA of cone = CSA of cone + Area of base

$$= \pi\left(\frac{r}{2}\right)(2l) + \pi\left(\frac{r}{2}\right)^2 = \pi r\left(l + \frac{r}{4}\right)$$

2. (a) Volume of right circular cone

$$= \frac{1}{3} \times \text{Area of base of cone} \times \text{Height of cone}$$

$$\Rightarrow \quad 924 = \frac{1}{3} \times \text{Area of base} \times 18$$

$$\therefore \text{Area of base} = \frac{924 \times 3}{18} = 154 \text{ cm}^2$$

3. (c) Given, volume of the cone = 1232 cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 1232$$

$$\Rightarrow \quad r^2 = \frac{1232 \times 3 \times 7}{22 \times 24}$$

$$\Rightarrow \quad r^2 = 49 \quad \Rightarrow \quad r = 7 \text{ cm}$$

TRICK

Slant height $(l) = \sqrt{h^2 + r^2}$

$\therefore$ Slant height $(l) = \sqrt{(24)^2 + (7)^2}$

$$= \sqrt{576 + 49} = \sqrt{625} = 25 \text{ cm}$$

$\therefore$ Area of curved surface $= \pi r l$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$$

4. (d) Let the radius of the base of a cone be r and height of cone be h.

$\therefore$ Radius of the sphere $= \frac{r}{2}$

According to the question,

Volume of cone = Volume of sphere

$$\therefore \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3$$

$$\Rightarrow \quad h = \frac{r}{2} \Rightarrow \frac{r}{h} = \frac{2}{1} \Rightarrow r : h = 2 : 1$$

5. (b) Given, slant height of cone, $l = 9$ cm
Let the radius of cone be r.

TRICK

Curved surface area of cone $= \pi rl$

Also given, curved surface area of cone $= 99\ cm^2$

$$\Rightarrow \quad 99 = \frac{22}{7} \times r \times 9$$

$$\Rightarrow \quad r = \frac{99 \times 7}{22 \times 9} = 3.5\,cm$$

$\therefore$ Diameter of cone $= 2r = 2 \times 3.5 = 7cm$

6. (c) Let the height of the cone be $3x$ cm and diameter be $2x$ cm.

$$\therefore \quad \text{Radius} = \frac{2x}{2} = x\,cm$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$1078 = \frac{1}{3} \times \frac{22}{7} \times x^2 \times 3x$$

$$\Rightarrow \quad 1078 = \frac{22}{7}x^3$$

$$\Rightarrow \quad x^3 = \frac{1078 \times 7}{22} = 343$$

$$\therefore \quad x = \sqrt[3]{343} = 7$$

$\therefore$ Height of cone $= 3x = 3 \times 7 = 21$ cm

7. (d) Given, slant height $(l) = 10$ cm
and circumference $= 50$ cm
$\Rightarrow \quad 2\pi r = 50$ cm
$\therefore \quad \pi r = 25$ cm
So, curved surface area $= \pi rl = 25 \times 10 = 250\ cm^2$

8. (c) Given, diameter of sphere $= 42$ cm

$$\therefore \quad \text{Radius of sphere, } r = \frac{42}{2} = 21\,cm$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 38808\ cm^3$$

9. (d) Let the radius of hemisphere be r cm.

TRICK

Total surface area of hemisphere $= 3\pi r^2$

$$\therefore \quad 1848 = 3 \times \frac{22}{7} \times r^2$$

$$\Rightarrow \quad r^2 = \frac{1848 \times 7}{3 \times 22} = 196$$

$$\Rightarrow \quad r = 14\ cm$$

10. (d) Given, diameter of hemisphere $= 28$ cm

$$\therefore \quad \text{Radius of hemisphere} = \frac{28}{2} = 14\,cm$$

$\therefore$ Curved surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 14 \times 14 = 1232\ cm^2$$

11. (a) Given, diameter of hemisphere $= 42$ cm

$$\therefore \quad \text{Radius of hemisphere} = \frac{42}{2} = 21\,cm$$

$\therefore$ Total surface area of hemisphere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times 21 \times 21 = 4158\ cm^2$$

12. (a) Let the radius of hemisphere be r cm.
Then, volume of hemisphere $= 19404\ cm^3$

$$\therefore \quad \frac{2}{3}\pi r^3 = 19404$$

$$\Rightarrow \quad \frac{2}{3} \times \frac{22}{7} \times r^3 = 19404$$

$$\Rightarrow \quad r^3 = \frac{19404 \times 3 \times 7}{2 \times 22} = 9261$$

$$\therefore \quad r = \sqrt[3]{21 \times 21 \times 21} = 21\,cm$$

$\therefore$ Total surface area of hemisphere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times 21 \times 21 = 4158\,cm^2$$

13. (d) Given, radius of sphere $(R) = 2r$

$$\therefore \text{ Volume of sphere } = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$$

14. (a) Given, radii of hemispherical balloon are
$r_1 = 6$ cm and $r_2 = 12$ cm.
The ratio of surface area of hemispherical balloon is

$$\frac{S_1}{S_2} = \frac{2\pi r_1^2}{2\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{6}{12}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 1:4$$

15. (b) Given, radius of sphere $= 13$ m
Curved surface area $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 13 \times 13 = \frac{14872}{7}\,m^2$$

Cost of electroplating $1\ m^2 = ₹\ 35$

$\therefore$ Cost of electroplating $\frac{14872}{7}\,m^2$

$$= ₹\ 35 \times \frac{14872}{7} = ₹\ 74360$$

16. (c) Let radii of spheres be R_1 and R_2 and volume be V_1 and V_2.

Given, $\quad \frac{V_1}{V_2} = \frac{64}{27} \Rightarrow \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{64}{27}$

$$\Rightarrow \frac{R_1^3}{R_2^3} = \frac{64}{27} \Rightarrow \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R_1}{R_2} = \frac{4}{3}$$

$\therefore$ The ratio of surface area of spheres

$$= \frac{4\pi R_1^2}{4\pi R_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$= 16 : 9$

17. (c) **Assertion (A):** Given, $\frac{r}{R} = \frac{1}{3} \Rightarrow R = 3r$

Let the radius of the first (smaller) cone be 'x' and the height of the cone will be $3x$.

$\therefore$ $l = \sqrt{h^2 + r^2} = \sqrt{(3x)^2 + x^2}$

$= \sqrt{10x^2} = \sqrt{10}x$

$\therefore$ CSA of first cone $= \pi rl$ sq. units

$= \pi x \sqrt{10}x = \pi x^2\sqrt{10}$

The radius and the height of the second (bigger) cone is $3x$.

$\therefore$ $l = \sqrt{(3x)^2 + (3x)^2} = \sqrt{9x^2 + 9x^2}$

$= \sqrt{18x^2} = 3\sqrt{2}x$

CSA of second cone $= \pi \times 3x \times 3\sqrt{2}x$ sq. units

$= 9\pi\sqrt{2}x^2$

$\therefore$ Ratio of the curved surface area

$= \pi x^2\sqrt{10} : 9\pi x^2\sqrt{2}$

$= \sqrt{10} : 9\sqrt{2} = \sqrt{5} : 9$

$\therefore$ Assertion (A) is true.

Reason (R): It is not always true to say that the ratio of curved surface area of two different cones cannot be in the same ratio of their radii.

Hence, Assertion (A) is true, but Reason (R) is false.

18. (a) **Assertion (A):** Given, volume of two spheres are in the ratio

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3}$$

$$\Rightarrow \quad \frac{125}{8} = \left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{5}{2}$$

$\therefore$ The ratio of the surface areas of two spheres are in the ratio

$$\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2}$$

$$= \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

19. (d) **Assertion (A):** Given, radius of metallic sphere is

$r = 4$ cm

TR!CK

Volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore$ Volume of metallic sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (4)^3$

$= 268.19 \text{ cm}^3$

$\because$ Density of metal $= 10 \text{ gm/cm}^3$

$\therefore$ Mass of shotput = Density $\times$ Volume

$= 10 \times 268.19$

$= 2681.9$ gm

$= \frac{2681.9}{1000}$ kg $\quad \left[\because 1 \text{ gm} = \frac{1}{1000}\text{kg}\right]$

$= 2.68 \text{ kg} \neq 2 \text{ kg}$

So, Assertion (A) is false.

Reason (R): It is true.

Hence, Assertion (A) is false but Reason (R) is true.

20. (d) **Assertion (A):** Given, diameter of sphere be 20 cm, original diameter, $D_1 = 20$ cm

$\therefore$ Original radius of sphere, $R_1 = \frac{20}{2} = 10$ cm

and original volume of sphere, $V_1 = \frac{4}{3}\pi R_1^3$

$= \frac{4}{3}\pi \times 10 \times 10 \times 10 \text{ cm}^3$

$= \frac{4}{3}\pi \times 1000 \text{ cm}^3$

New diameter, $D_2 = (100\% - 10\%)$ of 20 cm

$= \frac{90}{100} \times 20 = 18$ cm

New radius, $R_2 = \frac{18}{2} = 9$ cm

$\therefore$ New volume of sphere, $V_2 = \frac{4}{3}\pi R_2^3$

$= \frac{4}{3}\pi \times 9 \times 9 \times 9 \text{ cm}^3$

$= \frac{4}{3}\pi \times 729 \text{ cm}^3$

Now, change in volume $= \frac{V_1 - V_2}{V_1} \times 100$

$$= \frac{\frac{4\pi}{3}\times 1000 - \frac{4\pi}{3}\times 729}{\frac{4\pi}{3}\times 1000}\times 100$$

$$= \frac{4\pi(1000-729)\times 3\times 100}{3\times 1000\times 4\pi}$$

$$= \frac{271}{1000}\times 100 = 27.1\ \%$$

Assertion (A) is false.

Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

21. Let radius and height of a right circular cone be r and h respectively.

Then $r = \frac{r}{2}$ and $h = 2h$

TR!CK

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ Volume of cone $= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 (2h)$

$$= \frac{\pi}{3}\times\frac{r^2}{4}\times 2h$$

$$= \frac{\pi r^2 h}{6} = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

Thus, the new volume of right circular cone becomes half of original volume of a right circular cone.

Hence, new volume will remain changed.

22. Let r_1 and h_1 be the radius and height of a first cone.

Let r_2 and h_2 be the radius and height of a second cone.

Then $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

$$\therefore \frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{3}{1}\right)^2\left(\frac{1}{3}\right)$$

$$= 9\times\frac{1}{3} = \frac{3}{1}$$

23. Given, radius of the sphere is $r = 7$ cm

Then, surface area of the sphere is

$$S = 4\pi r^2 = 4\times\frac{22}{7}\times(7)^2 = 616\ \text{cm}^2$$

24. Given, radius of a hemisphere $r = 10$ cm.

Then, total surface area of hemisphere

$$= 3\pi r^2 = 3\times 3.14\times(10)^2$$
$$= 942\ \text{cm}^2$$

25. Let r_1 and r_2 be the radii of two hemispheres.

TR!CK

Volume of hemisphere $= \frac{2}{3}\pi r^3$

The ratio of volume of two hemispheres is

$$\frac{V_1}{V_2} = \frac{8}{27} \Rightarrow \frac{\frac{2}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3} = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$$

26. False.

Because it always be $r^2 + h^2 = l^2$.

27. Let r and h be the radius and height of an original cone.

Then, volume of cone, $V = \frac{1}{3}\pi r^2 h$

If $R = 2r$ and $H = \frac{h}{2}$, then

New volume becomes, $V' = \frac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi(2r)^2\times\frac{h}{2}$$

$$= \frac{1}{3}\pi\times 2r^2 h = 2\left(\frac{1}{3}\pi r^2 h\right)$$
$$= 2V$$

Hence, given statement is false.

28. Given, radius of solid sphere is $R = 24$ cm, and radius of small sphere is $r = 12$ cm

Let n small sphere be made from a solid sphere.

Then, $n \times$ volume of one small sphere

= volume of solid sphere

$$\therefore\quad n = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3 = \left(\frac{24}{12}\right)^3 = (2)^3 = 8$$

Hence, given statement is true.

29. Let r_1 and r_2 be the radii of two spheres, then

$$\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3}$$

$$\Rightarrow \quad \frac{27}{64} = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \frac{R_1}{R_2} = \frac{3}{4}$$

$$\therefore \frac{\text{Surface area of first sphere}}{\text{Surface area of second sphere}} = \frac{4\pi R_1^2}{4\pi R_2^2}$$

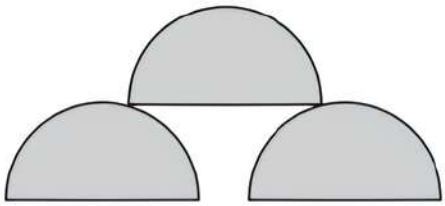
$$= \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Hence, given statement is false.

Case Study Based Questions

Case Study 1

Priyansh had three hemispherical shaped boxes. He thought of putting down his stationary in one of these boxes, so that it does not get lost. The radius of each hemispherical box is 7 cm.



On the basis of the above information, solve the following questions:

Q 1. The capacity of a box is:

a. 730 cm^3 b. 1500 cm^3

c. 718.67 cm^3 d. 1900 cm^3

Q 2. If any two box is melted and recasted into a single sphere, then radius of sphere will be:

a. 3.5 cm b. 7 cm

c. 14 cm d. 10.5 cm

Q 3. The base area of a box is:

a. 150 cm^2 b. 180 cm^2

c. 154 cm^2 d. 170 cm^2

Q 4. The total surface area of a box is:

a. 340 cm^2 b. 308 cm^2

c. 380 cm^2 d. 830 cm^2

Q 5. If the cost of painting a box per cm^2 is ₹ 2, then the total cost of painting a box is:

a. ₹ 740 b. ₹ 750

c. ₹ 730 d. ₹ 616

Solutions

1. (c) Given, radius, $r = 7$ cm

∴ The capacity of a box = Volume of hemispherical shaped box

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (7)^3 = \frac{2156}{3}$$

$$= 718.67 \text{ cm}^3$$

So, option (c) is correct.

2. (b) Given, 2× Volume of hemisphere = Volume of sphere

$$\Rightarrow \quad 2 \times \frac{2}{3} \times \pi \times (7)^3 = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad 7^3 = R^3$$

$$\Rightarrow \quad R = 7\text{cm}$$

Hence, radius of required sphere is 7cm.

So, option (b) is correct.

3. (c) The base area of a box = πr^2

$$= \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$$

So, option (c) is correct.

4. (b) The total surface area of a box = $2\pi r^2$

$$= 2 \times 154$$

$$= 308 \text{ cm}^2$$

So, option (b) is correct.

5. (d) Given, 1 cm^2 area is painted in ₹ 2.

Therefore, 308 cm^2 area is painted in ₹ 2 × 308

= ₹ 616.

So, option (d) is correct.

Case Study 2

During vaccation, two friends decided to visit Shimla. Due to peak days in Shimla, they did not get any room to stay it. So, they thought to buy a tent and set up in a park. They made a tent in the shape of cone, whose diameter is 14 cm and height is 22 cm.

On the basis of the above information, solve the following questions:

Q 1. How much volume of air is stored in a conical tent?

a. 1140 cm^3 b. 1132 cm^3

c. 1129.33 cm^3 d. 1134 cm^3

Q 2. The slant height of a cone is:

a. $\sqrt{521}$ cm b. $\sqrt{533}$ cm

c. $2\sqrt{533}$ cm d. $\sqrt{537}$ cm

Q 3. The base area covered by a conical tent is:

a. 150 cm^2 b. 153 cm^2

c. 154 cm^2 d. 159 cm^2

Q 4. The curved surface area of a conical tent is:
(Use $\sqrt{533} = 23.09$)

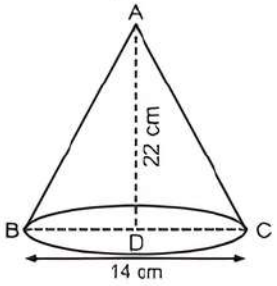
a. 508 cm^2 b. 507.98 cm^2
c. 509 cm^2 d. 512 cm^2

Q 5. If the cost of painting a tent is 50 paise per cm^2, then the cost of painting the tent is (approximate):

a. ₹ 260 b. ₹ 255
c. ₹ 254 d. ₹ 270

Solutions

1. (c) Given, diameter of cone, $d = 14$ cm
and height of cone, $h = 22$ cm



Therefore, radius of a cone, $r = \frac{d}{2} = \frac{14}{2} = 7$ cm

∴ Volume of air stored in a conical tent
= Volume of cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 22 \text{ cm}^3$$

$= 1129.33 \text{ cm}^3$

So, option (c) is correct.

2. (b)

TR!CK

Slant height of a cone is $l = \sqrt{r^2 + h^2}$

∴ The slant height of a cone $= \sqrt{(7)^2 + (22)^2}$

$= \sqrt{49 + 484} = \sqrt{533}$ cm

So, option (b) is correct.

3. (c) The base area covered by a conical tent is
A = Area of circle = πr^2

$$= \frac{22}{7} \times (7)^2 = 22 \times 7 = 154 \text{ cm}^2$$

So, option (c) is correct.

4. (b) The curved surface area of a conical tent $= \pi rl$

$$= \frac{22}{7} \times 7 \times \sqrt{533} \qquad [\because l = \sqrt{533} \text{ cm}]$$

$= 22 \times 23.09 = 507.98 \text{ cm}^2$

So, option (b) is correct.

5. (c) Given, 1 cm^2 is painted for 50 paise or ₹ 0.5.
∴ 507.98 cm^2 is painted for ₹ 507.98×0.5
= ₹ 253.99 ≈ ₹ 254

So, option (c) is correct.

Case Study 3

For decoration purpose, Sneha bought 100 orbeez balls and put it in a cylindrical shaped box. After filling it with water, the orbeez ball swell up and completely filled the cylindrical shaped box. Behind the orbeez ball packet, the change in volume of each orbeez ball was mentioned and which was 32% increase. Suppose the volume of all orbeez ball is 9900 cm^3.

On the basis of the above information, solve the following questions:

Q 1. The volume of each orbeez ball is:

a. 97 cm^3 b. 99 cm^3
c. 96 cm^3 d. 94 cm^3

Q 2. Volume of orbeez ball before swelling is:

a. 70 cm^3 b. 78 cm^3
c. 75 cm^3 d. 85 cm^3

Q 3. How many orbeez balls before swelling was needed to completely fill the cylindrical shaped box?

a. 132 b. 138 c. 134 d. 140

Q 4. What is the cubic radius of a orbeez ball before swelling up?

a. 17.30 cm^3 b. 16.50 cm^3
c. 17.70 cm^3 d. 17.89 cm^3

Q 5. If the change in volume of orbeez ball is increased to 48%, then the volume of orbeez ball after swell up will be:

a. 114 cm^3 b. 111 cm^3
c. 112 cm^3 d. 116 cm^3

Solutions

1. (b) Volume of 1 orbeez ball × 100 = Volume of all orbeez ball

∴ Volume of one orbeez ball $= \frac{9900}{100} = 99 \text{ cm}^3$

So, option (b) is correct.

2. (c) Let original volume of orbeez ball be x.
Change in volume of orbeez ball

$$= \frac{99 - x}{x} \times 100$$

Given, $32 = \frac{(99 - x) \times 100}{x}$

$\Rightarrow$ $32x = 9900 - 100x$

$\Rightarrow \quad 132\,x = 9900$

$\Rightarrow \quad x = \dfrac{9900}{132}$

$\therefore \quad x = 75\ \text{cm}^3$

So, option (c) is correct.

3. (a) $n \times$ Volume of 1 orbeez ball before swell up = Volume of container

$\Rightarrow \quad n \times 75\ \text{cm}^3 = 9900\ \text{cm}^3$

$\Rightarrow \quad n = \dfrac{9900}{75}$

$\therefore \quad n = 132$ balls

So, option (a) is correct.

4. (d) Volume of a orbeez ball before swell up $= 75\ \text{cm}^3$

$\Rightarrow \quad \dfrac{4}{3}\pi r^3 = 75$

$\Rightarrow \quad \dfrac{4}{3} \times \dfrac{22}{7} \times r^3 = 75$

$\Rightarrow \quad r^3 = \dfrac{75 \times 3 \times 7}{4 \times 22}$

$\therefore \quad r^3 = 17.89\ \text{cm}^3$

So, option (d) is correct.

5. (b) Let the volume of orbeez ball after swell up be x.

Then, $48 = \dfrac{x - 75}{75} \times 100$

$\Rightarrow \quad 48 = \dfrac{x - 75}{3} \times 4$

$\Rightarrow \quad 48 \times 3 = 4x - 300$

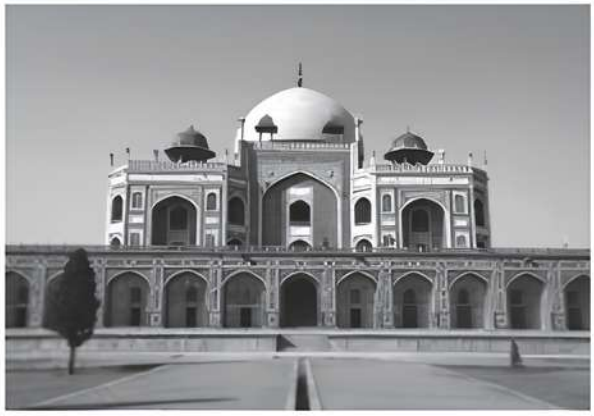
$\Rightarrow \quad 4x = 144 + 300 \Rightarrow 4x = 444\ \text{cm}^3$

$\therefore \quad x = 111\ \text{cm}^3$

So, option (b) is correct.

Case Study 4

Students of class IX were taken to an educational trip to Delhi. They were first shown Humayun's Tomb. It was the tomb of Mughal Emperor Humayun and is a great example of Mughal Architecture and it is an UNESCO approved World Heritage site. Since, it was a very old monument restoration and paint was going on. Maths teacher decided to ask some questions. He told them in the figure a dome can be seen, when radius of the dome is 15 m and in the monument there can be seen 2 pillars which resembles a cylinder whose radius and height is 28 m and 47 m.



On the basis of the above information, solve the following questions:

Q 1. Find the curved surface area of the dome.

Q 2. Find the volume of the dome.

Solutions

1. The dome resembles a hemispherical sphere
Given, radius = 15 m
Curved surface area of the hemispherical sphere
$= 2\pi r^2$
$= 2 \times \dfrac{22}{7} \times 15 \times 15 = \dfrac{9900}{7}\ \text{m}^2$
$= 1414.285\ \text{m}^2$

2. Volume of the dome = Volume of hemisphere
$= \dfrac{2}{3}\pi r^3$
$= \dfrac{2}{3} \times \dfrac{22}{7} \times 15 \times 15 \times 15 = \dfrac{49500}{7}$
$= 7071.43\ \text{m}^3$

Very Short Answer Type Questions

Q 1. The height and the slant height of a cone are 21 cm and 28 cm, respectively. Find the volume of the cone.

Q 2. The radius and vertical height of a cone are 5 cm and 12 cm, respectively. Find its lateral surface area.

Q 3. The radius of the base of a conical tent is 16 m. If $427\frac{3}{7}$ m^2 canvas is required to construct the tent, then find the slant height of the tent. $\left(\text{take } \pi = \dfrac{22}{7}\right)$

Q 4. If the curved surface area of a right circular cone is 3080 sq cm and its slant height is 35 cm, find its total surface area.

Q 5. The surface area of a sphere is 5544 cm^2. Find its diameter (in cm).

Q 6. A sphere and a hemisphere have the same volume. Find the ratio of their radii.

Q 7. Find the volume (in cm^3) of a sphere of diameter 7 cm.

Q 8. The volume of two hemispheres are in the ratio 27 : 125. Find the ratio of their radii.

Short Answer Type-I Questions

Q 1. How much ice-cream can be put into a cone with base radius 3.5 cm and height 12 cm?

Q 2. A conical tent is to accommodate 11 persons. Each person must have 4 m^2 of the space on the ground and 20 m^3 of air to breathe. Find the height of the cone.

Q 3. What is the radius and curved surface area of a cone made from a quadrant of a circle of radius 28 cm?

Q 4. The radius of the base and height of a right circular cone are in the ratio 5 : 12. If the volume of the cone is $314\frac{2}{7}$ cm^3, then find the slant height (in cm) of the cone.

Q 5. The diameter of a metallic ball is 5.6 cm. What is the mass of the ball, if the density of the metal is 7.6 g per cm^3?

Q 6. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Q 7. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

Q 8. The curved surface area of a right circular cone is twice that of another right circular cone. If the slant height of the second cone is twice that of the first cone, find the ratio of the radius of first cone to that of second cone.

Short Answer Type-II Questions

Q 1. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Q 2. A corn cob shaped somewhat like a cone, has the radius to its broadest end as 6 cm and length as 8 cm. If each 1 cm^2 of the surface of the cob carries an average of five grains, then find how many grains you would find on the entire cob?

Q 3. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2, what will be the cost of painting all these cones? *[NCERT Exemplar]*
(Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Q 4. The diameter is the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Q 5. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the:
(i) radius r' of the new sphere.
(ii) ratio of S and S'.

Q 6. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Q 7. A hemispherical dome of building needs to be painted. If the circumference of the base of the dome is 55.44 m, then find the cost of painting it at the rate of ₹12 per 100 cm^2.

Q 8. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Q 9. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 498.96. If the cost of white-washing is ₹ 2.00 per square metre, find the:
(i) inside surface area of the dome.
(ii) volume of the air inside the dome.

Q 10. Metallic spheres of radii 6 m, 8 m and 10 m, respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Long Answer Type Questions

Q 1. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m.
(i) How many students can sit in the tent, if a student on an average occupies $\frac{5}{7}$ m^2 on the ground?
(ii) Find the volume of the cone.

Q 2. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, then find:
(i) height of the cone.
(ii) slant height of the cone.
(iii) curved surface area of the cone.

Q 3. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (take $\pi = 3.14$)
[NCERT Exemplar]

Q 4. A right triangle with sides 6 cm, 8 cm and 10 cm, is revolved about the side 8 cm. Find the volume and the curved surface area of the solid so formed.

Q 5. A hemispherical dome open at base is made from sheet of fiber. If the diameter of hemispherical dome is 80 cm and $\frac{13}{170}$ part of sheet actually used was wasted in making the dome, then find the cost of dome at the rate of ₹ $\frac{35}{100}$ per cm^2.

Q 6. The volume of the two spheres are in the ratio 64 : 27. Find the difference of their surface areas, if the sum of their radii is 7 cm.

Solutions

Very Short Answer Type Questions

1. Given, height of a cone (h) = 21 cm
and their slant height (l) = 28 cm

TR!CK

$l^2 = r^2 + h^2$

$\therefore \quad r = \sqrt{l^2 - h^2}$

$= \sqrt{(28)^2 - (21)^2} = \sqrt{784 - 441}$

$= \sqrt{343} = 7\sqrt{7}$ cm

Hence, volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21$

$= 7546$ cm^3

2. Given, radius of cone (r) = 5 cm
and height of cone (h) = 12 cm

$\because$ Slant height (l) $= \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169} = 13$ cm

$\therefore$ Lateral surface area $= \pi rl = \frac{22}{7} \times 5 \times 13 = \frac{1430}{7}$

$= 204.3$ cm^2

3. Let slant height of conical tent be l

$\because$ Curved surface area of cone $= 427\frac{3}{7}$ m^2

$\Rightarrow \pi rl = \frac{2992}{7} \Rightarrow \frac{22}{7} \times 16 \times l = \frac{2992}{7}$

$\Rightarrow \quad 22 \times 16 \times l = 2992$

$\therefore \quad l = \frac{2992}{22 \times 16} = 8.5$ m

Hence, slant height of the tent is 8.5 m.

4. Given, slant height of cone, $l = 35$ cm
Let the radius of cone be r.
$\therefore$ Curved surface area of cone = πrl

$\Rightarrow \quad 3080 = \frac{22}{7} \times r \times 35$

$\Rightarrow \quad r = \frac{3080 \times 7}{22 \times 35} = 28$ cm

Now, total surface area of cone $= \pi rl + \pi r^2$

$= 3080 + \frac{22}{7} \times 28 \times 28$

$= 3080 + 2464 = 5544$ cm^2

5. Let the radius of a sphere be r.
$\therefore$ Surface area of sphere $= 4\pi r^2$

$\Rightarrow \quad 5544 = 4 \times \frac{22}{7} \times r^2$

$\Rightarrow \quad r^2 = \frac{5544 \times 7}{4 \times 22} \Rightarrow r^2 = 441$

$\Rightarrow \quad r = 21$ cm

$\therefore$ Diameter of sphere $= 2r = 2 \times 21 = 42$ cm

6. Let the radius of sphere be r_1 units and that of hemisphere be r_2 units.
According to the question,
Volume of sphere = Volume of hemisphere

$\Rightarrow \quad \frac{4}{3}\pi r_1^3 = \frac{2}{3}\pi r_2^3$

$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{2} \Rightarrow \frac{r_1}{r_2} = \frac{1}{\sqrt[3]{2}}$

$\therefore \quad r_1 : r_2 = 1 : \sqrt[3]{2}$

7. Given, diameter of sphere = 7 cm

$\therefore$ Radius of sphere, $(r) = \frac{7}{2}$ cm

$\therefore \quad$ Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{3}$

$= 179.67$ cm^3

8. Let 'r_1' and 'r_2' be the radii of two hemispheres respectively.
Given, ratio of volumes of hemispheres

$= \frac{V_1}{V_2} = \frac{27}{125}$

$\Rightarrow \quad \frac{\frac{2}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3} = \frac{27}{125}$

$\left[\because \text{volume of hemisphere} = \frac{2}{3}\pi r^3\right]$

$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{3}{5}\right)^3 \quad \Rightarrow \quad \frac{r_1}{r_2} = \frac{3}{5}$

Hence, the ratio of their radii is 3 : 5.

Short Answer Type-I Questions

1. Given, radius of cone $(r) = 3.5$ cm and height of cone $(h) = 12$ cm.

Now, volume of ice-cream in cone

$$= \text{Volume of the cone}$$

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12$$

$$= 154 \text{ cm}^3$$

Hence, the volume of ice-cream in cone is 154 cm^3.

2. Let the radius of the base of conical tent be 'r' and height be 'h'.

TR!CK

Area occupied by 11 persons is equal to the area of base of the cone.

Now, area of the base = Area occupied by 11 persons on the ground

$$\therefore \quad \pi r^2 = 4 \times 11 = 44 \text{ m}^2$$

So, volume of air in the tent

$$= \text{Volume of conical tent}$$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \text{Volume of air taken by 11 persons}$$

$$= 20 \times 11 = 220 \text{ m}^3$$

$$\Rightarrow \quad \frac{1}{3} \times 44 \times h = 220 \quad [\because \pi r^2 = 44 \text{ cm}^2]$$

$$\Rightarrow \quad h = \frac{220 \times 3}{44}$$

$$\therefore \quad h = 15 \text{ m}$$

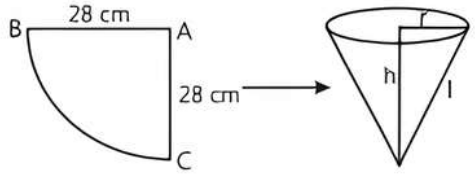
Hence, the height of the cone is 15 m.

3. Given, radius of quadrant of circle $(R) = 28$ cm.

Let 'r' and 'l' be the radius and slant height of cone.

TiP

When we join edge of quadrant AB and AC, it forms a cone and length of an arc BC becomes the circumference of the base of cone.



$\because$ Quadrant of circle is casted into a cone.

$\therefore$ Area of quadrant = CSA of cone

$$\Rightarrow \quad \frac{1}{4}\pi R^2 = \pi r l$$

$$\Rightarrow \quad \frac{1}{4} \times \pi \times (28)^2 = \pi r l$$

$$\Rightarrow \quad \frac{28 \times 28}{4} = r \times 28 \quad [\because l = R = 28 \text{ cm}]$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

Now, CSA of cone $= \pi r l$

$$= \frac{22}{7} \times 7 \times 28$$

$$= 616 \text{ cm}^2$$

Hence, the radius of cone is 7 cm and its curved surface area is 616 cm^2.

COMMON ERR!R

In these types of questions, students do not understand the concept of figure, what will be the radius and slant height of cone. So, regular practice will be required for these type of questions.

4. Let the radius of the base of the cone be $5x$ cm and its height be $12x$ cm.

$$\text{Volume of cone} = 314\frac{2}{7} \Rightarrow \frac{1}{3}\pi r^2 h = \frac{2200}{7}$$

$$\Rightarrow \quad (5x)^2 (12x) = \frac{2200 \times 3 \times 7}{7 \times 22}$$

$$\Rightarrow \quad x^3 = \frac{100 \times 3}{25 \times 12}$$

$$\Rightarrow \quad x = 1 \text{ cm}$$

$$\therefore \quad \text{Height} = 12x = 12 \times 1 = 12 \text{ cm}$$

$$\text{and} \quad \text{radius} = 5x = 5 \times 1 = 5 \text{ cm}$$

Now, slant height of cone,

$$l = \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$$

5. Given, diameter of the metallic ball = 5.6 cm

$\therefore$ Radius of the metallic ball $(r) = \frac{5.6}{2} = 2.8$ cm

$\therefore$ Volume of the metallic spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (2.8)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.8 \times 2.8 \times 2.8 \text{ cm}^3 = 91.99 \text{ cm}^3$$

Given, the density of the metal = 7.6 g per cm^3

$\therefore$ Mass of the ball = Volume × Density

$$= (91.99 \times 7.6) \text{ g} = 699.12 \text{ g}$$

Hence, the required mass of the ball is 699.12 g.

6. Given, diameter of hemispherical bowl = 10.5 cm

$\therefore$ Radius of hemispherical bowl (r)

$$= \frac{10.5}{2} \text{cm} = 5.25 \text{ cm}$$

Now, volume of the hemispherical bowl $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25 \text{ cm}^3$$

$$= 303 \text{ cm}^3 \text{ (Approx.)}$$

Hence, the hemispherical bowl can hold

$\frac{303}{1000}$ L = 0.303 L of milk. $[\because 1 \text{ cm}^3 = \frac{1}{1000} \text{ L}]$

7. Given, diameter of spherical ball = 4.2 cm

$\therefore$ Radius of spherical ball = 2.1 cm

So, volume of solid spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808 \text{ cm}^3$$

Now, amount of water displaced

$$= \text{Volume of solid spherical ball}$$

$$= 38.808 \text{ cm}^3$$

$$= 38.808 \text{ mL} \qquad [\because 1 \text{ cm}^3 = 1 \text{ mL}]$$

Hence, the amount of water displaced is 38.808 mL.

8. Let r and r' be the radii of first and second cone, l and l' be the slant heights, h and h' be the heights of first cone and second cone respectively.

Given, CSA of first cone = 2 × CSA of second cone

$\Rightarrow \quad \pi rl = 2 \times \pi r'l'$

$\Rightarrow \quad \pi rl = 2\pi r' \cdot 2l \qquad [\because l' = 2l, \text{ given}]$

$\Rightarrow \quad \frac{r}{r'} = \frac{2\pi \times 2l}{\pi l} = \frac{4}{1}$

$\therefore \quad r : r' = 4 : 1$

Hence, the ratio of the radius of first cone to that of second cone is 4 : 1.

Short Answer Type-II Questions

1. Given, radius of conical heap $(r) = \frac{10.5}{2}$ m

$= 5.25$ m

and height of conical heap $(h) = 3$ m

$\therefore$ Volume of the conical heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3 \text{ m}^3$

$= 86.625 \text{ m}^3$

TRICK

$l = \sqrt{h^2 + r^2}$

$\therefore \quad l = \sqrt{3^2 + (5.25)^2}$

$= \sqrt{9 + 27.5625} = \sqrt{36.5625}$

$= 6.05$ m (Approx.)

So, curved surface area of the cone $= \pi rl$

$= \frac{22}{7} \times 5.25 \times 6.05 \text{ m}^2$

$= 99.825 \text{ m}^2$

Hence, 99.825 m^2 of canvas is required.

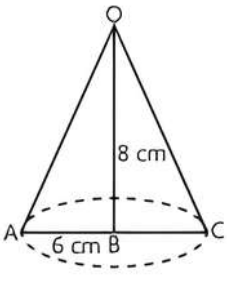
2. Given, radius of the broadest end $(r) = 6$ cm and length of cob $(h) = 8$ cm.

Let l be the slant height of the conical corn cob.

Then, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64}$

$= \sqrt{100} \text{ cm} = 10 \text{ cm}$



$\therefore$ Curved surface area of the corn cob $= \pi rl$

$= \frac{22}{7} \times 6 \times 10$

$= \frac{1320}{7} = 188.57 \text{ cm}^2$

Since, the grains of corn are found on the curved surface of the corn cob.

So, total number of grains on the corn cob

= curved surface area of the corn cob × number of grains of corn on 1 cm^2

$= 188.57 \times 5 = 942.85 = 943$ (Approx.)

Hence, the entire corn cob contain 943 grains.

3. Given, radius of cone

$(r) = \frac{40}{2}$ cm $= 20$ cm $= 0.20$ m

Height of cone $(h) = 1$ m

Slant height,

$l = \sqrt{h^2 + r^2} = \sqrt{1^2 + (0.2)^2} = \sqrt{1.04} = 1.02$ m

Now, curved surface area of 1 cone $= \pi rl$

$\therefore$ Curved surface area of 50 cones

$= 50 \times 3.14 \times 0.2 \times 1.02 \text{ m}^2 = 32.028 \text{ m}^2$

$\because$ Cost of painting an area of 1 m^2 = ₹ 12

$\therefore$ Cost of painting an area of 32.028 m^2

= ₹ 12 × 32.028

= ₹ 384.34 (Approx.)

Hence, the cost of painting all these cones is ₹ 384.34.

4. Let the diameter of the earth be '$2r$'.

Then, radius of the earth = r

But diameter of the moon $= \frac{2r}{4} = \frac{r}{2}$ [Given]

$\therefore \quad$ Radius of the moon $= \frac{r}{4}$

Now, volume of the earth $= \frac{4}{3}\pi r^3$ ...(1)

and volume of the moon $= \frac{4}{3}\pi\left(\frac{r}{4}\right)^3$ ...(2)

$\therefore \quad \frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\left(\frac{r}{4}\right)^3}$

[From eqs. (1) and (2)]

$= \frac{r^3}{\frac{r^3}{64}} = \frac{64}{1} = 64$

$\therefore$ Volume of the moon

$= \frac{1}{64} \times$ Volume of the earth

Hence, volume of the moon is $\frac{1}{64}$ of the volume of the earth.

5. (i) Volume of a sphere with radius $(r) = \frac{4}{3}\pi r^3$

$\therefore$ Volume of 27 such spheres

$= 27 \times \frac{4}{3}\pi r^3 = 36\pi r^3$

Volume of the new sphere with radius r'

$= \frac{4}{3}\pi r'^3$

$\because$ Volume of 27 solid iron spheres is equal to the volume of the new sphere.

$\therefore \quad 36\pi r^3 = \frac{4}{3}\pi r'^3$

$\Rightarrow \quad 27r^3 = r'^3$

$\Rightarrow \quad r' = \sqrt[3]{27r^3}$

$\Rightarrow \quad r' = 3r$

(ii) Surface area (S) of the sphere with radius $(r) = 4\pi r^2$

and surface area (S') of the sphere with radius (r')

$= 4\pi r'^2 = 4\pi (3r)^2 \qquad [\because r' = 3r]$

$= 36\pi r^2$

$\therefore \quad \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$

6. Let originally the diameter of the sphere be '$2r$'.
Then, radius of the sphere = r
$\therefore$ Surface area of the sphere = $4\pi r^2$

Now, 25% of diameter $= \frac{25}{100} \times 2r$

New diameter of the sphere

$$= 2r - 2r \times \frac{25}{100} = \frac{3r}{2}$$

$\therefore$ New radius of the sphere $= \frac{3r}{4}$

$\therefore$ Surface area of the new sphere

$$= 4\pi\left(\frac{3r}{4}\right)^2 = \frac{9\pi r^2}{4}$$

Decrease in surface area

$$= 4\pi r^2 - \frac{9\pi r^2}{4} = \frac{7\pi r^2}{4}$$

$\therefore$ Per cent decrease in curved surface area

$$= \frac{\frac{7\pi r^2}{4} \times 100}{4\pi r^2} = \frac{7}{16} \times 100 = \frac{175}{4} = 43.75$$

Hence, the curved surface area decreases by 43.75%.

7. Let radius of the dome be 'r' m.
Circumference of the base of the dome = 55.44 m [Given]

$\Rightarrow \quad 2\pi r = 55.44$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 55.44$

$\Rightarrow \quad r = \frac{55.44 \times 7}{44}$

$\Rightarrow \quad r = 8.82$ m

Curved surface area of the hemispherical dome

$$= 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times (8.82)^2 = 488.98 \text{ m}^2$$

Since, the cost of painting 100 cm^2 = ₹ 12

So, the cost of painting 1 cm^2 = ₹ $\frac{12}{100}$

$\therefore$ The cost of painting 1 m^2 = ₹ $\frac{12 \times 100 \times 100}{100}$

= ₹ 1200

Therefore, the cost of painting 488.98 m^2

= ₹ 488.98 × 1200

= ₹ 586776

Hence, the cost of painting the dome is ₹ 5,86,776.

8. Given, inner radius of the hemispherical tank (r) = 1 m
Thickness of the iron sheet = 1 cm = 0.01 m
$\therefore$ External radius of the tank (R)

= (1 + 0.01) m = 1.01 m

So, volume of the iron used to make the hemispherical tank $= \frac{2}{3}\pi(R^3 - r^3)$

$$= \frac{2}{3} \times \frac{22}{7} \times [(1.01)^3 - 1^3]$$

$$= \frac{2}{3} \times \frac{22}{7} \times (1.030 - 1)$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.030 \text{ m}^3 = 0.063 \text{ m}^3$$

Hence, the required volume of iron used is 0.063 m^3.

9. (i) Given, cost of white-washing per m^2 = ₹ 2
Total cost = ₹ 498.96
Inner surface area of the dome

$$= \frac{\text{Total cost}}{\text{Cost of white-washing per m}^2}$$

$$= \frac{498.96}{2} \text{m}^2 = 249.48 \text{ m}^2$$

(ii) Let radius of the dome be r m.
Then, surface area of hemispherical dome,

$2\pi r^2 = 249.48$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 249.48$

$\Rightarrow \quad r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69$

$\Rightarrow \quad r = 6.3$ m

$\therefore$ Volume of the air inside the dome

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3 \text{ m}^3$$

$$= 523.9 \text{ m}^3$$

10. Let $r_1 = 6$ m, $r_2 = 8$ m, $r_3 = 10$ m.
Let the radius of the resulting sphere be 'R'.
Volume of resulting sphere = Sum of volume of small spheres.

$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$

$$= \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$$

$$= \frac{4}{3}\pi(6^3 + 8^3 + 10^3)$$

$$= \frac{4}{3}\pi(216 + 512 + 1000)$$

$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(1728)$

$\Rightarrow \quad R^3 = 1728$

$\therefore \quad R = \sqrt[3]{1728} = 12$ m

Hence, the radius of the resulting sphere is 12 m.

Long Answer Type Questions

1. (i) Given, radius of the base of a conical tent = 5 m and area needs to sit a student on the ground $= \frac{5}{7}\text{m}^2$

$\therefore$ Area of the base of a conical tent = πr^2

$$= \left(\frac{22}{7} \times 5 \times 5\right)\text{m}^2$$

Now, number of students

$$= \frac{\text{Area of the base of a conical tent}}{\text{Area needs to sit a student on the ground}}$$

$$= \frac{\frac{22\times5\times5}{7}}{5/7}$$

$$= \frac{22}{7}\times5\times5\times\frac{7}{5} = 110$$

Hence, 110 students can sit in the conical tent.

(ii) Given, area of the cloth to form a conical tent $= 165\ m^2$

Radius of the base of a conical tent, $r = 5$ m

Curved surface area of a conical tent = Area of cloth to form a conical tent

$\Rightarrow \quad \pi rl = 165$

$\Rightarrow \quad \frac{22}{7}\times5\times l = 165$

$$l = \frac{165\times7}{22\times5} = \frac{33\times7}{22} = 10.5\text{m}$$

Now, height of a conical tent $= \sqrt{l^2 - r^2}$

$$= \sqrt{(10.5)^2 - (5)^2}$$

$$= \sqrt{110.25 - 25}$$

$$= \sqrt{85.25} = 9.23\text{ m}$$

Hence, volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times5\times5\times9.23$$

$$= 241.73\ m^3$$

2. Given, diameter of the base $(d) = 28$ cm

$\therefore$ Radius of the base $(r) = 14$ cm

$$\left[\because \text{radius} = \frac{\text{diameter}}{2}\right]$$

(i) Let h be the height of the cone.

Also given, volume of a right circular cone $= 9856\ cm^3$

TR!CK

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$\therefore \quad \frac{1}{3}\pi r^2 h = 9856$

$\Rightarrow \quad \frac{1}{3}\times\frac{22}{7}\times(14)^2\times h = 9856$

$\Rightarrow \quad h = \frac{9856\times3\times7}{22\times14\times14} = \frac{448\times3}{28} = 48\text{cm}$

(ii) We have, $h = 48$ cm and $r = 14$ cm

$\therefore \quad l = \sqrt{h^2 + r^2} = \sqrt{(48)^2 + (14)^2}$

$= \sqrt{2304 + 196} = \sqrt{2500}$

$\Rightarrow \quad l = 50$ cm

Hence, slant height of the cone is 50 cm.

(iii) Curved surface area of the cone $= \pi rl$

$$= \frac{22}{7}\times14\times50 = 44\times50 = 2200\ cm^2$$

3. Let r, h and l be the radius, height and slant height of the tent, respectively.

Given, $r = 6$ m and $h = 8$ m

We know that, $l^2 = h^2 + r^2$

$\Rightarrow \quad l = \sqrt{r^2 + h^2}$

[on taking positive square root]

$\Rightarrow \quad l = \sqrt{(6)^2 + (8)^2} \Rightarrow l = \sqrt{36 + 64}$

$\Rightarrow \quad l = \sqrt{100} = 10\text{m}$

Area of the canvas used for the tent

= Curved surface area of the cone

$= \pi rl = 3.14 \times 6 \times 10 = 188.4\ m^2$

$\therefore$ Length of tarpaulin required

$$= \frac{\text{Area of tarpaulin required}}{\text{Width of tarpaulin}} = \frac{188.4}{3}$$

[$\because$ width of tarpaulin $= 3$ m, given]

$= 62.8$ m

The extra material required for stitching margins and cutting

$$= 20\text{ cm} = 0.2\text{ m} \qquad \left[\because 1\text{cm} = \frac{1}{100}\text{m}\right]$$

Hence, the total length of tarpaulin required

$= 62.8 + 0.2 = 63$ m

4.

TiP

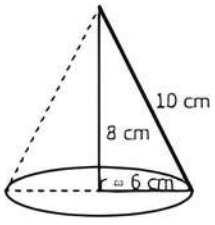
When a right-angled triangle is revolved about its side, then solid formed is a cone.

Here, triangle is revolved about side 8 cm.

$\therefore$ Radius of cone so formed $(r) = 6$ cm

Height of the cone so formed $(h) = 8$ cm

Slant height of cone $(l) = 10$ cm



TR!CK

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore \quad$ Volume of cone $= \frac{1}{3}\times\frac{22}{7}\times6^2\times8$

$$= \frac{1}{3} \times \frac{22}{7} \times 36 \times 8 \text{ cm}^3$$

$$= 301.71 \text{ cm}^3$$

and curved surface area of the cone

$$= \pi rl = \frac{22}{7} \times 6 \times 10$$

$$= 188.57 \text{ cm}^2$$

Hence, the volume and CSA of solid so formed are 301.71 cm^3 and 188.57 cm^2 respectively.

5. Given, diameter of hemispherical dome = 80 cm

$\therefore$ Radius of hemispherical dome (r) = 40 cm

Now, CSA of the dome = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 40 \times 40 = \frac{70400}{7} \text{cm}^2$$

Since, $\frac{13}{170}$ part of sheet was wasted.

So, area of sheet wasted $= \frac{13}{170} \times \frac{70400}{7}$

$$= \frac{915200}{1190} \text{cm}^2$$

$\therefore$ Total area of sheet $= \frac{70400}{7} + \frac{915200}{1190}$

$$= 10057.14 + 769.08$$

$$= 10826.22 \text{ cm}^2$$

Cost of sheet per square cm = ₹ $\frac{35}{100}$

$\therefore$ Total cost of sheet $= \frac{35}{100} \times 10826.22$

$$= ₹\ 3789.18$$

Hence, the total cost of dome at the rate of $\frac{35}{100}$ per cm^2 is ₹ 3789.18.

6. Let the radii of two spheres be r_1 cm and r_2 cm respectively.

Let the volumes of two spheres be V_1 and V_2 respectively.

Then, $\frac{V_1}{V_2} = \frac{64}{27}$

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow r_1 = \frac{4}{3} r_2 \quad \ldots(1)$$

Given, $r_1 + r_2 = 7$

$$\Rightarrow \frac{4}{3} r_2 + r_2 = 7 \Rightarrow \frac{7}{3} r_2 = 7$$

$$\Rightarrow r_2 = 7 \times \frac{3}{7} = 3 \text{cm}$$

$$\Rightarrow r_1 = 7 - 3 = 4 \text{ cm}$$

Let S_1 and S_2 be the surface areas of two spheres. Then,

$S_1 = 4\pi r_1^2 = 4\pi \times 4 \times 4 = 64\pi \text{ cm}^2$

and, $S_2 = 4\pi r_2^2 = 4\pi \times 3 \times 3 = 36\pi \text{ cm}^2$

$\therefore S_1 - S_2 = 64\pi - 36\pi = 28\pi \text{ cm}^2$

$$= 28 \times \frac{22}{7} \text{cm}^2 = 88 \text{ cm}^2$$

Chapter Test

Multiple Choice Questions

Q 1. A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is:

a. 1 : 2 b. 2 : 1 c. 4 : 1 d. $\sqrt{2}$: 1

Q 2. If the volumes of two cones are in the ratio 1 : 4 and radii of their bases are in the ratio 4 : 5, then the ratio of their heights is:

a. 5 : 4 b. 25 : 64 c. 3 : 2 d. 4 : 7

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Reason (R): Curved surface area is increased when diameter decreases.

Q 4. Assertion (A): The total surface area of a cone, whose radius is $\frac{r}{2}$ and slant height $2l$, is $\pi r\left(l + \frac{r}{4}\right)$.

Reason (R): Total surface area of cone is $\pi r(l + r)$ where r is radius and l is the slant height of cone.

Fill in the Blanks

Q 5. If the radius of a sphere is doubled, then the percentage increase in the surface area is ..... .

Q 6. The volume of a right circular cone of height 12 cm and base radius 6 cm is ..... .

True/False

Q 7. If the ratio of the volume of two spheres is 8 : 125, then the ratio of their surface area is 4 : 25.

Q 8. If surface area of a sphere is 346.5 cm^2, then radius of a sphere is 5.25 cm.

Case Study Based Question

Q 9. On Sumit's 15th birthday, Sumit's parent decided to invite all his friends on his birthday party and hired a magician for performing a magic show in the party. The magician was wearing a conical shape cap in which 30.8 sq. cm of cloth was used. Also the circumference of its base was 17.6 cm.

On the basis of the above information, solve the following questions:

(i) Find the slant height of the cone.

(ii) Find the height of the cone.

OR

How much cloth will be required for 15 caps (assuming that no cloth will be wasted)?

(iii) What is the volume (in cm^3) of the cone (leave the answer in terms of π)?

Very Short Answer Type Questions

Q 10. The height of a cone is 24 cm and the diameter of its base is 14 cm. Find the slant height of cone.

Q 11. Find the surface area of a sphere of radius 14 cm.

Short Answer Type-I Questions

Q 12. A solid sphere of radius 3 cm is melted and then cast into smaller spherical balls, each of diameter 0.6 cm. Find the number of small balls thus obtained.

Q 13. The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height of the cone.

Short Answer Type-II Questions

Q 14. A hemispherical dome of building needs to be painted. If the circumference of the base of the dome is 17.6 m, then find the cost of painting it, given that cost of painting is ₹5 per 100 cm^2.

Q 15. The radius and height of a cone are in the ratio 3 : 4. If its volume is 301.44 cm^3. What is its radius? What is its slant height? (use $\pi = 3.14$ and $\sqrt[3]{288} = 6.60$)

Long Answer Type Question

Q 16. A semicircular sheet of diameter 28 cm is bent to form an open conical cup. Find the capacity of the cup. $\left(\text{use } \sqrt{3} = 1.732\right)$