Chapter-7

Cube and Cube Roots

G6B5R2

7.1 Cube numbers

Look at the numbers 1, 8, 27, 64, 125 .....

$$1 = 1 \times 1 \times 1 = 1^3$$
$$8 = 2 \times 2 \times 2 = 2^3$$
$$27 = 3 \times 3 \times 3 = 3^3$$
$$64 = 4 \times 4 \times 4 = 4^3$$
$$125 = 5 \times 5 \times 5 = 5^3$$

The number obtained when a number multiplied by itself 3 times is known as cube of the number. Each of 1, 8, 27, 64, 125 are examples of some cube numbers or perfect cubes.

Here, 1 is the cube of 1
8 is the cube of 2
27 is the cube of 3
64 is the cube of 4
125 is the cube of 5

Do you know how many perfect cubes are there from 1 to 1000? Fill up the following table completely and you will get a clear idea.

Number	Cube	Cube Number
1	1^3	1
2	2^3	8
3	3^3	27
4	4^3	64
5	5^3	125
6	6^3	—
7	7^3	—
8	8^3	—
9	9^3	—
10	10^3	—

It would be easier for calculation, if you remember this table.

Fill up the table, you will see that there are only 10 cube numbers form 1 to 1000.

Let us identify the cube numbers –

Is 50 a cube number?

$50 = 2 \times 5 \times 5$. Here, the same number has not occurred as factor for three times. Therefore, it is not a cube number. To check, whether any number is cube number or not, find out its prime factors. In the prime factorisation of any number, if each factor appears three times then the number is a cube number or perfect cube. For example, prime factors of 1728 will be –

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$$\begin{aligned} 1728 &= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \\ &= 2^3 \times 2^3 \times 3^3 \\ &= (2 \times 2 \times 3)^3 \\ &= 12^3 \end{aligned}$$

i.e. $1728 = 12 \times 12 \times 12$

Hence, 1728 can be found by multiplying 12 by itself three times. i.e., 1728 is a cube number. 1728 is the cube of 12.

7.2 Square and cube numbers

Example 1 : Examine, whether 100 is a cube number or not .

$100 = 10 \times 10$

$$\begin{aligned} 100 &= 10 \times 10 \\ &= 2 \times 5 \times 2 \times 5 \\ &= 2 \times 2 \times 5 \times 5 \end{aligned}$$

Here, all factors of 100 are prime. But the factors are not the product of three same numbers.

Here, 2 and 5 are both present twice.

So, 100 is not a cube number.

But $100 = 10 \times 10$. Therefore, the number is a square number.

On the other hand, some square numbers are also cube numbers. For e.g., 64 is a cube number. Since $64 = 4 \times 4 \times 4 = 4^3$.

Also, $64 = 8 \times 8 = 8^2$

Therefore, 64 is a cube number as well as a square number.

Number	Square	Square number	cube	Cube number
1	1^2	1	1^3	1
2	2^2	4	2^3	8
3	3^2	9	3^3	27
4	4^2	16	4^3	64
5	5^2	25	5^3	125
6	6^2	36	6^3	216
7	7^2	49	7^3	343
8	8^2	64	8^3	512
9	9^2	81	9^3	729
10	10^2	100	10^3	1000
11	11^2	121	11^3	1331
12	12^2	144	12^3	1728
13	13^2	169	13^3	2197
14	14^2	196	14^3	2744
15	15^2	225	15^3	3375
16	16^2	256	16^3	4096
17	17^2	289	17^3	4913
18	18^2	324	18^3	5832
19	19^2	361	19^3	6859
20	20^2	400	20^3	8000

Observe the table given on left side :

1. **Some cube numbers are also square numbers.**

   E.g, 1, 64, 4096 etc.

   $64 = 8^2 = 4^3$

   $4096 = 64^2 = 16^3$

2. **Cubes of even numbers are even.**

   e.g. Cube of 2 is 8   Cube of 4 is 64
   Cube of 6 is 216   Cube of 18 is 5832
   etc.

3. **Cubes of odd numbers are odd.**

   e.g. Cube of 3 is 27   Cube of 5 is 125
   Cube of 7 is 343   Cube of 9 is 729
   Cube of 11 is 1331   etc.

4. **Now observe the following numbers –**

   $2^3 = 8$

   $12^3 = 1728$

   $22^3 = 10648$

When a number with 2 in its unit's place is cubed, the unit's place of the result is $2^3 = 8$. Thus, *those numbers who have 2 in the unit's place, their cubes have 8 in the unit place.*

Now, let us observe the unit place digits of the cubes of the numbers which have 8 in their unit place. $8^3 = 512$

$18^3 = 5832$

$28^3 = 21952$

What have you seen? 2 is in the unit place of the cubes of the numbers. Therefore, *those numbers which have 8 in their unit place, cubes of those numbers have 2 in their unit place*. Therefore, to remember it we use 2 $\rightleftarrows$ 8.

Similarly, we can verify

1 $\rightleftarrows$ 1,   3 $\rightleftarrows$ 7,   4 $\rightleftarrows$ 4,   5 $\rightleftarrows$ 5,   6 $\rightleftarrows$ 6,   9 $\rightleftarrows$ 9

If we remember the above relations, it would be easier to find cubes or cube numbers or cube roots of numbers .

7.3 Some amazing patterns

(i)Relation between cube numbers and square numbers.

$$1^3 = 1 = 1^2$$
$$1^3 + 2^3 = (1 + 2)^2 = 3^2$$
$$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2 = 6^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2 = 10^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 15^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = (1 + 2 + 3 + 4 + 5 + 6)^2 = 21^2$$

(Try yourself for further results)

(ii) Sum of consecutive odd numbers.

$$
\begin{aligned}
1 &= 1 = 1^3 \\
3 + 5 &= 8 = 2^3 \\
7 + 9 + 11 &= 27 = 3^3 \\
13 + 15 + 17 + 19 &= 64 = 4^3 \\
21 + 23 + 25 + 27 + 29 &= 125 = 5^3 \\
31 + 33 + 35 + 37 + 39 + 41 &= 216 = 6^3
\end{aligned}
$$

....

(For remaining parts try yourself)

Activity : Fill up the following table by yourself.

+	1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3	10^3	11^3	12^3
1^3												1729
2^3												
3^3												
4^3												
5^3												
6^3												
7^3												
8^3												
9^3										1729		
10^3									1729			
11^3												
12^3	1729											

From the table,

$$1729 = 12^3 + 1^3$$

$$1729 = 10^3 + 9^3$$

Expanding the table upto 16^3 or 24^3 it can be seen that.

$$16^3 + 2^3 = 15^3 + 9^3 = 4104$$

$$24^3 + 2^3 = 20^3 + 18^3 \quad \text{etc.}$$

Ramanujan is one of the India's renowned mathematicians. His full name is Srinivas Iyenger Ramanujan. This great mathematician of 19th century is known as 'Friend of Numbers' in all over the world. He devoted his entire life on observations and research on numbers.

Once Ramanujan had been admitted into a nursing home in London due to his illness. At that time his friend British mathematician, Professor Hardy, visited him to enquire about his health condition. Both of them were mathematicians. Obviously, their conversations were confined within mathematics. Hardy asked, "Friend, the number of the Taxi I travelled is 1729 . It is a dull number" Ramanujan's eyes were immediately lit up despite of being ill and bed ridden. He said, "My dear friend, this number is not dull, indeed it is a special number. It is the smallest number that can be expressed as the sum of two cubes in two different ways.".

i.c. $1729 = 12^3 + 1^3 \qquad = 10^3 + 9^3$

He discussed mathematics even on his death bed. **1729 is known as Ramanujan number**.

The 22nd December of every year, the birthday of Ramanujan is celebreted as '**National Mathematics day**' all over India. This day has been earmarked for his immense contribution in the field of mathematics, specially in the field of **Number Theory**.

Example 2 : Is 243 a perfect cube or a cube number? If not, find the smallest natural number by which 243 must be multiplied so that the product is a perfect cube.

Solution : Already you have learnt that a perfect cube is obtained when a number is multiplied by itself three times or factors could be grouped in triples . For example,

$$8 = 2 \times 2 \times 2$$

$$27 = 3 \times 3 \times 3$$

3	243
3	81
3	27
3	9
	3

Therefore, to check whether a number is a perfect cube, the prime factors must be grouped in triples.

$243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

In the above prime factorisation, 3 × 3 remains after grouping the 3's in triples. Therefore, 243 is not a perfect cube or cube number.

Notice that in the prime factorisation of any number, if each factor appears three times, then the number is a perfect cube. Therefore, the smallest number by which 243 should be multiplied to make a perfect cube is 3.

Example 3 : Is 2187 a perfect cube (or cube number) ? If not, by which smallest natural number 2187 should be divided so that the quotient would be a perfect cube?

Solution : The prime factors of 2187 are –

$2187 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times 3$

3	2187
3	729
3	243
3	81
3	27
3	9
	3

Here, in the above factorisation one 3 is left after grouping of 3's in triples two times.

Therefore, 2187 is not a perfect cube or a cube number.

To make it perfect cube or cube number, it should be divided by 3.

Therefore, 2187 must be divided by 3 to get a perfect cube.

Example 4 : Determine the smallest number by which 35000 be divided to get it a perfect cube.

Solution : $35000 = 35 \times 1000$

$= 5 \times 7 \times 10 \times 10 \times 10$

$= 5 \times 7 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5$

$= 5 \times 7 \times 2^3 \times 5^3$

Here, 5 and 7 each occurs once, therefore, to get a perfect cube 35000 must be divided by 5×7 = 35.

Exercise 7.1

1. Which of the following numbers are perfect cubes?

   (a) 500 (b) 1331 (c) 2025

   (d) 6859 (e) 2376 (f) 8000

2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

   (a) 675 (b) 256 (c) 100 (d) 72

3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

   (a) 2401 (b) 8192 (c) 6561 (d) 100,000

4. Find the smallest number by which each of the following numbers must be multiplied or divided to obtain a cube number.

   (a) 250 (b) 675 (c) 1372 (d) 3000 (e) 153664

7.4 Cube Roots

Consider the cube number 64. You know that $64 = 4 \times 4 \times 4 = 4^3$. i.e. 64 is the cube of 4. Conversely, we can say that 4 is the cube root of 64.

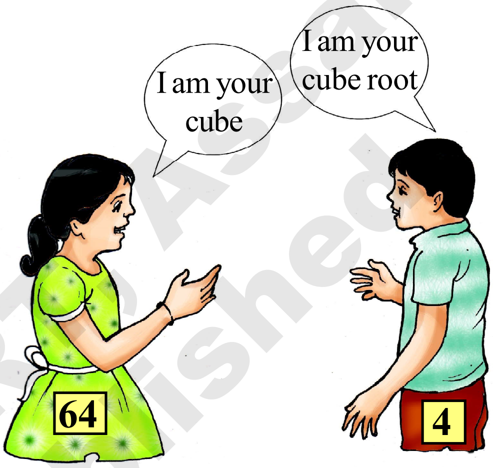
By prime factorisation method, we can say that

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^3 \times 2^3$$
$$= (2 \times 2)^3$$
$$= 4^3$$

You know that,
$a^m \times b^m = (a \times b)^m$

From above example we can write symbollically as

$$\sqrt[3]{64} = 4$$



The symbol $\sqrt[3]{\;}$ denotes cube root.

Consider the following table –

$1^3 = 1$ so, $\sqrt[3]{1} = 1$
$2^3 = 8$ so, $\sqrt[3]{8} = 2$
$3^3 = 27$ so, $\sqrt[3]{27} = 3$
$4^3 = 64$ so, $\sqrt[3]{64} = 4$

Complete yourself by writting cube root upto 1000.

7.4.1 Finding cube roots using prime factorisation method

Consider the number 13824. To find cube root of 13824, first find the prime factors.

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$$13824$$
$$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$
$$= 2^3 \times 2^3 \times 2^3 \times 3^3$$
$$= (2 \times 2 \times 2 \times 3)^3$$
$$= 24^3$$
$$\therefore \sqrt[3]{13842} = 24$$

Example 5 : Find the cube root of 15625

Solution :

$$15625$$
$$= \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$$
$$= 5^3 \times 5^3$$
$$= (5 \times 5)^3$$
$$= 25^3$$
$$\therefore \sqrt[3]{15625} = 25$$

5	15625
5	3125
5	625
5	125
5	25
	5

Try yourself

Find the cube roots by prime factorisation method.

(a) 512 (b) 27000 (c) 110592 (d) 46656 (e) 175616

7.4.2 Determination of cube roots of cube numbers by the method of assumption

Sometimes the cube root of a cube number can be find through estimation.

Step 1 : Consider a number whose cube root to be found, start making groups of three from the unit place of the number. Mark a bar above the group. If there are one or two digits left in the last (Left side), mark a bar above it also.

Step 2 : Now observe the last digit of the groups on the right side, Already, you know the rule of finding cubes (e.g $2 \rightleftarrows 8$, $1 \rightleftarrows 1$ etc.) Same rules can be applied for finding cube roots also, Write the cube roots using these rules.

Step 3 : Proceed applying the same rules for other groups too.

Step 4 : Observe the remaining one or two digit number in the left. Now you find a cube number which is very near to the one digit or two digit number. The cube root of the cube number is the cube root of the said group.

In this way, by dividing a number into groups and finding cube root of each group, we can find the cube root of the number. Remember, one thing that we can approximate the cube root of a number by using this method. Actually we shall use prime factorisation method to find cube roots.

Let us try to understand with the help of an example.

To find the cube root of 2744.

First, make groups of three digits starting from right as $\overline{2}\ \overline{744}$

There are three digits in the first group and one digit in the left. 4 is in the last place of the first group. Therefore, using the rule $4 \rightleftarrows 4$ we know that 4 is in the last place of the cube root.

Now, 2 is in the 2nd group. 1 is the cube number less than 2. i.e. $1^3 \neq 2 \neq 2^3$ or, $1 \neq 2 \neq 8$.

$\therefore$ 1 is the cube number less than 2. 1 will be in tenth place of the cube root of $\overline{2}\ \overline{744}$, i.e. the required cube root will be 14.

Example 6 : Find the cube root of 12167.

Solution : $\overline{12}\,\overline{167}$

By the rule $7 \rightleftarrows 3$, 3 will be in the unit's place of the cube root.

8 is the cube number which is less than 12 and 2 is the cube root of 8. Therefore, the approximate cube root of 12167 is 23. Of course $23^3 = 12167$. Hence the cube root of 12167 is 23.

Try yourself

Find the cube roots of the following through the method of assumption –

(a) 4096 (b) 9261 (c) 13824 (d) 15625

Exercise 7.2

1. Choose the correct option of the following questions –

   (i) Unit place of the cube of 23

   (a) 3 (b) 6 (c) 7 (d) 9

   (ii) Which of the following numbers is a perfect cube –

   (a) 243 (b) 216 (c) 392 (d) 8640

   (iii) Which of the following numbers is not a perfect cube –

   (a) 216 (b) 567 (c) 125 (d) 343

   (iv) Value of $\sqrt[3]{1000}$

   (a) 1 (b) 10 (c) 100 (d) 1000

   (v) Value of $\sqrt[3]{27} + \sqrt[3]{64} + \sqrt[3]{125}$

   (a) 10 (b) 11 (c) 12 (d) 13

2. Find the cube roots of the following by the prime factorisation method –

   (i) 125 (ii) 343 (iii) 2744 (iv) 10648 (v) 4096

   (vi) 35937 (vii) 216000 (viii) 9261 (ix) 21952 (x) 6859

What we have learnt

1. The number obtained by multiplying itself three times is known as cube numbers.
2. The cube of even numbers is even and cube of odd numbers is odd.
3. If 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 are in the unit place of any number, then 1, 8, 7, 4, 5, 6, 3, 2, 9 and 0 respectively are in the unit place of the cube of those number.
4. We can find out the cube root properly through prime factorisation method.

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