

LAWS OF MOTION

Newtonian Mechanics

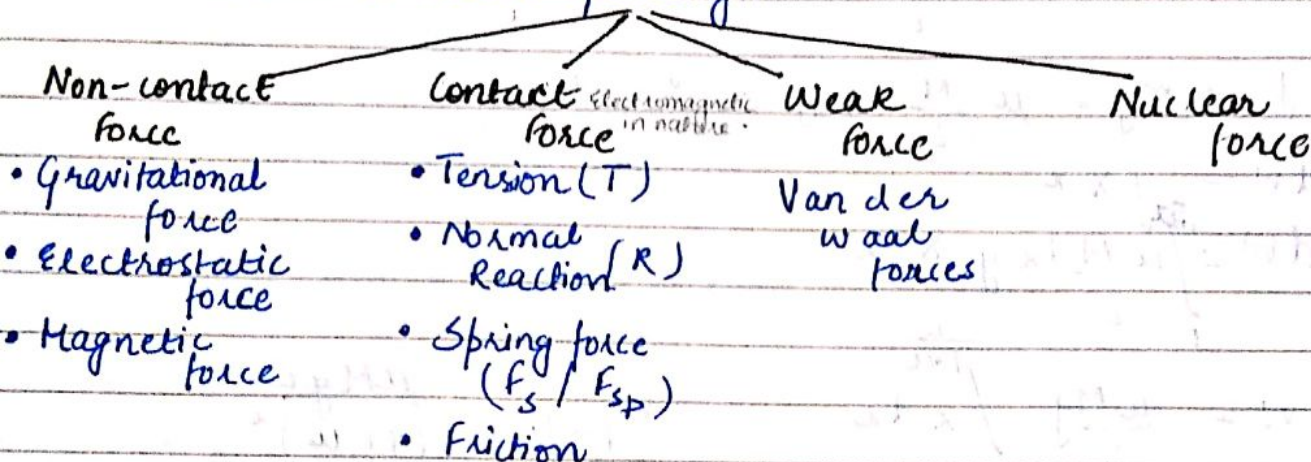
1642-1727

Force is that cause which changes the state of the body.

SI unit $\Rightarrow$ Newton

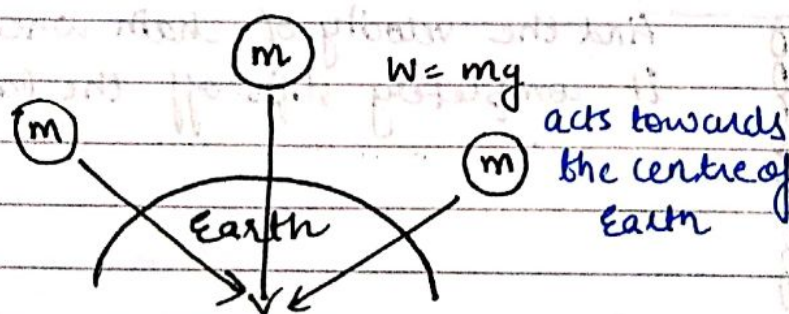
Cgs unit $\Rightarrow$ Dyne

Force is a vector quantity.



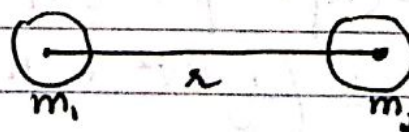
GRAVITATIONAL FORCE

Attraction between objects due to virtue of their masses.
(Weight)



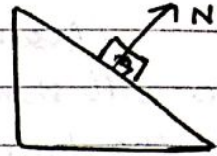
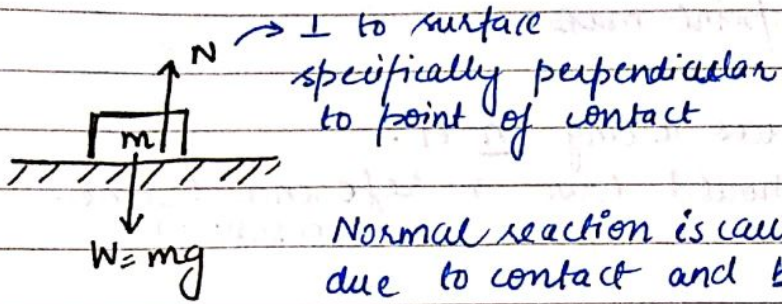
$$1 \text{ kgf} = 10 \text{ N} / 9.8 \text{ N}$$

$$F = \frac{G m_1 m_2}{r^2}$$



Spiral

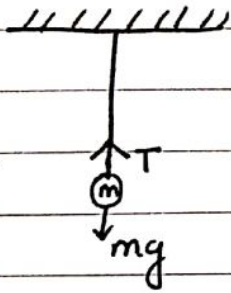
NORMAL REACTION (N)



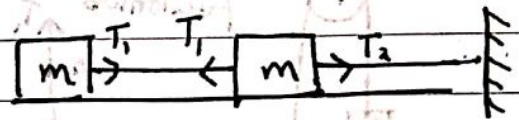
Normal reaction is caused due to contact and balances the weight of the body.

TENSION (T)

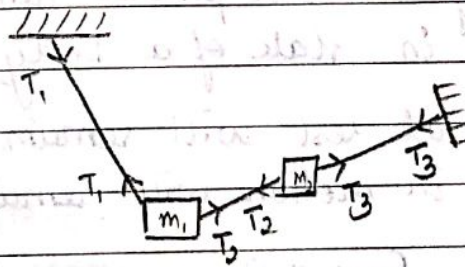
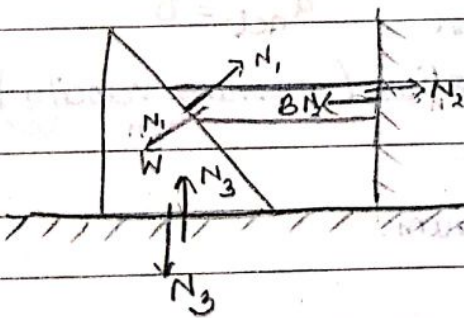
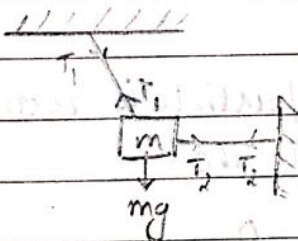
String / Rod / Chain



Direction of tension is away from point.
String change leads to tension change.

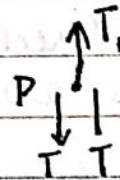
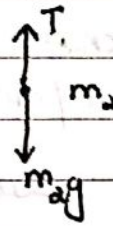
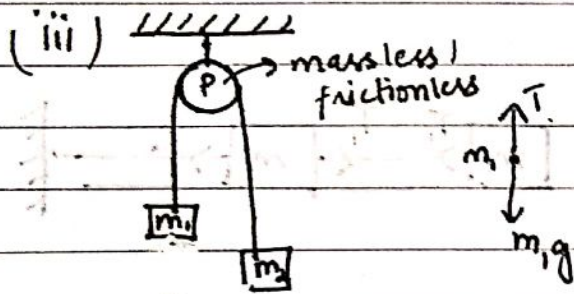
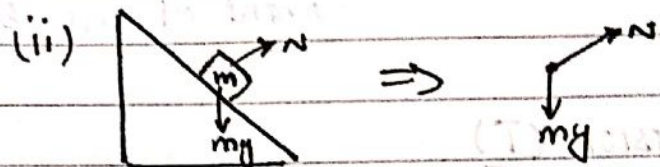
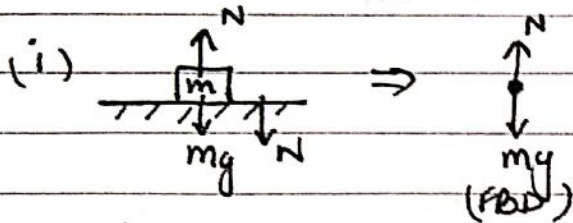


Tension remains same until string remains same as long as string is massless.



FREE BODY DIAGRAM (F.B.D.)

- Body is shown as point mass.
(Assumption)
- We will show all forces acting on it.
- For the force, we should have a reference frame.
(Observer)
Ground $\Rightarrow$ Inertial frame



NEWTON'S FIRST LAW OF MOTION

If the vector sum of all the forces acting on a particle is zero then and only then the particle remains unaccelerated.

(No change in state of a body)

$$F_{net} = 0$$

$$a_{net} = 0$$

A body at rest will remain at rest

A body in motion will remain in motion (constant velocity)

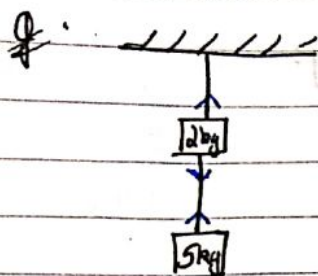
$$F_{net} = 0$$

called Equilibrium.

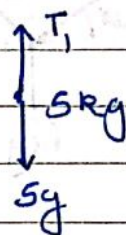
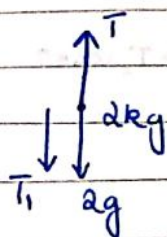
$$F_x = 0$$

$$F_y = 0$$

$$F_z = 0$$



Find Tension in both strings.
The whole system is in equilibrium.



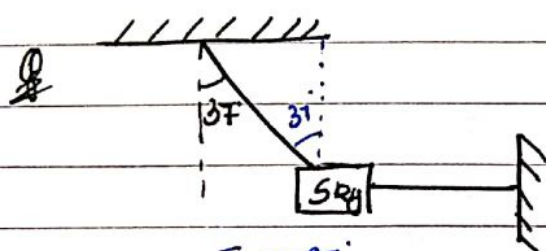
As system is in equilibrium,

$$T_1 = 5g$$

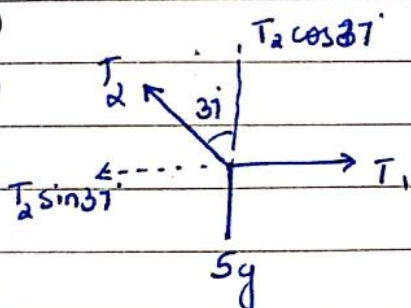
$$\Rightarrow T_1 = 50 \text{ N}$$

$$\Rightarrow T = 2g + T_1$$

$$\Rightarrow T = 20 + 50 = 70 \text{ N}$$



Find tension in both strings.
(The whole system is in eq^m)



$$\therefore T_2 \cos 37^\circ = 5g$$

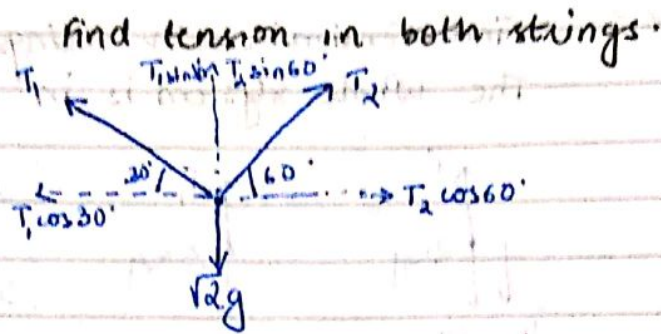
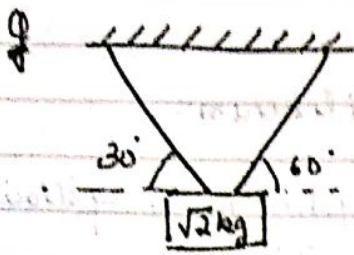
$$\Rightarrow T_2 \frac{4}{5} = 50$$

$$\Rightarrow T_2 = \frac{250}{4} = \underline{\underline{62.5 \text{ N}}}$$

$$\Rightarrow T_1 = T_2 \sin 37^\circ$$

$$\Rightarrow T_1 = \frac{250}{4} \times \frac{3}{5}$$

$$\Rightarrow T_1 = \underline{\underline{37.5 \text{ N}}}$$



find tension in both strings.

$$T_1 \cos 30^\circ = T_2 \cos 60^\circ$$

$$\Rightarrow T_1 \frac{\sqrt{3}}{2} = T_2 \frac{1}{2} \Rightarrow T_2 = \sqrt{3} T_1$$

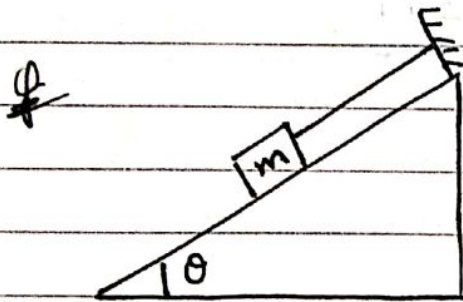
$$T_1 \sin 30^\circ + T_2 \sin 60^\circ = \sqrt{2}g$$

$$\Rightarrow T_1 \times \frac{1}{2} + T_2 \frac{\sqrt{3}}{2} = \sqrt{2} \times 10$$

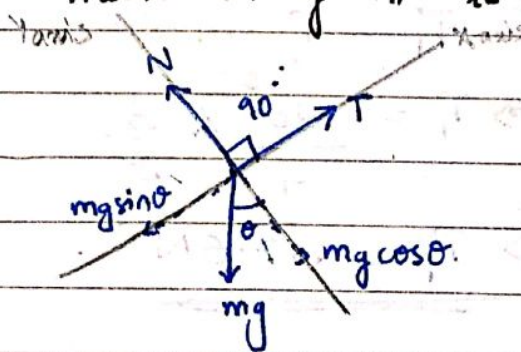
$$\Rightarrow T_1 + \sqrt{3} T_2 = 20\sqrt{2}$$

$$\Rightarrow T_1 + 3T_1 = 20\sqrt{2} \Rightarrow T_1 = 5\sqrt{2} \text{ N}$$

$$T_2 = 5\sqrt{6} \text{ N}$$



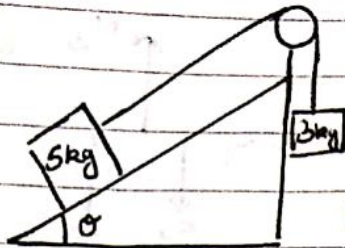
find T in string & N Reaction on mass m if ' m ' is in equilibrium.



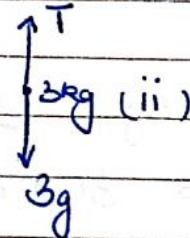
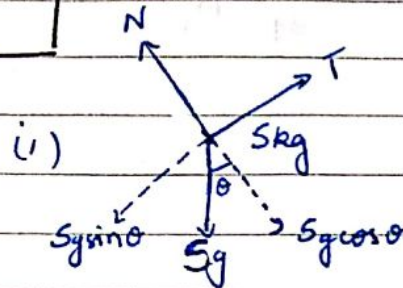
$$N = mg \cos \theta$$

$$T = mg \sin \theta$$

Q



If the system is in equilibrium, find θ , T and N .



$$T = 3g = 30 \text{ N} \quad [\text{from (ii)}]$$

Also,

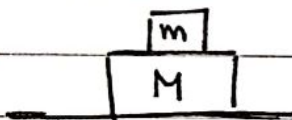
$$T = 5g \sin \theta$$

$$\Rightarrow 30 = 50 \sin \theta$$

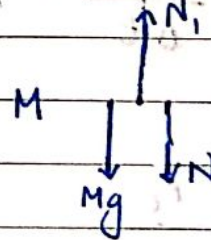
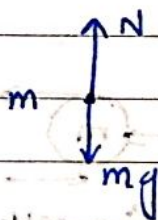
$$\therefore \sin \theta = \frac{3}{5} \quad \therefore \theta = 37^\circ$$

$$N = \frac{50 \times 4}{5} = 40 \text{ N}$$

Q



Find N between two blocks and N between surface and mass M .

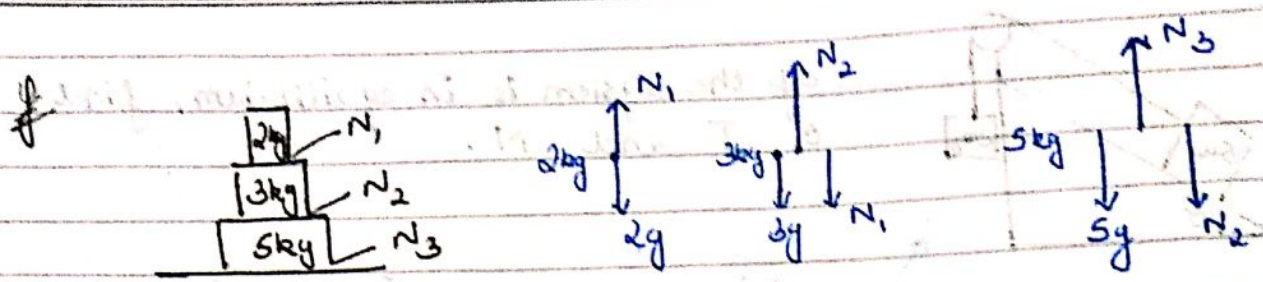


$$N = mg$$

$$N_1 = Mg + N$$

$$\Rightarrow N_1 = Mg + mg$$

$$\Rightarrow N_1 = (M + m)g$$



$$N_1 = 2g = \underline{20\text{ N}}$$

$$N_2 = 30 + 20 = \underline{50\text{ N}}$$

$$N_3 = 50 + 50 = \underline{100\text{ N}}$$

NEWTON'S SECOND LAW OF MOTION

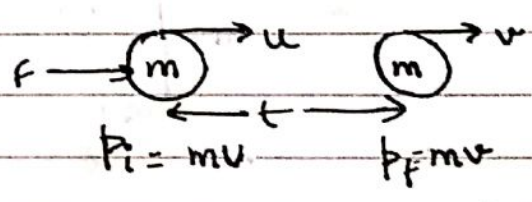
The acceleration of a particle as measured from an inertial frame is given by the vector sum of all forces acting on the particle divided by its mass.

Rate of change in momentum = Force applied

$$\frac{\Delta p}{\Delta t} = \text{force}$$

We know that,

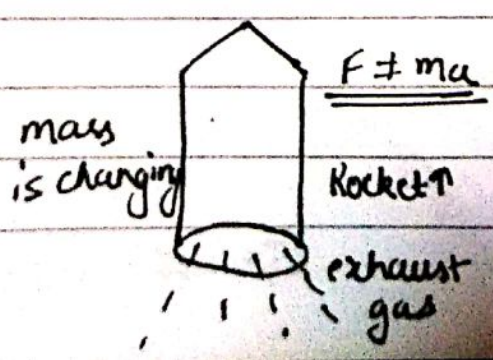
$$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{t}$$



$$F = \frac{mv_f - mu}{t}$$

$$F = m \left(\frac{v - u}{t} \right)$$

$$F = ma \quad] \text{ true as long as mass is constant}$$



$$F = \frac{\Delta p}{\Delta t} \quad \text{Always valid.}$$

Spiral

CASE
of Object speed. $v \approx c$ (speed of light)
mass changes (increases)

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Actual mass m_0 → Rest mass

eg when $v = \frac{c}{2}$, $m = \frac{2m}{\sqrt{3}}$

DIFFERENTIAL FORM OF NEWTON'S SECOND LAW

When mass is variable,

$$F = \frac{\Delta p}{\Delta t} = \frac{dp}{dt}$$

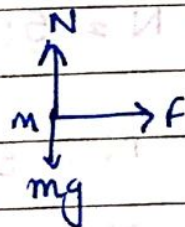
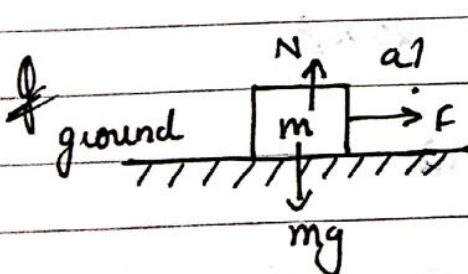
$$F = \frac{d(mv)}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt}$$

$$F = m \frac{dv}{dt} + v \frac{dm}{dt}$$

⇒ exact 2nd law of motion.

cause of force when velocity changes

cause of force when mass changes.

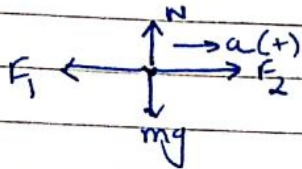
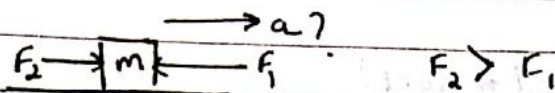


$$N = mg$$

$$F = ma$$

$$\Rightarrow a = \frac{F}{m}$$

Q

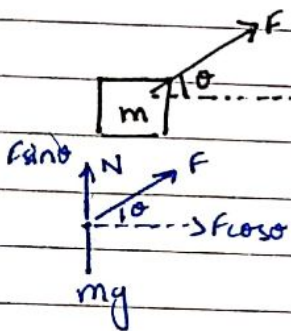


$$N = mg$$

$$F_2 - F_1 = ma$$

$$\Rightarrow a = \frac{F_2 - F_1}{m}$$

Q



Find N and a of body.

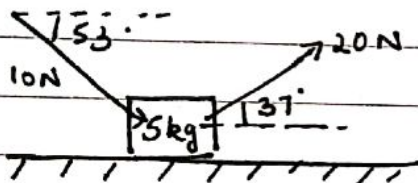
$$N + F \sin \theta = mg$$

$$\Rightarrow N = mg - F \sin \theta$$

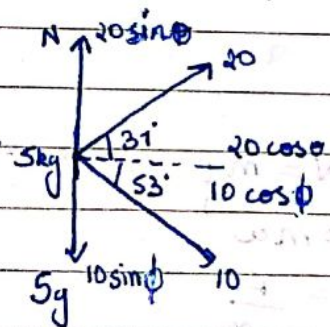
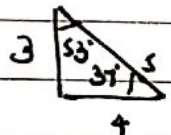
$$F \cos \theta = ma$$

$$\Rightarrow a = \frac{F \cos \theta}{m}$$

Q



Find N and a of body.



$$N + 20 \sin \theta = 50 + 10 \sin \phi$$

$$\Rightarrow N = 50 + 10 \times \frac{4}{5} - 20 \times \frac{3}{4}$$

$$\Rightarrow N = 50 + 8 - 12$$

$$\Rightarrow N = 46 \text{ N}$$

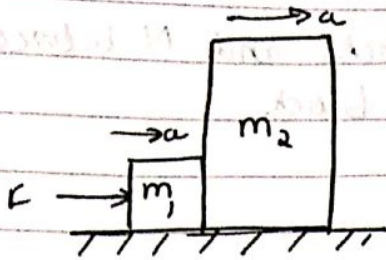
$$20 \cos \theta + 10 \cos \phi = 5a$$

$$\Rightarrow 5a = 20 \times \frac{4}{5} + 10 \times \frac{3}{4} = 16 + 6$$

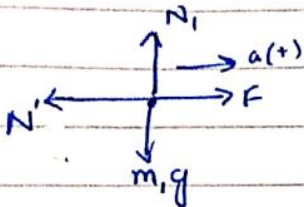
$$\Rightarrow a = \frac{22}{5} = 4.4 \text{ ms}^{-2}$$

Spiral

Q

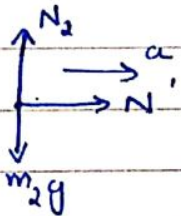


Find N between blocks & a of both blocks.



$$N_1 = m_1 g$$

$$F - N' = m_1 a \quad \text{--- (i)}$$



$$N_2 = m_2 g$$

$$N' = m_2 a \quad \text{--- (ii)}$$

Adding eqⁿ (i) & (ii).

$$F = (m_1 + m_2) a$$

$$\Rightarrow a = \frac{F}{(m_1 + m_2)}$$

$$N' = \frac{F m_2}{m_1 + m_2}$$

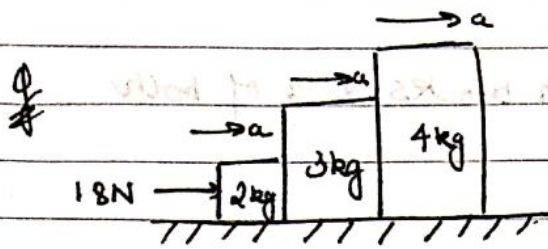
Shortcut:

Considering both bodies a system.

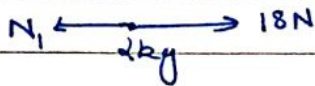
$$M_{\text{net}} = m_1 + m_2$$

$$F_{\text{net}} = m_{\text{net}} a$$

$$a = \frac{F}{m_1 + m_2}$$

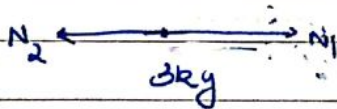


find a of all blocks and N between 4kg and 3kg block

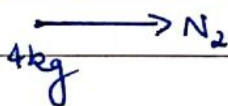


$$18 - N_1 = ma$$

$$\Rightarrow 18 - N_1 = 2a \quad \text{--- (i)}$$



$$N_1 - N_2 = 3a \quad \text{--- (ii)}$$



$$N_2 = 4a \quad \text{--- (iii)}$$

Adding all 3 eqⁿ,

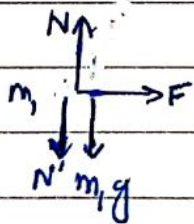
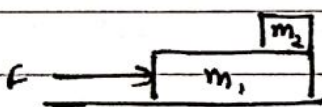
$$18 = 9a$$

$$\Rightarrow \underline{a = 2 \text{ ms}^{-2}}$$

$$N_2 = 4 \times 2 = \underline{8 \text{ N}}$$

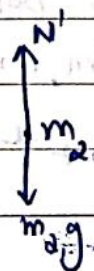
Find a of m_1 & m_2 .

(All surfaces are smooth)

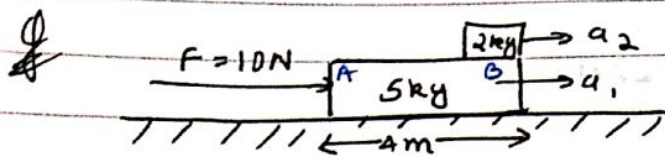


$$F = m_1 a$$

$$\Rightarrow \underline{a_1 = \frac{F}{m_1}}$$



$$\underline{a_2 = 0}$$



All surfaces are smooth.

Find the time in which 2kg falls off 5kg.

$$a_1 = \frac{10}{5} = 2 \text{ ms}^{-1}$$

$$a_2 = 0$$

$$a_{BA} = a_B - a_A \\ = 0 - 2 = -2$$

$$s_{BA} = -4 \text{ m}$$

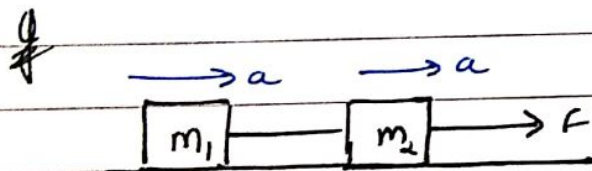
$$\mu_B = 0$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

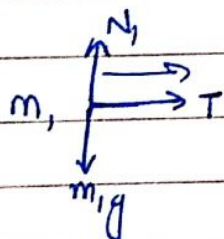
$$-4 = 0 + \frac{1}{2} \times (-2) \times t^2$$

$$t = 2 \text{ s}$$

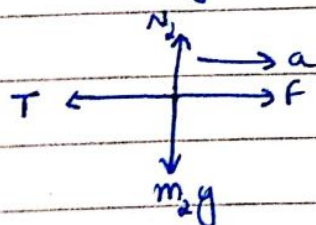


Find T in string.

Find a of m_1 & m_2



$$T = m_1 a \quad \text{--- (i)}$$



$$F - T = m_2 a \quad \text{--- (ii)}$$

Adding both eqn,

$$F = m_1 a + m_2 a$$

$$F = a(m_1 + m_2)$$

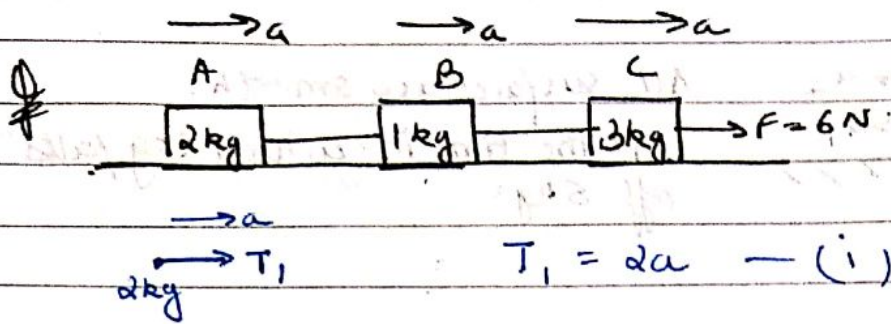
$$a = \frac{F}{m_1 + m_2}$$

$$T = \frac{F m_1}{m_1 + m_2}$$



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$$T_1 = 2a \quad \text{--- (i)}$$

$$T_2 - T_1 = a \quad \text{--- (ii)}$$

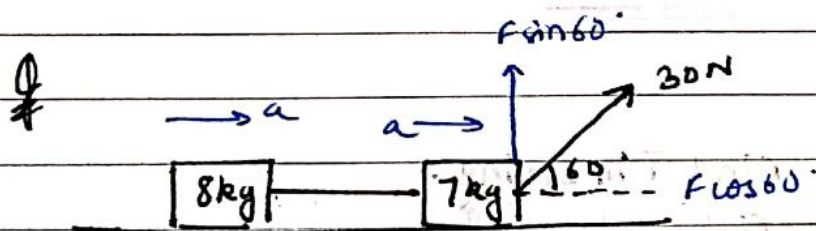
$$F - T_2 = 3a \quad \text{--- (iii)}$$

Adding 3 eqⁿ,

$$F = 6a$$

$$\Rightarrow 6 = 6a$$

$$\Rightarrow a = 1\text{ ms}^{-2}$$



$$T = 8a \quad \text{--- (1)}$$

$$F \cos 60^\circ - T = 7a$$

$$\Rightarrow 30 \cos \frac{1}{2} - T = 7a$$

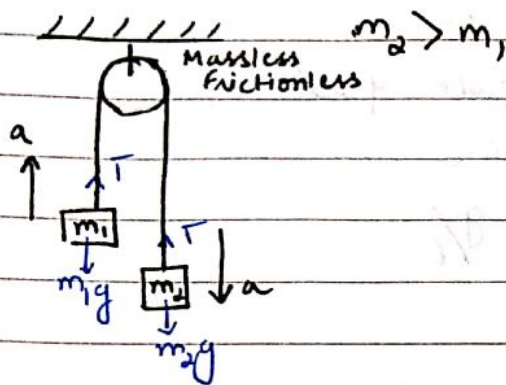
$$15 - T = 7a \quad \text{--- (2)}$$

Adding both eqⁿ,

$$15 = 15a$$

$$\Rightarrow a = 1\text{ ms}^{-2}$$

PULLEY SYSTEM / ATWOOD MACHINE



$$T - m_1g = m_1a \quad \text{---(i)}$$

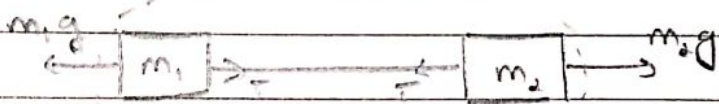
$$m_2g - T = m_2a \quad \text{---(ii)}$$

Adding both equations,

$$m_2g - m_1g = m_1a + m_2a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

System



$$\text{Net force} = m_2g - m_1g$$

$$\text{Net mass} = m_1 + m_2$$

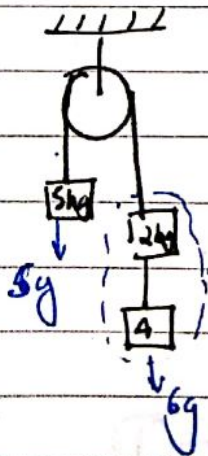
$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

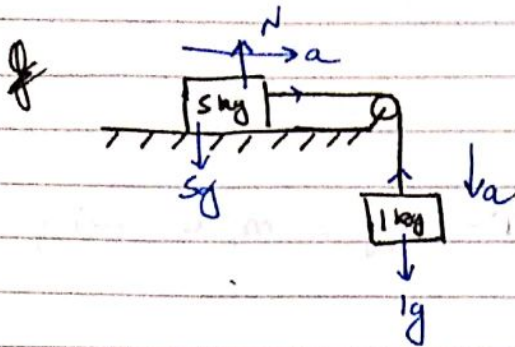
Find acceleration.

$$a = \frac{F_{\text{net}}}{M_{\text{net}}}$$

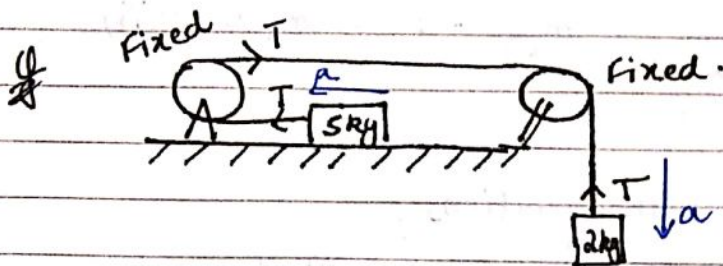
$$= \frac{6g - 5g}{11}$$

$$= \frac{10}{11} \text{ ms}^{-2}$$





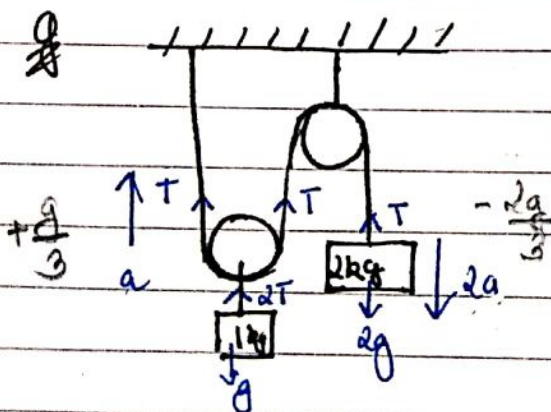
$$\begin{aligned}
 g - T &= a \\
 T &= 5a \\
 \text{Adding both equations,} \\
 g &= 6a \\
 a &= g/6
 \end{aligned}$$



if the direction of motion is upwards, acceleration is (+ve) else downwards is (-ve)

$$\begin{aligned}
 2g - T &= 2a \\
 T &= 5a \\
 \text{Adding both equations,} \\
 2g &= 7a \\
 a &= \frac{2g}{7}
 \end{aligned}$$

$$\begin{aligned}
 2g &= 7a \\
 a &= \frac{2g}{7}
 \end{aligned}$$



$$2g - T = 2 \times 2a = 4a \Rightarrow$$

$$2T - g = a \quad \text{--- (2)}$$

Adding both eqⁿ

$$3g = 9a$$

$$\Rightarrow a = g/3$$

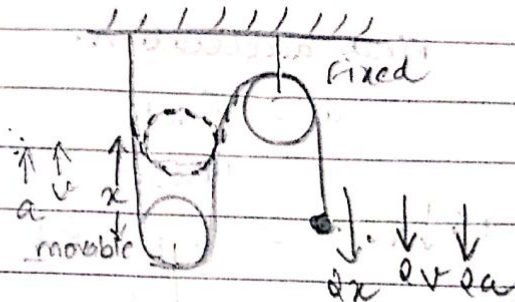
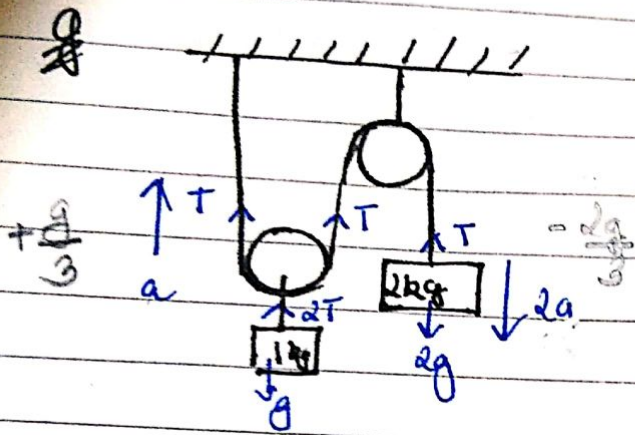
#

$$\begin{aligned}
 a &= a_1 + a_2 \quad (\text{with sign}) \\
 \Rightarrow a &= \frac{a_1 - a_2}{2}
 \end{aligned}$$

$$\begin{aligned}
 a_p &= \frac{a + 0}{2} \\
 \Rightarrow a_p &= \frac{a}{2} \\
 \text{If } a &= 2a \text{ then} \\
 a_p &= a
 \end{aligned}$$

Spiral

Adding both equations,
 $2g = 7a$
 $\therefore a = \frac{2g}{7}$



$$2g - T = 2 \times 2a = 4a \Rightarrow 4g - 2T = 8a \quad \text{--- (1)}$$

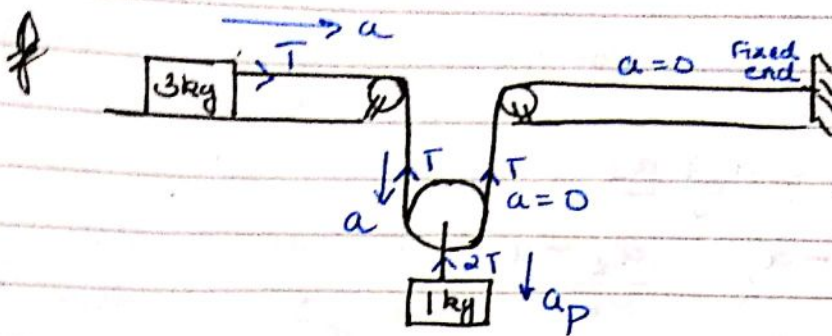
$$2T - g = a \quad \text{--- (2)}$$

Adding both eqⁿ

$$3g = 9a$$

$$\Rightarrow \underline{\underline{a = g/3}}$$

Spiral



$$a_p = \frac{a_1 + a_2}{2}$$

$$\Rightarrow -a_p = \frac{-a + 0}{2}$$

$$\therefore a_p = \frac{a}{2}$$

Let the acceleration of 3kg block be $2a$.

Acceleration of pulley = $\frac{a}{2}$.

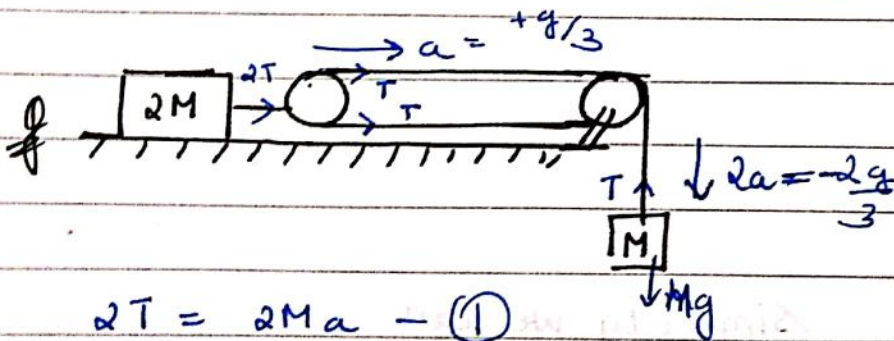
$$1g - 2T = a \quad \text{--- (1)}$$

$$T = 6a \Rightarrow 2T = 12a \quad \text{--- (2)}$$

Adding both equations,

$$g = 13a$$

$$\therefore a = \frac{g}{13}$$



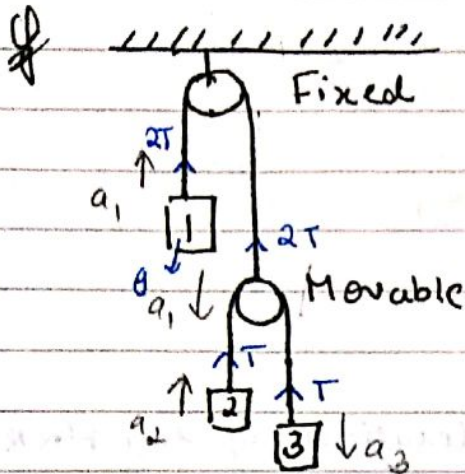
$$2T = 2Ma \quad \text{--- (1)}$$

$$Mg - T = 2Ma$$

$$\therefore 2Mg - 2T = 4Ma \quad \text{--- (2)}$$

$$2Mg = 6Ma$$

$$\Rightarrow a = \frac{g}{3}$$



$$a_p = \frac{a_1 + a_2}{2}$$

$$\Rightarrow -a_1 = \frac{a_2 - a_3}{2}$$

$$\Rightarrow -2a_1 = a_2 - a_3 \quad \text{--- (1)}$$

$$2T - g = a_1$$

$$3g - T = 3a_3$$

$$T - 2g = 2a_2$$

(2)

Taking eqn (1),

$$-2(2T - g) = \frac{T - 2g}{2} - \frac{3g - T}{3}$$

$$\Rightarrow -4T + 2g = \frac{3T - 6g - 6g + 2T}{6}$$

$$\Rightarrow -24T + 12g = 5T - 12g$$

$$\Rightarrow 24g = 29T$$

$$\Rightarrow T = \frac{24}{29}g$$

$$2 \times \frac{24}{29}g - g = a_1$$

$$\Rightarrow \frac{48g - 29g}{29} = a_1$$

$$\Rightarrow \frac{19g}{29} = a_1$$

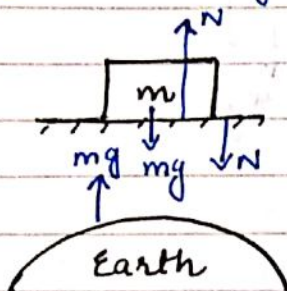
Similarly we can find a_2 & a_3

NEWTON'S THIRD LAW OF MOTION

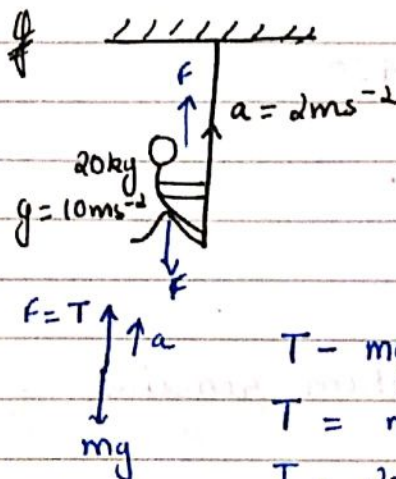
Every action has an equal and opposite reaction.

Action - reaction pair.

- ① Action & reaction acts on different bodies.
- ② Action - reaction pair are of same nature.
- ③ Force always exist in pairs.

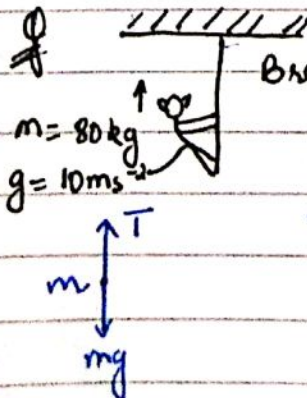


N and mg are not action-reaction pair.
 mg on block is paired with mg on earth.
 N on block is paired with N on surface.



Tension in string,

$$\begin{aligned} T - mg &= ma \\ T &= ma + mg \\ T &= 20 \times 10 + 20 \times 2 \\ &= \underline{240 \text{ N}} \end{aligned}$$



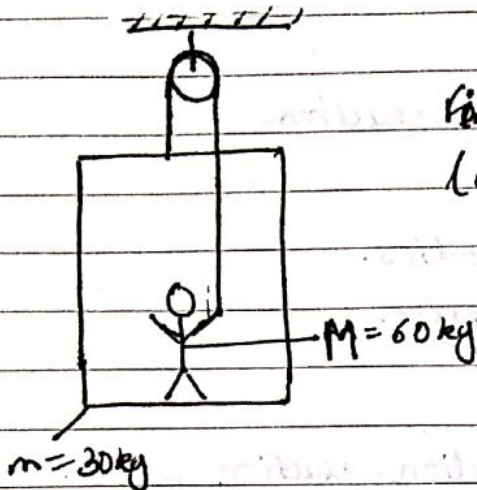
Break $T = 1000 \text{ N}$

With what maximum acceleration can the monkey safely climb the rope?

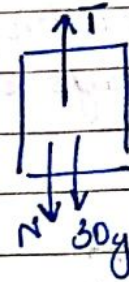
$$\begin{aligned} T - mg &= ma \Rightarrow T = ma + mg \\ T &\leq 1000 \text{ N} \quad \text{so } T_{\text{max}} = 1000 \text{ N} \\ \Rightarrow 80(a + 10) &= 1000 \\ \therefore a &= \frac{1000}{80} - 10 = \frac{20}{8} = \underline{2.5 \text{ ms}^{-2}} \end{aligned}$$

Spiral

Q



Find the force exerted on lift by man (weight of man) as shown by weighing machine.



$$T + N = 60g$$

$$T = N + 30g$$

$$\Rightarrow T - N = 30g$$

$$\therefore 2T = 90g$$

$$\Rightarrow T = 45g$$

$$\Rightarrow T = 450 \text{ N}$$

$$N = 600 - 450$$

$$= 150 \text{ N}$$

CONSERVATION OF MOMENTUM

In an isolated system, the total momentum remains constant or conserved.

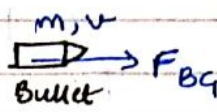
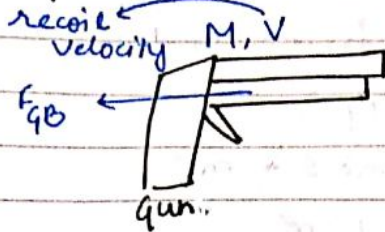
If $F_{\text{net}} = 0$ & $F_{\text{net}} = \frac{\Delta p}{\Delta t}$
then,

$$\Delta p = 0$$

change in momentum = 0

$\therefore p$ remains constant (conserved).

Gun and Bullet System



$$F_{net} = 0$$

$$\vec{F}_{BG} + \vec{F}_{GB} = 0$$

P is conserved.

$$P_{initial} = P_{final}$$

$$\Rightarrow 0 = \vec{P}_g + \vec{P}_b \quad \text{or} \quad \vec{P}_g = -\vec{P}_b$$

Q $M = 5 \text{ kg}$
 $V = ?$

$m = 0.2 \text{ kg}$
 $v = 200 \text{ ms}^{-1}$

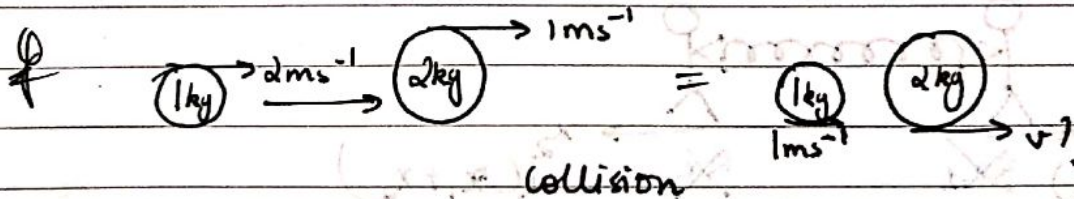
$$P_i = P_f$$

$$\Rightarrow 0 = MV - mu$$

$$\Rightarrow 5V - 40 = 0$$

$$\Rightarrow 5V = 40$$

$$\Rightarrow V = 8 \text{ ms}^{-1}$$



Collision

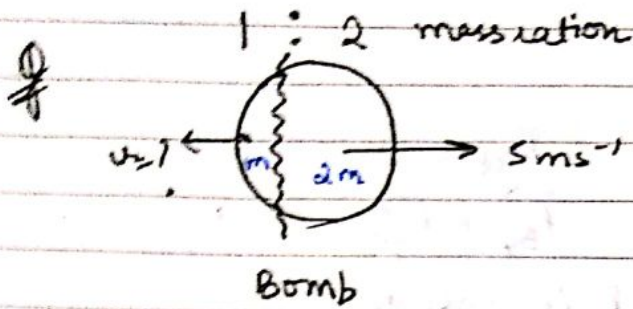
As there is no external force,

$$P_i = P_f$$

$$\Rightarrow 1 \times 2 + 2 \times 1 = 1 \times 1 + 2v$$

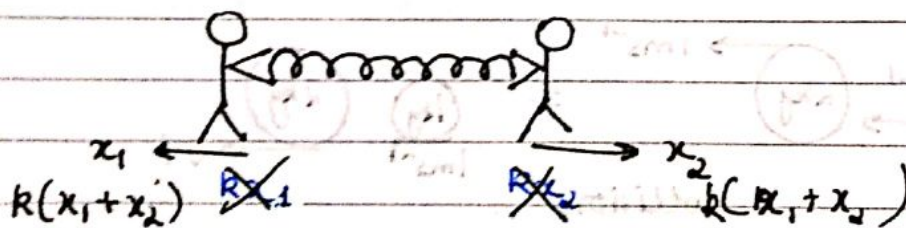
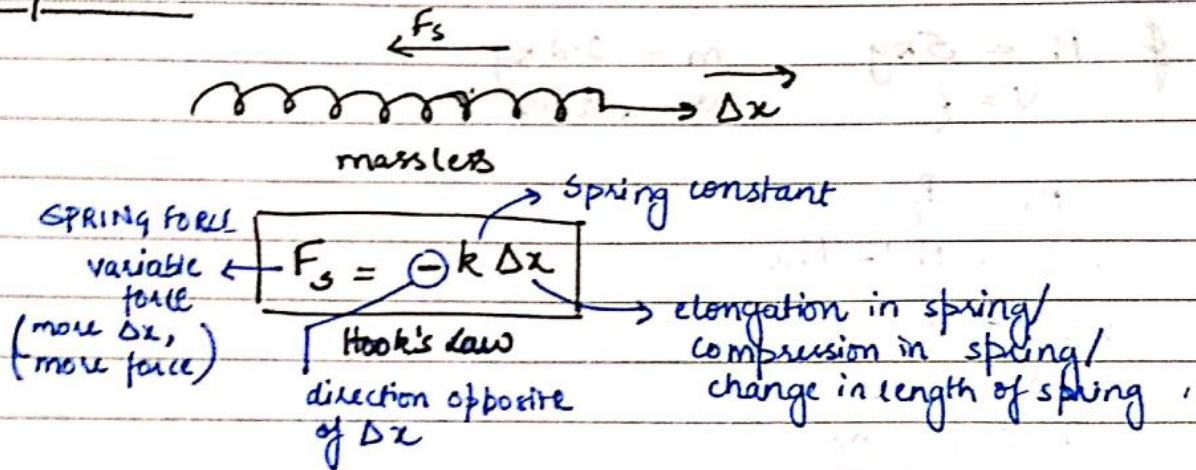
$$\Rightarrow 3 = 2v$$

$$\Rightarrow v = \frac{3}{2} \text{ ms}^{-1} = 1.5 \text{ ms}^{-1}$$

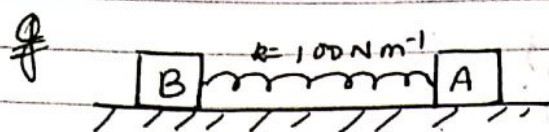


$F_{net} = 0$
 so p is conserved.
 $p_i = p_f$
 $\Rightarrow 0 = 2m \times 5 - m \times v$
 $\Rightarrow mv = 10m$
 $\Rightarrow v = 10ms^{-1}$

SPRING FORCE



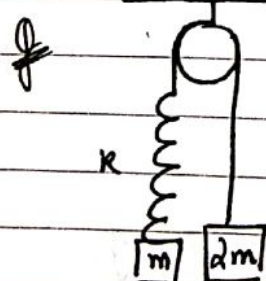
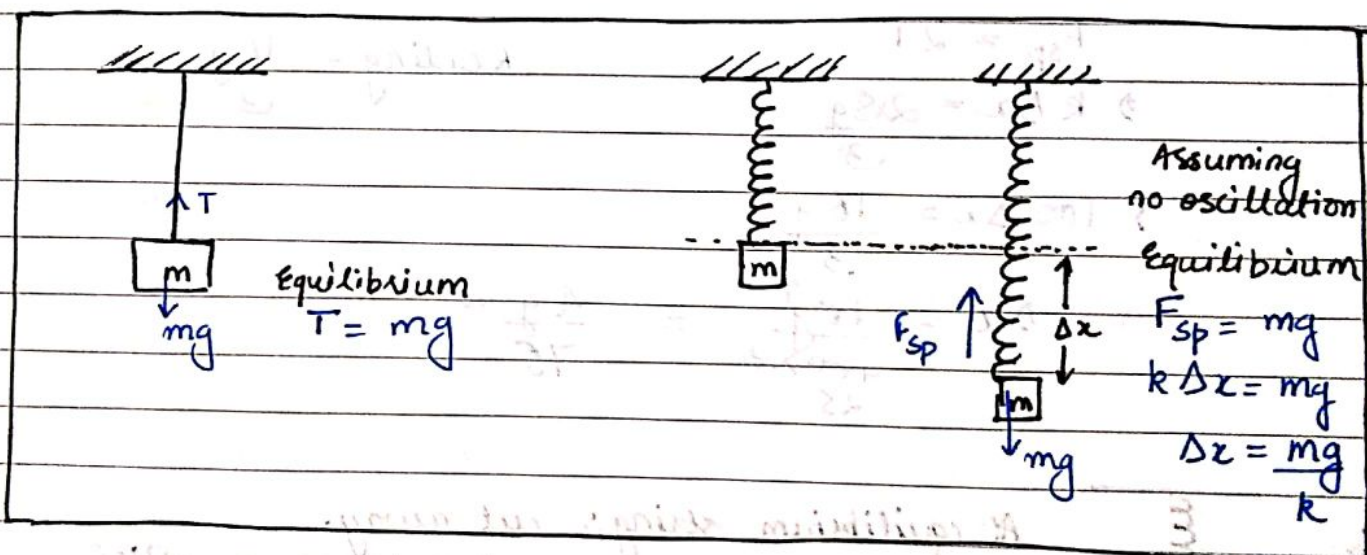
$F_{sp} = k(x_1 + x_2)$ Look at net compression or elongation.



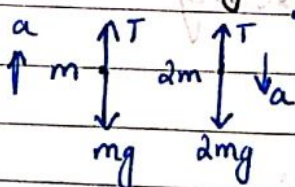
Find F_s if both blocks are displaced by 1m each
(a) in same direction
(b) in opposite direction

(a) Net change = 0
 $F_s = 0$

(b) Net elongation = 1 + 1 = 2m
 $F_s = k \Delta x$
 $= 100 \times 2$
 $= \underline{\underline{200 \text{ N}}}$



Find elongation of spring.



Let us assume a string instead of the spring

$$2mg - T = 2ma$$

$$T - mg = ma$$

$$\therefore mg = 3ma$$

$$a = g/3$$

$$T = mg + \frac{mg}{3} = \frac{4mg}{3}$$

$$F_{sp} = T = \frac{4mg}{3}$$

$$\therefore k \Delta x = \frac{4mg}{3}$$

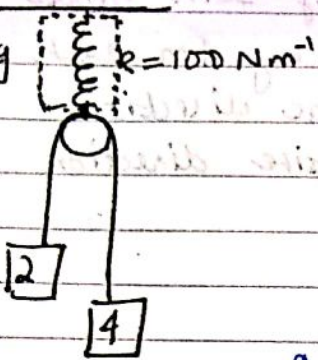
$$\Delta x = \frac{4mg}{3k}$$

Spiral



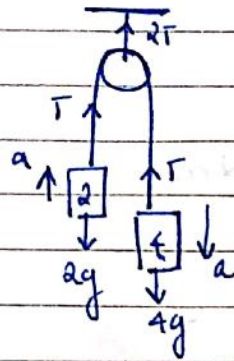
Spring Balance
weighing machine
Weight shown /
Reading / = Spring
force

Spring
Balance



{Elongation?
Reading?

Assuming a string,



$$4g - T = 4a$$

$$T - 2g = 2a$$

$$\therefore 2g = 6a$$

$$a = \frac{g}{3}$$

$$F_{sp} = 2T$$

$$\Rightarrow k \Delta x = 2 \times \frac{8g}{3}$$

$$\Rightarrow 100 \Delta x = \frac{16g}{3}$$

$$\therefore \Delta x = \frac{16g}{100 \times 3} = \frac{4g}{75} m$$

$$\text{Reading} = \frac{16g}{3}$$

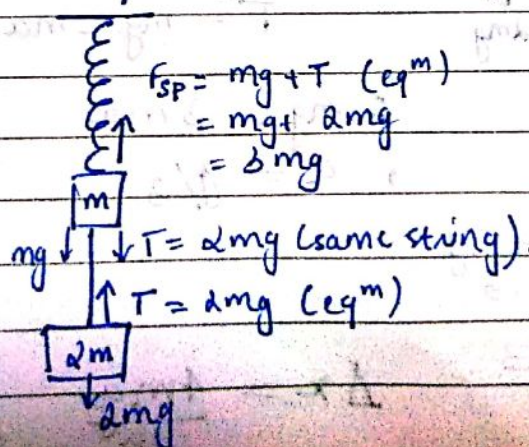
✂



At equilibrium, string is cut away.

Find the acceleration of both blocks JUST AFTER cutting the string

At equilibrium,



$$F_{sp} = mg + T \text{ (eqn)} \\ = mg + 2mg \\ = 3mg$$

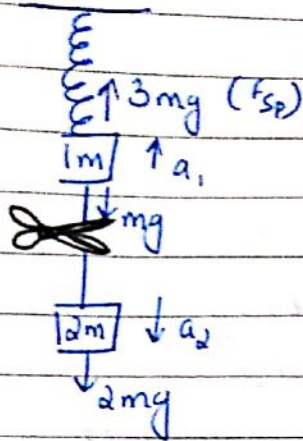
$$mg \downarrow \quad T = 2mg \text{ (same string)}$$

$$T = 2mg \text{ (eqn)}$$

$$2mg \downarrow$$

Just after cutting,
the tension becomes
zero but the spring
force remains the same.

Spiral



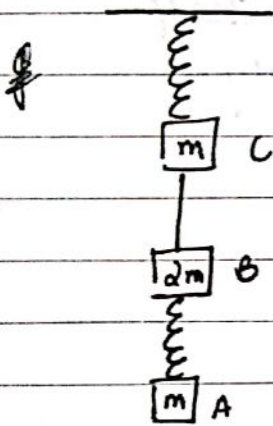
$$3mg - mg = ma_1$$

$$\therefore 2mg = ma_1$$

$$\therefore \underline{a_1 = 2g \text{ ms}^{-2}}$$

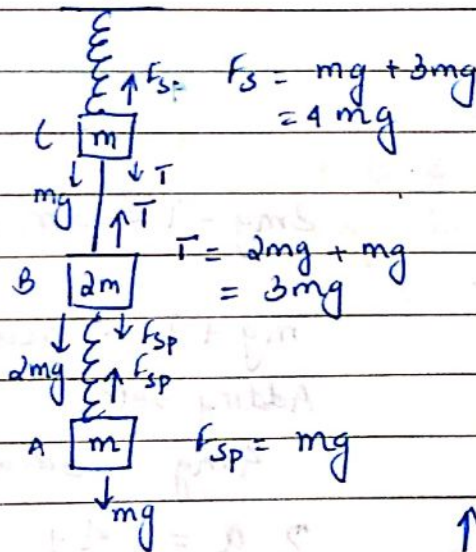
$$2mg = 2ma_2$$

$$\therefore \underline{a_2 = g \text{ ms}^{-2}}$$

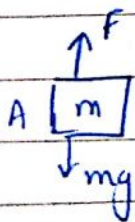


After system attains equilibrium, string is cut.
Find a_A , a_B , a_C just after cutting.

At equilibrium,

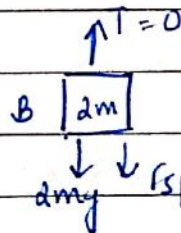


Just after cutting,
 $T = 0$ but
 F_s & F_{sp} are same.



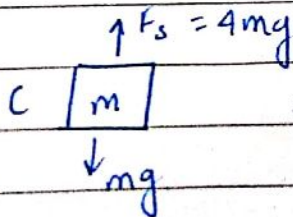
$$mg = mg = ma$$

$$\therefore \underline{a_A = 0}$$



$$3mg = 2ma$$

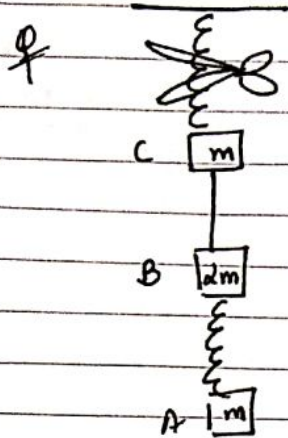
$$\therefore \underline{a_B = \frac{3g}{2}}$$



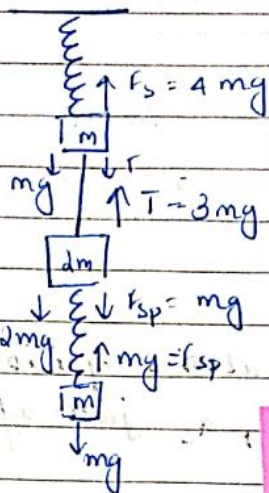
$$4mg - mg = ma$$

$$\therefore 3mg = ma$$

$$\therefore \underline{a_C = 3g}$$

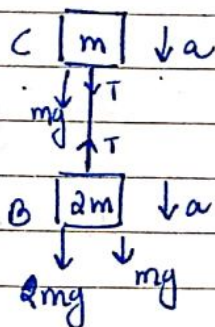
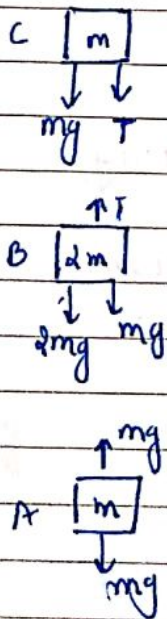


Spring is cut.
Find a_A , a_B & a_C .



If spring is cut $f_{sp} = 0$
Tension is a self adjusting force and it adjusts itself accordingly.

Same string & do mass connected then to acceleration same hoga. $\vec{a}$



$$3mg - T = 2ma$$

$$mg + T = ma$$

Adding both,

$$4mg = 3ma$$

$$\Rightarrow a = \frac{4g}{3}$$

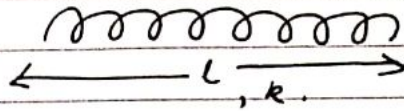
$$a_A = 0$$

$$a_A = 0$$

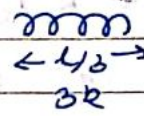
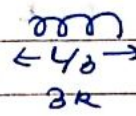
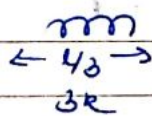
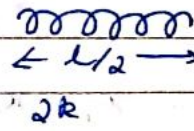
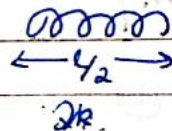
$$a_B = \frac{4g}{3}$$

$$a_C = \frac{4g}{3}$$

SPRING CONSTANT 'k'

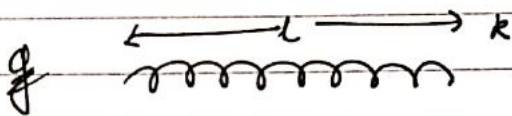


$$k \propto \frac{1}{L}$$

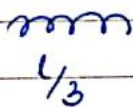


$$k = \frac{C}{L}$$

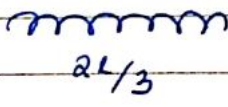
$$\Rightarrow k \times L = \text{constant}$$



(i) It is cut and ratio of two springs in length is 1:2.
Find new spring constant

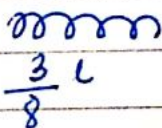


$$S.C., SK = 3k$$

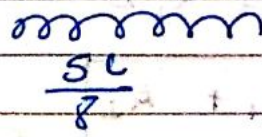


$$S.C. = \frac{3k}{2}$$

(ii) Spring is cut into ratio of 3:5.



$$S.C. = \frac{8k}{3}$$

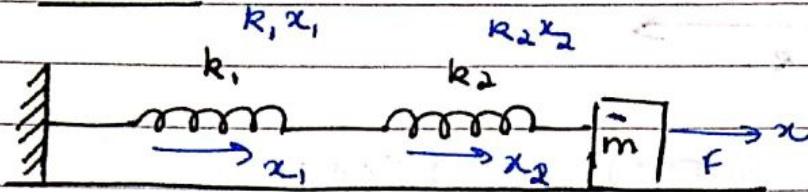


$$S.C. = \frac{8k}{5}$$

Spiral

COMBINATION OF SPRING

(i) SERIES



$$x = x_1 + x_2 \quad \text{--- (1)}$$

$$k_1 x_1 = k_2 x_2 = k_e x = F$$

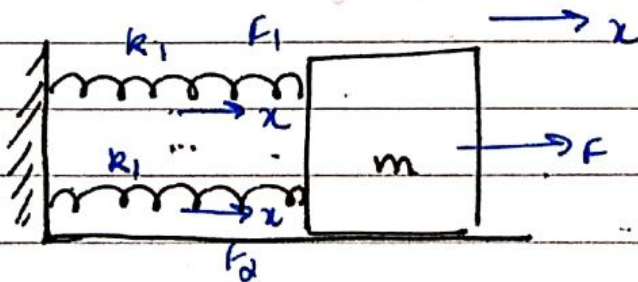
Taking eqⁿ (1)

$$x = x_1 + x_2$$

$$\Rightarrow \frac{F}{k_e} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\Rightarrow \boxed{\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}}$$

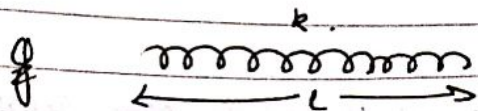
(ii) PARALLEL



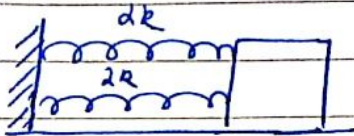
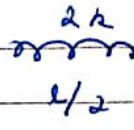
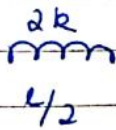
$$F = F_1 + F_2$$

$$\Rightarrow k_e x = k_1 x + k_2 x$$

$$\Rightarrow \boxed{k_e = k_1 + k_2}$$



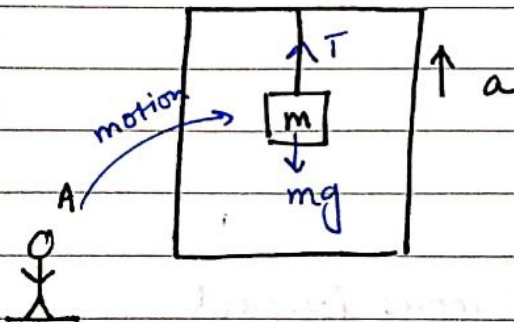
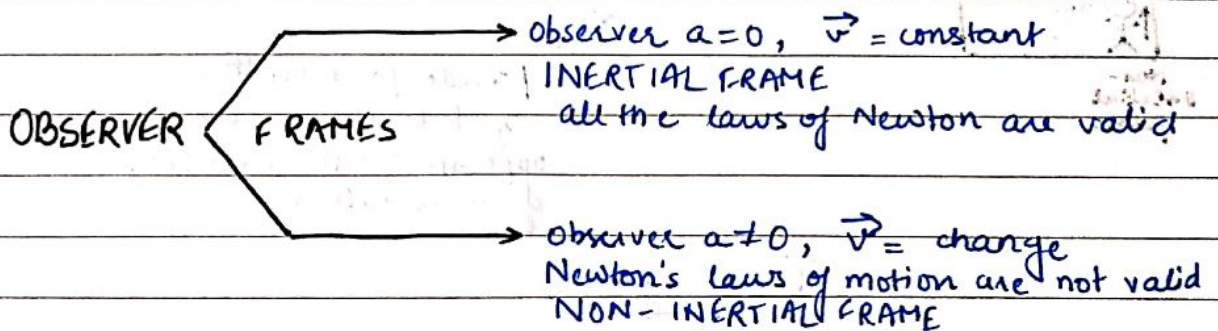
Broken in two parts & those two parts are joined in parallel. Find new 'k'.



$$R_c = k_1 + k_2$$

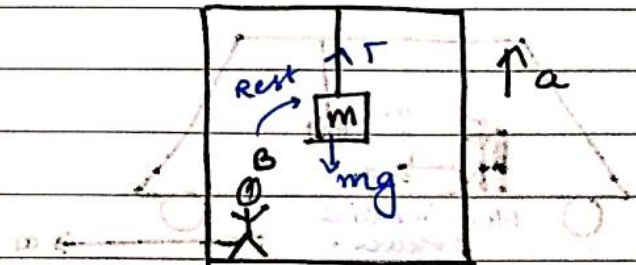
$$= 2k + 2k$$

$$= \underline{\underline{4k}}$$



$$T - mg = ma$$

$$\Rightarrow T = m(g + a)$$



$$T = mg$$

This answer is wrong.
The observer B is a non-inertial frame.

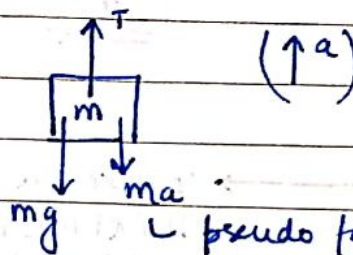
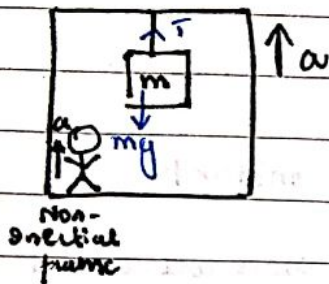
Newton's Laws hold only for INERTIAL OBSERVER.
To battle this Pseudo force was introduced.

PSEUDO FORCE

Pseudo force is used to apply Newton's laws of Motion to Non-Inertial frame. It is a fake force.

$$F_p = - m a$$

to be applied on the object opp. to the direction of acceleration. mass of object under study acceleration of observer

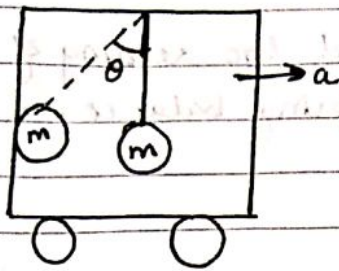


pseudo force on the object by the observer opposite to the direction of acceleration.

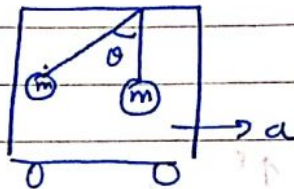
$$\underline{T = m(g + a)}$$



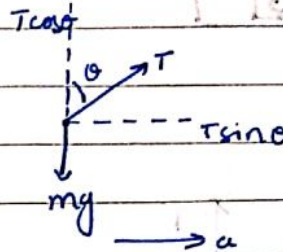
The pendulum moves back when the car moves forward due to the pseudo force of ma opposite to the direction of acceleration on it.



Find θ where m rests in equilibrium.



INERTIAL



$$T \cos \theta = mg \quad \text{--- (1)}$$

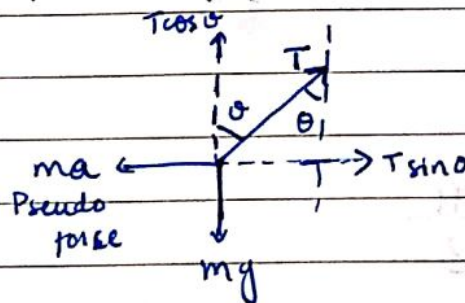
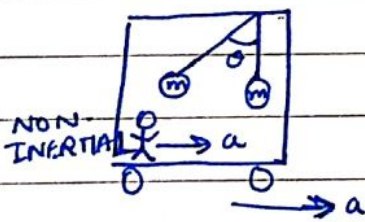
$$T \sin \theta = ma \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{T \sin \theta}{T \cos \theta} = \frac{ma}{mg}$$

$$\Rightarrow \tan \theta = \frac{a}{g}$$

$$\theta = \tan^{-1} \left(\frac{a}{g} \right)$$

OR



$$T \cos \theta = mg \quad \text{--- (1)}$$

$$T \sin \theta = ma \quad \text{--- (2)}$$

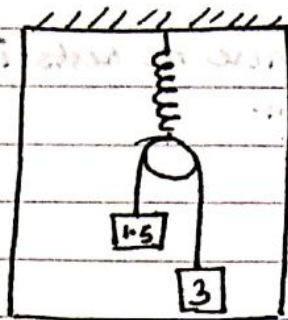
$$\frac{(2)}{(1)} \Rightarrow \frac{T \sin \theta}{T \cos \theta} = \frac{ma}{mg}$$

$$\Rightarrow \tan \theta = \frac{a}{g}$$

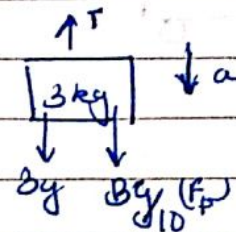
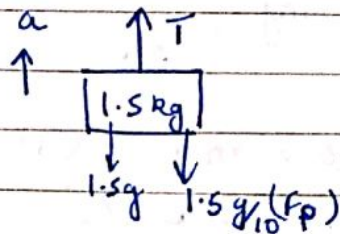
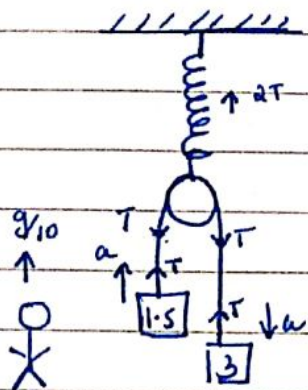
$$\theta = \tan^{-1} \left(\frac{a}{g} \right)$$

Spiral

Q



Find the reading of spring balance.



$$T - 1.5g - 1.5g/10 = 1.5a$$

$$-T + 3g + 3g/10 = 3a$$

$$1.5g + 1.5g/10 = 4.5a$$

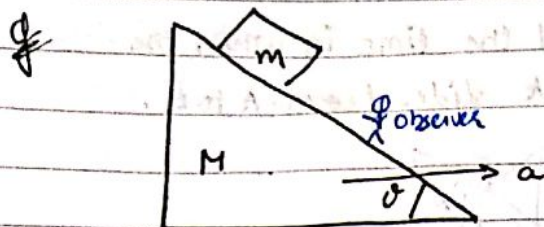
$$15 + 1.5 = 4.5a$$

$$16.5 = 4.5a$$

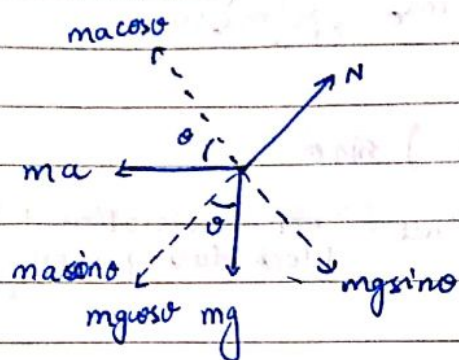
$$a = \frac{16.5}{4.5} = \frac{11}{3}$$

$$T = 30 + 3 - 11 = 22 \text{ N}$$

$$F_{sp} = 2T = 44 \text{ N}$$



Find acceleration of M such that m is at rest wrt M .



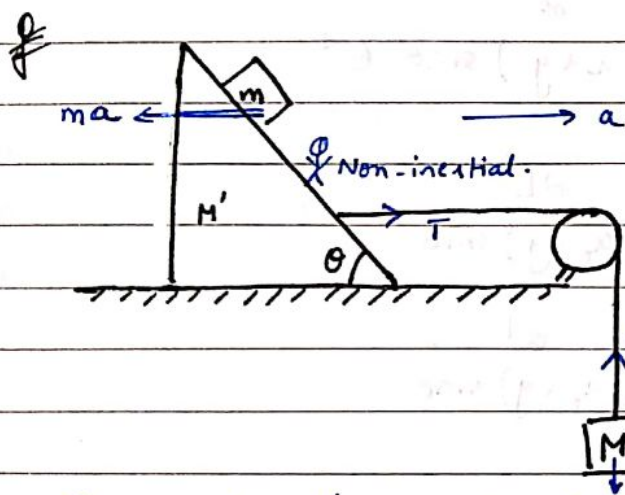
As the mass should be at rest,

$$N = m \sin \theta + m g \cos \theta$$

$$m a \cos \theta = m g \sin \theta$$

$$\Rightarrow a = g \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \underline{a = g \tan \theta}$$



What should be mass of hanging block ($M = ?$) for which ' m ' does not slide over M' .

The acceleration required for the mass m to stay at rest is $g \tan \theta$

Considering M' & m to be one system,

$$T = (M' + m) a \quad \text{--- (1)}$$

& for mass M

$$Mg - T = Ma \quad \text{--- (2)}$$

Adding (1) & (2),

$$Mg = (M' + m) a + Ma$$

$$\Rightarrow M(g - a) = (M' + m) a$$

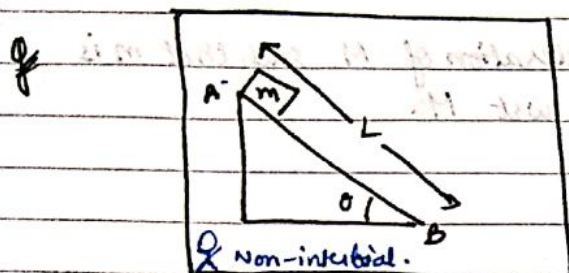
$$\Rightarrow M = \frac{(M' + m) a}{g - a}$$

Putting the value of $a = g \tan \theta$

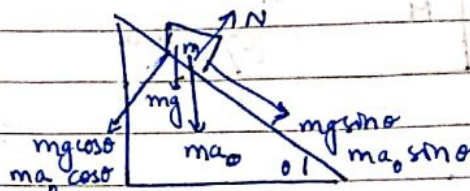
$$M = \frac{(M' + m) \tan \theta}{1 - \tan \theta}$$

or

$$M = \frac{(M' + m)}{\cot \theta - 1} \text{ Spiral}$$



Find the time in which the block slides from A to B.



Force which causes slipping = $(ma_0 + mg) \sin \theta$

& $(ma_0 + mg) \sin \theta = ma_{\text{net}}$ (a_{net} = net acceleration of block during slipping)

$\Rightarrow (a_0 + g) \sin \theta = a_{\text{net}}$

$u = 0 \text{ ms}^{-1}$

$s = L$

$a = (a_0 + g) \sin \theta$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow L = \frac{1}{2}(a_0 + g) \sin \theta t^2$

$\Rightarrow t^2 = \frac{2L}{(a_0 + g) \sin \theta}$

$\Rightarrow t = \sqrt{\frac{2L}{(a_0 + g) \sin \theta}}$

FRICTION

Friction is the force which opposes the relative motion of two bodies (or tendency of relative motion).

Friction force acts tangentially along the contact.

STATIC

when tendency of relative motion is there
(rest)

KINETIC

when relative motion is actually there
(motion)

Spiral

KINETIC FRICTION

- * Relative motion.
- * Constant friction force
 $f_R = \text{constant}$

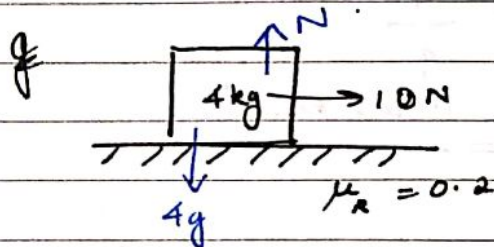
* $f_R \propto N$

$$f_R = \mu_R N$$

$\mu_R \rightarrow$ coefficient of kinetic friction.

depends on surface which are in contact
(determined experimentally)

- * direction is opposite to relative motion.



① Find friction force

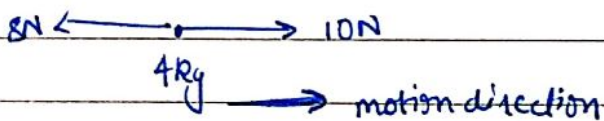
② $\vec{a}$ of 4kg block.

$$N = 4g = 40\text{N}$$

$$f_R = \mu_R N$$

$$= 0.2 \times 40$$

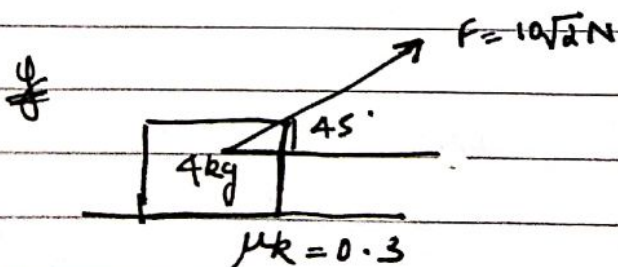
$$= 8\text{N (opposite the direction of motion)}$$



$$\therefore 10 - 8 = 4a$$

$$2 = 4a$$

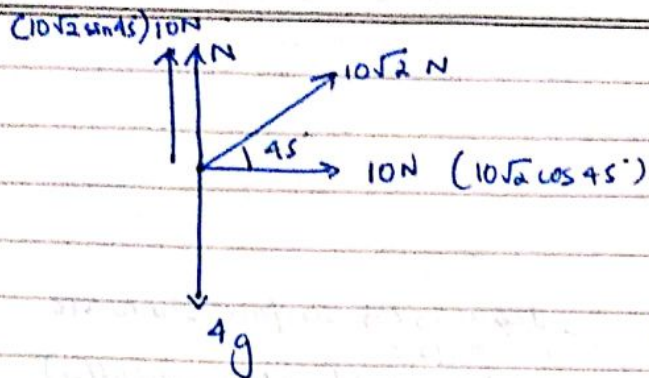
$$\therefore a = 0.5\text{ms}^{-2}$$



① Find N

② Find f_R

③ find a

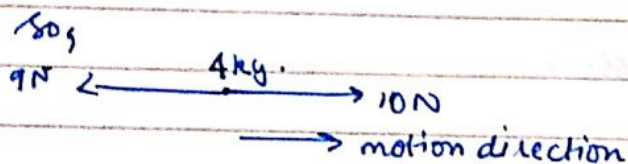


As the block is in vertical equilibrium

$$N + 10 = 4g$$

$$\therefore N = 40 - 10 = \underline{30 \text{ N}}$$

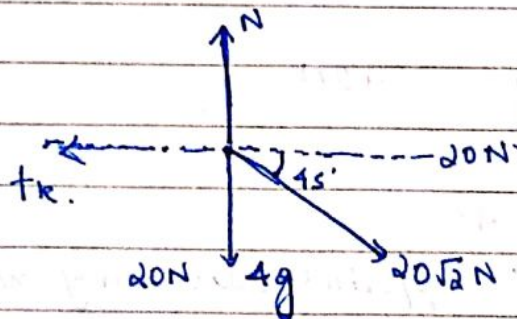
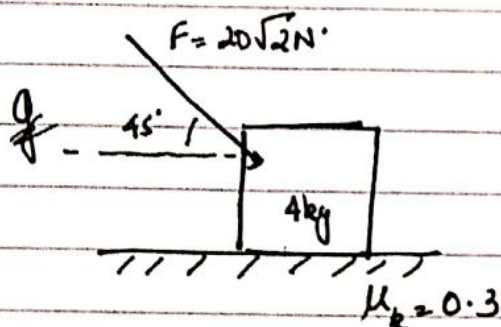
$$f_R = \mu_R N = 0.3 \times 30 = \underline{9 \text{ N}}$$



$$\therefore 10 - 9 = 4 \times a$$

$$\therefore a = \frac{1}{4}$$

$$\therefore a = \underline{0.25 \text{ ms}^{-2}}$$



$$N = 20 \text{ N} + 40 \text{ N}$$

$$\therefore N = \underline{60 \text{ N}}$$

$$f_R = 0.3 \times 60$$

$$= \underline{18 \text{ N}}$$

$$a = \frac{20 - 18}{4} = \underline{0.5 \text{ ms}^{-2}}$$

$F_{\text{limiting}} > F_{\text{kinetic}}$
(just about to move)

STATIC FRICTION

- * Tendency of relative motion (bodies at rest)
- * variable friction $\rightarrow$ self adjusting

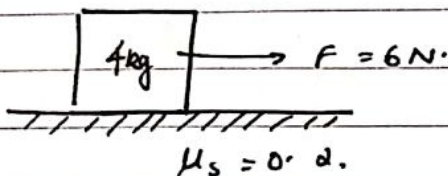
* $0 \leq f_s \leq f_{\text{limiting}}$

$$f_{\text{lim}} = \mu_s N$$

$\mu_s \rightarrow$ coefficient of static friction.

Body moves only if
 $F_{\text{applied}} > \text{Maximum static or } f_{\text{lim}}$
friction

Q



Find friction & $\vec{a}$:

$N = 40N$

If body is at rest,
Friction force = Applied force.

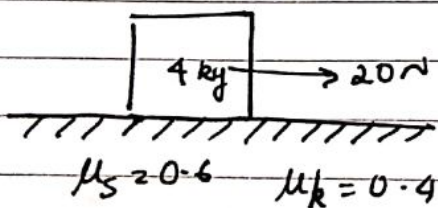
$$\begin{aligned} f_{\text{lim}} &= \mu_s N \\ &= 0.2 \times 40 \\ &= 8N \end{aligned}$$

As $f_{\text{applied}} < f_{\text{lim}}$, the body will not move.

$\vec{a} = 0$

Friction = 6N (opposite to applied force)

Q



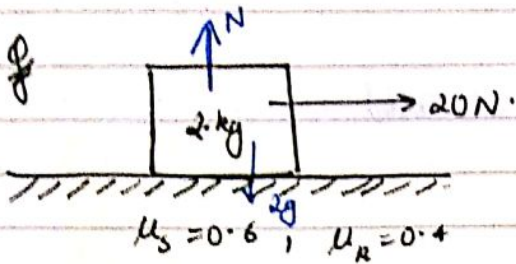
Find friction force and acceleration.

$$\begin{aligned} f_{\text{lim}} &= \mu_s N \quad (N = 40N) \\ &= 0.6 \times 40 \\ &= 24N \end{aligned}$$

As $f_{\text{applied}} < f_{\text{lim}}$, the body will not move.

Friction force = 20N

$\vec{a} = 0$



Find friction and $\vec{a}$.

$$N = 2g = 20 \text{ N}$$

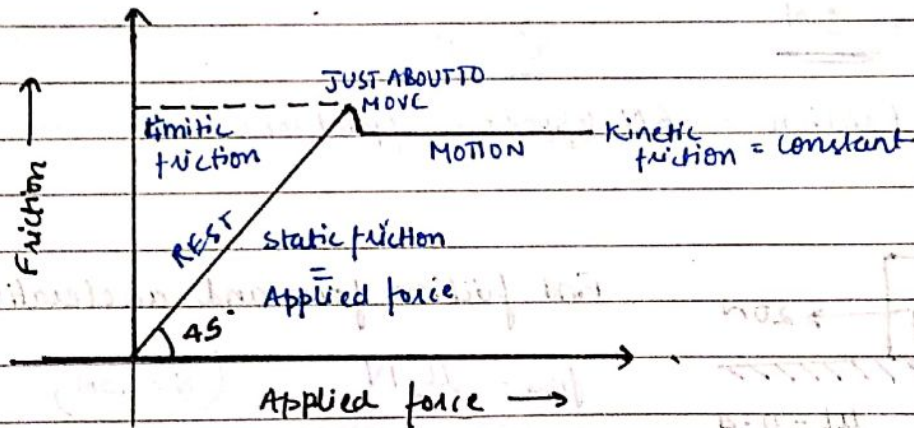
$$f_{\text{lim}} = \mu_s N = 0.6 \times 20 = 12 \text{ N}$$

As $f_{\text{app}} > f_{\text{lim}}$, body will move.

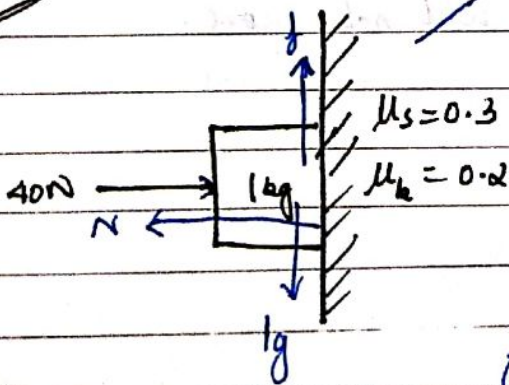
$$f_k = \mu_k N = 0.4 \times 20 = 8 \text{ N}$$

$$a = \frac{20 - 8}{2} = \frac{12}{2} = 6 \text{ ms}^{-2}$$

GRAPH OF FRICTION



IMP



So Friction = 10 N

Find force of friction.

In this case, since there is no horizontal motion, $N = 40 \text{ N}$

Also the motion of block is in downward direction and friction opposes motion so it opposes the downward slipping.

$$f_{\text{lim}} = \mu_s N = 0.3 \times 40 = 12 \text{ N}$$

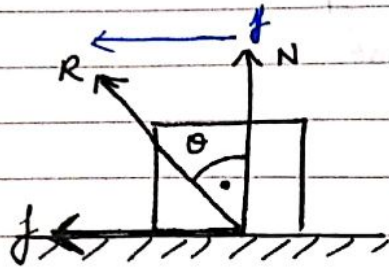
As the downward force is 10 N and is less than limiting friction, the block will not move.

ANGLE OF FRICTION

During contact between two surfaces, the contact forces are two.

- ① Normal Reaction
- ② Frictional force (in case of motion or tendency to move)

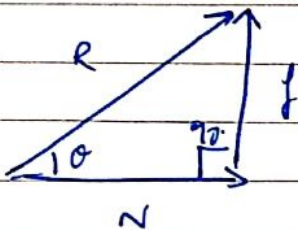
Let R be the RESULTANT CONTACT FORCE then,



If the object is just about to move,

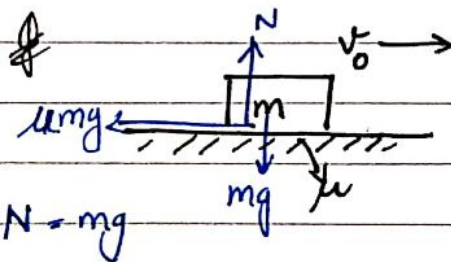
$$\text{Frictional force} = f_s = \mu N.$$

In this case, the angle formed by the resultant contact force and Normal Reaction is the angle of friction.



$$\tan \theta = \frac{f}{N} = \frac{\mu N}{N}$$

$$\boxed{\tan \theta = \mu}$$



Find the time after which it stops and also calculate displacement of block.

$$f_k = \mu N \\ = \mu mg$$

$$a = \frac{F}{m} = \frac{\mu mg}{m} = -\mu g$$

$$u = v_0$$

$$v = 0$$

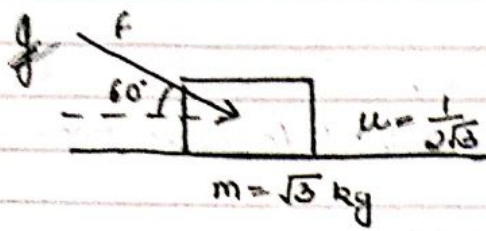
$$t = ?$$

$$v = u + at$$

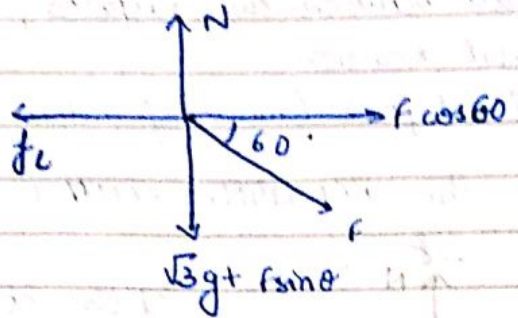
$$\Rightarrow 0 = v_0 - \mu g t$$

$$\Rightarrow t = \frac{v_0}{\mu g}$$

Spiral



Find maximum value of F such that block does not move.



$$N = \sqrt{3}g + F \sin 60^\circ$$

$$\Rightarrow N = 10\sqrt{3} + F \sin 60^\circ$$

For the block to not move,

$$F \cos 60^\circ = f_c \quad \& \quad f_c = \mu N$$

$$\text{So, } F \cos 60^\circ = \mu N$$

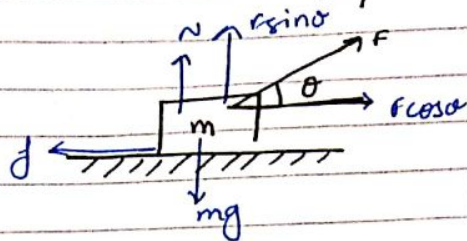
$$\Rightarrow F \cos 60^\circ = \frac{1}{2\sqrt{3}} (10\sqrt{3} + F \sin 60^\circ)$$

$$\Rightarrow F \cos 60^\circ = 5 + \frac{F}{4}$$

$$\Rightarrow \frac{F}{2} - \frac{F}{4} = 5 \quad \Rightarrow \frac{2F - F}{4} = 5$$

$$\therefore \underline{\underline{F = 20 \text{ N}}}$$

Q Which is easier, push or pull, when force is at an angle?

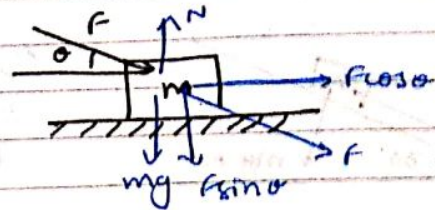


$$N + F \sin \theta = mg$$

$$N = mg - F \sin \theta$$

$$f = \mu N$$

$$f = \mu (mg - F \sin \theta)$$



$$N = mg + F \sin \theta$$

$$f = \mu N$$

$$f = \mu (mg + F \sin \theta)$$

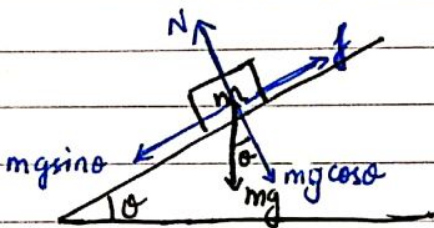
As friction is more in the push case, it is difficult to push.

ANGLE OF REPOSE

Angle of repose is the maximum angle of the incline at which the body stays at rest

OR

It is the maximum angle at which block is just about to slide.



$$N = mg \cos \theta$$

$$f = \mu N$$

$$= \mu mg \cos \theta$$

Make at which block will stay at rest,

$$f = mg \sin \theta$$

$$\Rightarrow \mu mg \cos \theta = mg \sin \theta$$

$$\Rightarrow \tan \theta = \mu$$

$$\boxed{\tan \theta_r = \mu}$$

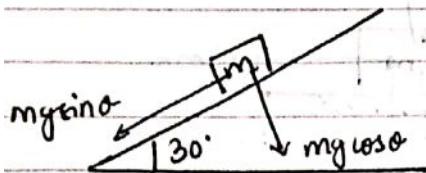
Spiral

Given $\theta < \theta_R$
No sliding
Body at rest
static friction = Applied force

$\theta = \theta_R$
Just about to slide.
Limiting friction $f_L = \mu_s N$

$\theta > \theta_R$
Body starts motion.
Friction $f_k = \mu_k N$

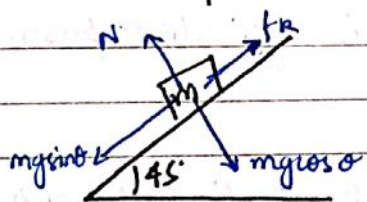
If angle of repose is 45° , find whether body is at rest or motion.
Calculate friction and $\vec{a}$.



$\theta_R = 45^\circ$
 $30^\circ < \theta_R$
No sliding.

static friction = applied force
= $mg \sin \theta$

If angle of repose is 37° , find if body is at rest or motion.
Calculate f .



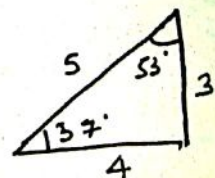
$\theta_R = 37^\circ$
 $45^\circ > \theta_R$
Body will slide.

$$\begin{aligned} mg \sin 45^\circ - f_k &= ma \\ \Rightarrow mg \sin 45^\circ - \mu N &= ma \\ \Rightarrow mg \sin 45^\circ - \mu mg \cos 45^\circ &= ma \\ \Rightarrow \frac{1}{\sqrt{2}} mg (1 - \mu) &= ma \end{aligned}$$

$$\Rightarrow a = \frac{1}{\sqrt{2}} (1 - \mu)g$$

$$\begin{aligned} f &= \mu N \\ &= \mu mg \cos 45^\circ \\ &= \frac{1}{\sqrt{2}} \mu mg \end{aligned}$$

μ_s	θ_R	$\tan \theta_R = \mu_s$
1	45°	
0.75 $(\frac{3}{4})$	37°	
1.33 $(\frac{4}{3})$	53°	



$$\theta = \theta_R$$

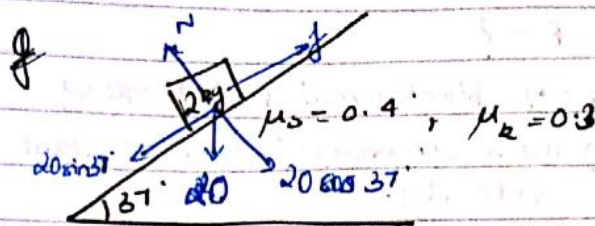
Just about to slide.

limiting friction $f_L = \mu_s N$

OR
ling
rot
Fu

$\theta > \theta_c$
Sliding starts
Motion.

Kinetic Friction $f_k = \mu_k N$



$$\mu_s = 0.4$$

$$N = 20 \cos 37^\circ \\ = 20 \times \frac{4}{5} = 16 \text{ N}$$

$$\text{Limiting friction} = 0.4 \times 16 \\ = 6.4 \text{ N}$$

Force which causes the mass to slip

$$= 20 \sin 37^\circ \\ = 20 \times \frac{3}{5} = 12 \text{ N}$$

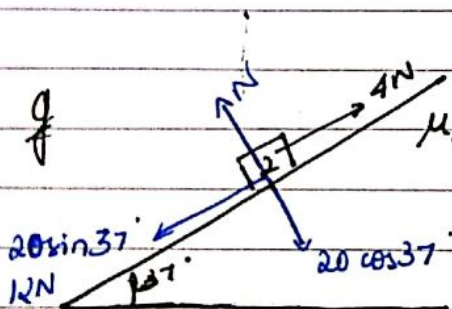
As the applied force > limiting friction, the mass will move.

$$\therefore \text{frictional force} = f_k \\ = \mu_k N \\ = 0.3 \times 16 \\ = 4.8 \text{ N}$$

$$m a_{\text{net}} = 12 - 4.8 \text{ N}$$

$$\rightarrow a = \frac{7.2}{2}$$

$$\therefore a = 3.6 \text{ ms}^{-2}$$



$\mu_s = 0.4, \mu_k = 0.3$ Find f and a .

$$\text{Net force causing block to slip} = 12 - 4 \\ = 8 \text{ N}$$

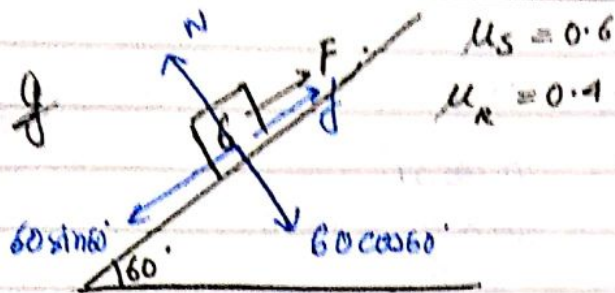
$$f_k = 0.4 \times 16 \\ = 6.4 \text{ N}$$

The object is in motion.

$$\therefore f_k = 6.4 \text{ N}$$

$$a = \frac{8 - 6.4}{2} = 0.8 \text{ ms}^{-2}$$

Spiral



$F = ?$

or (a) block remains stationary
(b) move downwards with constant velocity.

(c) move upwards with acceleration of 4 ms^{-2} .

$$N = 60 \cos 60^\circ$$

$$= 60 \times \frac{1}{2} = 30 \text{ N}$$

$$f_{\text{lim}} = \mu_s N$$

$$= 0.6 \times 30$$

$$= 18 \text{ N}$$

$$\text{Applied force} = 60 \sin 60^\circ - F \quad \text{or} \quad F = 60 \sin 60^\circ$$

(a) For the block to remain stationary,

$$60 \sin 60^\circ - F = f_{\text{lim}}$$

$$\Rightarrow \frac{60\sqrt{3}}{2} - F = 18$$

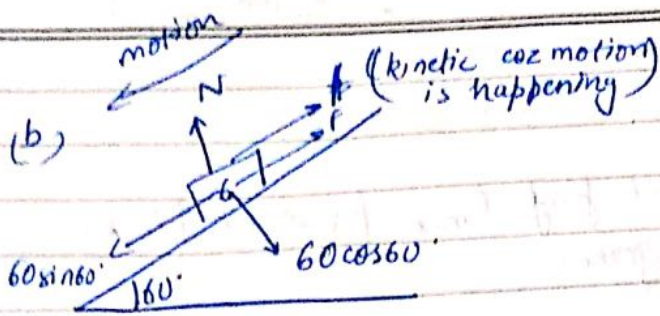
$$\Rightarrow F = 30\sqrt{3} - 18$$

$$\Rightarrow F = 6\sqrt{3}(5 - \sqrt{3}) \text{ N}$$

$$\Rightarrow F = 10.392 \times 3.268$$

$$\Rightarrow F = 35.072176$$

$$\Rightarrow F = \underline{\underline{35 \text{ N (approx)}}}$$



$$f_k = \mu_R N$$

$$= 0.4 \times 60 \times \frac{1}{2}$$

$$= 12 \text{ N}$$

Force which slides = $60 \sin 60$

$$= 52 \text{ N (approx)}$$

For constant velocity, $a = 0$.

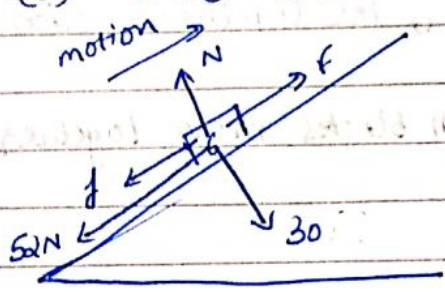
So,

$$60 \sin 60 - f_k - F = ma$$

$$\Rightarrow 52 - 12 - F = 0$$

$$\Rightarrow F = 52 - 12 = \underline{\underline{40 \text{ N}}}$$

(c) Since motion is happening, friction will be kinetic.

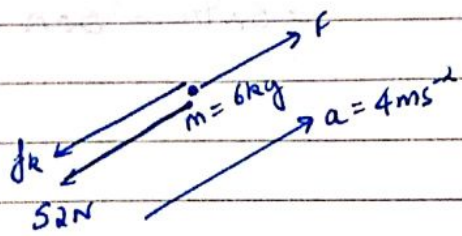


$$N = 30 \text{ N}$$

$$f_k = \mu_R N$$

$$= 0.4 \times 30$$

$$= 12 \text{ N}$$



$$\therefore F - (f_k + 52) = ma$$

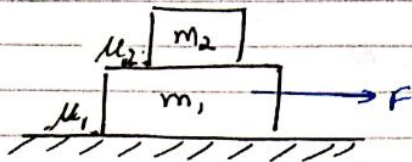
$$\Rightarrow F = 6 \times 4 + 12 + 52$$

$$= F = 24 + 12 + 52$$

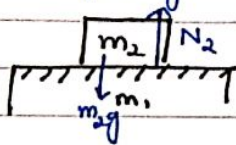
$$\Rightarrow F = \underline{\underline{88 \text{ N}}}$$

BLOCK ON BLOCK DIAGRAM.

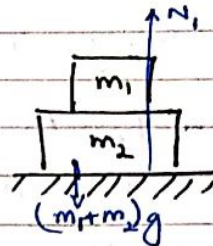
We will find $F_{\max}$ for which blocks move together



1 Find limiting on both surfaces.



$$N_2 = m_2 g$$

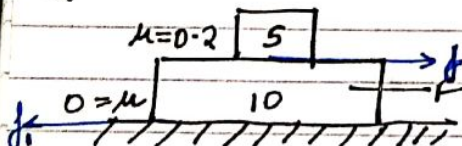


$$N_1 = (m_1 + m_2) g$$

2 Search that block which only has frictional force.
Find $a_{\max}$ of this block.

3 Use this $a_{\max}$ in other block and find $F_{\max}$ for together motion.

Find the maximum value of F such the both blocks move together

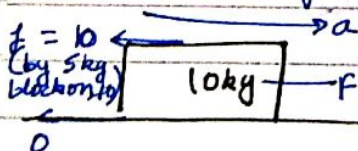


$$f_{\text{lim}} = 0$$

$$\begin{aligned} f_{\text{lim}} &= \mu N \\ &= 0.2 \times 50 \\ &= 10 \text{ N} \end{aligned}$$

$$\begin{aligned} N &= m_2 g \\ &= 5 \times 10 = 50 \text{ N} \end{aligned}$$

$$\text{So, } a_{\max} \text{ of } 5 \text{ kg block} = \frac{10}{5} = 2 \text{ ms}^{-2}$$



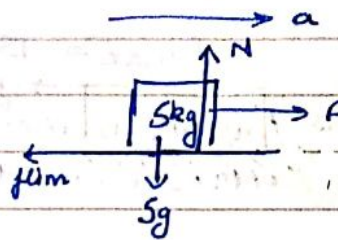
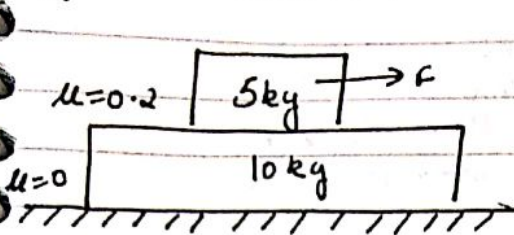
$$F - f = ma$$

$$\therefore F - 10 = 10 \times 2$$

$$\therefore F_{\max} = 30 \text{ N}$$

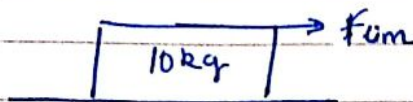
Spiral

* find $F_{\max}$ so that both blocks move together.



$$N = 5 \times 10 = 50 \text{ N}$$

$$f_{\lim} = \mu N = 0.2 \times 50 = 10 \text{ N}$$



$$f_{\lim} = 10 \text{ N}$$

$$a_{\max} = \frac{10}{10} = 1 \text{ m/s}^2$$

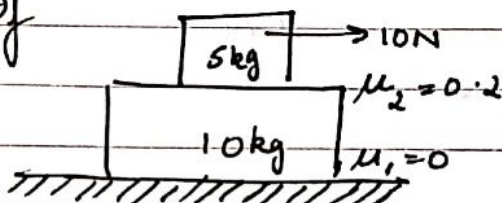
So, for the upper block,

$$F - f_{\lim} = ma$$

$$\therefore F_{\max} = 5 \times 1 + 10 = 15 \text{ N}$$

$$F_{\max} = 15 \text{ N}$$

if

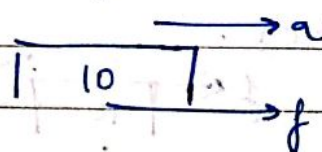
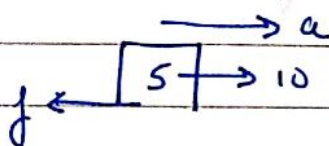


find a of both blocks

Blocks move together
 $\downarrow$
 a is same
 $\downarrow$
 No relative motion
 $\downarrow$
 Static friction (f)

As the force applied is less than $f_{\max}$, both blocks move together and acceleration will be the same.

Let the static friction b/w blocks be f .



$$10 - f = 5a$$

$$f = 10a$$

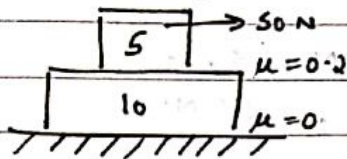
$$10 = 15a$$

$$\therefore a = \frac{10}{15} = \frac{2}{3} \text{ m/s}^2$$

Spiral

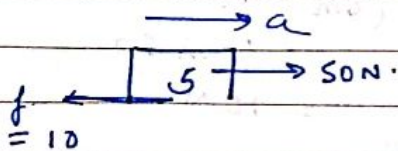
Both move separately
↓
 $a \neq 0$
↓
Relative motion
↓
Kinetic friction
 $f_k = \mu_k N$

Find a of both blocks & friction between them.

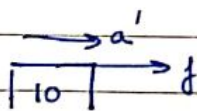


$$f_{\max} = 15 \text{ N}$$

As F_{applied} is more than $f_{\max}$, the blocks will not move together.

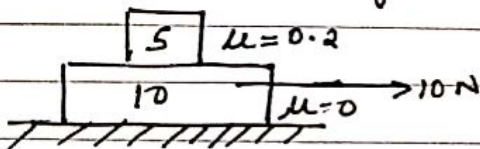


$$a = \frac{50 - 10}{5} = 8 \text{ ms}^{-2}$$



$$a' = \frac{10}{10} = 1 \text{ ms}^{-2}$$

Find acceleration of both blocks and friction b/w them.

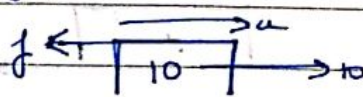
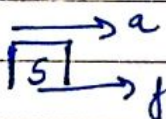


$$\begin{aligned} f_{\text{b/w blocks}} &= \mu N \\ &= 0.2 \times 50 \\ &= 10 \text{ N} \end{aligned}$$

$$a_{\max} \text{ for } 5 \text{ kg block} = \frac{10}{5} = 2 \text{ ms}^{-2}$$

$$\begin{aligned} f_{\max} \text{ for } 10 \text{ kg block} &= 10 \times 2 + 10 \\ &= 30 \text{ N} \end{aligned}$$

As the force applied is less than the maximum force, the blocks will move together.

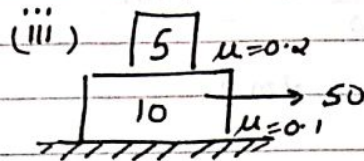
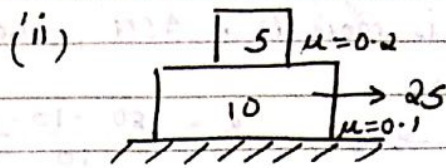
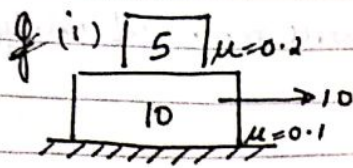


$$\begin{aligned} 10 - f &= 10a \\ f &= 5a \end{aligned}$$

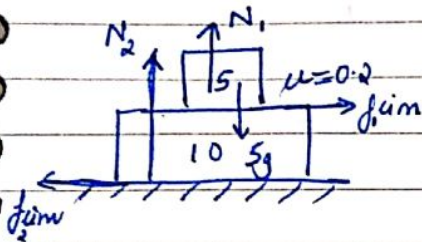
$$10 = 15a$$

$$a = \frac{2}{3} \text{ ms}^{-2}$$

$$\text{Force b/w blocks} = \frac{10}{3} \text{ N}$$



Find a of both blocks.



$$N_1 = 5g = 50 \text{ N}$$

$$f_{\text{lim}} = 0.2 \times 50 = 10 \text{ N}$$

$$N_2 = 150 \text{ N}$$

$$f_{\text{lim}} = 150 \times 0.1 = 15 \text{ N}$$

$f_{\text{lim}} = 10$ $a_{\text{max}} = \frac{10}{5} = 2 \text{ ms}^{-1}$

$f_{\text{max}} = 10 - 15 = -m \times a$
 $\Rightarrow f_{\text{max}} = 10 \times 2 + 10 + 15$
 $\Rightarrow \underline{f_{\text{max}} = 45 \text{ N}}$

As there is a limiting friction b/w ground & block this time,
 $15 \leq F \leq 45$ Both move together

(i) As the force applied is less than the limiting friction, the blocks are at rest. a of both block $= 0$

(ii) As the force is less than 45 N, the blocks will move together. Let the static friction be f .

$f = 5a$

$$25 - 15 - f = 10a$$

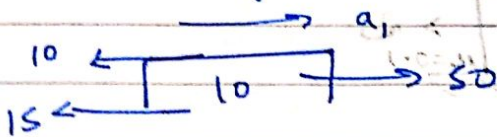
$$10 = 15a$$

$$\Rightarrow \underline{a = \frac{2}{3} \text{ ms}^{-1}}$$

$$\underline{f = \frac{10}{3} \text{ N}}$$

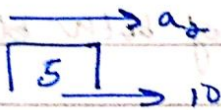
Spiral

(iii) As the force is more than 45N, the blocks will move separately.



$$a_1 = \frac{50 - 10 - 15}{10}$$

$$= \frac{25}{10} = 2.5 \text{ ms}^{-2}$$



$$a_2 = \frac{10}{5} = 2 \text{ ms}^{-2}$$