Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

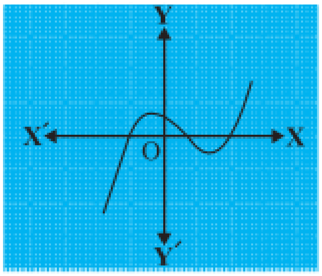
Section A

1. If two positive integers m and n can be expressed as $m = x^2y^5$ and $n = x^3y^2$, where x and y are prime numbers, then HCF(m, n) = **[1]**

    a) x^2y^2 b) x^2y^3

    c) x^3y^2 d) x^3y^3

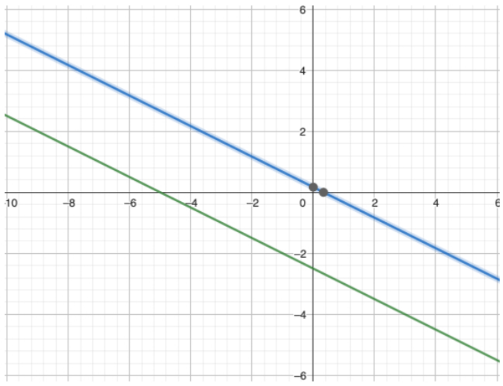
2. Find the number of zeroes of p(x) in the figure given below. **[1]**



    a) 3 b) 0

    c) 2 d) 1

3. The pair of equations x + 2y + 5 = 0 and –3x – 6y + 1 = 0 have **[1]**



a) a unique solution

b) infinitely many solutions

c) no solution

d) exactly two solutions

4. $4x^2$ - 2x - 3 = 0 have **[1]**

a) Real roots

b) Real and Distinct roots

c) No Real roots

d) Real and Equal roots

5. If 18, a, b, -3 are in A.P., then a + b = **[1]**

a) 7

b) 15

c) 19

d) 11

6. The point of intersection of the x-axis and y-axis is called **[1]**

a) ordinate

b) abscissa

c) quardant

d) origin

7. In what ratio does x-axis divide the line segment joining the points A(2, -3) and B(5, 6)? **[1]**

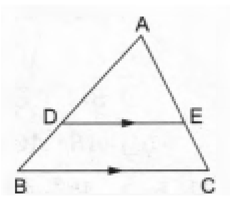
a) 1 : 2

b) 3 : 5

c) 2 : 1

d) 2 : 3

8. In a △ABC, if DE is drawn parallel to BC, cutting AB and AC at D and E respectively such that AB = 7.2 cm, AC = 6.4 cm and AD = 4.5 cm. Then, AE = ? **[1]**



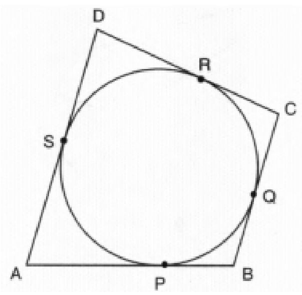
a) 4 cm

b) 5.4 cm

c) 3.2 cm

d) 3.6 cm

9. In Figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm, then x = **[1]**



a) 7 | b) 10
c) 9 | d) 8

10. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is **[1]**

a) 5 cm
b) $\sqrt{7}$ cm
c) 10 cm
d) $2\sqrt{7}$ cm

11. (1 + tan θ + sec θ) (1 + cotθ – cosecθ) = **[1]**

a) 0
b) 2
c) 1
d) –1

12. If secθ + tanθ = p, then the value of sinθ is **[1]**

a) $\frac{1-p^2}{p^2+1}$
b) $\frac{p^2-1}{p^2+1}$
c) None of these
d) $\frac{p^2+1}{p^2-1}$

13. Two men are on the same side of a tower. They observe the angles of elevation of the top of the tower as 30° and 45° respectively. If the height of the tower is 100m, then the distance between them is **[1]**

a) None of these
b) $100(\sqrt{3}+1)m$
c) $100(\sqrt{3}-1)m$
d) $100(1-\sqrt{3})m$

14. If θ is the angle (in degrees) of a sector of a circle of radius r, then area of the sector is **[1]**

a) $\frac{\pi r^2\theta}{360}$
b) $\frac{2\pi r\theta}{360}$
c) $\frac{\pi r^2\theta}{180}$
d) $\frac{2\pi r\theta}{180}$

15. If a chord of a circle of radius 28 cm makes an angle of 90^o at the centre, then the area of the major segment is **[1]**

a) 1456 cm^2
b) 1848 cm^2
c) 392 cm^2
d) 2240 cm^2

16. If a number x is chosen from the numbers 1, 2, 3 and a number y is selected from the numbers 1, 4, 9, then P(xy < 9) **[1]**

a) $\frac{7}{9}$
b) $\frac{5}{9}$
c) $\frac{2}{3}$
d) $\frac{1}{9}$

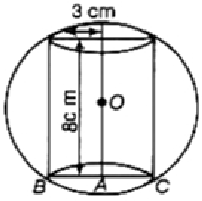
17. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought? **[1]**

a) 240
b) 750
c) 480
d) 40

18. The marks obtained by 9 students in Mathematics are 59, 46, 31, 23, 27, 40, 52, 35 and 29. The mean of the data is **[1]**

a) 30
b) 41
c) 23
d) 38

19. **Assertion (A):** In the given figure, a sphere circumscribes a right cylinder whose height is 8 cm and radius of the base is 3 cm. The ratio of the volumes of the sphere and the cylinder is 125 : 54 **[1]**



Reason (R): Ratio of their volume = $\frac{Volume\ of\ sphere}{Volume\ of\ cylinder}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Three consecutive terms 2k + 1, 3k + 3 and 5k - 1 form an AP than k is equal to 6. **[1]**

Reason (R): In an AP a, a + d, a + 2d, ... the sum to n terms of the AP be $S_n = \frac{n}{2}(2a + (n-1)d)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

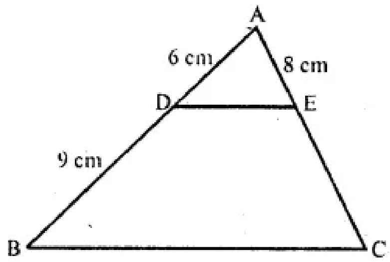
c) A is true but R is false.

d) A is false but R is true.

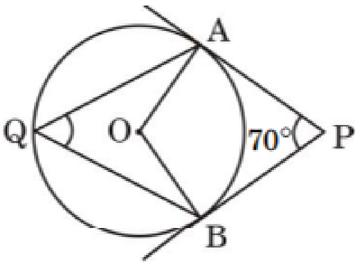
Section B

21. Find H.C.F. and L.C.M. of 56 and 112 by prime factorisation method. **[2]**

22. In the adjoining figure, find AC. **[2]**



23. In Figure, PA and PB are tangents to the circle with centre at O. If ∠APB = 70°, then find m ∠AQB. **[2]**



24. Prove the trigonometric identity: **[2]**

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

OR

Prove that: (sin α + cos α) (tan α + cot α) = sec α + cosec α

25. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector. **[2]**

OR

Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Section C

26. Renu has collected 8 U.S. stamps and 12 international stamps. She wants to display them in identical groups of U.S. and international stamps, with no stamps left over. What is the greatest number of groups Renu can display them in? **[3]**

27. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and other zero. **[3]**

28. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs.22 for a book kept for 6 days, while Anand paid Rs.16 for the book kept for four days. Find the fixed charges and charge for each extraday. **[3]**

OR

Find the four angles of a cyclic quadrilateral ABCD in which $\angle A=(2x-1)^{\circ}$,
$\angle B=(y+5)^{\circ} \angle C=(2y+15)^{\circ}$ and $\angle D=(4x-7)^{\circ}$

29. From an external point P,two tangents,PA and PB are drawn to a circle with centre O.At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA=10 cm,find the perimeter of the triangle PCD. **[3]**

OR

ABCD is a quadrilateral such that ∠D = 90°. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, Find r.

30. If sec $\alpha=\frac{5}{4}$ evaluate $\frac{1-\tan \alpha}{1+\tan \alpha}$, **[3]**

31. If the mean of the following frequency distribution is 18, find the missing frequency. **[3]**

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Section D

32. The product of Tanay's age (in years) five years ago and his age ten years later is 16. Determine Tanay's present age. **[5]**

OR

Solve for x:
$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$; $x \neq 1,2,3$

33. D and E are points on the sides AB and AC respectively of ΔABC such that $DE \| BC$ and divides ΔABC into two parts, equal in area, Find $\frac{BD}{AB}$. **[5]**

34. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy. **[5]**

OR

A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of ₹10 per dm^2.

35. Calculate the median from the following frequency distribution: **[5]**

Class	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45
Frequency	5	6	15	10	5	4	2	2

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII.

There are three sections of each class.

(i) Find total number of trees planted by primary 1 to 5 class students?

(ii) Find the total number of trees planted by the students of the school.

OR

Find the total no of trees planted by class 12^{th} students.

(iii) Find the total number of trees planted by class 10th student.

37. **Read the text carefully and answer the questions:** **[4]**

In an examination hall, students are seated at a distance of 2 m from each other, to maintain the social distance due to CORONA virus pandemic. Let three students sit at points A, B and C whose coordinates are (4, -3), (7, 3) and (8, 5) respectively.

(i) What is the distance between A and C?

(ii) If an invigilator at point 7, lying on the straight line joining B and C such that it divides the distance between them in the ratio of 1 : 2. Then what are the coordinates of I(invigilator)?

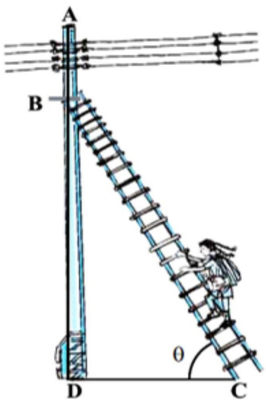
OR

What is the ratio in which B divides the line segment joining A and C?

(iii) What is the mid-point of the line segment joining A and C?

38. **Read the text carefully and answer the questions:** **[4]**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos \theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Find the length BD?

(ii) Find the length of ladder.

OR

If the height of pole and distance BD is doubled, then what will be the length of the ladder?

(iii) How far from the foot of the pole should she place the foot of the ladder?

Solution

Section A

1. **(a)** x^2y^2

   Explanation: $x^2y^5 = y^3(x^2y^2)$

   $x^3y^3 = x(x^2y^2)$

   Therefore HCF (m, n) is x^2y^2

2. **(a)** 3

   Explanation: The number of zeroes is 3 as the graph intersects the x-axis at three points.

3.

   (c) no solution

   Explanation: Given, equations are

   x + 2y + 5 = 0, and

   - 3x - 6y + 1 = 0.

   Comparing the equations with general form:

   $a_1x + b_1y + c_1 = 0$

   $a_2x + b_2y + c_2 = 0$

   Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$

   And $a_2 = -3$, $b_2 = -6$, $c_2 = 1$

   Taking the ratio of coefficients to compare

   $\frac{a_1}{a_2} = \frac{-1}{3}, \frac{b_1}{b_2} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

   So $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   This represents a pair of parallel lines.

   Hence, the pair of equations has no solution.

4.

   (b) Real and Distinct roots

   Explanation: $D = b^2 - 4ac$

   $D = (-2)^2 - 4 \times 4 \times (-3)$

   D = 4 + 48

   D = 52

   D > 0. Hence Real and Distinct roots.

5.

   (b) 15

   Explanation: 18, a, b - 3 are in A.P., then a - 18 = -3 - b

   $\Rightarrow$ a + b = -3 + 18

   $\Rightarrow$ a + b = 15

6.

   (d) origin

   Explanation: The point of intersection of the x-axis and y-axis is called an origin.

   The coordinates of the origin are (0, 0).

7. **(a)** 1 : 2

   Explanation: Let the x axis cut AB at P(x, 0) in the ratio K : 1

   Then $\frac{6k-3}{k+1} = 0 \Rightarrow 6k - 3 - 0 \Rightarrow 6k = 3 \Rightarrow k = \frac{1}{2}$

   required ratio = $\left(\frac{1}{2} : 1\right) = 1 : 2$

8. **(a)** 4 cm

Explanation: In △ABC, DE || BC

AB = 7.2 cm, AC = 6.4 cm, AD = 4.5 cm

Let AE = x cm

DE || BC

△ADE ∼ △ABC

$\frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{4.5}{7.2} = \frac{AE}{6.4}$

$\Rightarrow \mathrm{AE} = \frac{4.5\times6.4}{7.2} = 4.0 = 4\mathrm{cm}$

9.

(c) 9

Explanation: In the given figure,

ABCD is a quadrilateral circumscribe a circle and its sides AB, BC, CD and DA touch the circle at P, Q, R and S respectively

AB = x cm, BC = 7 cm, CR = 3 cm, AS = 5 cm

CR and CQ are tangents to the circle from C

CR = CQ = 3 cm

BQ = BC - CQ = 7 - 3 = 4 cm

BQ and BP are tangents from B

BP = BQ = 4 cm

AS and AP are tangents from A

AP = AS = 5 cm

AB = AP + BP = 5 + 4 = 9 cm

x = 9 cm

10.

(d) $2\sqrt{7}$ cm

Explanation: Radius of the circle =6 cm

and distance of the external point from the centre =8 cm

Length of tangent = $\sqrt{\{(8)^2 - (6)^2\}}$

$= \sqrt{(64 - 36)} = \sqrt{28}$

$= \sqrt{(4 \times 7)} = 2\sqrt{7}cm$

11.

(b) 2

Explanation: By applying formulae

$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}$

=$\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

= $\left(\frac{1+\cos\theta+\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

Multiplying both terms, we get

$= \frac{\sin\theta+\sin\theta\cos\theta+\sin^2\theta+\cos\theta+\cos^2\theta+\sin\theta\cos\theta-1-\cos\theta-\sin\theta}{\cos\theta\sin\theta}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$

$= \frac{1+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$

$= \frac{2\sin\theta\cos\theta}{\cos\theta\sin\theta}$

= 2

Therefore, (1 + tanθ + secθ)(1 + cotθ - cosecθ) = 2

12.

(b) $\frac{p^2-1}{p^2+1}$

Explanation: Given: secθ + tanθ = p

$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$ = p

$\Rightarrow \frac{1+\sin\theta}{\cos\theta}$ = p

Squaring both sides, we get

$\Rightarrow \frac{(1+\sin\theta)^2}{\cos^2\theta}$ = p^2

$\Rightarrow \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$ = p^2

$\Rightarrow \frac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}$ = p^2

$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta}$ = p^2

$\Rightarrow$ 1 + sinθ = p^2 (1 - sinθ)

$\Rightarrow$ 1 + sinθ = p^2 - p^2 sinθ

$\Rightarrow$ sinθ + p^2 sinθ = p^2 - 1

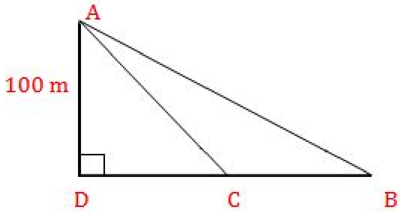
$\Rightarrow$ sinθ(1 + p^2) = p^2 - 1

$\Rightarrow$ sinθ = $\frac{p^2-1}{p^2+1}$

13.

(c) $100(\sqrt{3}-1)m$

Explanation:



Let AD be the height of the tower of 100 m.

In triangle ABD,

$\tan 30^\circ = \frac{\text{AD}}{\text{BD}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{\text{BD}}$

$\Rightarrow \text{BD} = 100\sqrt{3}$ m

Now, in triangle ACD,

$\tan 45^\circ = \frac{\text{AD}}{\text{CD}}$

$\Rightarrow 1 = \frac{100}{\text{CD}}$

$\Rightarrow \text{CD} = 100$ m

Therefore, required distance = BC = BD – CD = $100\sqrt{3} - 100 = 100\left(\sqrt{3}-1\right)$ m

14. **(a)** $\frac{\pi r^2\theta}{360}$

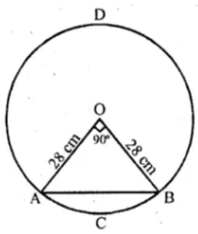
Explanation: $\frac{\pi r^2\theta}{360}$

15.

(d) 2240 cm^2

Explanation: A chord AB makes an angle of 90^o at the centre

Radius of the circle = 28 cm



Area of minor segment ACB

$= \pi r^2 \times \frac{\theta}{360^\circ}$ - area of $\triangle AOB$

$= \pi r^2 \times \frac{90^\circ}{360^\circ} - \frac{1}{2}\text{OA} \times \text{OB}$

$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28$

= 616 - 392

= 224 cm^2

$\therefore$ Area of the major segment ADB

= Area of circle - area of minor segment

$= \pi r^2 - 224 = \frac{22}{7} \times 28 \times 28 - 224$

= 2464 - 224

= 2240 sq. cm

16.

(b) $\frac{5}{9}$

Explanation: Numbers x = 1, 2, 3 and y = 1, 4, 9

Now xy = {1, 4, 9, 2, 8, 18, 3, 12, 27} = 9

$\therefore$ n = 9

and xy < 9 are 1, 2, 3, 4, 8

$\therefore$ m = 5

$\therefore$ P(xy < 9) = $\frac{5}{9}$

17.

(c) 480

Explanation: Given, the total number of sold tickets = 6000

Let she bought x tickets.

Then, the probability of her winning the first prize is given as,

$\Rightarrow \frac{x}{6000} = 0.08$ [given]

$\Rightarrow x = 0.08 \times 6000$

$\therefore x$ = 480

Hence, she bought 480 tickets.

18.

(d) 38

Explanation: Given: 59, 46, 31, 23, 27, 40, 52, 35 and 29

Mean = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

= $\frac{59+46+31+23+27+40+52+35+29}{9}$

= $\frac{342}{9}$

= 38

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: For 2k + 1, 3k + 3 and 5k - 1 to form an AP

(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)

k + 2 = 2k - 4

2 + 4 = 2k - k = k

k = 6

So, both assertion and reason are correct but reason does not explain assertion.

Section B

21. Using the factor tree we have,

56 = $2^3 \times 7$ and 112 = $2^4 \times 7$

Hence HCF is $2^3 \times 7$ = 56 and

LCM is $2^4 \times 7$ = 112

22. In the given figure in $\triangle$ABC, we have

$DE \| BC$. Let EC = x

$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{6}{9} = \frac{8}{x}$

$\Rightarrow x = \frac{9 \times 8}{6} = 12$

$\therefore$ AC = AE + EC = 8 + 12 = 20 cm

23. $\because$ AOPB is a quadrilateral

$\therefore \angle$APB + $\angle$AOB = 180° (opposite angles of quadrilateral are supplementary)

70 + $\angle$ AOB = 180°

$\angle AOB = 180 - 70 = 110°$

Now, $\angle AQB = \frac{1}{2}\angle AOB$ (angle on the circumference of the circle by same arc)

$\angle AQB = \frac{1}{2}\times\ 110 = 55$

$\angle AQB = 55$

24. We have,

$\mathrm{L.H.S} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$\Rightarrow \quad \mathrm{L.H.S} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$ [Multiplying and dividing by (1 - sinθ)]

$\Rightarrow \quad \mathrm{L.H.S} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} \quad \left[\because 1-\sin^2\theta = \cos^2\theta\right]$

$\Rightarrow \quad \mathrm{L.H.S} = \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2} = \frac{1-\sin\theta}{\cos\theta} = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$

= (secθ - tanθ) = R.H.S $\left[\because \frac{1}{\cos\theta} = sec\theta,\ \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

therefore, $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ = secθ - tanθ

Hence proved.

OR

LHS = (sin α + cos α) (tan α + cot α)

= (sin α + cos α) $\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$

= (sin α + cos α) $\left(\frac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha\cos\alpha}\right)$

= (sin α + cos α) $\frac{1}{\sin\alpha\cos\alpha}$ [∵ $\sin^2$ α + $\cos^2$ α = 1]

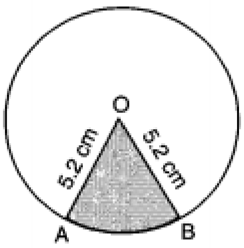
$= \frac{\sin\alpha}{\sin\alpha\cos\alpha} + \frac{\cos\alpha}{\sin\alpha\cos\alpha}$

$= \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha}$

= sec α+cosec α

= RHS

Hence, proved.

25. 

Let OAB be the given sector.

It is given that Perimeter of sector OAB = 16.4 cm

⇒OA + OB + arc AB = 16.4 cm

⇒5.2 + 5.2 + arc AB = 16.4

⇒arc AB = 6 cm

$\Rightarrow l = 6cm$

∴Area of sector OAB =$\frac{1}{2}lr = \frac{1}{2}\times 6\times 5.2\mathrm{cm}^2$ = 15.6 cm^2

OR

According to the question,

Radius of a circle $= r = 5cm$

Arc length $= l = 3.5cm$

∴ Area of sector $= \frac{1}{2}\times l\times r$

$= \frac{1}{2}\times 3.5\times 5$

= 8.7 cm^2

Section C

26. To make all the groups identical and find the greatest number of groups, we have to find the greatest number which can divide by 8 and 12 exactly.

That is nothing but H.C.F. of 8 and 12.

H.C.F of (8, 12) = 4

That is, 8 U.S stamps can be displayed in 4 groups at 2 stamps/group.

And 12 international stamps can be displayed in 4 groups at 3 stamps/group.

In this way, each of the 4 groups would have 2 U.S. stamps and 3 international stamps. And all the 4 groups would be identical.

Hence, the greatest number of groups can be made is 4.

27. Let $P(x) = 2x^2 + 3x + \lambda$

Its one zero is $\frac{1}{2}$ so $P(\frac{1}{2}) = 0$

$P(\frac{1}{2}) = 2 \times (\frac{1}{2})^2 + 3(\frac{1}{2}) + \lambda = 0$

$\Rightarrow 2 \times \frac{1}{4} + 3/2 + \lambda = 0$

$\Rightarrow \frac{1}{2} + \frac{3}{2} + \lambda = 0$

$\Rightarrow \frac{4}{2} + \lambda = 0$

$\Rightarrow 2 + \lambda = 0$

$\Rightarrow \lambda = -2$

Let the other zero be α

Then $\alpha + \frac{1}{2} = -\frac{3}{2}$

$\Rightarrow \alpha = -\frac{3}{2} - \frac{1}{2} = -\frac{4}{2} = -2$

28. Let the fixed charge for first three days be $Rs. x$

The additional charge for each day thereafter be $Rs. y$.

Then according to question, we have

$x + 4y = 22$ ...(i)

$x + 2y = 16$ ...(ii)

Subtracting equation (ii) from (i),

$2y = 6$

$\Rightarrow y = 3$

Substituting $y = 3$ in (ii),

$x + 2(3) = 16$

$\Rightarrow x + 6 = 16$

$\Rightarrow x = 10$

Thus, the fixed charge for first three days is $Rs. 10$.

and the additional charge for each day thereafter is $Rs. 3$.

OR

Since, we know that the sum of the opposite angles of a cyclic quadrilateral is 180°. In the cyclic quadrilateral ABCD, angles A and C and angles B and D form pairs of opposite angle.

$\therefore \angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

$\Rightarrow$2x - 1 + 2y + 15 =180 and y + 5 + 4x -7=180

$\Rightarrow$2x + 2y + 14 = 180 and 4x + y - 2 = 180

$\Rightarrow$ 2x + 2y =166 and 4x + y =182

$\Rightarrow$ x + y =83 .... (i)

and, 4x + y =182 .... (ii)

Subtracting equation (i) from equation (ii), we get

4x + y - (x + y) = 182 - 83

4x + y - x - y = 99

3x = 99 $\Rightarrow$ x = 33

Substitute the value of x = 33 in equation (i), we get

33+ y = 83

y = 83 - 33

y =50

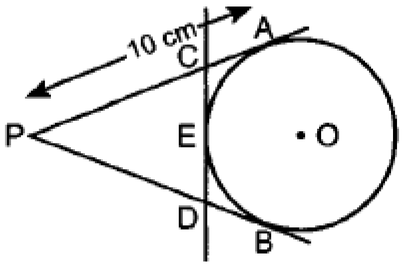
Therefore

$\angle A = (2 \times 33 - 1)^\circ = 65^\circ, \angle B = (y + 5)^\circ = (50 + 5)^\circ = 55^\circ$

$\angle C = (2y + 15)^\circ = (2 \times 50 + 15)^\circ = 115^\circ$ and $\angle D = (4 \times 33 - 7)^\circ = 125^\circ$

Thus $\angle A = 65^\circ$, $\angle B = 55^\circ$ $\angle C = 115^\circ$ and $\angle D = 125^\circ$

29. Given,



PA = 10 cm.

PA = PB [If P is external point] ...(i)[From an external point tangents drawn to a circle are equal in length]

If C is external point, then CA = CE

If D is external point, then

DB = DE ...(ii)

Perimeter of triangle △PCD

= PC + CD + PD

= PC + CE + ED + PD

= PC + CA + DB + PD

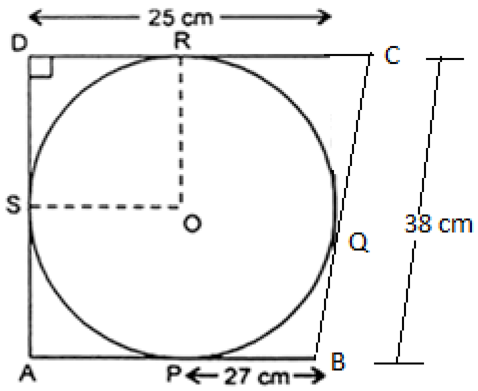
= PA + PB

= PA + PA

=2 PA

$= 2 \times 10 = 20\text{cm}$ [From (i)]

OR



Given that ABCD is a quadrilateral such that ∠D = 90°.

BC = 38 cm,CD = 25cm and BP = 27 cm

∴ From the figure,

BP = BQ = 27 cm [Tangents from an external point are equal]

Now, BC = 38

⇒ BQ + QC = 38

⇒ 27 + QC = 38

⇒ QC = 38 - 27

⇒ QC = 11 cm

∴ QC = 11 cm = CR [Tangents from an external point are equal]

CD = 25 cm

CR + RD=25

⇒ 11+ RD = 25

⇒ RD = 25-11

⇒ RD = 14 cm

Also,

RD = DS = 14 cm [Tangents from an external point are equal]

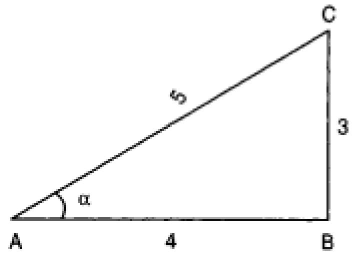
OR and OS are radii of the circle.

From tangents R and S, ∠ORD = ∠OSD = 90°

Thus, ORDS is a square.

OR = DS = 14 cm

Hence, the radius of the circle, r = OR = 14 cm

30. 

We have,

$\sec\alpha = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{5}{4}$

So,Let us draw a right triangle ABC, $\angle B = 90^\circ$ such that

hypotenuse = AC = 5 units, Base = AB = 4 units, and$\angle BAC$= α.

Applying Pythagoras theorem in ΔABC, we get

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 5^2 = 4^2 + BC^2$

$\Rightarrow \quad BC^2 = 5^2 - 4^2 = 9$

$\Rightarrow \quad BC = \sqrt{9} = 3$

$\therefore \quad \tan\alpha = \frac{BC}{AB} = \frac{3}{4}$

Now, we have, $\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1-\frac{3}{4}}{1+\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{1}{7}$

therefore, $\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1}{7}$

31.

Class interval	**Frequency f_i**	**Mid-value x_i**	**$f_i x_i$**
11-13	3	12	36
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
	$\sum f_i$ = 40 + f		$\sum f_i x_i$ = 704 + 20 f

let the missing frequency is 'f '.

we know that , Mean = $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow$ 18 = $\frac{704+20f}{40+f}$

$\Rightarrow$ 18(40 + f) = 704 + 20f

$\Rightarrow$ 720 + 18f = 704 + 20f

$\Rightarrow$ 2f = 16

$\Rightarrow$ f = 8

Section D

32. Let the present age of Tanay be x years

By the question,

(x - 5)(x + 10) = 16

or, $x^2 + 5x - 50 = 16$

or, $x^2 + 5x - 66 = 0$

or, $x^2 + 11x - 6x - 66 = -66$

x(x + 1)-16(x - l11) = 0

(x + 11) (x -6) = 0

=-11, 6

Rejecting x = - 11, as age cannot be negative.

∴ Present age of Tanay is 6 years.

OR

Given, $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$

$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{x-3+x-1}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{2x-4}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{2(x-2)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{2}{(x-1)(x-3)}=\frac{2}{3}$

(x - 1) (x - 3) = 3

x^2 - 4x + 3 = 3

x^2 -4x = 0

x(x - 4) = 0

x = 0 , x - 4= 0

x = 0, x = 4

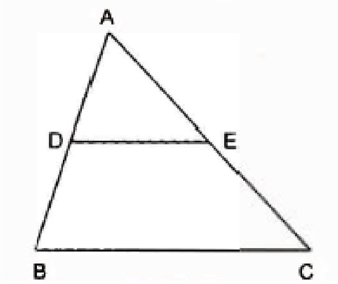
33. We have,

Area (ΔADE) = Area (trapezium BCED)

⇒ Area (ΔADE)+ Area (ΔADE) = Area (trapezium BCED) + Area (ΔADE)

⇒ 2 Area (ΔADE)= Area (Δ ABC) ...(i)

In ΔADE and ΔABC, we have



$DE \| BC, therefore, \angle ADE = \angle B$ [Corresponding angles]

and, ∠A = ∠A [Common]

∴ ΔADE~ ΔABC

Since for similar triangles ,the ratio of the areas is the square of the ratio of their corresponding sides.Therefore,

$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta ABC)}=\frac{AD^2}{AB^2}$

$\Rightarrow \quad \frac{\text{Area }(\Delta ADE)}{2\text{ Area }(\Delta ADE)}=\frac{AD^2}{AB^2}$

$\Rightarrow \quad \frac{1}{2}=\left(\frac{AD}{AB}\right)^2$

$\Rightarrow \quad \frac{AD}{AB}=\frac{1}{\sqrt{2}}$

$\Rightarrow \quad AB=\sqrt{2}AD$

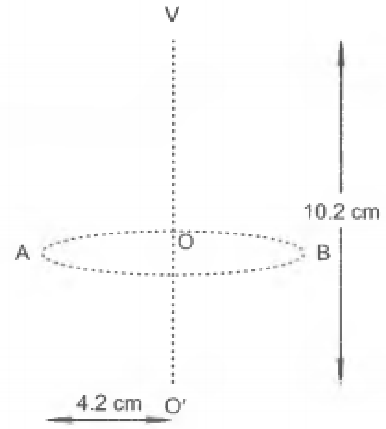
$\Rightarrow \quad AB=\sqrt{2}(AB-BD)$

$\Rightarrow \quad (\sqrt{2}-1)AB=\sqrt{2}BD \Rightarrow \frac{BD}{AB}=\frac{\sqrt{2}-1}{\sqrt{2}}=\frac{2-\sqrt{2}}{2}$

34. We have, VO' = 10.2cm, OA = OO' = 4.2cm

Let r be the radius of the hemisphere and h be the height of the conical part of the toy.

Then, r = OA = 4.2 cm, h = VO = VO' - OO' = (10.2 - 4.2) cm = 6 cm



Also, radius of the base of the cone = OA = r = 4.2 cm

$\therefore$ Volume of the wooden toy = Volume of the conical part + Volume of the hemispherical part

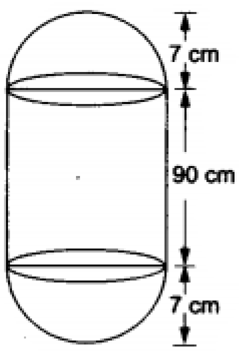
$= \left(\frac{1}{3}\pi r^2 h + \frac{2\pi}{3} r^3\right)$ cm^3

$= \frac{\pi r^2}{3}$ (h + 2r) cm^3

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times (6 + 2 \times 4.2)$ cm^3

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 14.4$ cm^2 = 266.11 cm^3

OR



Radius of each hemispherical end = 7 cm.

Height of each hemispherical part = its radius = 7 cm.

Height of the cylindrical part = (104 - 2 $\times$ 7) cm = 90 cm.

Area of surface to be polished = 2(curved surface area of the hemisphere) + (curved surface area of the cylinder)

= [2 $(2\pi r^2)$ + $2\pi rh$] sq units

$= \left[\left(4 \times \frac{22}{7} \times 7 \times 7\right) + \left(2 \times \frac{22}{7} \times 7 \times 90\right)\right]$ cm^2

= (616 + 3960) cm^2 = 4576 cm^2

= $\left(\frac{4576}{10\times10}\right)$ dm^2 = 45.76 dm^2 [$\because$ 10 cm = 1 dm].

$\therefore$ cost of polishing the surface of the solid

= ₹(45.76 $\times$ 10) = ₹ 457.60.

35. Calculation of median:

Class interval	Frequency(f_i)	Cumulative frequency
5 - 10	5	5
10- 15	6	11
15 - 20	15	26
20 - 25	10	36
25 - 30	5	41
30 - 35	4	45
35 - 40	2	47

40 - 45	2	49

Now,N = 49 ⇒ $\frac{N}{2}$ = 24.5.

Thus, the median class is 15 - 20.

∴ l = 15, h = 5, f = 15, c.f. = 11

Median, M = l + $\left\{h \times \frac{\left(\frac{N}{2}-cf\right)}{f}\right\}$

= 15 + $\left(5 \times \frac{(24.5-11)}{15}\right)$

= 15 + $\left(5 \times \frac{13.5}{15}\right)$

= 15 + 4.5 = 19.5

Hence, the median of frequency distribution is 19.5

Section E

36. **Read the text carefully and answer the questions:**

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class.

(i) Each class has 3 section

class 1 plants = 3 trees

class 2 plants = 6 trees

class 3 plants = 9 trees

∴ 3, 6, 9, ...

The no of trees planted by each class is in AP.

$S_n = \frac{n}{2}$ {2a + (n - 1)d}

$S_5 = \frac{5}{2}$ {2 × 3 + (5 - 1)3}

$S_5 = \frac{5}{2}$ {6 + 12}

$S_5 = \frac{5}{2} \times 18$

S_5 = 45

∴ class 1 to 5 students plant 45 trees.

(ii) $S_n = \frac{n}{2}$ {2a + (n - 1)d}

$S_{12} = \frac{12}{2}$ {2 × 3 + (12 - 1)3}

S_{12} = 6 {6 + 33}

S_{12} = 6 × 39

S_{12} = 234

∴ total no of trees planted by school = 234

OR

∵ Class 12^{th} has 3 sections and each section plants 12 trees.

∴ total no of trees = 12 × 3

= 36 trees.

(iii) 30

37. **Read the text carefully and answer the questions:**

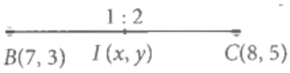
In an examination hall, students are seated at a distance of 2 m from each other, to maintain the social distance due to CORONA virus pandemic. Let three students sit at points A, B and C whose coordinates are (4, -3), (7, 3) and (8, 5) respectively.

(i) The distance between A and C

$= \sqrt{(8-4)^2 + (5+3)^2} = \sqrt{4^2 + 8^2}$

$= \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$ units

(ii) Let the coordinates of I be (x, y)



Then, by section formula,

x = $\frac{1\times8+2\times7}{1+2} = \frac{8+14}{3} = \frac{22}{3}$

and y = $\frac{1\times5+2\times3}{1+2} = \frac{5+6}{3} = \frac{11}{3}$

Thus, the coordinates of I is $\left(\frac{22}{3}, \frac{11}{3}\right)$

OR

Let B divides the line segment joining A and C in the ratio k : 1. Then, the coordinates of B will be $\left(\frac{8k+4}{k+1}, \frac{5k-3}{k+1}\right)$.

Thus, we have $\left(\frac{8k+4}{k+1}, \frac{5k-3}{k+1}\right)$ = (7, 3)

$\Rightarrow \frac{8k+4}{k+1}$ = 7 and $\frac{5k-3}{k+1}$ = 3

Consider, $\frac{8k+4}{k+1}$ = 7 ⇒ 8k + 4 = 7k + 7 ⇒ k = 3

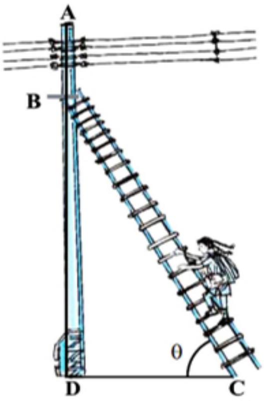
Hence, the required ratio is 3 : 1.

(iii) The mid-point of A and C

$= \left(\frac{8+4}{2}, \frac{5-3}{2}\right)$ = (6, 1)

38. **Read the text carefully and answer the questions:**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that cos θ = $\frac{\sqrt{3}}{2}$, to reach the required position.



(i) Length BD = AD - AB = 10 - 2.5 = 8.5

(ii) The length of ladder BC

In △BDC

$\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^o$

sin $30^o = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{8.5}{BC}$

$\Rightarrow BC = 2 \times 8.5$ = 17 m

OR

If the height of pole and distance BD is doubled, then the length of the ladder is

sin 30^o = $\frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{17}{BC}$

$\Rightarrow BC = 2 \times 17$ = 34 m

(iii) Distance between foot of ladder and foot of wall CD

In $\triangle$BDC

cos 30^o = $\frac{CD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{17}$

$\Rightarrow CD = 8.5\sqrt{3}$ m