11. PHOTOGRAMMETRIC SURVEYING

SCALE A VERTICAL PHOTOGRAPH

Scale $S = \frac{\text{Map distance}}{\text{Ground distance}} = \frac{f}{H-h}$

H = height of exposure station (or the air plane) above the mean sea level.

h = Height of ground above MSL

f = Focal length of camera

- If A and B are two points on ground having elevations h_a and h_b above MSL, then Average scale of line joining A and B is given by.

$$S = \frac{f}{H - \left(\frac{h_a + h_b}{2}\right)} \quad \text{where, } \frac{h_a + h_b}{2} = h_{av}.$$

- Datum scale

$$S = \frac{f}{H}$$

- Scale of a photograph

$$S_h = \frac{l}{L}$$

$$\frac{\text{Photo scale}}{\text{Map scale}} = \frac{\text{Photo distance}}{\text{Map distance}}$$

where, l = distance in Photograph

L = distance in ground

- **Computation of length of the line between points of different elevations from measurement on a vertical photograph.**

(i) If A and B be two ground point having elevations h_a and h_b above MSL and coordinates (X_a, Y_a) and (X_b, Y_b)

(ii) Let a and b be the position of corresponding points in photograph and (x_a, y_a) and (x_b, y_b) be the corresponding coordinates.

then $$\frac{x_a}{X_a} = \frac{y_a}{Y_a} = \frac{f}{H-h_a}$$

$$\frac{x_b}{X_b} = \frac{y_b}{Y_b} = \frac{f}{H-h_b}$$

where, $X_a = \frac{H-h_a}{f}.x_a$

$$X_b = \frac{H-h_b}{f}.x_b$$

$$Y_a = \frac{H-h_a}{f}.y_a$$

$$Y_b = \frac{H-h_b}{f}.y_b$$

The length between AB is given by

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$

- **Relief displacement on a Vertical Photograph**
  When the ground is not horizontal the scale of the photograph varies from point to point. The ground relief is shown in perspective on the photograph. Every point on the Photograph is therefore, displaced from true orthography position. This displacement is called relief displacement.
- **Relief displacement**

$$d = \frac{Rfh}{H(H-h)}$$

$$d = \frac{r \times h}{H} = \frac{r_o \times h}{H-h}$$

(i) The relief displacement increases as the distance from the principal point increases.

(ii) $d \alpha \frac{1}{H}$

SCALE OF A TILTED PHOTOGRAPH

$$S_h = \frac{f \sec t - mn \sin t}{H-h}$$

$$S_h = \frac{f \sec t - y^1 \sin t}{H-h}$$

where, $y' = -x \sin q + y \cancel{c} \cos q + f \tan t$

$\theta = 180 - s$

s = Swing

t = tilt

f = Focal length

H = Flying height above datum

h = high of ground above datum.

It can be seen that the tilt and relief displacements tend to cancel in the upper part of the photograph while they are cumulative to the lower part.

OVERLAP IN THE PHOTOGRAPHS

Longitudinal overlap = 55 to 65%

Lateral Overlap = 15 to 35%

for maximum rectangular area, to be covered by one photograph, the rectangle should have the dimension in the flight to be one-half the dimension normal to the direction of flight.

W = 2B W = 1.22H W= width of ground % overlap ≈ 60% in longitudinal direction.

NUMBER OF PHOTOGRAPHS TO COVER A GIVEN AREA

$$N = \frac{A}{a}$$

A = Total area to be photographed

a = net ground area covered by each photography

N = number of photographs required.

$a = L \times W$

$L = (1 - P_1)s.l.$

$W = (1 - P_w)s.l.$

$a = l.ws^2(1 - P_1)(1 - P_w)$

Where, l = length of photograph in direction of flight

W = width of photograph.

P_l = %lap in longitudinal direction

P_w = lap in longitudinal direction

S = Scale of Photograph = $\frac{H}{f}$

If instead of total area A, the rectangular dimensions $L_1 \times L_2$ (Parallel and Transverse to flight) are given then, the number of photograph required are given as follows.

Let L_1 = Dimension of area parallel to the direction flight

L_2 = Dimension of area Transverse the direction of flight

N_1 = Number of Photographs in each strip

N_2 = Number of strips required.

N = Total number of photographs to cover the whole area.

$N = N_1 \times N_2$

$$N_1 = \frac{L_1}{(1-P_l)s \times l} + 1$$

$$N_2 = \frac{L_2}{(1-P_w)s.l} + l$$

- Interval between exposures

$$T = \frac{3600 \times L}{V}$$

V = ground speed of airplane KMPH.
L = ground distance covered by each photograph in the direction of flight = $(1 - P_l)\, s.l.$.......in Km

PHOTOGRAMMETRY

(i) **Terrestrial photogrammetry:** Photographs are taken from a fixed position on or near the ground.

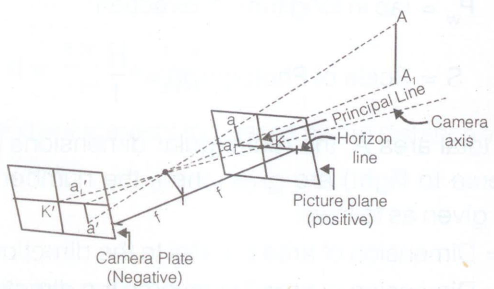
(ii) **Aerial photogrammetry:** Photographs are taken from a camera mounted in an aircraft flying over the area.

Phototheodolite: It is a combination of "theodolite and a terrestrial camera. Important parts are:

(i) Camera Box of a fixed focus type.
(ii) Hollow rectangular frame consist of two cross hair.
(iii) Photographic plate
(iv) Theodolite

- Important Definitions

(i) **Camera Axis:** Line passing through centre of camera lens perpendicular both to camera plate (Negative) and picture plane (photograph).



(ii) **Picture Plane:** Positive plane, perpendicular to camera axis.

(iii) **Principal point:** K or K′ point on intersection of camera axis with either picture plane or the camera plate.

(iv) **Focal length (f):** Perpendicular distance from centre of camera lens to either to picture plane or camera plate. It satisfy the relation.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{u+v}{u.v} \Rightarrow f = \frac{uv}{u+v}$$

(v) **Nodal point:** Nodal point is either of two points on the optical axis of a lens so located that when all object distances are measured from one point, and all image distances are measured from other. They satisfy the simple lens relation.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

(vi) **Principal plane:** It is a plane which contain principal line and optical axis.

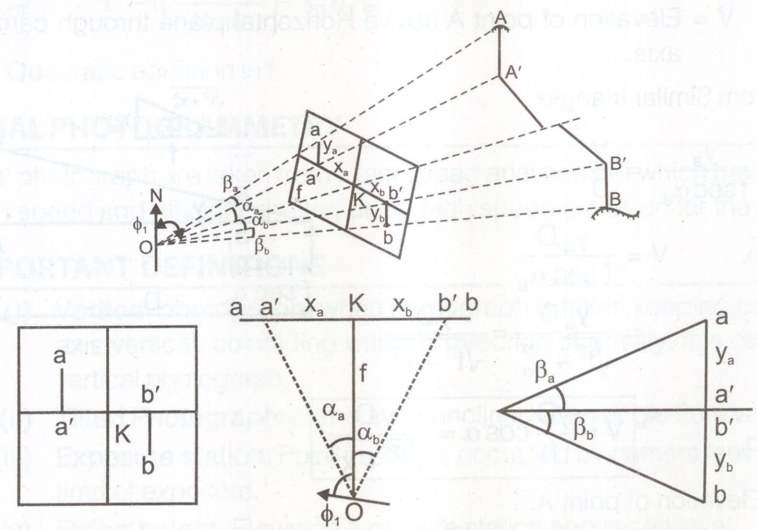
(vii) **Oblique photograph:** Photograph taken from air with axis of camera tilted from vertical are called oblique photograph, these are of two type

(a) **Low Oblique photograph:** An oblique photograph that does not show the horizon is called low oblique photograph.

(b) **High Oblique photograph:** If tilt is more upto such that horizon is shown in the photograph, it is called high oblique photograph.

(viii) **Convergent photograph:** Low oblique photographs which are taken with two cameras exposed simultaneously at successive exposure stations, with their axes tilted at a fixed inclination from vertical, so that forward exposure of first station from a stereo pair with backward exposure of next station, these photographs are called 'Convergent Photographs'.

HORIZONTAL AND VERTICAL ANGLES FROM TERRESTRIAL PHOTOGRAPH



Photograph Plan Elevation

$Oa' = f \sec \alpha_a$

$Ob' = f \sec \alpha_b$

$x_a = f \tan \alpha_a \; y_a = Oa' \tan\beta_a = f \sec \alpha_a . \tan\beta_a$

$x_b = f \tan \alpha_b \; y_b = Ob' \tan\beta_b = f \sec \alpha_b . \tan\beta_b$

$$\boxed{\tan\alpha_a = \frac{x_a}{f}, \quad \tan\alpha_b = \frac{x_b}{f}} \qquad \boxed{\tan\beta_a = \frac{y_a}{f\sec\alpha_a}, \quad \tan\beta_b = \frac{y_b}{f\sec\alpha_b}}$$

Angle ϕ_1 is magnetic bearing of camera axis (or principal vertical plane.)

Azimuth of line Ok = ϕ_1

Azimuth of line OA = $\phi_1 - \alpha_a$ (OA is left to OK)

Azimuth of line OB = $\phi_1 + \alpha_b$ (OA is right to OK)

So, $\boxed{\text{Azimuth of a line} = \text{Camera azimuth} + \alpha}$

ELEVATION OF A POINT BY PHOTOGRAPHIC MEASUREMENT

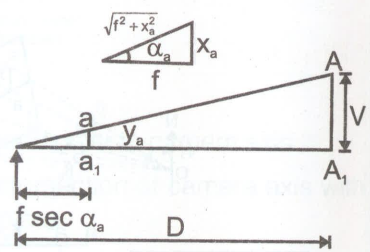
Consider Point A

$$\tan\alpha_a = \frac{x_a}{f}$$

$$\tan\beta_a = \frac{y_a}{Oa_1} = \frac{y_a}{f\sec\alpha_a} = \frac{y_a}{f}\cos\alpha_a.$$

If V = Elevation of point A above Horizontal plane through camera axis.

From Similar triangle



$$\frac{y_a}{f\sec\alpha_a} = \frac{V}{D}$$

So, $$V = \frac{y_a.D}{f\sec\alpha_a}$$

$$= \frac{y_a.D}{\sqrt{f^2+x_a^2}} = \frac{y.D}{\sqrt{f^2+x^2}}$$

So, $$\boxed{V = \frac{yD}{f}\cos\alpha = \frac{y.D}{\sqrt{f^2+x^2}}}$$

Elevation of point A.

$$h = H_C + V + C$$

Where, H_C = Elevation of camera

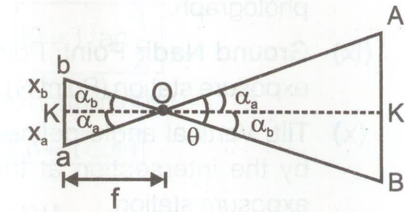
V = Elevation of point A

C = Correction for curvature and refraction.

$$\boxed{h = H_C + V + C}$$

- **Determination of focal length of the lens**

Take two points A and B. Measure angle θ very accurately from a theodolite



$\angle AOB = \theta$

$ak = x_a, \; bk = x_b$

$$\tan\alpha_a = \frac{x_a}{f} \quad \tan\alpha_b = \frac{x_b}{f}$$

$$\tan\theta = \tan(\alpha_a + \alpha_b)$$

$$= \frac{\tan\alpha_a + \tan\alpha_b}{1 - \tan\alpha_a.\tan\alpha_b}$$

$$= \frac{\frac{x_a}{f} + \frac{x_b}{f}}{1 - \frac{x_a}{f}.\frac{x_b}{f}} = \frac{(x_a + x_b)f}{f^2 - x_a.x_b}$$

$$f^2\tan\theta = x_a.x_b\tan\theta - f(x_a + x_b) = 0$$

or $$f^2 - f\left(\frac{x_a + x_b}{\tan\theta}\right) - x_a x_b = 0$$

Quadratic equation in f.

ARIAL PHOTOGRAMMETRY

Arial photograph are taken from a fast speed arial camera which have very high speed and efficient shutter, using high speed emulsion for the film.

IMPORTANT DEFINITIONS

(i) **Vertical photograph:** when photograph is taken keeping camera axis vertical, coinciding with the direction of gravity, it is called a vertical photograph.

(ii) **Tilted Photograph:** camera axis inclined at an angle from vertical.

(iii) **Exposure station:** Point in space, occupied by camera lens at the time of exposure.

(iv) **Flying height:** Elevation exposure station above sea level.

(v) **Flight line:** Line drawn on the map to represent the track of the aircraft.

(vi) **Focal length:** distance from front Nodal point of lens to plane of photograph (OK)

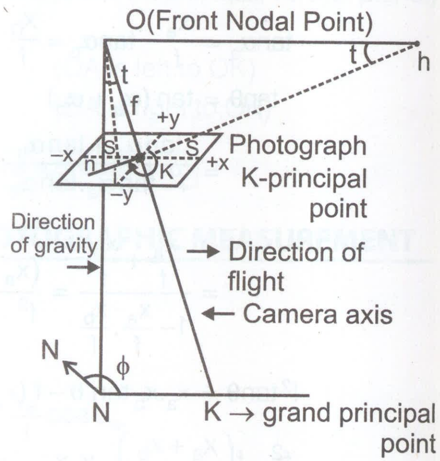
(vii) **Principal Point:** Point where perpendicular dropped from the front nodal point strikes the photograph (K).

(viii) **Nadir Point:** Nadir point is point where plumb line intersects photograph.

(ix) **Ground Nadir Point:** Point on the ground vertically beneath the exposure station (Point N).

(x) **Tilt:** Vertical angle defined by the intersection at the exposure station.

$\angle KON = t = \text{tilt}$



(xi) **Principal Plane:** Plane defined by lens (O) ground Nadir Point (N) Principal point produced on ground (K).

(xii) **Principal Line:** Intersection of principal plane with plane of photograph (line NK)

(xiii) **Isocentre:** Isocentre is the point in which the bisector of angle of tilt meets the photograph. Oi is the bisector and i is isocentre - distance $ki = f \tan^{t/2}$.

(xiv) **Swing:** Angle measured in plane of photograph from + y axis clockwise to Nadir point.

(xv) **Azimuth of principal plane:** Clockwise horizontal angle measured about the ground nadir point from the ground survey north meridian to the principal plane of the photograph.

(xvi) **Horizon point (h):** Intersection of principal line with the horizontal line through the perspective centre. Such as point h in figure is horizon point.

(xvii) **Axis of Tilt:** Axis of tilt is a line in photograph plane perpendicular to principal line at the isocentre such as $i_1, i\, i_2$ in figure. The plane of photograph is tilted about axis.

RELATION BETWEEN PRINCIPAL POINT, PLUMB POINT AND ISOCENTRE

(i) NK = distance of nadir point from principal point.

$$\boxed{\frac{NK}{KO} = \tan t} \Rightarrow \boxed{NK = KO \tan t = f \tan t}$$

(ii) Ki = distance of the isocentre from principal point.

$$\boxed{\frac{Ki}{KO} = \tan\frac{t}{2}} \Rightarrow \boxed{Ki = f \tan\frac{t}{2}}$$

(iii) Kh = distance of principal point to horizon point.

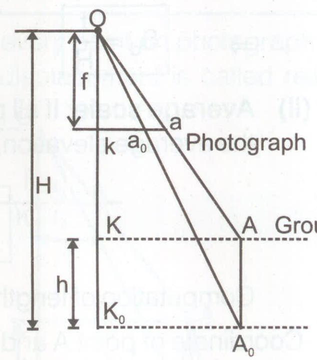
$$\boxed{Kh = KO.\cot t}\ \boxed{Kh = f \cot t}$$

SCALE OF A VERTICAL PHOTOGRAPH

Scale varies for points of different elevation, scale will be constant only when elevation of all points are same.

If all the points are on the same elevation.

Then



Scale $\boxed{S = \dfrac{\text{Map distance}}{\text{Ground distance}} = \dfrac{ka}{KA}}$

or $S = \dfrac{ka}{KA} = \dfrac{Ok}{OK} = \dfrac{f}{H-h}$

$\Rightarrow \boxed{S = \dfrac{f}{H-h}}$

Case 2: If A and B are two points having elevation h_a and h_b respectively above mean sea level.

Scale of photograph at elevation h_a.

$$\boxed{\frac{ka}{K_a.A} = \frac{ok}{OK_a} = \frac{f}{H-h_a}}$$

Scale of photograph at elevation h_b.

$$\boxed{\frac{kb}{BK_b} = \frac{ok}{OK_b} = \frac{f}{H-h_b}}$$

So in general scale of photograph for different elevations (h).

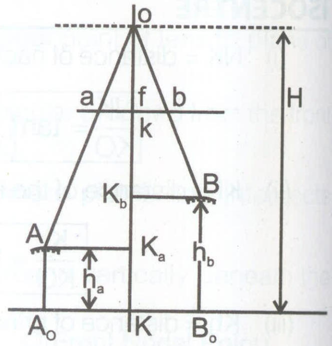
$$\boxed{S_h = \frac{f}{H-h}}$$

This can be represented by representative fraction (R_h) also.

$$R_h = \frac{1}{\left(\frac{H-h}{f}\right)}$$

$$\uparrow$$

$$R_h = \frac{1}{SH} = \frac{\frac{1}{f}}{H-h} = \frac{1}{\left(\frac{H-h}{f}\right)}$$



DIFFERENT SCALES

(i) **Datum scale:** if all points are projected at mean sea level.

Datum scale $S_d = \frac{ka}{KA_O} = \frac{ok}{OK} = \frac{f}{H}$

$$\Rightarrow \quad S_d = \frac{f}{H}$$

(ii) **Average scale:** If all points are projected on a plane representing the average elevation.

$$S_{av} = \frac{f}{H-h_{av}}$$

Computation of length of line between points of different elevation

Coordinate of point A and B on ground in plan.

$A - X_a, Y_a$

$B - X_b, Y_b$

Corresponding points on photograph

$a - x_a, y_a$

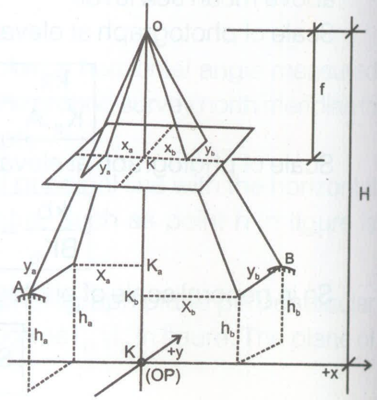
$b - x_b, y_b$

For point (A) from similar triangles.

$$\frac{OK}{OK_a} = \frac{x_a}{X_a} = \frac{y_a}{Y_a} = \frac{f}{H-h_a}$$

For Point B.

$$\frac{OK}{Ok_b} = \frac{x_b}{X_b} = \frac{y_b}{Y_b} = \frac{f}{H-h_b}$$



So,

$$X_a = \frac{H-h_a}{f}.x_a \qquad Y_a = \frac{H-h_a}{f}.y_a$$

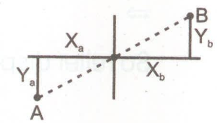
$$X_b = \frac{H-h_b}{f}.x_b \qquad Y_b = \frac{H-h_b}{f}.y_b$$

So, In general coordinate X and Y of any point.

$$X = \frac{H-h}{f}.x \qquad Y = \frac{H-h}{f}.y$$

Length between two points A and B is given by

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$



$X_a, X_b, Y_a, Y_b \rightarrow$ should be given with proper sign.

RELIEF DISPLACEMENT

Due to different elevation of different points, every point on photograph is displaced from their original position. This displacement is called relief displacement.

$r = K_a$

$r_o = K_{a_o}$

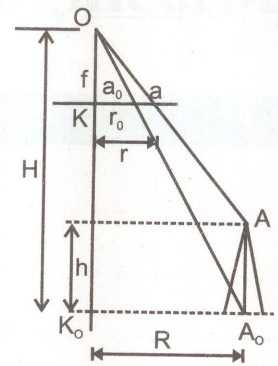
$R = K_oA_o$

aa_o is called relief displacement.

$aa_o = r - r_o$

from similar triangle.

$$\frac{r}{R} = \frac{f}{H-h} \Rightarrow r = \frac{fR}{H-h} \qquad \text{(i)}$$



$$\frac{r_o}{R} = \frac{f}{H} \Rightarrow r_o = \frac{fR}{H} \qquad \text{(ii)}$$

So relief displacement

$$d = r - r_o = \frac{fR}{H-h} - \frac{fR}{H} = \frac{fR}{H(H-h)}(H-H+h)$$

$$\Rightarrow \quad d = \frac{f.R.h}{H.(H-h)} \qquad \text{(iii)}$$

But $$R = \left(\frac{H-h}{f}\right) r$$

So, $$d = \frac{fh}{H(H-h)} \times \frac{(H-h)}{f} . r$$

So, $$\boxed{d = \frac{rh}{H}} \qquad \text{(iv)}$$

from $$\frac{\text{(i)}}{\text{(ii)}} \quad \frac{r}{r_O} = \frac{H}{H-h}$$

$$\Rightarrow \quad \boxed{\frac{r}{H} = \frac{r_O}{H-h}}$$

So relief displacement

$$\boxed{d = \frac{rh}{H} = \frac{r_O h}{H-h}} \qquad \text{(v)}$$

If relief displacement is known then height of an object

$$\boxed{h = \frac{dH}{r}}$$

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