# Ch- (R-H criteria) Stability Criteria

$r(t) \longrightarrow$ [ SYSTEM ] $\longrightarrow c(t)$

For stable system,

$$c(t)\big|_{t\to\infty} = \lim_{t\to\infty} c(t) = \text{finite} \rightarrow \text{stable}$$

$$= \infty \rightarrow \text{unstable}$$

→ For bounded i/p ⟶ output should be bounded

BIBO → stable

For an LTI system, system is said to be stable if it satisfies following conditions:-

1. i/p is bounded, o/p must be bounded
2. If i/p signal, system is zero the output must be zero irrespective of all initial conditions

- Stability are classified in 3 ways:-

1. Conditional stable system

- Here system is stable for certain range of system component

For eg:- $0 < k < 100$ where $k$ is parameter of system components

(2) Absolutely stable system

- Here the system is stable for all the values of system components marginal or limitedly stable system

- A linear time invariant system is said to be stable if for the bounded input, output maintains constant amplitude and frequency of oscillation [sustained oscillation]

- The non-repeated poles on ~~real~~ imaginary axis gives constant amplitude and f.o.o and system is marginal stable.

## * Techniques used to check stability

(1) Routh-Hurwitz criteria [RH criteria]

(2) Root locus

(3) Bode plot ⎤ → frequency response

(4) Polar plot and nyquist plot ⎦
f: 0 to ∞      f: -∞ to ∞

(5) Nicholus chart

$\boxed{\text{Stability}}$

↓

CL poles location

CE: $1 + GH(s) = 0$

1) RH criteria

↓

CE: $1 + GH(s) = 0$

2) RL
3) BP
4) PP & NP

} $GH(s) = OLTF$

Purpose:-

① To find out close loop system stability

② To find no. of CL poles lies on right, left and imaginary axis of s-plane.

③ The main purpose of RH criteria is to find no. of poles right hand side on s-plane.

④ For finding range of k value for close loop system stability.

⑤ To find k value

⑥ To become a system marginal stable or undamped system

⑦ To find relative stability. By using relative stability concept we can find system time constant and settling time.

⑧ To find close loop system stability by using RH criteria required characteristics equation i.e. $1+GH(s)=0$ and roots of C.E. gives close loop poles location.

From C.E.

The $n^{th}$ order general form of C.E.

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \dots\dots + a_n s^0 = 0$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $s^n$ | $a_0$ | $a_2$ | $a_4$ | $a_6$ | $\dots\dots$ | $a_n$ |
| $s^{n-1}$ | $a_1$ | $a_3$ | $a_5$ | $a_7$ | $\dots\dots$ | $a_{n-1}$ |
| $s^{n-2}$ | $\frac{a_1 a_2 - a_0 a_3}{a_1}$ | $\frac{a_1 a_4 - a_0 a_5}{a_1}$ | $\frac{a_1 a_6 - a_0 a_7}{a_1}$ | | | |
| $s^{n-3}$ | | | | | | |
| $\vdots$ | | | | | | |
| $s^0$ | $a_n$ | | | | | |

* Conditions for system stability

- A system is said to be stable if all the co-efficients in first column of RH table have same sign and no co-efficient is zero.
- The no. of sign changes in 1st column ≐ no. of close loops poles lies in right hand side of s-plane & system becomes unstable.

Q:- Check the system stability for following using RH criteria of characteristics eq$^n$ given:-

① $as + b = 0$

$$\begin{array}{c|c|c|l} s^1 & a & + & - \\ s^0 & b & + & - \end{array}$$

- System is stable if $a>0, b>0$ OR $a<0, b<0$

② $as^2 + bs + c = 0$

$$\begin{array}{c|cc|c|l} s^2 & a & c & + & - \\ s^1 & b & & + & - \\ s^0 & c & & + & - \end{array}$$

- System is stable if $a, b, c > 0$ OR $a, b, c < 0$

③ $as^3 + bs^2 + cs^1 + d = 0$

$$\begin{array}{c|cc|c} s^3 & a & c & + \\ s^2 & b & d & + \\ s^1 & \frac{bc-ad}{b} & & + \\ s^0 & d & & + \end{array}$$

- System is stable if $a, b, d > 0$

$$\frac{bc-ad}{b} > 0$$

$\boxed{cb > ad} \rightarrow S$ $\qquad cb < ad \longrightarrow$ US

$cb = ad \longrightarrow$ MS $\rightarrow$ then there will be f.O.O.

If it is marginal stable system. then we have to also find frequency of oscillation.

$$\boxed{cb = ad} \rightarrow MS$$

$bs^2 + ds^0 = 0$ $\quad s^2 \longrightarrow$ column see

$s^2 = -\frac{d}{b}$

$s = \pm j\sqrt{\frac{d}{b}}$

$j\omega_n = \pm j\sqrt{\frac{d}{b}}$

$$\boxed{\omega_n = \sqrt{\frac{d}{b}}} \text{ rad/sec.}$$

Short-cut

(Outer product) ad

$$as^3 + bs^2 + cs + d = 0$$

cb (Inner product)

For stable
Inner product co-efficient > Outer product co-efficient

$cb > ad \longrightarrow$ S

$cb < ad \longrightarrow$ US

$cb = ad \longrightarrow$ ms

For f.o.o,

$\rightarrow bs^2 + d = 0$

(4) $s^3 + 8s^2 + 4s + 32 = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 4 |
| $s^2$ | 8 | 32 |
| $s^1$ | 0 | |
| $s^0$ | 32 | |

MS

$8s^2 + 32 = 0$

$s^2 = -4$

$s = \pm j2$

$j\omega = \pm j2$

$$\boxed{\omega = 2 \text{ rad/sec.}}$$

(7) $s^2 + 10s + 10 = 0$

$$\begin{array}{c|cc} s^2 & 1 & 10 \\ s^1 & 10 & \\ s^0 & 10 & \end{array}$$

stable

(5) $s + 10 = 0$

$$\begin{array}{c|cc} s^1 & 1 & + \\ s^0 & 10 & + \end{array}$$

Stable

(6) $s^2 + 25 = 0$

$$\begin{array}{c|cc} s^2 & 1 & 25 \\ s^1 & 0 & \\ s^0 & 25 & \end{array}$$

ms

$w = 5$ rad/sec.

(8) $s^3 + 25s^2 + 8s + 10 = 0$

$25 \cdot 8 > 10$

$200 > 10 \rightarrow$ stable

(9) $s^4 + 2s^3 + 3s^2 + 2s + 1 = 0$

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 1 \\ s^3 & 2 & 2 & \\ s^2 & 2 & 1 & \\ s^1 & 1 & & \\ s^0 & 1 & & \end{array}$$

stable

(10) $s^4 + 2s^3 + 3s^2 + s + 2 = 0$

| | | | | |
|---|---|---|---|---|
| $s^4$ | 1 | 3 | 2 | + |
| $s^3$ | 2 | 1 | | + |
| $s^2$ | $5/2$ | 2 | | + |
| $s^1$ | $-3/5$ | | | − |
| $s^0$ | 2 | | | + |

unstable

No. of sign changes = 2 = No. of poles on R.H.S.

(11) $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$

Difficulty

| | | | | |
|---|---|---|---|---|
| $s^4$ | 1 | 2 | 8 | + |
| $s^3$ | 2 | 4 | | + |
| $s^2$ | $\cancel{0}\ \varepsilon$ | 8 | | + |
| $s^1$ | $\cancel{0}\ \frac{4\varepsilon-16}{\varepsilon}$ | | | − |
| $s^0$ | 8 | | | + |

Difficulty no. 1

- Whenever any one element ins zero in $1^{st}$ column then replace that zero by smallest +ve constant $\varepsilon$ and continue Routh's stability.
- Finally for $\varepsilon \to 0$ check no. of sign changes

$$\lim_{\varepsilon \to 0} \frac{4\varepsilon - 16}{\varepsilon} = \lim_{\varepsilon \to 0} 4 - \frac{16}{\varepsilon} = -\infty$$

No. of sign changes = 2 = no. of poles on RHS

unstable

12) $s^5+s^4+2s^3+2s^2+3s+15=0$

| | | | | |
|---|---|---|---|---|
| $s^5$ | 1 | 2 | 3 | + |
| $s^4$ | 1 | 2 | 15 | + |
| $s^3$ | $0(\varepsilon)$ | $-12$ | | + |
| $s^2$ | $\frac{2\varepsilon+12}{\varepsilon}$ | 15 | | + |
| $s^1$ | A | | | − |
| $s^0$ | 15 | | | + |

unstable

$$A = \frac{-12(\varepsilon+6)-15\varepsilon}{\varepsilon+6}$$

No. of sign changes = 2 = no. of poles lies on RHS of s-plane

$$\lim_{\varepsilon\to 0} \frac{2\varepsilon+12}{\varepsilon} = +\infty$$

$$\lim_{\varepsilon\to 0} \frac{-12(\varepsilon+6)-15\varepsilon}{\varepsilon+6} = \frac{-12(\cancel{6})}{\cancel{6}} = -12$$

13) $s^5+s^4+3s^3+3s^2+2s+2=0$

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 3 | 2 |
| $s^4$ | 1 | 3 | 2 |
| $s^3$ | $0(\varepsilon)$ | $0(\varepsilon)$ | |
| $s^2$ | | | |
| $s^1$ | | | |
| $s^0$ | | | |

$\rightarrow$ rows of zeros

Difficulty no:-2

- Whenever in the RH table row of zeros occurs we require to form an auxiliary eqn by using row above the row of zero and differentiate the A.E. and replace zeros by co-efficient by differentiated A.E. and continue Routh's stability.

| | | | | |
|---|---|---|---|---|
| + | $s^5$ | 1 | 3 | 2 |
| + | $s^4$ | 1 | 3 | 2 |
| + | $s^3$ | $\cancel{0}\ 4$ | $\cancel{0}\ 6$ | |
| + | $s^2$ | $\frac{3}{2}$ | 2 | |
| + | $s^1$ | $2/3$ | | |
| + | $s^0$ | 2 | | |

→ A.E. (auxiliary equation)

→ row of zeros

$$A.E. = s^4 + 3s^2 + 2 \Rightarrow 4^{th} \text{ order}$$

4 poles are symmetrical @ origin.

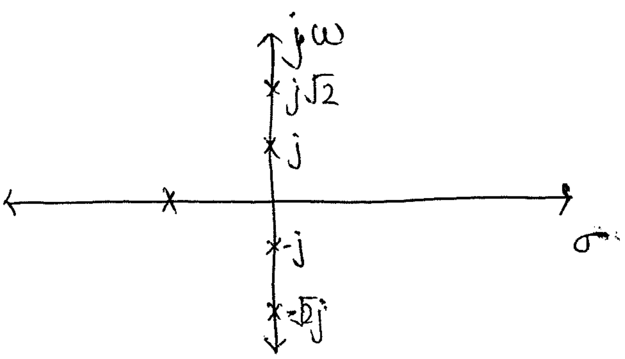
$$\frac{d\,A.E.}{ds} = 4s^3 + 6s$$

$$A.E.: s^4 + 3s^2 + 2 = 0$$

$$(s^2+2)(s^2+1) = 0$$

$$s^2 + 2 = 0 \quad ; \quad s^2 + 1 = 0$$

$$s = \pm j\sqrt{2} \quad ; \quad s = \pm j$$




MS

- In Routh's tabular form row of zero occurs means poles must be symmetrical about the origin.
- The auxiliary eq^n must consists ~~of~~ even power of s term. bcz. roots of A.E. must be symmetrical about origin.

Whenever in Routh's tabular form only one roz (rows of zeros) and all the co-efficient of 1st column are positive then system is marginal stable system because poles must lies on imaginary axis which are non-repeated.

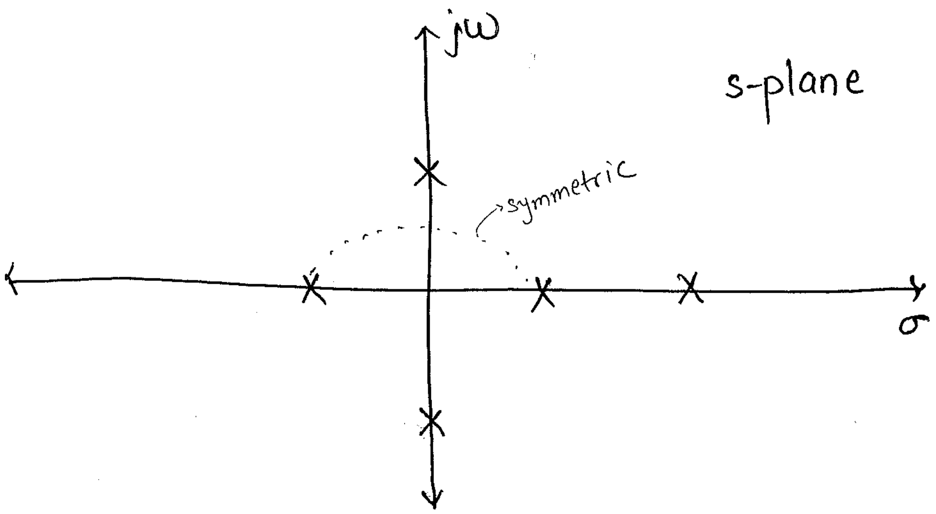
**NOTE:**

only once → 1 → ROZ → poles lies on jω axis → non-repeated → marginal stable

more than 1 ROZ → poles lies on jω axis → repeated

*** Concept**

| | | | | |
|---|---|---|---|---|
| $s^5$ | − | | | ① Non-symmetrical |
| $s^4$ | + | | | → auxiliary eqⁿ — $4^{th}$ order AE |
| $s^3$ | 0 | 0 | 0 | → ROZ — 4 poles are symmetrical @ origin |
| $s^2$ | + | | | |
| $s^1$ | + | | | |
| $s^0$ | − | | | ② Symmetrical |



## NOTE :-

- The sign change occurs below the A.E. there must be a symmetrical pole in left to the pole placed in right

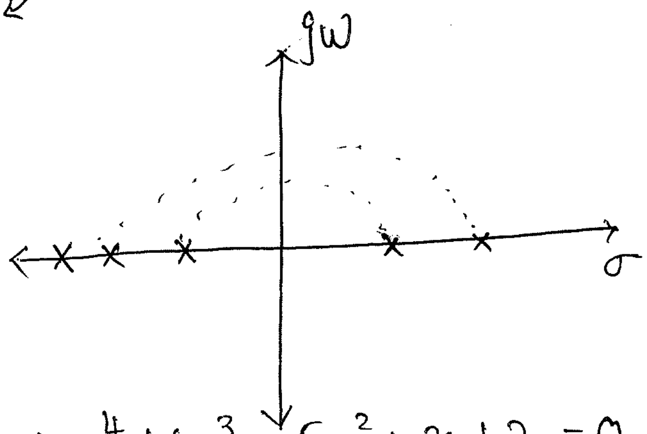
- The sign change occurs above the A.E. there must be non-symmetrical pole in ~~right~~ left ~~of~~ to the pole placed in right.

Q:-

| | | | | |
|---|---|---|---|---|
| $s^5$ | + | | | |
| $s^4$ | + | | | → A.E :- 4th order, 4 poles are sym. @ origin |
| $s^3$ | 0 | 0 | 0 | → ROZ |
| $s^2$ | + | | | |
| $s^1$ | − | | | |
| $s^0$ | + | | | |

non-symmetric

① ② symmetrical pole.




Q:- $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

| | | | | | | |
|---|---|---|---|---|---|---|
| + | $s^6$ | 1 | 4 | 5 | 2 | |
| + | $s^5$ | 3 | 6 | 3 | | |
| + | $s^4$ | 2 | 4 | 2 | | → A.E ① = $2s^4 + 4s^2 + 2$ 4th order |
| + | $s^3$ | $\cancel{0}$ 8 | $\cancel{0}$ 8 | | | → ROZ |
| + | $s^2$ | 2 | 2 | | | |
| + | $s^1$ | $\cancel{0}$ 4 | $\cancel{0}$ | | | → ROZ |
| + | $s^0$ | 2 | | | | |

$2s^4 + 4s^2 + 2$

$\frac{dAE}{ds} = 8s^3 + 8s$

② A.E. $= 2s^2 + 2$ 2nd order

$\frac{dA.E.}{ds} = 4s.$

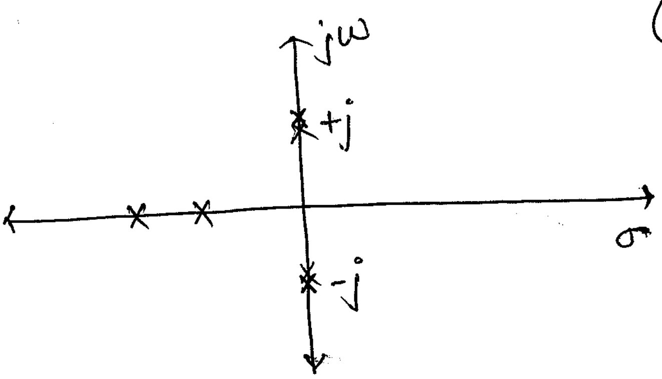
For higher order A.E.

$\Rightarrow$ $4^{th}$ order

$A.E. = 2s^4 + 4s^2 + 2 = 0$

$4^{th}$ order A.E. $\rightarrow$ 4 poles are symmetric

$\hookrightarrow$ 2 times repeated poles on imaginary axis bcz. 2 times Rows of zeros occurs




$$2s^4 + 4s^2 + 2 = 0$$
$$s^4 + 2s^2 + 1 = 0$$
$$(s^2+1)^2 = 0$$
$$s = \pm j, \; s = \pm j$$

unstable

- Whenever many times row of zero occurs & all the co-efficient in $1^{st}$ column are positive then the system is unstable bcz. poles lies on imaginary axis which are repeated.

Q:- $s^4 + s^3 - s - 1 = 0$

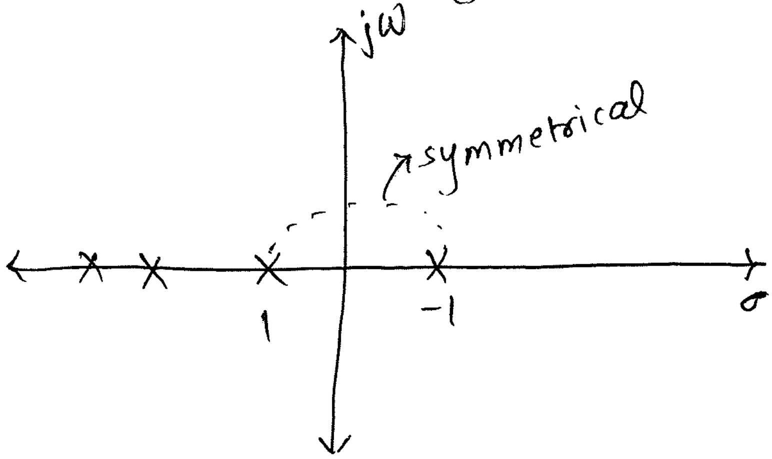
| | | | | |
|---|---|---|---|---|
| + | $s^4$ | 1 | 0 | -1 |
| + | $s^3$ | 1 | -1 | |
| + | $s^2$ | 1 | -1 | |
| + | $s^1$ | $\cancel{0}$ 2 | $\cancel{0}$ | $\rightarrow$ R.O.Z$_1$ |
| ↓ | $s^0$ | -1 | | |

$A.E._1 = s^2 - 1$

$\frac{dA.E_1}{ds} = 2s$

A.E. → $2^{nd}$ order

2 poles are symmetric @ origin

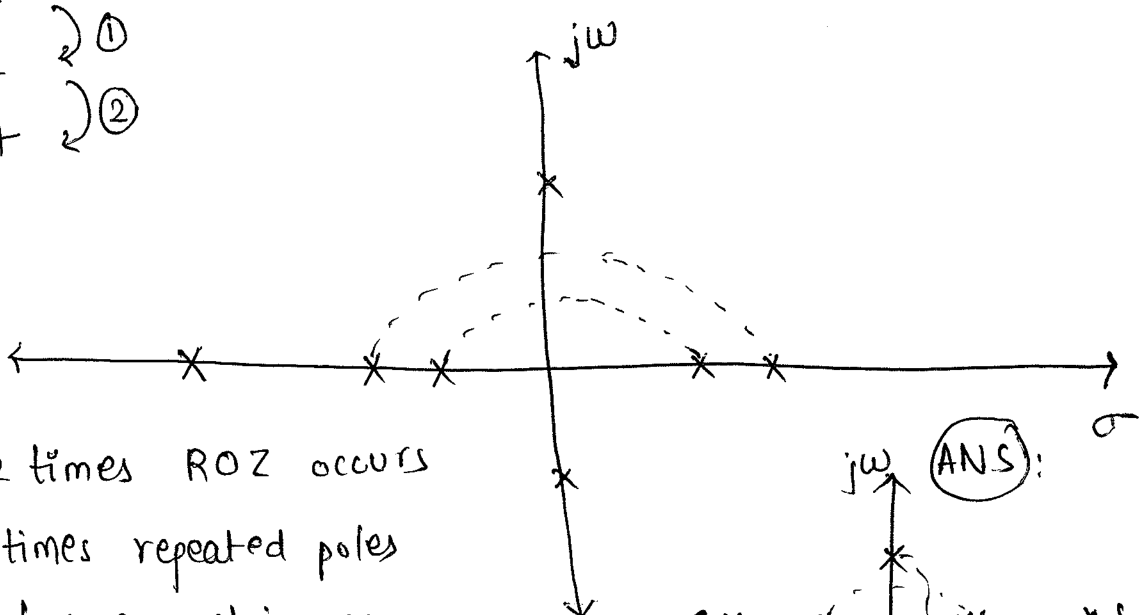



Q:- Identify the poles lies on real axis, imaginary axis, R.H. side & L.H side to the given Routh's tabular form?.

| | | | |
|---|---|---|---|
| $s^7$ | + | | |
| $s^6$ | + | | |
| $s^5$ | 0 | 0 | 0 → $ROZ_1$ |
| $s^4$ | + | | |
| $s^3$ | 0 | 0 | → $ROZ_2$ |
| $s^2$ | + | | |
| $s^1$ | − | | |
| $s^0$ | + | | |

① (between $s^2$ and $s^1$), ② (between $s^1$ and $s^0$)

$A.E._1 = s^6 \rightarrow 6^{th}$ order



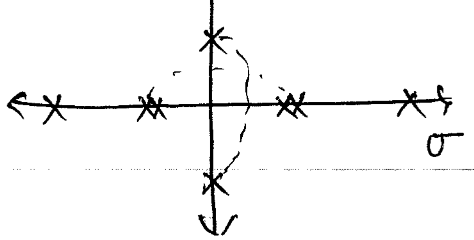

Now, 2 times ROZ occurs

So, 2 times repeated poles

has symmetric so,

on real axis poles are repeated

ANS:



RHS $\rightarrow$ 2

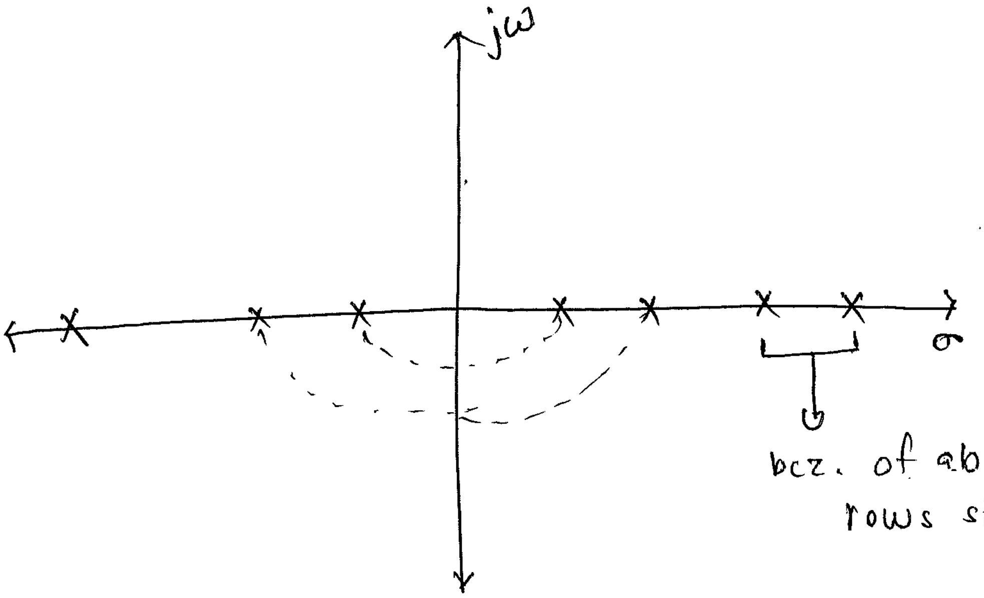
LHS $\rightarrow$ 3

$j\omega$ axis $\rightarrow$ 2

Q:-

| | | | |
|---|---|---|---|
| $s^7$ | + | | 1 |
| $s^6$ | − | | 2 |
| $s^5$ | + | | |
| $s^4$ | + | | $\rightarrow$ A.E. |
| $s^3$ | 0 | 0 | $\rightarrow$ ROZ |
| $s^2$ | + | | 1 |
| $s^1$ | − | | 2 |
| $s^0$ | + | | |

$s^4 \rightarrow 4^{th}$ order
4 poles are symmetric w.r.t. origin



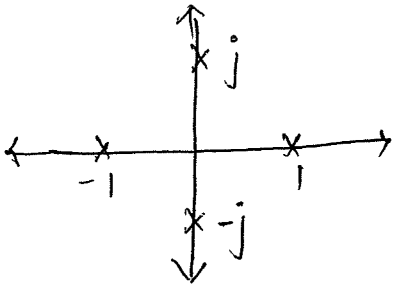

bcz. of above are. rows sign changes

R.H.S. = 4

L.H.S. = 3

$j\omega$ = 0

Q:- Identify Routh's tabular form to given poles location in s-plane.




Sol$^n$:- 4 poles are symmetrical @ origin

- 4$^{th}$ order A.E.

- 1 time sign changes below ROZ

- No sign changes above ROZ

- 1 ROZ

| | | |
|---|---|---|
| $s^4$ | + | |
| $s^3$ | 0 | 0 |
| $s^2$ | + | |
| $s^1$ | + | |
| $s^0$ | − | |

Conventional method:-

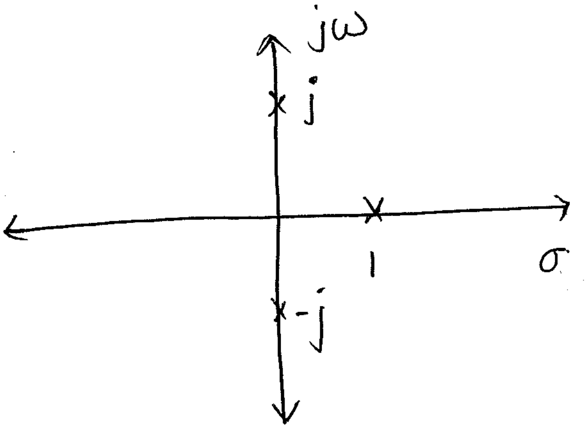
CE: $(s+1)(s-1)(s^2+1)=0$

$(s^2-1)(s^2+1)=0$

$s^4-1=0$

| | | | | |
|---|---|---|---|---|
| $+s^4$ | 1 | 0 | -1 | → A.E. |
| $+s^3$ | $\cancel{0}\,4$ | 0 | | → ROZ |
| $+s^2$ | $\cancel{0}\,\varepsilon$ | -1 | | |
| $+s^1$ | $\frac{4}{\varepsilon}$ | | | |
| $-s^0$ | -1 | | | |

Q:- Fill in the blank:




2 poles are symmetrical @origin

2nd order of AE

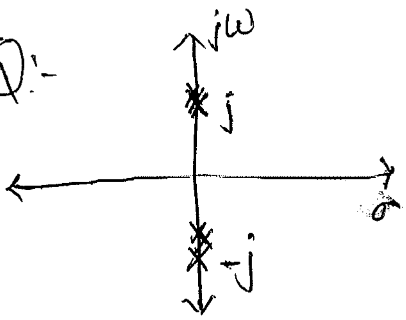
0 time sign changes below AE

1 time sign changes above ROZ

1 ROZ

| | |
|---|---|
| $s^3$ | + |
| $s^2$ | − |
| $s^1$ | 0 |
| $s^0$ | − |

Q:-




6 poles are symmetric @ origin

6th order of AE

0 time sign changes below AE

0 time sign changes above ROZ

3 ROZ

unstable

| | | | | |
|---|---|---|---|---|
| $s^6$ | + | | | |
| $s^5$ | 0 | 0 | 0 | → ROZ |
| $s^4$ | + | | | |
| $s^3$ | + | | | |
| $s^2$ | + | | | |
| $s^1$ | + | | | |
| $s^0$ | + | | | |

$(s^2+1)^3=0$

$s^6+1+3s^4+3s^2=0$

$s^6+3s^4+3s^2+1=0$

$$
\begin{array}{c|cccc}
s^6 & 1 & 3 & 3 & 1 \\
s^5 & \cancel{0}\,6 & \cancel{0}\,12 & \cancel{0}\,6 & \\
s^4 & 1 & 2 & 1 & \\
s^3 & \cancel{0}\,4 & \cancel{0}\,4 & & \\
s^2 & 1 & 1 & & \\
s^1 & \cancel{0}\,2 & & & \\
s^0 & 1 & & &
\end{array}
$$

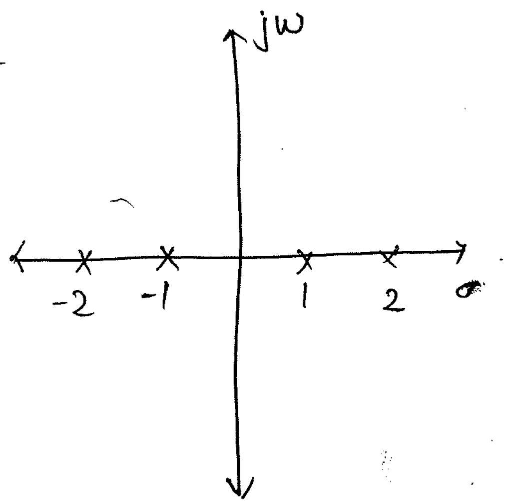
$\rightarrow ROZ_1$

$\rightarrow ROZ_2$

$\rightarrow ROZ_3$

A.E. = $6^{th}$ order
↓
6 poles are symmetrical @ origin

Q:-




4 poles are symmetric atori

4 order of A.E.

2 time sign changes below AE

0 time sign changes above RO

1 ROZ

unstable

Q:- Find the range of k value for system stability. Find the k value for marginal system or oscillation undamped system. Find natural value of oscillation when system is marginal stable.

① $s^3 + 8s^2 + 4s + K = 0$

$$
\begin{array}{cc|cc}
+ & s^3 & 1 & 4 \\
+ & s^2 & 8 & K \\
+ & s^1 & \frac{32-K}{8} & \\
+ & s^0 & K &
\end{array}
$$

For stable system,

① $K > 0$

② $\frac{32-K}{8} > 0$

$\Rightarrow K < 32$

(S) $0 < K < 32$

(US) $K < 0$ & $K > 32$

(MS) $\frac{32-K}{8} = 0$ $K_{mar.} = 32$

$8s^2 + K_{mar.} = 0$

$8s^2 = -32$

$s^2 = -4$

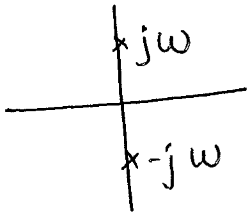
$s = \pm 2j$

$j\omega = \pm j2$

$\boxed{\omega_n = 2\ rad/sec.}$

$s = \sigma + j\omega$

$s = j\omega$




For sinusoidal, put $\sigma = 0$

$\therefore s = j\omega$

(2) $2s^3 + 5s^2 + 10s + (k+5) = 0$

| | | |
|---|---|---|
| $s^3$ | 2 | 10 |
| $s^2$ | 5 | $K+5$ |
| $s^1$ | $\frac{50-2k-10}{5}$ | |
| $s^0$ | $K+5$ | |

For stable, $\frac{50-10-2k}{5} > 0$

$40 > 2k$

$k < 20$

$k+5 > 0$

$K > -5$

$-5 < K < 20$

For marginal stable, $50 - 2k - 10 = 0$

$40 - 2k = 0$

$2k = 40$

$\boxed{K_{ma.} = 20}$

For unstable system,

$\frac{40-2k}{5} < 0$, $k+5$ be any value

$40 < 2k$

$k > 20$

$5s^2 + k + 5 = 0$

$5s^2 + 20 + 5 = 0$

$5s^2 = 25$

$s^2 = -5$

$s = \pm\sqrt{5}\,j$

$j\omega_n = \pm\sqrt{5}\,j$

$\omega_n = +\sqrt{5}\ rad/sec.$

Q:- Repeat the above problem for finding K for system stability

$$GH(s) = \frac{K}{s(s+2)(s+4)(s+6)}$$

C.E. :- $1 + GH(s) = 0$

$$1 + \frac{K}{s(s+2)(s+4)(s+6)} = 0$$

$$s(s+2)(s+4)(s+6) + K = 0$$

$$(s^2+2s)(s^2+10s+24) + K = 0$$

$$s^4+10s^3+24s^2+2s^3+20s^2+48s+K = 0$$

$$s^4+12s^3+44s^2+48s+K = 0$$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 44 | K |
| $s^3$ | 12 | 48 | |
| $s^2$ | 40 | K | |
| $s^1$ | $\frac{1920-12K}{40}$ | | |
| $s^0$ | K | | |

$K > 0$ $\qquad$ $1920 - 12K > 0$

$$12K < 1920$$

$$K < 160$$

For stable $\quad 0 < K < 160$

For ms, $K_{ma} = 160$

$$40s^2 + K = 0$$

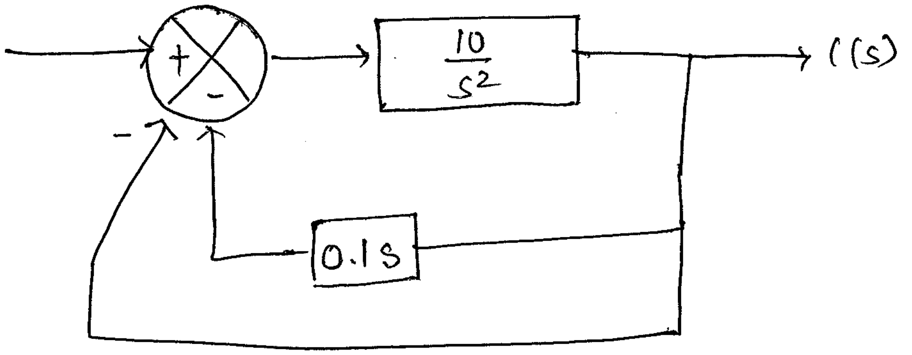
$$s^2 = -4$$

$$s = \pm 2j$$

$j\omega_n = +2j$

$\omega_n = 2$ rad/sec.

Q:- Check the system stability for given block diagram




$$\Delta = 1 - \left[ -\frac{10}{s^2} \cdot 0.1s - \frac{10}{s^2} \right] = 0$$

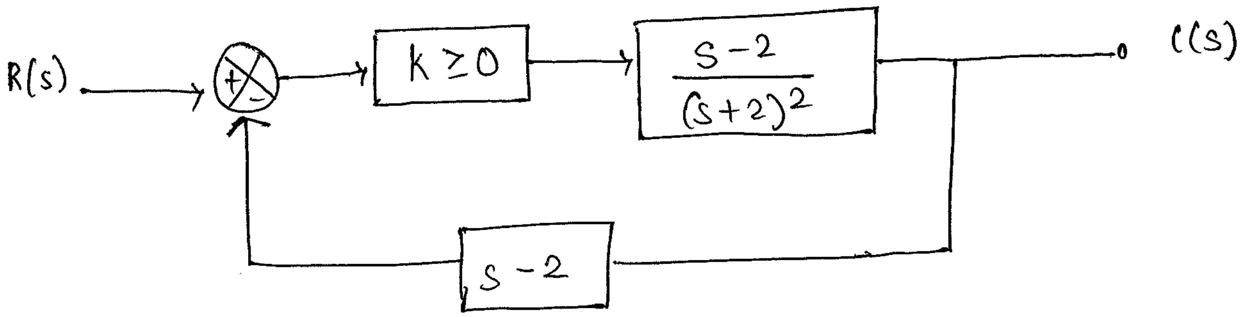
$$\Delta = 1 + \frac{1}{s} + \frac{10}{s^2} = 0 \rightarrow \text{C.F.}$$

$$s^2 + s + 10 = 0$$

| | | |
|---|---|---|
| $s^2$ | 1 | 10 |
| $s^1$ | 1 | |
| $s^0$ | 10 | |

stable

Q:- Find the range of k values to the given system for system stability.




$$\Delta = 1 + \frac{s-2 \; (s-2)K}{(s+2)^2} = 0$$

$$(s+2)^2 \, 1 + K(s^2 - 4s + 4) = 0$$

$s^2+4s+4+s^2k-4ks+4k=0$

$s^2k-4ks+(4k+1)=0$

$s^2-4s+\frac{4k+1}{k}=0$

| | | |
|---|---|---|
| $s^2$ | $1$ | $4+1/k$ |
| $s^1$ | $-4$ | |
| $s^0$ | $4+1/k$ | |

$s^2(1+k)+4(1-k)s+4(k+1)$

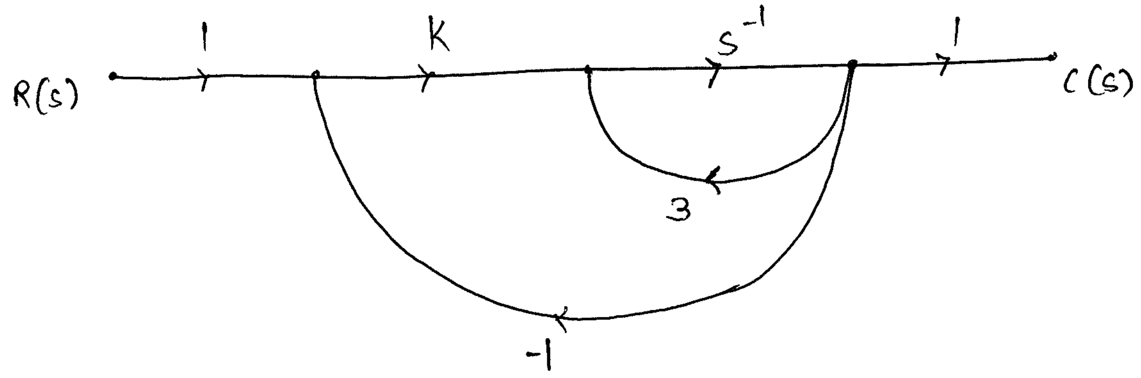
| | | |
|---|---|---|
| $+\ s^2$ | $1+k$ | $4(k+1)$ |
| $+\ s^1$ | $4(1-k)$ | |
| $+\ s^0$ | $4(k+1)$ | |

| | | |
|---|---|---|
| $1+k>0$ | $4(1-k)>0$ | $4(k+1)>$ |
| $k>-1$ × | $1>k$ | $k>-1$ × |
| | $k<1$ ✓ | |

For stable,

$\therefore \boxed{0\leq k<1}$

Q:-




$\Delta=1-3s^{-1}+ks^{-1}=0$

$1-\frac{3}{s}+\frac{k}{s}=0$

$s-3+k=0$

| | | |
|---|---|---|
| $+\ s^1$ | $1$ | |
| $+\ s^0$ | $k-3$ | |

For stable :-

$\boxed{K>3}$

Q:- Find the k and b values so that $GH(s) = \frac{k(s+1)}{s^3+bs^2+3s+1}$ oscillates with frequency 2rad/s.

CE: $1 + GH(s) = 0$

$$1 + \frac{k(s+1)}{s^3+bs^2+3s+1} = 0$$

$$s^3+bs^2+3s+1 + k(s+1) = 0$$

$$s^3+bs^2+(3+k)s+1+k = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 3+k \\ s^2 & b & 1+k \\ s^1 & \frac{3b+kb-1-k}{b} & \\ s^0 & 1+k & \end{array}$$

for f.o.o.

$$bs^2 + 1+k = 0$$

$$s^2 = -\frac{(1+k)}{b}$$

$$s = j\sqrt{\frac{k+1}{b}}$$

$$j\omega = j\sqrt{\frac{k+1}{b}}$$

$$\omega = \sqrt{\frac{k+1}{b}}$$

$$2 = \sqrt{\frac{k+1}{b}}$$

$$b4 = k+1$$

$$\boxed{b = \frac{k+1}{4}}$$

Now, for marginal stable

Inner product of CE = Outer product of CE

$$b(3+k) = 1+k$$

$$b = \frac{1+k}{3+k}$$

$$\frac{\cancel{k+1}}{4} = \frac{\cancel{1+k}}{3+k}$$

$$\boxed{k=1}, \quad \boxed{b = 1/2}$$

Q:- The loop gain of the system is

$GH(s) = \dfrac{k}{s(s+1)(s+2)}$. The value of k for which system just becomes unstable is ___.

$1 + GH(s) = 0$

$$1 + \frac{k}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2) + K = 0$$

$$(s^2+s)(s+2) + k = 0$$

$$s^3 + s^2 + 2s^2 + 2s + k = 0$$

$$s^3 + 3s^2 + 2s + k = 0$$

I.P. = O.P. (marginal)

$$\boxed{6 = k}$$

Q:- A system has $GH(s) = \dfrac{k}{s^3+8s^2+4s}$. For what value of k the system will produce continuous oscillation.

(sustained oscillation / continous / damped oscillation → m.s.)

C.E. = $1 + GH(s) = 0$

$$1 + \frac{K}{s^3+8s^2+4s} = 0$$

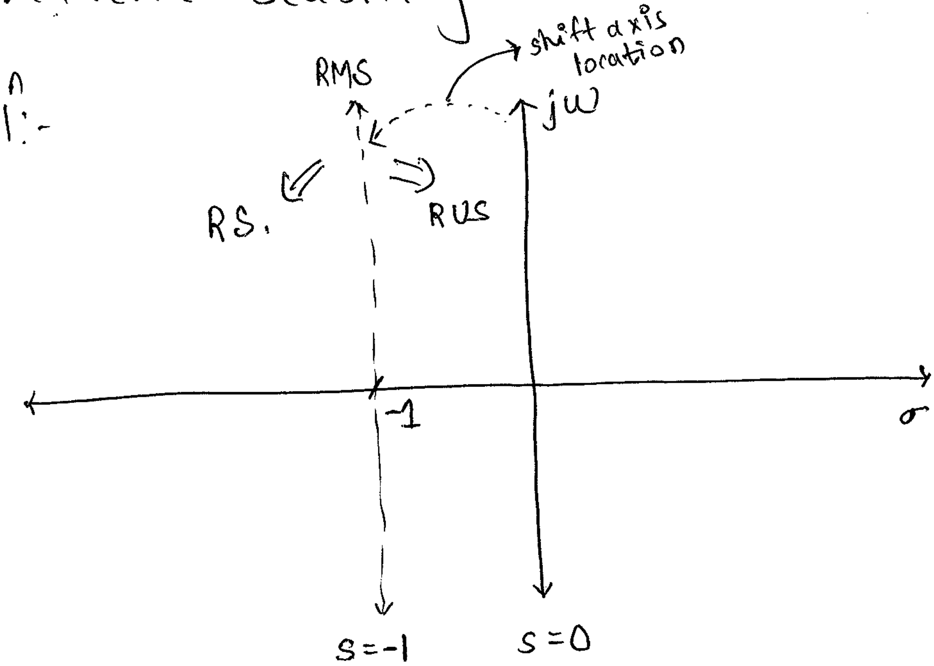
$$s^3 + 8s^2 + 4s + K = 0$$

$\boxed{k = 32}$ (I.P. = O.P.)

## *Relative stability

- It is applicable for stable system only. A system has $GH(s) = \frac{2}{s(s+1)(s+2)}$ and $H(s) = 1$ Determine it's relative stability about the line $s = -1$.

Sol^n:-




$s+1=0$

$z = 0$

$s = -1$

$s+1 = 0$

$z = s+1$

$$G(s) = \frac{2}{s(s+1)(s+2)}$$

C.E. : $1 + GH(s) = 0$

$$1 + \frac{2}{s(s+1)(s+2)} = 0$$

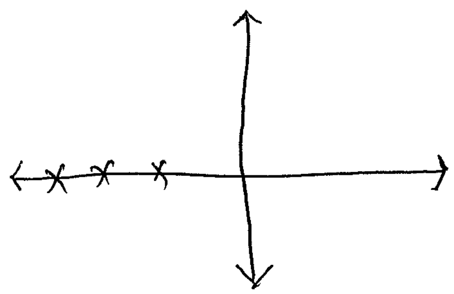
$$s^3 + 3s^2 + 2s + 2 = 0$$

| | | |
|---|---|---|
| $+ s^3$ | 1 | 2 |
| $+ s^2$ | 3 | 2 |
| $+ s^1$ | 4/3 | |
| $+ s^0$ | 2 | |

w.r.t. $s = 0$

$\Downarrow$

stable



w.r.t. $z = 0$

$GH(z) = ?$

Put $s = z - 1$

$$G(z) = \frac{2}{(z-1)\,z\,(z+1)} \quad = \text{OLTF}$$

$H(z) = 1$

CE : $1 + GH(z) = 0$

$$1 + \frac{2}{z(z-1)(z+1)} = 0$$

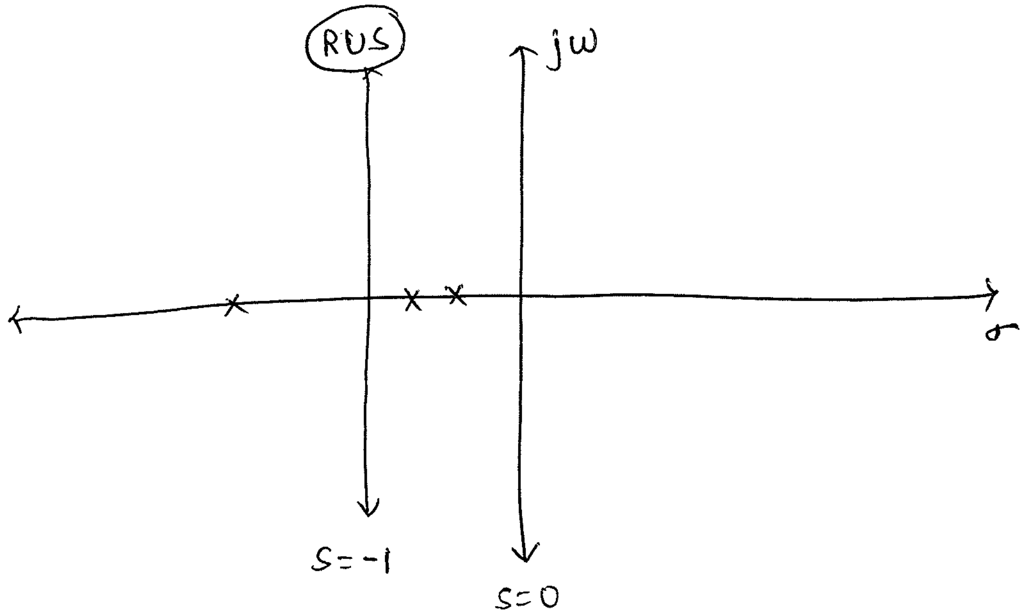
$$z(z^2-1) + 2 = 0$$

$$z^3 - z + 2 = 0$$

unstable but relatively

| | | | | |
|---|---|---|---|---|
| + | $z^3$ | $1$ | $-1$ | |
| + | $z^2$ | $\cancel{0}\ \varepsilon$ | $2$ | + ① |
| − | $z^1$ | $\frac{-\varepsilon-2}{\varepsilon}$ | | − |
| + | $z^0$ | $2$ | | + ② |

2 poles RHS $z = 0$, $s = -1$



## Limitations:-

1. The exact location of poles can't be determined
2. RH criteria is not applicable for exponential, sine or cosine term because it gives infinite series
3. RH criteria is applicable to finite no. of terms

## NOTE:-

By using RH criteria, we can get approximate sol$^n$ of the exponential term.

Q:- Find the value of K for the system stability to the given system.

$$GH(s) = \frac{k e^{-s\tau}}{s(s+1)}$$

CE : $1 + GH(s) = 0$

$$1 + \frac{k e^{-s\tau}}{s(s+1)} = 0$$

$$s(s+1) + k e^{-s\tau} = 0$$

From series expansion : $e^{-s\tau} = 1 - s\tau + \cancel{\frac{(s\tau)^2}{2!} - \frac{(s\tau)^3}{3!} + \ldots\ldots}$

Neglecting higher order terms

$$s^2 + s + k[1 - s\tau] = 0$$

$$s^2 + s(1 - k\tau) + k = 0$$

$$\begin{array}{c|cc} s^2 & 1 & k \\ s^1 & 1-k\tau & \\ s^0 & k & \end{array}$$

$$1 - k\tau > 0 \quad , \quad k > 0$$

$$1 > k\tau$$

$$k < \frac{1}{\tau}$$

For stable system : $\boxed{0 < k < 1/\tau}$