Mathematical Logic

EXERCISE 1.1 [PAGES 6 - 8]

Exercise 1.1 | Q 1.01 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$5 + 4 = 13$

Solution: It is a statement which is false, hence its truth value is 'F'.

Exercise 1.1 | Q 1.02 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$x – 3 = 14$

Solution: It is an open sentence, hence it is not a statement.

Exercise 1.1 | Q 1.03 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Close the door.

Solution: It is an imperative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.04 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Zero is a complex number.

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.05 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Please get me breakfast.

Solution: It is an imperative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.06 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Congruent triangles are similar.

Solution: It is a statement which is true, hence its truth value is ‘T’.

Exercise 1.1 | Q 1.07 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$x^2 = x$

Solution: It is an open sentence, hence it is not a statement.

Exercise 1.1 | Q 1.08 | Page 8

State which of the following is the statement. Justify. In case of a statement, state its truth value.

A quadratic equation cannot have more than two roots.

Solution: It is a statement which is true, hence its truth value is ‘T’.

Exercise 1.1 | Q 1.09 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Do you like Mathematics?

Solution: It is an interrogative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.1 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

The sunsets in the west

Solution: It is a statement which is true, hence its truth value is ‘T’.

Exercise 1.1 | Q 1.11 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

All real numbers are whole numbers.

Solution: It is a statement which is false, hence its truth value is ‘F’.

Exercise 1.1 | Q 1.12 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Can you speak in Marathi?

Solution: It is an interrogative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.13 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$x^2 - 6x - 7 = 0$, when $x = 7$

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.14 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

The sum of cube roots of unity is zero.

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.15 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

It rains heavily.

Solution: It is an open sentence, hence it is not a statement.

Exercise 1.1 | Q 2.1 | Page 7

Write the following compound statement symbolically.

Nagpur is in Maharashtra and Chennai is in Tamil Nadu.

Solution: Let p: Nagpur is in Maharashtra.

Let q: Chennai is in Tamil Nadu.

Then the symbolic form of the given statement is $p \wedge q$.

Exercise 1.1 | Q 2.2 | Page 7

Write the following compound statement symbolically.

Triangle is equilateral or isosceles.

Solution: Let p: Triangle is equilateral.

Let q: Triangle is isosceles.

Then the symbolic form of the given statement is $p \vee q$.

Exercise 1.1 | Q 2.3 | Page 7

Write the following compound statement symbolically.

The angle is right angle if and only if it is of measure 90°.

Solution: Let p: The angle is right angle.

Let q: It is of measure 90°

Then the symbolic form of the given statement is $p \leftrightarrow q$.

Exercise 1.1 | Q 2.4 | Page 7

Write the following compound statement symbolically.

Angle is neither acute nor obtuse.

Solution: Let p: Angle is acute.

Let q: Angle is obtuse.

Then the symbolic form of the given statement is $\sim p \wedge \sim q$.

Exercise 1.1 | Q 2.5 | Page 7

Write the following compound statement symbolically.

If Δ ABC is right-angled at B, then $m\angle A + m\angle C = 90°$

Solution: Let p: Δ ABC is right-angled at B.

Let q: $m\angle A + m\angle C = 90°$

Then the symbolic form of the given statement is $p \rightarrow q$.

Exercise 1.1 | Q 2.6 | Page 7

Write the following compound statement symbolically.

Hima Das wins gold medal if and only if she runs fast.

Solution: Let p: Hima Das wins gold medal

Let q: She runs fast.

Then the symbolic form of the given statement is $p \leftrightarrow q$.

Exercise 1.1 | Q 2.7 | Page 7

Write the following compound statement symbolically.

x is not irrational number but is a square of an integer.

Solution: Let p: x is not irrational number

Let q: It is a square of an integer

Then the symbolic form of the given statement is $p \wedge q$.

[**Note:** If p: x is irrational number, then the symbolic form of the given statement is $\sim p \wedge q$.]

Exercise 1.1 | Q 3.1 | Page 7

Write the truth values of the following.

4 is odd or 1 is prime.

Solution: Let p: 4 is odd.

q: 1 is prime.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of both p and q are F.

∴ The truth value of $p \vee q$ is F ....[$F \vee F \equiv F$]

Exercise 1.1 | Q 3.2 | Page 7

Write the truth values of the following.

64 is a perfect square and 46 is a prime number.

Solution: Let p: 64 is a perfect square.

q: 46 is a prime number.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively. ∴ The truth value of $p \wedge q$ is F ....[$T \wedge F \equiv F$]

Exercise 1.1 | Q 3.3 | Page 7

Write the truth values of the following.

5 is a prime number and 7 divides 94.

Solution: Let p: 5 is a prime number.

q: 7 divides 94.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively.

∴ The truth value of $p \wedge q$ is F ....[$T \wedge F \equiv F$]

Exercise 1.1 | Q 3.4 | Page 7

Write the truth values of the following.

It is not true that $5-3i$ is a real number.

Solution: Let p: $5-3i$ is a real number.

Then the symbolic form of the given statement is $\sim p$.

The truth values of p is F.

∴ The truth values of $\sim p$ is T ....[$\sim$ F ≡ T]

Exercise 1.1 | Q 3.5 | Page 7

Write the truth value of the following.

If $3 \times 5 = 8$ then $3 + 5 = 15$.

Solution: Let p: $3 \times 5 = 8$

q: $3 + 5 = 15$

Then the symbolic form of the given statement is $p \to q$.

The truth values of both p and q are F.

∴ The truth value of $p \to q$ is T ....[$F \to F \equiv T$]

Exercise 1.1 | Q 3.6 | Page 7

Write the truth value of the following.

Milk is white if and only if sky is blue.

Solution: Let p: Milk is white.

q: Sky is blue

Then the symbolic form of the given statement is $p \leftrightarrow q$.

The truth values of both p and q are T.

∴ The truth value of $p \leftrightarrow q$ is T ....[$T \leftrightarrow T \equiv T$]

Exercise 1.1 | Q 3.7 | Page 7

Write the truth values of the following.

24 is a composite number or 17 is a prime number.

Solution: Let p: 24 is a composite number.

q: 17 is a prime number.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of both p and q are T.

∴ The truth value of $p \vee q$ is T ....[$T \vee T \equiv T$]

Exercise 1.1 | Q 4.1 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$p \vee (q \wedge r)$

Solution: Truth values of p and q are T and truth values of r and s are F.

$p \vee (q \wedge r) \equiv T \vee (T \wedge F)$

$\equiv T \vee F \equiv T$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.2 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$(p \to q) \vee (r \to s)$

Solution: Truth values of p and q are T and truth values of r and s are F.

$(p \to q) \vee (r \to s) \equiv (T \to T) \vee (F \to F)$

$\equiv T \vee T \equiv T$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.3 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$(q \wedge r) \vee (\sim p \wedge s)$

Solution: Truth values of p and q are T and truth values of r and s are F.

$(q \wedge r) \vee (\sim p \wedge s) \equiv (T \wedge F) \vee (\sim T \wedge F)$
$\equiv F \vee (F \wedge F)$
$\equiv F \vee F \equiv F$
Hence the truth value of the given statement is false.

Exercise 1.1 | Q 4.4 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$(p \to q) \wedge \sim r$

Solution: Truth values of p and q are T and truth values of r and s are F.

$(p \to q) \land (\sim r) \equiv (T \to T) \land (\sim F)$
$\equiv T \land T \equiv T$
Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.5 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$(\sim r \leftrightarrow p) \to \sim q$

Solution: Truth values of p and q are T and truth values of r and s are F.

$(\sim r \leftrightarrow p) \to (\sim q) \equiv (\sim F \leftrightarrow T) \to (\sim T)$
$\equiv (T \leftrightarrow T) \to F$
$\equiv T \to F \equiv F$
Hence the truth value of the given statement is false.

Exercise 1.1 | Q 4.6 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$[\sim p \land (\sim q \land r)] \lor [(q \land r) \lor (p \land r)]$

Solution: Truth values of p and q are T and truth values of r and s are F.

$[\sim p \land (\sim q \land r)] \lor [(q \land r) \lor (p \land r)]$
$\equiv [\sim T \land (\sim T \land F)] \lor [(T \land F) \lor (T \land F)]$
$\equiv [F \land (F \land F)] \lor [F \lor F]$
$\equiv (F \land F) \lor F$
$\equiv F \lor F \equiv F$
Hence the truth value of the given statement is false.

Exercise 1.1 | Q 4.7 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$[(\sim p \land q) \land \sim r] \lor [(q \to p) \to (\sim s \lor r)]$

Solution: Truth values of p and q are T and truth values of r and s are F.

$[(\sim p \land q) \land (\sim r)] \lor [(q \to p) \to (\sim s \lor r)]$
$\equiv [(\sim T \land T) \land (\sim F)] \lor [(T \to T) \to (\sim F \lor F)]$
$\equiv [(F \land T) \land T] \lor [T \to (T \lor F)]$
$\equiv (F \land T) \lor (T \to T)$
$\equiv F \lor T \equiv T$
Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.8 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$\sim [(\sim p \wedge r) \vee (s \rightarrow \sim q)] \leftrightarrow (p \wedge r)$

Solution: Truth values of p and q are T and truth values of r and s are F.

$\sim [(\sim p \wedge r) \vee (s \rightarrow \sim q)] \leftrightarrow (p \wedge r)$
$\equiv \sim [(\sim T \wedge F) \vee (F \rightarrow \sim T)] \leftrightarrow (T \wedge F)$
$\equiv \sim [(F \wedge F) \vee (F \rightarrow F)] \leftrightarrow F$
$\equiv \sim (F \vee T) \leftrightarrow F$
$\equiv \sim T \leftrightarrow F$
$\equiv F \leftrightarrow F \equiv T$
Hence the truth value of the given statement is true.

Exercise 1.1 | Q 5.1 | Page 7

Write the negation of the following.

Tirupati is in Andhra Pradesh.

Solution: The negation of the given statement is:

Tirupati is not in Andhra Pradesh.

Exercise 1.1 | Q 5.2 | Page 7

Write the negation of the following.

3 is not a root of the equation $x^2 + 3x - 18 = 0$

Solution: The negation of the given statement is:

3 is a root of the equation $x^2 + 3x - 18 = 0$

Exercise 1.1 | Q 5.3 | Page 7

Write the negation of the following.

$\sqrt{2}$ is a rational number.

Solution: The negation of the given statement is:

$\sqrt{2}$ is not a rational number.

Exercise 1.1 | Q 5.4 | Page 7

Write the negation of the following.

Polygon ABCDE is a pentagon.

Solution: The negation of the given statement is:

Polygon ABCDE is not a pentagon.

Exercise 1.1 | Q 5.5 | Page 7

Write the negation of the following.

7 + 3 > 5

Solution: 7 + 3 ≯5

EXERCISE 1.2 [PAGE 13]

Exercise 1.2 | Q 1.01 | Page 13

Construct the truth table of the following statement pattern.

$[(p \to q) \land q] \to p$

Solution: Here are two statements and three connectives.

∴ There are 2 × 2 = 4 rows and 2 + 3 = 5 columns in the truth table.

p	q	$p \to q$	$(p \to q) \land q$	$[(p \to q) \land q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Exercise 1.2 | Q 1.02 | Page 13

Construct the truth table of the following statement pattern.

$(p \land \sim q) \leftrightarrow (p \to q)$

Solution:

p	q	$\sim q$	$p \land \sim q$	$p \to q$	$(p \land \sim q) \leftrightarrow (p \to q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

Exercise 1.2 | Q 1.03 | Page 13

Construct the truth table of the following statement pattern.

(p ∧ q) ↔ (q ∨ r)

Solution:

p	q	r	p ∧ q	q ∨ r	(p ∧ q) ↔ (q ∨ r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	F	T
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	F	T

Exercise 1.2 | Q 1.04 | Page 13

Construct the truth table of the following statement pattern.

p → [~ (q ∧ r)]

Solution:

p	q	r	q ∧ r	~ (q)∧ r)	p → [~ (q ∧ r)]
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

Exercise 1.2 | Q 1.05 | Page 13

Construct the truth table of the following statement pattern.

$\sim p \wedge [(p \vee \sim q) \wedge q]$

Solution:

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge q$	$\sim p \wedge [p \vee \sim q] \wedge q$
T	T	F	F	T	T	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	T	F	F

Exercise 1.2 | Q 1.06 | Page 13

Construct the truth table of the following statement pattern.

$(\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p)$

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$(\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

Exercise 1.2 | Q 1.07 | Page 13

Construct the truth table of the following statement pattern.

$(q \rightarrow p) \vee (\sim p \leftrightarrow q)$

Solution:

p	q	$\sim p$	$q \rightarrow p$	$\sim p \leftrightarrow q$	$(q \rightarrow p) \vee (\sim p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

Exercise 1.2 | Q 1.08 | Page 13

Construct the truth table of the following statement pattern.

$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \land q) \rightarrow r]$

Solution:

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \land q$	$(p \land q) \rightarrow r$	$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \land q) \rightarrow r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Exercise 1.2 | Q 1.09 | Page 13

Construct the truth table of the following statement pattern.

$p \rightarrow [\sim (q \land r)]$

Solution:

p	q	r	$q \land r$	$\sim (q) \land r)$	$p \rightarrow [\sim (q \land r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

Exercise 1.2 | Q 1.1 | Page 13

Construct the truth table of the following statement pattern.

$(p \vee \sim q) \rightarrow (r \wedge p)$

Solution:

p	q	r	$\sim q$	$p \vee \sim q$	$r \wedge p$	$(p \vee \sim q) \rightarrow (r \wedge p)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

Exercise 1.2 | Q 2.01 | Page 13

Using truth table, prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

Solution:

1	2	3	4	5	6
p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \wedge \sim p$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	F	F	F

The entries in columns 4 and 6 are identical

$\therefore \sim p \wedge q \equiv (p \vee q) \wedge \sim p$

Exercise 1.2 | Q 2.02 | Page 13

Using the truth table prove the following logical equivalence.

$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Solution:

1	2	3	4	5	6	7
p	q	$\sim p$	$p \vee q$	$\sim (p \vee q)$	$\sim p \wedge q$	$\sim (p \vee q) \vee (\sim p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

The entries in columns 3 and 7 are identical.

$\therefore \sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Exercise 1.2 | Q 2.03 | Page 13

Using the truth table prove the following logical equivalence.

$p \leftrightarrow q \equiv \sim [(p \vee q) \wedge \sim (p \wedge q)]$

Solution:

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$	$\sim [(p \vee q) \wedge \sim (p \wedge q)]$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

The entries in columns 3 and 8 are identical.

$\therefore p \leftrightarrow q \equiv \sim [(p \vee q) \wedge \sim (p \wedge q)]$

Exercise 1.2 | Q 2.04 | Page 13

Using the truth table prove the following logical equivalence.

$p \rightarrow (q \rightarrow p) \equiv \sim p \rightarrow (p \rightarrow q)$

Solution:

1	2	3	4	5	6	7
p	q	q → p	p → (q → p)	~ p	p → q	~ p → (p → q)
T	T	T	T	F	T	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

The entries in columns 4 and 7 are identical.
∴ p → (q → p) ≡ ~ p → (p → q)

Exercise 1.2 | Q 2.05 | Page 13

Using the truth table prove the following logical equivalence.

(p ∨ q) → r ≡ (p → r) ∧ (q → r)

Solution:

1	2	3	4	5	6	7	8
p	q	r	p ∨ q	(p ∨ q) → r	p → r	q → r	(p → r) ∧ (q → r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 are identical.
∴ (p ∨ q) → r ≡ (p → r) ∧ (q → r)

Exercise 1.2 | Q 2.06 | Page 13

Using the truth table prove the following logical equivalence.

p → (q ∧ r) ≡ (p → q) ∧ (p → r)

Solution:

1	2	3	4	5	6	7	8
p	q	r	q ∧ r	p → (q ∧ r)	p → q	p → r	(p → q) ∧ (p → r)
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 are identical.

∴ p → (q ∧ r) ≡ (p → q) ∧ (p → r)

Exercise 1.2 | Q 2.07 | Page 13

Using the truth table prove the following logical equivalence.

p → (q ∧ r) ≡ (p ∧ q) (p → r)

Solution:

1	2	3	4	5	6	7	8
p	q	r	q ∧ r	p → (q ∧ r)	(p ∧ q)	(p → r)	(p ∧ q) (p → r)
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 are identical.

∴ p → (q ∧ r) ≡ (p ∧ q) (p → r)

Exercise 1.2 | Q 2.08 | Page 13

Using the truth table prove the following logical equivalence.

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Solution:

1	2	3	4	5	6	7	8
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

The entries in columns 5 and 8 are identical.
$\therefore p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Exercise 1.2 | Q 2.09 | Page 13

Using the truth table prove the following logical equivalence.

$[\sim (p \vee q) \vee (p \vee q)] \wedge r \equiv r$

Solution:

1	2	3	4	5	6	7
p	q	r	$p \vee r$	$\sim (p \vee q)$	$\sim (p \vee q) \vee (p \vee q)$	$[\sim (p \vee q) \vee (p \vee q)] \wedge r$
T	T	T	T	F	T	T
T	T	F	T	F	T	F
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F

F	F	T	F	T	T	T
F	F	F	F	T	T	F

The entries in columns 3 and 7 are identical.

∴ $[\sim (p \vee q) \vee (p \vee q)] \wedge r \equiv r$

Exercise 1.2 | Q 2.1 | Page 13

Using the truth table proves the following logical equivalence.

$\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Solution:

1	2	3	4	5	6	7	8	9
p	q	~ p	~ q	p ↔ q	~ (p ↔ q)	p ∧ ~ q	q ∧ ~ p	(p ∧ ~ q) ∨ (q ∧ ~ p)
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

The entries in columns 6 and 9 are identical.
∴ $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Exercise 1.2 | Q 3.01 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$(p \wedge q) \rightarrow (q \vee p)$

Solution:

p	q	p ∧ q	q ∨ p	(p ∧ q) → (q ∨ p)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

All the entries in the last column of the above truth table are T.
∴ $(p \wedge q) \rightarrow (q \vee p)$ is a tautology.

Exercise 1.2 | Q 3.02 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$(p \to q) \leftrightarrow (\sim p \vee q)$

Solution:

p	q	~ p	p → q	~ p ∨ q	(p → q) ↔ (~ p ∨ q)
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

All the entries in the last column of the above truth table are T.
∴ $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology.

Exercise 1.2 | Q 3.03 | Page 13

Discuss the statement pattern, using truth table : $\sim(\sim p \wedge \sim q) \vee q$

Solution: Consider the statement pattern: $\sim (\sim p \wedge \sim q) \vee q$

Thus the truth table of the given logical statement: $\sim(\sim p \wedge \sim q) \vee q$

p	q	~p	~q	~p∧~q	~(~p ∧ ~q)	~(~p ∧ ~q) ∨ q
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

The above statement is **contingency**.

Exercise 1.2 | Q 3.04 | Page 13

Examine whether the following logical statement pattern is a tautology, contradiction, or contingency.

$[(p \to q) \wedge q] \to p$

Solution: Consider the statement pattern : $[(p \to q) \wedge q] \to p$

No. of rows = 2n = 2 × 2 = 4

No. of column = m + n = 3 + 2 = 5

Thus the truth table of the given logical statement :

$[(p \rightarrow q) \land q] \rightarrow p$

p	**q**	**$p \rightarrow q$**	**$(p \rightarrow q) \land q$**	**$[(p \rightarrow q) \land q] \rightarrow p$**
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

From the above truth table we can say that given logical statement: $[(p \rightarrow q) \land q] \rightarrow p$ is contingency.

Exercise 1.2 | Q 3.05 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$[(p \rightarrow q) \land \sim q] \rightarrow \sim p$

Solution:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \land \sim q$	$[(p \rightarrow q) \land \sim q] \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore [(p \rightarrow q) \land \sim q] \rightarrow \sim p$ is a tautology.

Exercise 1.2 | Q 3.06 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$(p \leftrightarrow q) \land (p \rightarrow \sim q)$

Solution:

p	q	~ q	p ↔ q	p → ~ q	(p ↔ q) ∧ (p → ~ q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T

The entries in the last column of the above truth table are neither all T nor all F.
∴ (p ↔ q) ∧ (p → ~ q) is a contingency.

Exercise 1.2 | Q 3.07 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

~ (~ q ∧ p) ∧ q

Solution:

p	q	~ q	~ q ∧ p	~ (~ q ∧ p)	~ (~ q ∧ p) ∧ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	F

The entries in the last column of the above truth table are neither all T nor all F.

∴ ~ (~ q ∧ p) ∧ q is a contingency.

Exercise 1.2 | Q 3.08 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

(p ∧ ~ q) ↔ (p → q)

Solution:

p	q	~ q	p ∧ ~ q	p → q	(p ∧ ~ q) ↔ (p → q)
T	T	F	F	T	F
T	F	T	T	F	F

F	T	F	F	T	F
F	F	T	F	T	F

All the entries in the last column of the above truth table are F.
∴ (p ∧ ~ q) ↔ (p → q) is a contradiction.
[Note: Answer in the textbook is incorrect]

Exercise 1.2 | Q 3.09 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

(~ p → q) ∧ (p ∧ r)

Solution:

p	q	r	~ p	~ p → q	p ∧ r	(~ p → q) ∧ (p ∧ r)
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

The entries in the last column of the above truth table are neither all T nor all F.
∴ (~ p → q) ∧ (p ∧ r) is a contingency.

Exercise 1.2 | Q 3.1 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

[p → (~ q ∨ r)] ↔ ~ [p → (q → r)]

Solution:

p	q	r	~ q	~ q ∨ r	p → (~ q ∨ r)	q → r	p → (q → r)	~[p → (q → r)]	[p → (~ q ∨ r)] ↔ [p → (q → r)]
T	T	T	F	T	T	T	T	F	F
T	T	F	F	F	F	F	F	T	F

T	F	T	T	T	T	T	T	F	F
T	F	F	T	T	T	T	T	F	F
F	T	T	F	T	T	T	T	F	F
F	T	F	F	F	T	F	T	F	F
F	F	T	T	T	T	T	T	F	F
F	F	F	T	T	T	T	T	F	F

All the entries in the last column of the above truth table are F.
$\therefore$ $[p \rightarrow (\sim q \vee r)] \leftrightarrow \sim [p \rightarrow (q \rightarrow r)]$ is a contradiction.

EXERCISE 1.3 [PAGES 17 - 18]

Exercise 1.3 | Q 1.1 | Page 17

If A = {3, 5, 7, 9, 11, 12}, determine the truth value of the following.

$\exists$ x $\in$ A such that x – 8 = 1

Solution: Clearly x = 9 $\in$ A satisfies x − 8 = 1. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 1.2 | Page 17

If A = {3, 5, 7, 9, 11, 12}, determine the truth value of the following.

$\forall$ x $\in$ A, x^2 + x is an even number

Solution: For each x $\in$ A, x^2 + x is an even number. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 1.3 | Page 17

If A = {3, 5, 7, 9, 11, 12}, determine the truth value of the following.

$\exists$ x $\in$ A such that $x^2 < 0$

Solution: There is no x $\in$ A which satisfies $x^2 < 0$. So the given statement is false, hence its truth value is F.

Exercise 1.3 | Q 1.4 | Page 17

If A = {3, 5, 7, 9, 11, 12}, determine the truth value of the following.

$\forall$ x $\in$ A, x is an even number

Solution: x = 3 $\in$ A, x = 5 $\in$ A, x = 7 $\in$ A, x = 9 $\in$ A, x = 11 $\in$ A do not satisfy x is an even number. So the given statement is false, hence its truth value is F.

Exercise 1.3 | Q 1.5 | Page 17

If A = {3, 5, 7, 9, 11, 12}, determine the truth value of the following.

$\exists\ x \in A$ such that $3x + 8 > 40$

Solution: Clearly $x = 11 \in A$ and $x = 12 \in A$ satisfies $3x + 8 > 40$. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 1.6 | Page 17

If A = {3, 5, 7, 9, 11, 12}, determine the truth value of the following.

$\forall\ x \in A,\ 2x + 9 > 14$

Solution: For each $x \in A$, $2x + 9 > 14$. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 2.01 | Page 17

Write the dual of the following.

$p \vee (q \wedge r)$

Solution: The dual of the given statement pattern is:

$p \wedge (q \vee r)$

Exercise 1.3 | Q 2.02 | Page 17

Write the dual of the following.

$p \wedge (q \wedge r)$

Solution: The dual of the given statement pattern is:

$p \vee (q \vee r)$

Exercise 1.3 | Q 2.03 | Page 17

Write the dual of the following.

$(p \vee q) \wedge (r \vee s)$

Solution: The dual of the given statement pattern is:

$(p \wedge q) \vee (r \wedge s)$

Exercise 1.3 | Q 2.04 | Page 17

Write the dual of the following.

$p \wedge \sim q$

Solution: The dual of the given statement pattern is:

$p \vee \sim q$

Exercise 1.3 | Q 2.05 | Page 17

Write the dual of the following.

$(\sim p \vee q) \wedge (\sim r \wedge s)$

Solution: The dual of the given statement pattern is:

$(\sim p \wedge q) \vee (\sim r \vee s)$

Exercise 1.3 | Q 2.06 | Page 17

Write the dual of the following.

$\sim p \wedge (\sim q \wedge (p \vee q) \wedge \sim r)$

Solution: The dual of the given statement pattern is:

$\sim p \vee (\sim q \vee (p \wedge q) \vee \sim r)$

Exercise 1.3 | Q 2.07 | Page 17

Write the dual of the following.

$[\sim (p \vee q)] \wedge [p \vee \sim (q \wedge \sim s)]$

Solution: The dual of the given statement pattern is:

$[\sim (p \wedge q)] \vee [p \wedge \sim (q \vee \sim s)]$

Exercise 1.3 | Q 2.08 | Page 17

Write the dual of the following.

$c \vee \{p \wedge (q \vee r)\}$

Solution: The dual of the given statement pattern is:

$t \wedge \{p \vee (q \wedge r)\}$

Exercise 1.3 | Q 2.09 | Page 17

Write the dual of the following.

$\sim p \vee (q \wedge r) \wedge t$

Solution: The dual of the given statement pattern is:

$\sim p \wedge (q \vee r) \vee c$

Exercise 1.3 | Q 2.1 | Page 17

Write the dual of the following.

(p ∨ q) ∨ c

Solution: The dual of the given statement pattern is:

(p ∧ q) ∧ t

Exercise 1.3 | Q 3.1 | Page 18

Write the negation of the following.

x + 8 > 11 or y – 3 = 6

Solution: Let p: x + 8 > 11,

q: y − 3 = 6

Then the symbolic form of the given statement is p∨q.

Since ~ (p v q) ≡ ~ p ∧ ~ q, the negation of the given statement is:

x + 8 ≯ 11 and y − 3 ≠ 6.

OR

x + 8 ≤ 11 and y − 3 ≠ 6.

Exercise 1.3 | Q 3.2 | Page 18

Write the negation of the following.

11 < 15 and 25 > 20

Solution: Let p: 11 < 15,

q: 25 > 20

Then the symbolic form of the given statement is p∧q.

Since ~ (p ∧ q) ≡ ~ p ∨ q, the negation of the given statement is:

11 ≥ 15 or 25 ≤ 20

Exercise 1.3 | Q 3.3 | Page 18

Write the negation of the following.

Quadrilateral is a square if and only if it is a rhombus.

Solution: Let p: Quadrilateral is a square.

q: It is a rhombus.

Then the symbolic form of the given statement is p↔q.

Since $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$, the negation of the given statement is:
Quadrilateral is a square but it is not a rhombus or quadrilateral is a rhombus but it is not a square.

Exercise 1.3 | Q 3.4 | Page 18

Write the negation of the following.

It is cold and raining.

Solution: Let p: It is cold.

q: It is raining.

Then the symbolic form of the given statement is $p \wedge q$.

Since $\sim (p \wedge q) \equiv \sim p \vee q$, the negation of the given statement is:

It is not cold or not raining.

Exercise 1.3 | Q 3.5 | Page 18

Write the negation of the following.

If it is raining then we will go and play football.

Solution: Let p: It is raining.

q: We will go.

r: We play football.

Then the symbolic form of the given statement is $p \rightarrow (q \wedge r)$.

Since $\sim [p \rightarrow (q \wedge r)] \equiv p \wedge \sim (q \wedge r) \equiv p \wedge (\sim q \vee \sim r)$, the negation of the given statement is:

It is raining and we will not go or not play football.

Exercise 1.3 | Q 3.6 | Page 18

Write the negation of the following.

$\sqrt{2}$ is a rational number.

Solution: The negation of the given statement is:

$\sqrt{2}$ is not a rational number.

Exercise 1.3 | Q 3.7 | Page 18

Write the negation of the following.

All-natural numbers are whole numbers.

Solution: The negation of the given statement is:

Some natural numbers are not whole numbers.

Exercise 1.3 | Q 3.8 | Page 18

Write the negation of the following.

$\forall$ n $\in$ N, $n^2 + n + 2$ is divisible by 4.

Solution: The negation of the given statement is:

$\exists$ n $\in$ N, such that $n^2 + n + 2$ is not divisible by 4.

Exercise 1.3 | Q 3.9 | Page 18

Write the negation of the following.

$\exists$ x $\in$ N such that $x – 17 < 20$

Solution: The negation of the given statement is:

$\forall$ x $\in$ N, $x - 17 \geq 20$

Exercise 1.3 | Q 4.1 | Page 18

Write converse, inverse and contrapositive of the following statement.

If $x < y$ then $x^2 < y^2$ $(x, y \in R)$

Solution: Let p: $x < y$,

q: $x^2 < y^2$

Then the symbolic form of the given statement is $p \to q$.

Converse: $q \to p$ is the converse of $p \to q$.

i.e. If $x^2 < y^2$, then $x < y$.

Inverse: $\sim p \to \sim q$ is the inverse of $p \to q$.

i.e. If $x \geq y$, then $x^2 \geq y^2$.

OR

If $x \nless y$, then $x^2 \nless y^2$.

Contrapositive: $\sim q \to p$ is the contrapositive of $p \to q$

i.e. If $x^2 \geq y^2$, then $x \geq y$.

OR

If $x^2 \nless y^2$, then $x \nless y$.

Exercise 1.3 | Q 4.2 | Page 18

Write converse, inverse and contrapositive of the following statement.

A family becomes literate if the woman in it is literate.

Solution: Let p: The woman in the family is literate.

q: A family becomes literate.

Then the symbolic form of the given statement is $p \to q$.

Converse: $q \to p$ is the converse of $p \to q$.

i.e. If a family becomes literate, then the woman in it is literate.

Inverse: $\sim p \to \sim q$ is the inverse of $p \to q$.

i.e. If the woman in the family is not literate, then the family does not become literate.

Contrapositive: $\sim q \to \sim p$ is the contrapositive of $p \to q$.

i.e. If a family does not become literate, then the woman in it is not literate.

Exercise 1.3 | Q 4.3 | Page 18

Write converse, inverse and contrapositive of the following statement.

If surface area decreases then pressure increases.

Solution: Let p: The surface area decreases.

q: The pressure increases.

Then the symbolic form of the given statement is $p \to q$.

Converse: $q \to p$ is the converse of $p \to q$.

i.e. If the pressure increases, then the surface area decreases.

Inverse: $\sim p \to \sim q$ is the inverse of $p \to q$.

i.e. If the surface area does not decrease, then the pressure does not increase.

Contrapositive: $\sim q \to \sim p$ is the contrapositive of $p \to q$.

i.e. If the pressure does not increase, then the surface area does not decrease.

Exercise 1.3 | Q 4.4 | Page 18

Write converse, inverse and contrapositive of the following statement.

If voltage increases then current decreases.

Solution: Let p: Voltage increases.

q: Current decreases.

Then the symbolic form of the given statement is $p \to q$.

Converse: $q \to p$ is the converse of $p \to q$.

i.e. If current decreases, then voltage increases. **Inverse:** $\sim p \to \sim q$ is the inverse of $p \to q$.

i.e. If voltage does not increase, then-current does not decrease.

Contrapositive: $\sim q \to p$, is the contrapositive of $p \to q$.

i.e. If current does not decrease, then voltage does not increase.

EXERCISE 1.4 [PAGE 21]

Exercise 1.4 | Q 1.1 | Page 21

Using the rule of negation write the negation of the following with justification.

$\sim q \to p$

Solution: The negation of is $\sim q \to p$ is

$\sim (\sim q \to p) \equiv \sim q \wedge \sim p$ ...(Negation of implication)

Exercise 1.4 | Q 1.2 | Page 21

Using the rule of negation write the negation of the following with justification.

$p \wedge \sim q$

Solution: The negation of $p \wedge \sim q$ is

$\sim (p \wedge \sim q) \equiv \sim p \vee \sim (\sim q)$ ....(Negation of conjunction)

$\equiv \sim p \vee q$ ....(Negation of negation)

Exercise 1.4 | Q 1.3 | Page 21

Using the rule of negation write the negation of the following with justification.

$p \vee \sim q$

Solution: The negation of $p \vee \sim q$ is

$\sim (p \vee \sim q) \equiv \sim p \wedge \sim (\sim q)$ ....(Negation of disjunction)

$\equiv \sim p \wedge q$ ....(Negation of negation)

Exercise 1.4 | Q 1.4 | Page 21

Using the rule of negation write the negation of the following with justification.

$(p \vee \sim q) \wedge r$

Solution: The negation of $(p \lor \sim q) \land r$ is

$\sim [(p \lor \sim q) \land r] \equiv (p \lor \sim q) \lor \sim r$ ....(Negation of conjunction)

$\equiv [\sim p \land \sim (\sim q)] \lor \sim r$ ......(Negation of disjunction) $\equiv (\sim p \land q) \lor \sim r$ ....(Negation of negation)

Exercise 1.4 | Q 1.5 | Page 21

Using the rule of negation write the negation of the following with justification.

$p \to (p \lor \sim q)$

Solution: The negation of $p \to (p \lor \sim q)$ is

$\sim [p \to (p \lor \sim q)] \equiv p \land \sim (p \lor \sim q)$ ....(Negation of implication)

$\equiv p \land [\sim p \land \sim (\sim q)]$ .....(Negation of disjunction)

$\equiv p \land (\sim p \land q)$ ....(Negation of negation)

Exercise 1.4 | Q 1.6 | Page 21

Using the rule of negation write the negation of the following with justification.

$\sim (p \land q) \lor (p \lor \sim q)$

Solution: The negation of $\sim (p \land q) \lor (p \lor \sim q)$ is

$\sim [\sim (p \land q) \lor (p \lor \sim q)] \equiv \sim [\sim (p \land q)] \land \sim (p \lor \sim q)$ ....(Negation of disjunction)

$\equiv \sim [\sim (p \land q)] \land [\sim p \land \sim (\sim q)]$ ....(Negation of disjunction)

$\equiv (p \land q) \land (\sim p \land q)$ ....(Negation of negation)

Exercise 1.4 | Q 1.7 | Page 21

Using the rule of negation write the negation of the following with justification.

$(p \lor \sim q) \to (p \land \sim q)$

Solution: The negation of $(p \lor \sim q) \to (p \land \sim q)$ is

$\sim [(p \lor \sim q) \to (p \land \sim q)] \equiv (p \lor \sim q) \land \sim (p \land \sim q)$ ....(Negation of implication)

$\equiv (p \lor \sim q) \land [\sim p \lor \sim (\sim q)]$ ....(Negation of conjunction)

$\equiv (p \lor \sim q) \land (\sim p \lor q)$ ....(Negation of negation)

Exercise 1.4 | Q 1.8 | Page 21

Using the rule of negation write the negation of the following with justification.

$(\sim p \lor \sim q) \lor (p \land \sim q)$

Solution: The negation of $(\sim p \vee \sim q) \vee (p \wedge \sim q)$ is

$\sim [(\sim p \vee \sim q) \vee (p \wedge \sim q)] \equiv \sim (\sim p \vee \sim q) \wedge \sim (p \wedge \sim q)$ ....(Negation of disjunction)

$\equiv [\sim (\sim p) \wedge \sim (\sim q)] \wedge [\sim p \vee \sim (\sim q)]$ ...(Negation of disjunction and conjunction)

$\equiv (p \wedge q) \wedge (\sim p \vee q)$ ....(Negation of negation)

Exercise 1.4 | Q 2.1 | Page 21

Rewrite the following statement without using if ...... then.

If a man is a judge then he is honest.

Solution: Since $p \rightarrow q \equiv \sim p \vee q$, the given statement can be written as:

A man is not a judge or he is honest.

Exercise 1.4 | Q 2.2 | Page 21

Rewrite the following statement without using if ...... then.

It 2 is a rational number then $\sqrt{2}$ is irrational number.

Solution:

Since $p \rightarrow q \equiv \sim p \vee q$, the given statement can be written as:

2 is not a rational number or $\sqrt{2}$ is irrational number.

Exercise 1.4 | Q 2.3 | Page 21

Rewrite the following statement without using if ...... then.

It $f(2) = 0$ then $f(x)$ is divisible by $(x - 2)$.

Solution: Since $p \rightarrow q \equiv \sim p \vee q$, the given statement can be written as:

$f(2) \neq 0$ or $f(x)$ is divisible by $(x - 2)$.

Exercise 1.4 | Q 3.1 | Page 21

Without using the truth table show that $P \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Solution: L.H.S $= p \leftrightarrow q$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ ....(Biconditional Law)

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ ....(Conditional Law)

$\equiv [\sim p \wedge (\sim q \vee p)] \vee [q \wedge (\sim q \vee p)]$ ....(Distributive Law)

$\equiv [(\sim p \wedge \sim q)] \vee (\sim p \wedge p)] \vee [(q \wedge \sim q) \vee (q \wedge p)]$ ....(Distributive Law)

$\equiv [(\sim p \wedge \sim q) \vee F] \vee [F \vee (q \wedge p)]$ ....(Complement Law)

$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$ ....(Identity Law)

$\equiv (\sim p \wedge \sim q) \vee (p \wedge q)$ ....(Commutative Law)

$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ ....(Commutative Law)

$\equiv$ R.H.S.

Exercise 1.4 | Q 3.2 | Page 21

Without using truth table prove that:

$(p \vee q) \wedge (p \vee \sim q) \equiv p$

Solution: L.H.S. = $(p \vee q) \wedge (p \vee \sim q)$

$\equiv p \vee (q \wedge \sim q)$ ....(Distributive Law)

$\equiv p \vee F$ ....(Complement Law)

$\equiv p$ ....(Identity Law)

= R.H.S.

Exercise 1.4 | Q 3.3 | Page 21

Without using truth table prove that:

$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

Solution: L.H.S. = $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$

$\equiv [(p \vee \sim p) \wedge q] \vee (p \wedge \sim q)$ ....(Distributive Law)

$\equiv (T \wedge q) \vee (p \wedge \sim q)$ ....(Complement Law)

$\equiv q \vee (p \wedge \sim q)$ ....(Identity Law)

$\equiv (q \vee p) \wedge (q \vee \sim q)$ ....(Distributive Law)

$\equiv (q \vee p) \wedge T$ ....(Complement Law)

$\equiv q \vee p$ ....(Identity Law)

$\equiv p \vee q$ ......(Commutative Law)

= R.H.S.

Exercise 1.4 | Q 3.4 | Page 21

Without using truth table prove that:

$\sim [(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge (\sim p \vee q)$

Solution: L.H.S. = $\sim [(p \vee \sim q) \rightarrow (p \wedge \sim q)]$

$\equiv (p \vee \sim q) \rightarrow (p \wedge \sim q)$ ....(Negation of implication)

$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim (\sim q)]$ ......(Negation of conjunction)

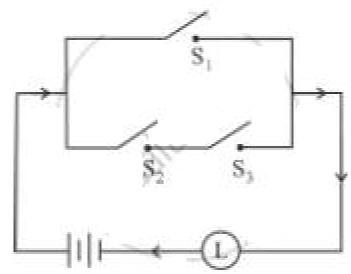
$\equiv (p \vee \sim q) \wedge (\sim p \vee q)$ ....(Negation of negation)

= R.H.S.

EXERCISE 1.5 [PAGES 29 - 30]

Exercise 1.5 | Q 1.1 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim$ p: the switch S_1' is closed or the switch S_1 is open

$\sim$ q: the switch S_2' is closed or the switch S_2 is open

$\sim$ r: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

$p \vee (q \wedge r) \equiv l$

l is generally dropped and it can be expressed as:

$p \vee (q \wedge r)$

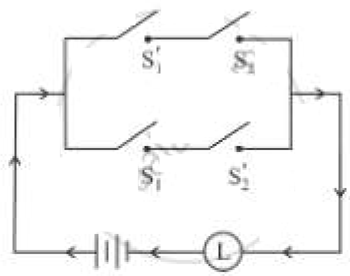
Input-Output Table

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
1	1		1	1
1	1		0	1

1	0		0	1
1	0		0	1
0	1		1	1
0	1		0	0
0	0		0	0
0	0		0	0

Exercise 1.5 | Q 1.2 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

~ p: the switch S_1' is closed or the switch S_1 is open

~ q: the switch S_2' is closed or the switch S_2 is open

~ r: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

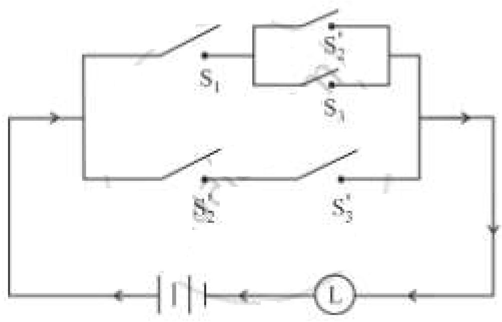
$(\sim p \wedge q) \vee (p \wedge \sim q)$

Input-Output Table

p	q	~ p	~ q	~ p ∧ q	p ∧ ~ q	(~ p ∧ q) ∨ (p ∧ ~ q)
1	1	0	0	0	0	0
1	0	0	1	0	1	1
0	1	1	0	1	0	1
0	0	1	1	0	0	0

Exercise 1.5 | Q 1.3 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

~ p: the switch S_1' is closed or the switch S_1 is open

~ q: the switch S_2' is closed or the switch S_2 is open

~ r: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

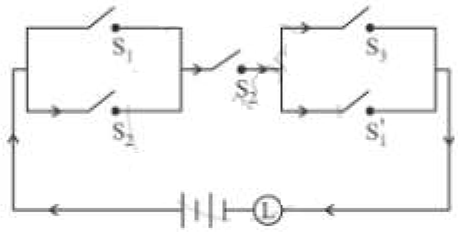
$[p \wedge (\sim q \vee r)] \vee (\sim q \wedge \sim r)$

Input-Output Table

p	q	r	~q	~r	~q∨r	p∧(~q∨r)	~q∧~r	[p∧(~q∨r)] ∨ (~q∧~r)
1	1				1	1	0	1
1	1				0	0	0	0
1	0				1	1	0	1
1	0				1	1	1	1
0	1				1	0	0	0
0	1				0	0	0	0
0	0				1	0	0	0
0	0				1	0	1	1

Exercise 1.5 | Q 1.4 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

~ p: the switch S_1' is closed or the switch S_1 is open

~ q: the switch S_2' is closed or the switch S_2 is open

~ r: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

$(p \vee q) \wedge q \wedge (r \vee \sim p)$

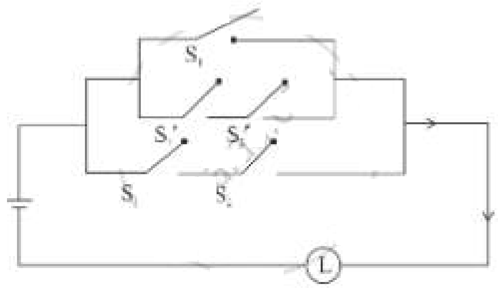
Input-Output Table

p	q	r	~p	p∨q	r∨~p	(p∨q)∧q∧(r∨~p)
1	1		0	1	1	1
1	1		0	1	0	0
1	0		0	1	1	0
1	0		0	1	0	0
0	1		1	1	1	1
0	1		1	1	1	1
0	0		1	0	1	0
0	0		1	0	1	0

[**Note:** Answer in the textbook is incorrect.]

Exercise 1.5 | Q 1.5 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim$ p: the switch S_1' is closed or the switch S_1 is open

$\sim$ q: the switch S_2' is closed or the switch S_2 is open

$\sim$ r: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

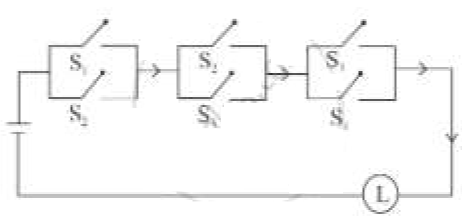
$[p \vee (\sim p \wedge \sim q)] \vee (p \wedge q)$

Input-Output Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$p \wedge q$	$[p \vee (\sim p \wedge \sim q)] \vee (p \wedge q)$
1	1	0	0	0	1	1	1
1	0	0	1	0	1	0	1
0	1	1	0	0	0	0	0
0	0	1	1	1	1	0	1

Exercise 1.5 | Q 1.6 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

~ p: the switch S_1' is closed or the switch S_1 is open

~ q: the switch S_2' is closed or the switch S_2 is open

~ r: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

$(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$

Input-Output Table

p	q	r	p∨q	q∨r	r∨p	(p∨q)∧(q∨r)∧(r∨p)
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	1	1	1	1
1	0	0	1	0	1	0
0	1	1	1	1	1	1
0	1	0	1	1	0	0
0	0	1	0	1	1	0
0	0	0	0	0	0	0

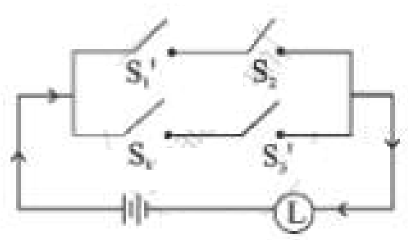
Exercise 1.5 | Q 2.1 | Page 30

Construct the switching circuit of the following:

$(\sim p \wedge q) \vee (p \wedge \sim r)$

Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
~ p: the switch S_1' is closed or the switch S_1 is open
~ q: the switch S_2' is closed or the switch S_2 is open
~ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



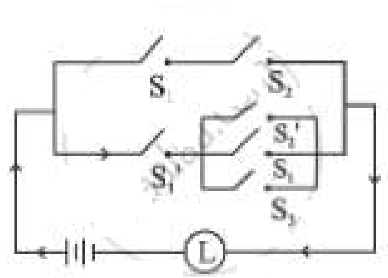
Exercise 1.5 | Q 2.2 | Page 30

Construct the switching circuit of the following:

$(p \wedge q) \vee [\sim p \wedge (\sim q \vee p \vee r)]$

Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
$\sim$ p: the switch S_1' is closed or the switch S_1 is open
$\sim$ q: the switch S_2' is closed or the switch S_2 is open
$\sim$ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



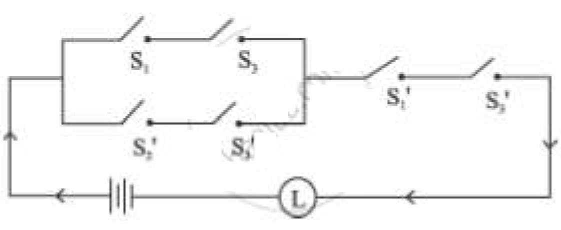
Exercise 1.5 | Q 2.3 | Page 30

Construct the switching circuit of the following:

$(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)$

Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
$\sim$ p: the switch S_1' is closed or the switch S_1 is open
$\sim$ q: the switch S_2' is closed or the switch S_2 is open
$\sim$ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.4 | Page 30

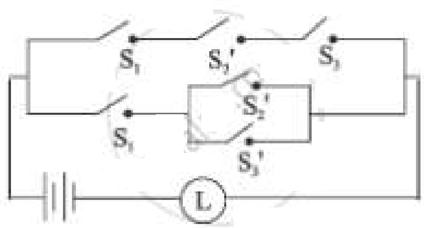
Construct the switching circuit of the following:

$(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee \sim r)]$

Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed

r: the switch S_3 is closed
~ p: the switch S_1' is closed or the switch S_1 is open
~ q: the switch S_2' is closed or the switch S_2 is open
~ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



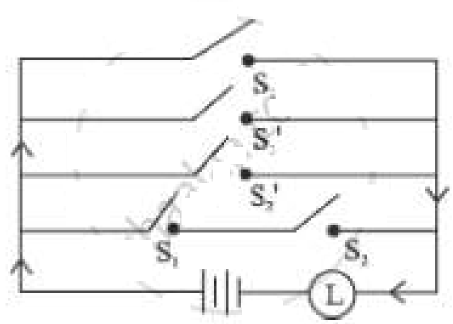
Exercise 1.5 | Q 2.5 | Page 30

Construct the switching circuit of the following:

p ∨ (~ p) ∨ (~ q) ∨ (p ∧ q)

Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
~ p: the switch S_1' is closed or the switch S_1 is open
~ q: the switch S_2' is closed or the switch S_2 is open
~ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.6 | Page 30

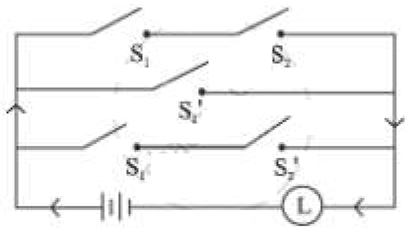
Construct the switching circuit of the following:

(p ∧ q) ∨ (~ p) ∨ (p ∧ ~ q)

Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
~ p: the switch S_1' is closed or the switch S_1 is open

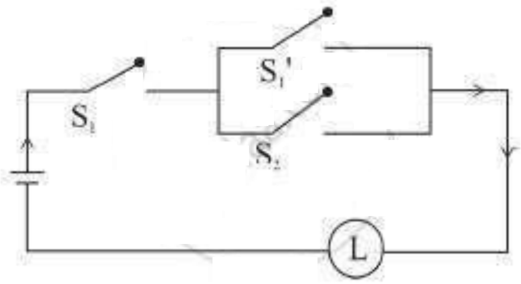
$\sim$ q: the switch S_2' is closed or the switch S_2 is open
$\sim$ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 3.1 | Page 30

Give an alternative equivalent simple circuit for the following circuit:



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim$ p: the switch S_1' is closed or the switch S_1 is open

Then the symbolic form of the given circuit is

$p \wedge (\sim p \vee q)$.

Using the laws of logic, we have,

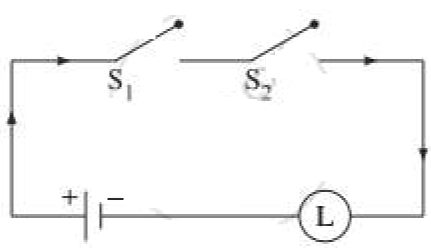
$p \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim p) \vee (p \wedge q)$ ....(By Distributive Law)

$\equiv F \vee (p \wedge q)$ ....(By Complement Law)

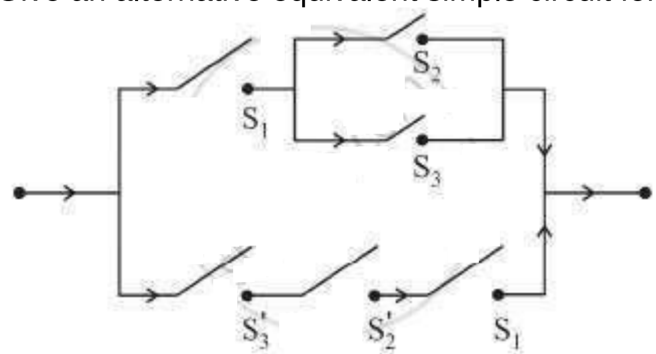
$\equiv p \wedge q$ ....(By Identity Law)

Hence, the alternative equivalent simple circuit is:



Exercise 1.5 | Q 3.2 | Page 30

Give an alternative equivalent simple circuit for the following circuit:



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

~ q: the switch S_2' is closed or the switch S_2 is open

~ r: the switch S_3' is closed or the switch S_3 is open.

Then the symbolic form of the given circuit is:

$[p \wedge (q \vee r)] \vee (\sim r \wedge \sim q \wedge p)$

Using the laws of logic, we have

$[p \wedge (q \vee r)] \vee (\sim r \wedge \sim q \wedge p)$

$\equiv [p \wedge (q \vee r)] \vee [\sim (r \vee q) \wedge p]$ ....(By De Morgan's Law)

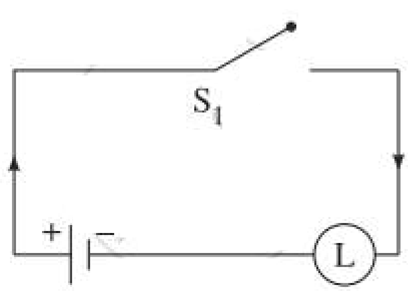
$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim (q \vee r)]$ ......(By Commutative Law)

$\equiv p \wedge [(q \vee r) \vee \sim (q \vee r)]$ ....(By Distributive Law)

$\equiv p \wedge T$ ....(By Complement Law)

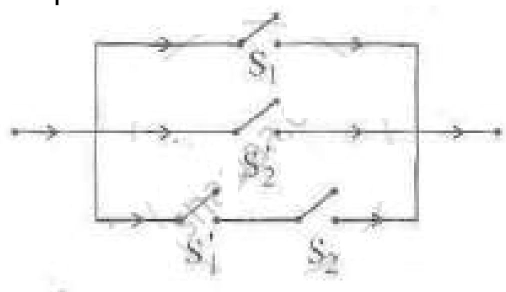
$\equiv p$ ....(By Identity Law)

Hence, the alternative equivalent simple circuit is



Exercise 1.5 | Q 4.1 | Page 30

find the symbolic fom of the following switching circuit, construct its switching table and interpret it.



Solution: Let

p: The switch S_1 is closed,
q: The switch S_2 is closed.

Switching circuit is (pv~q)v(~p∧q)

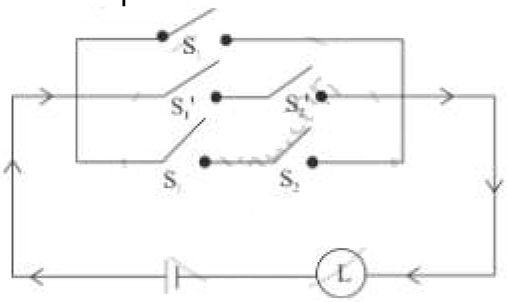
The switching table

p	q	~p	~q	pv~q	~p∧ q	(pv~q)v(~p∧q)
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	1	0	1

From the last column of switching table we conclude that the current will always flow through the circuit.

Exercise 1.5 | Q 4.2 | Page 30

Write the symbolic form of the following switching circuit construct its switching table and interpret it.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

~ p: the switch S_1‘ is closed or the switch S_1 is open.

~ q: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given circuit is:

p ∨ (~ p ∧ ~ q) ∨ (p ∧ q)

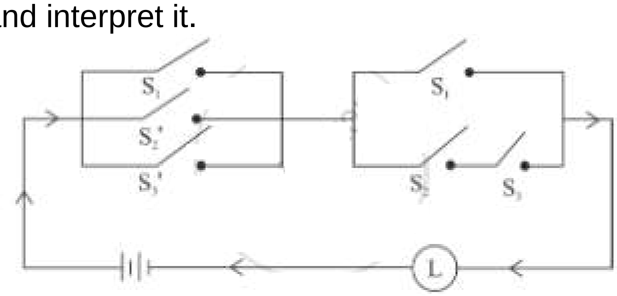
Switching Table

p	q	~p	~q	~p∧~q	p∧q	p∨(~p∧~q)∨(p∧q)
1	1	0	0	0	1	1
1	0	0	1	0	0	1
0	1	1	0	0	0	0
0	0	1	1	1	0	1

Since the final column contains ‘0’ when p is 0 and q is ‘1’, otherwise it contains ‘1’. Hence, the lamp will not glow when S_1 is OFF and S_2 is ON, otherwise, the lamp will glow.

Exercise 1.5 | Q 4.3 | Page 30

Write the symbolic form of the following switching circuit construct its switching table and interpret it.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

~ q: the switch S_2' is closed or the switch S_2 is open

~ r: the switch S_3' is closed or the switch S_3 is open

Then the symbolic form of the given circuit is:

[p ∨ (~ q) ∨ (~ r)] ∧ [p ∨ (q ∧ r)]

Switching Table

p	q	r	~q	~r	p∨(~q)∨(~r)	q∧r	p∨(q∧r)	Final column
					(I)		(II)	(I) ∧ (II)
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1

1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

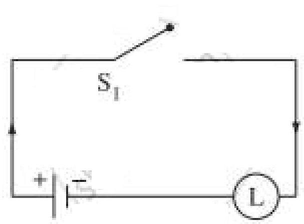
From the switching table, the 'final column' and the column of p are identical. Hence, the lamp will glow which S_1 is 'ON'.

Exercise 1.5 | Q 5.1 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$p \vee (q \wedge \sim q)$

Solution: Using the laws of logic, we have,
$p \vee (q \wedge \sim q)$
$\equiv p \vee F$ ....(By Complement Law)
$\equiv p$ ....(By Identity Law)
Hence, the simple logical expression of the given expression is p.
Let p: the switch S_1 is closed
Then the corresponding switching circuit is:



Exercise 1.5 | Q 5.2 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$

Solution: Using the laws of logic, we have,

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$

$\equiv [\sim P \wedge (q \vee \sim q)] \vee (p \wedge \sim q)$ ....(By Distributive Law)

$\equiv (\sim p \wedge T) \vee (p \wedge \sim q)$ ....(By Complement Law)

$\equiv \sim p \vee (p \wedge \sim q)$ ....(By Identity Law)

$\equiv (\sim p \vee p) \wedge (\sim p \wedge \sim q)$ ....(By Distributive Law)

$\equiv T \wedge (\sim p \wedge \sim q)$ ....(By Complement Law)

≡ ~ p ∨ ~ q ....(By Identity Law)

Hence, the simple logical expression of the given expression is ~ p ∨ ~ q.

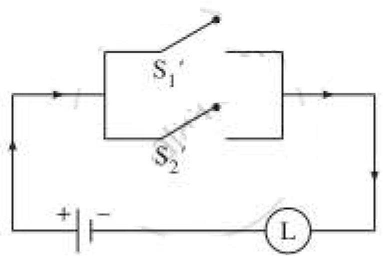
Let p: the switch S_1 is closed

q: the switch S_2 is closed

~ p: the switch S_1' is closed or the switch S_1 is open

~ q: the switch S_2' is closed or the switch S_2 is open.

Then the corresponding switching circuit is:



Exercise 1.5 | Q 5.3 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

[p ∨ (~ q) ∨ (~ r)] ∧ [p ∨ (q ∧ r)]

Solution: Using the laws of logic, we have,

[p ∨ (~ q) ∨ (~ r)] ∧ [p ∨ (q ∧ r)]

≡ [p ∨ {~ (q ∧ r)}] ∧ [p ∨ (q ∧ r)] ......(By De Morgan's Law)

≡ p ∨ [~ (q ∧ r) ∧ (q ∧ r)] ....(By Distributive Law)

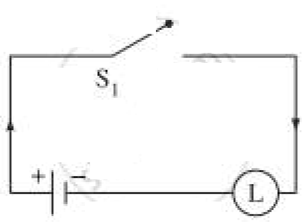
≡ p ∨ F ....(By Complement Law)

≡ p ....(By Identity Law)

Hence, the simple logical expression of the given expression is p.

Let p: the switch S_1 is closed

Then the corresponding switching circuit is:



Exercise 1.5 | Q 5.4 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$

Solution: Using the laws of logic, we have,

$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$

$\equiv (p \wedge \sim p \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$ ....(By Commutative Law)

$\equiv (F \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$ ....(By Complement Law)

$\equiv F \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$ ....(By Identity Law)

$\equiv (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$ ....(By Identity Law)

$\equiv (\sim p \vee p) \wedge (q \wedge r)$ ......(By Distributive Law)

$\equiv T \wedge (q \wedge r)$ ....(By Complement Law)

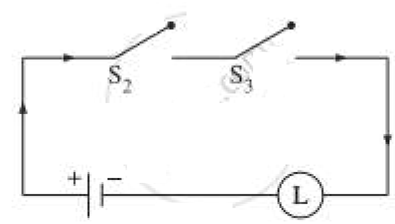
$\equiv q \wedge r$ ....(By Identity Law)

Hence, the simple logical expression of the given expression is $q \wedge r$.

Let q: the switch S_2 is closed

r: the switch S_3 is closed.

Then the corresponding switching circuit is:



MISCELLANEOUS EXERCISE 1 [PAGES 32 - 35]

Miscellaneous Exercise 1 | Q 1.1 | Page 32

Select and write the correct answer from the given alternative of the following question:

If $p \wedge q$ is false and $p \vee q$ is true, then __________ is not true.

1. $p \vee q$
2. **$p \leftrightarrow q$**
3. $\sim p \vee \sim q$
4. $q \vee \sim p$

Solution: If $p \wedge q$ is false and $p \vee q$ is true, then **$p \leftrightarrow q$** is not true.

Miscellaneous Exercise 1 | Q 1.2 | Page 32

Select and write the correct answer from the given alternative of the following question:

$(p \wedge q) \rightarrow r$ is logically equivalent to ________ .

1. **$p \rightarrow (q \rightarrow r)$**
2. $(p \wedge q) \rightarrow \sim r$
3. $(\sim p \vee \sim q) \rightarrow \sim r$
4. $(p \vee q) \rightarrow r$

Solution: $(p \wedge q) \rightarrow r$ is logically equivalent to **$p \rightarrow (q \rightarrow r)$**.

Miscellaneous Exercise 1 | Q 1.3 | Page 32

Select and write the correct answer from the given alternative of the following question:

Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is ________ .

1. $(p \wedge q) \rightarrow (p \vee q)$
2. $\sim (p \vee q) \rightarrow (p \wedge q)$
3. **$(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$**
4. $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$

Solution: Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is **$(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$**.

Miscellaneous Exercise 1 | Q 1.4 | Page 32

Select and write the correct answer from the given alternative of the following question:

If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are ________.

1. T, T
2. **T, F**
3. F, T
4. F, F

Solution: If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are **T, F**.

Miscellaneous Exercise 1 | Q 1.5 | Page 32

Select and write the correct answer from the given alternative of the following question:

The negation of inverse of $\sim p \rightarrow q$ is ________.

1. **$q \wedge p$**
2. $\sim p \wedge \sim q$
3. $p \wedge q$

4. $\sim q \rightarrow \sim p$

   Solution: The negation of inverse of $\sim p \rightarrow q$ is $\mathbf{q \wedge p}$.

   Miscellaneous Exercise 1 | Q 1.6 | Page 32

   Select and write the correct answer from the given alternative of the following question:

   The negation of $p \wedge (q \rightarrow r)$ is ________.

1. $\sim p \wedge (\sim q \rightarrow \sim r)$
2. $p \vee (\sim q \vee r)$
3. $\sim p \wedge (\sim q \rightarrow \sim r)$
4. **$\sim p \vee (\sim q \wedge \sim r)$**

   Solution: $\forall$ x $\in$ A, x + 6 $\geq$ 9

   Miscellaneous Exercise 1 | Q 1.7 | Page 32

   Select and write the correct answer from the given alternative of the following question:

   If A = {1, 2, 3, 4, 5} then which of the following is not true?

1. $\exists$ x $\in$ A such that x + 3 = 8
2. $\exists$ x $\in$ A such that x + 2 < 9
3. **$\forall$ x $\in$ A, x + 6 $\geq$ 9**
4. $\exists$ x $\in$ A such that x + 6 < 10

   Solution: $\forall$ x $\in$ A, x + 6 $\geq$ 9

   Miscellaneous Exercise 1 | Q 2.1 | Page 33

   Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

   4! = 24.

   Solution: It is a statement which is true, hence its truth value is ‘T’.

   Miscellaneous Exercise 1 | Q 2.2 | Page 33

   Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

   π is an irrational number.

   Solution: It is a statement which is true, hence its truth value is ‘T’.

   Miscellaneous Exercise 1 | Q 2.3 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

India is a country and Himalayas is a river.

Solution: It is a statement which is false, hence its truth value is ‘F’. ....[$T \wedge F \equiv F$]

Miscellaneous Exercise 1 | Q 2.4 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

Please get me a glass of water.

Solution: It is an imperative sentence, hence it is not a statement.

Miscellaneous Exercise 1 | Q 2.5 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

$\cos^2\theta - \sin^2\theta = \cos 2\theta$ for all $\theta \in R$.

Solution: It is a statement which is true, hence its truth value is ‘T’.

Miscellaneous Exercise 1 | Q 2.6 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

If x is a whole number then $x + 6 = 0$.

Solution: It is a statement which is false, hence its truth value is ‘F’.

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 3.1 | Page 33

Write the truth value of the following statement:

$\sqrt{5}$ is an irrational but $3\sqrt{5}$ is a complex number.

Solution: Let p: 5 is an irrational.

q: 35 is a complex number.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively.

$\therefore$ The truth value of $p \wedge q$ is F. ....[$T \wedge F \equiv F$]

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 3.2 | Page 33

Write the truth value of the following statement:

$\forall$ n $\in$ N, n^2 + n is even number while n^2 – n is an odd number.

Solution: Let p: $\forall$ n $\in$ N, n^2 + n is an even number.

q: $\forall$ n $\in$ N, n^2 − n is an odd number.

Then the symbolic form of the given statement is p∧q.

The truth values of p and q are T and F respectively. ∴ The truth value of p∧q is F.

..........[T∧F ≡ F].

Miscellaneous Exercise 1 | Q 3.3 | Page 33

Write the truth value of the following statement:

$\exists$ n $\in$ N such that n + 5 > 10.

Solution: $\exists$ n $\in$ N, such that n + 5 > 10 is a true statement, hence its truth value is T.

(All n ≥ 6, where n ∈ N, satisfy n + 5 > 10).

Miscellaneous Exercise 1 | Q 3.4 | Page 33

Write the truth value of the following statement:

The square of any even number is odd or the cube of any odd number is odd.

Solution: Let p: The square of any even number is odd.

q: The cube of any odd number is odd.

Then the symbolic form of the given statement is p∨q.

The truth values of p and q are F and T respectively.

∴ The truth value of p∨q is T. ....[F∨T ≡ T]

Miscellaneous Exercise 1 | Q 3.5 | Page 33

Write the truth value of the following statement:

In ΔABC if all sides are equal then its all angles are equal.

Solution: Let p: ABC is a triangle and all its sides are equal.

q: It's all angles are equal.

Then the symbolic form of the given statement is p→q.

If the truth value of p is T, then the truth value of q is T.

∴ The truth value of p→q is T ....[T→T ≡ T].

Miscellaneous Exercise 1 | Q 3.6 | Page 33

Write the truth value of the following statement:

$\forall$ n $\in$ N, n + 6 > 8.

Solution: $\forall$ n $\in$ N, n + 6 > 8 is a false statement, hence its truth value is F.

(n = 1 $\in$ N, n = 2 $\in$ N do not satisfy n + 6 > 8).

Miscellaneous Exercise 1 | Q 4.1 | Page 33

If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of the following statement:

$\exists$ x $\in$ A such that x + 8 = 15

Solution: Clearly x = 7 $\in$ A satisfies x + 8 = 15. So the given statement is true, hence its truth value is T.

Miscellaneous Exercise 1 | Q 4.2 | Page 33

If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of the following statement:

$\forall$ x $\in$ A, x + 5 < 12.

Solution: There is no x $\in$ A which satisfies x + 5 < 12. So the given statement is false, hence its truth value is F.

Miscellaneous Exercise 1 | Q 4.3 | Page 33

If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of the following statement:

$\exists$ x $\in$ A, such that x + 7 ≥ 11.

Solution: Clearly x = 1 $\in$ A, x = 2 $\in$ A and x = 3 $\in$ A satisfies x + 7 ≥ 11. So the given statement is true, hence its truth value is T.

Miscellaneous Exercise 1 | Q 4.4 | Page 33

If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, determine the truth value of the following statement:

$\forall$ x $\in$ A, 3x ≤ 25.

Solution: x = 9 $\in$ A does not satisfy 3x ≤ 25 So the given statement is false, hence its truth value is F.

Miscellaneous Exercise 1 | Q 5.1 | Page 33

Write the negation of the following:

$\forall$ n $\in$ A, n + 7 > 6.

Solution: The negation of the given statement is:

$\exists$ n $\in$ A, such that n + 7 $\leq$ 6.

OR

$\exists$ n $\in$ A, such that n + 7$\ngtr$ 6.

Miscellaneous Exercise 1 | Q 5.2 | Page 33

Write the negation of the following:

$\exists$ x $\in$ A, such that x + 9 $\leq$ 15.

Solution: The negation of the given statement is:

$\forall$ x $\in$ A, x + 9 > 15.

Miscellaneous Exercise 1 | Q 5.3 | Page 33

Write the negation of the following:

Some triangles are equilateral triangle.

Solution: The negation of the given statement is:

All triangles are not equilateral triangles.

Miscellaneous Exercise 1 | Q 6.1 | Page 33

Construct the truth table of the following:

$p \rightarrow (q \rightarrow p)$

Solution:

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Miscellaneous Exercise 1 | Q 6.2 | Page 33

Construct the truth table of the following:

$(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$

Solution:

p	q	~p	~q	~p ∨ ~q	p∧q	~ (p∧q)	(~p ∨ ~q) ↔ [~ (p∧q)]
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

Miscellaneous Exercise 1 | Q 6.3 | Page 33

Construct the truth table of the following:

~ (~p ∧ ~q) ∨ q

Solution:

p	q	~p	~q	~p ∧ ~q	~ (~P ∧ ~q)	~ (~p ∧ ~q) ∨ q
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 6.4 | Page 33

Construct the truth table of the following:

[(p ∧ q) ∨ r] ∧ [~r ∨ (p ∧ q)]

Solution:

p	q	r	p∧q	(p∧q) ∨ r	~r	~r ∨ (p∧q)	[(p∧q) ∨ r] ∧ [~r ∨ (p∧q)]
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	F	F	F	F	T	T	F

Miscellaneous Exercise 1 | Q 6.5 | Page 33

Construct the truth table of the following:

$[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Solution:

p	q	r	~p	~p ∨ q	q→r	(~pvq) ∧ (q→r)	p→r	[(~pvq) ∧ (q→r)] → (p→r)
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Miscellaneous Exercise 1 | Q 7.1 | Page 33

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$

Solution:

p	q	~ p	~ q	p → q	(p → q) ∧ ~ q	[(p → q) ∧ ~ q] → ~ p
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore [(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology.

Miscellaneous Exercise 1 | Q 7.2 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency:

[(p ∨ q) ∧ ~p] ∧ ~q

Solution:

p	q	~p	~q	pvq	(pvq) ∧ ~p	[(pvq) ∧ ~p] ∧ ~q
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	T	T	T	F	F	F

All the entries in the last column of the above truth table are F.

∴ [(p ∨ q) ∧ ~p] ∧ ~q is a contradiction.

Miscellaneous Exercise 1 | Q 7.3 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency:

(p → q) ∧ (p ∧ ~q)

Solution:

p	q	~q	p → q	p ∧ ~q	(p → q) ∧ (p ∧ ~q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

All the entries in the last column of the above truth table are F.
∴ (p → q) ∧ (p ∧ ~q) is a contradiction.

Miscellaneous Exercise 1 | Q 7.4 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

[p → (q → r)] ↔ [(p ∧ q) → r]

Solution:

p	q	r	q → r	p → (q → r)	p ∧ q	(p ∧ q) → r	[p → (q → r)] ↔ [(p ∧ q) → r]
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

All the entries in the last column of the above truth table are T.
∴ [p → (q → r)] ↔ [(p ∧ q) → r] is a tautology.

Miscellaneous Exercise 1 | Q 7.5 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

[(p ∧ (p → q)] → q

Solution:

p	q	p → q	p ∧ (p → q)	[p ∧ (p → q)] → r
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

All the entries in the last column of the above truth table are T.
∴ [(p ∧ (p → q)] → q is a tautology.

Miscellaneous Exercise 1 | Q 7.6 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$

Solution:

p	q	~p	~q	p ∧ q	~p ∧ q	p ∨ ~q	~p ∧ ~q	(I) ∨ (II) ∨ (III) ∨ (IV)
				(I)	(II)	(III)	(IV)	
T	T	F	F	T	F	T	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	T	T	T

All the entries in the last column of the above truth table are T.
∴ $(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

Miscellaneous Exercise 1 | Q 7.7 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$

Solution:

p	q	r	~p	~q	p ∨ ~q	~p ∧ q	(p ∨ ~q) ∨ (~p ∧ q)	(I) ∧ r
							(I)	
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

The entries in the last column are neither all T nor all F.
∴ $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$ is a contingency.

Miscellaneous Exercise 1 | Q 7.8 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$(p \rightarrow q) \vee (q \rightarrow p)$

Solution:

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore (p \rightarrow q) \vee (q \rightarrow p)$ is a tautology.

Miscellaneous Exercise 1 | Q 8.1 | Page 34

Determine the truth values of p and q in the following case:

$(p \vee q)$ is T and $(p \wedge q)$ is T

Solution:

p	q	$p \vee q$	$p \wedge q$
T	**T**	**T**	**T**
T	F	T	F
F	T	T	F
F	F	F	F

Since $p \vee q$ and $p \wedge q$ both are T, from the table, the truth values of both p and q are T.

Miscellaneous Exercise 1 | Q 8.2 | Page 34

Determine the truth values of p and q in the following case:

$(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F

Solution:

p	q	$p \vee q$	$(p \vee q) \rightarrow q$
T	T	T	T

T	**F**	**T**	**F**
F	T	T	T
F	F	F	T

Since the truth values of (p ∨ q) is T and (p ∨ q) → q is F, from the table, the truth values of p and q are T and F respectively.

Miscellaneous Exercise 1 | Q 8.3 | Page 34

Determine the truth values of p and q in the following case:

(p ∧ q) is F and (p ∧ q) → q is T

Solution:

p	q	p ∧ q	(p ∧ q) → q
T	T	T	T
T	**F**	**F**	**T**
F	**T**	**F**	**T**
F	**F**	**F**	**T**

Since the truth values of (p ∧ q) is F and (p ∧ q) → q is T, from the table, the truth values of p and q are either T and F respectively or F and T respectively or both F.

Miscellaneous Exercise 1 | Q 9.1 | Page 34

Using the truth table, prove the following logical equivalence :

p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)

Solution:

1	2	3	4 A	5	6	7 B	8
p	q	p ↔ q	p ∧ q	~p	~q	~p ∧ ~q	A V B
T	T	**T**	T	F	F	F	**T**
T	F	**F**	F	F	T	F	**F**
F	T	**F**	F	T	F	F	**F**
F	F	**T**	F	T	T	T	**T**

By column number 3 and 8

p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)

Miscellaneous Exercise 1 | Q 9.2 | Page 34

Using truth table, prove the following logical equivalence :

$(p \land q) \to r \equiv p \to (q \to r)$

Solution:

1	2	3	4	5	6	7
p	q	r	p∧q	(p∧q)→r	q→r	p→(q→r)
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

The entries in columns 5 and 7 are identical.

$\therefore (p \land q) \to r \equiv p \to (q \to r)$.

Miscellaneous Exercise 1 | Q 10.1 | Page 34

Using rules in logic, prove the following:

$p \leftrightarrow q \equiv \sim(p \land \sim q) \lor \sim(q \land \sim p)$

Solution: By the rules of negation of biconditional,

$\sim (p \leftrightarrow q) \equiv (p \land \sim q) \lor (q \land \sim p)$

$\therefore \sim [(p \land \sim q) \land (q \land \sim p)] \equiv p \leftrightarrow q$

$\therefore \sim (p \land \sim q) \land \sim (q \land \sim p) \equiv p \leftrightarrow q$ ....(Negation of disjunction)

$\therefore p \leftrightarrow q \equiv \sim(p \land \sim q) \land \sim(q \land \sim p)$.

Miscellaneous Exercise 1 | Q 10.2 | Page 34

Using rules in logic, prove the following:

$\sim p \land q \equiv (p \lor q) \land \sim p$

Solution: $(p \lor q) \land \sim p$

$\equiv (p \land \sim p) \lor (q \land \sim p)$ .....(Distributive Law)

$\equiv F \lor (q \land \sim p)$ ....(Complement Law)

$\equiv q \land \sim p$ ....(Identity Law)

$\equiv \sim p \land q$ ....(Commutative Law)

$\therefore \sim p \land q \equiv (p \lor q) \land \sim p$

Miscellaneous Exercise 1 | Q 10.3 | Page 34

Using rules in logic, prove the following:

$\sim (p \lor q) \lor (\sim p \land q) \equiv \sim p$

Solution: $\sim (p \lor q) \lor (\sim p \land q)$

$\equiv (\sim p \land \sim q) \lor (\sim p \land q)$ ....(Negation of disjunction)

$\equiv \sim p \land (\sim q \lor q)$ ....(Distributive Law)

$\equiv \sim p \land T$ ....(Complement Law)

$\equiv \sim p$ ....(Identity Law)

$\therefore \sim (p \lor q) \lor (\sim p \land q) \equiv \sim p$

Miscellaneous Exercise 1 | Q 11.1 | Page 34

Using the rules in logic, write the negation of the following:

$(p \lor q) \land (q \lor \sim r)$

Solution: The negation of $(p \lor q) \land (q \lor \sim r)$ is

$\sim [(p \lor q) \land (q \lor \sim r)]$

$\equiv \sim(p \lor q) \lor \sim(q \lor \sim r)$ .....(Negation of conjunction)

$\equiv (\sim p \land \sim q) \lor [\sim q \land \sim(\sim r)]$ ....(Negation of disjunction

$\equiv (\sim p \land \sim q) \lor (\sim q \land r)$ ....(Negation of negation)

$\equiv (\sim q \land \sim p) \lor (\sim q \land r)$ ....(Commutative law)

$\equiv (\sim q) \land (\sim p \lor r)$ ....(Distributive Law)

Miscellaneous Exercise 1 | Q 11.2 | Page 34

Using the rules in logic, write the negation of the following:

$p \land (q \lor r)$

Solution: The negation of $p \land (q \lor r)$ is

$\sim [p \land (q \lor r)]$

$\equiv \sim p \lor \sim(q \lor r)$ ....(Negation of conjunction)

$\equiv \sim p \lor (\sim q \land \sim r)$ ....(Negation of disjunction)

Miscellaneous Exercise 1 | Q 11.3 | Page 34

Using the rules in logic, write the negation of the following:

$(p \to q) \land r$

Solution: The negation of $(p \to q) \land r$ is

$\sim [(p \to r) \land r]$

$\equiv \sim (p \to q) \lor (\sim r)$ ....(Negation of conjunction)

$\equiv (p \land \sim q) \lor (\sim r)$ ....(Negation of implication)

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 11.4 | Page 34

Using the rules in logic, write the negation of the following:

$(\sim p \land q) \lor (p \land \sim q)$

Solution: The negation of $(\sim p \land q) \lor (p \land \sim q)$ is

$\sim [(\sim p \land q) \lor (p \land \sim q)]$

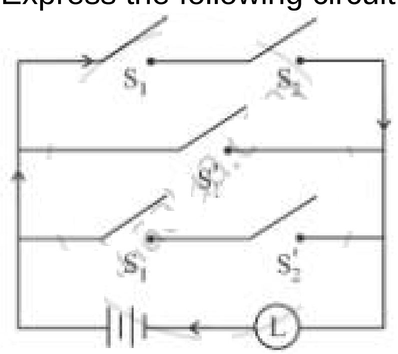
$\equiv \sim (p \land q) \land \sim(p \land \sim q)$ ....(Negation of disjunction)

$\equiv [\sim (\sim p) \lor \sim q] \land [\sim p \lor \sim(\sim q)]$ ....(Negation of conjunction)

$\equiv (p \lor \sim q) \land (\sim p \lor q)$ ....(Negation of negation)

Miscellaneous Exercise 1 | Q 12.1 | Page 34

Express the following circuit in the symbolic form. Prepare the switching table:



Solution:

Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim$p: the switch S_1' is closed or the switch S_1 is open

$\sim$q: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given circuit is:

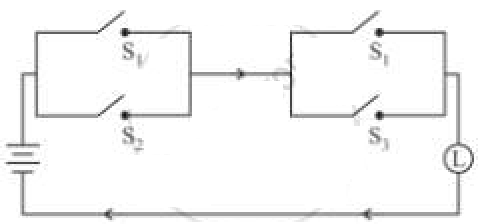
(p∧q) ∨ (~p) ∨ (p ∧ ~q)

Switching Table

p	q	~p	~q	p∧q	p ∧ ~q	(p∧q) ∨ (~p) ∨ (p ∧ ~q)
1	1	0	0	1	0	1
1	0	0	1	0	1	1
0	1	1	0	0	0	1
0	0	1	1	0	0	1

Miscellaneous Exercise 1 | Q 12.2 | Page 34

Express the following circuit in the symbolic form. Prepare the switching table:



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

Then the symbolic form of the given statement is:

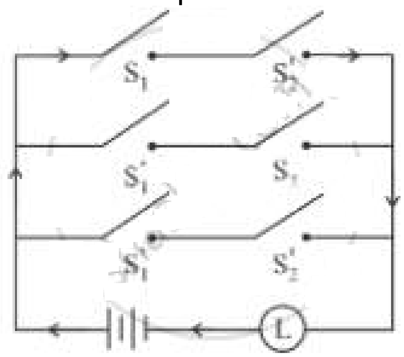
(pvq) ∧ (pvq)

Switching Table

p	q	r	pvq	pvr	(pvq) ∧ (pvq)
1	1	1	1	1	1
1	1	0	1	1	1
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	0	0	0

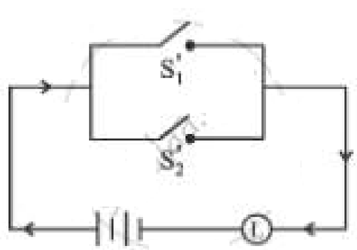
Miscellaneous Exercise 1 | Q 13.1 | Page 34

Simplify the following so that the new circuit has a minimum number of switches. Also, draw the simplified circuit.



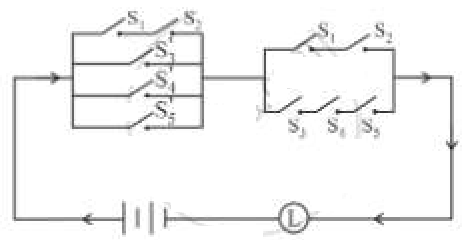
Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
~p: the switch S_1' is closed or the switch S_1 is open
~q: the switch S_2' is closed or the switch S_2 is open.
Then the given circuit in symbolic form is:
$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$
Using the laws of logic, we have,
$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$
$\equiv (p \wedge \sim q) \vee [(\sim p \wedge q) \vee (\sim p \wedge \sim q)]$ ....(By Associative Law)
$\equiv (p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)]$ ....(By Distributive Law)
$\equiv (p \wedge \sim q) \vee (\sim p \wedge T)$ ....(By Complement Law)
$\equiv (p \wedge \sim q) \vee \sim p$ ....(By Identity Law)
$\equiv (p \vee \sim p) \wedge (\sim q \vee \sim p)$ ....(By Distributive Law)
$\equiv T \wedge (\sim q \vee \sim p)$ ....(By Complement Law)

$\equiv \sim q \vee \sim p$ ....(By Identity Law)
$\equiv \sim p \vee \sim q$ ....(By Commutative Law)
Hence, the simplified circuit for the given circuit is:



Miscellaneous Exercise 1 | Q 13.2 | Page 34

Simplify the following so that the new circuit has a minimum number of switches. Also, draw the simplified circuit.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

s: the switch S_4 is closed

t: the switch S_5 is closed

~p: the switch S_1' is closed or the switch S_1 is open

~q: the switch S_2' is closed or the switch S_2 is open

~r: the switch S_3' is closed or the switch S_3 is open

~s: the switch S_4' is closed or the switch S_4 is open

~t: the switch S_5' is closed or the switch S_5 is open.

Then the given circuit in symbolic form is:

$[(p \wedge q) \vee \sim r \vee \sim s \vee \sim t] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$

Using the laws of logic, we have,

$[(p \wedge q) \vee \sim r \vee \sim s \vee \sim t] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$

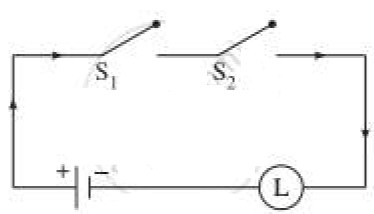
$\equiv [(p \wedge q) \vee \sim (r \wedge s \wedge t)] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$ ....(By De Morgan's Law)

$\equiv (p \wedge q) \vee [\sim (r \wedge s \wedge t) \wedge (r \wedge s \wedge t)]$ ......(By Distributive Law)

$\equiv (p \wedge q) \vee F$ ....(By Complement Law)

$\equiv p \wedge q$ ....(By Identity Law)

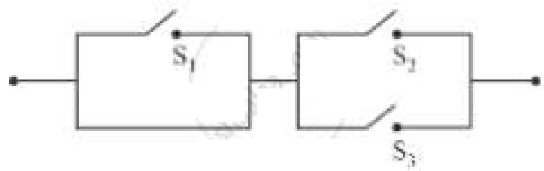
Hence, the alternative simplified circuit is:



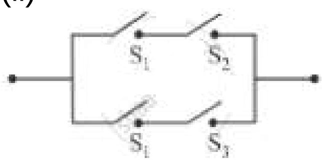
Miscellaneous Exercise 1 | Q 14.1 | Page 35

Check whether the following switching circuits are logically equivalent - Justify.

(i)



(ii)



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

The symbolic form of the given switching circuits is $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ respectively.

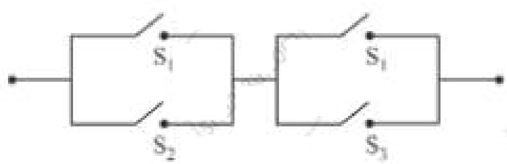
By Distributive Law, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Hence, the given switching circuits are logically equivalent.

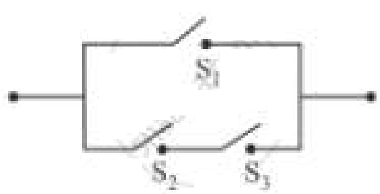
Miscellaneous Exercise 1 | Q 14.2 | Page 35

Check whether the following switching circuits are logically equivalent - Justify.

(i)



(ii)



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

The symbolic form of the given switching circuits are

$(p \vee q) \wedge (p \vee r)$ and $p \vee (q \wedge r)$

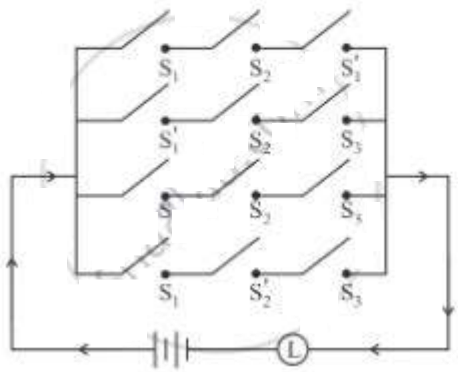
By Distributive Law,

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Hence, the given switching circuits are logically equivalent.

Miscellaneous Exercise 1 | Q 15 | Page 35

Give alternative arrangement of the switching following circuit, has minimum switches.



Solution: Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S3 is closed
~p: the switch S_1' is closed or the switch S_1 is open.
~q: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given circuit is:
$(p \land q \land \sim p) \lor (\sim p \land q \land r) \lor (p \land q \land r) \lor (p \land \sim q \land r)$

Using the laws of logic, we have,

$(p \land q \land \sim p) \lor (\sim p \land q \land r) \lor (p \land q \land r) \lor (p \land \sim q \land r)$

$\equiv (p \land \sim p \land q) \lor (\sim p \land q \land r) \lor (p \land q \land r) \lor (p \land \sim q \land r)$ ....(By Commutative Law)

$\equiv (F \land q) \lor (\sim p \land q \land r) \lor (p \land q \land r) \lor (p \land \sim q \land r)$ ....(By Complement Law)
$\equiv F \lor (\sim p \land q \land r) \lor (p \land q \land r) \lor (p \land \sim q \land r)$ ....(By Identity Law)

$\equiv (\sim p \land q \land r) \lor (p \land q \land r) \lor (p \land \sim q \land r)$ ....(By Identity Law)

$\equiv [(\sim p \lor p) \land (q \land r)] \lor (p \land \sim q \land r)$ ....(By Distributive Law)

$\equiv [T \land (q \land r)] \lor (p \land \sim q \land r)$ ......(By Complement Law)

$\equiv (q \land r) \lor (p \land \sim q \land r)$ ....(By Identity Law)

$\equiv [q \lor (p \land \sim q)] \land r$ ....(By Distributive Law)

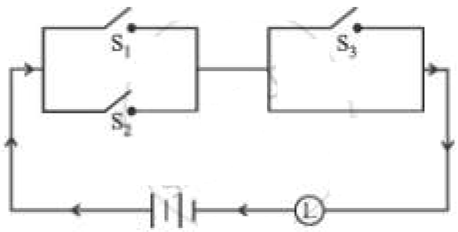
$\equiv [(q \lor p) \land (q \lor \sim q)] \land r$ ....(By Distributive Law)

$\equiv (q \lor p) \land T] \land r$ ....(By Complement Law)

$\equiv (q \lor p) \land r$ ....(By Identity Law)

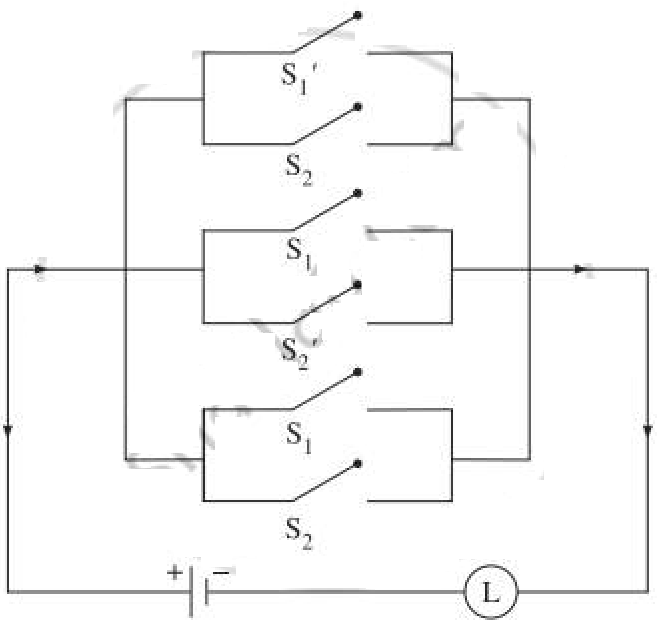
$\equiv (p \vee q) \wedge r$ ....(By Commutative Law)

$\therefore$ the alternative arrangement of the new circuit with minimum switches is:



Miscellaneous Exercise 1 | Q 16 | Page 35

Simplify the following so that the new circuit.



Solution:

Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim$p: the switch S_1' is closed or the switch S_1 is open

$\sim$q: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given switching circuit is:

$(\sim p \vee q) \vee (p \vee \sim q) \vee (p \vee q)$

Using the laws of logic, we have,

$(\sim p \vee q) \vee (p \vee \sim q) \vee (p \vee q)$

$\equiv (\sim p \vee q \vee p \vee q) \vee (p \vee q)$

≡ [(~p ∨ p) ∨ (q ∨ q)] ∨ (p ∨ q) ......(By Commutative Law)

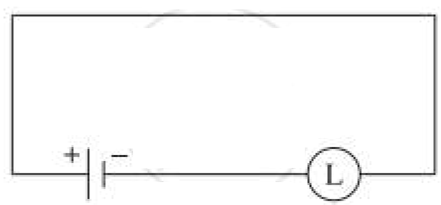
≡ (T ∨ T) ∨ (p ∨ q) ....(By Complement Law)

≡ T ∨ (p ∨ q) ....(By Identity Law)

≡ T ....(By Identity Law)

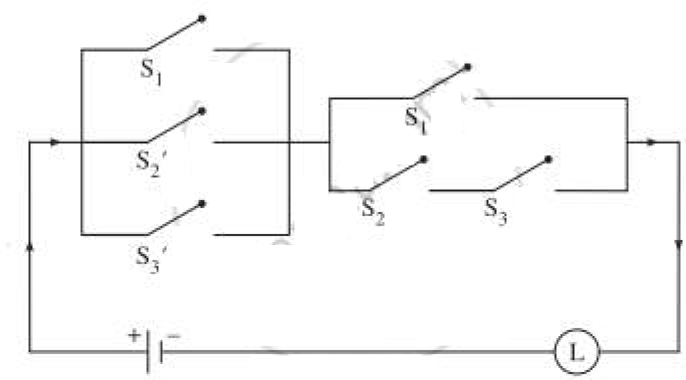
∴ the current always flows whether the switches are open or closed. So, it is not necessary to use any switch in the circuit.

∴ the simplified form of the given circuit is:



Miscellaneous Exercise 1 | Q 17 | Page 35

Represent the following switching circuit in symbolic form and construct its switching table. Write your conclusion from the switching table.



Solution:

Let p: the switch S_1 is closed
q: the switch S_2 is closed
r: the switch S_3 is closed
 ~ q: the switch S_2' is closed or the switch S_2 is open
~ r: the switch S_3' is closed or the switch S_3 is open
Then the symbolic form of the given circuit is:
[p ∨ (~ q) ∨ (~ r)] ∧ [p ∨ (q ∧ r)]

Switching Table

p	q	r	~q	~r	p∨(~q)∨(~r)	q∧r	p∨(q∧r)	Final column
					(I)		(II)	(I) ∧ (II)
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1
1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

From the table, the 'final column' and the column of p are identical. Hence, the given circuit is equivalent to the simple circuit with only one switch S_1.

∴ the simplified form of the given circuit is: