Chapter 9. Factoring

Ex. 9.5

Answer 1CU.

The objective is to give the description of a binomial that is the difference of two squares.

The Binomial is the difference of two terms, each of which is a perfect square.

For example $x^2-9=x^2-3x^2$

General form is $\boxed{a^2-b^2}$.

Answer 2CU.

Consider the binomial $4x^2-25$.

$4x^2-25=2\cdot 2x^2-5\cdot 5$

$=2^2\cdot x^2-5^2$ (Because $x\cdot x=x^2$)

$=(2x+5)(2x-5)$ (Because $a^2-b^2=(a+b)(a-b)$)

$4x^2-25=(2x+5)(2x-5)$

Therefore,

$\boxed{4x^2-25=(2x+5)(2x-5)}$.

Answer 3CU.

Consider the polynomial $3n^2-48$

$3n^2-48=3\cdot n^2-3\cdot 16$ (Because $3\cdot 16=48$)

$=3\left(n^2-16\right)$ (Take the GCF as common)

$=3\cdot\left(n^2-4^2\right)$ (Because $16=4\cdot 4$)

Factors of difference of squares is

$a^2-b^2=(a+b)(a-b)$

$3n^2-48=3\left(n^2-4^2\right)$

$=3(n+4)(n-4)$ (Factor the difference of squares)

Therefore, difference of square pattern can be used to factor $3n^2-48$

Therefore,

$\boxed{3n^2-48=3\cdot(n+4)(n-4)}$.

Answer 4CU.

Consider the Binomial $64x^2+16y^2$.

Manuel factoring as $64x^2+16y^2$

$=16\left(4x^2+y^2\right)$

Jessica factoring as $64x^2+16y^2$

$=16\left(4x^2+y^2\right)$

$=16(2x+y)(2x-y)$

Manuel is correct since 16 is the GCF of $64x^2$ and $16y^2$.

So he takes it as common factor.

$64x^2+16y^2=16\left(4x^2+y^2\right)$

Jessica written $4x^2+y^2$ as $(2x+y)(2x-y)$.

Jessica used difference of two squares for $4x^2+y^2$ it is wrong, it can be used for the binomial of the form a^2-b^2 only.

Answer 5CU.

Consider the polynomial n^2-81

The objective is to factor the given polynomial

Difference the squares is

$$a^2-b^2=(a+b)(a-b)$$

$$n^2-81=n^2-9\cdot9 \quad (9\cdot9=81)$$

$$=n^2-9^2 \quad (9^2=9\cdot9)$$

$$=(n+9)(n-9) \text{ (Factor the difference of squares)}$$

Therefore,

$\boxed{n^2-81=(n+9)(n-9)}$.

Answer 6CU.

Consider the binomial $4-9a^2$

The objective is to factor the given binomial.

Since difference of squares is

$$a^2-b^2=(a+b)(a-b)$$

$$4-9a^2=2\cdot2-3\cdot3a^2 \quad (2\cdot2=4, 3\cdot3=9)$$

$$=2^2-3^2a^2 \quad (2\cdot2=2^2, 3\cdot3=3^2)$$

$$=(2+3a)(2-3a) \text{ (Factor the difference of squares)}$$

Therefore, the factorization of $\boxed{4-9a^2}$ is $\boxed{(2+3a)(2-3a)}$.

Answer 7CU.

Consider the binomial $2x^5-98x^3$

The objective is to factor the given binomial.

$$2x^5-98x^3=2x^3\left(x^2-49\right)$$

$$=2x^3\left(x^2-(7)^2\right) \text{ (Factor the } GCF \text{ of } x^2, 49\text{)}$$

$$=2x^3\left(x^2-7^2\right) \text{ (Simplify)}$$

$$=2x^3(x+7)(x-7) \text{ (Since } a^2-b^2=(a+b)(a-b)\text{)}$$

$$=2\cdot x^3(x+7)(x-7)$$

Therefore, the factorization of $\boxed{2x^5-98x^3}$ is $\boxed{2x^3(x+7)(x-7)}$.

Answer 8CU.

Consider the binomial $32x^4 - 2y^4$.

The objective is to factor the given binomial.

Since the difference of squares is

$$a^2 - b^2 = (a+b)(a-b)$$

$$32x^4 - 2y^4 = 2^4 \cdot (x^2)^2 - 2(y^2)^2$$

$$= 2^4 \cdot 2(x^2)^2 - 2(y^2)^2$$ (Because $2^5 = 2^4 \cdot 2, x^4 = (x^2)^2, y^4 = (y^2)^2$)

$$= 2^4 \cdot 2\left((x^2)^2 - (y^2)^2\right)$$ (Take common 2)

$$= 2^4 \cdot 2\left((x^2 + y^2)(x^2 - y^2)\right)$$ (Since $a^2 - b^2 = (a+b)(a-b)$)

Therefore, the factorization of $\boxed{32x^4 - 2y^4}$ is $\boxed{2^4 \cdot 2 \cdot (x^2 + y^2)(x^2 - y^2)}$.

Answer 9CU.

Consider the polynomial is $4t^2 - 27$.

The objective is to factor the given polynomial.

$$4t^2 - 27 = 2 \cdot 2 \cdot t^2 - 27$$ (Since $2 \cdot 2 = 4$)

$$= 2 \cdot 2 \cdot t^2 - 3 \cdot 9 \quad (27 = 3 \cdot 9)$$

$$= 2 \cdot 2 \cdot t \cdot t - 3 \cdot 3 \cdot 3 \quad (t^2 = t \cdot t, 3 \cdot 3 = 9)$$

$$= (2t)^2 - 3 \cdot 3^2$$ (Simplify)

It is not factored.

Therefore, $4t^2 - 27$ is not factored, since $4t^2, 27$ have no common factors.

Therefore, $\boxed{4t^2 - 27}$ is prime.

Answer 10CU.

Consider the polynomial $x^3-3x^2-9x+27$

The objective is to factors the given polynomial.

$x^3-3x^2-9x+27=\left(x^3-3x^2\right)+\left(-9x+27\right)$

$=x^2(x-3)-9(x-3)$ (Factor the GCF)

$=\left(x^2-9\right)(x-3)$ (By distributive $(b+c)a=ba+ca$)

$=\left(x^2-3^2\right)(x-3)$ $\left(3^2=9\right)$

$=(x+3)(x-3)(x-3)$ (Because $a^2-b^2=(a+b)(a-b)$)

Therefore, the factorization of $\boxed{x^3-3x^3-9x+27}$ is $\boxed{(x+3)(x-3)(x-3)}$.

Answer 11CU.

Consider the equation $4y^2=25$

$4y^2-25=25-25$ (Subtract 25 on each side)

$4y^2-25=0$

The objective is to find the solution set of given equation.

$4y^2-25=0$

$(2y)^2-5^2=0$ (Factor the GCF of $4y^2,25$)

$(2y+5)(2y-5)=0$ (Since $a^2-b^2=(a+b)(a-b)$)

The zero product property is if

$ab=0$ then

$a=0$ (or)

$b=0$ or both.

By zero product property,

$2y+5=0$

Or, $2y-5=0$

Now solve each equation separately.

$2y+5=0$

$2y+5-5=0-5$ (Subtract 5 on each side)

$2y=-5$

$\frac{2y}{2}=\frac{-5}{2}$ (Divide by 2 on both sides)

$y=\frac{-5}{2}$

$2y-5=0$

$2y-5+5=0+5$ (Add 5 on each side)

$2y=5$

$\frac{2y}{2}=\frac{5}{2}$ (Divide by 2 on both sides)

$y=\frac{5}{2}$

The solution set is $\left\{\frac{-5}{2},\frac{5}{2}\right\}$.

Check:- To check the proposed solution set, substitute each solution in the given set.

For $y = \frac{-5}{2}$,

$$4y^2 - 25 = 0$$

$4\left(\frac{-5}{2}\right)^2 - 25 = 0$ (Put $y = \frac{-5}{2}$)

$4\left(\frac{25}{4}\right) - 25 = 0$ (Simplify)

$\frac{100}{4} - 25 \cdot \frac{4}{4} = 0$ (Equating the denominators)

$$\frac{100 - 100}{4} = 0$$

$0 = 0$ True

For $y = \frac{5}{2}$,

$$4y^2 - 25 = 0$$

$4\left(\frac{5}{2}\right)^2 - 25 = 0$ (Put $y = \frac{5}{2}$)

$$4\left(\frac{25}{4}\right) - 25 = 0$$

$\frac{100}{4} - 25 \cdot \frac{4}{4} = 0$ (Equating the denominators)

$\frac{100 - 100}{4} = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set of given equation is $\boxed{\left\{\frac{-5}{2}, \frac{5}{2}\right\}}$.

Answer 12CU.

Consider the equation

$$17-68k^2=0$$

The objective is to find the solution set of given equation.

$$17-68k^2=0$$

$17\left(1-4k^2\right)=0$ (Factor the GCF of $17,-68k^2$)

$$17=0$$

Or, $1-4k^2=0$ (Using zero product property)

$17=0$ is no solution set of given equation.

<u>Therefore, the given equation has no solution set. It is a prime</u>.

Answer 13CU.

Consider the equation

$$x^2-\frac{1}{36}=0$$

$\frac{36}{36}\cdot x^2-\frac{1}{36}=0$ (Equating the denominators)

$\frac{36x^2-1}{36}=0$ (Simplify)

$\frac{36x^2-1}{36}\cdot 36=0.36$ (Multiplying 36 on each side)

$$36x^2-1=0$$

The objective is to find the solution set of given equation.

$$36x^2-1=0$$

$6^2\cdot x^2-1^2=0$ (Because $36=6^2, 1=1^2$)

$(6x)^2-1^2=0$ (Simplify)

$(6x+1)(6x-1)=0$ (Since $(a+b)(a-b)=a^2-b^2$)

$$6x+1=0$$

Or, $6x-1=0$ (Using zero product property)

Now solve each equation separately.

$6x - 1 = 0$

$6x + 1 - 1 = 0 - 1$ (Subtract 1 on each side)

$6x = -1$

$\frac{6x}{6} = -\frac{1}{6}$ (Divide by 6 on both sides)

$x = -\frac{1}{6}$.

$6x - 1 = 0$

$6x - 1 + 1 = 0 + 1$ (Add 1 on each side)

$6x = 1$

$\frac{6x}{6} = \frac{1}{6}$ (Divide by 6 on each side)

$x = \frac{1}{6}$

The solution set is $\left\{-\frac{1}{6}, \frac{1}{6}\right\}$.

Check:- To check the proposed solution, substitute the solutions in given equation.

For $x=-\frac{1}{6}$,

$$36x^2-1=0$$

$$36\left(-\frac{1}{6}\right)^2-1=0 \text{ (Put } x=\frac{-1}{6}\text{)}$$

$$36\left(\frac{1}{36}\right)-1=0$$

$$\frac{36}{36}-1\cdot\frac{36}{36}=0 \text{ (Equating the denominators)}$$

$$\frac{36-36}{36}=0 \text{ (Simplify)}$$

$0=0$ True

For $x=\frac{1}{6}$,

$$36x^2-1=0$$

$$36\left(\frac{1}{6}\right)^2-1=0 \text{ (Put } x=\frac{1}{6}\text{)}$$

$$\frac{36}{36}-1\cdot\frac{36}{36}=0 \text{ (Equating the denominators)}$$

$$\frac{36-36}{36}=0 \text{ (Simplify)}$$

$0=0$ True

Therefore, the solution set is $\boxed{\left\{-\frac{1}{6},\frac{1}{6}\right\}}$.

Answer 14CU.

Consider the equation $121a=49a^3$

$$121a-49a^3=49a^3-49a^3 \text{ (Subtract } 49a^3 \text{ on both sides)}$$

$$121a-49a^3=0$$

The objective is to find the solution set of given equation.

$$121a-49a^3=0$$

$a\left(121-49a^2\right)=0$ (Simplify)

$a\left(11^2-7^2a^2\right)=0$ (Since $121=11^2, 49=7^2$)

$a\left((11+7a)(11-7a)\right)=0$ (Since $a^2-b^2=(a+b)(a-b)$)

$a=0$

Or, $11+7a=0$

Or, $11-7a=0$ (Using zero product)

Now solve each equation separately.

$a=0$

$11+7a=0$

$11+7a-11=0-11$ (Subtract 11 on both sides)

$74a=-11$

$\frac{7a}{7}=-\frac{11}{7}$ (Divide by 7 on each side)

$a=\frac{-11}{7}$

$11-7a=0$

$11-7a-11=0-11$ (Subtract 11 on both side)

$-7a=-11$ (Simplify)

$7a=11$

$\frac{7a}{7}=\frac{11}{7}$ (Divide by 7 on each side)

$a=\frac{11}{7}$

The solution set is $\left\{0,\frac{-11}{7},\frac{11}{7}\right\}$.

Check: To check the proposed solution, substitute the solutions in given equation.

For $a=0$,

$121a - 49a^3 = 0$

$121(0) - 49(0)^3 = 0$ (Put $a = 0$)

$0 - 0 = 0$ True

For $a = -\frac{11}{7}$,

$121a - 49a^3 = 0$

$121\left(\frac{-11}{7}\right) - 49\left(-\frac{11}{7}\right)^3 = 0$ (Put $a = \frac{-11}{7}$)

$\frac{-1331}{7} + \frac{65,219}{343} = 0$

$\frac{49}{49}\left(-\frac{1331}{7}\right) + \frac{65219}{343} = 0$ (Equating the denominators)

$\frac{-65219 + 65219}{343} = 0$

$0 = 0$ True

For $a = \frac{11}{7}$,

$121a - 49a^3 = 0$

$121\left(\frac{11}{7}\right) - 49\left(\frac{11}{7}\right)^3 = 0$ (Put $a = \frac{11}{7}$)

$\frac{1331}{7} - \frac{65219}{343} = 0$ (Simplify)

$\frac{49}{49} \cdot \frac{1331}{7} - \frac{65219}{343} = 0$ (Simplify)

$\frac{65219}{343} - \frac{65219}{343} = 0$

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{0, \frac{-11}{7}, \frac{11}{7}\right\}}$.

Answer 16PA.

Consider the binomial $x^2 - 49$.

The objective is to factor the given binomial.

Since the difference of squares is

$$a^2 - b^2 = (a+b)(a-b)$$

$x^2 - 49 = x^2 - 7^2$ (Since $49 = 7^2$)

$= (x+7)(x-7)$ (Since $a^2 - b^2 = (a+b)(a-b)$)

Therefore, $x^2 - 49 = (x+7)(x-7)$

Therefore, the factorization of $\boxed{x^2 - 49}$ is $\boxed{(x+7)(x-7)}$.

Answer 17PA.

Consider the binomial $n^2 - 36$

The objective is to factor the given binomial.

Since the difference of squares is

$$a^2 - b^2 = (a+b)(a-b)$$

$n^2 - 36 = n^2 - 6^2$ (Since $36 = 6^2$)

$= (n+6)(n-6)$ (Because $a^2 - b^2 = (a+b)(a-b)$)

Therefore, $n^2 - 36 = (n+6)(n-6)$

Therefore, the factorization of $\boxed{n^2 - 36}$ is $\boxed{(n+6)(n-6)}$.

Answer 18PA.

Consider the binomial $81 + 16k^2$.

The objective is to factor the given binomial.

Since the difference of squares is

$$a^2 - b^2 = (a+B)(a-b)$$

$81 + 16k^2 = 9^2 + 4^2 \cdot k^2$ (Since $81 = 9^2, 16 = 4^2$)

It is not factored.

Therefore, $81 + 16k^2$ is not factored. Since $81, 16k^2$ have no common factors and also

$$a^2 - b^2 = (a+b)(a-b)$$

Here the difference of squares does not exist.

Therefore, $\boxed{81 + 16k^2}$ is a prime.

Answer 19PA.

Consider the binomial $25-4p^2$

The objective is to factor the given polynomial.

Since the difference of squares is

$a^2-b^2=(a+b)(a-b)$

$25-4p^2=5^2-2^2\cdot p^2$ (Since $25=5^2, 4=2^2$)

$=(5+2p)(5-2p)$ (Since $a^2-b^2=(a+b)(a-b)$)

Therefore, the factorization of $\boxed{25-4p^2}$ is $\boxed{(5+2p)(5-2p)}$.

Answer 20PA.

Consider the polynomial $-16+49h^2$.

The objective is to factor the given polynomial.

Since the difference of squares

$a^2-b^2=(a+n)(a-b)$

$-16+49h^2$ can be written as $49h^2-16$ (Because $-b+a=a-b$)

$49h^2-16=7^2\cdot h^2-4^2$ (Because $49=7^2, 16=4^2$)

$=(7h+4)(7h-4)$ (Since $a^2-b^2=(a+b)(a-b)$)

Therefore, $49h^2-16=(7h+4)(7h-4)$

Therefore, the factorization of $\boxed{49h^2-16}$ is $\boxed{(7h+4)(7h-4)}$.

Answer 21PA.

Consider the binomial $-9r^2+121$

The objective is to factor the given polynomial.

Since the difference of squares is

$a^2-b^2=(a+b)(a-b)$

$-9r^2+121=-3^2\cdot r^2+11^2$ (Since $9=3^2, 121=11^2$)

$=11^2-3^2r^2$ (Since $-a+b=b-a$)

$=(11+3r)(11-3r)$ (Because $a^2-b^2=(a+b)(a-b)$)

Therefore, $-9r^2+121=(11+3r)(11-3r)$

Therefore, the factorization of $\boxed{-9r^2+121}$ is $\boxed{(11+3r)(11-3r)}$.

Answer 22PA.

Consider the binomial $100c^2 - d^2$.

The objective is to find factor the given polynomial.

Since the difference of squares is

$a^2 - b^2 = (a+b)(a-b)$

$100c^2 - d^2 = 10^2 \cdot c^2 - d^2$ (Since $100 = 10^2, d = d^2$)

$= (10c)^2 - d^2$ (Simplify)

$= (10c+d)(10c-d)$ (Since $(a+b)(a-b) = a^2 - b^2$)

Therefore, $100c^2 - d^2 = (10c+d)(10c-d)$

Therefore, the factorization of $\boxed{100c^2 - d^2}$ is $\boxed{(10c+d)(10c-d)}$.

Answer 23PA.

Consider the polynomial $9x^2 - 10y^2$.

The objective is to find factor the given polynomial.

$9x^2 - 10 \cdot y^2 = 3 \cdot 3 \cdot x^2 - 2 \cdot 5 \cdot y^2$ (Since $9 = 3 \cdot 3, 10 = 2 \cdot 5$)

$= 3 \cdot 3 \cdot x \cdot x - 2 \cdot 5 \cdot y \cdot y$ (Because $x^2 = x \cdot x, y^2 = y \cdot y$)

$= (3x)^2 - 2 \cdot 5(y)^2$ (Simplify)

It is not factored.

Therefore, $9x^2 - 10y^2$ is not factored. Since $9x^2, 10y^2$ have no common factors.

Therefore, $\boxed{9x^2 - 10y^2}$ is prime.

Answer 24PA.

Consider the polynomial $144a^2-49b^2$.

The objective is to factor the given polynomial.

$144a^2-49b^2=12\cdot12a^2-49b^2 \quad (144=12\cdot12)$

$=12\cdot12\cdot a^2-7\cdot7\cdot b^2 \quad (49=7\cdot7)$

$=12\cdot12\cdot a\cdot a-7\cdot7\cdot b\cdot b$

$\left(a^2=a\cdot a, b^2=b\cdot b\right)$

$=(12a)^2-(7b)^2$ (Simplify)

$=(12a+7b)(12a-7b)$

(Since $a^2-b^2=(a+b)(a-b)$)

Therefore, $144a^2-49b^2=(12a+7b)(12a-7b)$

Therefore, the factorization of $\boxed{144a^2-49b^2}$ is $\boxed{(12a+7b)(12a-7b)}$.

Answer 25PA.

Consider the polynomial $169y^2-36z^2$.

The objective is to factor the given polynomial.

$169y^2-36z^2=13\cdot13y^2-36z^2$ (Since $169=13\cdot13$)

$=13\cdot13\cdot y^2-6\cdot6\cdot z^2$ (Because $36=6^2$)

$=13\cdot13\cdot y^2-6\cdot6\cdot z^2$ (Because $y^2=y\cdot y, z^2=z\cdot z$)

$=(13y)^2-(6z)^2$ (Simplify)

$=(13y+6z)(13y-6z)$ (Since $a^2-b^2=(a+b)(a-b)$)

Therefore,

$169y^2-36z^2=(13y+6z)(13y-6z)$

Therefore, the factorization of

$\boxed{169y^2-36z^2=(13y+6z)(13y-6z)}$.

Answer 26PA.

Consider the polynomial $8d^2-18$

The objective is to find factors of polynomial.

Since the difference of squares is

$$a^2-b^2=(a+b)(a-b)$$

$$8d^2-18=2\cdot2\cdot2\cdot d^2-18 \text{ (Because } 8=2\cdot2\cdot2\text{)}$$

$$=2\cdot2^2\cdot d^2-18 \text{ (Simplify)}$$

$$=2\cdot2^2\cdot d^2-2\cdot9 \text{ (Because } 18=2\cdot9\text{)}$$

$$=2\cdot2^2\cdot d^2-2\cdot3\cdot3 \text{ (Since } 9=3\cdot3\text{)}$$

$$=2\left(2^2\cdot d^2-3^2\right) \text{ (Simplify)}$$

$$=2\left((2d)^2-(3)^2\right) \text{ (Simplify)}$$

$$=2((2d+3)(2d-3)) \text{ (Since } a^2-b^2=(a+b)(a-b)\text{)}$$

$$=2\cdot(2d+3)(2d-3)$$

Therefore,

$$8d^2-18=2\cdot(2d+3)(2d-3)$$

Therefore, the factorization of $\boxed{8d^2-18}$ is $\boxed{2\cdot(2d+3)(2d-3)}$.

Answer 27PA.

Consider the polynomial $3x^2-75$

The objective is to factor the given polynomial.

$3x^2-75=3\cdot x\cdot x-75$ (Since $x^2=x\cdot x$)

$=3\cdot x\cdot x-15\cdot 5$ (Because $75=15\cdot 5$)

$=3\cdot x\cdot x-3\cdot 5\cdot 5$ (Because $15=3\cdot 5$)

$=3(x\cdot 2-5\cdot 5)$ (Simplify)

$=3\left((x)^2-(5)^2\right)$

$=3\left((x+5)(x-5)\right)$ (Since $a^2-b^2=(a+b)(a-b)$)

$=3\cdot(x+5)(x-5)$

Therefore,

$3x^2-75=3\cdot(x+5)(x-5)$

Therefore, the factorization of $\boxed{3x^2-75}$ is $\boxed{3(x+5)(x-5)}$.

Answer 28PA.

Consider the polynomial $8z^2-64$.

The objective is to factors the given polynomial.

Since the difference of squares is

$a^2-b^2=(a+b)(a-b)$

$8z^2-64=2\cdot 2\cdot 2\cdot z^2-64$ (Since $8=2\cdot 2\cdot 2$)

$=2\cdot 2^2\cdot z^2-64$ (Simplify)

$=2\cdot 2^2\cdot z^2-2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2$

(Since $64=2\cdot 32, 32=2\cdot 2\cdot 2\cdot 2\cdot 2$)

$=2\cdot 2^2-z^2-2\cdot 2^5$

It is not factored.

Therefore, $8z^2-64$ is not factored, since $8z^2, 64$ have no common factors.

Therefore, $\boxed{8z^2-64}$ is prime.

Answer 29PA.

Consider the polynomial $4g^2-50$.

The objective is to factor the given polynomial.

Since the difference of squares is

$a^2-b^2=(a+b)(a-b)$.

$4g^2-50=2\cdot2\cdot g^2-50$ (Since $4=2\cdot2$)

$=2^2\cdot g^2-2\cdot25$ (Simplify, $50=2\cdot25$)

It is not factored.

Therefore, $4g^2-50$ is not factored. Since $4g^2,50$ have no common factors.

Therefore, $\boxed{4g^2-50}$ is a prime.

Answer 30PA.

Consider the polynomial $18a^4-72a^2$

The objective is to factor the given polynomial.

Since the difference of squares is

$a^2-b^2=(a+b)(a-b)$

$18a^4-72a^2=9\cdot2\cdot a^4-72a^2$ (Since $18=9\cdot2$)

$=3\cdot3\cdot2\cdot a^4-72a^2$ (Because $3\cdot3=9$)

$=3\cdot3\cdot2\cdot a^2\cdot a^2-72a^2$ (Because $a^4=a^2\cdot a^2$)

$=3\cdot3\cdot2\cdot a^2\cdot a^2-8\cdot9\cdot a^2$ (Because $72=8\cdot9$)

$=3^2\cdot2\cdot a^2\cdot a^2-8\cdot9\cdot a^2$ (Simplify)

$=(3a)^2\cdot2\cdot a^2-2\cdot4\cdot3\cdot3\cdot a^2$ (Because $9=3\cdot3,8=2\cdot4$)

$=(3a)^2\cdot2\cdot a^2-2\cdot a^2\cdot2\cdot2\cdot3\cdot3$

(Because $4=2\cdot2$)

$=(3a)^2\cdot2a^2-2\cdot a^2\cdot2^2\cdot3^2$ (Simplify)

$=(3a)^2\,2a^2-2\cdot a^2\cdot(2\cdot3)^2$ (Simplify)

$=2a^2\left((3a)^2-(6)^2\right)$ (Take common $2a^2$)

$=2a^2\left((3a+6)(3a-6)\right)$ (Since $a^2-b^2=(a+b)(a-b)$)

$=2a^2\cdot(3a+6)(3a-6)$ (Simplify)

Therefore, the factorization of $\boxed{18a^4-72a^2}$ is $\boxed{2a^2\cdot(3a+6)(3a-6)}$.

Answer 31PA.

Consider the polynomial $20x^3 - 45xy^2$

The objective is to factors the given polynomial.

Since the difference of squares is

$$a^2 - b^2 = (a-b)(a+b)$$

$$20x^2 - 45xy^2 = 2\cdot 5x^3 - 9\cdot 5x\cdot y^2$$ (Since $20 = 2\cdot 5, 45 = 9\cdot 5$)

$$= 2\cdot 5\cdot x\cdot x^2 - 3\cdot 3\cdot 5\cdot x\cdot y^2$$ (Because $x^3 = x\cdot x^2, 9 = 3\cdot 3$)

$$= 5x\left(2x^2 - 3^2\cdot y^2\right)$$ (Simplify)

It is not factored.

Therefore, $20x^3 - 45xy^2$ is not factored. Since $20x^3 - 45xy^2$ have no common factors.

Therefore, $\boxed{20x^3 - 45xy^2}$ is prime.

Answer 33PA.

Consider the polynomial $(a+b)^2 - c^2$

The objective is to factor the given polynomial.

Since the difference of squares is

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^2 - c^2 = (a+b+c)((a-b)-c)$$ (Factor the difference of squares)

$$= (a+b+c)(a+b-c)$$

Therefore, the factorization of $\boxed{(a+b)^2 - c^2}$ is $\boxed{(a+b+c)(a+b-c)}$.

Answer 34PA.

Consider the equation $25x^2 = 36$

$$25x^2 - 36 = 36 - 36$$ (Subtract 36 on each side)

$$25x^2 - 36 = 0$$

The objective is to find the solution set of given equation.

$$25x^2 - 36 = 0$$

$$5^2\cdot x^2 - 6^2 = 0$$ (Because $25 = 5^2, 36 = 6^2$)

$(5x)^2 - 6^2 = 0$ (Simplify)

$(5x+6)(5x-6) = 0$ (Since $a^2 - b^2 = (a+b)(a-b)$)

The zero product property is if

$ab = 0$ then

$a = 0$

Or, $b = 0$ or both.

By the zero product property.

$5x + 6 = 0$

Or, $5x - 6 = 0$

Now solve each equation separately.

$5x + 6 = 0$

$5x + 6 - 6 = 0 - 6$ (Subtract 6 on both sides)

$5x = -6$

$\frac{5x}{5} = \frac{-6}{5}$ (Divide by 5 on each side)

$x = \frac{-6}{5}$

$5x - 6 = 0$

$5x - 6 + 6 = 0 + 6$ (Add 6 on each side)

$5x = 6$

$\frac{5x}{5} = \frac{6}{5}$ (Divide by 5 on both sides)

$x = \frac{6}{5}$

The solution set is $\left\{\frac{-6}{5}, \frac{6}{5}\right\}$.

Check:- To check the proposed solution set, substitute each solution in given equation.

For $x = \frac{-6}{5}$,

$25x^2 - 36 = 0$

$25\left(-\frac{6}{3}\right)^2-36=0$ (Put $x=-\frac{6}{5}$)

$25\left(\frac{36}{25}\right)-36=0$

$\frac{900}{25}-36\cdot\frac{25}{25}=0$ (Equating the denominators)

$\frac{900-900}{25}=0$ (Simplify)

$0=0$ True

For $x=\frac{6}{5}$

$25x^2-36=0$

$25\left(\frac{6}{5}\right)^2-36=0$ (Put $x=\frac{6}{5}$)

$25\left(\frac{36}{25}\right)-36=0$

$\frac{900}{25}-36\cdot\frac{25}{25}=0$ (Equating the denominators)

$\frac{900-900}{25}=0$ (Simplify)

$0=0$ True

For $x=\frac{6}{5}$

$25x^2-36=0$

$25\left(\frac{6}{5}\right)^2-36=0$ (Put $x=\frac{6}{5}$)

$25\left(\frac{36}{25}\right)-36=0$

$$\frac{900}{25} - 36 \cdot \frac{25}{25} = 0$$ (Equating the denominators)

$$\frac{900 - 900}{25} = 0$$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{-\frac{6}{5}, \frac{6}{5}\right\}}$.

Answer 35PA.

Consider the equation $9y^2 = 64$

The objective is to find solution set of given equation.

$$9y^2 - 64 = 0$$

$$3^2 \cdot y^2 - 8^2 = 0 \quad \left(9 = 3^2, 64 = 8^2\right)$$

$$(3y)^2 - 8^2 = 0$$

$(3y+8)(3y-8) = 0$ (Since $a^2 - b^2 = (a+b)(a-b)$)

The zero product property is if

$ab = 0$ then

$a = 0$

Or, $b = 0$ or both.

By zero product property,

$3y + 8 = 0$

Or, $3y - 8 = 0$

Now solve each equation separately.

$$3y + 8 = 0$$

$3y + 8 - 8 = 0 - 8$ (Subtract 8 on each side)

$$3y = -8$$

$\frac{3y}{3} = \frac{-8}{3}$ (Divide by 3 on both sides)

$$y = \frac{-8}{3}$$

$3y-8=0$

$3y-8+8=0+8$ (Add 8 on each side)

$3y=8$

$\frac{3y}{3}=\frac{8}{3}$ (Divide by 3 on each side)

$y=\frac{8}{3}$.

The solution set is $\left\{\frac{-8}{3},\frac{8}{3}\right\}$.

Check:- To check the proposed solution set, substitute each solution in given equation.

For $y = -\frac{8}{3}$,

$$9y^2 - 64 = 0$$

$$9\left(-\frac{8}{3}\right)^2 - 64 = 0 \text{ (Put } y = \frac{-8}{3}\text{)}$$

$$9\left(\frac{64}{9}\right) - 64 = 0$$

$$\frac{576}{9} - 64 \cdot \frac{9}{9} = 0 \text{ (Equating the denominators)}$$

$$\frac{576 - 576}{9} = 0 \text{ (Simplify)}$$

$0 = 0$ True

For $y = \frac{8}{3}$,

$$9y^2 - 64 = 0$$

$$9\left(\frac{8}{3}\right)^2 - 64 = 0 \text{ (Put } y = \frac{8}{3}\text{)}$$

$$9\left(\frac{64}{9}\right) - 64 = 0$$

$$\frac{576}{9} - 64 \cdot \frac{9}{9} = 0 \text{ (Equating the denominators)}$$

$$\frac{576 - 576}{9} = 0 \text{ (Simplify)}$$

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{-\frac{8}{3}, \frac{8}{3}\right\}}$.

Answer 36PA.

Consider the equation

$$12 - 27n^2 = 0$$

The objective is to find solution set of given equation.

$$12 - 27n^2 = 0$$

$$3 \cdot 4 - 3 \cdot 9n^2 = 0 \quad (12 = 3 \cdot 4, 27 = 3 \cdot 9)$$

$3\cdot 4-3\cdot 9n^2=0$ $(12=3\cdot 4, 27=3\cdot 9)$

$3\cdot 2^2-3\cdot 3^2\cdot n^2=0$ $\left(4=2^2, 9=3^2\right)$

$3\left(2^2-(3n)^2\right)=0$ (Take common ' 3' on each side)

$3(2+3n)(2-3n)=0$ (Since $a^2-b^2=(a+b)(a-b)$)

The zero product property, if

$ab=0$ then

$a=0$

Or, $b=0$ or both.

By zero product property,

$2+3n=0$

Or, $2-3n=0$ (Since $3\neq 0$)

Now solve each equation separately.

$2+3n=0$

$2+3n-2=0-2$ (Subtract 2 on each side)

$3n=-2$

$\frac{3n}{3}=-\frac{2}{3}$ (Divide by 3 on both sides)

$n=\frac{-2}{3}$

$2-3n=0$

$2-3n-2=0-2$ (Subtract 2 on both sides)

$-3n=-2$

$\frac{3n}{3}=\frac{2}{3}$ (Divide by 3 on each side)

$n=\frac{2}{3}$

The solution set is $\left\{\frac{-2}{3},\frac{2}{3}\right\}$.

Check:- To check the proposed solution set, substitute n by $\frac{-2}{3}, \frac{2}{3}$ in the given equation.

For $n = \frac{-2}{3}$,

$$12 - 27n^2 = 0$$

$$12 - 27\left(-\frac{2}{3}\right)^2 = 0$$ (Put $n = \frac{-2}{3}$)

$$12 - \frac{108}{9} = 0$$

$$\frac{9}{9} \cdot 12 - \frac{108}{9} = 0$$ (Equating the denominators)

$$\frac{108 - 108}{9} = 0$$

$0 = 0$ True

For $n = \frac{2}{3}$,

$$12 - 27n^2 = 0$$

$$12 - 27\left(\frac{2}{3}\right)^2 = 0$$ (Put $n = \frac{2}{3}$)

$$12 - 27\left(\frac{4}{9}\right) = 0$$ (Simplify)

$$12 - \frac{108}{9} = 0$$

$$12 \cdot \frac{9}{9} - \frac{108}{9} = 0$$ (Equating the denominators)

$$\frac{108 - 108}{9} = 0$$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{-\frac{2}{3}, \frac{2}{3}\right\}}$.

Answer 37PA.

Given equation is

$50-8a^2=0$

The objective is to find solution set of given equation.

$50-8a^2=0$

$2\cdot 25-2\cdot 4\cdot a^2=0$ (Since $50=2\cdot 25, 8=2\cdot 4$)

$2\cdot 5^2-2\cdot 2^2\cdot a^2=0$ (Because $25=5^2, 4=2^2$)

$2\left(5^2-2^2a^2\right)=0$ (Take common 2)

$2\left((5+2a)(5-2a)\right)=0$ (Since $a^2-b^2=(a+b)(a-b)$)

The zero product property is if

$ab=0$ then

$a=0$

Or, $b=0$ or both.

By zero product property,

$5+2a=0$

Or, $5-2a=0$

But $2\neq 0$

Now solve each equation separately.

$5+2a=0$

$5+2a-5=0-5$ (Subtract 5 on each side)

$2a=-5$

$\frac{2a}{2}=-\frac{5}{2}$ (Divide by 2 on both sides)

$a=\frac{-5}{2}$

$5-2a=0$

$5-2a-5=0-5$ (Subtract 5 on each side)

$-2a=-5$

$2a=5$ (Simplify)

$\frac{2a}{2}=\frac{5}{2}$ (Divide by 2 on each side)

$a = \frac{5}{2}$.

The solution set is $\left\{\frac{-5}{2}, \frac{5}{2}\right\}$.

Check:- To check the proposed solution set, substitute each solution in given equation.

For $a = \frac{-5}{2}$,

$50 - 8a^2 = 0$

$50 - 8\left(\frac{-5}{2}\right)^2 = 0$ (Put $a = \frac{-5}{2}$)

$50 - 8\left(\frac{25}{4}\right) = 0$ (Simplify)

$50 - \frac{200}{4} = 0$

$50.\frac{4}{4} - \frac{200}{4} = 0$ (Equating the denominators)

$\frac{200}{4} - \frac{200}{4} = 0$ (Simplify)

$0 = 0$ True

For $a = \frac{5}{2}$,

$50 - 8a^2 = 0$

$50 - 8\left(\frac{5}{2}\right)^2 = 0$ (Put $a = \frac{5}{2}$)

$50 - 8\left(\frac{25}{4}\right) = 0$

$50 - \frac{200}{4} = 0$ (Simplify)

$\frac{4}{4} - \frac{200}{4} = 0$ (Equating the denominators)

$\frac{200}{4} - \frac{200}{4} = 0$

$0 = 0$ True

Therefore, the solution set is $\boxed{\left\{\frac{-5}{2},\frac{5}{2}\right\}}$.

Answer 38PA.

Consider the equation

$$w^2-\frac{4}{49}=0$$

$$\frac{49}{49}\cdot w^2-\frac{4}{49}=0$$ (Equating the denominators)

$$\frac{49w^2-4}{49}=0$$

$$49w^2-4=0$$ (Simplify)

The objective is to find solution set of given equation.

$$49w^2-4=0$$

$$7\cdot 7\cdot w^2-2\cdot 2=0 \quad (49=7\cdot 7, 4=2\cdot 2)$$

$$7^2w^2-2^2=0$$ (Simplify)

$$(7w)^2-2^2=0$$

$$(7w+2)(7w-2)=0$$ (Since $a^2-b^2=(a+b)(a-b)$)

The zero product property if

$ab=0$ then

$a=0$

Or, $b=0$ or both.

By zero product property,

$$7w+2=0$$

Or, $7w-2=0$

Now solve each equation sepertely.

$$7w+2=0$$

$$7w+2-2=0-2$$ (Subtract 2 on both sides)

$$7w=-2$$

$$\frac{7w}{-}=\frac{-2}{-}$$ (Divide by 7 on each side)

7 7

$w = \frac{-2}{7}$

$7w - 2 = 0$

$7w - 2 + 2 = 0 + 2$ (Add 2 on each side)

$7w = 2$

$\frac{7w}{7} = \frac{2}{7}$ (Divide by 7 on both sides)

$w = \frac{2}{7}$

The solution set is $\left\{\frac{-2}{7}, \frac{2}{7}\right\}$.

Check:- To check the proposed solution, substitute the each solution in the given equation.

For $w = \frac{-2}{7}$,

$49w^2 - 4 = 0$

$49\left(\frac{-2}{7}\right)^2 - 4 = 0$

$\frac{196}{49} - 4 \cdot \frac{49}{49} = 0$ (Equating the denominators)

$\frac{196 - 196}{49} = 0$ (Simplify)

$0 = 0$ True

For $w = \frac{2}{7}$,

$49w^2 - 4 = 0$

$49\left(\frac{2}{7}\right)^2 - 4 = 0$ (Put $w = \frac{2}{7}$)

$49\left(\frac{9}{49}\right) - 4 = 0$

$\frac{196}{49} - 4 \cdot \frac{49}{49} = 0$ (Equating the denominators)

$\frac{196 - 196}{49} = 0$ (Simplify)

Therefore, the solution set is $\boxed{\left\{\frac{-2}{7}, \frac{2}{7}\right\}}$.

Answer 39PA.

Consider the equation

$$\frac{81}{100} - p^2 = 0$$

$$\frac{81}{100} - p^2 \cdot \frac{100}{100} = 0$$ (Equating the denominators)

$$\frac{81 - 100p^2}{100} = 0$$ (Simplify)

$$81 - 100p^2 = 0$$

The objective is to find solution set of given equation.

$$81 - 100p^2 = 0$$

$$9 \cdot 9 - 10 \cdot 10p^2 = 0 \quad (81 = 9 \cdot 9, 100 = 10 \cdot 10)$$

$$9^2 - 10^2 p^2 = 0 \quad \left(9 \cdot 9 = 9^2, 10 \cdot 10 = 10^2\right)$$

$$\left(9^2 - (10p)^2\right) = 0$$ (Simplify)

$$(9 + 10p)(9 - 10p) = 0$$ (Since $a^2 - b^2 = (a+b)(a-b)$)

The zero product property is if

$ab = 0$ then

$a = 0$

Or, $b = 0$ or both.

By zero product property,

$$9 + 10p = 0$$

Or, $9 - 10p = 0$

Now solve each equation separately.

$$9 + 10p = 0$$

$$9 + 10p - 9 = 0 - 9$$ (Subtract 9 on each side)

$$10p = -9$$

$$\frac{10p}{10} = -\frac{9}{10}$$ (Divide by 10 on both sides)

$$p = \frac{-9}{10}$$

$$9 - 10p = 0$$

$$9 - 10p - 9 = 0 - 9$$ (Subtract 9 on each side)

$9-10p-9=0-9$ (Subtract 9 on each side)

$-10p=-9$

$10p=9$

$\frac{10p}{10}=\frac{9}{10}$ (Divide by 10 on each side)

$p=\frac{9}{10}$

The solution set is $\left\{\frac{-9}{10},\frac{9}{10}\right\}$.

Check:- To check the proposed solution, substitute each solution in the given equation.

For $p=-\frac{9}{10}$,

$81-100p^2=0$

$81-100\left(-\frac{9}{10}\right)^2=0$ (Put $p=\frac{-9}{10}$)

$81-100\left(\frac{81}{100}\right)=0$

$81\cdot\frac{100}{100}-\frac{8100}{100}=0$ (Equating the denominators)

$\frac{8100}{100}-\frac{8100}{100}=0$ (Simplify)

$0=0$ True

For $p=\frac{9}{10}$,

$81-100p^2=0$

$81-100\left(\frac{9}{10}\right)^2=0$ (Put $p=\frac{9}{10}$)

$81-100\left(\frac{81}{100}\right)=0$

$81-\frac{8100}{100}=0$ (Simplify)

$81\cdot\frac{100}{100}-\frac{8100}{100}=0$ (Equating the denominators)

$$\frac{8100}{100}-\frac{8100}{100}=0$$

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{-9}{10},\frac{9}{10}\right\}}$.

Answer 40PA.

Consider the equation

$$36-\frac{1}{9}r^2=0$$

$$36\cdot\frac{9}{9}-\frac{1}{9}\cdot r^2=0 \text{ (Equating the denominators)}$$

$$\frac{324}{9}-\frac{1}{9}r^2=0$$

$$\frac{324-r^2}{9}=0 \text{ (Take } LCM(9,9)=9\text{)}$$

$$324-r^2=0 \text{ (Simplify)}$$

The objective is to find solution set of given equation.

$$324-r^2=0$$

$$18\cdot18-r^2=0 \quad (324=18\cdot18)$$

$$18^2-r^2=0 \text{ (Simplify)}$$

$$(18+r)(18-r)=0$$

The zero product property is if

$ab=0$ then

$a=0$

Or, $b=0$ or both.

By zero product property,

$$18+r=0$$

Or, $18-r=0$

Now solve each equation separately.

$$18+r=0$$

$$18+r-18=0-18 \text{ (Subtract 18 on each side)}$$

$18 + r - 18 = 0 - 18$ (Subtract 18 on each side)

$r = -18$

$18 - r = 0$

$18 - r - 18 = 0 - 18$ (Subtract 18 on each side)

$r = 18$.

The solution set is $\{-18, 18\}$.

Check:- To check the proposed solution set, substitute each solution in given equation.

For $r = -18$,

$324 - r^2 = 0$

$324 - (-18)^2 = 0$ (Put $r = -18$)

$324 - 324 = 0$ (Simplify)

$0 = 0$ True

For $r = 18$,

$324 - r^2 = 0$

$324 - (18)^2 = 0$ (Put $r = 18$)

$324 - 324 = 0$

$0 = 0$ True

Therefore, the solution set is $\boxed{\{-18, 18\}}$.

Answer 41PA.

Consider the equation

$\frac{1}{4}x^2 - 25 = 0$

$\frac{1}{4}x^2 - 25 \cdot \frac{4}{4} = 0$ (Equating the denominators)

$\frac{x^2}{4} - \frac{100}{4} = 0$ (Simplify)

$x^2 - 100 = 0$

The objective is to find solution set of given equation.

$x^2 - 100 = 0$

$x^2 - 10 \cdot 10 = 0$ $\left(100 = 10^2\right)$

$x^2 - 10^2 = 0$ (Simplify)

$(x-10)(x+10) = 0$ (Since $a^2 - b^2 = (a+b)(a-b)$)

The zero product property is if

$ab = 0$ then

$a = 0$

Or, $b = 0$ or both.

By zero product property,

$x - 10 = 0$

Or, $x + 10 = 0$

Now solve each equation separately.

$x - 10 = 0$

$x - 10 + 10 = 0 + 10$ (Add 10 on each side)

$x = 10$

$x + 10 = 0$

$x + 10 - 10 = 0 - 10$ (Add 10 on each side)

$x = -10$.

The solution set is $\{10, -10\}$.

Check:- To check the proposed solution set, substitute each solution in given equation.

For $x^2 - 100 = 0$

$(10)^2 - 100 = 0$ (Put $x = 10$)

$100 - 100 = 0$

$0=0$ True

For $x=-10$,

$x^2-100=0$

$(-10)^2-100=0$

$100-100=0$

$0=0$ True

Therefore, the solution set is $\boxed{\{10,-10\}}$.

Answer 42PA.

Consider the equation $12d^3-147d=0$

The objective is to find the solution set of given equation

$$12d^3-147d=0$$

$$3\cdot4\cdot d\cdot d^2-3\cdot49\cdot d=0$$

$$3d\left[4d^2-49\right]=0 \qquad \left[\text{Factor the GCF } \left(12d^3,147d\right)=3d\right]$$

$$3d\left[2\cdot2d^2-7\cdot7\right]=0 \qquad \left[\text{Since } 2\cdot2=4, 7\cdot7=49\right]$$

$$3d\left[2^2d^2-7^2\right]=0 \qquad \left[\text{Since } x\cdot x=x^2\right]$$

$$3d\left[(2d)^2-7^2\right]=0$$

The difference of squares property is $a^2-b^2=(a+b)(a-b)$

$$3d[2d+7][2d-7]=0$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$3d=0 \text{ or } 2d+7=0 \text{ or } 2d-7=0$$

Now solve each equation completely

$$3d = 0$$
$$d = 0$$
$$2d + 7 = 0$$
$$2d + 7 - 7 = 0 - 7 \qquad [\text{Subtract 7 on both sides}]$$
$$2d = -7$$
$$\frac{2d}{2} = \frac{-7}{2} \qquad [\text{divide with 2 on both sides}]$$
$$d = \frac{-7}{2}$$
$$2d - 7 = 0$$
$$2d - 7 + 7 = 0 + 7 \qquad [\text{Add 7 on both sides}]$$
$$d = \frac{7}{2}$$
$$\frac{2d}{2} = \frac{7}{2} \qquad [\text{Divide with 2 on both sides}]$$
$$d = \frac{7}{2}$$

The solution set of given equation is $\left\{0, \frac{-7}{2}, \frac{7}{2}\right\}$

Answer 43PA.

Consider the equation

$$18n^3 - 50n = 0$$

The objective is to find the solution set of given equation.

$$18n^3 - 50n = 0$$
$$2n \cdot 9n^2 - 2n \cdot 25 = 0$$
$$2n\left(9n^2 - 25\right) = 0$$ (Factor the GCF of $18n^3, 50n$)
$$2n \cdot \left((3n)^2 - 5^2\right) = 0$$ (Simplify)
$$2n(3n+5)(3n-5) = 0$$ (Since $a^2 - b^2 = (a+b)(a-b)$)

The zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

By zero product property,

$2n = 0$

Or, $3n + 5 = 0$

Or, $3n - 5 = 0$

Solve each equation separately.

$2n = 0$

$\frac{2n}{2} = \frac{0}{2}$ (Divide with 2 on each side)

$n = 0$

$3n + 5 = 0$ (Simplify)

$\frac{3n}{3} = \frac{-5}{3}$ (Divide with 3 on both sides)

$n = \frac{-5}{3}$

$3n - 5 = 0$

$3n - 5 + 5 = 0 + 5$ (Add 5 on each side)

$3n = 5$

$\frac{3n}{3} = \frac{5}{3}$ (Divide with 3 on both sides)

$n = \frac{5}{3}$

The solution set is $\left\{0, \frac{-5}{3}, \frac{5}{3}\right\}$.

Check:- To check the proposed solution set, substitute each solution in the given set.

For $n = 0$

$18n^3 - 50n = 0$

$18(0)^3 - 50(0) = 0$ (Put $n = 0$)

$0 - 0 = 0$ (Simplify)

$0=0$ True

For $n=\frac{-5}{3}$,

$18n^3-50n=0$

$18\left(\frac{-5}{3}\right)^3-50\left(\frac{-5}{3}\right)=0$ (Put $n=\frac{-5}{3}$)

$18\cdot\frac{-5}{3}-\frac{-5}{3}\cdot\frac{-5}{3}-50\frac{-5}{3}=0$

$3\cdot 3\cdot 2\cdot\frac{-5}{3}\cdot\frac{-5}{3}\cdot\frac{-5}{3}+\frac{250}{3}$ (Simplify)
$=0$

$-\frac{250}{3}+\frac{250}{3}=0$ (Simplify)

$0=0$ True

For $n=\frac{5}{3}$,

$18n^3-50n=0$

$18\cdot\left(\frac{5}{3}\right)^3-50\left(\frac{5}{3}\right)=0$

$3\cdot 3\cdot 2\cdot\frac{5}{3}\cdot\frac{5}{3}\cdot\frac{5}{3}-50\cdot\frac{5}{3}=0$ (Simplify)

$\frac{250}{3}-\frac{250}{3}=0$

$0=0$ True.

Therefore, the solution set of given equation is $\boxed{\left\{0,\frac{-5}{3},\frac{5}{3}\right\}}$.

Answer 44PA.

Consider the equation $x^3 - 4x = 12 - 3x^2$

The objective is to find the solution set of given equation

$$x^3 - 4x = 12 - 3x^2$$

$$x^3 - 4x + 3x^2 = 12 - 3x^2 + 3x^2 \quad \left[\text{Add } 3x^2 \text{on both sides}\right]$$

$$x^3 + 3x^2 - 4x = 12$$

$$x^3 + 3x^2 - 4x - 12 = 12 - 12 \quad [\text{Subtract 12 on both sides}]$$

$$x^3 + 3x^2 - 4x - 12 = 0$$

$$\left(x^3 - 4x\right) + \left(3x^2 - 4\right) = 0 \quad \left[\begin{array}{l}\text{Group the terms having} \\ \text{common factors}\end{array}\right]$$

$$\left(x \cdot x^2 - 4 \cdot x\right) + \left(3 \cdot x^2 - 3 \cdot 4\right) = 0$$

$$x\left(x^2 - 4\right) + 3\left(x^2 - 4\right) = 0 \quad [\text{Factor of GCF}]$$

$$(x+3)\left(x^2 - 4\right) = 0 \quad \left[\text{By distributive } (b+c)a = ba + ca\right]$$

$$(x+3)\left(x^2 - 2^2\right) = 0 \quad \left[\text{Since } 2^2 = 4\right]$$

The difference of squares property is $a^2 - b^2 = (a+b)(a-b)$

$$x + 3 = 0 \text{ or } x + 2 = 0 \text{ or } x - 2 = 0$$

Now solve each equation completely

$$x + 3 = 0$$

$$x + 3 - 3 = 0 - 3 \quad [\text{Subtract 3 on both sides}]$$

$$x = -3$$

$$x + 2 = 0$$

$$x + 2 - 2 = 0 - 2 \quad [\text{Subtract 2 on both sides}]$$

$$x = -2$$

$$x - 2 = 0$$

$$x - 2 + 2 = 0 + 2 \quad [\text{Add 2 on both}]$$

$$x = 2$$

The solution set of given equation is $\{-3, -2, 2\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify.

Given equation is $x^3 - 4x = 12 - 3x^2$

$$(-3)^3 - 4(-3) = 12 - 3(-3)^2 \qquad [\text{put } x = 0]$$
$$-8 + 8 = 12 - 12 \qquad [\text{Simplify}]$$
$$0 = 0 \quad \text{True}$$

For $x = -2$

$$x^3 - 4x = 12 - 3x^2$$
$$(-2)^3 - 4(-2) = 12 - 3(-2)^2 \qquad [\text{Put } x = -2]$$
$$8 + 8 = 12 - 12 \qquad [\text{Simplify}]$$

$$0 = 0 \ \text{True}$$

For $x = 2$,

$$x^3 - 4x = 12 - 3x^2$$
$$(2)^3 - 4(2) = 12 - 3(2)^2 \qquad [\text{put } x = 2]$$
$$8 - 8 = 12 - 12 \qquad [\text{Simplify}]$$
$$0 = 0 \ \text{True}$$

Therefore, the solution set of given equation is $\{-3, -2, 2\}$

Answer 45PA.

Consider the equation $36x-16x^3=9x^2-4x^4$

The objective is to find the solution set of given equation

$$36x-16x^3=9x^2-4x^4 \quad \left[\text{Add } 4x^2 \text{ on both sides}\right]$$

$$36x+16x^3+4x^4=9x^2 \quad \left[\text{Combine like terms}\right]$$

$$36x-16x^3+4x^4-9x^2=9x^2-9x^2 \quad \left[\text{Subtract } 9x^2 \text{ on both sides}\right]$$

$$4x^4-16x^3-9x^2+36x=0 \quad \left[\text{Combine like terms}\right]$$

$$36x-16x^3=9x^2-4x^4 \quad \left[\text{Add } 4x^2 \text{ on both sides}\right]$$

$$36x+16x^3+4x^4=9x^2 \quad \left[\text{Combine like terms}\right]$$

$$36x-16x^3+4x^4-9x^2=9x^2-9x^2 \quad \left[\text{Subtract } 9x^2 \text{ on both sides}\right]$$

$$4x^4-16x^3-9x^2+36x=0 \quad \left[\text{Combine like terms}\right]$$

$$x\left(4x^2-9\right)\left(x-4\right)=0 \quad \left[\text{Factor of GCF}\right]$$

$$x\left[(2x)^2-3^2\right]\left(x-4\right)=0 \quad \left[\begin{array}{l}\text{By difference of squares}\\ a^2-b^2=(a+b)(a-b)\end{array}\right]$$

$$x(2x+3)(2x-3)(x-4)=0$$

The zero product property is of $ab=0$ then $a=0$ or $b=0$ or both

$$x=0 \text{ or } 2x+3=0 \text{ or } 2x-3=0 \text{ or } x-4=0$$

Now solve each equation separately

$$x = 0$$

$$2x + 3 = 0$$

$$2x + 3 - 3 = 0 - 3 \qquad \text{[Subtract 3 on both sides]}$$

$$2x = -3$$

$$\frac{2x}{2} = \frac{-3}{2} \qquad \text{[Divide with 2 on both sides]}$$

$$x = \frac{-3}{2}$$

$$2x - 3 = 0$$

$$2x - 3 + 3 = 0 + 3 \qquad \text{[Add 3 on both sides]}$$

$$2x = 3$$

$$\frac{2x}{2} = \frac{3}{2} \qquad \text{[Divide with 2 on both sides]}$$

$$x = \frac{3}{2}$$

$$x - 4 = 0$$

$$x - 4 + 4 = 0 + 4 \qquad \text{[Add 4 on both sides]}$$

$$x = 4$$

The solution set of given equation is $\left\{0, \frac{-3}{2}, \frac{3}{2}, 4\right\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify.

Given equation is $36x-16x^3=9x^2-4x^4$

$$36(0)-16(0)^3=9(0)-4(0)^4 \qquad [\text{put } x=0]$$
$$0-0=0-0$$
$$0=0 \quad \text{True}$$

For $x=\frac{-3}{2}$

$$36x-16x^3=9x^2-4x^4$$
$$36\left(\frac{-3}{2}\right)-16\left(\frac{-3}{2}\right)^3=9\left(\frac{-3}{2}\right)^2-4\left(\frac{-3}{2}\right)^4 \qquad \left[\text{Put } x=\frac{-3}{2}\right]$$
$$18(-3)-16\left(\frac{-27}{8}\right)=9\cdot\frac{9}{4}-4\cdot\frac{81}{16} \qquad [\text{Simplify}]$$
$$-54+54=\frac{81}{4}-\frac{81}{4} \qquad [\text{Simplify}]$$

$0=0$ True

For $x=4$.

$$36x-16x^3=9x^2-4x^4$$
$$36(4)-16(4^3)=9(4)^2-4(4)^4 \qquad [\text{put } x=4]$$
$$144-16\cdot(64)=9\cdot(16)-4(256) \qquad [\text{Simplify}]$$
$$-944=-944 \quad \text{True}$$

Therefore, the solution set of given equation is $\left\{0,\frac{-3}{2},\frac{3}{2},4\right\}$

Answer 46PA.

The objective is to show that $a^2 - b^2 = (a+b)(a-b)$ algebraically

Consider

$$\begin{aligned} a^2 - b^2 &= a^2 - ab + ab - b^2 && [\text{add and subtract } ab] \\ &= a \cdot a - ab + a \cdot b - b \cdot b && [\text{Since; } a^2 = a \cdot a, b^2 = b \cdot b] \\ &= a(a-b) + b(a-b) && [\text{Factor GCF}] \\ &= (a+b)(a-b) && [\text{Since; by distributive } (b+c) = a = ba + ca] \end{aligned}$$

Therefore, $\boxed{a^2 - b^2 = (a+b)(a-b)}$

Answer 47PA.

Consider that the basic breaking strength *b* in pounds for a natural fiber line is determined by the formula

$900c^2 = b$, where *c* is the circumference of the line in inches.

The objective is to find the circumference of natural line would have 3600 pounds of breaking strength.

Given $b = 3600$

$$\begin{aligned} 900c^2 &= b \\ 900c^2 &= 3600 && [\text{put } b = 3600] \\ 900c^2 - 3600 &= 3600 - 3600 && [\text{Subtract 3600 on both sides}] \\ 900c^2 - 3600 &= 0 \\ 30 \cdot 30c^2 - 60 \cdot 60 &= 0 && [900 = 30 \cdot 30, 3600 = 60 \cdot 60] \\ 30^2 c^2 - 60^2 &= 0 \\ (30c)^2 - 60^2 &= 0 \end{aligned}$$

The different of squares property is $a^2 - b^2 = (a+b)(a-b)$

$$(30c + 60)(30c - 60) = 0 \qquad [\text{By difference of squares property}]$$

By zero product property is of $ab = 0$ then $a = 0$ or $b = 0$ or both.

Now solve each equation separately.

$$30c + 60 = 0$$

$$30c + 60 - 60 = 0 - 60 \qquad [\text{Subtract 60 on both sides}]$$

$$\frac{30c}{30} = \frac{-60}{30} \qquad [\text{Divide with 30 on both sides}]$$

$$c = -2$$

$$30c - 60 = 0$$

$$30c - 60 + 60 = 0 + 60 \qquad [\text{Add 60 on both sides}]$$

$$30c = 60$$

$$\frac{30c}{30} = \frac{60}{30} \qquad [\text{Divide with 30 on both sides}]$$

$$c = 2$$

Since length always positive we consider $c = 2$

Therefore, the circumference of natural line is $\boxed{2 \text{ in}}$

Answer 48PA.

Consider that the formula for the pressure difference *p* above and below a wing is described by the formula $p = \frac{1}{2}dv_1^2 - \frac{1}{2}dv_2^2$

Where *d* is the density of the air.

v_1 is the velocity of the air passing above

v_2 is the velocity of the air passing below

The objective is the write the given formula in factored form.

$$p = \frac{1}{2}dv_1^2 - \frac{1}{2}dv_2^2$$

$$= \frac{1}{2}d\left[v_1^2 - v_2^2\right] \qquad \left[\text{Factor the CFF } \frac{1}{2}d\right]$$

The difference of squares property is $a^2 - b^2 = (a+b)(a-b)$

$$p = \frac{1}{2}d(v_1 + v_2)(v_1 - v_2) \qquad [\text{By difference of squares}]$$

The given formula in factored form is $\boxed{p = \frac{1}{2}d(v_1 + v_2)(v_1 - v_2)}$

Answer 49PA.

Consider that, of a car skids on dry concrete, police can use the formula, $\frac{1}{2u}s^2 = d$ to approximate the speed of a vehicle in miles per hour given the length *d* of the skid marks in feet

Consider that the length of the skid marks on dry concrete are 54 feet long.

The objective is find the speed of can travelling when the breaks were applied,

Here $d = 54 \text{ feet}$

$$\frac{1}{24}s^2 = d$$

$$\frac{1}{24}s^2 = 54 \qquad [d = 54]$$

$$24 \cdot \frac{1}{24}s^2 = 54 \cdot 24 \qquad [\text{Multiply with 24 on both sides}]$$

$$s^2 = 1296$$

The square root property is if $n > 0, x^2 = n$ then $x = \pm\sqrt{n}$

$$s^2 = 1296$$

$$s = \pm\sqrt{1296} \qquad [\text{By square root property}]$$

$$s = \pm\sqrt{(36)^2}$$

$$s = \pm 36$$

Since speed always positive, consider $s = 36$

Therefore, the speed of car traveling when the breaks were applied is $\boxed{36 \text{ mph}}$

Answer 51PA.

Consider that,

a and *b* are real numbers such that $a=b, a\neq 0, b\neq 0$

(1) $a=b$ (Given)

(2) $a^2=ab$ (Multiply each side by *a*)

(3) $a^2-b^2=ab-b^2$ (Subtract b^2 on each side)

(4) $(a+b)(a-b)=b(a-b)$ (Factor)

(5) $a+b=b$ (Divide each side by $a-b$)

(6) $a+a=b$ (Substitute property $a=b$)

(7) $2a=a$ (Combine like terms)

(8) $2=1$ (divide each side by *a*)

The objective is to find the flow in above proof

The flow is in line 5

Since $a=b$ then $a-b=0$

Dividing by $a-b$ means dividing with zero. which is undefined.

Answer 53PA.

Consider the polynomial $25b^2-1$

The objective is to find the factorization of given polynomial

$$\begin{aligned} 25b^2-1 &= 5\cdot 5\cdot b^2-1 && [\text{Since; } 25=5\cdot 5] \\ &= 5^2b^2-1 && [x^2=x\cdot x] \\ &= (5b)^2-1 && [a^m\cdot b^m=(ab)^m] \\ &= (5b)^2-1^2 && [1^2=1\cdot 1=1] \end{aligned}$$

Since the difference of squares property is

$$a^2-b^2=(a+b)(a-b)$$

$$\begin{aligned} 25b^2-1 &= (5b)^2-1^2 \\ &= (5b+1)(5b-1) \end{aligned}$$

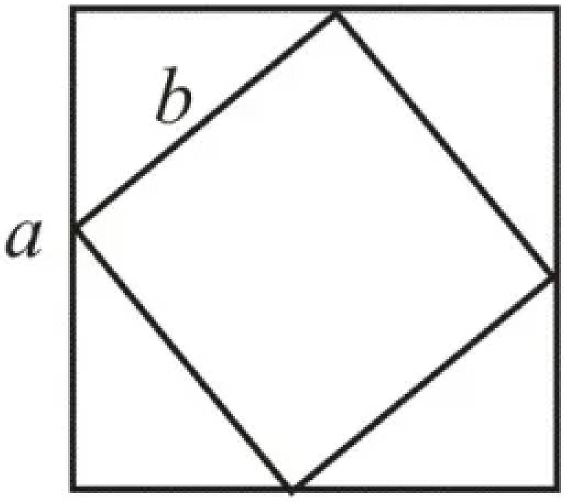
Therefore, the factored form of $25b^2-1$ is $\boxed{(5b+1)(5b-1)}$

Answer 54PA.

Consider that the area between the two squares is 17 square inches.

Also the sum of the perimeters of the two squares is 68 inches.

The objective is the find the length of the side of large square.



Let the length of large square = *a*

Length of short square = *b*

Area of the large square $= a^2$

Area of the short square $= b^2$

Area between two squares = Area of large square – Area of short square

$$= a^2 - b^2$$

Given that $a^2 - b^2 = 17$

Perimeter of large square $= 4a$

Perimeter of short square $= 4a$

Sum of perimeters of square $= 68$

$4a + 4b = 68$

$4(a+b) = 4 \cdot 17$

$a + b = 17$ ...... (1)

$a^2 - b^2 = 17$

$(a+b)(a-b) = 17$ ..... (2)

From (1), $(a+b)(a-b) = 17$

$17(a-b) = 17$ $\quad$ [Substitute $a + b = 17$]

$\frac{17(a-b)}{17} = \frac{17}{14}$ $\quad$ [Divide with 17 on both sides]

$a - b = 1$

$\Rightarrow a = 1 + b$

$a - b = 1$

$a + b = 17$

$1 + b + b = 17$ $\quad$ [Put $a = 1 + b$]

$2b = 17 - 1$

$2b = 16$

$\frac{2b}{2} = \frac{16}{2}$ $\quad$ [Divide with 2 on both sides]

$b = 8$

$a = 1 + b$

$= 1 + 8$

$= 9$

Therefore, the length of large square is $\boxed{9 \text{ inches}}$

Answer 55MYS.

Consider the trinomial $2n^2+5n+7$.

The objective is to factor the given polynomial

Compare $2n^2+5n+7$ with ax^2+bx+c.

Here $a=2$,

$b=5$,

$c=7$

Now find two numbers m,n such that the sum is

$b=5$ and the product is

$$ac=2\cdot 7$$
$$=14$$

Since $m+n=5$ and

$mn=14$ positive.

For this, first list all the factors of 14 and choose one pair in those whose sum is 5.

Factors of 14	Sum of factors
1.14	15
2.7	9

There are no prime factors whose sum is 5.

Therefore, $2n^2+5n+7$ cannot be factored using integers.

Since a polynomial that cannot be written as a product of two polynomials with integral co-efficient is called a prime polynomial.

Therefore, $\boxed{2n^2+5n+7}$ has no solution.

Answer 56MYS.

Consider the trinomial $6x^2-11x+4$.

The objective is to factors of given trinomial.

Compare $6x^2-11x+4$ with ax^2+bx+c.

Here $a=6$,

$b = -11,$

$c = 4$

Now find two numbers m, n such that whose sum is

$b = -11$ and whose product is

$ac = 6 \cdot 4$

$= 24$

Since $m + n = -11$ negative and

$mn = 24$ positive.

So, m and n must be both negative.

Now list all factors of 24, choose in those one pair whose sum is -11.

Factors of 24	Sum of factors
$-1. -24$	-25
$-2. -12$	-14
$-3. -8$	-11
$-4. -6$	-10

The correct factors are $-3, -8$.

$6x^2 - 11x + 4 = 6x^2 - 3x - 8x + 4$ (Because $-11 = -3 - 8$)

$= 2 \cdot 3 \cdot x^2 - 3 \cdot x - 2 \cdot 4 \cdot x + 4$

$(6 = 2 \cdot 3, 8 = 2 \cdot 4)$

$= 3x(2x - 1) - 4(2x - 1)$ (Group all terms with their common factors)

$= (3x - 4)(2x - 1)$ (By distributive)

Therefore,

$6x^2 - 11x + 4 = (3x - 4)(2x - 1)$

Check:- To check the solution, using $FOIL$ method, by multiplying two factors.

$(3x-4)(2x-1) = \overset{F}{3x \cdot 2x} - \overset{O}{1 \cdot 3x} - \overset{I}{4 \cdot 2x} + \overset{L}{4 \cdot 1}$

($FOIL$ method)

$= 6x^2 - 3x - 8x + 4$ (Simplify)

$= 6x^2 - 11x + 4$ True

Therefore, the factorization of $\boxed{6x^2 - 11x + 4}$ is $\boxed{(3x-4)(2x-1)}$.

Answer 57MYS.

Consider the trinomial $21p^2 + 29p - 10$.

The objective is to factors the given trinomial.

Compare $21p^2 + 29p - 10$ with $ax^2 + bx + c$.

Here $a = 21,$

$b = 29,$

$c = -10$

Now find two numbers m, n such that whose sum is

$b = 29$ and whose product is

$ac = 21 \cdot -10$

$= -210$

Since $m + n = 29$ positive and

$mn = -210$ negative.

So, either m (or) n negative but not both.

Now list all the factors of -210, choose in those one pair whose sum is 29.

Factors of -210	Sum of factors
$1, -210$	-209
$-1, 210$	209

-2.105	103
$2.-105$	-103
$3.-70$	-67
-3.70	67
$6.-35$	-29
-6.35	29
-7.30	23
$7.-30$	-23
-10.21	11
$10.-21$	-11

The correct factors are $-6,35$.

$21p^2+29p-10=21p^2-6p+35p-10$

(Because $29=-6+35$)

$=7\cdot 3p^2-2\cdot 3p+7\cdot 5p-2\cdot 5$

$(21=7\cdot 3,6=2\cdot 3,35=7\cdot 5,10=2.5)$

$=3p(7p-2)+5(7p-2)$

(Group all terms with common factors)

$=(3p+5)(7p-2)$

(By distributive)

Check:- To check the solution, multiplying two factors by using $FOIL$ method.

$(3p+5)(7p-2)=3p\cdot\overset{F}{7}p-2\cdot\overset{O}{3}p+5\cdot\overset{I}{7}p-5\cdot\overset{L}{2}$

($FOIL$ method)

$=21p^2-6p+35p-10$

(Simplify)

$=21p^2+29p-10$

True

Therefore, the factorized of $\boxed{21p^2+29p-10}$ is $\boxed{(3p+5)(7p-2)}$.

Answer 58MYS.

Consider the equation

$y^2+18y+32=0$

The objective is to solve the given equation. For this first find the factors of given equation and then use zero product property.

Compare $y^2+18y+32$ with ax^2+bx+c.

Here $a=1$,

$b=18$,

$c=32$

Now find two numbers m,n such that whose sum is

$b=18$ and whose product is

$ac=1\cdot 32$

$\quad =32$

Since $m+n=18$ positive and

$mn=32$ also positive.

Now list all the factors of 32, choose one pair in those whose sum is 18.

Factors of 32	Sum of factors

1.32	32
2.16	18
4.8	12

The correct factors are $2,16$.

Therefore, $y^2+18y+32=y^2+2y+16y+32$ $(18=2+16)$

$=y^2+2y+2\cdot 8y+8\cdot 4$ (Because $16=2\cdot 8, 32=8\cdot 4$)

$=y^2+2y+2\cdot 2\cdot 4y+2\cdot 4\cdot 4$

(Because $8=2\cdot 4$)

$=y(y+2)+16(y+2)$ (Group all terms with common factors)

$=(y+16)(y+2)$

Now $y^2+18y+32=0$

$(y+16)(y+2)=0$ (Factors)

$y+16=0$

Or, $y+2=0$ (Using zero product property)

Now solve each equation separately.

$y+16=0$

$y+16-16=0-16$ (Subtract 16 on both sides)

$y=-16$

$y+2=0$

$y+2-2=0-2$ (Subtract 2 on each side)

$y=-2$

The solution set is $\{-16,-2\}$.

Check:- To check the solution, substitute y by $-16,-2$ in the given equation.

For $y=-16$,

$y^2+18y+32=0$

$(-16)^2+18(-16)+32=0$ (Put $y=-16$)

$256-288+32=0$ (Simplify)

$288-288=0$

$0=0$ True

For $y=-2$,

$y^2+18y+32=0$

$(-2)^2+18(-2)+32=0$ (Put $y=-2$)

$4-36+32=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{-16,-2\}}$.

Answer 59MYS.

Consider the equation

$k^2-8k=-15$

$k^2-8k+15=-15+15$ (Add 15 on each side)

$k^2-8k+15=0$

The objective is to solve the given equation. For this, first find factors and then use zero product property.

Compare $k^2-8k+15$ with ax^2+bx+c.

Here $a=1,$

$b=-8,$

$c=15$

Now find two numbers m,n such that whose sum is

$b=-8$ and whose product is

$$ac=1\cdot 15$$
$$=15$$

Since $m+n=-8$ negative and

$mn=15$ positive.

So, m and n must be both negative.

Now list all the factors of 15 and choose one pair in those whose sum is -8.

Factors of 15	Sum of factors
$-1.-15$	-16
$-3.-5$	-8

The correct factors are $-3,-5$.

$k^2-8k+15=k^2-3k-5k+15$

(Because $-8=-3-5$)

$=k^2-3k-5k+3\cdot 5$

($15=3\cdot 5$)

$=k(k-3)-5(k-3)$

(Group of all terms with common factors)

$=(k-5)(k-3)$

Therefore, $k^2 - 8k + 15 = 0$

$(k-5)(k-3) = 0$ (Factors)

$k - 5 = 0$

Or, $k - 3 = 0$ (Using zero product property)

Now solve each equation separately.

$k - 5 = 0$

$k - 5 + 5 = 0 + 5$ (Add 5 on each side)

$k = 5$

$k - 3 = 0$

$k - 3 + 3 = 0 + 3$ (Add 3 on each side)

$k = 3$

The solution set is $\{5,3\}$.

Check:- To check the solution, substitute k by $5,3$ in the given equation.

For $k = 5$,

$k^2 - 8k + 15 = 0$

$(5)^2 - 8(5) + 15 = 0$ (Put $k = 5$)

$25 - 40 + 15 = 0$

$0 = 0$ True

For $k = 3$,

$k^2 - 8k + 15 = 0$

$(3)^2 - 8(3) + 15 = 0$ (Put $k = 3$)

$9 - 24 + 15 = 0$

$0 = 0$ True

Therefore, the solution set is $\boxed{\{5,3\}}$.

Answer 60MYS.

Consider the equation $b^2 - 8 = 2b$

The objective is to find the solution set of given equation

$$\begin{aligned} b^2 - 8 &= 2b \\ b^2 - 8 - 2b &= 2b - 2b \quad [\text{Subtract } 2b \text{ on both sides}] \\ b^2 - 2b - 8 &= 0 \quad [\text{Combine like terms}] \end{aligned}$$

First factor $b^2 - 2b - 8$

Compare $b^2 - 2b - 8$ with $x^2 + ax + c$

Here $a = -2, c = -8$

$$\begin{aligned} b^2 - 2b - 8 &= (b+m)(b+n) \\ &= b^2 + (m+n)b + mn \end{aligned}$$

Now find two numbers *m*, *n* such that $m + n = -2$ and $mn = -8$

Since $m + n, mn$ are negative then one of *m* or *n* must be negative but not both.

List all the factors of $mn = -8$ in choose a pair whose sum is -2

Factors of -8	Sum of factors
$-1 \cdot 8$	7
$1 \cdot -8$	-7
$-2 \cdot 4$	2
$2 \cdot -4$	✓ -2

The correct factors are $2, -4$

$$b^2 - 2b - 8 = (b+m)(b+n)$$
$$= (b+2)(b-4) \qquad [m = 2, n = -4]$$

$$b^2 - 2b - 8 = 0$$
$$(b+2)(b-4) = 0$$

The zero product property is if $ab = 0$ then $a = 0$ or $b = 0$ or both $b + 2 = 0$ or $b - 4 = 0$

Now solve each equation completely

$$b + 2 = 0$$
$$b + 2 - 2 = 0 - 2 \qquad [\text{Subtract 2 on both sides}]$$
$$b = -2$$

$$b - 4 = 0$$
$$b - 4 + 4 = 0 + 4 \qquad [\text{add 4 on both sides}]$$
$$b = 4$$

The solution set is $\{-2, 4\}$

Check:

To check the proposed solution set substitute each solution in the given equation and verify

Given equation is $b^2 - 8 = 2b$

$$(-2)^2 - 8 = 2(-2) \qquad [\text{Put } b = -2]$$
$$4 - 8 = -4$$
$$-4 = -4 \text{ True}$$

$$b^2 - 8 = 2b$$
$$4^2 - 8 = 2(4) \qquad [\text{Put } b = 4]$$
$$16 - 8 = 8$$
$$8 = 8 \text{ True}$$

Therefore, the solution set of given equation is $\boxed{\{-2, 14\}}$

Answer 62MYS.

Consider the inequality $6 \le 3d - 12$

The objective is to solve the given inequality

To solve the given inequality

$$6 \le 3d - 12$$

$$6 + 12 \le 3d - 12 + 12 \quad [\text{Add 12 on both sides}]$$

$$18 \le 3d$$

$$\frac{18}{3} \le \frac{3d}{3} \quad [\text{divide with 3 on both sides}]$$

$$6 \le d$$

$$d \ge 6$$

Thus the solution is $d \ge 6$

Check:

To check the solution take one number less than 6 and one number greater than 6 and substitute these in given equality and verify.

For $d = 5$,

$$6 \le 3d + 12$$

$$6 \le 3(5) - 12 \quad [\text{Put } d = 5]$$

$$6 \le 15 - 12$$

$$6 \le 3 \text{ False}$$

Thus, 5 is not in the solution set

For $d = 7$,

$$6 \le 3d - 12$$

$$6 \le 3(7) - 12 \quad [\text{Put } d = 7]$$

$$6 \le 21 - 12$$

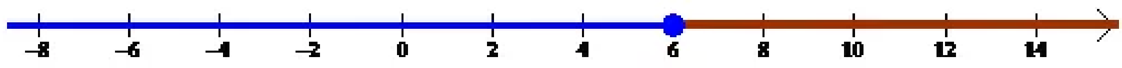
$$6 \le 9 \quad \text{True}$$

Thus $d = 7$ is in the solution set

The graph $d \geq 6$ is as below

In the graph the inequality symbols < or > are indicated by open circles and the inequality symbols $\geq$ or $\leq$ are indicated by closed circles.

The open circles indicate that the particular values are not included in the solution where as closed circles indicate that the particular values are included in the solution.



Thus, the solution is $\boxed{d \leq 6}$

Answer 63MYS.

Consider the inequality $-5x+10r>2$

The objective is to solve the given inequality

To solve the given inequality

$$-5+10r>2$$

$$-5+10r+5>2+5 \quad [\text{Add 5 on both sides}]$$

$$\frac{10r}{10}>\frac{7}{10} \quad [\text{divide with 10 on both sides}]$$

$$r>\frac{7}{10}$$

Thus the solution is $r>\frac{7}{10}$

Check:

To check the solution take one number less than $\frac{7}{10}$ and one number greater than $\frac{7}{10}$ and substitute these in given equality and verify.

For $r = \frac{6}{10}$,

$$-5 + 10r > 2$$

$$-5 + 10 \cdot \frac{6}{10} > 2 \qquad \left[\text{Put } x = \frac{6}{10}\right]$$

$$-5 + 6 > 2 \qquad [\text{Simplify}]$$

$$1 > 2 \text{ False}$$

Thus, $\frac{6}{10}$ is not in the solution set

For $r = \frac{8}{10}$,

$$-5 + 10r > 2$$

$$-5 + 10 \cdot \frac{8}{10} > 2 \qquad \left[\text{Put } r = \frac{8}{10}\right]$$

$$-5 + 8 > 2 \qquad [\text{Simplify}]$$

$$3 > 2 \quad \text{True}$$

Thus $\frac{8}{10}$ is in the solution set

The solution set is $r > \frac{7}{10}$

The graph $r > \frac{7}{10}$ is as below

In the graph the inequality symbols < or > are indicated by open circles and the inequality symbols $\leq$ or $\geq$ are indicated by closed circles.

The open circles indicate that the particular values are not included in the solution where as closed circles indicate that the particular values are included in the solution.

Thus, the solution is $\boxed{r > \frac{7}{10}}$

Answer 64MYS.

Consider the inequality $13x - 3 < 23$

The objective is to solve the given inequality

To solve the given inequality

$$13x - 3 < 23 \quad [\text{Add 3 on both sides}]$$

$$13x - 3 + 3 < 23 + 3$$

$$13x < 23$$

$$\frac{13x}{13} < \frac{26}{13} \quad [\text{Divide with 13 on both sides}]$$

$$x < 2$$

Thus, the solution is $x < 2$

Check:

To check the solution take one number less than 2 and one number greater than 2, substitute these in the given inequality and verify.

Take $x=1$,

$$13x-3<23$$
$$13(1)-3<23 \quad [\text{put } x=1]$$
$$13-3<23$$
$$10<23 \text{ True}$$

Thus $x=1$ is in the solution set.

Take $x=3$,

$$13x-3<23$$
$$13(3)-3<23 \quad [\text{Put } x=3]$$
$$39-3<23$$
$$36<23 \text{ False}$$

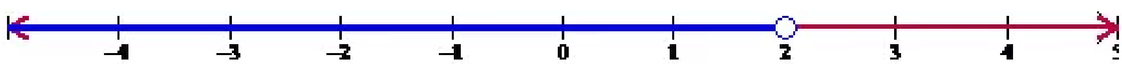
Thus $x=3$ is not in the solution set.

Thus the solution is $x<2$

The graph $x<2$ is as below

In the graph the inequality symbols < or > are indicated by open circles and the inequality symbols $\le$ or $\ge$ are indicated by closed circles.

The open circles indicate that the particular values are not included in the solution where as closed circles indicated that the particular values are included in the solution.



Thus the solution is $\boxed{x<2}$

Answer 65MYS.

Consider the expression $(x+1)(x+1)$

The objective is to find the product

$$(x+1)(x+1)=(x+1)^2 \qquad [\text{Since } x\cdot x=x^2]$$

Also the perfect squares property is $(a+b)^2=a^2+2ab+b^2$

Here $a=x, b=1$

$$\begin{aligned}(x+1)(x+1)&=(x+1)^2\\&=x^2+2\cdot x\cdot 1+1^2\\&=x^2+2x+1\end{aligned}$$

Therefore, $(x+1)(x+1)=\boxed{x^2+2x+1}$

Answer 66MYS.

Consider the polynomial $(x-6)(x-6)$

The objective is to factor the given product.

Since $(a-b)^2=a^2-2ab+b^2$

$$(x-6)(x-6)=(x-6)^2 \qquad (x\cdot x=x^2)$$

Compare $(x-6)^2$ with $(a-b)^2$

$a=x, b=6$

$$\begin{aligned}(a-b)^2&=-a^2-2ab+b^2\\(x-6)^2&=x^2-2\cdot x\cdot 6+6^2 \qquad [a=x,b=6]\\&=x^2-12x+36\,[\text{Simplify}]\end{aligned}$$

Therefore, $\boxed{(x-6)^2=x^2-12x+36}$

Answer 67MYS.

Consider the polynomial $(x+8)^2$

The objective is to factor the given product.

Since $(a+b)^2 = a^2 + 2ab + b^2$

Here $a = x, b = 8$

Thus, $(x+8)^2 = x^2 + 2\cdot x\cdot 8 + 8^2$

$= x^2 + 16x + 64 \qquad (\text{Simplify})$

Therefore, the given product is $\boxed{x^2 + 16x + 64}$

Answer 68MYS.

Consider the polynomial $(3x-4)(3x-4)$

The objective is to find the given product

Since $(a-b)^2 = a^2 - 2ab + b^2$

$(3x-4)(3x-4) = (3x-4)^2$

Compare $(3x-4)^2$ with $(a-b)^2$

Here $a = 3x, b = 4$

$$
\begin{aligned}
(3x-4)^2 &= (3x)^2 - 2\cdot 3x\cdot 4 + 4^2 \\
&= 3^2\cdot x^2 - 24x + 16 \qquad \left[\text{Since; } (ab)^m = a^m b^m\right] \\
&= 9x^2 - 24x + 16
\end{aligned}
$$

Therefore, $\boxed{(3x-4)(3x-4) = 9x^2 - 24x + 16}$

Answer 69MYS.

Consider the polynomial $(5x-2)^2$

The objective is to find the given product

Since $(a-b)^2 = a^2 - 2ab + b^2$

Compare $(5x-2)^2$ with $(a-b)^2$

Here $a = 5x, b = 2$

$$(a-b)^2 = -a^2 - 2ab + b^2$$

$$(5x-2)^2 = (5x)^2 - 2\cdot 5x\cdot 2 + 2^2 \qquad [a = 5x, b = 2]$$

$$= 5^2\cdot x^2 - 20x + 4 \qquad \left[\text{Since; } (ab)^m = a^m b^m\right]$$

$$= 25x^2 - 20x + 4$$

Therefore, $\boxed{(5x-2)^2 = 25x^2 - 20x + 4}$

Answer 70MYS.

Consider the polynomial $(7x+3)^2$

The objective is to find the given product

Since $(a+b)^2 = a^2 + 2ab + b^2$

Here $a = 7x, b = 3$

Then

$$(7x+3)^2 = (7x)^2 + 2\cdot 7x\cdot 3 + 3^2$$

$$= 7^2\cdot x^2 + 42x + 9 \qquad \left[\text{Since; } (ab)^m = a^m\cdot b^m\right]$$

$$= 49\cdot x^2 + 42x + 9 \qquad \left[\text{Since; } 7^2 = 49\right]$$

Therefore, the factored form of $(7x+3)^2$ is $\boxed{49x^2 + 42x + 9}$