CHAPTER 5 POLYNOMIALS

- **Polynomials :** An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + ... + a_n x^n$, where $a_n \neq 0$, is called a polynomial in x of degree n.
  Here $a_0, a_1, a_2, ...a_n$ are real numbers and each power of x is a non-negative integer.
  e.g.

(i) $2x + 7$ is a polynomial in x of degree 1.

(ii) $2y^2 - 5y + 7$ is a polynomial in y of degree 2.

(iii) $3u^3 + \frac{3}{7}u^2 - 8u + \sqrt{7}$ is a polynomial in u of degree 3.

(iv) $5t^4 - \frac{2}{7}t^3 + \sqrt{3}t^2 + \frac{3}{8}$ is a polynomial in t of degree 4.

(v) $\left(\sqrt{x}+5\right)$, $\frac{1}{x+3}$, $\frac{5}{x^2-3x+1}$ etc. are not polynomials.

- **Polynomials of Various Degrees :**

(1) Linear Polynomial : A polynomial of degree 1 is called a linear polynomial.
A linear polynomial is of the form $p(x) = ax + b$, where $a \neq 0$

e.g. $(3x-7), \left(\sqrt{2}x+5\right), \left(x-\frac{7}{3}\right)$ etc.

(2) Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial.
It is of the form $p(x) = ax^2 + bx + c$, where $a \neq 0$

E.g. $(2x^2 + 7x - 9)$, $\left(3x^2 - \sqrt{2}x + 7\right)$, $\left(y^2 - 7y + \sqrt{5}\right)$ etc.

- **Byquadratic Polynomial :** A polynomial of degree 4 is called a biquadratic polynomial.
  It is of the form $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ where $a \neq 0$

E.g. $(3x^4 + 7x^3 - 4x^2 + 6x + 11)$, $(4t^4 - 7t^3 + 6t^2 - 11t + 9)$ etc.

(3) Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial.
It is of the form $P(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$

E.g. $(4x^3 - 2x^2 + 7x + 9)$, $\left(2\sqrt{2}y^3 - 5y^2 - 8\right)$ etc.

Value of a Polynomial at a given point :
If $P(x)$ is a polynomial in x and if α is any real number, then the value obtained by putting $x = \alpha$ in $P(x)$ is called the value of $P(x)$ at $x = \alpha$
The value of $P(x)$ at $x = \alpha$ is denoted by $p(\alpha)$.

e.g. Let $p(x) = 3x^2 - 2x + 7$. then

$p(2) = (3 \times 2^2 - 2 \times 2 + 7)$
$= (12 - 4 + 7)$
$= 15$

$p(-1) = [3 \times (-1)^2 - 2(-1) + 7]$
$= (3+2+7)$
$= 12$

Zeros of a Polynomial : A real number α is called a zero of the polynomial $p(x)$, if $p(\alpha) = 0$

Note : 1. If α and β are the zeros of $p(x) = ax^2 + bx + c, a \neq 0$, then.

(i) $\alpha + \beta = -\frac{b}{a}$

(ii) $\alpha\beta = \frac{c}{a}$

(2) A quadratic polynomial whose zeros are α and β is given by $p(x) = \{x^2 - (\alpha + \beta)x + \alpha\beta\}$

(3) If α, β and γ are the zeros of $p(x) = ax^3 + bx^2 + cx + d$, then,

(i) $\alpha + \beta + \gamma = -\frac{b}{a}$

(ii) $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a}$

(iii) $\alpha\beta\gamma = -\frac{d}{a}$

(4) A cubic polynomial whose zeros are α, β and γ is given by $p(x) = \{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$

Factor Theorem : The Condition that $(x - a)$ is a factor of a polynomial $f(x)$, if and only if $f(a) = 0$

Thus, $(x - a)$ is a factor of $f(x) \Leftrightarrow f(a) = 0$.

Remarks : (i) $(x + a)$ is a factor of polynomiual $p(x)$ if and only if $p(-a) = 0$

(ii) $(ax - b)$ is a factor of a polynomial $p(x)$, if $p\left(\frac{b}{a}\right) = 0$

(iii) $(ax + b)$ is a factor of a polynomial $p(x)$, iff $p\left(-\frac{b}{a}\right) = 0$

(iv) $(x - a)(x - b)$ are factors of a polynomial $p(x)$ iff $p(a) = 0$ and $p(b) = 0$.

- **Remainder Theorem :** If a polynomial $f(x)$ of degree $n \geq 1$, is divided by $(x - a)$, then the remainder is $f(a)$.

e.g. Let $f(x) = x^3 + 3x^2 - 5x + 4$ be divided by $(x - 1)$. Find the remainder.

Sol. Remainder $= f(1)$

$= 1^3 + 3 \times 1^2 - 5 \times 1 + 4$

$= 3$

Important Results :

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(ii) $(x^n + a^n)$ is divisible by $(x + a)$ only when n is odd.

(iii) $(x^n - a^n)$ is divisible by $(x + a)$ only for even values of n.

(iv) $(x^n + a^n)$ is never divisible by $(x - a)$

H.C.F & L.C.M of Polynomials :

Divisor : A polynomial $p(x)$ is called a divisor of another polynomial $f(x) = p(x).g(x)$ for some polynomial $g(x)$.

H.C.F. or (G.C.D.) of Polynomials : A polynomial $h(x)$ is called the H.C.F. or G.C.D of two or more given polynomials, if $h(x)$ is a polynomial of heighest degree dividing each one of the given polynomials.

Remark : The coefficient of heighest degree term in H.C.F is always taken as positive.

e.g. What is the HCF of $(x + 3)^2 (x - 2)^3$ and $(x - 1)(x + 3)(x - 2)^2$?

Sol. $p(x) = (x + 3)^2 (x - 2)^3$

$q(x) = (x - 1)(x + 3)(x - 2)^2$

We see that $(x + 3)(x - 2)^2$ is such a polynomial that is a common divisor and whose degree is heighest among all common divisors.

- **L.C.M. of Polynomials :** A polynomial $p(x)$ is called the L.C.M. of two or more given polynomials, if it is a polynomial of smallest degree which is divided by each one of the given polynomials.

e.g. Find the L.C.M of $(x-3)(x+4)^2$ and $(x-3)^3(x+4)$:

Sol : $p(x) = (x-3)(x+4)^2$

$q(x) = (x-3)^3(x+4)$

we make a polynomial by taking each factor of $p(x)$ and $q(x)$.

If a factor is common in both, then we take the factor which has highest degree in $p(x)$ and $q(x)$.

$\therefore$ LCM $= (x-3)^3(x+4)^2$

Note : For any two polynomials $p(x)$ and $q(x)$

$p(x) \times q(x)$ = (Their H.C.F.) × (Their L.C.M.)

- **Factorisation of Polynomials :** To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorisation.

- **Formulae for Factorisation :**

(i) $(x+y)^2 = x^2 + y^2 + 2xy$

(ii) $(x-y)^2 = x^2 + y^2 - 2xy$

(iii) $(x+y)^2 + (x-y)^2 = 2(x^2 + y^2)$

(iv) $(x+y)^2 - (x-y)^2 = 4xy$

(v) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(vi) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(vii) $x^2 - y^2 = (x+y)(x-y)$

(viii) $(x^3 + y^3) = (x+y)(x^2 + y^2 - xy)$

(ix) $(x^3 - y^3) = (x-y)(x^2 + y^2 + xy)$

(x) $(x+y+z)^2 = (x^2 + y^2 + z^2 + 2(xy + yz + zx))$

(xi) $(x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= \frac{1}{2}(x+y+z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

(xii) $x^2 + y^2 + z^2 - xy - yz - zx$

$= \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$

(xiii) $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$

Exercise

1. If $f(x)$ is divided by $(3x+5)$, the remainder is :
(a) $f\left(\frac{3}{5}\right)$ (b) $f\left(-\frac{3}{5}\right)$
(c) $f\left(\frac{5}{3}\right)$ (d) $f\left(-\frac{5}{3}\right)$

2. If $(x^{11}+1)$ is divided by $(x+1)$, the remainder is :
(a) 0 (b) 2
(c) 11 (d) 12

3. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x-2)$, the remainder is :
(a) 3 (b) - 3
(c) 2 (d) 0

4. If $(x-2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :
(a) 2 (b) - 2
(c) 1 (d) - 1

5. The value of λ for which the expression $x^3 + x^2 - 5x + \lambda$ will be divisible by $(x-2)$ is :
(a) 2 (b) - 2
(c) - 3 (d) 4

6. If $(x+1)$ and $(x-2)$ be the factors of $x^3 + (a+1)x^2 - (b-2)x - 6$, then the value of a and b will be :
(a) 2 and 8 (b) 1 and 7
(c) 5 and 3 (d) 3 and 7

7. The polynomial $(x^4 - 5x^3 + 5x^2 - 10x + 24)$ has a factor as :
(a) $x+4$ (b) $x-2$
(c) $x+2$
(d) None of these

8. $(x^{29} - x^{25} + x^{13} - 1)$ is divisible by :
(a) both $(x-1)$ & $(x+1)$
(b) $(x-1)$ but not by $(x+1)$
(c) $(x+1)$ but not by $(x-1)$
(d) neither $(x-1)$ nor $(x+1)$

9. The value of expression $(9x^2 + 12x + 7)$ for $x = -\frac{4}{3}$ is :
(a) 7 (b) 0
(c) -7 (d) 18

10. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x-6)$. The values of P and q are :
(a) p = - 2, q = -6 (b) p = 2, q = -6
(c) p = - 2, q = 6 (d) p = 2, q = 6

11. If $(x-a)$ is a factor of $(x^3 - 3x^2a + 2a^2x + b)$, then the value of b is :
(a) 0 (b) 2
(c) 1 (d) 3

12. If $x^{100} + 2x^{99} + K$ is divisible by $(x+1)$, then the value of K is :
(a) -3 (b) 2
(c) -2 (d) 1

13. If the polynomial $f(x)$ is such that $f(-1) = 0$, then a factor of $f(x)$ is :
(a) -1 (b) $x-1$
(c) $x+1$ (d) $-1-x$

14. If $x^3 + 5x^2 + 10K$ leaves remainder $-2x$ when divided by x^2+2, then the value of k is :
(a) -2 (b) 1
(c) -1 (d) 2

15. Which of the following is a polynomial ?
(a) $x^2 - 3x + 2\sqrt{x} + 7$
(b) $\sqrt{x} - \frac{1}{\sqrt{x}}$
(c) $x^{7/2} - x + x^{3/2}$
(d) None of these

16. If α and β are the zeros of $x^2 + 3x + 7$, then the vaue of $(\alpha + \beta)$ is :
(a) -3 (b) 3
(c) 7 (d) -7

17. If α and β are the zeros of $2x^2 + 3x - 10$, then the value of $\alpha\beta$ is :

(a) $-\frac{5}{2}$ (b) 5

(c) - 5 (d) $-\frac{3}{2}$

18. If common factor of $x^2 + bx + c$ and $x^2 + mx + n$ is $(x + a)$, then the value of a is :

(a) $\frac{c-n}{b-m}$ (b) $\frac{c-n}{b+m}$

(c) $\frac{c-n}{m-b}$ (d) $\frac{c+1}{b-m}$

19. $(x^4 + 5x^3 + 6x^2)$ is equal to :
(a) $x(x+3)(x^2+2)$
(b) $x^2(x+3)(x+2)$
(c) $x^2(x-2)(x-3)$
(d) $x(x^2+3)(x+2)$

20. The factors of $(x^4 + 625)$ are :
(a) $(x^2 - 25)(x^2 + 25)$
(b) $(x^2 + 25)(x^2 + 25)$
(c) $(x^2 - 10x + 25)(x^2 + 5x + 24)$
(d) do not exist

21. The factors of $(x^4 + 4)$ are :
(a) $(x^2 + 2)^2$
(b) $(x^2 + 2)(x^2 - 2)$
(c) $(x^2 + 2x + 2)(x^2 - 2x + 2)$
(d) None of these

22. $(x + y)^3 - (x - y)^3$ can be factorized as :
(a) $2y(3x^2 + y^2)$
(b) $2x(3x^2 + y^2)$
(c) $2y(3y^2 + x^2)$
(d) $2x(x^2 + 3y^2)$

23. The H.C.F. of $x^2 - xy - 2y^2$ and $2x^2 - xy - y^2$ is :
(a) $(x + y)$ (b) $(x - y)$
(c) $(2x - 3y)$
(d) None of these

24. The H.C.F. of $(x^3 + x^2 + x + 1)$ and $(x^4 - 1)$ is:
(a) $(x^2 - 1)(x^2 + 1)$
(b) $(x + 1)(x^2 - 1)$
(c) $(x + 1)(x^2 + 1)$
(d) $(x^2 + 1)(x + 1)(x^3 + 1)$

25. The L.C.M of the polynomials X and Y, where $X = (x + 3)^2 (x - 2)(x + 1)^2$ and $Y = (x + 1)^2 (x + 3)(x + 4)$ is given by :
(a) $(x - 2)(x + 4)(x + 3)^2 (x + 1)^2$
(b) $(x + 1)(x - 2)(x + 3)(x + 4)$
(c) $(x - 2)(x + 1)(x + 3)^2 (x + 4)$
(d) $(x - 2)(x + 1)^2 (x + 3)(x + 4)$

26. The L.C.M of $(x + 2)^2 (x - 2)$ and $(x^2 - 4x - 12)$ is :
(a) $(x + 2)(x - 2)$
(b) $(x + 2)(x - 2)(x - 6)$
(c) $(x + 2)(x - 2)^2$
(d) $(x + 2)^2 (x - 2)(x - 6)$

27. The H.C.F. of $(x^2 - 4)$, $(x^2 - 5x - 6)$ and $(x^2 + x - 6)$ is :
(a) 1 (b) $(x - 2)$
(c) $(x + 2)$ (d) $(x^2 + x - 6)$

28. The H.C.F of $2(x^2 - y^2)$ and $5(x^3 - y^3)$ is :
(a) $2(x^2 - y^2)$ (b) $(x - y)$
(c) $(x + y)$ (d) $(x^2 + y^2)$

29. The L.C.M of $2(x^2 - 3x + 2)$ and $(x^3 - 4x^2 + 4x)$ is :
(a) $x(2x^2 + 1)(x^2 + 2)$
(b) $x(2x + 1)(x - 2)^2$
(c) $x(2x^2 + 1)(x - 1)^2$
(d) $x(2x + 1)(x^2 - 1)$

30. The L.C.M of $(a^3 + b^3)$ and $(a^4 - b^4)$ is:
(a) $(a^3 + b^3)(a^2 + b^2)(a - b)$
(b) $(a^3 + b^3)(a + b)(a^2 + b^2)$
(c) $(a + b)(a^2 + ab + b^2)(a^3 + b^3)$
(d) $(a^3 + b^3)(a^2 - b^2)(a - b)$

31. If Polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x-2)$, the same remainder are obtained. Find the value of a :

(a) 3 (b) - 9
(c) - 3 (d) - 5

32. If the polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, the remainders are 5 and 19 respectively. The values of a and b are:

(a) a = 8, b = 5 (b) a = 5, b = 6
(c) a = 6, b = 8 (d) a = 5, b = 8

33. Factories : $(x^8 + x^4 y^4 + y^8)$

(a) $(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$
(b) $(x^2 + xy - y^2)(x^4 - x^4y^4 + y^4)$
(c) $(x^2 + xy + y^2)^2(x^4 - x^2y^2 + y^4)$
(d) $(x^2 - xy + y^2)^2(x^4 - x^4y^4 - y^4)$

34. Factorise : $\left(x^6 + \frac{y^6}{27}\right)$

(a) $\left(x^2 + \frac{y^2}{3}\right)\left(x^4 + \frac{x^2y^2}{3} + \frac{x^2y^6}{9}\right)$
(b) $\left(x^2 + \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$
(c) $\left(x^2 - \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{x^2y^4}{9}\right)$
(d) $\left(x^2 - \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$

35. Factorise : $(x^4 + x^2 + 25)$

(a) $(x^2 + 3x + 5)(x^2 + 3x - 5)$
(b) $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$
(c) $(x^2 + x + 5)(x^2 - x + 5)$
(d) None of these

Hints and Solutions :

1.(d) $3x+5=0 \Rightarrow x=-\frac{5}{3}$

So, remainder is $f\left(-\frac{5}{3}\right)$

2.(a) Remainder $= f(-1)$
$= (-1)^{11} + 1$
$= -1 + 1$
$= 0$

3.(b) Remainder $= f(2)$
$= 2^4 - 3(2)^3 + 2(2)^2 - 5\times 2 + 7$
$= 16 - 24 + 8 - 10 + 7$
$= -3$

4.(d) Since $(x-2)$ is a factor of $f(x)$
$= x^2 + 3qx - 2q$
$\therefore\ f(2) = 0 \Rightarrow 2^2 + 3q\times 2 - 2q = 0$
$\Rightarrow\ 4q = -4 \Rightarrow q = -1$

5.(b) $(x-2)$ is a factor of polynomial
$f(x) = x^3 + x^2 - 5x + \lambda$
$\therefore\ f(2) = 0 \Rightarrow 2^3 + 2^2 - 5\times 2 + \lambda = 0$
$\Rightarrow\ 12 - 10 + \lambda = 0 \Rightarrow \lambda = -2$

6.(b) Since $(x+1)$ & $(x-2)$ are the factors of
$f(x) = x^3 + (a+1)x^2 - (b-2)x - 6$
$\therefore\ f(-1) = 0$ and $f(2) = 0$
or $-1 + (a+1) + (b-2) - 6 = 0$
and $8 + 4(a+1) - (b-2)\times 2 - 6 = 0$
or $a + b = 8$ ......(i)
and $2a - b = -5$ ......(ii)
(i) + (ii) $3a = 3 \Rightarrow a = 1$
From equation (i)
$b = 8 - 1 = 7$
$\therefore\ a = 1$ & $b = 7$

7.(b) Since $x = 2$ makes the given expression zero, so, $(x-2)$ is its factor.

8.(b) Since $x = 1$ makes $x^{29} - x^{25} + x^{13} - 1$ zero, so $(x-1)$ is its factor. And $x = -1$ does not make it zero so $(x+1)$ is not its factor.

9.(a) $f(x) = 9x^2 + 12x + 7$

$\therefore\ f\left(-\frac{4}{3}\right) = 9\left(-\frac{4}{3}\right)^2 + 12\left(-\frac{4}{3}\right) + 7$

$= 16 - 16 + 7 = 7$

10.(c)

$$\begin{array}{r|l} & x \\ x^2-2x-3 & x^3-2x^2+px-q \\ & x^3-2x^2-3x \\ & -\quad +\quad + \\ \hline & (p+3)x-q \end{array}$$ $\Leftarrow$ remainder

$\therefore\ (p+3)x - q = x - 6$
$\therefore\ p + 3 = 1$ and $q = 6$
or $p = -2$ and $q = 6$

11.(a) let $f(x) = x^3 - 3x^2a + 2a^2x + b$
$\because\ (x-a)$ is a factor of $f(x)$
$\therefore\ f(a) = 0 \Rightarrow a^3 - 3a^3 + 2a^3 + b = 0$
$\Rightarrow b = 0$

12.(d) $\because\ x^{100} + 2x^{99} + k$
$= f(x)$ (let) is divisible by $(x+1)$
$\therefore\ f(-1) = 0$
$\Rightarrow\ 1 - 2 + k = 0 \Rightarrow k = 1$

13.(c) Since $x = -1$ makes $f(x)$ zero, So $(x+1)$ is its factor.

14.(b)

$$\begin{array}{r|l} & x+5 \\ x^2+2 & x^3+5x^2+10k \\ & x^3+2x \\ & -\quad - \\ \hline & 5x^2-2x+10k \\ & 5x^2\quad\quad +10 \\ & -\quad\quad - \\ \hline & -2x+10k-10 \end{array}$$ $\Leftarrow$ Remainder

but given, remainder $= -2x$
$\therefore\ -2x + 10k - 10 = -2x$
$\Rightarrow 10k = 10$
$\Rightarrow\ k = 1$

15.(d) For polynomial, each power of x must be a non-negative integer.

16.(a) $\alpha+\beta=-\frac{b}{a}=-\frac{3}{1}=-3$

17.(c) $\alpha\beta=\frac{c}{a}=-\frac{10}{2}=-5$

18.(a) Let $f(x) = x^2 + bx + c$
and $g(x) = x^2 + mx + n$
$\because$ $(x + a)$ is a common factor of $f(x)$ and $g(x)$
$\therefore$ $f(-a) = 0$ and $g(-a) = 0$
or $a^2 - ba + c = 0$ and $a^2 - ma + n = 0$
$\Rightarrow$ $a^2 = ab - c$...(i) and $a^2 = ma - n$ .....(ii)
$\therefore$ from (i) and (ii)
$ab - c = ma - n$

$\Rightarrow a(b-m) = c - n$ or $a = \frac{c-n}{b-m}$

19.(b) $x^4 + 5x^3 + 6x^2 = x^2(x^2 + 5x + 6)$
$= x^2(x^2 + 3x + 2x + 6)$
$= x^2[x(x+3) + 2(x+3)]$
$= x^2(x+3)(x+2)$

21.(c) $x^4 + 4 = (x^2)^2 + (2)^2 + 4x^2 - 4x^2$
$= (x+2)^2 - (2x)^2$
$= (x^2 + 2x + 2)(x^2 - 2x + 2)$

22.(a) Using formuale, $a^3 - b^3$
$= (a-b)(a^2 + b^2 + ab)$
$\therefore$ $(x+y)^3 - (x-y)^3 = [(x+y) - (x-y)] + [(x+y)^2 + (x-y)^2 + (x+y)(x-y)]$
$= 2y[2(x^2 + y^2) + (x^2 - y^2)]$
$= 2y(3x^2 + y^2)$

23.(d) $x^2 - xy - 2y^2 = (x^2 - y^2) - (xy + y^2)$
$= (x+y)(x-y) - y(x+y)$
$= (x+y)(x-y-y)$
$= (x+y)(x-2y)$
$2x^2 - xy - y^2 = (x^2 - xy) + (x^2 - y^2)$
$= x(x-y) + (x+y)(x-y)$
$= (x-y)(x+x+y)$
$= (x-y)(2x+y)$
Clearly, no factor is common,
So, H.C.F = 1

24.(c) $x^3 + x^2 + x + 1 = x^2(x+1) + 1(x+1)$
$= (x+1)(x^2+1)$
$x^4 - 1 = (x^2-1)(x^2+1)$
$= (x+1)(x-1)(x^2+1)$
$\therefore$ Required H.C.F $= (x+1)(x^2+1)$

26.(d) $x^2 - 4x - 12 = x^2 - 6x + 2x - 12$
$= x(x-6) + 2(x-6)$
$= (x+2)(x-6)$
and other is $(x+2)^2(x-2)$
$\therefore$ L.C.M $= (x+2)^2(x-2)(x-6)$

27.(a) $x^2 - 4 = (x+2)(x-2)$
$x^2 - 5x - 6 = x^2 - 6x + x - 6$
$= (x-6)(x+1)$
and $x^2 + x - 6 = x^2 + 3x - 2x - 6$
$= (x+3)(x-2)$
Clearly, ther is no common factor.
So, H.C.F = 1.

28.(b) $2(x^2 - y^2) = 2(x-y)(x+y)$
and $5(x^3 - y^3) = 5(x-y)(x^2 + y^2 + xy)$
$\therefore$ H.C.F. $= (x-y)$

29.(b) $2x^2 - 3x + 2 = 2x^2 - 4x + x - 2 = 2x(x-2) + 1(x-2) = (x-2)(2x+1)$
$x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x-2)^2$
$\therefore$ L.C.M. $= x(x-2)^2(2x+1)$

30.(a) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^4 - b^4 = (a-b)(a+b)(a^2+b^2)$
$\therefore$ L.C.M. $= (a-b)(a+b)(a^2 - ab + b^2)(a^2+b^2)$
$= (a-b)(a^3+b^3)(a^2+b^2)$

31.(c) $f(x) = 2x^3 + ax^2 + 3x - 5$
$g(x) = x^3 + x^2 - 2x + a$
By remainder theorem,
$f(2) = 2(2)^3 + a(2)^2 + 3\times2 - 5$
$= 17 + 4a$
and, $g(2) = 2^3 + (2)^2 - 2\times2 + a$
$= 8 + a$
$\therefore$ $17 + 4a = 8 + a$

$\Rightarrow 3a = -9$ or $a = -3$

32.(d) By remainder theorem,

$f(1) = 5$ ....(i) $[\because x - 1 = 0 \Rightarrow x = 1]$

and $f(-1) = 19$ .....(ii) $[\because x + 1 = 0 \Rightarrow x = -1]$

Now, from (i) $1 - 2 + 3 - a + b = 5$

or $b - a = 3$ ...... (iii)

from (ii) $1 + 2 + 3 + a + b = 19$

or $a + b = 13$ ......(iv)

(iii) + (iv) $2b = 16$ or $b = 8$

Now from (iv), $a = 13 - 8 = 5$

$\therefore a = 5,\ b = 8$

33.(a) $x^8 + x^4y^4 + y^8$

$= x^8 + 2x^4y^4 + y^8 - x^4y^4$

$= (x^4 + y^4)^2 - (x^2y^2)^2$

$= (x^4 + y^4 + x^2y^2)(x^4 + y^4 - x^2y^2)$

$= [(x^2 + y^2)^2 - (xy)^2](x^4 - x^2y^2 + y^4)$

$= (x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

34.(b) $x^6 + \frac{y^6}{27} = (x^2)^3 + \left(\frac{y^2}{3}\right)^3$

$= \left(x^2 + \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$

35.(b) $x^4 + x^2 + 25 = (x^2)^2 + (5)^2 + 10x^2 - 9x^2$

$= (x^2 + 5)^2 - (3x)^2$

$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

Answer -key

LEVEL - 1

1. (d)	2. (a)	3. (b)
4. (d)	5. (b)	6. (b)
7. (b)	8. (b)	9. (a)
10. (c)	11. (a)	12. (d)
13. (c)	14. (b)	15. (d)
16. (a)	17. (c)	18. (a)
19. (b)	20. (d)	21. (c)
22. (a)	23. (d)	24. (c)
25. (a)	26. (d)	27. (a)
28. (b)	29. (b)	30. (a)
31. (c)	32. (d)	33. (a)
34. (b)	35. (b)	