Chapter 15

Statistics

Miscellaneous Exercise

Q. 1 The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Answer:

Let us assume the remaining two observations to be x and y respectively such that,

Observations: 6, 7, 10, 12, 12, 13*, x, y*.

Mean, $\overline{x} = \frac{6+7+10+12+2+13+x+y}{8} = 9$

= 60 + x + y = 72

= x + y = 12... (1)

Variance = 9.25 = $\frac{1}{n}\sum_{i=1}^{8}(x_i - \overline{x})^2$

9.25 = $\frac{1}{8}[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2 \times 9(x + y) + 2 \times (9)^2]$

9.25 = $\frac{1}{8}[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18(12) + 162]$

9.25 = $\frac{1}{8}[48 + x^2 + y^2 - 216 + 162]$

9.25 = $\frac{1}{8}[x^2 + y^2 - 6]$

$x_2 + y_2$ = 80… (2)

From (1), we obtain

$x_2 + y_2 + 2xy = 144$… (3)

From (2) and (3), we obtain

$2xy = 64$… (4)

Substituting (4) from (2), we obtain

$x_2 + y_2 – 2xy = 80 – 64 = 16$

$= x – y = \pm 4$… (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 4$, when $x – y = 4$

$x = 4$ and $y = 8$, when observations are 4 and 8.

Q. 2 The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

Answer:

Let us assume the remaining two observations be x and y

The given observations in the question are 2, 4, 10, 12, 14, x, y

Mean, $\overline{x} = \frac{2+4+10+12+14+x+y}{7} = 8$

$= 56 = 42 + x + y$

$= x + y = 14$… (1)

Variance $= 16 = \frac{1}{n}\sum_{i=1}^{7}(x_i - \overline{x})^2$

$16 = \frac{1}{7}[(-6)^2 + (-4)^2 + (x_i - \overline{x})^2 + (6)^2 + x^2 + y^2 - 2 \times 8\ (x+y) + 2 \times (8)^2]$

$16 = \frac{1}{7}[(-6)^2 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)]$

$16 = \frac{1}{7}[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)]$

$16 = \frac{1}{7}[108 + x^2 + y^2 - 224 + 128]$

$16 = \frac{1}{7}[12 + x^2 + y^2]$

$= x_2 + y_2 = 112 - 12 = 100$

$x_2 + y_2 = 100$ .... (2)

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ … (3)

From (2) and (3), we obtain

$2xy = 196 - 100$

$= 2xy = 96$… (4)

Substituting (4) from (2), we obtain

$x_2 + y - 2xy = 100 - 96$

$= x - y = \pm 2$ …(5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 6$ when $x - y = 2$

$x = 6$ and $y = 8$ when $x - y = -2$

Thus, the remaining observation are 6 and 8

Q. 3 The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Answer:

Let us assume the observations be x_1, x_2, x_3, x_4, x_5 and x6

It is given in the question that,

Mean = 8 and Standard deviation = 4

Mean, $\overline{x} = \frac{x_1+x_2+x_3+x_4+x_5+x_6}{6} = 8$

Now, according to question if each observation is multiplied by 3 and the resulting observations are y_i then, we have:

$y_i = 3x_i$ *i.e.,* $x_i = \frac{1}{3} y_i$, for I = 1 to 6

New mean, $\overline{y} = \frac{y_1+y_2+y_3+y_4+y_5+y_6}{6}$

$= \frac{3(x_1+x_2+x_3+x_4+x_5+x_6)}{6}$

$= 3 \times 8$

$= 24$

Standard deviation, σ = $\sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i - \overline{x})^2}$

$$(4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \overline{x})^2$$

$$\frac{1}{6}\sum_{i=1}^{6}(x_i - \overline{x})^2 = 96 \ldots (2)$$

From (1) and (2), it can be observed that,

$$\overline{y} = 3\overline{x}$$

$$\overline{x} = \frac{1}{3}\overline{y}$$

Substituting the value of xi and $\overline{x}$ in (2), we obtain

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}y\right)^2 = 96$$

$$= \quad \sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}y\right)^2 = 864$$

Therefore, variance of new observation = $(\frac{1}{6} \times 864) = 144$

Hence, the standard deviation of new observation is $\sqrt{144} = 12$

Q. 4 Given that is the mean and is the variance of n observations x_1, x_2, ..., x_n.

Prove that the mean and variance of the observations ax1, ax_2, ax_3, ..., a_n is and, respectively,

Answer:

The given observations in the question are $x_1, x_2, \ldots x_n$

And, variance = σ^2

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_i\,(x_i - \overline{x})^2 \ldots$ (1)

If each observation is multiplied by a and the new observation are Y_i, then

y_i = ax i.e., $X_i = \frac{1}{a} Y_i$

$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n} ax_i = \frac{a}{n}\sum_{i=1}^{n} x_i = a\overline{x} \quad (\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i)$

Therefore, mean of the observation, $ax_1, ax_2, \ldots ax_n$, is

Substituting the value of x_i and $\overline{x}$ in (1), we obtain

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} \left(\frac{1}{a} y_i - \frac{1}{a} y\right)^2$

$= a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \overline{y})^2$

Thus, the variance of the observation, $ax_1, ax_2 \ldots ax_n$, is $a^2\sigma^2$

Q. 5 A

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect.

Calculate the correct mean and standard deviation in each of the following cases:

If wrong item is omitted

Answer:

Total number of observations (n) = 20

Also, incorrect mean = 20

And, incorrect standard deviation = 2

Thus, incorrect sum of observations = 200

Hence, correct sum of observations = 200 – 8

= 192

= 10.1

Thus,

= 2080 – 64

= 2016

= 2.02

Q. 5 B

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect.

Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted.

(ii) If it is replaced by 12

Answer:

(i) Number of observation (n) = 20

Incorrect mean = 10

Incorrect standard deviation = 2

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$= \sum_{i-1}^{20} x_{i=200}$$

incorrect sum of observations = 200

Also, correct sum of observations = 200 – 8 = 192

Correct mean = $\frac{Correct\ sum}{19} = \frac{192}{19} = 10.1$

Standard deviation σ = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n}(x)\right)^2}$

$$= \sqrt{\frac{1}{n}\frac{1}{4}x^2 - (x)^2}$$

= 2 = $\sqrt{\frac{1}{20}Incorrect\ \sum_{i=1}^{n} {x_i}^2 - (10)^2}$

= 4 = $\frac{1}{20}Incorrect\ \sum_{k=1}^{n} x^2 - 100$

= Incorrect $\sum_{H=1}^{n} x^2 = 2080$

Correct $\sum {x_1}^2 = Incorret\ \sum_{k=1}^{n} {x_i}^2 - (8)^2$

= 2080- 64

= 2016

Correct standard deviation = $\sqrt{\frac{Correct \sum x^2}{n} - (Correct\ mean)^2}$

= $\sqrt{\frac{2016}{19} - (10.1)^2}$

= $\sqrt{1061.1 - 102.1}$

= $\sqrt{4.09}$

= 0.02

(ii) When 8 is replaced by 12,

Incorrect sum of observation = 200

Correct sum of observation = 200 – 8 + 12 = 204

Correct mean = $\frac{Correct\ sum}{20} = \frac{204}{20} = 10.2$

Standard deviation σ =

$\sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - (\overline{x})^2}$

= 2 = $\sqrt{\frac{1}{20} Incoeerct\ \sum_{i=1}^{n} {x_i}^2 - (10)^2}$

= 4 = $\frac{1}{20} Incorrect\ \sum_{i=1}^{n} {x_i}^2 - 100$

= Incorrect $\sum_{i=1}^{n} {x_i}^2 = 2080$

Correct $\sum_{i=1}^{n} {x_i}^2 = Incorrect\ \sum_{i=1}^{n} {x_i}^2 - (8)^2 + (12)^2$

= 2080 – 64 + 144

= 2160

Correct standard deviation = $\frac{Correct \sum {x_i}^2}{n} - Correct\ mean\ ^2$

$= \sqrt{\frac{2160}{20} - (10.2)^2}$

$= \sqrt{108 - 104.04}$

= 13.96

= 1.98

Q. 6 The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	1520	

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Answer:

Standard deviation of mathematics = 12

Also, standard deviation of physics = 15

And, standard deviation of chemistry = 20

The coefficient of variation (c.v.) is given by $\frac{Standard\ deviation}{Mean} \times 100$

C.V. (in mathematics) = $\frac{12}{42} \times 100 = 28.57$

C.V. (in physics) = $\frac{15}{32} \times 100 = 46.87$

C.V. (in chemistry) = $\frac{20}{40.9} \times 100 = 48.89$

The subject with greater C.V. is more variable than others.

Therefore, the highest variability in marks is in chemistry and the lowest variability in marks is in mathematics.

Q. 7 The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on, it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Answer:

Total number of observations (n) = 100

Incorrect mean, $(\overline{x})$ = 20

And, Incorrect standard deviation (σ) = 3

$= 20 + \frac{1}{100}\sum_{i=1}^{100} x_i$

$= \sum_{i=1}^{100} x_i = 20 \times 100 = 2000$

Thus, incorrect sum of observations is 2000

Now, correct sum of observations = 2000 – 21 – 21 – 18= 2000 – 60= 1940

Correct mean $= \frac{Correct\ sum}{100-3} = \frac{1940}{97} = 20$

Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2}$

$$= \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - (\overline{x})^2}$$

$= 3 = \sqrt{\frac{1}{100} \times Incorrect \sum {x_i}^2 - (20)^2}$

$= \text{Incorrect} \sum {x_i}^2 = 100(9 + 400) = 40900$

Correct $\sum_{i=1}^{n} x_t{}^2 = Incorrect\ \sum_{i=1}^{n} x_i{}^2 - (21)^2 - (21)^2 - (18)^2$

= 40900 – 441 – 441 – 324

= 39694

Correct standard deviation = $\sqrt{\frac{Correct\ \sum x_i{}^2}{n} - (Correct\ mean\ )^2}$

$= \sqrt{\frac{39694}{97} - (20)^2}$

$= \sqrt{9.216}$

= 3.036