REAL NUMBERS

★ INTRODUCTION

"God gave us the natural number, all else is the work of man". It was exclaimed by L. Kronecker (1823-1891), the reputed German Mathematician. This statement reveals in a nut shell the significant role of the universe of numbers played in the evolution of human thought.

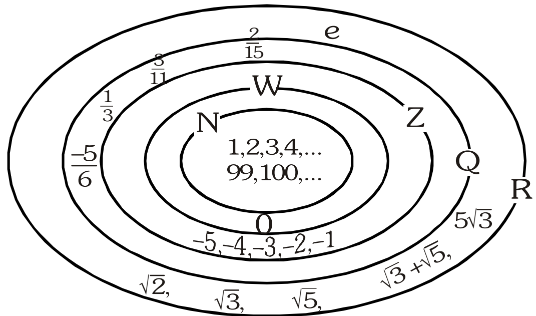
N : The set of natural numbers,

W : The set of whole numbers,

Z : The set of Integers,

Q : The set of rationals,

R : The set of Real Numbers.



★ HISTORICAL FACTS

Richard Dedekind
(1831-1916)

Dedekind was the first modern mathematician to publish in 1872 the mathematically rigorous definition of irrational numbers. He gave explanation of their place in the real Numbers System. He was able to demonstrate the completeness of the real number line. He filled in the 'holes' in the system of Rational numbers with irrational Numbers. This innovation has made Richard Dedekind an immortal figure in the history of Mathematics.

Srinivasa Ramanujan
(1887-1920)

Srinivasa Ramanujan (1887-1920) was one of the most outstanding mathematician that India has produced. He worked on history of Numbers and discovered wonderul properties of numbers. He stated intuitively many complicated result in mathematics. Once a great mathematician Prof. Hardy come to India to see Ramanujan. Prof. Hardy remarked that he has travelled in a taxi with a rather dull number viz. 1729. Ramanujan jumped up and said, Oh! No. 1729 is very interesting number. It is the smallest number which can be expressed as the sum of two cubes in two different ways.

viz $1729 = 1^3 + 12^3$,

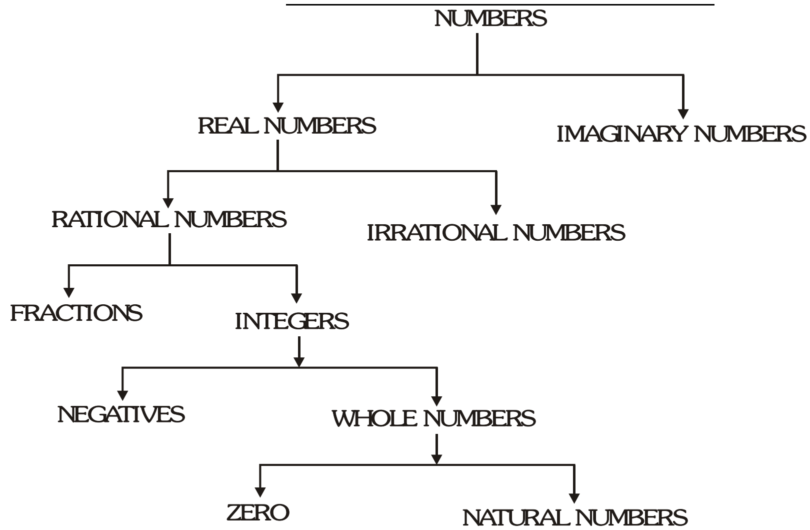
$1729 = 9^3 + 10^3$,

$\Rightarrow$ $1729 = 1^3 + 12^3 = 9^3 + 10^3$

★ RECALL

In our day to day life, we deal with different types of numbers which can be broadly classified as follows.

CLASSIFICATION OF NUMBERS



(i) Natural numbers (N) : N = {1, 2, 3, 4...∞}

Remark :
- (i) The set N is infinite i.e. it has unlimited members.
- (ii) N has the smallest element namely '1'.
- (iii) N has no largest element. i.e., give me any natural number, we can find the bigger number from the given number.
- (iv) N does not contain '0' as a member. i.e. '0' is not a member of the set N.

(ii) Whole numbers (W) : W = {0, 1, 2, 3, 4...∞}

Remark :
- (i) The set of whole number is infinite (unlimited elements).
- (ii) This set has the smallest members as '0'. i.e. '0' the smallest whole number. i.e., set W contain '0' as a member.
- (iii) The set of whole numbers has no largest member.
- (iv) Every natural number is a whole number.
- (v) Non-zero smallest whole number is '1'.

(iii) Integers (I or Z) : I or Z = {– ∞... –3, –2, –1, 0, +1, +2, +3 ...+ ∞}
Positive integers : {1, 2, 3...}, **Negative integers :** {.... –4, –3, –2, –1}

Remark :
- (i) This set Z is infinite.
- (ii) It has neither the greatest nor the least element.
- (iii) Every natural number is an integer.
- (iv) Every whole number is an integer.
- (iv) The set of non-negative integer = {0, 1, 2, 3, 4,....}
- (v) The set of non-positive integer = {......–4, – 3, – 2, –1, 0}

(iv) Rational numbers :– These are real numbers which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Ex. $\frac{2}{3}, \frac{37}{15}, \frac{-17}{19}$, –3, 0, 10, 4.33, 7.123123123.........

Remark :
(i) Every integer is a rational number.
(ii) Every terminating decimal is a rational number.
(iii) Every recurring decimal is a rational number.
(iv) A non- terminating repeating decimal is called a recurring decimal.
(v) Between any two rational numbers there are an infinite number of rational numbers. This property is known as the density of rational numbers.
(vi) If a and b are two rational numbers then $\frac{1}{2}(a+b)$ lies between a and b.

$a < \frac{1}{2}(a+b) < b$

n rational numbers between two different rational numbers a and b are :

$a + \frac{(b-a)}{n+1};\ a + \frac{2(b-a)}{n+1};\ a + \frac{3(b-a)}{n+1}; a + \frac{4(b-a)}{n+1}; ....a + \frac{n(b-a)}{n+1};$

(vii) Every rational number can be represented either as a terminating decimal or as a non-terminating repeating (recurring) decimals.
(viii) Types of rational numbers :– (a) Terminating decimal numbers and
(b) Non-terminating repeating (recurring) decimal numbers

(v) **Irrational numbers :–** A number is called irrational number, if it can not be written in the form $\frac{p}{q}$, where p & q are integers and $q \neq 0$. All Non-terminating & Non-repeating decimal numbers are Irrational numbers.

Ex. $\sqrt{2}, \sqrt{3}, 3\sqrt{2}, 2+\sqrt{3}, \sqrt{2+\sqrt{3}}$, π, e, etc...

(vi) **Real numbers :–** The totality of rational numbers and irrational numbers is called the set of real numbers i.e. rational numbers and irrational numbers taken together are called real numbers.
Every real number is either a rational number or an irrational number.

✭ NATURE OF THE DECIMAL EXPANSION OF RATIONAL NUMBERS

Theorem-1 : Let x be a rational number whose decimal expansion terminates. Then we can express x in the form $\frac{p}{q}$, where p and q are co-primes, and the prime factorisation of q is of the form 2^m Ч 5^n, where m,n are non-negative integers.

Theorem-2 : Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form 2^m Ч 5^n, where m,n are non-negative integers . Then, x has a decimal expansion which terminates.

Theorem-3 : Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form 2^m Ч 5^n, where m,n are non-negative integers . Then, x has a decimal expansion which is non-terminating repeating.

Ex. (i) $\frac{189}{125} = \frac{189}{5^3} = \frac{189}{2^0 \times 5^3}$

we observe that the prime factorisation of the denominators of these rational numbers are of the form 2^m Ч 5^n, where m,n are non-negative integers. Hence, $\frac{189}{125}$ has terminating decimal expansion.

(ii) $\frac{17}{6} = \frac{17}{2 \times 3}$

we observe that the prime factorisation of the denominator of these rational numbers are not of the form 2^m Ч 5^n, where m,n are non-negative integers. Hence $\frac{17}{6}$ has non-terminating and repeating decimal expansion.

(iii) $\frac{17}{8} = \frac{17}{2^3 \times 5^0}$

So, the denominator 8 of $\frac{17}{8}$ is of the form 2^m Ч 5^n, where m,n are non-negative integers.

Hence $\frac{17}{8}$ has terminating decimal expansion.

(iv) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Clearly, 455 is not of the form 2^m Ч 5^n. So, the decimal expansion of $\frac{64}{455}$ is non-terminating repeating.

★ PROOF OF IRRATIONALITY OF $\sqrt{2}, \sqrt{3}, \sqrt{5}, ...$

Ex.1 **Prove that $\sqrt{2}$ is not a rational number or there is no rational whose square is 2.**

(CBSE (outside Delhi) 2008).

Sol. Let us find the square root of 2 by long division method as shown below.

	1.414215
1	2.000000000000
+1	1
24	100
4	96
281	400
+1	281
2824	11900
+4	11296
28282	60400
+2	56564
282841	383600
+1	282841
2828423	10075900
3	8485269
28284265	159063100
+5	141421325
28284270	17641775

$\sqrt{2} = 1.414215$

Clearly, the decimal representation of $\sqrt{2}$ is neither terminating nor repeating.

We shall prove this by the method of contradiction.

If possible, let us assume that $\sqrt{2}$ is a rational number.

Then $\sqrt{2} = \frac{a}{b}$ where a, b are integers having no common factor other than 1.

$\Rightarrow \left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2$ (squaring both sides)

$2 = \frac{a^2}{b^2}$

$a^2 = 2b^2$

$\Rightarrow$ 2 divides a^2

$\Rightarrow$ 2 divides a

Therefore let a = 2c for some integer c.

$\Rightarrow a^2 = 4c^2$

$\Rightarrow$ $2b^2 = 4c^2$

$\Rightarrow$ $b^2 = 2c^2$

$\Rightarrow$ 2 divides b^2

$\Rightarrow$ 2 divides b

Thus, 2 is a common factor of a and b.

But, it contradicts our assumption that a and b have no common factor other than 1.

So, our assumption that $\sqrt{2}$ is a rational, is wrong.

Hence, $\sqrt{2}$ is irrational.

Ex.2 Prove that $\sqrt[3]{3}$ is irrational.

Sol. Let $\sqrt[3]{3}$ be rational = $\frac{p}{q}$, where p and q $\in$ Z and p, q have no common factor except 1 also q > 1.

$\therefore \frac{p}{q} = \sqrt[3]{3}$

Cubing both sides

$$\frac{p^3}{q^3} = 3$$

Multiply both sides by q^2

$$\frac{p^3}{q} = 3q^2$$, Clearly L.H.S is rational since p, q have no common factor.

$\therefore$ p^3, q also have no common factor while R.H.S. is an integer.

$\therefore$ L.H.S $\neq$ R.H.S which contradicts our assumption that $\sqrt[3]{3}$ is Irrational.

Ex.3 Prove that $2 + \sqrt{3}$ is irrational. [Sample paper (CBSE) 2008]

Sol. Let $2 + \sqrt{3}$ be a rational number equals to r

$\therefore$ $2 + \sqrt{3} = r$

$\sqrt{3} = r - 2$

Here L.H.S is an irrational number while R.H.S. r – 2 is rational. $\therefore$ L.H.S $\neq$ R.H.S

Hence it contradicts our assumption that $2 + \sqrt{3}$ is rational.

$\therefore$ $2 + \sqrt{3}$ is irrational.

Ex.4 Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Sol. Let $\sqrt{2} + \sqrt{3}$ be rational number say 'x' $\Rightarrow x = \sqrt{2} + \sqrt{3}$

$x^2 = 2 + 3 + 2\sqrt{3} \cdot \sqrt{2} = 5 + 2\sqrt{6}$

$\Rightarrow$ $x^2 = 5 + 2\sqrt{6} \Rightarrow \sqrt{6} = \frac{x^2 - 5}{2}$

As x, 5 and 2 are rationals $\Rightarrow \frac{x^2 - 5}{2}$ is a rational number.

$\Rightarrow$ $\sqrt{6} = \frac{x^2 - 5}{2}$ is a rational number

Which is contradication of the fact that $\sqrt{6}$ is a irrational number.

Hence our supposition is wrong $\Rightarrow \sqrt{2} + \sqrt{3}$ is an irrational number.

★ EUCLID'S DIVISION LEMMA OR EUCLID'S DIVISION ALGORITHM

For any two positive integers **a** and **b** there exist unique integers q and r such that

a = bq + r, where 0 á r < b.

Let us consider a = 217, b = 5 and make the division of 217 by 5 as under :

```
                 Dividend
                    ↓
Divisor ⟶  5) 217 (43 ⟵ Quotient
               20
              ----
               17
               15
              ----
                2 ⟵ Remainder
```

i.e.

Dividend	=	Divisor	Ч	Quotient	+	Remainder
(a)	=	(b)	Ч	(q)	+	(r)

e.g. (i) Consider number 23 and 5, then :

23 = 5 Ч 4 + 3

Comparing with a = bq + r

we get, a = 23, b = 5, q = 4, r = 3 and $0 \leq r < b$ (as $0 < 3 < 5$)

(ii) Consider positive integers 18 and 4

18 = 4 Ч 4 + 2

For 18 (= a) and 4 (= b) we have q = 4, r = 2 and $0 \leq r < b$

In the relation a = bq + r, where $0 \leq r < b$ is nothing but a statement of the long division of number a by b in which q is the quotient obtained and r is the remainder.

Ex. 5 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m +1 for some interger m.

Sol. Let **a** and **b** are two positive integers such that **a** is greater than **b**, then :

a = bq + r; where q and r are also positive integers and $0 \leq r < b$

Taking b = 3, we get:

a = 3q7 + r; where $0 \leq r < 3$

$\Rightarrow$ The value of positive integer **a** will be

3q + 0, 3q + 1 or 3q + 2

i.e., 3q, 3q + 1 or 3q + 2.

Now we have to show that the square of positive integers 3q, 3q + 1 and 3q + 2 can be expressed as 3m or 3m + 1 for some integer m.

$\therefore$ **Square of 3q** = $(3q)^2$

= $9q^2 = 3(3q^2) = 3m$; where m is some integer and $m = 3q^2$

Square of 3q + 1 = $(3q + 1)^2$

= $9q^2 + 6q + 1$

= $3(3q^2 + 2q) + 1 = 3m + 1$ for some integer and $m = 3q^2 + 2q$.

Square of 3q + 2 = $(3q + 2)^2$

= $9q^2 + 12q + 4$

= $9q^2 + 12q + 3 + 1$

= $3(3q^2 + 4q + 1) + 1 = 3m + 1$ for some integer and $m = 3q^2 + 4q + 1$.

$\therefore$ The square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Ex. 6 Show that one and only one out of n; n + 2 or n + 4 is divisible by 3, where n is any positive integer.

Sol. Consider any two positive integers **a** and **b** such that **a** is greater than **b**, then according to Euclid's division algorithm –

$a = bq + r$; where q and r positive integers and $0 \le r < b$

Let a = n and b = 3, then

$a = bq + r \Rightarrow n = 3q + r$; where $0 \le r < 3$.

r = 0 $\Rightarrow n = 3q + 0 = 3q$

r = 1 $\Rightarrow n = 3q + 1$

and **r = 2** $\Rightarrow n = 3q + 2$

If n = 3q; **n is divisible by 3**

If n = 3q + 1; then n + 2 = 3q + 1 + 2

= 3q + 3; which is divisible by 3

$\Rightarrow$ **n + 2 is divisible by 3**

If n = 3q + 2; then n + 4 = 3q + 2 + 4

= 3q + 6; which is divisible by 3

$\Rightarrow$ **n + 4 is divisible by 3**

Hence, if n is any positive integer, then one and only one out of n, n + 2 or n + 4 is divisible by 3.

★ APPLICATION OF EUCLID'S DIVISION LEMMA FOR FINDING H.C.F OF POSITIVE INTEGERS

Algorithm :

Consider positive integers 418 and 33

Step.(a) Taking bigger number (418) as a and smaller number (33) as b.

Express the numbers as a = bq + r

418 = 33 Ч 12 + 22

Step.(b) Now taking the divisor 33 and remainder 22, apply the Euclid's division method to get.

33 = 22 Ч 1 + 11 [Expressing as a = bq + r]

Step.(c) Again with new divisior 22 and new remainder 11, apply the Euclid's division algorithm to get

22 = 11 Ч 2 + 0

Step.(d) Since, the remainder = 0 so we can not proceed further.

Step.(e) The last divisor is 11 and we say H.C.F. of 418 and 33 = 11

Ex.7 Use Euclid's algorithm to find the HCF of 4052 and 12576.

Sol. Using a = bq + r, where $0 \le r < b$.

Clearly, 12576 > 4052 [a = 12576 , b = 4052]

$\Rightarrow$ 12576 = 4052 Ч 3 + 420

$\Rightarrow$ 4052 = 420 Ч 9 + 272

$\Rightarrow$ 420 = 272 Ч 1 + 148

$\Rightarrow$ 272 = 148 Ч 1 + 124

$\Rightarrow$ 148 = 124 Ч 1 + 24

$\Rightarrow$ 124 = 24 Ч 5 + 4

$\Rightarrow$ 24 = 4 Ч 6 + 0

The remainder at this stage is 0. So, the divisor at this stage, i.e., 4 is the HCF of 12576 and 4052.

Ex.8 Find the HCF of 1848, 3058 and 1331.

Sol. Two numbers 1848 and 3058, where 3058 > 1848

3058 = 1848 Ч 1 + 1210

1848 = 1210 Ч 1 + 638 [Using Euclid's division algorithm to the given number 1848 and 3058]

1210 = 638 Ч 1 + 572

638 = 572 Ч 1 + 66

572 = 66 Ч 8 + 44

66 = 44 Ч 1 + 22

44 = $\boxed{22}$ Ч 2 + 0

Therefore HCF of 1848 and 3058 is 22.

HCF (1848 and 3058) = 22

Let us find the HCF of the numbers 1331 and 22.

1331 = 22 Ч 60 + 11

22 = $\boxed{11}$ Ч 2 + 10

∴ HCF of 1331 and 22 is 11

⇒ HCF (22, 1331) = 11

Hence the HCF of the three given numbers 1848, 3058 and 1331 is 11.

HCF (1848, 3058, 1331) = 11

Ex.9 Using Euclid's division, find the HCF of 56, 96 and 404 [Sample paper (CBSE)-2008]

Sol. Using Euclid's division algorithm, to 56 and 96.

96 = 56 Ч 1 + 40

56 = 40 Ч 1 + 16

40 = 16 Ч 2 + 8

16 = $\boxed{8}$ Ч 2 + 0

Now to find HCF of 8 and 404

We apply Euclid's division algorithm to 404 and 8

404 = 8 Ч 50 + 4

8 = $\boxed{4}$ Ч 2 + 0

Hence 4 is the HCF of the given numbers 56, 96 and 404.

★ THE FUNDAMENTAL THEOREM OF ARITHMETIC

Statement – "Every composite number can be factorised as a product of prime numbers in a unique way, except for the order in which the prime numbers occur."

e.g. (i) 90 = 2 Ч 3 Ч 3 Ч 5 = 2 Ч 3^2 Ч 5

(ii) 432 = 2 Ч 2 Ч 2 Ч 2 Ч 3 Ч 3 Ч 3 = 2^4 Ч 3^3

(iii) 12600 = 2 Ч 2 Ч 2 Ч 3 Ч 3 Ч 5 Ч 5 Ч 7 = 2^3 Ч 3^2 Ч 5^2 Ч 7

In general, a composite number is expressed as the product of its prime factors written in ascending order of their values.

COMPETITION WINDOW

NUMBER OF FACTORS OF A NUMBER

To get the number of factors (or divisors) of a number N, express N as

$N = a^p.b^q.c^r.d^s....$ (a,b,c,d are prime numbers and p,q,r,s are indices)

Then the number of total divisors or factors of N = (p + 1) (q + 1) (r + 1) (s + 1)....

Eg. $540 = 2^2 \times 3^3 \times 5^1$

∴ total number of factors of 540 = (2 + 1) (3 + 1) (1 + 1) = 24

SUM OF FACTORS OF A NUMBER

The sum of all factors of N = $\frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(b-1)(c-1)(d-1)}$

Eg. $270 = 2 \times 3^3 \times 5$

∴ Sum of factors of 270 = $\frac{(2^{1+1}-1)(3^{3+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)} = \frac{3 \times 80 \times 24}{1 \times 2 \times 4} = 720$

PRODUCT OF FACTORS

The product of factors of composite number N = $N^{n/2}$, where n is the total number of factors of N.

Eg. $360 = 2^3 \times 3^2 \times 5^1$

∴ No. of factors of 360 = (3 + 1) (2 + 1) (1 + 1) = 24

Thus, the product of factors = $(360)^{24/2} = (360)^{12}$

NUMBER OF ODD FACTORS OF A NUMBER

To get the number of odd factors of a number N, express N as

$N = (p_1^a \times p_2^b \times p_3^c \times .....) \times (e^x)$

(where p_1, p_2, p_3,....are the odd prime factors and e is the even prime factor)

Then the total number of odd factors = (a + 1) (b + 1) (c + 1).....

Eg. $90 = 2^1 \times 3^2 \times 5^1$

∴ Total number of odd factors of 90 = (2 + 1) (1 + 1) = 6

NUMBER OF EVEN FACTORS OF A NUMBER

Number of even factors of a number = Total number of factors – Total number of odd factors.

NUMBER OF WAYS TO EXPRESS A NUMBER AS A PRODUCT OF TWO FACTORS

Let n be the number of total factors of a composite number.

Case-1 : **If the composite number is not a perfect square** then number of ways of expressing the composite number as a product of two factors = $\frac{n}{2}$

Case-2 : **If the composite number is a perfect square then**

(a) Number of ways of expressing the composite number as a product of two factors = $\frac{n+1}{2}$

(b) Number of ways of expressing the composite number as a product of two distinct factors = $\frac{(n-1)}{2}$

★ USING THE FUNDAMENTAL THEOREM OF ARITHMETIC TO FIND H.C.F AND L.C.M.

For any two numbers a and b,

(a) L.C.M. (Least common multiple) = Product of each prime factor with highest powers

$$\text{L.C.M. } (a,b) = \frac{\text{Product of the numbers or } (a \times b)}{\text{H.C.F } (a,b)}$$

(b) H.C.F. (Highest common factor) = Product of common prime factor with lowest powers.

$$\text{H.C.F. } (a,b) = \frac{\text{Product of the numbers or } (a \times b)}{\text{L.C.M } (a,b)}$$

Remark : The above relations hold only for two numbers.

COMPETITION WINDOW

For any three positive integers p,q,r –

$\text{HCF}(p,q,r) \times \text{LCM}(p,q,r) \neq p \times q \times r$

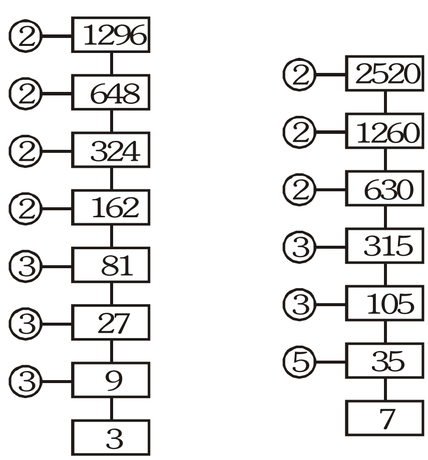
However, the following results hold good for three positive integers p,q and r :

$$\text{LCM}(p,q,r) = \frac{p \cdot q \cdot r \cdot \text{HCF}(p,q,r)}{\text{HCF}(p,q) \cdot \text{HCF}(q,r) \cdot \text{HCF}(p,r)} \; ; \; \text{HCF}(p,q,r) = \frac{p \cdot q \cdot r \cdot \text{LCM}(p,q,r)}{\text{LCM}(p,q) \cdot \text{LCM}(q,r) \cdot \text{LCM}(p,r)} \; ;$$

Ex.10 Find the L.C.M and H.C.F. of 1296 and 2520 by applying the fundamental theorem of arithmetic method i.e. using the prime factorisation method.

Sol. $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^4 \times 3^4$

$2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$



L.C.M. $= 2^4 \times 3^4 \times 5 \times 7 = 45360$

H.C.F. $= 2^3 \times 3^2 = 72$

Ex.11 Given that H.C.F. (306, 657) = 9. Find L.C.M. (306, 657)

Sol. H.C.F. (306, 657) = 9 means H.C.F. of 306 and 657 = 9

Required L.C.M. (306, 657) means required L.C.M. of 306 and 657.

For any two positive integers;

$$\text{their L.C.M.} = \frac{\text{Product of the number}}{\text{Their H.C.F.}}$$

$$\text{i.e., L.C.M. } (306, 657) = \frac{306 \times 657}{9} = 22{,}338$$

Ex.12 Given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100).

Sol. L.C.M. (150, 100) = 300

$\Rightarrow$ L.C.M. of 150 and 100 = 300

Since, the product of number 150 and 100 = 150 Ч 100

And , we know : $\text{H.C.F. (150, 100)} = \dfrac{\text{Product of 150 and 100}}{\text{L.C.M(150,100)}} = \dfrac{150 \times 100}{300} = 50$

Ex.13 Explain why 7 Ч 11 Ч 13 + 13 are composite numbers:

Sol. (i) Let 7 Ч 11 Ч 13 + 13 = (7 Ч 11 + 1) Ч 13

= (77 + 1) Ч 13 = 78 Ч 13 $\Rightarrow$ 7 Ч 11 Ч 13 + 13 = 2 Ч 3 Ч 13 Ч 13

= 2 Ч 3 Ч 13^2 is a composite number as powers of prime occur.

COMPETITION WINDOW

HCF AND LCM OF FRACTIONS

HCF of Fractions : The greatest common fraction is called the HCF of the given fractions.

$$\text{HCF of fractions} = \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$$

For example : The HCF of $\frac{4}{3}, \frac{4}{9}, \frac{2}{15}, \frac{36}{21} = \dfrac{\text{HCF of } 4,4,2,36}{\text{LCM of } 3,9,15,21} = \dfrac{2}{315}$

LCM of Fractions : The least possible number of fraction which is exactly divisible by all the given fractions is called the LCM of the fractions.

$$\text{LCM of fractions} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$$

For example : The LCM of $\frac{4}{3}, \frac{4}{9}, \frac{2}{15}, \frac{36}{21} = \dfrac{\text{LCM of } 4,4,2,36}{\text{HCF of } 3,9,15,21} = \dfrac{36}{3} = 12$

HCF AND LCM OF DECIMALS

HCF

Step-1 : First of all equate the number of places in all the numbers by using zeros, wherever required.

Step-2 : Then considering these numbers as integers find the HCF of these numbers.

Step-3 : Put the decimal point in the resultant value as many places before the right most digit as that of in the every equated number.

Ex. Find the HCF of 0.0005, 0.005, 0.15, 0.175, 0.5 and 3.5.

Sol. 0.0005 $\Rightarrow$ 5 0.0050 $\Rightarrow$ 50 0.1500 $\Rightarrow$ 15 0.1750 $\Rightarrow$ 1750

0.5000 $\Rightarrow$ 5000 3.5000 $\Rightarrow$ 35000

Then the HCF of 5, 50, 1500, 1750, 5000 and 35000 is 5. So, the HCF of the given numbers is 0.0005.

LCM

Step-1 : First of all equate the number of places in all the given numbers by putting the minimum possible number of zeros at the end of the decimal numbers, wherever even required.

Step-2 : Now consider the equated numbers as integers and then find the LCM of these numbers.

Step-3 : Put the decimal point in the LCM of the number as many places as that of in the equated numbers.

Ex. Find LCM of 1.8, 0.54 and 7.2.

Sol. $\left.\begin{matrix} 1.8 \\ 0.54 \\ 7.2 \end{matrix}\right\} \rightarrow \left.\begin{matrix} 1.80 \\ 0.54 \\ 7.20 \end{matrix}\right\} \rightarrow \begin{matrix} 180 \\ 54 \\ 720 \end{matrix}$; Now the LCM of 180, 54 and 720 is 2160. Therefore the required LCM is 21.60.

★ SYNOPSIS

1. **Euclid Division Algorithm :** Given any two positive integers a and b, $b \neq 1$, $a > b$ and a is not divisible by b, there exists two (unique) integers q and r such that

$$a = b\,q + r, \text{ where } r < b$$

2. **Prime Factorization Theorem :** Every composite number can be expressed as a product of prime factors, and this decomposition is unique, apart from the order of factors.

(The fundamental Theorem of Arithmetic)

i.e. given any composite number x, we can find unique prime factors p_1, p_2, p_3.........p_n such that

$x = p_1$ Ч p_2 Ч p_3 Ч .... Ч P_n

3. **HCF and LCM of two numbers :** Let a, b be given numbers, Let each of these is expressed as a product of prime factors.

   (i) The product of the smaller powers of the common prime numbers is the HCF. *

   (ii) The product of the prime numbers is either or both of these expression taken with greater power is the required LCM.

   (iii) HCF (a, b) Ч LCM (a,b) = a Ч b

4. Rational Numbers $\frac{p}{q}, q \neq 0$ has a terminating decimal expansion if the prime factors of q are only 2's and 5's or both.

5. Let $x = \frac{p}{q}$ be a rational number such that the prime factorization of q is of the form $2^n \cdot 5^m$ where n, m are non-negative integers, then x has a decimal expansion which terminates.

6. A rational number $\frac{p}{q}, q \neq 0$ has terminating repeating decimal expansion if the prime factors of q are other than 2 and 5 or both.

7. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \cdot 5^m$, where n and m are non negative integers, then x has a decimal expansion which is non-terminating repeating.

8. **Irrational Numbers :** $\sqrt{2}, \sqrt{3}, \sqrt{5}, 3\sqrt{3}, \sqrt{2} + \sqrt{3}, \pi, e$ are all irrational numbers. Numbers which are expressed as non-terminating and non-repeated decimals are called the irrational numbers.

9. Real Numbers are a combination of the rational numbers and the irrational numbers.

SOLVED NCERT EXERCISE

EXERCISE : 1.1

1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255.

Sol. (i) **135 and 225.** Start with the larger integer, that is, 225. Apply the division lemma to 225 and 135, to get.

225 = 135 Ч 1 + 90

Since the remainder 90 ≠ 0, we apply the division lemma to 135 and 90 to get

138 = 90 Ч 1 + 45

We consider the new divisior 90 and the new remainder 45, and apply the division lemma to get

90 = 45 Ч 2 + 0

The remainder has now become zero, so our procedure stops.

Since the divisor at this stage is 45, the HCF of 225 and 135 is 45.

[Rest Try Yourself]

2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3 or 6q + 5, where q is some integer.

Sol. Let us start with taking a, where a is any positive odd integer. We apply the division algorithm, with a and b = 6. Since $0 \le r < 6$, the possible remainders are 0, 1, 2, 3, 4, 5. That is, a can be 6q or 6q + 1, or 6q + 2, or 6q + 3, or 6q + 4, or 6q + 5, where q is the quotient. However, since a is odd, we do not consider the cases 6q, 6q + 2 and 6q + 4 (since all the three are divisible by 2). Therefore, any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5.

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. Hint : Find HCF of 616 & 32

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol. Let a be any odd positive integer. We apply the division lemma with a and b = 3. Since $0 \le r < 3$, the possible remainders are 0, 1 and 2. That is, a can be 3q, or 3q + 1, or 3q + 2, where q is the quotient.

Now, $(3q)^2 = 9q^2$

which can be written in the form 3m, since 9 is divisible by 3.

Again, $(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

which can be written in the form 3m + 1 since $9q^2 + 6q$, i.e., $3(3q^2 + 2q)$ is divisible by 3.

Lastly, $(3q + 2)^2 = 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1 = 3(3q^2 + 4q + 1) + 1$

which can be written in the form 3m + 1, since $9q^2 + 12q + 3$, i.e., $3(3q^2 + 4q + 1)$ is divisible by 3.

Therefore, the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

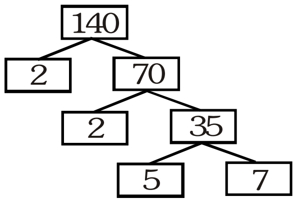
Sol. Try Yourself

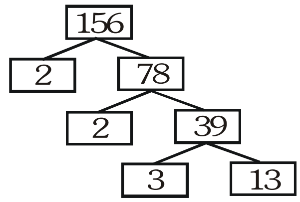
EXERCISE : 1.2

1. Express each number as product of its prime factors :

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Sol. (i) 140 (ii) 156





So, $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

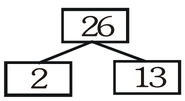
So, $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

[Rest Try Yourself]

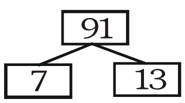
2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol.(i) 26 and 91



So, $26 = 2 \times 13$



So, $91 = 7 \times 13$

Therefore, LCM $(26, 91) = 2 \times 7 \times 13 = 182$

HCF $(26, 91) = 13$

Verification LCM × HCF $= 182 \times 13 = 2366$ and $26 \times 91 = 2366$

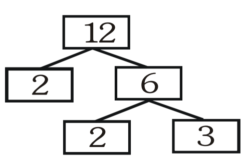
i.e., LCM × HCF = product of two numbers.

[Rest Try Yourself]

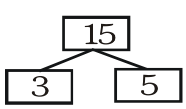
3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

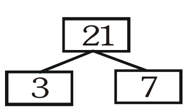
Sol.(i) 12, 15 and 21



So, $12 = 2 \times 2 \times 3 = 2^2 \times 3$



So, $15 = 3 \times 5$



So, $21 = 3 \times 7$

Therefore, HCF $(12, 15, 21) = 3$; LCM $= (12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420$ **[Rest Try Yourself]**

4. **Given that HCF (306, 657) = 9, find LCM (306, 657).**

Sol. $\text{LCM}(306, 657) = \dfrac{306 \times 657}{\text{HCF}(306, 657)} = \dfrac{306 \times 657}{9} = 22338.$

5. **Check whether 6^n can end with the digit 0 for any natural number n.**

Sol. If the number 6^n, for any natural number n, ends with digit 0, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime number 5. This is not possible because $6^n = (2 \times 3)^n = 2^n \times 3^n$; so the only primes in the factorisation of 6^n are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 6^n. So, there is no natural number n for which 6^n ends with the digit zero.

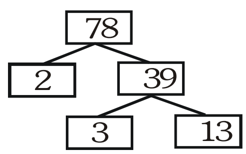
6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. (i) $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13$

$= 78 \times 13 = (2 \times 3 \times 13) \times 13$

So, $78 = 2 \times 3 \times 13 = 2 \times 3 \times 13^2$



Since, $7 \times 11 \times 13 + 13$ can be expressed as a product of primes, therefore, it is a composite number.

(ii) [Try yourself]

7. **There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes wil they meet again at the starting point?**

Sol. [Hint : Take LCM of 18 and 12]

EXERCISE : 1.3

1. **Prove that $\sqrt{5}$ is irrational.**

Sol. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find coprime integers a and b ($\neq 0$) such that

$\sqrt{5} = \dfrac{a}{b}$

$\Rightarrow \sqrt{5}\, b = a$

Squaring on both sides, we get

$5b^2 = a^2$

Therefore, 5 divides a^2.

Therefore, 5, divides a

So, we can write a = 5c for some integer c.

Substituting for a, we get

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

This means that 5 divides b^2, and so 5 divides b.

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction arose because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

2. **Prove that $3 + 2\sqrt{5}$ is irrational.**

Sol. Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational.

That is, we can find coprime integers a and b (b ≠ 0) such that $3 + 2\sqrt{5} = \frac{a}{b}$

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \quad \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \quad \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational, and so $\frac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

3. **Prove that the following are irrationals :**

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Sol. **[Try yourself]**

EXERCISE : 1.4

1. **Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.**

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Sol. (i) $\frac{13}{3125} = \frac{13}{5^5}$

Hence, $q = 5^5$, which is of the form $2^n 5^m$ $(n = 0, m = 5)$. So, the rational number $\frac{13}{3125}$ has a terminating decimal expansion.

(ii) $\frac{17}{8} = \frac{17}{2^3}$

Hence, $q = 2^3$, which is of the form $2^n 5^m$ $(n = 3, m = 0)$. So, the rational number $\frac{17}{8}$ has a terminating decimal expansion.

(iii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Hence, q = 5 Ч 7 Ч 13, which is not of the form $2^n 5^m$. So, the rational number $\frac{64}{455}$ has a non-terminating repeating decimal expansion.

[Rest Try Yourself]

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. (i) $\frac{13}{3125}$

$= \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{416}{10^5} = 0.00416$

(ii) $\frac{17}{8} = \frac{17}{2^3}$

$= \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.125$

[Rest Try Yourself]

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120 1200 12000 120000....

(iii) $43.\overline{123456789}$

Sol. (i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form $\frac{p}{q}$.

43.123456789

$= \frac{43123456789}{1000000000}$

$= \frac{43123456789}{10^9}$

$= \frac{43123456789}{(2 \times 5)^9}$

$= \frac{43123456789}{2^9\, 5^9}$

Hence, $q = 2^9\, 5^9$

The prime factorization of q is of the form $2^n\, 5^m$, where n = 9, m = 9.

(ii) 0.120 1200 12000 120000....

Since, the decimal expansion is neither terminating nor non-terminating repeating, therefore, the given real number is not rational.

(iii) Try Yourself

EXERCISE-1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOOSE THE CORRECT ONE

1. $\sqrt{2}$ is –

(A) an integer
(B) A rational number
(C) An irrational number
(D) None of these

2. $\frac{1}{\sqrt{3}}$ is –

(A) A rational number
(B) An irrational number
(C) A whole number
(D) None of these

3. $7\sqrt{3}$ is –

(A) An irrational
(B) A natural number
(C) A rational number
(D) None of these

4. $5-\sqrt{3}$ is –

(A) An integer
(B) A rational number
(C) An irrational number
(D) None of these

5. $\pi = \frac{\text{Circumference of the circle}}{\text{Diameter of the circle}}$

(A) A rational number
(B) A whole number
(C) A positive interger
(D) None of these

6. HCF (p,q) Ч LCM (p,q) =

(A) p + q
(B) $\frac{p}{q}$
(C) p Ч q
(D) p^q

7. HCF (p,q,r) · LCM (p,q,r) =

(A) $\frac{pq}{r}$
(B) $\frac{qr}{p}$
(C) p,q,r
(D) None of these

8. If $\sqrt[3]{32} = 2^x$ then x is equal to

(A) 5
(B) 3
(C) $\frac{3}{5}$
(D) $\frac{5}{3}$

9. 0.737373...=

(A) $(0.73)^3$
(B) $\frac{73}{100}$
(C) $\frac{73}{99}$
(D) None of these

10. If p is a positive prime integer, then $\sqrt{p}$ is –

(A) A rational number
(B) An irrational number
(C) A positive integer
(D) None of these

11. LCM of three numbers 28, 44, 132 is –

(A) 528
(B) 231
(C) 462
(D) 924

12. If a is a positive integer and p be a prime number and p divides a^2, then

(A) a divides p
(B) p divides a
(C) p^2 divides a
(D) None of these

13. Evaluate $\sqrt[3]{\left(\frac{1}{64}\right)^{-2}}$

(A) 4 (B) 16 (C) 32 (D) 64

14. If $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$, $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ then the value of a + b is –

(A) 14 (B) – 14 (C) $8\sqrt{3}$ (D) – $\sqrt{3}$

15. If x = $0.\overline{16}$, then 3x is –

(A) $0.\overline{48}$ (B) $0.4\overline{9}$ (C) $0.\overline{5}$ (D) 0.5

16. Find the value of x then $\left(\frac{3}{5}\right)^{2x-3} = \left(\frac{5}{3}\right)^{x-3}$

(A) x = 2 (B) x = – 2 (C) x = 1 (D) x = – 1

17. $1.\overline{3}$ is equal to –

(A) 3/4 (B) 2/3 (C) 4/3 (D) 2/5

18. The product of $4\sqrt{6}$ and $3\sqrt{24}$ is –

(A) 124 (B) 134 (C) 144 (D) 154

19. If x = $(7 + 4\sqrt{3})$, then the value of $x^2 + \frac{1}{x^2}$ is –

(A) 193 (B) 194 (C) 195 (D) 196

20. If 16 Ч $8^{n+2} = 2^m$, then m is equal to –

(A) n + 8 (B) 2n + 10 (C) 3n + 2 (D) 3n + 10

OBJECTIVE — ANSWER KEY — EXERCISE -1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	A	C	D	C	D	D	C	B	D	B	B	B	A
Que.	16	17	18	19	20										
Ans.	A	C	C	B	D										

EXERCISE-2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. Show that the product of two numbers 60 and 84 is equal to the product of their HCF and LCM.

2. The product of two numbers is 396 Ч 576 and their LCM is 6336. Find their HCF.

3. Without actually performing the long division, state whether the following rational numbers have a terminating decimal expansion or a non-terminating repeating decimal expansion :

   (i) $\frac{1}{7}$ (ii) $\frac{1}{11}$ (iii) $\frac{22}{7}$ (iv) $\frac{3}{5}$ (v) $\frac{7}{20}$ (vi) $\frac{2}{13}$ (vii) $\frac{27}{40}$ (viii) $\frac{13}{125}$ (ix) $\frac{23}{7}$ (x) $\frac{42}{100}$

4. Write down the decimal expansions of the following rational numbers :

   (i) $\frac{241}{2^3 5^2}$ (ii) $\frac{19}{256}$ (iii) $\frac{25}{1600}$ (iv) $\frac{9}{30}$ (v) $\frac{133}{2^3\ 5^4}$

5. Show that 5309 and 3072 are prime to each other.

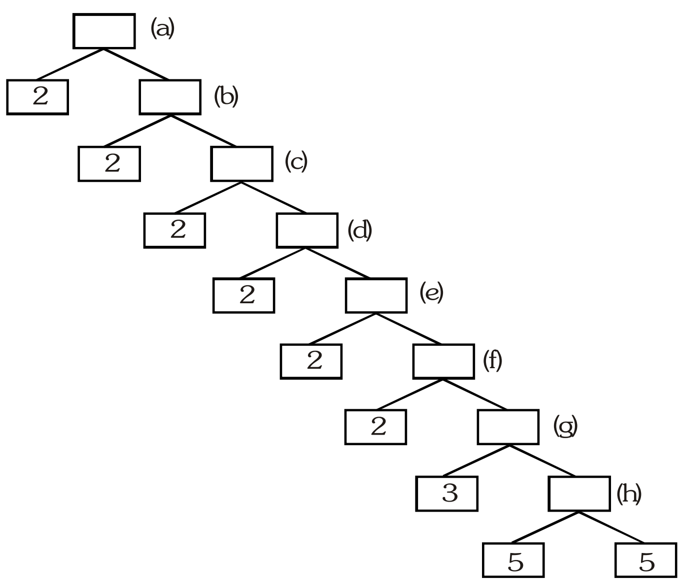
6. The HCF of two numbers is 119 and their LCM is 11781. If one of the numbers is 1071, find the other.

7. The LCM of two numbers is 2079 and their HCF is 27. If one of the numbers is 189, find the other.

8. Find the prime factorization of the following numbers :

   (i) 10000 (ii) 2160 (iii) 396 (iv) 4725 (v) 1188

9. Find the missing numbers in the following factorisation :



10. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion :

    (i) $\frac{11}{125}$ (ii) $\frac{19}{128}$ (iii) $\frac{32}{405}$ (iv) $\frac{15}{3200}$ (v) $\frac{29}{2401}$

11. Write down the decimal expansions of the following rational numbers :

    (i) $\frac{5}{8}$ (ii) $\frac{12}{125}$ (iii) $\frac{13}{625}$ (iv) $\frac{7}{64}$ (v) $\frac{7}{8}$

Short Answer Type Questions

12. Use Euclid's algorithm to find the HCF of 4052 and 12576.
13. Find the HCF 84 and 105, using Euclid's algorithm.
14. Find the HCF of 595 and 107, using Euclid's algorithm.
15. Find the HCF of 861 and 1353, using Euclid's algorithm.
16. Find the HCF of 616 and 1300, using Euclid's algorithm.
17. Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.
18. Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.
19. Show that one and only one out of n, n + 2 or n + 4 is divisible by 3, where n is any positive integer.
20. Find the greatest length which can be contained exactly in 10 m 5 dm 2 cm 4 mm and 12m 7dm 5cm 2mm.
21. Find the greatest measure which is exactly contained in 10 litres 857 millilitres and 15 litres 87 millilitres.
22. Consider the number 4^n, where n is a natural number. Check whether there is any value of $n \in N$ for which 4^n ends with the digit zero.
23. Find the LCM and HCF of 6 and 20 by the prime factorisation method.
24. Find the HCF of 12576 and 4052 by using the prime factorisation method.
25. Find the HCF and LCM of 6, 72 and 120 using the prime factorisation method.
26. Find the prime factors of the following numbers :

    (i) 1300 (ii) 1365 (iii) 3456
27. Find the LCM and HCF of 18, 24, 60, 150.
28. Find the HCF and LCM of 60, 32, 45, 80, 36, 120.
29. Split 4536 and 18711 into their prime factors and hence find their LCM and HCF.
30. Prove that $\sqrt{5}$ is irrational.
31. Prove that $\sqrt{7}$ is irrational.
32. Prove that $\frac{1}{\sqrt{3}}$ is irrational.
33. Prove that $3\sqrt{5}$ is irrational.
34. Prove that $3-\sqrt{3}$ is irrational.
35. Prove that $7+\sqrt{2}$ is irrational.
36. Prove that $5-\sqrt{5}$ is irrational.
37. Prove that $3\sqrt{2}$ is irrational.
38. Use Euclid's division lemma to find the HCF of

    (i) 13281 and 15844 (ii) 1128 and 1464 (iii) 4059 and 2190

    (iv) 10524 and 12752 (v) 10025 and 14035

39. What is the greatest number by which 1037 and 1159 can both be divided exactly?

40. Find the greatest number which both 2458090 and 867090 will contain an exact number of times.

41. Find the greatest weight which can be contained exactly in 3 kg 7 hg 8 dag 1 g and 9kg 1hg 5dag 4g.

42. Find the LCM of the following using prime factorization method. :

(i) 72, 90, 120

(ii) 24, 63, 70

(iii) 455, 117, 338

(iv) 225, 240, 208

(v) 2184, 2730, 3360

43. Prove that $\sqrt{13}$ is irrational.

44. Prove that $2\sqrt{2}$ is irrational.

45. Prove that $\frac{1}{\sqrt{5}}$ is irrational.

46. Prove that $7+\sqrt{3}$ is irrational.

47. Prove that $8-\sqrt{2}$ is irrational.

REAL NUMBERS | **ANSWER KEY** | EXERCISE-2 (X)-CBSE

- **Very Short Answer Type Questions**

2. 36.

3. (i) Non-terminating repeating ; (ii) Non-terminating repeating ; (iii) Non-terminating repeating
(iv) Terminating ; (v) Terminating ; (vi) Non-terminating repeating ; (vii) Terminating ; (viii) Terminating
(ix) Non-terminating repeating ; (x) Terminating

4. (i) 1.205 ; (ii) 0.07421875 ; (iii) 0.015625 ; (iv) 0.3 ; (v) 0.0266 **6.** 1309 **7.** 297

8. (i) 2^4 Ч 5^4 ; (ii) 2^4 Ч 3^3 Ч 5 ; (iii) 2^2 Ч 3^2 Ч 11 ; (iv) 3^3 Ч 5^2 Ч 7 ; (v) 2^2 Ч 3^3 Ч 11

9. (a) 4800 ; (b) 2400 ; (c) 1200 ; (d) 600 ; (e) 300 ; (f) 150 ; (g) 75 ; (h) 25

10. (i) Terminating ; (ii) Terminating ; (iii) Non-terminating repeating ; (iv) Terminating ; (v) Non-terminating repeating

11. (i) 0.625 ; (ii) 0.96 ; (iii) 0.0208 ; (iv) 0.109375 ; (v) 0.875

- **Short Answer Type Questions**

12. 4 **13.** 21 **14.** 119 **15.** 123 **16.** 4 **20.** 4 mm **21.** 141 millilitres **22.** No **23.** 60, 2
24. 4 **25.** 6, 360 **26.** (i) 2^2 Ч 5^2 Ч 13 ; (ii) 3 Ч 5 Ч 7 Ч 13 ; (iii) 2^7 Ч 3^3 **27.** 1800, 6 **28.** 1, 1440
29. 149688, 567 **38.** (i) 233 ; (ii) 24 ; (iii) 3 ; (iv) 4 ; (v) 2005. **39.** 61 **40.** 10 **41.** 1 hg 9 dag 9 g

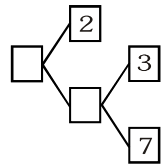
42. (i) 360 ; (ii) 2520 ; (iii) 106470 ; (iv) 46800 ; (v) 43680

EXERCISE-3 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD (CBSE) QUESTIONS

QUESTIONS CARRYING 1 MARK

1. If $\frac{p}{q}$ is a rational number ($q \neq 0$), what is condition of q so that the decimal representation of $\frac{p}{q}$ is terminating? [Delhi-2008]
2. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. [AI-2008]
3. Complete the missing entries in the following factor tree :



[Foreign-2008]

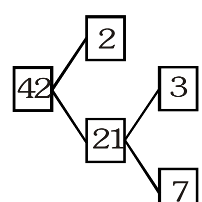
4. The decimal expansion of the rational number $\frac{43}{2^4 . 5^3}$, will terminate after how many places of decimals? [Delhi-2009]
5. Find the [HCF Ч LCM] for the numbers 100 and 190. [AI-2009]
6. Find the [HCF Ч LCM] for the numbers 105 and 120. [AI-2009]
7. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion. [Foreign-2009]
8. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number. [Foreign-2009]

QUESTIONS CARRYING 3 MARKS

9. Show that $5 - 2\sqrt{3}$ is an irrational number. [Delhi-2008]
10. Show that $2 - \sqrt{3}$ is an irrational number. [Delhi-2008]
11. Show that $5 + 3\sqrt{2}$ is an irrational number. [Delhi-2008]
12. Prove that $\sqrt{3}$ is an irrational number. [Delhi -2009/AI-2008]
13. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m. [Foreign-2008/AI-2008]
14. Prove that $\sqrt{2}$ is an irrational number. [Delhi-2009/AI-2008]
15. Prove that $\sqrt{5}$ is an irrational number. [Delhi-2009/AI-2008]
16. Prove that $3 + \sqrt{2}$ is an irrational number. [AI-2009]
17. Prove that $5 - 2\sqrt{3}$ is an irrational number. [AI-2008]
18. Prove that $3 + 5\sqrt{2}$ is an irrational number. [AI-2009]
19. Show that the square of any positive odd integers is of the form 8m + 1, for some integer m. [Foreign-2009]
20. Prove that $7 + 3\sqrt{2}$ is not a rational number. [Foreign-2009]

REAL NUMBERS **ANSWER KEY** EXERCISE-3 (X)-CBSE

1. $q = 2^n$ Ч 5^m, where n and m are whole numbers.
2. $\sqrt{2} = 1.41.......$, $\sqrt{3} = 1.73.....$
   $\therefore$ One rational no. between $\sqrt{2}$ and $\sqrt{3}$ is 1.5.
3.



4. After 4 decimal; $\frac{43}{2^4 . 5^3} = \frac{43}{2000} = 0.0215$
5. HCF Ч LCM = 100 Ч 190 = 19000 6. HCF Ч LCM = 105 Ч 120 = 12600
7. $\frac{51}{1500} = \frac{17}{500}$; 500 = 2^2 Ч 5^3 ($2^m \cdot 5^n$). So, it has terminating expansion. 8. Other number = $\frac{9 \times 360}{45} = 72$

EXERCISE-4 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

1. The greatest possible number with which when we divide 37 and 58, leaves the respective remainder of 2 and 3, is –

(A) 2 (B) 5 (C) 10 (D) None of these

2. The largest possible number with which when 60 and 98 are divided, leaves the remainder 3 in each case, is –

(A) 38 (B) 18 (C) 19 (D) None of these

3. The largest possible number with which when 38, 66 and 80 are divided the remainders remain the same is –

(A) 14 (B) 7 (C) 28 (D) None of these

4. What is the least possible number which when divided by 24, 32 or 42 in each case it leaves the remainder 5?

(A) 557 (B) 677 (C) 777 (D) None of these

5. In Q.No. 4, how many numbers are possible between 666 and 8888?

(A) 10 (B) 11 (C) 12 (D) 13

6. What is the least number which when divided by 8, 12 and 16 leaves 3 as the remainder in each case, but when divided by 7 leaves no remainder?

(A) 147 (B) 145 (C) 197 (D) None of these

7. What is the least possible number which when divided by 18, 35 or 42 leaves 2, 19, 26 as the remainders respectively?

(A) 514 (B) 614 (C) 314 (D) None of these

8. What is the least possible number which when divided by 2, 3, 4, 5, 6 leaves the remainders 1, 2, 3, 4, 5 respectively?

(A) 39 (B) 48 (C) 59 (D) None of these

9. In Q.No. 8, what is the least possible 3 digit number which is divisible by 11?

(A) 293 (B) 539 (C) 613 (D) None of these

10. How many numbers lie between 11 and 1111 which when divided by 9 leave a remainder of 6 and when divided by 21 leave a remainder of 12?

(A) 18 (B) 28 (C) 8 (D) None of these

11. If x divides y (written as $x|y$) and $y|z$, $(x, y, z \in z)$ then –

(A) $x|z$ (B) $z|y$ (C) $z|x$ (D) None of these

12. If $x|y$, where $x > 0$, $y > 0$ $(x, y \in z)$ then –

(A) $x < y$ (B) $x = y$ (C) $x \le y$ (D) $x \ge y$

13. If $a|b$, then gcd of a and b is –

(A) a (B) b (C) ab (D) Can't be determined

14. If gcd of b and c is g and $d|b$ & $d|c$, then –

(A) $d = g$ (B) $g|d$ (C) $d|g$ (D) None of these

15. If $x, y \in R$ and $|x| + |y| = 0$, then –

(A) $x > 0, y < 0$ (B) $x < 0, y > 0$ (C) $x = 0, y = 0$ (D) None of these

16. If a, b, c $\in$ R and $a^2 + b^2 + c^2 = ab + bc + ca$, then –
(A) a = b = c (B) a = b = c = 0 (C) a,b,c are distinct (D) None of these

17. If x, y $\in$ R and $x < y \Rightarrow x^2 > y^2$ then –
(A) $x > 0$ (B) $y > 0$ (C) $x < 0$ (D) $y < 0$

18. If x, y $\in$ R and $x > y \Rightarrow |x| > |y|$, then–
(A) $x > 0$ (B) $y > 0$ (C) $x < 0$ (D) $y < 0$

19. If x, y $\in$ R and $x > y \Rightarrow |x| < |y|$, then–
(A) $x < 0$ (B) $x > 0$ (C) $y > 0$ (D) $y < 0$

20. π and e are –
(A) Natural numbers (B) Integers (C) Rational numbers (D) Irrational numbers.

21. If a, b $\in$ R and a < b, then –
(A) $\frac{1}{a} < \frac{1}{b}$ (B) $\frac{1}{a} > \frac{1}{b}$ (C) $a^2 > b^2$ (D) Nothing can be said

22. If x is a non-zero rational number and xy is irrational, then y must be –
(A) a rational number (B) an irrational number (C) non-zero (D) an integer

23. The arithmetical fraction that exceeds it's square by the greatest quantity is –
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) None of these

24. If x and y are rational numbers such that $\sqrt{xy}$ is irrational, then $\sqrt{x} + \sqrt{y}$ is –
(A) Rational (B) Irrational (C) Non-real (D) None of these

25. If x and y are positive real numbers, then –
(A) $\sqrt{x} + \sqrt{y} > \sqrt{x+y}$ (B) $\sqrt{x} + \sqrt{y} < \sqrt{x+y}$
(C) $\sqrt{x} + \sqrt{y} = \sqrt{x+y}$ (D) None of these

26. If $(\sqrt{2} + \sqrt{3})^2 = a + b\sqrt{6}$, where a, b $\in$ Q, then –
(A) a = 5, b = 6 (B) a = 5, b = 2 (C) a = 6, b = 5 (D) None of these

27. If x $\in$ R, then $|x| =$
(A) x (B) – x (C) max {x, –x} (D) min {x, – x}

28. $\frac{15}{\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{125}}$ is equal to –
(A) $\sqrt{5}(5 + \sqrt{2})$ (B) $\sqrt{5}(2 + \sqrt{2})$ (C) $\sqrt{5}(\sqrt{2} + 1)$ (D) $\sqrt{5}(3 + \sqrt{2})$

29. $\sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}}$ is equal to –
(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{\sqrt{2}}$ (C) $\frac{\sqrt{2}}{\sqrt{3}}$ (D) $\sqrt{6}$

30. The expression $\frac{\sqrt{3} - 1}{2\sqrt{2} - \sqrt{3} - 1}$ is equal to –
(A) $\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$ (B) $\sqrt{6} - \sqrt{4} + \sqrt{3} - \sqrt{2}$
(C) $\sqrt{6} - \sqrt{4} - \sqrt{3} + \sqrt{2}$ (D) None of these

31. If x, y, z are real numbers such that $\sqrt{x-1}+\sqrt{y-2}+\sqrt{z-3} = 0$ then the values of x, y, z are respectively

(A) 1, 2, 3 (B) 0, 0, 0

(C) 2, 3, 1 (D) None of these

32. if a, b, c $\in$ R and a > b $\Rightarrow$ ac < bc, then –

(A) $c \geq 0$ (B) $c \leq 0$

(C) $c > 0$ (D) $c < 0$

33. If a, b, c $\in$ R and ac = bc $\Rightarrow$ a = b, then –

(A) $c \geq 0$ (B) $c \leq 0$

(C) $c = 0$ (D) $c \neq 0$

34. Between any two distinct rational numbers –

(A) There lie infinitely many rational numbers.

(B) There lies only one rational number.

(C) There lie only finitely many numbers.

(D) There lie only rational numbers.

35. The total number of divisors of 10500 except 1 and itself is –

(A) 48 (B) 50

(C) 46 (D) 56

36. The sum of the factors of 19600 is –

(A) 54777 (B) 33667

(C) 5428 (D) None of these

37. The product of divisors of 7056 is –

(A) $(84)^{48}$ (B) $(84)^{44}$

(C) $(84)^{45}$ (D) None of these

38. The number of odd factors (or divisors) of 24 is –

(A) 2 (B) 3 (C) 1 (D) None of theser

39. The number of even factors (or divisors) of 24 is –

(A) 6 (B) 4 (C) 8 (D) None of these

40. In how many ways can 576 be expressed as a product of two distinct factors?

(A) 10 (B) 11 (C) 21 (D) None of these

OBJECTIVE — ANSWER KEY — EXERCISE -4

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	A	B	D	A	B	C	B	A	A	C	A	C	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	D	B	A	D	D	B	B	B	A	B	C	D	D	A
Que.	31	32	33	34	35	36	37	38	39	40					
Ans.	A	D	D	A	C	A	C	A	A	A					

COMPETITION WINDOW

COMPLEX NUMBERS

The idea of complex numbers was introduced, so that all algebraic equations could have solutions. Over the real numbers, the square root of negative number is not defined.

Leonhard Euler for the first time introduced the symbol iota (i) in 1748, (i is the first letter of Latin word 'imaginarius'] for $\sqrt{-1}$ with the property $i^2 = -1$.

$i = \sqrt{-1}$ so $i^2 = -1$.

Imaginary Numbers : Square root of a negative number is called imaginary number. e.g. $\sqrt{-1}$, $\sqrt{-2}$, $\sqrt{-9/4}$ etc. $\sqrt{-2}$ can be written as

$\sqrt{-2} = \sqrt{2} \times \sqrt{-1} = \sqrt{2}i$

Remark :

1. If a, b are positive real numbers, then $\sqrt{-a} \times \sqrt{-b} = -\sqrt{ab}$
2. For any two real numbers $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is not valid if a and b both are negative.
3. For any positive real number a, we have $\sqrt{-a} = \sqrt{-1 \times a} = \sqrt{-1} \times \sqrt{a} = i\sqrt{a}$

E.g

1. $\sqrt{-144} = \sqrt{-1 \times 144} = \sqrt{-1} \times \sqrt{144} = 12i$
2. $\sqrt{-4} \times \sqrt{-\frac{9}{4}} = 2i\left(\frac{3i}{2}\right) = 3i^2 = -3$
3. $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9} = 5i + 6i + 6i = 17i$

Integral powers of i : We have $i = \sqrt{-1}$ so $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

For any $n \in N$, we have

$i^{4n} = 1$,

$i^{4n+1} = i$,

$i^{4n+2} = -1$,

$i^{4n+3} = -i$

E.g.

1. $i^{35} = i^3 = -i$
2. $i^{-999} = \frac{1}{i^{999}} = \frac{1}{i^3} = \frac{i}{i^4} = \frac{i}{1} = i$
3. $\left[i^{19} + \left(\frac{1}{i}\right)^{25}\right]^2 = \left[i^{19} + \frac{1}{i^{25}}\right]^2 = \left[i^3 + \frac{1}{i}\right]^2 = \left[-i + \frac{i^3}{i^4}\right]^2 = [-i + i^3]^2 = (-i - i)^2 = 4i^2 = -4$

Complex Numbers : If a, b are two real numbers, then a number of the form a + ib is called a complex number. e.g. 7 + 2i, − 1 + i, 3 − 2i etc.

If z = a + ib is a complex number, then 'a' is called the real part of z (Re (z)) and 'b' is called the imaginary part of z(Im(z)).

Equality of complex numbers : Two complex numbers $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are equal if $a_1 = a_2$ and $b_1 = b_2$.

Algebra of complex numbers : Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ then

(i) $z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$

(ii) $z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i(b_1 - b_2)$

(iii) $z_1 \cdot z_2 = (a_1 + ib_1) \cdot (a_2 + ib_2) = (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)$

(iv) $\frac{z_1}{z_2} = \frac{(a_1 + ib_1)}{(a_2 + ib_2)} = \frac{(a_1a_2 + b_1b_2)}{a_2^2 + b_2^2} + i\frac{(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$

Multiplicative Inverse of a complex number : Corresponding to every non-zero complex number $z = a + ib$, there exists a complex number $z^{-1} = x + iy$ such that

$$z \cdot z^{-1} = 1 \ (z \neq 0)$$

$$z^{-1} = \frac{1}{z} = \frac{1}{a+ib} = \frac{a-ib}{(a+ib)(a-ib)} = \frac{a-ib}{a^2+b^2}$$

Conjugate of a complex number : Let $z = a + ib$ be a complex number. Then the conjugate of z is denoted by $\bar{z}$ and is equal to $a - ib$.

Thus $z = a + ib \Rightarrow \bar{z} = a - ib$

E.g if $z = 3 + 4i \Rightarrow \bar{z} = 3 - 4i$

Modulus of a complex number : The modulus of a complex number $z = a + ib$ is denoted by $|z|$ and is defined as

$$|z| = \sqrt{a^2+b^2}$$

$|z|$ is also called the absolute value of z.

EXERCISE-5 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

1. The value of i^{457} is –

(A) 1 (B) – 1 (C) i (D) – i

2. The value of $i^{37} + \frac{1}{i^{67}}$ is –

(A) 1 (B) – 1 (C) 2i (D) – 2

3. The value of $\left(i^{41} + \frac{1}{i^{257}}\right)^9$ is –

(A) 1 (B) 0 (C) – 1 (D) 2

4. The value of $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(A) – 8 (B) – 6 (C) 6 (D) 8

5. The value of the expression $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$ is –

(A) – 1 (B) 1 (C) i (D) – i

6. The value of $1 + i^2 + i^4 + i^6 + i^8 + \ldots\ldots + i^{20}$ is –

(A) – 1 (B) 1 (C) 0 (D) i

7. The standard form of (1 + i) (1 + 2i) is –

(A) 3 + i (B) – 3 + i (C) 1 – 3i (D) – 1 + 3i

8. The standard form of $\frac{(1+i)(1+\sqrt{3}i)}{(1-i)}$ is –

(A) $-\sqrt{3} + i$ (B) $\sqrt{3} - i$ (C) $1 - i\sqrt{3}$ (D) $1 + i\sqrt{3}$

9. The standard form of $\frac{3-4i}{(4-2i)(1+i)}$ is –

(A) $\frac{1}{4} + \frac{3}{4}i$ (B) $\frac{1}{4} - \frac{3}{4}i$ (C) $\frac{3}{4} + \frac{1}{4}i$ (D) $\frac{3}{4} - \frac{1}{4}i$

10. If (x + iy) (2 – 3i) = 4 + i, then real values of x and y are –

(A) x = 5, y = 14 (B) $x = \frac{13}{5}, y = \frac{14}{13}$

(C) $x = \frac{5}{13}, y = \frac{14}{13}$ (D) None of these

11. If $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$, then real values of x and y are –

(A) x = 3, y = – 1 (B) x = – 1, y = 3

(C) x = 1, y = – 2 (D) x = – 1, y = – 3

12. The conjugate of 4 – 5i is –

(A) 4 + 5i (B) – 4 – 5i (C) – 4 + 5i (D) 4 – 5i

13. The conjugate of $\frac{1}{3+5i}$ is –

(A) $\frac{1}{34}(3+5i)$ (B) 3 + 5i (C) $\frac{1}{3-5i}$ (D) $\frac{34}{3-5i}$

14. The conjugate of $\frac{(1+i)(2+i)}{3+i}$ is –

(A) $\frac{3}{5}+\frac{4}{5}i$ (B) $\frac{3}{5}-\frac{4}{5}i$ (C) $-\frac{3}{5}-\frac{4}{5}i$ (D) $\frac{3}{5}+\frac{4}{5}i$

15. The multiplicative inverse of 1 – i is –

(A) 1 + i (B) $\frac{1}{1+i}$ (C) $\frac{1}{2}+\frac{1}{2}i$ (D) None of these

16. The multiplicative inverse of $(1+i\sqrt{3})^2$ is –

(A) $-\frac{1}{8}-\frac{i\sqrt{3}}{8}$ (B) $(1-i\sqrt{3})^2$ (C) $\frac{1}{8}+\frac{i\sqrt{3}}{8}$ (D) None of these

17. The value of $2x^3 + 2x^2 - 7x + 72$, when $x = \frac{3-5i}{2}$ is –

(A) 4 (B) – 4 (C) 2 (D) 0

18. The value of $x^4 + 4x^3 + 6x^2 + 4x + 9$, when $x = -1+i\sqrt{2}$ is –

(A) 12 (B) 10 (C) 14 (D) 8

19. If $a + ib = \frac{c+i}{c-i}$, where c is real, then $a^2 + b^2$ =

(A) i (B) 1 (C) – 1 (D) 0

20. If $(x + iy)^{1/3} = a + ib$, x, y, a b ∈ R, then $\frac{x}{a}+\frac{y}{b}$ =

(A) 4 (B) $4(a^2 + b^2)$ (C) $4(a^2 - b^2)$ (D) $(a^2 - b^2)$

OBJECTIVE — ANSWER KEY — EXERCISE -5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	B	A	A	B	D	A	B	C	A	A	A	B	C
Que.	16	17	18	19	20										
Ans.	A	A	A	B	C										