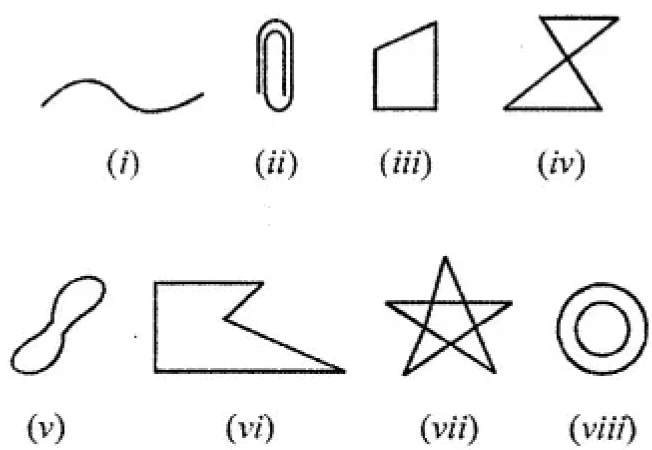
CHAPTER – 13

UNDERSTANDING QUADRILATERALS

Exercise 13.1

1. Some figures are given below.



Classify each of them on the basis of the following:
(a) Simple curve
(b) Simple closed curve
(c) Polygon
(d) Convex polygon
(e) Concave polygon

Solution:-

The given figure are classified as,

(a) Figure (i), Figure (ii), Figure (iii), Figure (v) and Figure (vi) are Simple curves.

Simple curve is a curve that does not cross itself.

(b) Figure (iii), Figure (v) and Figure (vi) are Simple closed curves.

In simple closed curves the shapes are closed by line-segments or by a curved line.

(c) Figure (iii) and Figure (vi) are Polygons.

A Polygon is any 2-dimensional shape formed with straight lines.

(d) Figure (iii) is a Convex polygon.

In a convex polygon, every diagonal of the figure passes only through interior points of the polygon.

(e) Figure (vi) is a Concave polygon.

In a concave polygon, at least one diagonal of the figure contains points that are exterior to the polygon.

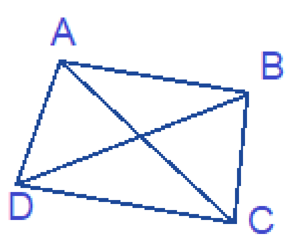
2. How many diagonals does each of the following have?

(a) A convex quadrilateral

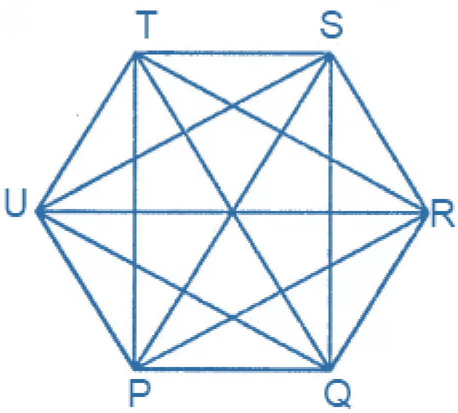
(b) A regular hexagon

Solution:

(a) A convex quadrilateral has two diagonals.



(b) A regular hexagon has 9 diagonals as shown.



3. Find the sum of measures of all interior angles of a polygon with the number of sides:

(i) 8

(ii) 12

Solution:

From the question it is given that,

(i) 8

We know that,

Sum of measures of all interior angles of 8 sided polygons = $(2n - 4) \times 90^o$

Where, $n = 8$

$= ((2 \times 8) - 4) \times 90^o$

$= (16 - 4) \times 90^o$

$= 12 \times 90^o$

$= 1080^o$

(ii) 12

We know that,

Sum of measures of all interior angles of 12 sided polygons = $(2n - 4) \times 90^\circ$

Where, $n = 12$

$= ((2 \times 12) - 4) \times 90^\circ$

$= (24 - 4) \times 90^\circ$

$= 20 \times 90^\circ$

$= 1800^\circ$

4. Find the number of sides of a regular polygon whose each exterior angles has a measure of

(i) 24°

(ii) 60°

(iii) 72°

Solution:-

(i) The number of sides of a regular polygon whose each exterior angles has a measure of 24°

Let us assume the number of sides of the regular polygon be n,

Then, $n = \frac{360^\circ}{24^\circ}$

$n = 15$

Therefore, the number of sides of a regular polygon is 15.

(ii) The number of sides of a regular polygon whose each exterior angles has a measure of 60°

Let us assume the number of sides of the regular polygon be n,

Then, $n = \frac{360^\circ}{60^\circ}$

n = 6

Therefore, the number of sides of a regular polygon is 6.

(iii) The number of sides of a regular polygon whose each exterior angles has a measure of 72°

Let us assume the number of sides of the regular polygon be n,

Then, n = $\frac{360°}{72°}$

n = 5

Therefore, the number of sides of a regular polygon is 5.

5. Find the number of sides of a regular polygon if each of its interior angles is

(i) 90°

(ii) 108°

(iii) 165°

Solution:-

(i) The number of sides of a regular polygon whose each interior angles has a measure of 90°

Let us assume the number of sides of the regular polygon be n,

Then, we know that 90° = $\left(\frac{(2n-4)}{n}\right) \times 90°$

$\frac{90°}{90°} = \frac{(2n-4)}{n}$

$1 = \frac{(2n-4)}{n}$

2n – 4 = n

By transposing we get,

2n – n = 4

n = 4

Therefore, the number of sides of a regular polygon is 4.

So, it is a Square.

(ii) The number of sides of a regular polygon whose each interior angles has a measure of 108°

Let us assume the number of sides of the regular polygon be n,

Then, we know that $108^\circ = \left(\frac{(2n - 4)}{n}\right) \times 90^\circ$

$\frac{108^\circ}{90^\circ} = \frac{(2n - 4)}{n}$

$\frac{6}{5} = \frac{(2n - 4)}{n}$

By cross multiplication,

5(2n – 4) = 6n

10n – 20 = 6n

By transposing we get,

10n – 6n = 20

4n = 20

n = $\frac{20}{4}$

n = 5

Therefore, the number of sides of a regular polygon is 5.

So, it is a Pentagon.

(iii) The number of sides of a regular polygon whose each interior angles has a measure of 165°

Let us assume the number of sides of the regular polygon be n,

Then, we know that $165^{o} = \left(\frac{(2n - 4)}{n}\right) \times 90^{o}$

$\frac{165°}{90°} = \frac{(2n - 4)}{n}$

$\frac{11}{6} = \frac{(2n - 4)}{n}$

By cross multiplication,

6(2n – 4) = 11n

12n – 24 = 11n

By transposing we get,

12n – 11n = 24

n = 24

Therefore, the number of sides of a regular polygon is 24.

6. Find the number of sides in a polygon if the sum of its interior angles is:

(i) 1260^{o}
(ii) 1980^{o}
(iii) 3420^{o}
Solution:-

(i) We know that,

Sum of measures of all interior angles of polygons = (2n – 4) × 90^{o}

Given, interior angle = 1260^{o}

1260 = (2n – 4) × 90^{o}

$\frac{1260}{90}$ = 2n – 4

$14 = 2n - 4$

By transposing we get,

$2n = 14 + 4$

$2n = 18$

$n = \frac{18}{2}$

$n = 9$

Therefore, the number of sides in a polygon is 9.

(ii) We know that,

Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Given, interior angle = 1980^o

$1980 = (2n - 4) \times 90^o$

$\frac{1980}{90} = 2n - 4$

$22 = 2n - 4$

By transposing we get,

$2n = 22 + 4$

$2n = 26$

$n = \frac{26}{2}$

$n = 13$

Therefore, the number of sides in a polygon is 13.

(ii) We know that,

Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Given, interior angle = 3420^o

$3420 = (2n - 4) \times 90^{o}$

$\frac{3420}{90} = 2n - 4$

$38 = 2n - 4$

By transposing we get,

$2n = 38 + 4$

$2n = 42$

$n = \frac{42}{2}$

$n = 21$

Therefore, the number of sides in a polygon is 21.

7. If the angles of a pentagon are in the ratio 7: 8: 11: 13: 15, find the angles.

Solution:-

From the question it is given that,

The angles of a pentagon are in the ratio 7: 8: 11: 13: 15

We know that, Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^{o}$

Given, n = 5

$= ((2 \times 5) - 4) \times 90^{o}$

$= (10 - 4) \times 90^{o}$

$= 6 \times 90^{o}$

$= 540^{o}$

Let us assume the angles of the pentagon be 7a, 8a, 11a, 13a and 15a.

Then, $7a + 8a + 11a + 13a + 15a = 540^o$

$54a = 540^o$

$a = \frac{540}{54}$

$a = 10^o$

Therefore, the angles are $7a = 7 \times 10 = 70^o$

$8a = 8 \times 10 = 80^o$

$11a = 11 \times 10 = 110^o$

$13a = 13 \times 10 = 130^o$

$15a = 15 \times 10 = 150^o$

8. The angles of a pentagon are x^o, $(x - 10)^o$, $(x + 20)^o$, $(2x - 44)^o$ and $(2x - 70)^o$ Calculate x.

Solution:-

From the question it is given that, angles of a pentagon are x^o, $(x - 10)^o$, $(x + 20)^o$, $(2x - 44)^o$ and $(2x - 70)^o$

We know that, Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Where, $n = 5$

$= ((2 \times 5) - 4) \times 90^o$

$= (10 - 4) \times 90^o$

$= 6 \times 90^o$

$= 540^o$

Then, $x^o + (x - 10)^o + (x + 20)^o + (2x - 44)^o + (2x - 70)^o = 540$

$x + x - 10^o + x + 20^o + 2x - 44^o + 2x - 70^o = 540^o$

$7x + 20^{o} – 124^{o} = 540^{o}$

$7x – 104^{o} = 540^{o}$

By transposing we get,

$7x = 540^{o} + 104^{o}$

$7x = 644^{o}$

$x = \frac{644°}{7}$

$x = 92^{o}$

Therefore, the value of x is 92^{o}.

9. The exterior angles of a pentagon are in ratio 1: 2: 3: 4: 5. Find all the interior angles of the pentagon.

Solution:-

From the question it is given that, the exterior angles of a pentagon are in ratio 1: 2: 3: 4: 5.

We know that, sum of exterior angles of pentagon is equal to 360^{o}.

So, let us assume the angles of the pentagon be 1a, 2a, 3a, 4a and 5a.

$1a + 2a + 3a + 4a + 5a = 360^{o}$

$15a = 360^{o}$

$a = \frac{360°}{15}$

$a = 24^{o}$

Therefore, the angles of pentagon are, $1a = 1 \times 24 = 24^{o}$

$2a = 2 \times 24 = 48^{o}$

$3a = 3 \times 24 = 72^{o}$

$4a = 4 \times 24 = 96^o$

$5a = 5 \times 24 = 120^o$

Then, interior angles of the pentagon are, $180^o – 24^o = 156^o$

$180^o – 48^o = 132^o$

$180^o – 72^o = 108^o$

$180^o – 96^o = 84^o$

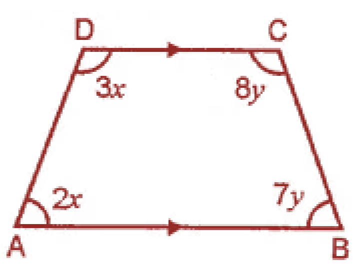
$180^o – 120^o = 60^o$

10. In a quadrilateral ABCD, AB || DC. If ∠A: ∠D = 2:3 and ∠B: ∠C = ∠7: 8, find the measure of each angle.

Solution:-

From the question it is given that,

In a quadrilateral ABCD, AB || DC. If ∠A: ∠D = 2:3 and ∠B: ∠C = ∠7: 8,



Then, $\angle A + \angle D = 180^o$

Let us assume the angle $\angle A = 2a$ and $\angle D = 3a$

$2a + 3a = 180^o$

$5a = 180^o$

$a = \frac{180^\circ}{5}$

$a = 36^o$

Therefore, $\angle A = 2a = 2 \times 36^o = 72^o$

$\angle D = 3a = 3 \times 36^o = 108^o$

Now, $\angle B + \angle C = 180^o$

Let us assume the angle $\angle B = 7b$ and $\angle C = 8b$

$7b + 8b = 180^o$

$15b = 180^o$

$b = \frac{180^\circ}{15}$

$b = 12^o$

Therefore, $\angle B = 7b = 7 \times 12^o = 84^o$

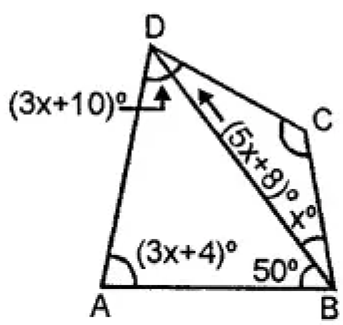
$\angle C = 8b = 8 \times 12^o = 96^o$

11. From the adjoining figure, find

(i) x

(ii) ∠DAB

(iii) ∠ADB



Solution:-

(i) From the given figure,

ABCD is a quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360^o$

$(3x + 4) + (50 + x) + (5x + 8) + (3x + 10) = 360^o$

$3x + 4 + 50 + x + 5x + 8 + 3x + 20 = 360^o$

$12x + 72 = 360^o$

By transposing we get,

$12x = 360^o – 72$

$12x = 288$

$x = \frac{288}{12}$

$x = 24$

(ii) $\angle DAB = (3x + 4)$

$= ((3 \times 24) + 4)$

$= 72 + 4$

$= 76^o$

Therefore, $\angle DAB = 76^o$

(iii) Consider the triangle ABD,

We know that, sum of interior angles of triangle is equal to 180^o,

$\angle DAB + \angle ABD + \angle ADB = 180^o$

$76^o + 50^o + \angle ADB = 180^o$

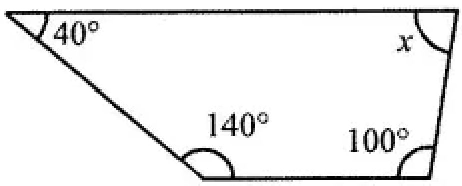
$\angle ADB + 126^o = 180^o$

$\angle ADB = 180^o – 126^o$

Therefore, $\angle ADB = 54^o$

12. Find the angle measure x in the following figures:

(i)



Solution:-

From the given quadrilateral three angles are 40°, 100° and 140°

We have to find the value of x,

We know that, sum of four angles of quadrilateral is equal to 360°.

So, $40° + 100° + 140° + x = 360°$

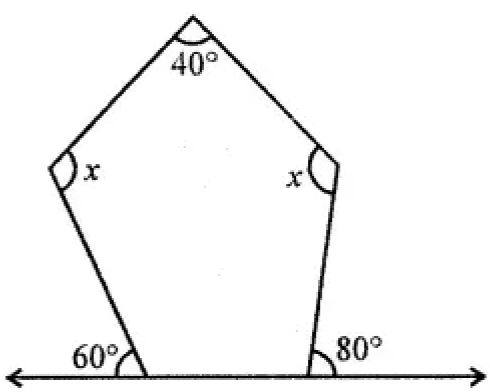
$280° + x = 360°$

$x = 360° – 280°$

$x = 80°$

Therefore, the value of x is 80°.

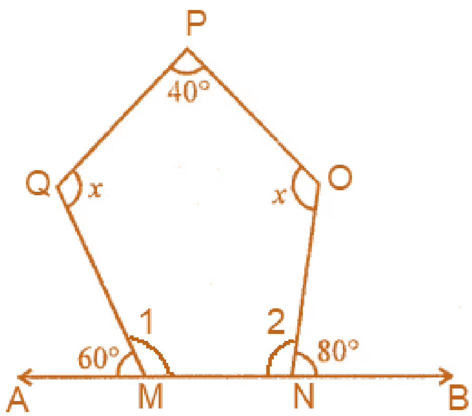
(ii)



Solution:-

From the given figure,

Let MNOPQ is a pentagon,



We know that, sum of angles linear pair is equal to 180^o

So, $\angle 1 + 60^o = 180^o$

$\angle 1 = 180^o - 60^o$

$\angle 1 = 120^o$

And $\angle 2 + 80^o = 180^o$

$\angle 2 = 180^o - 80^o$

$\angle 2 = 100^o$

Also we know that, Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Where, n = 5

$= ((2 \times 5) - 4) \times 90^o$

$= (10 - 4) \times 90^o$

$= 6 \times 90^o$

$= 540^o$

Then, $\angle M + \angle N + \angle O + \angle Q + \angle P = 540^o$

$120^o + 100^o + x + x + 40^o = 540^o$

$260^\circ + 2x = 540^\circ$

By transposing we get,

$2x = 540^\circ – 260^\circ$

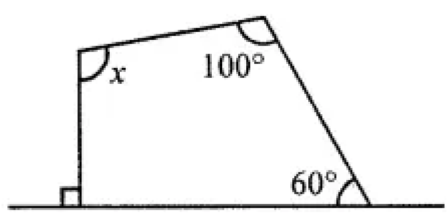
$2x = 280^\circ$

$x = \frac{280^\circ}{2}$

$x = 140^\circ$

Therefore, the value of x is 140°.

(iii)



Solution:-

From the given quadrilateral angles are 60° and 100°,

We know that, sum of angles linear pair is equal to 180°

So, another angle is $180^\circ – 90^\circ = 90^\circ$

We have to find the value of x,

We know that, sum of four angles of quadrilateral is equal to 360°.

So, $60^\circ + 100^\circ + 90^\circ + x = 360^\circ$

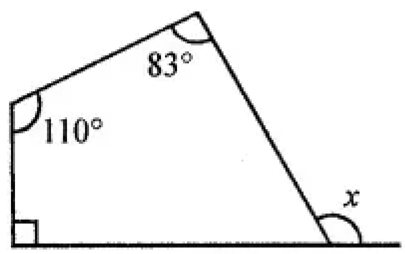
$250^\circ + x = 360^\circ$

$x = 360^\circ – 250^\circ$

$x = 110^\circ$

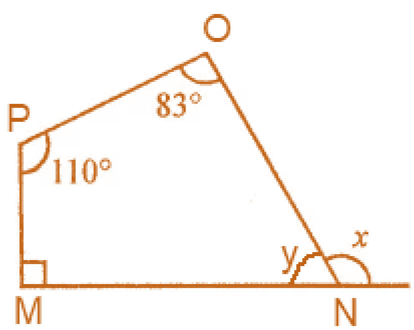
Therefore, the value of x is 110°.

(iv)



Solution:-

We know that, sum of four angles of quadrilateral is equal to 360^o.



Consider Quadrilateral MNOP,

$\angle M + \angle N + \angle O + \angle P = 360^o$

$90^o + y + 83^o + 110^o = 360^o$

$283^o + y = 360^o$

$y = 360^o - 283^o$

$y = 77^o$

Therefore, the value of y is 110^o

We know that, sum of angles linear pair is equal to 180^o

So, $y + x = 180^o$

$77^o + x = 180^o$

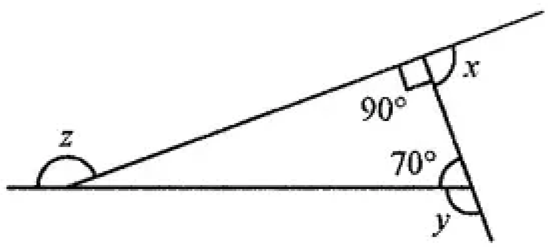
By transposing we get,

$x = 180^o – 77^o$

$x = 103^o$

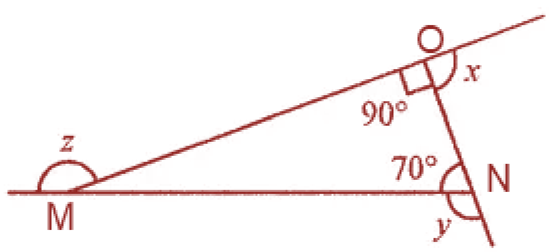
Therefore, the value of x 103^o.

13: (i) In the given figure, find x + y + z.



Solution:-

From the figure,



Consider the triangle MNO,

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle M + \angle N + \angle O = 180^o$

$\angle M + 70^o + 90^o = 180^o$

$160^o + \angle M = 180^o$

$\angle M = 180^o – 160$

$\angle M = 20^o$

We know that, sum of angles linear pair is equal to 180^o

So, $x + 90 = 180^o$

By transposing we get,

$x = 180^o – 90^o$

$x = 90^o$

Therefore, the value of x is 90^o.

Then, $y + 70^o = 180^o$

By transposing we get,

$y = 180^o – 70^o$

$y = 110^o$

Therefore, the value of y is 110^o.

Similarly, $z + 20 = 180^o$

By transposing we get,

$z = 180^o – 20^o$

$z = 160^o$

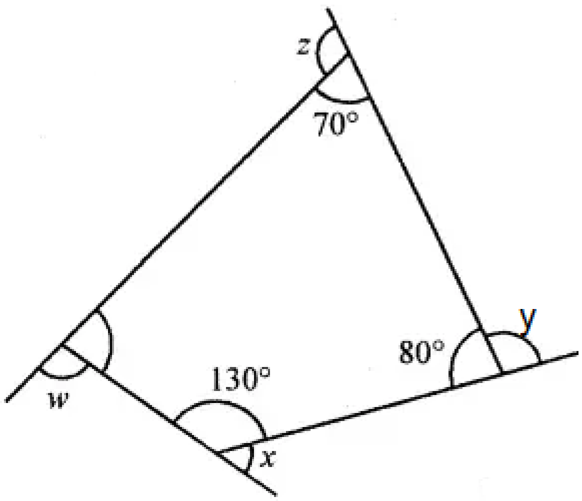
Therefore, the value of z is 160^o.

Hence, $x + y + z$

$= 90^o + 110^o + 160^o$

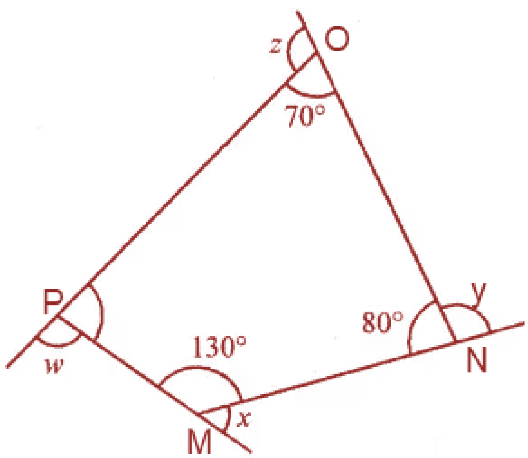
$= 360^o$

(ii) In the given figure, find x + y + z + w



Solution:-

Let MNOP is a quadrilateral,



We know that, sum of four angles of quadrilateral is equal to 360°.

∠M + ∠N + ∠O + ∠P = 360°

130° + 80° + 70° + ∠P = 360°

280° + ∠P = 360°

∠P = 360° – 280°

∠P = 80°

We know that, sum of angles linear pair is equal to 180°

So, $x + 130^o = 180^o$

By transposing we get,

$x = 180^o – 130^o$

$x = 50^o$

Therefore, the value of x is 50^o.

Then, $y + 80^o = 180^o$

By transposing we get,

$y = 180^o – 80^o$

$y = 100^o$

Therefore, the value of y is 100^o.

Similarly, $z + 70 = 180^o$

By transposing we get,

$z = 180^o – 70^o$

$z = 110^o$

Therefore, the value of z is 110^o.

Similarly, $w + 80 = 180^o$

By transposing we get,

$z = 180^o – 80^o$

$z = 100^o$

Therefore, the value of z is 110^o.

Hence, $x + y + z + w$

$= 50^o + 100^o + 110^o + 100$

$= 360^o$

14. A heptagon has three equal angles each of 120^o and four equal angles. Find the size of equal angles.

Solution:-

From the question it is given that,

A heptagon has three equal angles each of 120^o

Four equal angles =?

We know that, Sum of measures of all interior angles of polygons = $(2n – 4) \times 90^o$

Where, n = 7

$= ((2 \times 7) – 4) \times 90^o$

$= (14 – 4) \times 90^o$

$= 10 \times 90^o = 900^o$

Sum of 3 equal angles = $120^o + 120^o + 120^o = 360^o$

Let us assume the sum of four equal angle be 4x,

So, sum of 7 angles of heptagon = 900^o

Sum of 3 equal angles + Sum of 4 equal angles = 900^o

$360^o + 4x = 900^o$

By transposing we get,

$4x = 900^o – 360^o$

$4x = 540^o$

$x = \frac{540^\circ}{4}$

$x = 134^o$

Therefore, remaining four equal angle measures 135^o each.

15. The ratio between an exterior angle and the interior angle of a regular polygon is 1: 5. Find

(i) The measure of each exterior angle

(ii) The measure of each interior angle

(iii) The number of sides in the polygon.

Solution:-

From the question it is given that,

The ratio between an exterior angle and the interior angle of a regular polygon is 1: 5

Let us assume exterior angle be y

And interior angle be 5y

We know that, sum of interior and exterior angle is equal to 180^o,

$y + 5y = 180^o$

$6y = 180^o$

$y = \frac{180°}{6}$

$y = 30^o$

(i) The measure of each exterior angle = y = 30^o

(ii) The measure of each interior angle = 5y = $5 \times 30^o = 150^o$

(iii) The number of sides in the polygon

The number of sides of a regular polygon whose each interior angles has a measure of 150^o

Let us assume the number of sides of the regular polygon be n,

Then, we know that $150^o = \left(\frac{(2n - 4)}{n}\right) \times 90^o$

$\frac{150°}{90°} = \frac{(2n-4)}{n}$

$\frac{5}{3} = \frac{(2n-4)}{n}$

By cross multiplication,

3(2n – 4) = 5n

6n – 12 = 5n

By transposing we get,

6n – 5n = 12

n = 12

Therefore, the number of sides of a regular polygon is 12.

16. Each interior angle of a regular polygon is double of its exterior angle. Find the number of sides in the polygon.

Solution:-

From the question it is given that,

Each interior angle of a regular polygon is double of its exterior angle.

So, let us assume exterior angle be y

Interior angle be 2y,

We know that, sum of interior and exterior angle is equal to 180^o,

$y + 2y = 180^o$

$3y = 180^o$

$y = \frac{180°}{3}$

$y = 60^o$

Then, interior angle = 2y = 2 × 60° = 120°

The number of sides of a regular polygon whose each interior angles has a measure of 120°

Let us assume the number of sides of the regular polygon be n,

Then, we know that $120^\circ = \left(\frac{(2n-4)}{n}\right) \times 90^\circ$

$$\frac{120^\circ}{90^\circ} = \frac{(2n-4)}{n}$$

$$\frac{4}{3} = \frac{(2n-4)}{n}$$

By cross multiplication,

3(2n – 4) = 4n

6n – 12 = 4n

By transposing we get,

6n – 4n = 12

2n = 12

$n = \frac{12}{2}$

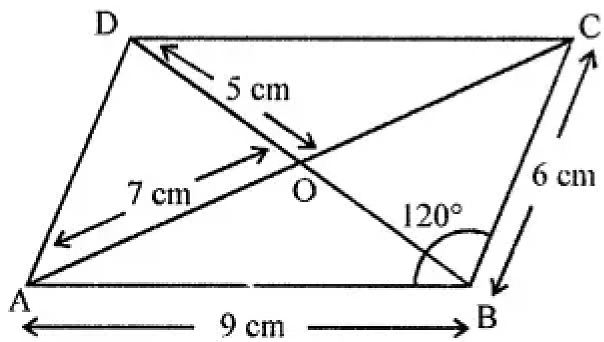
n = 6

Therefore, the number of sides of a regular polygon is 6.

Exercise 13.2

1. In the given figure, ABCD is a parallelogram. Complete each statement along with the definition or property used.

(i) AD = ………..
(ii) DC = ………..
(iii) ∠DCB = ………..
(iv) ∠ADC = ………..
(v) ∠DAB = ………..
(vi) OC = ………..
(vii) OB = ………..
(viii) m ∠DAB + m ∠CDA = ………..



Solution:-

From the given figure,

(i) AD = 6 cm … [because opposite sides of parallelogram are equal]

(ii) DC = 9 cm … [because opposite sides of parallelogram are equal]

(iii) ∠DCB = 60^o

(iv) ∠ADC = ∠ABC = 120^o

(v) ∠DAB = ∠DCB = 60^o

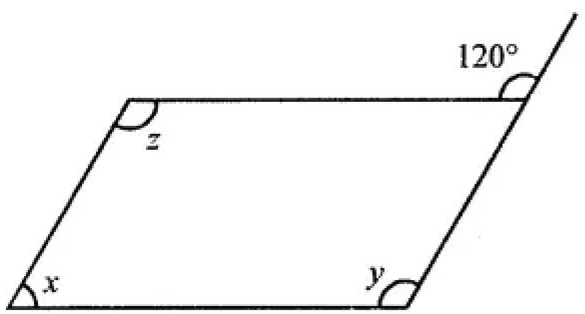
(vi) OC = AO = 7 cm

(vii) OB = OD = 5 cm

(viii) m ∠DAB + m ∠CDA = 180º

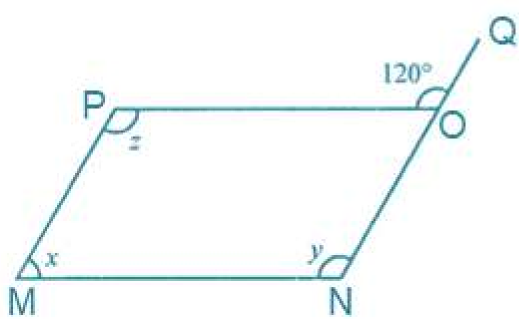
2. Consider the following parallelograms. Find the values of x, y, z in each.

(i)



Solution:-

(i) Consider parallelogram MNOP



From the figure, ∠POQ = 120º

We know that, sum of angles linear pair is equal to 180º

So, ∠POQ + ∠PON = 180º

120º + ∠PON = 180º

∠PON = 180º – 120º

∠PON = 60°

Then, ∠M = ∠O = 60°

… [Because opposite angles of parallelogram are equal]

∠POQ = ∠MNO

120° = 120° … [because corresponding angels are equal]

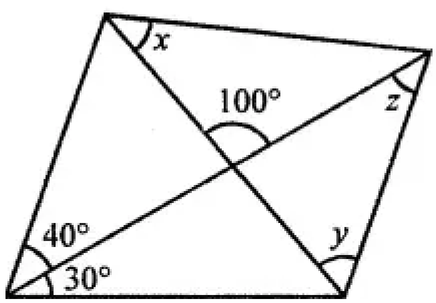
Hence, y = 120°

Also, z = y

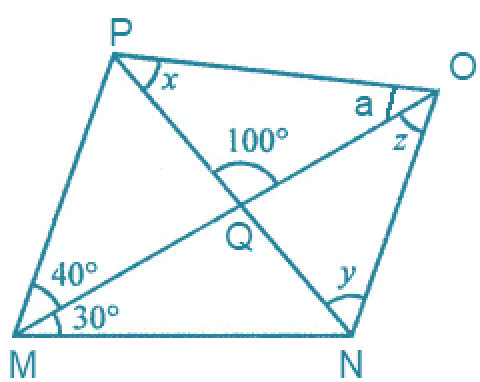
120° = 120°

Therefore, x = 60°, y = 120° and z = 120°

(ii)



Solution:-



From the figure, it is given that $\angle PQO = 100^o$, $\angle OMN = 30^o$, $\angle PMO = 40^o$.

Then, $\angle NOM = \angle OMP$ … [because alternate angles are equal]

So, $z = 40^o$

Now, $\angle NMO = \angle POM$ … [because alternate angles are equal]

So, $\angle NMO = a = 30^o$

Consider the triangle PQO,

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle P + \angle Q + \angle O = 180^o$

$x + 100^o + 30^o = 180^o$

$x + 130^o = 180^o$

$x = 180^o - 130^o$

$x = 50^o$

Then, exterior angle $\angle OQP = y + z$

$100^o = y + 40^o$

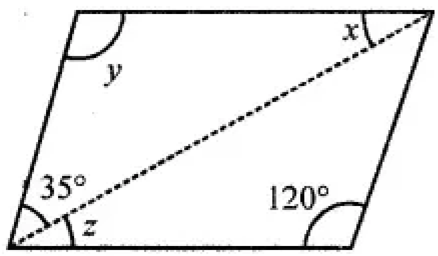
By transposing we get,

$y = 100^o - 40^o$

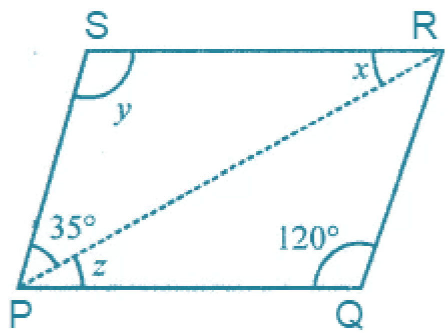
$y = 60^o$

Therefore, the value of $x = 50^o$, $y = 60^o$ and $z = 40^o$.

(iii)



Solution:-



From the above figure,

$\angle SPR = \angle PRQ$

$35^o = 35^o$ … [because alternate angles are equal]

Now consider the triangle PQR,

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle RPQ + \angle PQR + \angle PRQ = 180^o$

$z + 120^o + 35^o = 180^o$

$z + 155^o = 180^o$

$z = 180^o - 155^o$

$z = 25^o$

Then, $\angle QPR = \angle PRQ$

$Z = x$

$25^\circ = 25^\circ$ … [because alternate angles are equal]

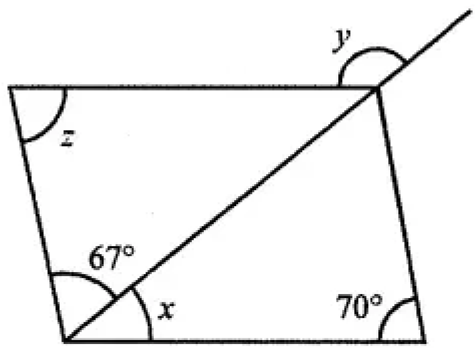
We know that, in parallelogram opposite angles are equal.

So, $\angle S = \angle Q$

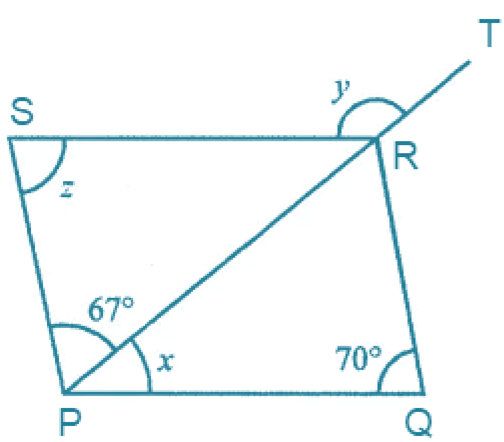
$y = 120^\circ$

Therefore, value of $x = 25^\circ$, $y = 120^\circ$ and $\angle z = 25^\circ$.

(iv)



Solution:-



From the above figure, it is given that $\angle SPR = 67^\circ$ and $\angle PQR = 70^\circ$

$\angle SPR = \angle PRQ$

$67^\circ = 67^\circ$ … [because alternate angles are equal]

Now, consider the triangle PQR

We know that, sum of measures of interior angles of triangle is equal to 180°.

$\angle RPQ + \angle PQR + \angle PRQ = 180^\circ$

$x + 70^\circ + 67^\circ = 180^\circ$

$x + 137^\circ = 180^\circ$

$x = 180^\circ - 137^\circ$

$x = 43^\circ$

Then, $\angle PSR = \angle PQR$

We know that, in parallelogram opposite angles are equal.

$Z = 70^\circ$

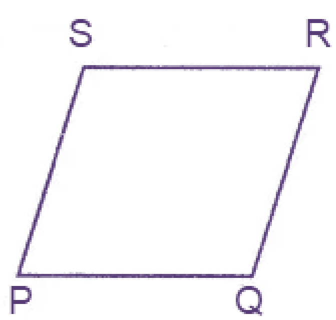
Also we know that, exterior angle $\angle SRT = \angle PSR + \angle SPR$

$y = 70^\circ + 67^\circ$

$y = 137^\circ$

Therefore, value of $x = 43^\circ$, $y = 137^\circ$ and $z = 70^\circ$

3. Two adjacent sides of a parallelogram are in the ratio 5: 7. If the perimeter of a parallelogram is 72 cm, find the length of its sides. Solution:-



Consider the parallelogram PQRS,

From the question it is given that, two adjacent sides of a parallelogram are in the ratio 5: 7.

Perimeter of parallelogram = 72 cm

2(SP + RQ) = 72 cm

SP + RQ = $\frac{72}{2}$

SP + RQ = 36 cm

Let us assume the length of side SP = 5y and RQ = 7y,

5y + 7y = 36

12y = 36

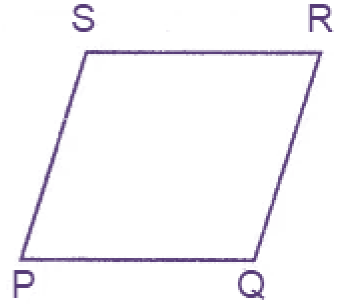
y = $\frac{36}{12}$

y = 3

Therefore, SP = 5y = 5 × 3 = 15 cm

RQ = 7y = 7 × 3 = 21 cm

4. The measure of two adjacent angles of a parallelogram is in the ratio 4: 5. Find the measure of each angle of the parallelogram. Solution:-



Consider the parallelogram PQRS,

From the question it is given that, the measure of two adjacent angles of a parallelogram is in the ratio 4: 5.

So, $\angle P : \angle Q = 4 : 5$

Let us assume the $\angle P = 4y$ and $\angle Q = 5y$.

Then, we know that, $\angle P + \angle Q = 180^\circ$

$4y + 5y = 180^\circ$

$9y = 180^\circ$

$y = \frac{180^\circ}{9}$

$y = 20^\circ$

Therefore, $\angle P = 4y = 4 \times 20^\circ = 80^\circ$ and $\angle Q = 5y = (5 \times 20^\circ) = 100^\circ$

In parallelogram opposite angles are equal,

So, $\angle R = \angle P = 80^\circ$

$\angle S = \angle Q = 100^\circ$

5. Can a quadrilateral ABCD be a parallelogram, give reasons in support of your answer?

(i) $\angle A + \angle C = 180^\circ$?

(ii) AD = BC = 6 cm, AB = 5 cm, DC = 4.5 cm?

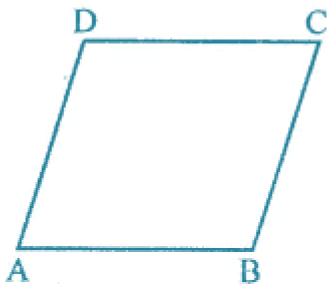
(iii) $\angle B = 80^\circ$, $\angle D = 70^\circ$?

(iv) $\angle B + \angle C = 180^\circ$?

Solution:-

From the question it is given that, quadrilateral ABCD can be a parallelogram.

We know that in parallelogram opposite sides are equal and opposite's angles are equal.



So, AB = DC and AD = BC also $\angle A = \angle C$ and $\angle B = \angle D$.

(i) $\angle A + \angle C = 180^o$

From the above condition it may be a parallelogram and may not be a parallelogram.

(ii) AD = BC = 6 cm, AB = 5 cm, DC = 4.5 cm

From the above dimension not able to form parallelogram.

Because AB $\neq$ DC

(iii) $\angle B = 80^o$, $\angle D = 70^o$

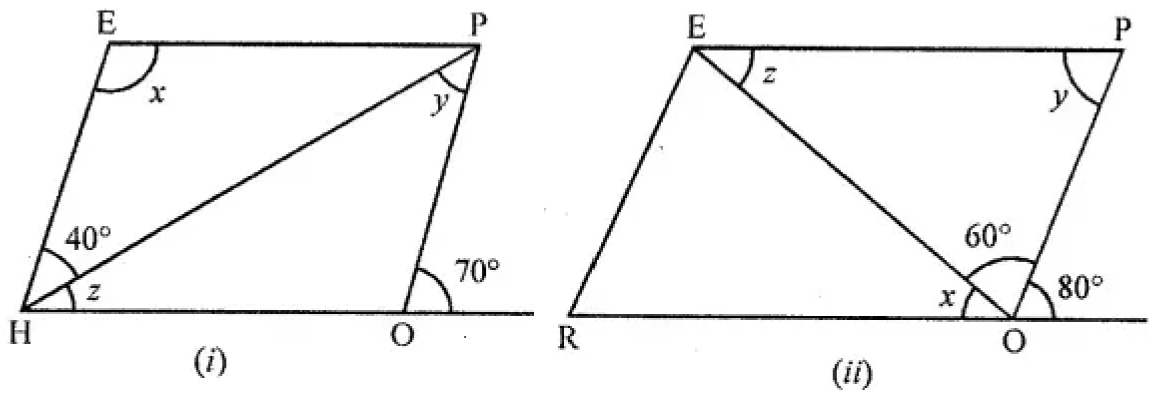
From the above dimension not able to form parallelogram.

Because $\angle B \neq \angle D$

(iv) $\angle B + \angle C = 180^o$

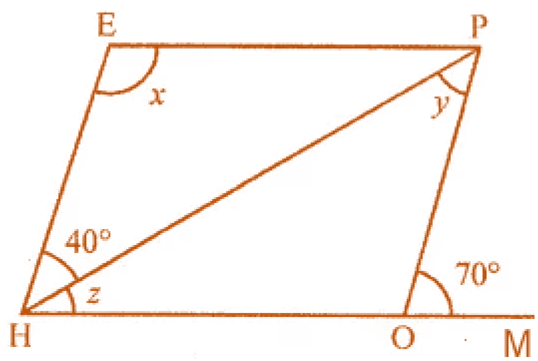
From the above condition it may be a parallelogram and may not be a parallelogram.

6. In the following figures, HOPE and ROPE are parallelograms. Find the measures of angles x, y and z. State the properties you use to find them.



Solution:

(i) Consider the parallelogram HOPE



We know that, sum of interior and exterior angle is equal to 180°,

$\angle HOP + \angle POM = 180^\circ$

$\angle HOP + 70^\circ = 180^\circ - 70^\circ$

$\angle HOP = 110^\circ$

Then, $\angle HEP = \angle HOP$

$x = 110^\circ$ … [because in parallelogram opposite angles are equal]

$\angle OPH = \angle PHE$

$y = 40^\circ$ … [because alternate angles are equal]

Now, consider the triangle HOP

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle PHO + \angle HOP + \angle OPH = 180^o$

$z + 110^o + 40^o = 180^o$

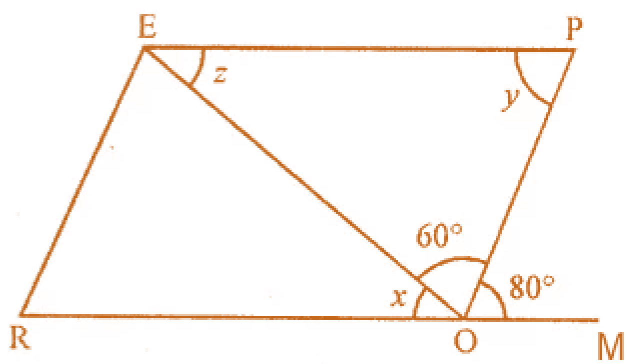
$z + 150^o = 180^o$

$z = 180^o - 150^o$

$z = 30^o$

Therefore, value of $x = 110^o$, $y = 40^o$ and $z = 30^o$.

(ii) Consider the parallelogram ROPE



From the figure, it is given that $\angle POM = 80^o$ and $\angle POE = 60^o$.

Then, $\angle OPE = \angle POM$

$y = 80^o$ … [because alternate angles are equal]

We know that, angles on the same straight line are equal to 180^o.

$\angle ROE + \angle EOP + POM = 180^o$

$x + 60^o + 80^o = 180^o$

$x + 140^o = 180^o$

By transposing we get,

$x = 180^o - 140^o$

$x = 40^o$

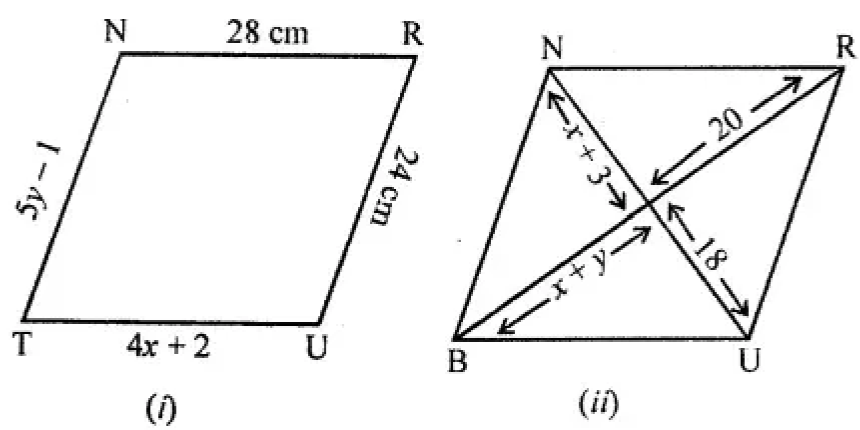
Then, ∠ROE = ∠OEP

$x = z$

$40^o = z$

Therefore, value of $x = 40^o$, $y = 80^o$ and $z = 40^o$.

7. In the given figure TURN and BURN are parallelograms. Find the measures of x and y (lengths are in cm).



Solution:-

(i) Consider the parallelogram TURN

We know that, in parallelogram opposite sides are equal.

So, TU = RN

$4x + 2 = 28$

By transposing,

$4x = 28 – 2$

$4x = 26$

$x = \frac{26}{4}$

x = 6.5 cm and NT = RU

5y – 1 = 24

5y = 24 + 1

5y = 25

$y = \frac{25}{5}$

y = 5

Therefore, value of x = 6.5 cm and y = 5 cm.

(ii) Consider the parallelogram BURN,

BO = OR

x + y = 20 … [equation (i)]

UO = ON

x + 3 = 18

x = 18 – 3

x = 15

Substitute the value of x in equation (i),

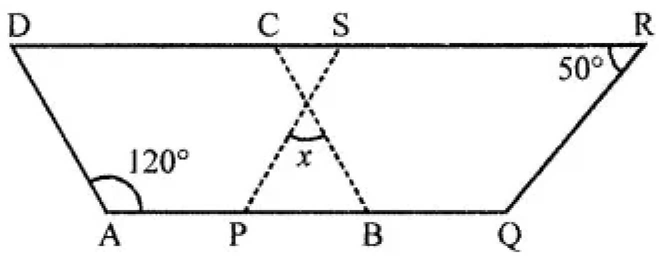
15 + y = 20

y = 20 – 15

y = 5

Therefore, value of x = 15 and y = 5.

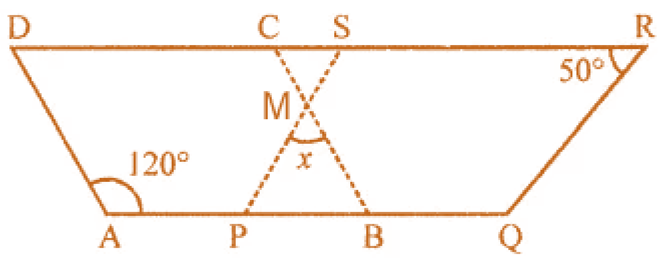
8. In the following figure, both ABCD and PQRS are parallelograms. Find the value of x.



Solution:-

From the figure it is given that, ABCD and PQRS are two parallelograms.

$\angle A = 120^\circ$ and $\angle R = 50^\circ$



We know that, $\angle A + \angle B = 180^\circ$

$120^\circ + \angle B = 180^\circ$

$\angle B = 180^\circ - 120^\circ$

$\angle B = 60^\circ$

In parallelogram opposite angles are equal,

Then, consider the triangle MPB

We know that, sum of measures of interior angles of triangle is equal to 180°.

$\angle PMB + \angle P + \angle B = 180^\circ$

$x + 50^\circ + 60^\circ = 180^\circ$

$x + 110^\circ = 180^\circ$

By transposing we get,

$x = 180^o - 110^o$

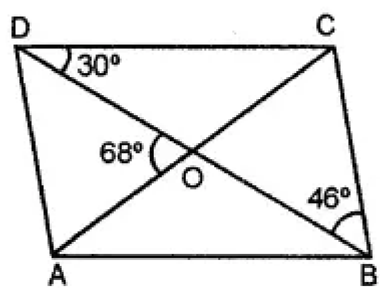
Therefore, value of $x = 70^o$

9. In the given figure, ABCD, is a parallelogram and diagonals intersect at O. Find:

(i) ∠CAD

(ii) ∠ACD

(iii) ∠ADC



Solution:-

From the figure it is given that,

∠CBD = 46°, ∠AOD = 68° and ∠BDC = 30°

(i) ∠CBD = ∠BDA = 46° … [alternate angles are equal]

Consider the ΔAOD,

We know that, sum of measures of interior angles of triangle is equal to 180°.

∠AOD + ∠ODA + ∠DAO = 180°

68° + 46° + ∠DAO = 180°

∠DAO + 114° + 180°

∠DAO = 180° – 114°

$\angle DAO = 66^o$

Therefore, $\angle CAD = 66^o$

(ii) We know that, sum of angles on the straight line are equal to 180^o,

$\angle AOD + \angle COD = 180^o$

$68^o + \angle COD = 180^o$

$\angle COD = 180^o - 68^o$

$\angle COD = 112^o$

Now consider ΔCOD,

$\angle COD + \angle ODC + \angle DCO = 180^o$

$112^o + 30^o + \angle DCO = 180^o$

$\angle DCO + 142^o = 180^o$

By transposing we get,

$\angle DCO = 180^o - 142^o$

$\angle DCO = 38^o$

Therefore, $\angle ACD = 38^o$

(iii) $\angle ADC = \angle ADO + \angle ODC$

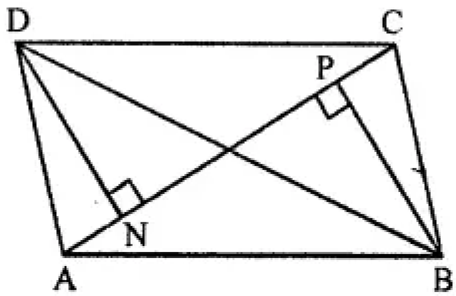
$\angle ADO = \angle OBC = 46^o$ … [alternate angles are equal]

Then, $\angle ADC = 46^o + 30 = 76^o$

10. In the given figure, ABCD is a parallelogram. Perpendiculars DN and BP are drawn on diagonal AC. prove that:

(i) $\Delta DCN \cong \Delta BAP$

(ii) AN = CP



Solution:-

From the figure it is given that,

ABCD is a parallelogram

Perpendiculars DN and BP are drawn on diagonal AC

We have to prove that, (i) $\Delta DCN \cong \Delta BAP$, (ii) $AN = CP$

So, consider the ΔDCN and ΔBAP

$AB = DC$ … [opposite sides of parallelogram are equal]

$\angle N = \angle P$ … [both angles are equal to 90°]

$\angle BAP = \angle DCN$ … [alternate angles are equal]

Therefore, $\Delta DCN \cong \Delta BAP$ … [AAS axiom]

Then, $NC = AP$

Because, corresponding parts of congruent triangle.

So, subtracting NP from both sides we get,

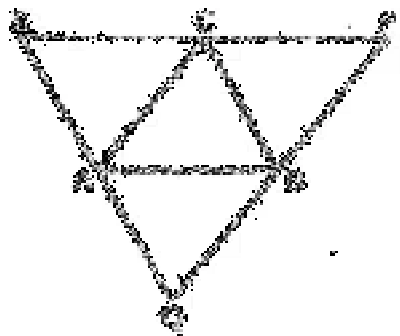
$NC - NP = AP - NP$

$AN = CP$

Hence it is proved that, $\Delta DCN \cong \Delta BAP$ and $AN = CP$.

11. In the given figure, ABC is a triangle. Through A, B and C lines are drawn parallel to BC, CA and AB respectively, which forms a ΔPQR.

Show that 2(AB + BC + CA) = PQ + QR + RP.



Solution:-

From the figure it is given that,

Through A, B and C lines are drawn parallel to BC, CA and AB respectively.

We have to show that 2(AB + BC + CA) = PQ + QR + RP

Then, AB ∥ RC and AR ∥ CB

Therefore, ABCR is a parallelogram.

So, AB = CR … [equation (i)]

CB = AR … [equation (ii)]

Similarly, ABPC is a parallelogram.

AB ∥ CP and PB ∥ CA

AB = PC … [equation (iii)]

AC = PB … [equation (iv)]

Similarly, ACBQ is a parallelogram

AC = BQ … [equation (v)]

AQ = BC … [equation (vi)]

By adding all the equation, we get,

AB + AB + BC + BC + AC + AC = PB + PC + CR + AR + BQ +BC

2AB + 2BC + 2AC = PQ + QR + RP

By taking common we get,

2(AB + BC + AC) = PQ + QR + RP

Exercise 13.3

1. Identify all the quadrilaterals that have
(i) four sides of equal length
(ii) four right angles.

Solution:-

(i) The quadrilaterals that have four sides of equal length are square and rhombus.

(ii) The quadrilaterals that have four right angles are square and rectangle.

2. Explain how a square is
(i) a quadrilateral
(ii) a parallelogram
(iii) a rhombus
(iv) a rectangle.

Solution:-

(i) A square is a quadrilateral because it has four equal sides and four angles whose sum is equal to 360^o.

(ii) A square is a parallelogram because it has opposite sides equal and opposite are parallel.

(iii) A square is a rhombus because it's all four sides have equal length.

(iv) A square is a rectangle because it's opposite sides are equal and parallel and each angle are equal to 90^o.

3. Name the quadrilaterals whose diagonals
(i) bisect each other

(ii) are perpendicular bisectors of each other
(iii) are equal.

Solution:-

(i) The quadrilaterals whose diagonals are bisect each other are rectangle, square, rhombus and parallelogram.

(ii) The quadrilaterals whose diagonals are perpendicular bisectors of each other are square and rhombus.

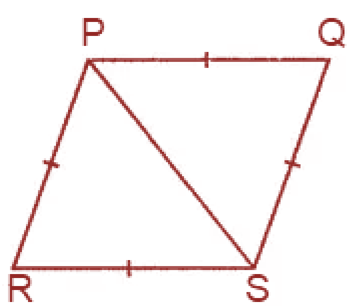
(iii) The quadrilaterals whose diagonals equal are square and rectangle.

4. One of the diagonals of a rhombus and its sides are equal. Find the angles of the rhombus.

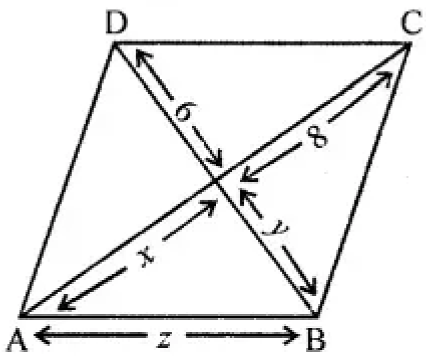
Solution:-

From the question it is given that, one of the diagonals of a rhombus and its sides are equal.

Therefore, the angles of the rhombus are 60° and 120°.



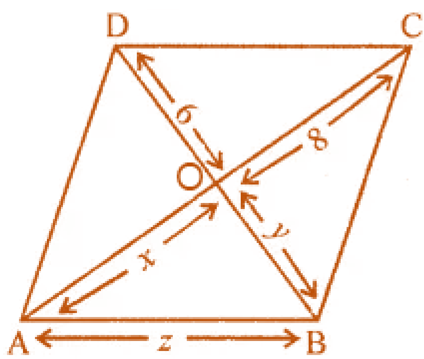
5. In the given figure, ABCD is a rhombus, find the values of x, y and z



Solution:-

From the figure,

ABCD is a rhombus



Then, the diagonals of rhombus bisect each other at right angles.

So, AO = OC

x = 8 cm

Therefore, AO = 8 cm

And BO = OD

y = 6 cm

Therefore, BO = 6 cm

Consider the ΔAOB, it is a right angled triangle.

By Pythagoras theorem,

$AB^2 = AO^2 + BO^2$

$AB^2 = 8^2 + 6^2$

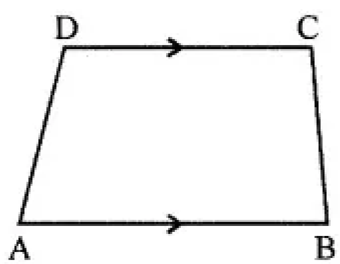
$AB^2 = 64 + 36$

$AB^2 = 100$

$AB = \sqrt{100}$

AB = 10 cm

6. In the given figure, ABCD is a trapezium. If ∠A: ∠D = 5: 7, ∠B = (3x + 11) ° and ∠C = (5x – 31) °, then find all the angles of the trapezium.



Solution:-

From the given figure,

ABCD is a trapezium

∠A: ∠D = 5: 7, ∠B = (3x + 11) ° and ZC = (5x – 31) °

Then, ∠B + ∠C = 180° … [because co – interior angle]

(3x + 11) ° + (5x – 31) ° = 180°

3x + 11 + 5x – 31 = 180°

8x – 20 = 180°

By transposing we get,

$8x = 180^{o} + 20$

$8x = 200^{o}$

$x = \frac{200^{\circ}}{8}$

$x = 25^{o}$

Then, $\angle B = 3x + 11$

$= (3 \times 25) + 11$

$= 75 + 11$

$= 86^{o}$

$\angle C = 5x - 31$

$= (5 \times 25) - 31$

$= 125 - 31 = 94^{o}$

Let us assume the angles $\angle A = 5y$ and $\angle D = 7y$

We know that, sum of co – interior angles are equal to 180^{o}.

$\angle A + \angle D = 180^{o}$

$5y + 7y = 180^{o}$

$12y = 180^{o}$

$y = \frac{180^{\circ}}{12}$

$y = 15^{o}$

Then, $\angle A = 5y = (5 \times 15) = 75^{o}$

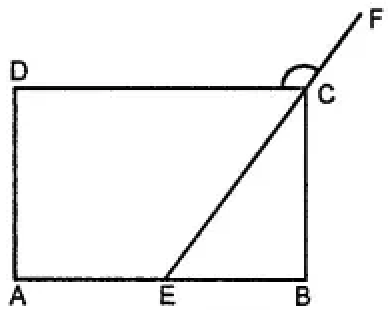
$\angle D = 7y = (7 \times 15) = 105^{o}$

Therefore, the angles are $\angle A = 75^{o}$, $\angle B = 86^{o}$, $\angle C = 94^{o}$ and $\angle D = 105^{o}$.

7. In the given figure, ABCD is a rectangle. If ∠CEB: ∠ECB = 3: 2 find

(i) ∠CEB,

(ii) ∠DCF



Solution:-

From the question it is given that,

ABCD is a rectangle

∠CEB: ∠ECB = 3: 2

We have to find, (i) ∠CEB and (ii) ∠DCF

Consider the ΔBCE,

$\angle B = 90^\circ$

Therefore, $\angle CEB + \angle ECB = 90^\circ$

Let us assume the angles be 3y and 2y

$3y + 2y = 90^\circ$

$5y = 90^\circ$

$y = \frac{90^\circ}{5}$

$y = 18^\circ$

Then, $\angle CEB = 3y = 3 \times 18 = 54^\circ$

$\angle CEB = \angle ECD$

$54^o = 54^o$ … [alternate angles are equal]

We know that, sum of linear pair angles equal to 180^o

$\angle ECD + DCF = 180^o$

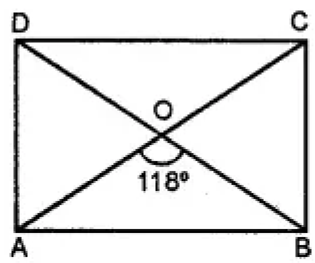
$54^o + \angle DCF = 180^o$

By transposing we get,

$\angle DCF = 180^o – 54^o$

$\angle DCF = 126^o$

8. In the given figure, ABCD is a rectangle and diagonals intersect at O. If ∠AOB = 118°, find



(i) ∠ABO

(ii) ∠ADO

(iii) ∠OCB

Solution:-

From the figure it is given that,

ABCD is a rectangle and diagonals intersect at O.

$\angle AOB = 118^o$

(i) Consider the ΔAOB,

$\angle OAB = \angle OBA$

Let us assume $\angle OAB = \angle OBA = y^o$

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle OAB + \angle OBA + \angle AOB = 180^o$

$y + y + 118^o = 180^o$

$2x + 118^o = 180^o$

By transposing we get,

$2y = 180^o - 118^o$

$2y = 62^o$

$y = \frac{62}{2}$

$y = 31^o$

So, $\angle OAB = \angle OBA = 31^o$

Therefore, $\angle ABO = 31^o$

(ii) We know that sum of liner pair angles is equal to 180^o.

$\angle AOB + \angle AOD = 180^o$

$118^o + \angle AOD = 180^o$

$\angle AOD = 180^o - 118^o$

$\angle AOD = 62^o$

Now consider the ΔAOD,

Let us assume the $\angle ADO = \angle DAO = x$

$\angle AOD + \angle ADO + \angle DAO = 180^o$

$62^o + x + x = 180^o$

$62^o + 2x = 180^o$

By transposing we get,

$2x = 180^o – 62$

$2x = 118^o$

$x = \frac{118°}{2}$

$x = 59^o$

Therefore, $\angle ADO = 59^o$

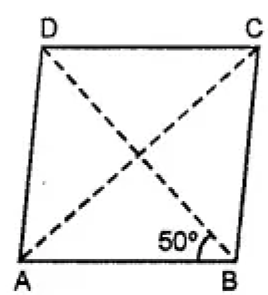
(iii) $\angle OCB = \angle OAD = 59^o$ … [because alternate angles are equal]

9. In the given figure, ABCD is a rhombus and ∠ABD = 50^o. Find:

(i) ∠CAB

(ii) ∠BCD

(iii) ∠ADC

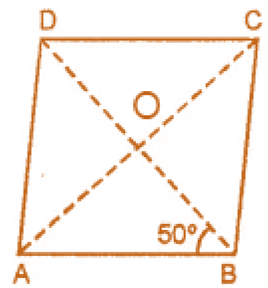


Solution:-

From the figure it is given that,

ABCD is a rhombus

$\angle ABD = 50^o$



(i) Consider the ΔAOB,

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle OAB + \angle BOA + \angle ABO = 180^o$

$\angle OAB + 90^o + 50^o = 180^o$

By transposing we get,

$\angle OAB + 140^o = 180^o$

$\angle OAB = 180^o - 140^o$

$\angle OAB = 40^o$

Therefore, $\angle CAB = 40^o$

(ii) $\angle BCD = 2\ \angle ACD$

$= 2 \times 40^o$

$= 80^o$

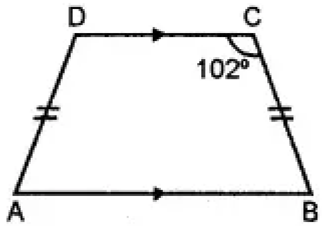
(iii) Then, $\angle ADC = 2\ \angle BDC$

$\angle ABD = \angle BDC$ because alternate angles are equal

$= 2 \times 50^o$

$= 100^o$

10. In the given isosceles trapezium ABCD, ∠C = 102°. Find all the remaining angles of the trapezium.



Solution:-

From the figure, it is given that,

Isosceles trapezium ABCD,

$\angle C = 102^\circ$

$AB \parallel CD$

We know that sum of adjacent angles is equal to 180°.

So, $\angle B + \angle C = 180^\circ$

$\angle B + 102^\circ = 180^\circ$

$\angle B = 180^\circ - 102^\circ$

$\angle B = 78^\circ$

Then, $AD = BC$

So, $\angle A = \angle B$

$78^\circ = 78^\circ$

Sum of all interior angles of trapezium is equal to 360°.

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$78^\circ + 78^\circ + 102^\circ + \angle D = 360^\circ$

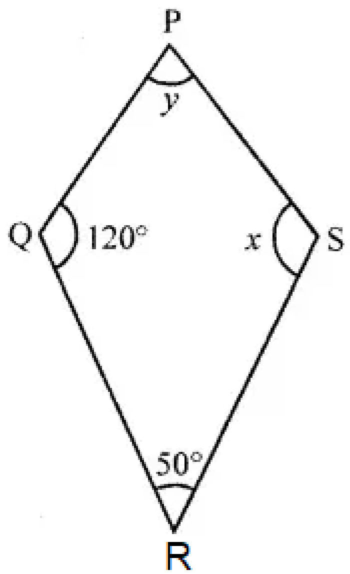
$258 + \angle D = 360^\circ$

By transposing we get,

$\angle D = 360^o - 258^o$

$\angle D = 102^o$

11. In the given figure, PQRS is a kite. Find the values of x and y.



Solution:-

From the figure it is given that,

PQRS is a kite.

$\angle Q = 120^o$

$\angle R = 50^o$

Then, $\angle Q = \angle S$

So, $x = 120^o$

We know that sum of all angles of Rhombus is equal to 360^o.

$\angle P + \angle Q + \angle R + \angle S = 360^o$

$y + 120^o + 50^o + 120^o = 360^o$

$y + 290^o = 360^o$

By transposing we get,

$y = 360^o - 290^o$

$y = 70^o$

Therefore, the value of $x = 120^o$ and $y = 70^o$

Mental Maths

Question 1: Fill in the blanks:
(i) The curves which have different beginning and end points are called …………
(ii) A curve which does not cross itself at any point is called a …………
(iii) A simple closed curve made up entirely of line segments is called a …………
(iv) A polygon in which each interior angle is less than 180° is called a …………
(v) 10 sided polygon is called …..
(vi) A polygon which has all its sides of equal length and all its angles of equal measure is called a …………
(vii) Sum of all exterior angles of a polygon is …………
(vi) A polygon which has all its sides of equal length and all its angles of equal measure is called a …………
(vii) Sum of all exterior angles of a polygon is …………
(viii) Sum of all interior angles of a n-sided polygon is …………
(ix) The adjacent angles of a parallelogram are …………
(x) If an angle of a parallelogram is a right angle, then it is called a …………
(xi) If two adjacent sides of a parallelogram are equal, then it is called a …………
(xii) It two adjacent sides of a rectangle are equal, then it is called a …………
(xiii) The diagonal of a rhombus bisect each other at …………
(xiv) A quadrilateral in which one pair of opposite sides is parallel is called a …………
(xv) A quadrilateral in which two pairs of adjacent sides are equal is called a …………

(xvi) If two non-parallel sides of a trapezium are equal then it is called …………

Solution:

(i) The curves which have different beginning
and end points are called open curves.

(ii) A curve which does not cross itself
at any point is called a simple curve.

(iii) A simple closed curve made up entirely
of line segments is called a polygon.

(iv) A polygon in which each interior angle
is less than 180° is called a convex polygon.

(v) 10 sided polygon is called decagon.

(vi) A polygon which has all its sides of equal length
and all its angles of equal measure is called a regular polygon.

(vii) Sum of all exterior angles of a polygon is 360°.

(viii) Sum of all interior angles of a n-sided polygon is
$(n - 2) \times 180°$ or $(2n - 4) \times 90°$.

(ix) The adjacent angles of a parallelogram are supplementary.

(x) If an angle of a parallelogram is a right angle,
then it is called a rectangle.

(xi) If two adjacent sides of a parallelogram are equal,
then it is called a rhombus.

(xii) It two adjacent sides of a rectangle are equal,
then it is called a square.

(xiii) The diagonal of a rhombus bisect each other at right angles.

(xiv) A quadrilateral in which one pair of
opposite sides is parallel is called a trapezium.

(xv) A quadrilateral in which two pairs of
adjacent sides are equal is called a kite.

(xvi) If two non-parallel sides of a trapezium
are equal then it is called an isosceles trapezium.

Question 2: State whether the following statements are true (T) or false (F):
(i) The curves which have same beginning and end points are called open curves.
(ii) The region of the plane that lies inside the curve is called interior of curve.
(iii) A polygon in which atleast one interior angle is greater than 180° is called convex polygon.
(iv) 6 sided polygon is called hexagon.
(v) Sum of all interior angles of a quadrilateral is 180°.
(vi) Each interior angle of a n-sided regular polygon is $\frac{(2n-4)\times 90°}{n}$
(vii) The diagonals of a parallelogram bisect each other at right angles.
(viii) The opposite angles of a parallelogram are of equal measure.
(ix) The diagonals of a rhombus bisect the angles of rhombus.
(x) The diagonals of a square are not equal.
(xi) Co-interior angles of a parallelogram are supplementary.
(xii) The diagonals of a kite bisect at right angles.
(xiii) All rectangles are squares.
(xiv) All rhombuses are parallelograms.
(xv) All squares are rhombuses and also rectangles.
(xvi) All squares are not parallelograms.
(xvii) All kites are rhombuses.
(xviii) All rhombuses are kites.
(xix) All parallelograms are trapeziums.
(xx) All squares are trapeziums.
Solution:
(i) The curves which have same beginning
and end points are called open curves. False
Correct:
It is called a closed curve.
(ii) The region of the plane that lies inside

the curve is called interior of curve. True
(iii) A polygon in which at least one interior angle is
greater than 180° is called convex polygon. False
Correct:
It is called a concave polygon.
(iv) 6 sided polygon is called hexagon. True
(v) Sum of all interior angles of a quadrilateral is 180°. False
Correct:
The sum is 360°.
(vi) Each interior angle of a n-sided regular polygon is $\frac{(2n-4)\times 90^\circ}{n}$. True
(vii) The diagonals of a parallelogram bisect each other at right angles.
False
(viii)The opposite angles of a parallelogram are of equal measure. True
(ix) The diagonals of a rhombus bisect the angles of rhombus. True
(x) The diagonals of a square are not equal.
False
Correct:
Diagonals are equal.
(xi) Co-interior angles of a parallelogram are supplementary. True
(xii) The diagonals of a kite bisect at right angles. False
(xiii) All rectangles are squares. False
Correct:
Some rectangle whose sides are equal are squares.
(xiv) All rhombuses are parallelograms. True
(xv) All squares are rhombuses and also rectangles. True
(xvi) All squares are not parallelograms. False
Correct:
All squares are parallelograms.
(xvii) All kites are rhombuses. False
Correct: A kite with all sides equal is a rhombus.
(xviii) All rhombuses are kites. True

(xix) All parallelograms are trapeziums. True
(xx) All squares are trapeziums. True

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 19):
Question 3: Sum of all interior angles of a 11-sided polygon is
(a) 1620°
(b) 1440°
(c) 1260v
(d) none of these
Solution:
Sum of all interior angles of an 11-sided polygon is
= (2 × n – 4) × 90°
= (2 × 11 – 4) × 90°
= 18 × 90 = 1620 (a)

Question 4: If each interior angle of a regular polygon is 144°, then number of sides of polygon is
(a) 8
(b) 9
(c) 10
(d) 11
Solution:
Each interior angle of a regular polygon is 144°
Then $\frac{2n-4}{n}$ × 90° = 144°
⇒ $\frac{2n-4}{n} = \frac{144°}{90°}$
⇒ 10n – 20° = 8n
⇒ 10n – 8n = 20°
⇒ 2n = 20°

$\Rightarrow$ n = 10
$\therefore$ It is 10-sided polygon. (c)

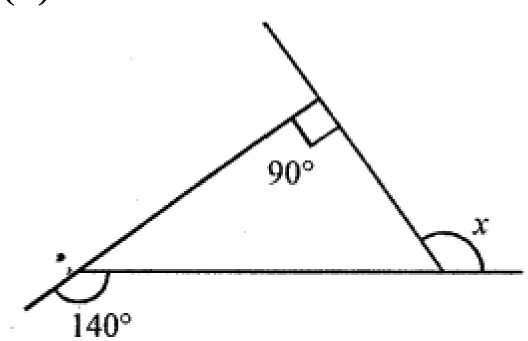
Question 5: If the sum of all interior angles of a polygon is 1260°, then number of sides of polygon is
(a) 6
(b) 7
(c) 8
(d) 9
Solution:
Sum of all interior angles of a polygon = 1260°
$\therefore$ $(2n - 4) \times 90° = 1260°$
$\Rightarrow 2n - 4 = \frac{1260°}{90°}$
$\Rightarrow 2n = 14 + 4 = 18$
$\Rightarrow n = \frac{18}{2} = 9$
$\therefore$ Polygon is 9-sided. (d)

Question 6: The sum of all exterior angles of a pentagon is
(a) 590°
(b) 360°
(c) 180°
(d) none of these
Solution:
Sum of exterior angles of a pentagon = 360° (b)

Question 7: If the ratio between an exterior and interior angle of a regular polygon is 1: 5, then the number of sides of the polygon is

(a) 11
(b) 12
(c) 13
(d) 14
Solution:
Ratio between exterior angle and interior angle
of a regular polygon = 1 : 5
But sum of angles = 180°
∴ Exterior angle = $\frac{180°}{1+5} \times 1$
$= \frac{180°}{6} = 30°$
∴ Number of sides = $\frac{360°}{30}$ =12 (b)
(Sum of exterior angles = 360°)

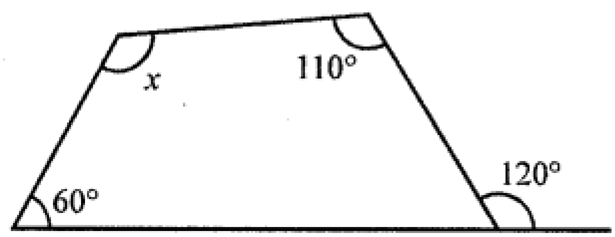
Question 8: In the given figure, the value of x is
(a) 140°
(b) 50°
(c) 130°
(d) 40°



Solution:
In the given figure,
Sum of exterior angles of a triangle = 360°
∴ 140° + x + 90° = 360°

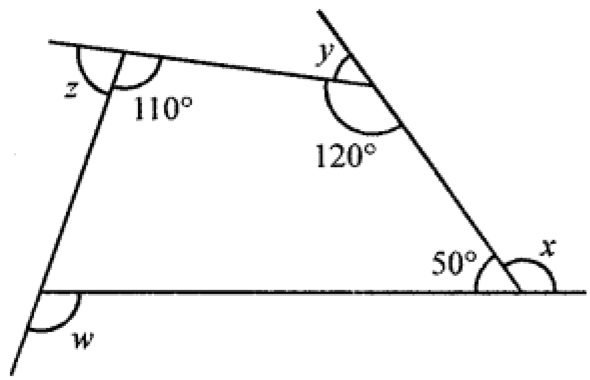
$\Rightarrow x + 230° = 360°$
$\therefore x = 360° – 230° = 130°$ (c)

Question 9: In the given figure, the value of x is
(a) 120°
(b) 130°
(c) 140°
(d) 150°



Solution:
In the given figure,
Sum of angles of a quadrilateral = 360°
$\therefore 60° + (180° – 120°) + 110° + x = 360°$
$\Rightarrow 60° + 60° + 110° + x = 360°$
$230° + x = 360°$
$\therefore x = 360° - 230° = 130°$ (b)

Question 10: In the given figure, the value of x + y + z + w is
(a) 180°
(b) 270°
(c) 300°
(d) 360°



Solution:

In the given figure,

Sum of exterior angles of a quadrilateral = 360°

∴ x + y + z + w = 360° (d)

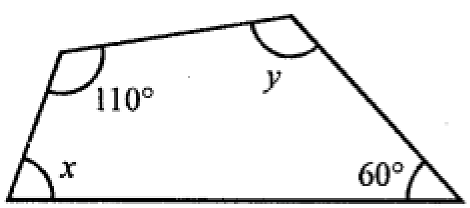
Question 11: In the given figure, the value of x + y is

(a) 180°

(b) 190°

(c) 170°

(d) 160°



Solution:

In the given figure,

Sum of interior angles of a quadrilateral = 360°

60° + y + 110° + x = 360°

⇒ x + y + 170° = 360°

⇒ x + y = 360° – 170°

x + y = 190° (b)

Question 12: The lengths of two adjacent sides of a parallelogram are in the ratio 1: 2. If the perimeter of parallelogram is 60 cm, then length of its sides are

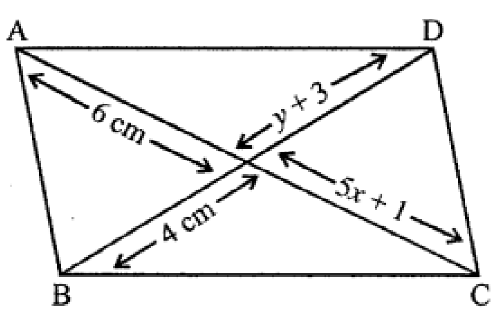
(a) 6 cm, 12 cm
(b) 8 cm, 16 cm
(c) 9 cm, 18 cm
(d) 10 cm, 20 cm

Solution:

Ratio in the length of two adjacent sides of a parallelogram = 1 : 2
Perimeter = 60 cm
∴ Sum of two adjacent sides = $\frac{60}{2}$ = 30 cm
Let first side = x, then second side = 2x
∴ x + 2x = 30 ⇒ 3x = 30
x = $\frac{30}{2}$ = 10 cm
First side = 10 cm
and second side = 10 × 2 = 20 cm (d)

Question 13: In the given figure, ABCD is a parallelogram, the values of x and y respectively are

(a) 1 cm, 1 cm
(b) 2 cm, 1 cm
(c) 1 cm, 2 cm
(d) 2 cm, 2 cm



Solution:

In the given figure, ABCD is a parallelogram

∵ Diagonals of a parallelogram bisect each other

∴ AO = OC and BO = OD

∴ 6 = 5x + 1

⇒ 5x = 6 – 1 = 5

⇒ $x = \frac{5}{5}$

and y + 3 = 4

⇒ y = 4 – 3 = 1

∴ x = 1, y = 4 (a)

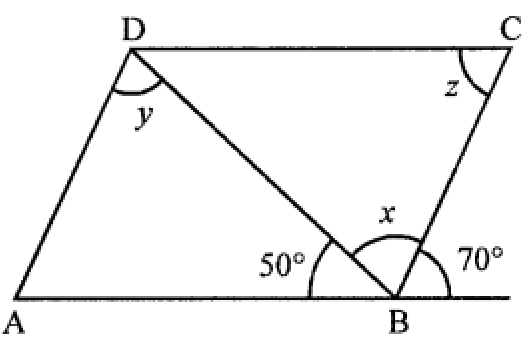
Question 14: In the given figure, ABCD is a parallelogram, the values of x, y and z respectively are

(a) 60°, 60°, 70°

(b) 60°, 70°, 60°

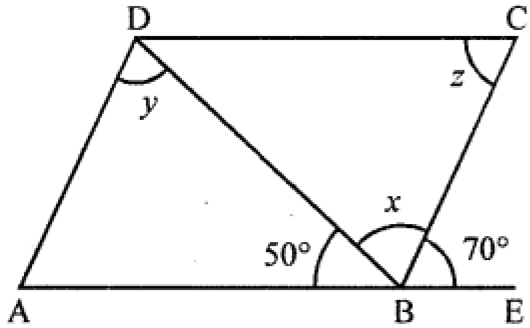
(c) 70°, 60°, 60°

(d) none of these



Solution:

In the given figure,



ABCD is a parallelogram, BD is its one diagonal

$\angle ABD + \angle DBC + \angle CBE = 180°$

(Angles on one side of a line)

$\Rightarrow 50° + x + 70° = 180°$

$x + 120° = 180°$

$\therefore x = 180° - 120° = 60°$

But $y = x$ (Alternate angles)

$\therefore y = 60°$

$z = 70°$ (Alternate angles)

$\therefore x = 60°, y = 60°, z = 70°$ (a)

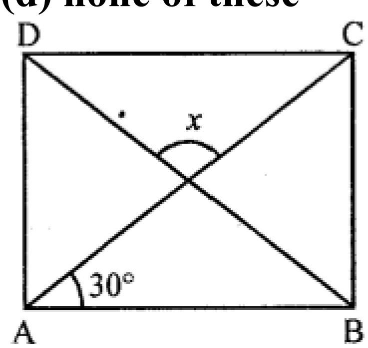
Question 15: In the given figure, ABCD is a rectangle, the value of angle x is

(a) 60°

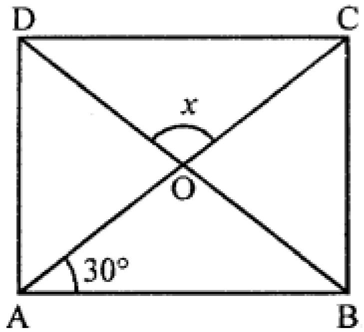
(b) 90°

(c) 120°

(d) none of these



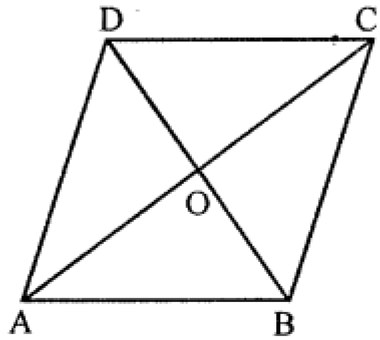
Solution:
In the given figure, ABCD is a rectangle



∴ ∠OBA = ∠OAB = 30°
In ΔAOB,
∠AOB = ∠COD (Vertically opposite angles)
∠AOB = x
∠AOB + ∠OBA + ∠OAB = 180° (Angles of a triangle)
⇒ x + 30° + 30° = 180°
⇒ x = 180°- 30°- 30° = 120°
∴ x = 120° (c)

Question 16: In a rhombus ABCD, the diagonals AC and BD are respectively 8 cm and 6 cm. The length of each side of the rhombus is

(a) 7 cm
(b) 5 cm
(c) 6 cm
(d) 8 cm
Solution:
In rhombus ABCD
Diagonals AC and BD are 8 cm and 6 cm



∴ AC = 8 cm and BD = 6 cm

∵ Diagonals of a rhombus bisect each other at right angles

AO = OC = $\frac{8}{2}$ = 4 cm,

BO = OD = $\frac{6}{2}$ = 3 cm

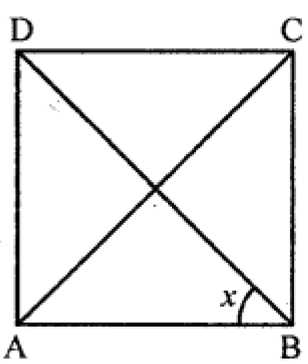
∴ In right ΔAOB

AB = $\sqrt{AO^2 + BO^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25}$ = 5 cm

Each side of rhombus = 5 cm (b)

Question 17: In the given figure, ABCD is a square, the value of angle x is

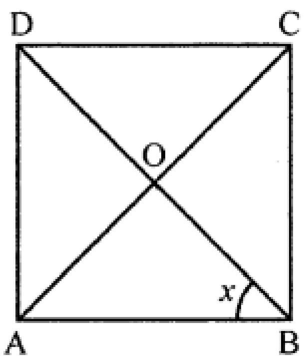
(a) 30°
(b) 45°
(c) 60°
(d) not possible to find



Solution:

In the given figure,

ABCD is a square whose diagonals AC and BD
bisect each other at O.



∵ Diagonals of a square bisect the opposite angles.

∴ $x = \frac{1}{2} \times \angle B = \frac{1}{2} \times 90° = 45°$ (b)

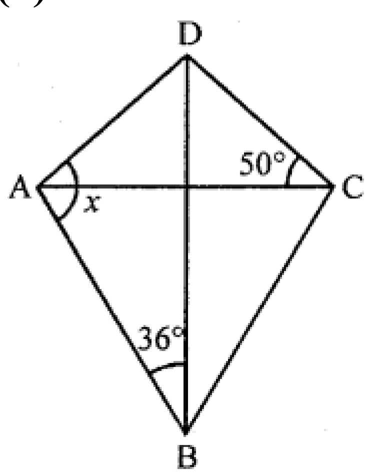
Question 18: In the given figure, ABCD is a kite, the value of angle x is

(a) 86°

(b) 100°

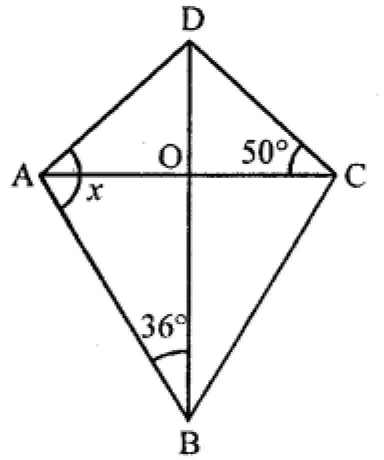
(c) 104°

(d) none of these



Solution:

In the given figure, ABCD is a kite whose
diagonals AC and BD intersect at O at right angles.



In ΔOAB, $\angle O = 90°$

$\therefore \angle OAB + \angle ABO = 90°$

$\Rightarrow \angle OAB + 36° = 90°$

$\Rightarrow \angle OAB = 90° - 36° = 54°$

But $\angle OAD = \angle OCD = 50°$

$x = \angle DAO + \angle AOB$

$\Rightarrow x = 50° + 54° = 104°$ (c)

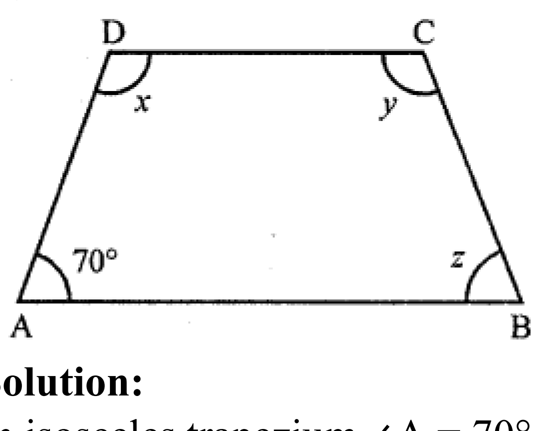
Question 19: In the given figure, ABCD is an isosceles trapezium. The values of x, y and z respectively are

(a) 110°, 110°, 70°

(b) 110°, 70°, 110°

(c) 70°, 110°, 110°

(d) none of these



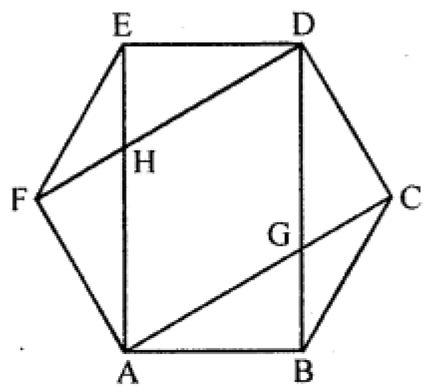
Solution:

In isosceles trapezium $\angle A = 70°$

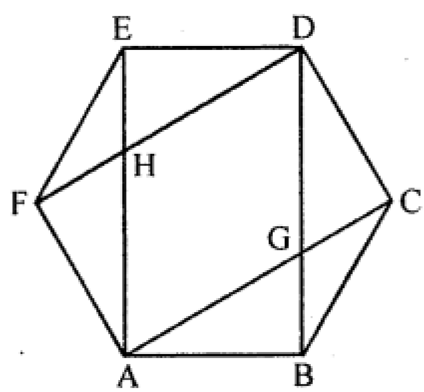
But $\angle B = \angle A = 70° \Rightarrow z = 70°$
But $x + 70° = 180°$
$\Rightarrow x = 180° - 70° = 110°$
But $y = x = 110°$
$\therefore$ 110°, 110°, 70° (a)

Higher Order Thinking Skills (Hots)

Question 1: In the given figure, ABCDEF is a regular hexagon. Prove that quadrialterals ABDE and ACDF are parallelograms. Also prove that quadrilateral AGDH is a parallelogram.



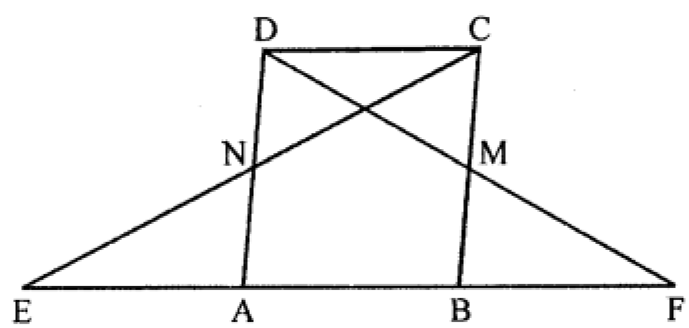
Solution:
In the given figure, ABCDEF is a regular hexagon.
AC, AE, DF, DB are joined.



To prove: ABDE, ACDF and AGDH are ||gm

Proof: In ΔBCD,
$BC = CD$
$\therefore \angle CBD = \angle CDB = 30°$ ($\because \angle C = 120°$ angle of hexagon)
But $\angle B = \angle D$ (Angles of a regular hexagon)
$\therefore \angle B - \angle CBD = \angle D - \angle CDB$
$\Rightarrow \angle ABD = \angle BDE = 90°$
Similarly, $\angle FAB = \angle AED = 90°$
$\therefore \angle ABD + \angle BDE = 90° + 90° = 180°$
But they are cointerior angle.
$\therefore AB \parallel DE$
$\therefore ABDE$ is a ||gm
similarly we can prove that ACDF is a parallelogram.
$\because AC \parallel DF$ and $BD \parallel AC$
$\therefore AGDH$ is a parallelogram.

Question 2: In the given figure, ABCD is a parallelogram and M, N are the mid-points of sides BC, AD respectively. Prove that EA = AB = BF.

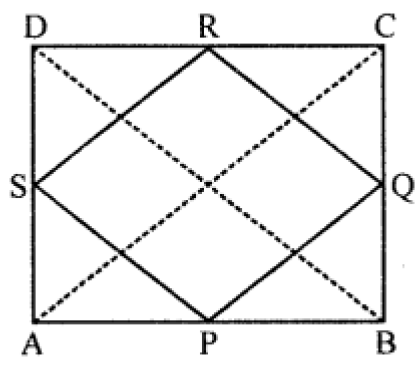


Solution:
In the given figure,
ABCD is a parallelogram M and N are
midpoints of the sides BC and AD respectively.
To prove: EA = AB = BF.
Proof: In ΔAFD,

M is the midpoint of BC and BC || AD
∴ B is mid-point of AF
∴ AB = BF …(i)
Similarly in ΔEBC,
N is the midpoint of AD and AD || BC
∴ A is the midpoint of EB
∴ EA = AB …(ii)
From (i) and (ii),
EA = AB = BF

Question 3: Prove that the quadrilateral formed by joining the mid-points of the adjacent sides of a rectangle is a parallelogram.

Solution:
Given : ABCD is a rectangle.
P, Q, R and S are the mid points of the sides
AB, BC, CD and DA respectively.
PQ, QR, RS and SP are joined.
To prove: PQRS is a parallelogram.
Construction: Join AC and BD.



Proof: In ΔABC,
P and Q are mid points of AB and BC respectively
∴ PQ || AC and PQ = $\frac{1}{2}$AC …(i)
Similarly in ΔADC

SR mid points of AD and CD respectively

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC …(ii)

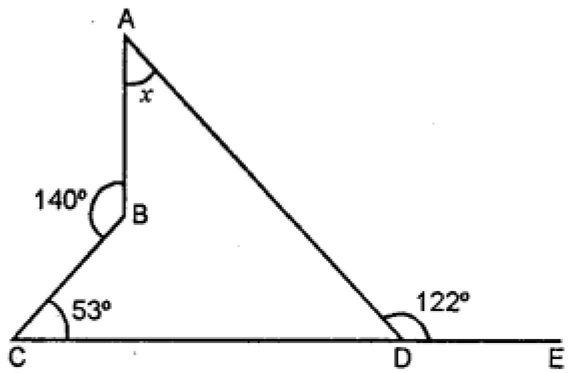
From (i) and (ii),

SR || PQ and SR = PQ

$\therefore$ PQRS is a parallelogram.

Check Your Progress

Question 1: From the given diagram, find the value of x.



Solution:

Reflex angle B = 360 – 140 = 220°

Also, ∠ADC = 180 – 122 = 58° (Linear pair)

Now, ABCD is a quadrilateral

∴ ∠A + ∠B + ∠C + ∠D = 360°

⇒ x + 220 + 53 + 58 = 360

or x = 360 – 331 =29°

Question 2: If two angles of a quadrilateral are 77° and 51°, and out of the remaining two angles, one angle is 10° smaller than the other, find these angles.

Solution:

Two angles of a quadrilateral are 77° and 51°

Sum of angles of a quadrilateral = 360°

∴ Sum of other two angles = 360° – (77° + 51°)

= 360° – 134° = 226°

Let one angle among there two angles = x

Then other angle = x – 10°

∴ x + x – 10 = 226°

⇒ 2x = 226° + 10° = 236°

$\Rightarrow x = \frac{236°}{2} = 118°$

$\therefore$ One angle = 118°

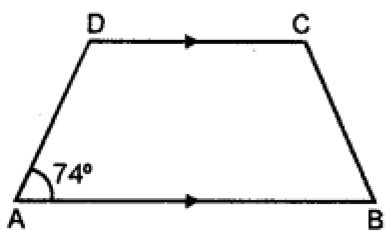
and other angle =118 – 10 = 108°

Question 3: In the given figure, AB || DC, ∠A = 74° and ∠B: ∠C = 4: 5. Find

(i) ∠D

(ii) ∠B

(iii) ∠C



Solution:

∠A = 74° ∠B = 4x, ∠C = 5x.

As AB || CD

$\therefore$ 4x + 5x= 180°, 9x= 180°

$\Rightarrow$ x = 20°

$\therefore$ ∠B = 80° and ∠C = 100°

ABCD is a quadrilateral

$\therefore$ ∠A + ∠B + ∠C + ∠D = 360°

$\Rightarrow$ ∠D = 360° – (74° + 80° + 100°)

= 360°- 254 = 106°

Therefore, ∠D = 106° ∠B = 80°, ∠C = 100°

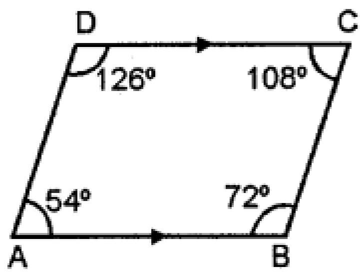
Question 4: In quadrilateral ABCD, ∠A: ∠B: ∠C: ∠D = 3: 4: 6: 7. Find all the angles of the quadrilateral. Hence, prove that AB and DC are parallel. Is BC also parallel to AD?

Solution:

Let, four angles of quadrilateral be 3x, 4x, 6x, 7x.

$\therefore 3x + 4x + 6x + 7x = 360$

$20x = 360 \Rightarrow x = 18$



$\therefore \angle A = 3x = 3 \times 18 = 54°$

$\angle B = 4x = 4 \times 18 = 72°$

$\angle C = 6x = 6 \times 18 = 108°$

$\angle D = 7x = 7 \times 18 = 126°$

As, $\angle A + \angle D = 54 + 126 = 180°$

$\therefore AB \parallel CD$

(If two angles on the same side of transversal are supplementary, then lines are parallel)

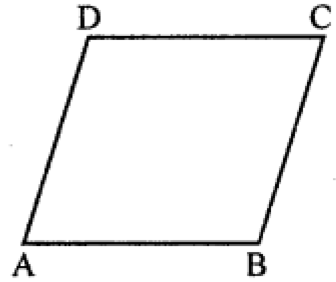
But $\angle A + \angle B$ $(54 + 72) \neq 180$

$\therefore$ BC is not parallel to AD.

Question 5: One angle of a parallelogram is two-third of the other. Find the angles of the parallelogram.

Solution:

In a parallelogram, one angle is $\frac{2}{3}$ of the other.



Let one angle = x

Then second = $\frac{2}{3}$x

But x + $\frac{2}{3}$x = 180°

⇒ $\frac{5}{3}$ = 180°

⇒ x = $\frac{180° \times 3}{5}$ = 108°

∴ ∠A = x = 108°

∠B = $\frac{2}{3}$x = 108° × $\frac{2}{3}$ =72°

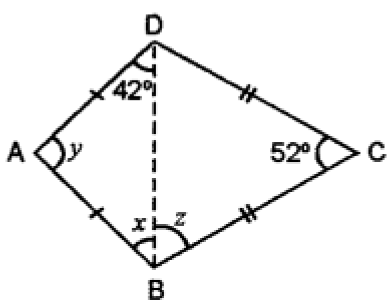
But ∠C = ∠A and ∠D = ∠B

(Opposite angles of a parallelogram)

∴∠C = ∠A = 108° and ∠D = ∠B = 72°

Hence, ∠A = 108°, ∠B = 72°, ∠C = 108°, ∠D = 72°

Question 6: In the given figure, ABCD is a kite. If ∠BCD = 52° and ∠ADB = 42°, find the values of x, y, and z.



Solution:

Join BD.

In Δ ABD,

AB = AD (Given)

∴ ∠ABD = ∠ADB (angles opposite to equal sides are equal)

⇒ x = 42°

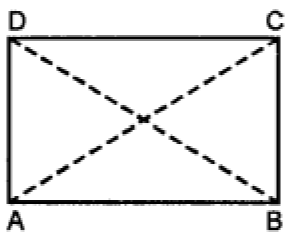
In Δ BCD,

BC = CD (given)

∴ ∠BDC = ∠DBC = z

$\therefore z + z + 52 = 180° \Rightarrow 2z = 128$
$\Rightarrow z = 64°$
ABCD is quadrilateral
$\therefore \angle A + \angle B + \angle C + \angle D = 360°$
$\Rightarrow y + (x + z) + 52 + (42 + z) = 360°$
$\Rightarrow y + 106 + 52 + 106 = 360$
$\Rightarrow y = 360 – 264 = 96°$

Question 7: In the given figure, ABCD is a rectangle. Prove that AC = BD.

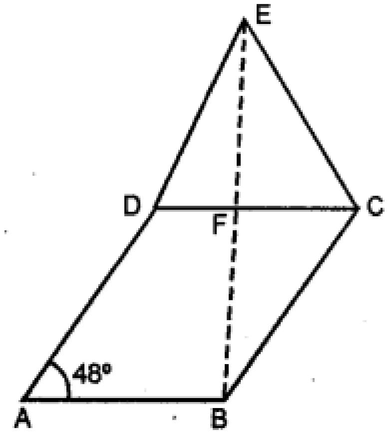


Solution:
In Δ ABC and ΔABD

BC = AD	(opposite sides of rectangle)
$\angle B = \angle A$	(each 90°)
AB = AB	(common)
$\therefore$ ΔABC = ΔABD	(S.A.S.)
$\therefore$ AC ≅ BD	(c.p.c.t.)

Question 8: In the given figure, ABCD is a rhombus and EDC is an equilateral triangle. If ∠DAB = 48°, find

(i) ∠BEC
(ii) ∠DEB
(iii) ∠BFC



Solution:
ABCD is a rhombus
∴ AB = BC = CD = DA
Also, EDC is an equilateral Δ
∴ ED = DC = CE …(ii)
From (i) and (ii)
We get, BC = CE.
In Δ BCE, ∠BCE = 60 + 48 = 108
Also, BC = EC
∴ ∠BEC = ∠EBC = x
⇒ x + x + ∠BCE = 180°
⇒ 2x = 180 – 108 = 72°
⇒ x = 36°
∴ ∠BEC = x = 36°.

(ii) ∠DEB = ∠DEC – ∠BEC
= 60° – 36° = 24°

(iii) In Δ DEF,
∠D = 60°, ∠DEF = 24°, ∠DFE = y.
60 + 24 + 7 = 180°
y = 180 – 84 = 96°
∠BFC = ∠DFE = 96° (Vertically opposite angles)

Question 9: Find the number of sides of a regular polygon if each of its interior angle is 168°.

Solution:

Each interior angle of a regular polygon = 168°

Let number of sides = n, then

$\frac{2n-4}{n} \times 90° = 168°$

$\frac{2n-4}{n} = \frac{168°}{90°} = \frac{28}{15}$

∴ 30n – 60 = 28n

⇒ 30n – 28n = 60

⇒ 2n = 60

⇒ n = $\frac{60}{2}$ = 30

∴ Number of sides of the polygon = 30

Question 10: If the sum of interior angles of polygon is 3780° find the number of sides.

Solution:

Sum of interior angles of polygon = (2n – 4) × 90

∴ 3780 = (2n – 4) × 90 ⇒ (2n – 4) = $\frac{3780}{90}$

⇒ 2n – 4 = 42

⇒ 2n = 46

⇒ n = 23

Question 11: The angles of a hexagon are (2x + 5)°, (3x – 5)°, (x + 40)°, (2x + 20)°, (2x + 25)° and (2x + 35)°. Find the value of x.

Solution:

Number of sides in hexagon = 6.

Sum of interior angles = (2 × 6 – 4) × 90 = 720°

∴ (2x + 5) + (3x – 5) + (x + 40) + (2x + 20)

$+ (2x + 25) + (2x + 35) = 720$

$\Rightarrow 12x + 120 = 720$

$\Rightarrow 12x = 720 - 120$

$\Rightarrow 12x = 600$

$\Rightarrow x = 50.$

Question 12: Two angles of a polygon are right angles and every other angle is 120°. Find the number of sides of the polygon.

Solution:

Let the number of sides = n

$\therefore 2 \times 90 + (n - 2) \times 120 = (2n - 4) \times 90$

$\Rightarrow 180 + 120n - 240 = 180n - 360$

$\Rightarrow 120n - 60 = 180n - 360$

$\Rightarrow 60n = 300$

$\Rightarrow n = 5$

Question 13: The sum of interior angles of a regular polygon is twice the sum of its exterior angles. Find the number of sides of the polygon.

Solution:

Sum of interior angles = $(2n - 4) \times 90$

Sum of exterior angles = 360

According to the condition,

$(2n - 4) \times 90 = 2 \times 360$

$\Rightarrow 2n - 4 = 8$

$\therefore n = 6$

Question 14: An exterior angle of a regular polygon is one- fourth of its interior angle. Find the number of sides in the polygon.

Solution:

Let measure of interior angle = $x°$

Then exterior angle = $\frac{1}{4}x°$

$\therefore x + \frac{1}{4}x = 180° \Rightarrow \frac{5}{4}x = 180$

$\Rightarrow x = 180 \times \frac{4}{5} \Rightarrow x = 144°$

Therefore, each interior angle is 144°

$144 = \frac{(2n-4)}{4} \times 90$

$\Rightarrow 144n = 180n - 360$

$\Rightarrow 180n - 144n = 360$

$\Rightarrow 36n = 360$

$\Rightarrow n = 10$