Statistics

NOTES

In this chapter we will learn about statistics and probability.

Statistics

Statistics is the branch of Mathematics which deals with data collected for specific purpose.

Central Tendencies of Data

The central tendency gives us an idea that represents the entire data. There are three types of central tendencies which are:

- Mean
- Median
- Mode

Mean

It is also known as arithmetic mean of the given observations and is equal to ratio of sum of all the observations and total number of observations, i.e.,

$$Mean = \frac{Sum\ of\ all\ the\ observations}{Total\ number\ of\ observations}$$

If $x_1, x_2, ---x_n$ are n observations then its mean is

$$A.M. = \bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum xi}{n}$$

Arithmetic Mean for Frequency Distribution

Let $f_1, f_2, ... f_n$ be corresponding frequencies of $x_1, x_2, x_3, ... x_n$ then

$$A.M. = \frac{x_1 f_1 + x_2 f_2 + ... + x_n f_n}{f_1 + f_2 + ...f_n} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$$

Arithmetic Mean for Grouped Data

For a classified data, we take the class marks $x_1, x_2, ..., x_n$ of the classes as variabies, then arithmetic by

(i) Direct method is $A.M. = \frac{\sum_{i=1}^{n} xf}{\sum_{i=1}^{n} f}$

(ii) Deviation method is $A.M. = A_1 + \left(\frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \right) \times h$

Where A_1 = assumed mean, $d_i = x_i$ deviation $= x_i - A_1$

f_i = frequency, h = width of interval

(iii) Step deviation method is $A.M. = A_1 + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

Where A_1 = assumed mean; u_i = step deviation $= \frac{x_i - A}{h}$ and h = width of interval

Median

The median is the middle most value of a distribution i.e. median of a distribution is the value of the variable which divides it into two equal parts.

For a distribution, when observation are arranged in either ascending or descending order.

- If number of observations (n) is odd, then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.
- If number of observations (n) is even, then is the value of arithmetic observation mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation i.e.

  $$Mean = \frac{\left(\frac{n}{2}\right)^{th} observation + \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2}.$$

Median of a Continuous Frequency Distribution

$$Median = l + \left[\frac{\frac{N}{2} - c}{f} \right] x\ h^{\text{f}}$$

Where, $N = \sum f_i$

l = lower limit of median class/

h = width of median class,

f = frequency of median class

c = cumulative frequency of the class preceding the median class

Mode

Mode is the value that occurs the most frequently in a data set or mode is a way of capturing important information about a random variable in a single quantity. The mode is generally different from the mean and median. The following formula is generally used for calculating mode of a continuous frequency distribution.

$$\text{Mode} = x_k + h\left[\frac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}}\right]$$

Where, $x_k =$ lower limit of the modal class interval.

$f_k =$ frequency of the modal class

$f_{k-1} =$ frequency of the class preceding the modal class

$f_{k+1} =$ frequency of the class succeeding the modal class

$h =$ width of the class interval

Probability

The word 'probability' in one of the most commonly used word in our day to day life. Like probably today it will rain, probably India will win the world cup etc. In Mathematics the concept of probability really originated in the beginning of eighteenth century in problems involving the game of chance.

Terms Related to Probability

Some of the terms to the concept of probability are as follows:

Event

The outcomes of a random experiment is called its elementary event. When a coin is tossed/ head and tail are the only possible outcomes that is why a head is an elementary event.

Compound Events

When two or more than two elementary events occur with a random experiment, it is said to be compound event. When we throw a die/ then getting an odd number is a compound event.

Equally Likely Events

A given number of events are said to be equally likely, if none of them is expected to occur in preference to the others.

Possible Outcomes

The total number of the events which are possible to occur is called possible outcomes. When two dice are thrown, the possible outcomes are: {(1, 1), (1, 2) (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Favourable Outcomes

The outcomes which satisfy the given condition of chance are called favourable outcomes.

When an unbiased die is thrown, the number obtained greater than 2 can be 3, 4, 5, 6 which are the favourable outcomes.

Probability of an Event

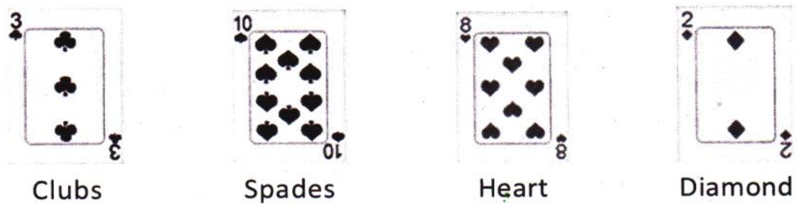
Let an event is denoted by E, the number of favourable events is denoted by n(e) and number of all possible outcomes denoted by n (S), then $P(E) = \frac{n(E)}{n(S)}$.

Drawing a Card

We know that there are 52 cards in a deck of playing cards.

It has four types, which are Spades, Clubs, Hearts and Diamonds, All are equally divided. It means there are 13 spades, 13 clubs, 13 hearts and 13 diamonds are in a deck of cards.

In 52 cards, there are two colours viz. red and black which are also equally divided. It means there are 26 red cards and 26 black cards. Hearts and diamonds are red cards and spades and clubs are black cards.



Number and Face Cards

The card on which numbers are written are called number cards.

Kings, queens and jacks are called face cards, therefore, total number of face cards are 12, out of which 6 are red face cards and 6 are black face cards.

- **Example:**

Find the mean of the following data:

Class-Interval	Frequency
0-10	7
10-20	8
20-30	12
30-40	13
40-50	10

(a) 25.60 (b) 24.80

(c) 23.30 (d) 27.20

(e) None of these

Ans. (d)

Solution: $A.M. = \frac{7\times5+8\times15+12\times25+13\times35+45\times10}{7+8+12+13+10}$

$$= \frac{35+120+300+455+450}{50} = \frac{1360}{50} = 27.20$$

- **Example:**

Find the median of daily wages from the following frequency distribution.

Daily Wages (In Rs.)	100-150	150-200	200-250	250-300	300-350
Frequency	6	3	5	20	10

(a) 250 (b) 260

(c) 270 (d) 280

(e) None of these

Ans. (c)

Explanation:

Class Interval	**Frequency (f_i)**	**Cf**
100-150	6	6
150-200	3	9
200-250	5	14
250-300	20	34
300-350	10	44
	$\sum f_i = 44 = N$	

$$Median = 250 + \left\{50x\left(\frac{22-14}{20}\right)\right\} = 270$$

- **Example:**

Find the mode for the following frequency distribution:

Class Interval	**Frequency**	**Class Interval**	**Frequency**
0-10	5	40-50	28
10-20	8	50-60	20
20-30	7	60-70	10
30-40	12	70-80	10

(a) 46.67 (b) 45.24

(c) 42.26 (d) 43.34

(e) None of these

Ans. (a)

Explanation: From table we find that 40-50 is modal class $x_k = 40,\ h = 10,\ f_k = 28,\ f_{k-1} = 12,\ f_{k+1} = 20$ then by using formula, $M = x_k + \left[h\ x \frac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}}\right] = 40 + \left[\frac{28-12}{2\ x\ 28\ -\ 12\ -20}\right] \times 10 = 46.67$

- **Example:**

In the single throw of a die, what will be the probability of getting a number which is less than 7?

Solution: Sample space = {1, 2, 3, 4, 5, 6}, here n(E) = 6 and n(S) = 6

$$\therefore \quad p(E) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1.$$ So. it is a sure event.

- **Example:**

A coin is tossed twice, the probability of getting at least a head is:

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(e) None of these

Ans. (b)

Explanation: Here, $p(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$

Since, S = (HT, TH, HH, TT) $\Rightarrow$ n(S) = 4

And E = (HT, TH, HH) $\Rightarrow$ n(E) =3