⇒ Signals and Systems:

**Analysis**

① Introduction

② L.T.I. systems

**Approximation**

① Fourier series

**Transformation**

① C.T.F.T

② L.T.

③ D.T.F.T.

④ Z.T.

⑤ D.F.T.

⇒ **Books:**

① Openhium & Nawab.

② Haykins & Vanveen - IES

③ S&S by Hsu & Ranjan (TMH).

* **Signal:**

⇒ Signal is an indication about which some amount of information is conveyed.

⇒ Randomnes is a information i.e. the things which we are dont know is signal.

⇒ The operations that are perform on the signals are:

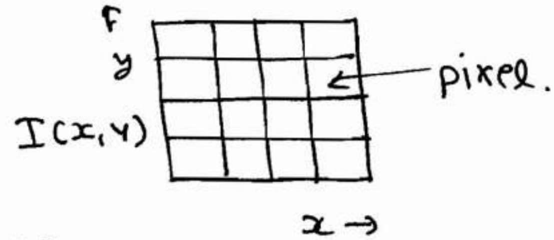
① Enhancing. ④ Filtering.

## ☆ Characteristics of a Signal:

⇒ ① More than one independent variable.

e.g. : ① Speech → 1D (time)

② Image → 2D




③ T.V. picture → 3D

$I(x, y, t)$.

⇒ ② Randomness:

⇒ More the Randomness more the information.

$$I = \log_2 \frac{1}{P_i} = -\log_2 P_i$$

$\therefore \; P_i = 1/8 \Rightarrow I = 3$ bits

more randm $\Rightarrow P_i = \frac{1}{32} \Rightarrow I = 5$ bits.

⇒ ③ Bandwidth:

⇒ In order to know the BW of the channel we should know the BW of signal that's why the BW is one of the chara. of signal.

HCf:

## ☆ Types of signal:

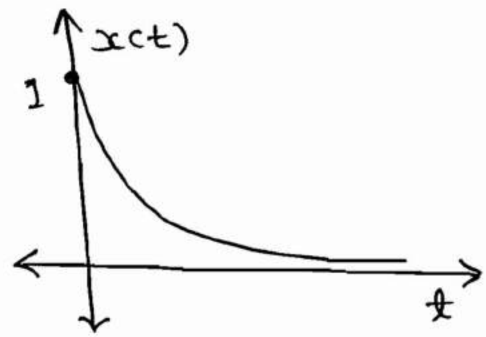
### ① Continuous time signal:

=> A signal which occurs for continuum value of time. is called continuous signal.

=> At any instant the amplitude of the signal is known.

=> Continuous both in time and amp. is Continuous signal.

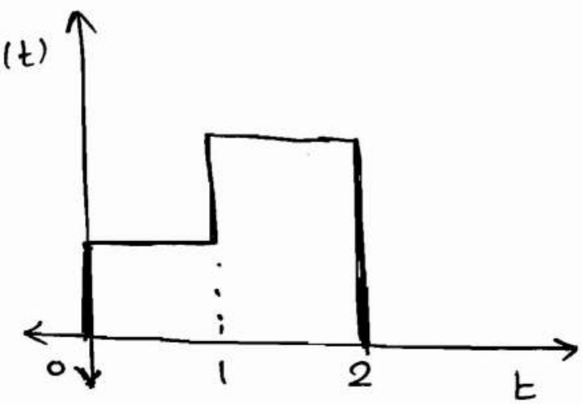
e.g. $x(t) = e^{-3t}.u(t)$




| Amp | Time | signal Type |
|---|---|---|
| C | C | conti. signal |
| C | D | Discrete signal |
| D | C | quantizer |
| D | D | Digital signal |

=> Whenever there is sudden changes in the ~~signal~~ level (or) amp is called Discontinuous signal.

e.g. y(t) signal.




at $t=0,1,2$ there are sudden change in amp. so, at $t=0,1,2,$ y(t) the amp. is not defined i.e. discontinuous

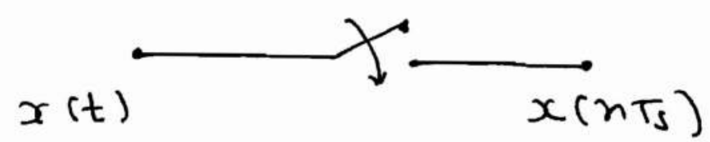
## (2) Discrete signal :-

⇒ A signal which is continuous in amplitude but discrete in time (integer values of time index). is called discrete signal.

⇒ BW will be less than continuous signal.

⇒ Used in concept of multiplexing which is very easy.

⇒ Sampling:

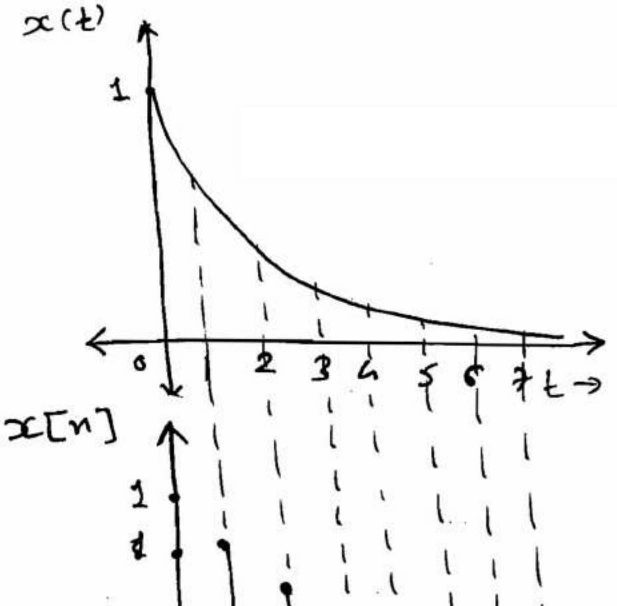



$t = nT_s \quad , \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$t = nT_s$

$T_s =$ Sampling time.

⇒ $x(t) = e^{-3t} . u(t)$.




$x[nT_s] = e^{-3nT_s} . u[nT_s]$

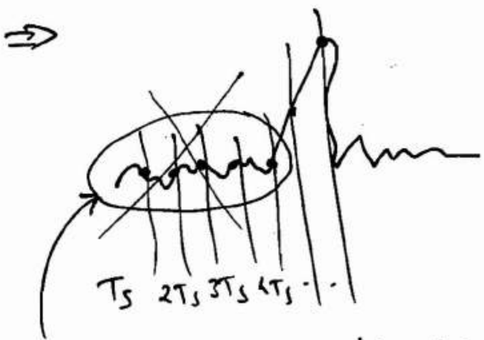
Let, $T_s = 1$ sec

$x[n] = e^{-3n} . u[n]$.

$x[nT_s] \rightarrow x[n]$

for simplicity

$x[nT_s - T_s] \rightarrow x[n-1]$

⇒ 

$x[nT_s] \rightarrow x[n]$ for simplicity.

So, we can't use $T_s = 1$ sec every time.

Almost amplitude are same for all the sample.

⇒ The purpose of Sampling is multiplexing.

⇒ Every discrete signal is not the Sampled version of the continuous signal Some of the discrete signals are Predefine.

$\boxed{\text{NOTE:}}$

⇒ Amp. conversion from C→D ⇒ Quantizer

⇒ Time Conversion from C→D ⇒ Sampling.

③ Digital Signal:

⇒ A signal which discrete in time and amplitude (quantized amp.) is called Digital signal.

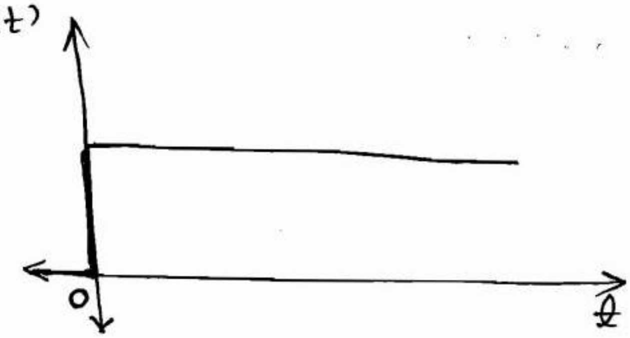
⇒ Quantizer is used to convert the continuous amp. signal into discrete amp. signal.

⇒ Sampler is used to convert continuous time into discrete time signal.

# * Some Standard Continuous signal :-

① **Unit Step Function:**

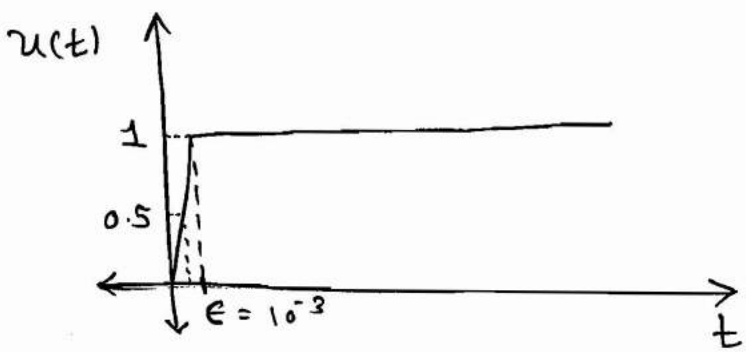
⇒ $u(t) = 1, \quad t > 0$
$\quad\;\; = 0, \quad t < 0$




⇒ At $t=0$ there is a discontinuity i.e. why are not defining $u(t)$ at $t=0$.

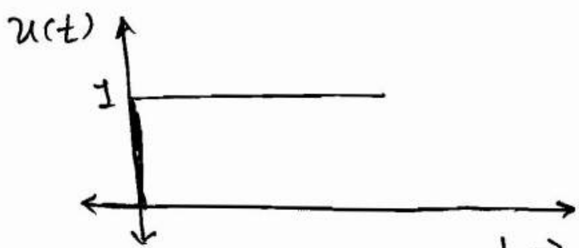
Transient & Bounded amp.

⇒ But practically $u(t)$ signal is as follow in Lab. :

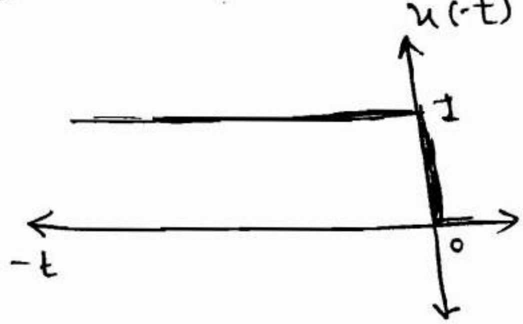



⇒ Mathematically $u(0) = 1/2$.

① $u(t)$




② $u(-t)$.




⇒ $u(t) + u(-t)$


$u(t) + u(-t) = 1$ at $t = 0$.

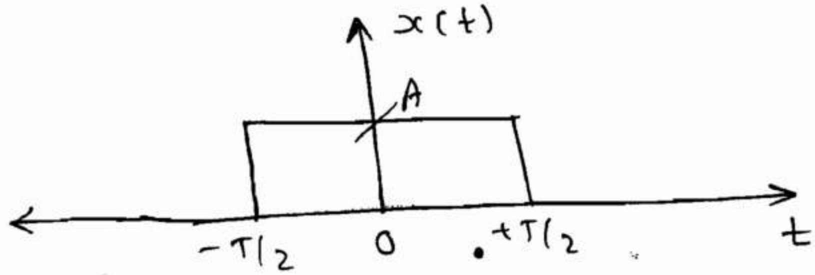
$\therefore u(0) + u(0) = 1$

② **Rectangular function :-**

$\Rightarrow$ $A\,rect\,(t/T)$ (or) $A\,\pi\,(t/T)$.

$\Rightarrow$ $A\,rect\,(t/T) = A \;;\; -\frac{T}{2} < t < T/2$

$= 0 \;;\;$ elsewhere

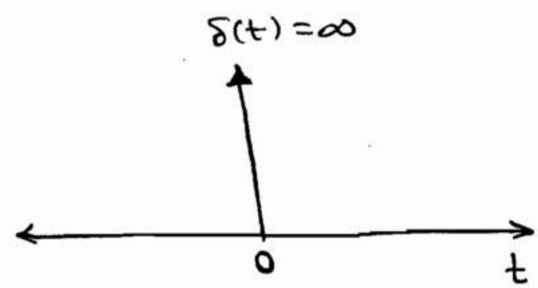



③ **Continuous impulse function (or) Dirac delta function:**

$\Rightarrow$ $\delta(t)$.

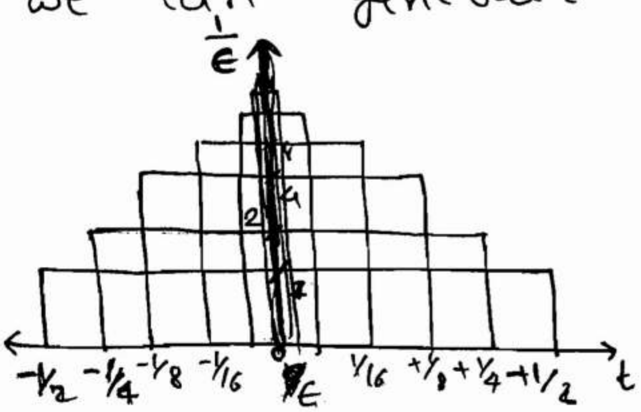
$\Rightarrow$ $\delta(t) = \infty \;;\; t=0 \;\; (t \to 0)$

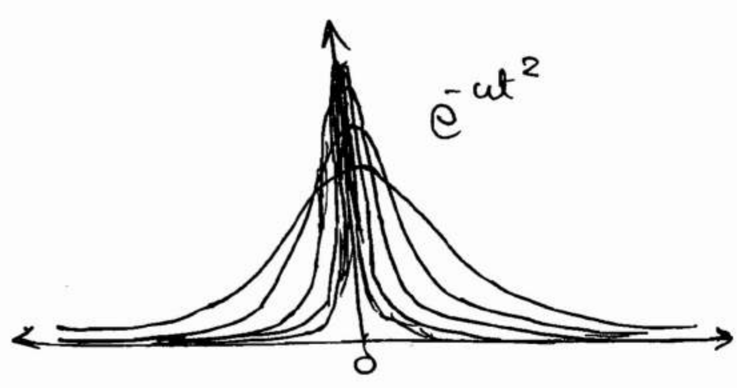
$= 0 \;;\; t \neq 0$




$\Rightarrow$ Used as approximation of Standard signals.

$\Rightarrow$ By approximate unit area rectangular $f^n$ we can generate unit impulse $f^n$.



$\Rightarrow$ ~~$\delta(t) \neq 3t$~~ $x(t) = 3\delta(t)$.

i.e. amp. $\neq 3$ but amp. $= \infty$.

but $\boxed{\text{area under } \delta(t) = 3.}$

## * Properties of $\delta(t)$ fn:

(i) $\int_{-\infty}^{+\infty} \delta(t)\,dt = 1.$

Area Concept are used in Continuous $\delta$ fn. (or) cont. impulse fn.

(ii) $\delta(\alpha t) = \frac{1}{|\alpha|} \cdot \delta(t).$

$\alpha \to$ Scaling factor.

(iii) Product (or) Sampling: IMP

$\to x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$ if $x(t)$ is cont$^n$ at $t = t_0$.

$t_0 \to$ time shift.

e.g. ① $\cos t \cdot \delta(t - \pi)$. $\Rightarrow t_0 = \pi$

$= \cos \pi$

$= -1.$

② $t \cdot \delta(t) = t_0 \cdot \delta(t)$

as $t_0 = 0$

$= 0 \cdot \delta(t)$

$\boxed{t \cdot \delta(t) = 0}$

(iv) $\delta(t) = \delta(-t).$

## (V) Shifting:

$$\int_{t_1}^{t_2} x(t).\ \delta(t-t_0)\, dt = x(t_0)\ ;\ t_1 \le t_0 \le t_2$$
$$= 0\ ;\ \text{elsewhere.}$$

Shift should be lies within the limit.

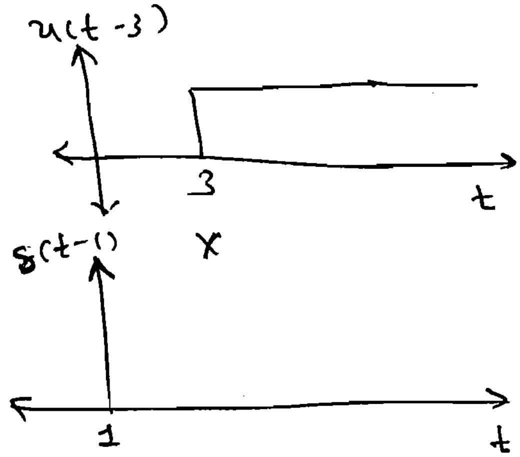
e.g. ① $\int_0^{\infty} (t + \cos\pi t)\ \delta(t-1)\ dt$

Soln: here, $t_0 = 1$.

$= 1 + \cos\pi = 1 - 1 = 0.$

② $\int_0^{\infty} \cos t.\ u(t-3)\ .\ \delta(t-1).\ dt = 0.$

Soln:




= 0

no overlap.

So, Ans is 0.

$u(t-3).\ \delta(t-1) = 0.$

③ $\int_{-\infty}^{\infty} x(2-t).\ \delta(4-t)\ dt.$

Soln:

$\delta(4-t) = \delta(t-4)$ $[\because \delta(t) = \delta(-t)]$.

so, $t_0 = 4$.

$= x(2-t_0).$

$= x(2-4)$

$= x(-2).$

④ $$\int_{0}^{\infty} e^{(t-3)} \cdot \delta(3t-9) \cdot dt$$

$\underline{\underline{\text{Sol}^n}}$: $\delta(3t-9) = \delta[3(t-3)] = \frac{1}{3}\,\delta(t-3).$

So, $t_0 = 3$

$$= \frac{1}{3} \cdot e^{(t_0-3)}$$

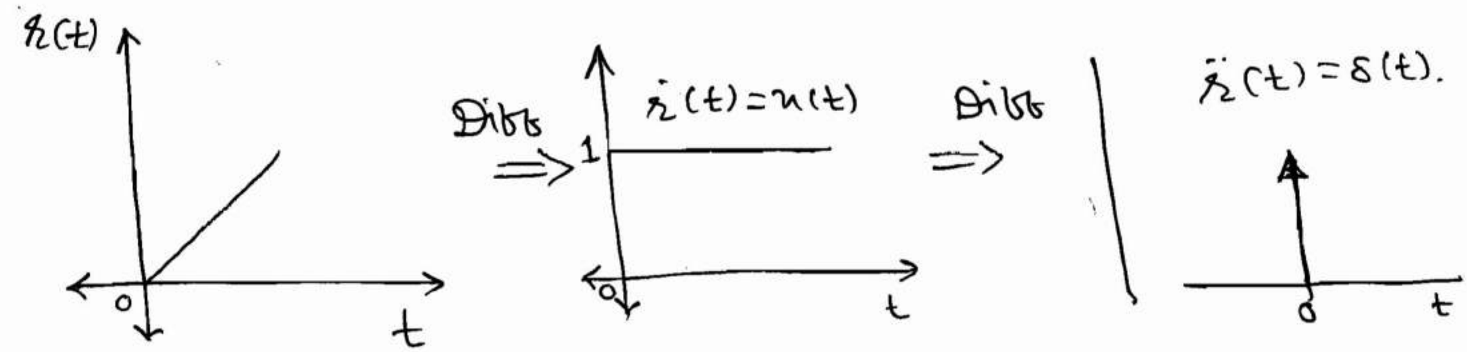
$$= 1/3 \cdot e^{(3-3)}$$

$$= 1/3 \,//$$

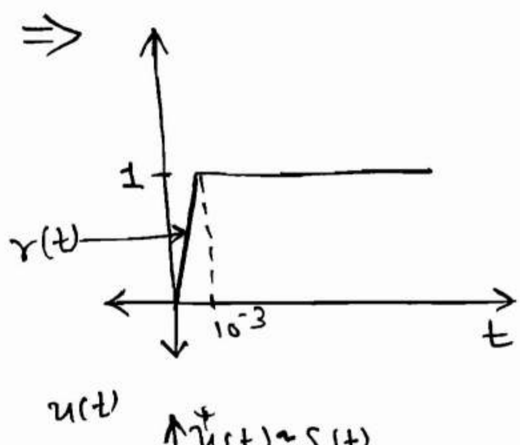
## ③ Unit Ramp:

$\Rightarrow$ $r(t) = t \;;\; t > 0$

$\quad\quad = 0 \;;\; t < 0$

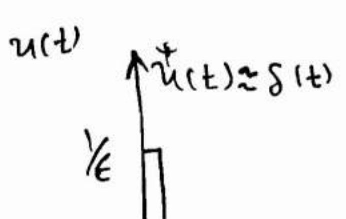



$\Rightarrow$




$$\delta(t) = \frac{d}{dt}u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt.$$

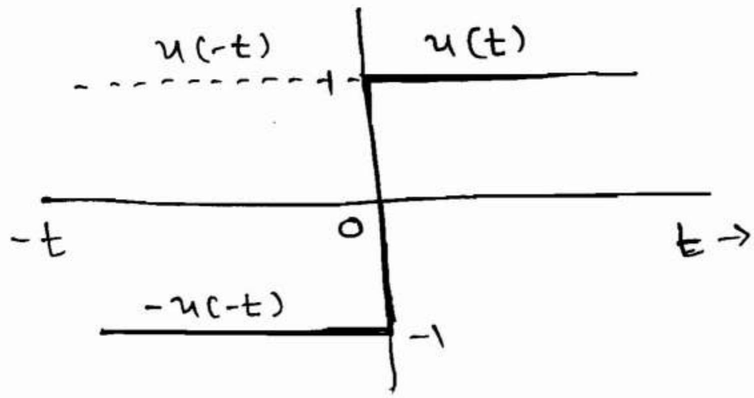


⇒ The function which do not posses higher ✓ derivative are singularity $f^n$.

e.g. $r(t)$ & $u(t)$.

## ④ Signum function:

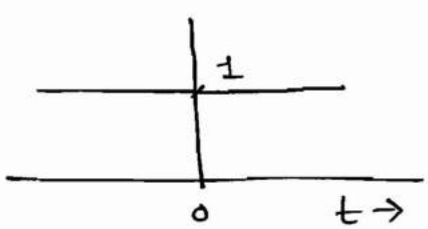
⇒




⇒ For any odd $f^n$ amplitude is zero at origin.

$$\text{sign}(t) = u(t) - u(-t)$$

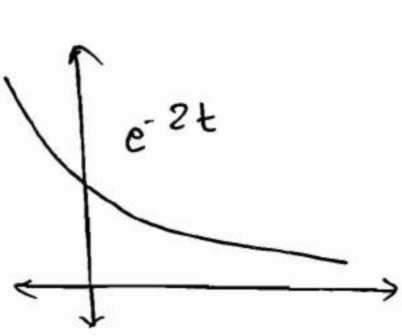
$$\text{sign}(t) = 2u(t) - 1 .$$ Put $u(-t)$

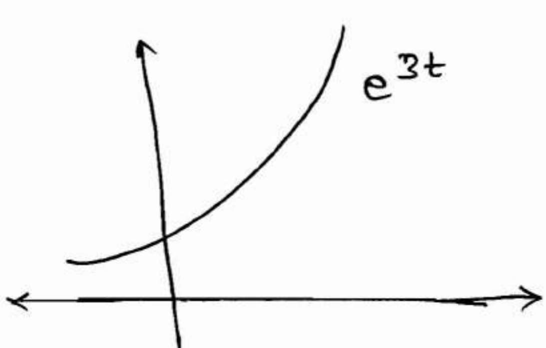



$u(t) + u(-t) = 1$ for $t=0$. only.

## ⑤ Exponentials:

① Real exp. $e^{(\sigma t)}$ :



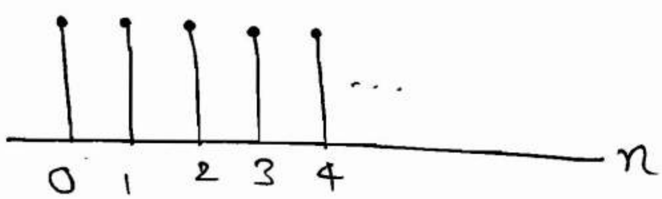
② Complex sinusoid $e^{\pm j\omega_0 t}$.

③ Complex exp.

$$e^{st} = e^{(\sigma + j\omega)t}.$$

⑥ Unit Step sequence:

⇒




$$u[n] = 1 \; ; \; n \geq 0$$
$$= 0 \; ; \; n < 0.$$

⇒ It is not a Sample version of $u(t)$.

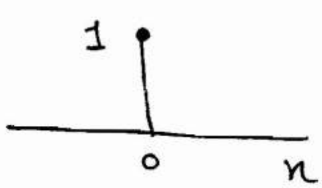
because $u(t)$ = discontinuous at $t = 0$.

but $\underline{u[n] = 1}$ at $\underline{n=0}$.

✓

⑦ Discrete Impulse: (or) Kronecker delta:

⇒




$$\delta[n] = 1 \; ; \; n = 0.$$
$$= 0 \; ; \; n \neq 0.$$

⇒ $\boxed{\delta(t) = \frac{d}{dt} u(t).}$

$\boxed{u(t) = \int_{-\infty}^{t} \delta(\tau).d\tau.}$

→ Differentiation ⇒ Difference

$t \qquad n$

$\nabla x_n = x_n - x_{n-1}.$

$\boxed{\delta[n] = u[n] - u[n-1]}$

⇒ Integration ⇒ Summation.

$t \qquad n$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

put $n - k = m$.

$$u[n] = \sum^{0} \delta[n-m]$$

$\Rightarrow$ ~~$\delta[kn]/z$~~

$$\boxed{\delta[kn] = \delta[n].}$$

e.g.: $\delta[4n] = \begin{cases} 1 ; & 4n = 0 \Rightarrow n = 0. \\ 0 ; & \end{cases}$

## ☆ Transformation of a Signal:-

① Time - Scaling:-

$\Rightarrow \quad x(t) \longrightarrow x(at)$ — Used in recording.

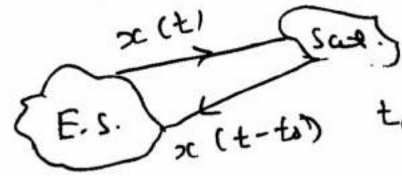
$\quad\quad x[n] \longrightarrow x[mn].$

$x(t) =$ K r.p.m. (60 min).

$x(2t) =$ K r.p.m. (30 min).

② Time - Shift:

$\Rightarrow x(t - t_0) \mid x[n - n_0]$ — Propogation delay.




$t_0$ = P.D.

③ Time - reversal:

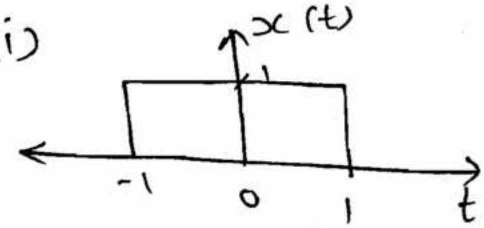
$\Rightarrow x[-t] \mid x[-n].$

$\Rightarrow$ All ready data are stored. we want to access it one more time we use time reversal. i.e. if we ~~we~~ are hearing a music song. and after completion at first time, inorder to re hear it we ~~take~~ rewind it. it is nothing but the time - reversal.

④ Amplitude Scaling:

## ① Time- Scaling:

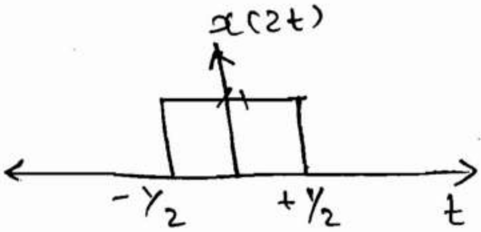
⇒ (i)




$x(t) = 1 \quad ; \quad -1 < t < 1$

$\quad\;\; = 0 \quad ; \quad$ elsewher

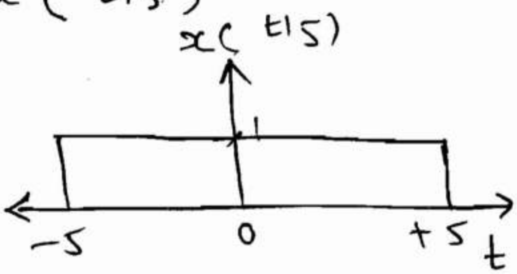
⇒ $x(2t)$




$x(2t) = 1 \quad ; \quad -1 < 2t < 1$

$x(2t) = 1 \quad ; \quad -\frac{1}{2} < t < \frac{1}{2}$

⇒ $x(t/5)$



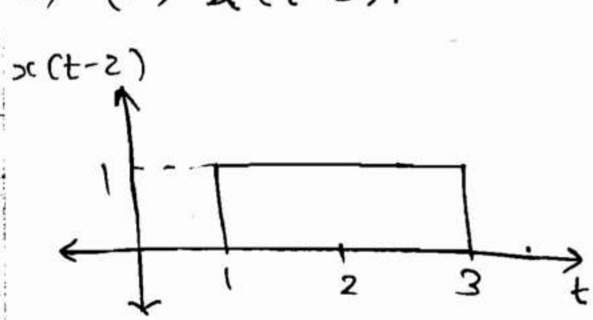

$x(t/5) = 1 \quad ; \quad -5 < t < 5.$

⇒

| | | |
|---|---|---|
| $a > 1$ | ⇒ | Compression of $x(t)$. |
| $a < 1$ | ⇒ | Expansion of $x(t)$. |

## ② Time- Shifting:

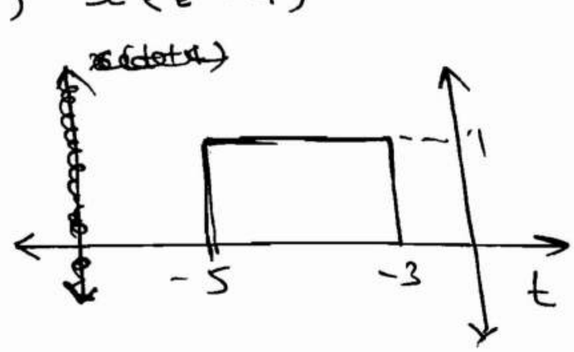
⇒ (i) $x(t-2)$.




$\boxed{t_0 > 0}$

→ Right Shift

(ii) $x(t+4)$




$\boxed{t_0 < 0}$

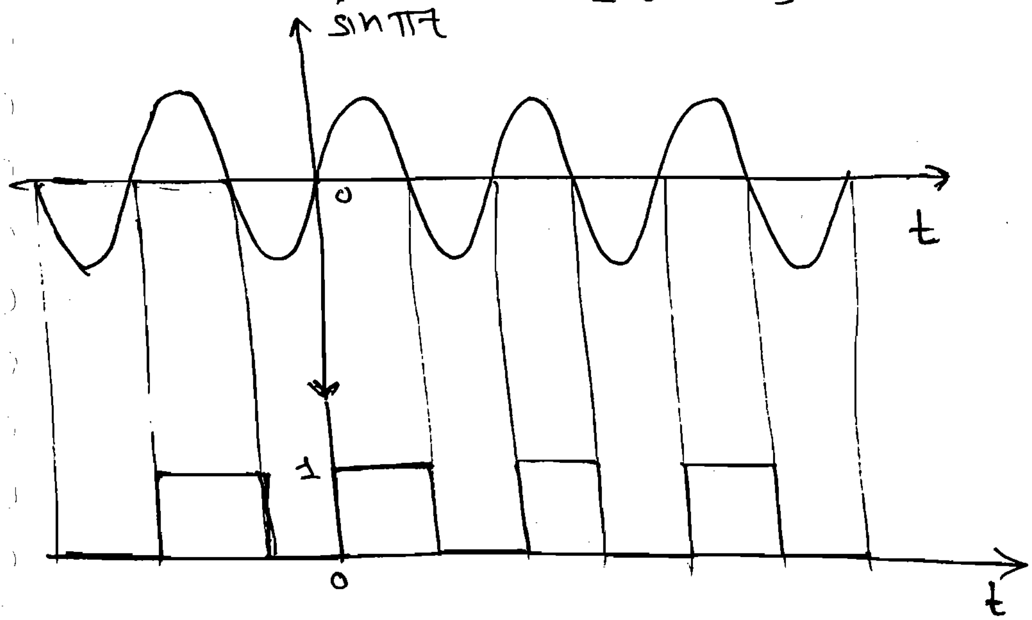
→ Left Shift

⇒ In Real-time, time advance is not possible.
i.e. we can not predict any output before giving a input.

$\boxed{Q-1}$ Draw the following signals:

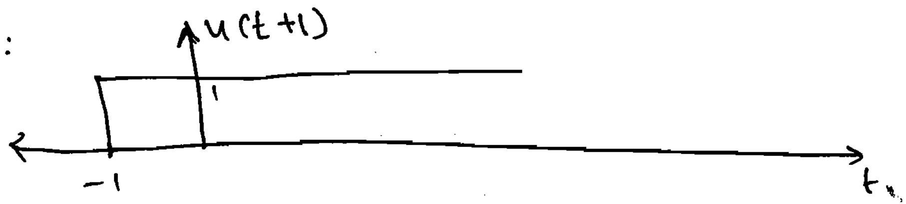
① $u[\sin \pi t]$.

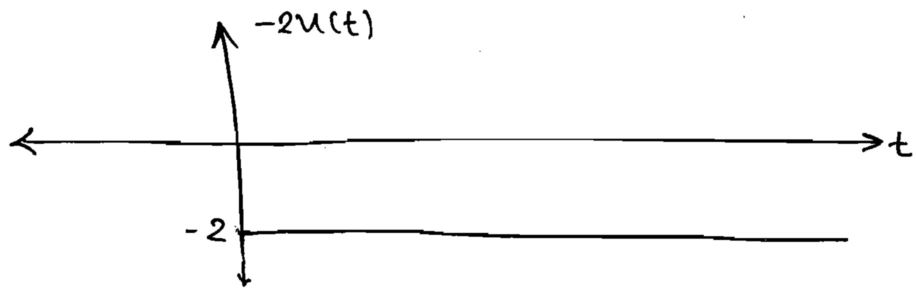
<u>Soln</u>: $u[\sin \pi t] = 1 \quad ; \quad \sin \pi t > 0.$
$\qquad\qquad\qquad = 0 \quad ; \quad \sin \pi t < 0.$

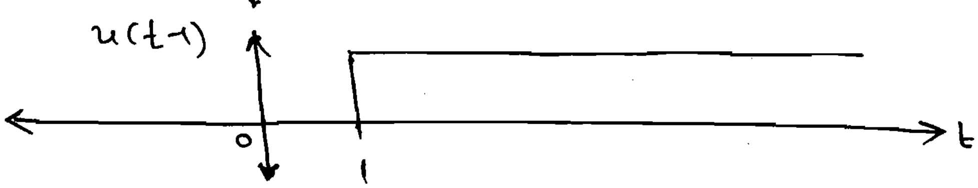



② $x(t) = u(t+1) - 2u(t) + u(t-1)$.

<u>Soln</u>:



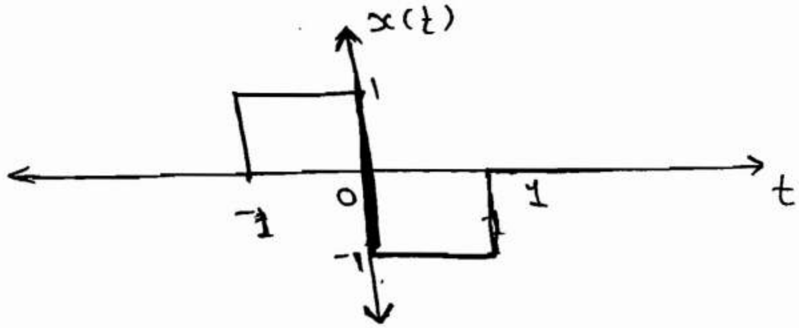




(or)

$\Rightarrow x(t) = 1.u(t+1) - 2u(t) + 1u(t-1).$

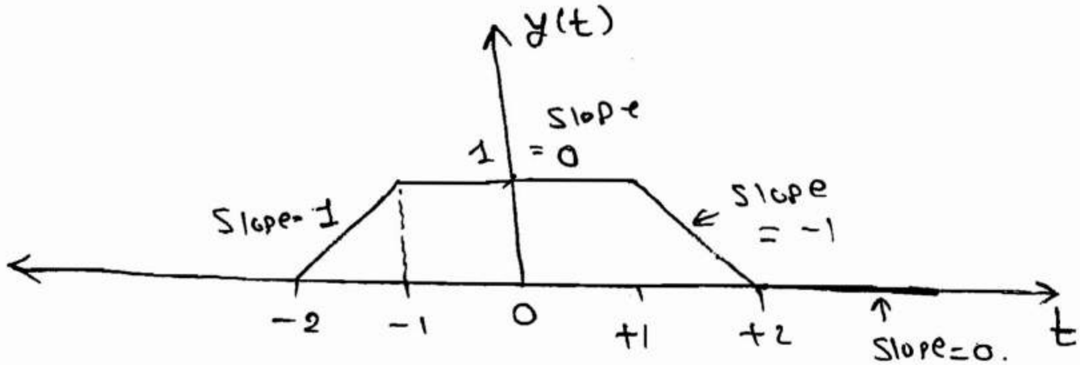
$t = -1 \qquad 0 \qquad 1$




② $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2).$

Sol^n: $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2).$

$-2 \qquad -1 \qquad +1 \qquad +2.$




$\Rightarrow$

$-\infty < t < -2 \Rightarrow$ Slope $= 0.$

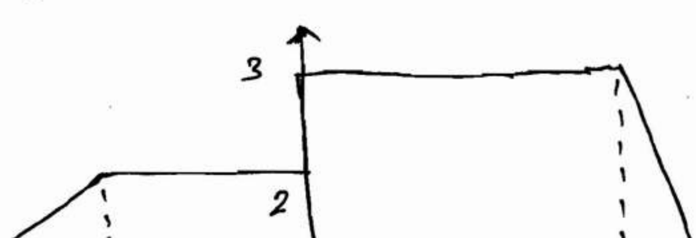
$-2 < t < -1 \Rightarrow$ Slope $= 0 + 1 = 1.$

$-1 < t < +1 \Rightarrow$ Slope $= 1 - 1 = 0.$

$1 < t < 2 \Rightarrow$ Slope $= 0 - 1 = -1.$

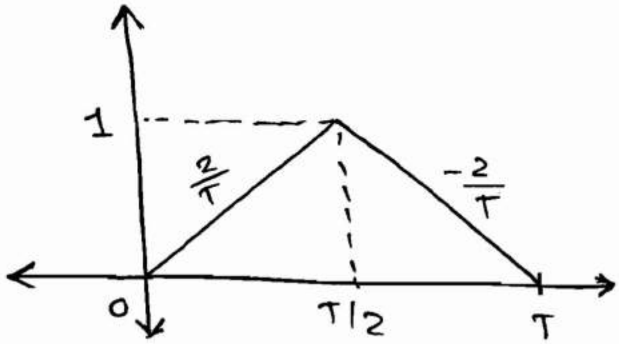
$2 < t < \infty \Rightarrow$ Slope $= -1 + 1 = 0.$

* Write the following signal in terms of singularity f^ns:



**Sol$^n$:** $y(t) = 1.r(t+4) - 1.r(t+2) + u(t) - 3r(t-4) + 3r(t-5)$.

② 




**Sol$^n$:** $y(t) = \frac{2}{T}r(t) - \frac{4}{T}r(t-\frac{T}{2}) + \frac{2}{T}r(t-T)$.

③ If $x(t) = 0$ for $t<3$. For what range of $t$ following f$^n$ are said to zero.

① $x(1-t) + x(2-t) = 0$ for $t$?

② $x(1-t) \cdot x(2-t) = 0$ for $t$?

**Sol$^n$:** $x(t)=0$ for $t<3$.


$\rightarrow$ $x(1-t) = 0$ ; $1-t<3$

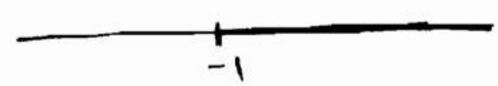
$-t<2$

$x(1-t) = 0$ ; $t>-2$


$\rightarrow$ $x(2-t) = 0$ ; $2-t<3$

$-1<t$

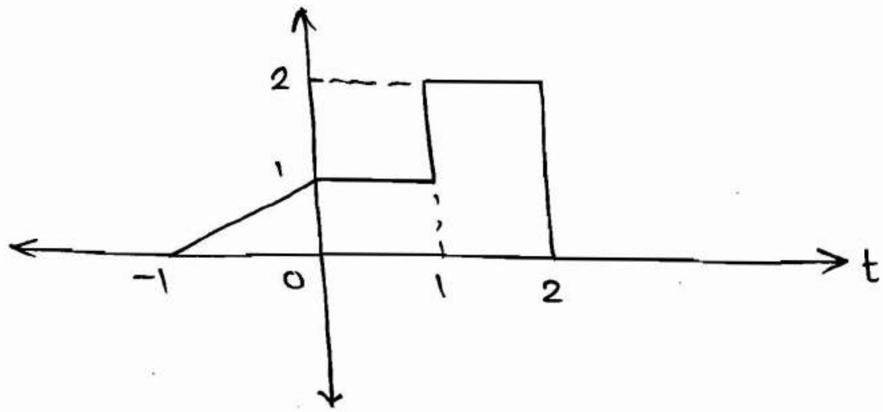
$x(2-t) = 0$ ; $t>-1$.




① $x(1-t) + x(2-t) = 0$ for $t>-1$.

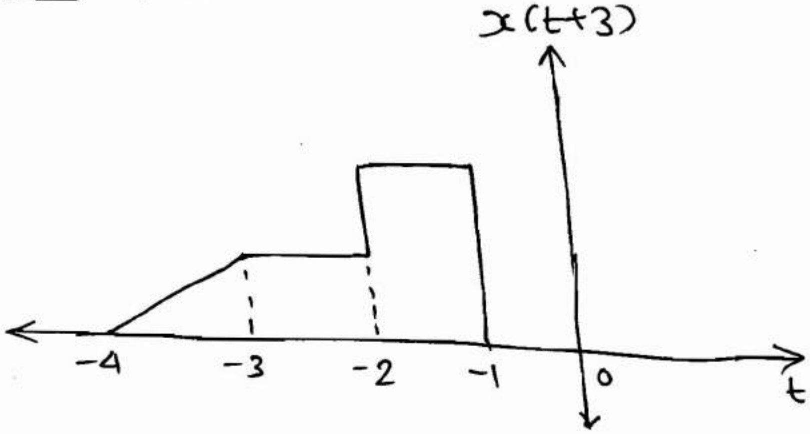
② $x(1-t) \cdot x(2-t) = 0$ for $t>-2$.

* For the signal $x(t)$ shown in figure. Draw the following signals:




<u>Sol<sup></sup></u>

Sol$^n$: (1) $x(t+3)$




* $x(at+b)$.

$\Rightarrow$ Method: -(1): $x(t) = x\left(a\,(t \pm b/a)\right)$.

(i) Time - scaling $x(at)$.

(ii) Shift - $x(at)$ by $\pm b/a$.

$\Rightarrow$ Method : -(2) :

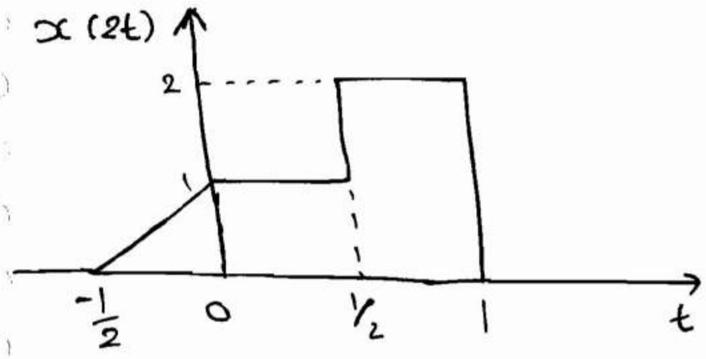
$$x(t) \longrightarrow x(t \pm b) \longrightarrow x(at \pm b).$$

(i) Time shift by b i.e $x(t \pm b)$.

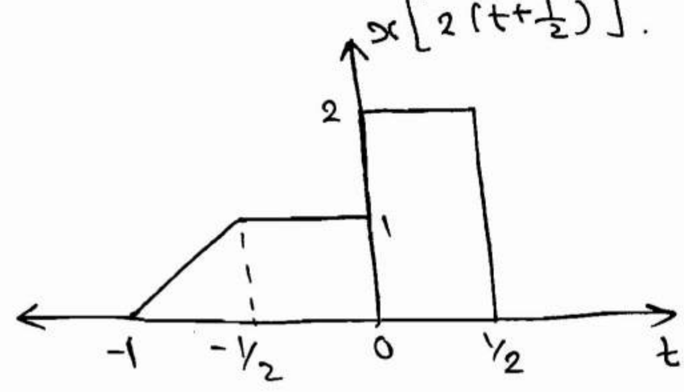
(ii) Time - scaling $x(t+b)$ by $x(at \pm b)$.

(2) $x(2t+1)$.

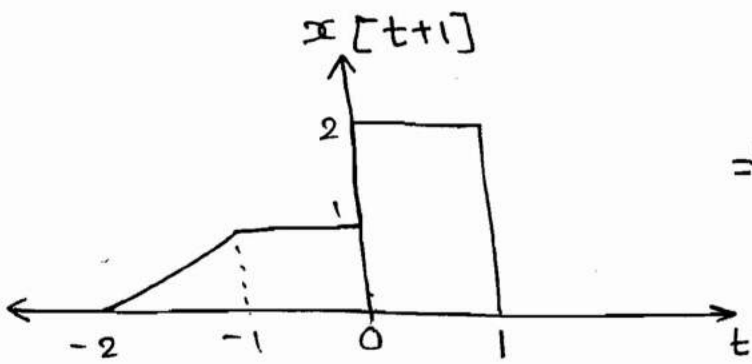
Sol$^n$: Method -1 : $x\left[2\left(t+\frac{1}{2}\right)\right]$.



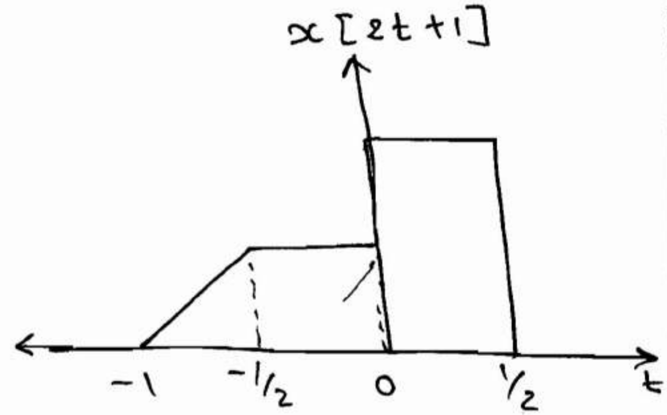

$\Rightarrow$




$\Rightarrow$ Method -2: $x[2t+1]$

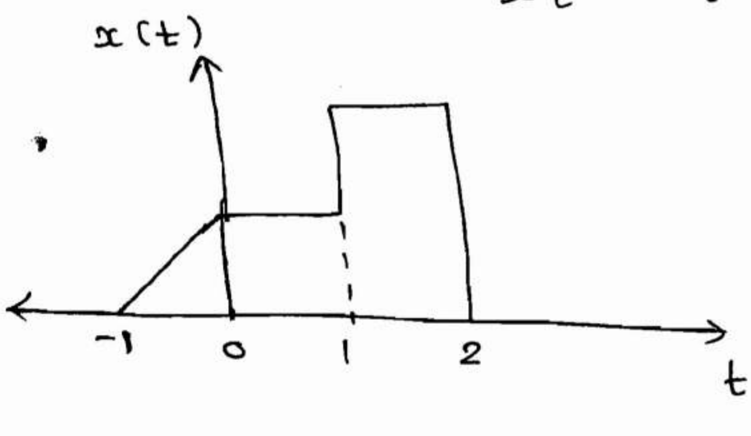



$\Rightarrow$

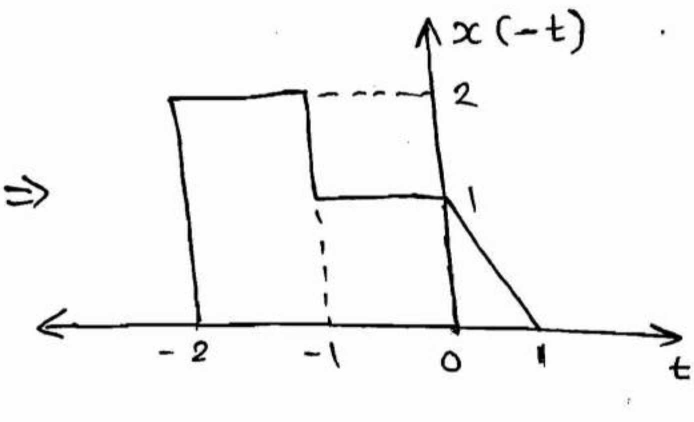


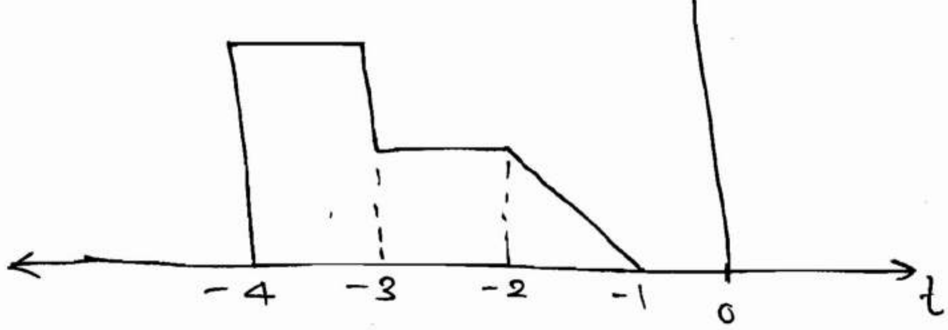

(3) $x(-t-2)$.

$x[-(t+2)]$



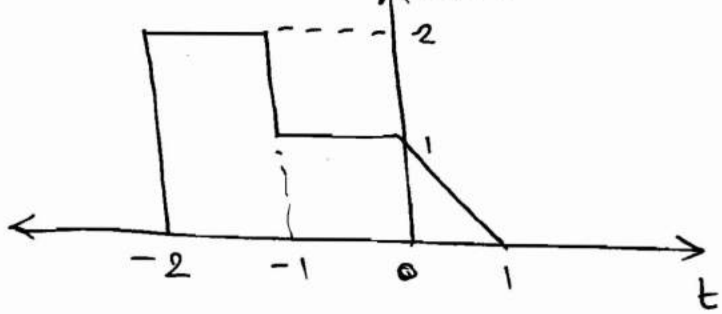

$\Rightarrow$

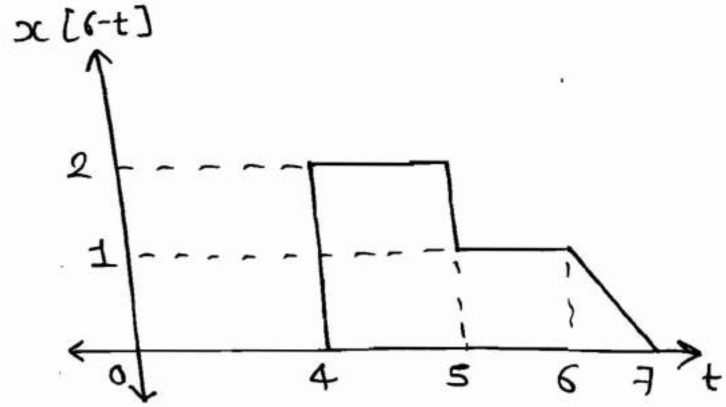


(4) $x[6-t]$.

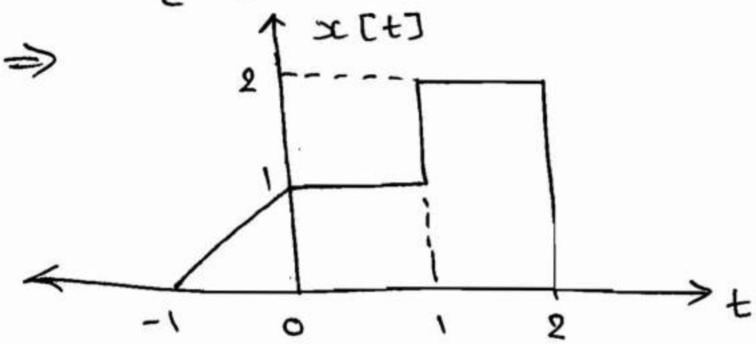
Sol$^n$: $x[-[t-6]]$.

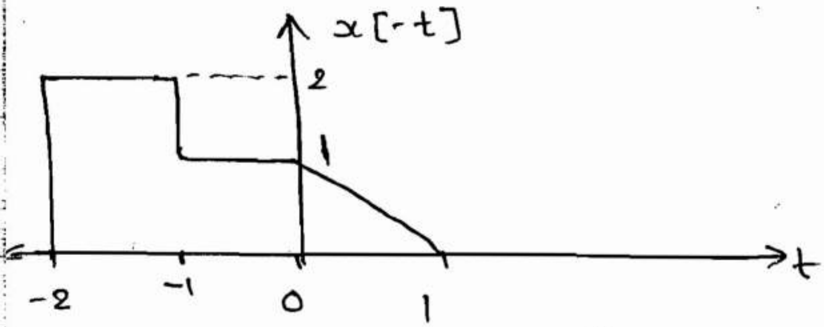


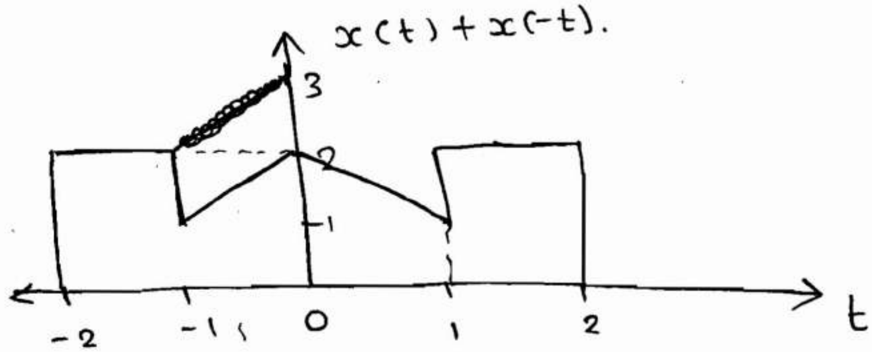




(5) $[x[t) + x(-t)]u(t)$.

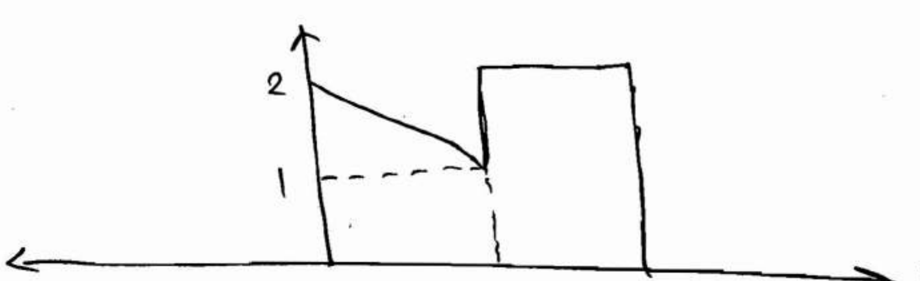
⇒




$\times u(t)$



* $x(1 - t/3) = x\left[-\frac{1}{3}(t-3)\right]$.

$\Rightarrow x(t) \xrightarrow{T.R.} x(-t) \xrightarrow{T.S.} x(-t/3) \xrightarrow{R.S.} x\left((t-3)\cdot -\frac{1}{3}\right)$.

Can be Interchange

Can not be Interchange

$\Rightarrow$ $y(t) = x(t-t_0)$

$y(\alpha t) = x(\alpha t - t_0)$ ✓

$= x(\cancel{\alpha}(t-t_0))$

$y(t) = x(\alpha t)$.

$y(t-t_0) = x(\alpha(t-t_0))$ ✓

$= x(\cancel{\alpha t} - t_0)$.

Q Given, $x[n] = (6-n)[u(n) - u(n-6)]$.

Draw the following signals

(i) $x[n+3]$

(ii) $x[6-n]$

(iii) $x[3n+1]$

(iv) $x\left[\frac{n}{2}\right]$

(v) $x[n-1]\cdot\delta[n-3]$

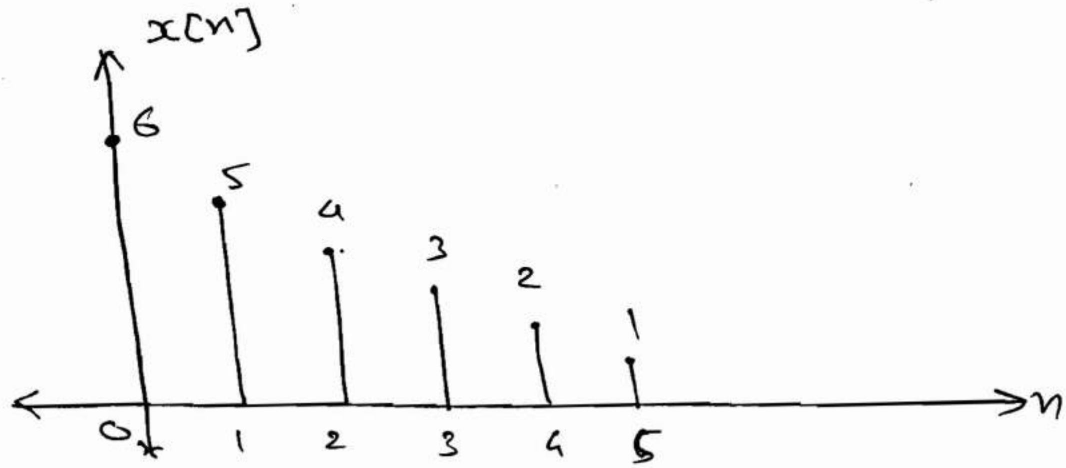
Soln:

$u[n]$

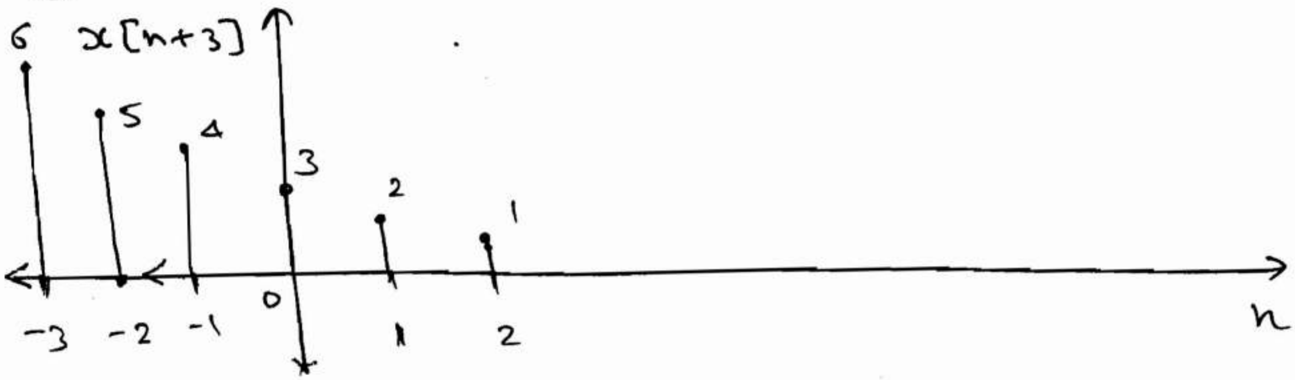

$-$ $u[n-6]$


$u[n] - u[n-6]$.

$\Rightarrow$




① $x[n+3]$:




② $x[6-n]$.

$\Rightarrow \quad x[n] : \quad 0 \le n \le 5$

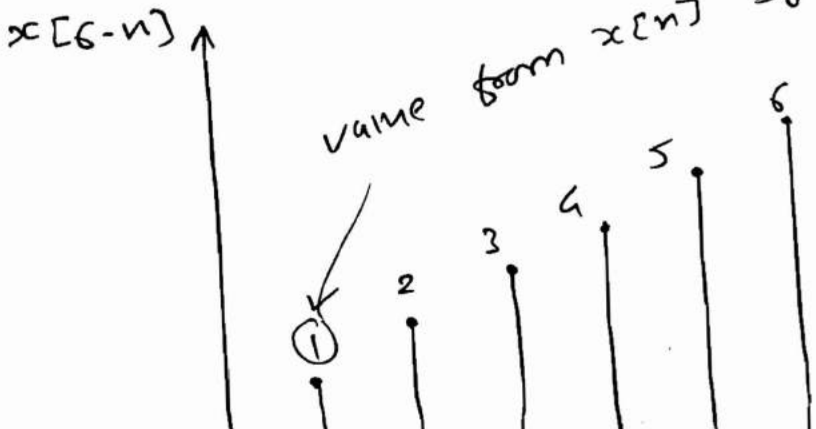
$x[6-n] : \quad 0 \le 6-n \le 5$

$-6 \le -n \le -1$

$6 \ge n \ge 1$

$x[6-n] : 1 \le n \le 6.$

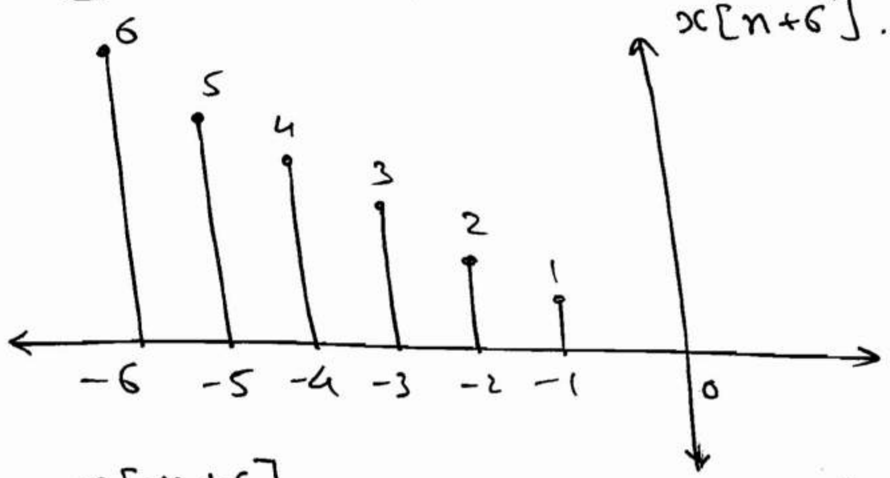
value from $x[n]$ by substituting $n$ into $x[6-n]$



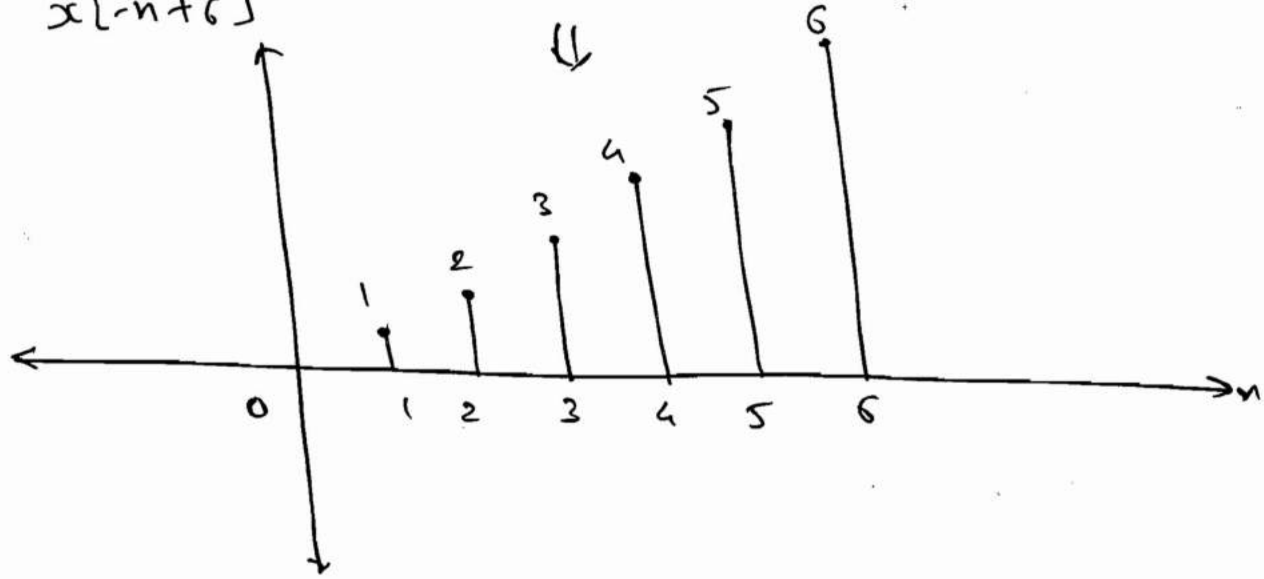
Method-2:

① shift left by 6

② Time reversel.




⇓




③ $x[3n+1]$.

$\Rightarrow \quad x\left[\cancel{3\left(n+\frac{1}{3}\right)}\right] \qquad \cancel{n_0 = -1/3}$.

NOTE: Method-I i.e. first scaling and then shifting may be fails in discrete time domain. because $n$ is integer. so, always go through by following two methods.

Method-I: $x[n]: \quad 0 \le n \le 5$.

$\therefore x[3n+1]: \quad 0 \le 3n+1 \le 5$

Valid index,

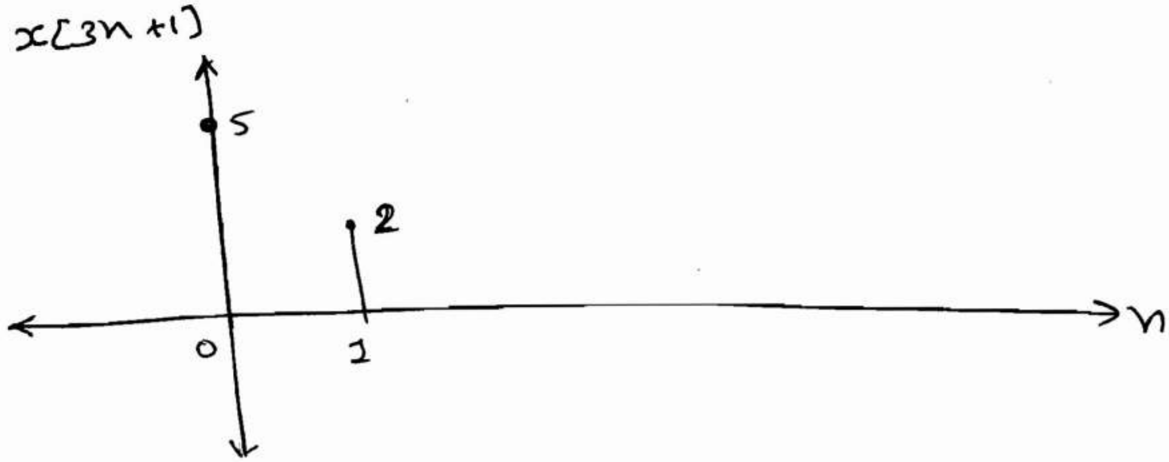
$$-0.33 \leq n \leq 1.33.$$

but $n$ is always integer.

So, valid $n$ in range $(-0.33, 1.33)$

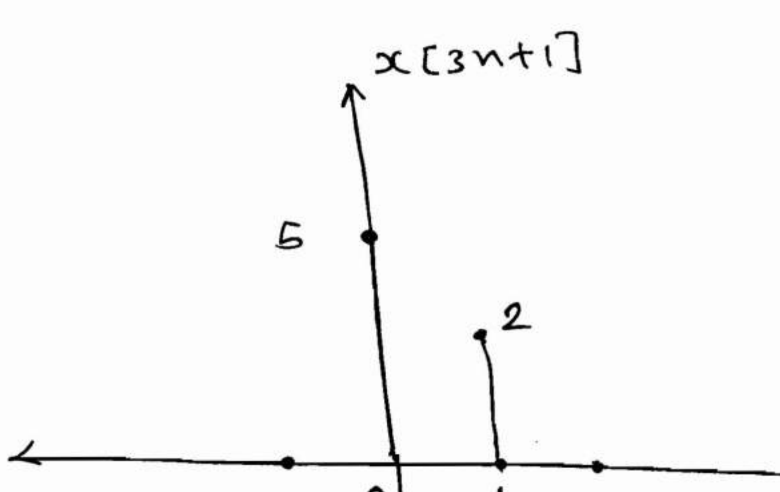
is $n=0$ & $n=1$.

∴




**Method - II:**

→ ① shifting ② scaling.

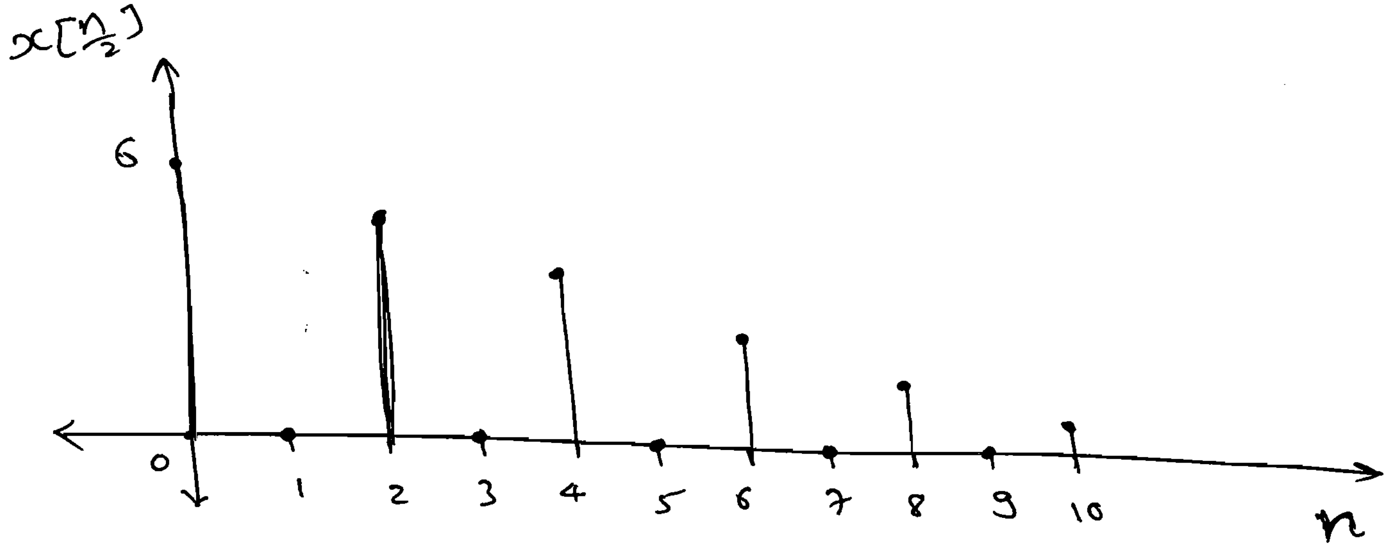

$\cancel{-\frac{1}{3}}, 0, \cancel{\frac{1}{3}}, \cancel{\frac{2}{3}}, 1, \cancel{\frac{4}{3}}$

④ $x[\frac{n}{2}]$.

$\Rightarrow$ $x[n]$ : $0 \le n \le 5$

$x[\frac{n}{2}]$ : $0 \le \frac{n}{2} \le 5$

$0 \le n \le 10$.




## * Interpolation and Decimation.

$\Rightarrow$ In cont$^n$ time domain,

$x[at] \rightarrow$ Scaling property,

a is scaling factor.

if $a > 1 \Rightarrow$ Compression

$a < 1 \Rightarrow$ Expansion.

$\Rightarrow$ In ~~con~~ discrete time domain,

$x[mn] \rightarrow$ scaling property.

if $m > 1 \Rightarrow$ Decimation.

$m < 1 \Rightarrow$ Interpollation.

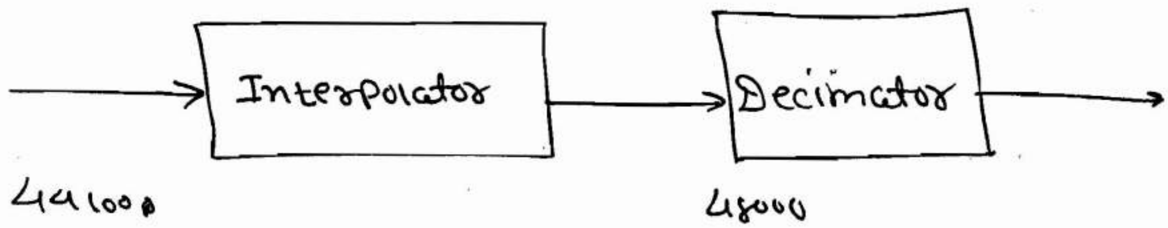
$\Rightarrow$ In above ~~expa~~ example we can say

Sample zero.

⇒ <u>Multi</u> - <u>Rate</u> <u>DSP</u> :-

⇒ C.D. : $f_s = 44.1$ KHz.

D.A.T. : $f_s = 48$ KHz.



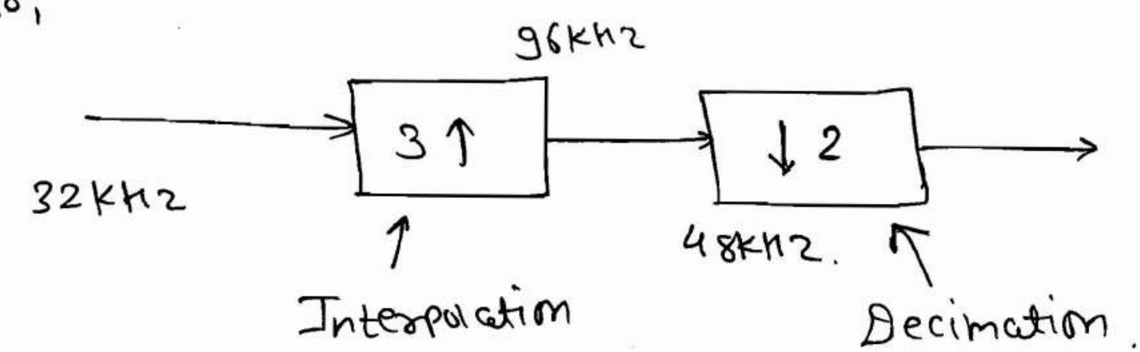

→ Let, $f_{s_1}$ : 32 KHz

$f_{s_2}$ : 48 KHz.

Now, we want to convert 32K S.R.

to the 48 K S.R.

So,




$$\frac{48}{32} = 3/2 \Rightarrow x\left[\frac{2}{3}n\right].$$

<u>NOTE</u>:

⇒ Conversion of one sampling rate to another sampling rate is multiRate DSP interpolator.

→ whenever we want to perform this operations simultaneously first do Interpolation (up sampling) and then Decimation (Down sampling).

i.e. if $x\left[\frac{an}{b}\right]$ → then first do $x\left[\frac{n}{b}\right]$ and then $x\left[\frac{an}{b}\right]$.

→ In interpolation no. of samples increases and in Decimation no. of samples decreases.

(5) $x[n-1]\,\delta[n-3]$.

$\Rightarrow \quad x[n-1]\,\delta[n-3]$

$n_0 = 3$.

∴ by sifting property of δ

$x[3-1]\cdot\delta[n-3]$.

$= \cancel{4}\,\delta[n-3]$.

$\boxed{a}$ If $x[n] = 1 - \sum_{k=4}^{\infty} \delta[n-1-k]$. such that $x[n] = u[Mn - n_0]$ find $M$ & $n_0$.

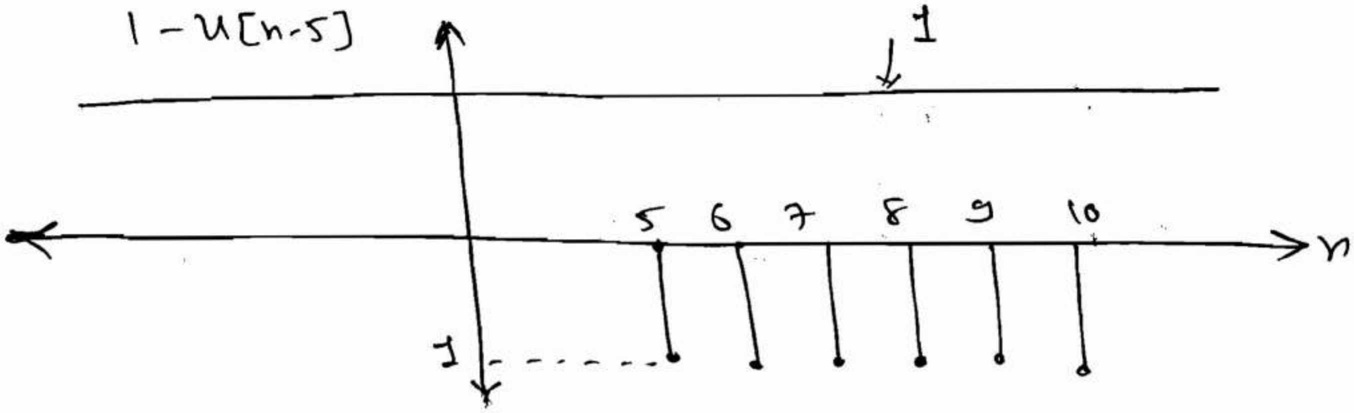
Soln:

$x[n] = 1 - [\delta[n-5] + \delta[n-6] + \delta[n-7] + \dots]$.

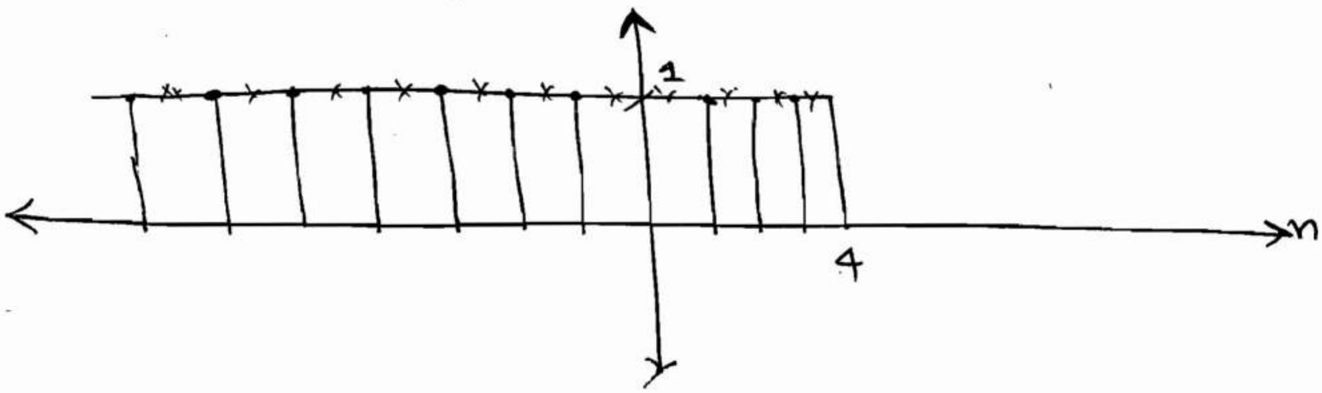
$x[n = 1 - u[n-5]$

$\Rightarrow$


$\Rightarrow$




$\Downarrow$




So, $u[-n+4]$.

So, $m = -1$, $n_0 = -4$.

## ☆ Classification of signals: $x(t)$, $x[n]$.

### ① Energy and power signal:

⇒ **Energy signal:**

→ If Energy of a signal is finite then it is called Energy ~~sing~~ signal i.e

$0 < E < \infty$.

⇒ For cont$^n$ time signal,

$$\boxed{E_{x(t)} = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \cdot dt.} = \text{finite.}$$

⇒ For discrete time signal,

$$\boxed{E_{x[n]} = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2} = \text{finite.}$$

⇒ For Complex $\underline{|x(t)|^2 = x(t) \cdot x^*(t)}$.

⇒ **Power signal:**

⇒ If avg. power of a signal is finite i.e. $0 < P < \infty$ then it is called power signal.

$$P_{avg\,x(t)} = \lim \frac{1}{\;} \int^{T} |x(t)|^2 \cdot dt.$$

For discrete time signal,

$$\boxed{P_{avg\,x[n]} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x(n)|^2}$$

$n$: $-\infty$ to $+\infty$

: $-N$ to $+N$

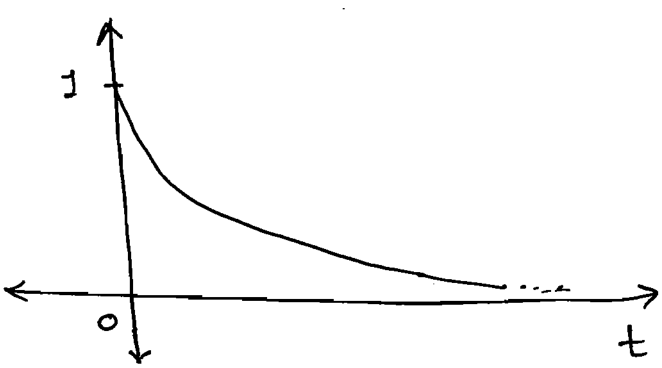
$-N$ to $-1$    $0$    $1$ to $N$

So, $2N+1$.

$\Rightarrow$ The physically possible (or) Practically possible signal is said to be the Energy signal if following condn is satisfied:

$$\boxed{\left.\begin{array}{l} \text{As } t \to \pm\infty \Rightarrow \underline{t \to \pm\infty} \\ \underline{\text{amp} \to 0} \end{array}\right\} \text{Energy signal}}$$

* $\underline{x(t) = e^{-t} \cdot u(t)}$.




T

$$= \lim_{T \to \infty} \left[ \frac{e^{-2t}}{-2} \right]_0^T$$

$$= \lim_{T \to \infty} \frac{1 - e^{-2T}}{2}$$

$$= \frac{1}{2} . = \text{finite,}$$

So, Energy signal.

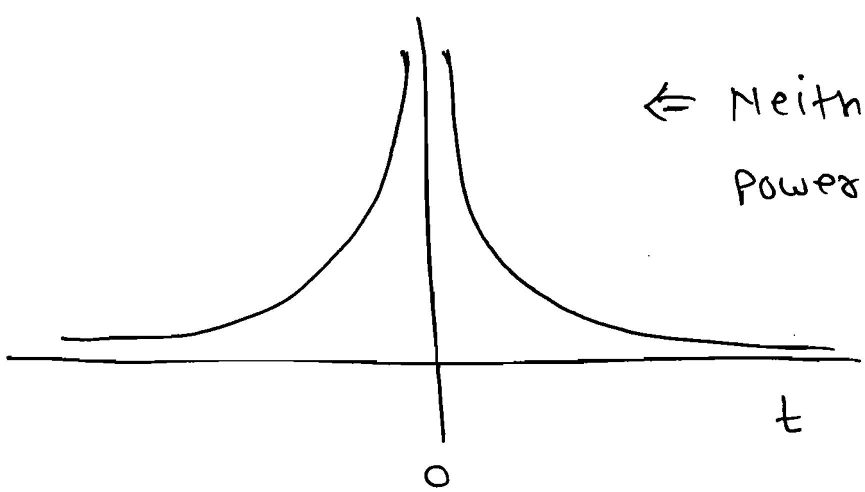
$$\Rightarrow \quad P_{av} = \lim_{T \to \infty} \frac{E}{2T} = \frac{1}{\infty} = 0.$$

So, An Energy signal should has zero avg. Power.

→ over the signal is existing it should maintain the finite amplitude level.

* $y(t) = \left| \frac{1}{t} \right|$.

⇒




⇐ Neither Energy nor Power signal.

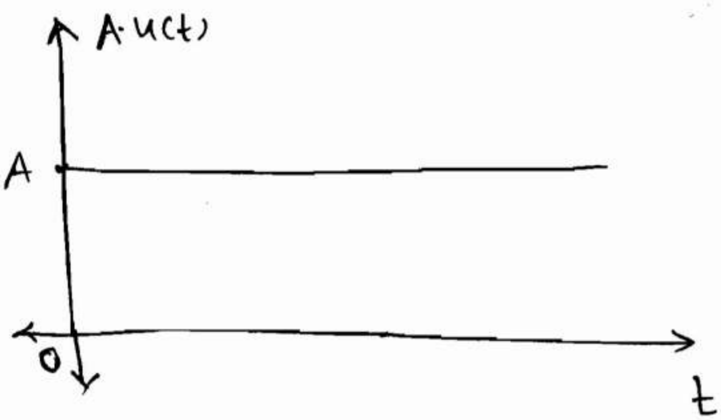
⇒ at $t = 0$ its amplitude is $\infty$.

⇒ at $t = \infty$ it's amplitude is ZERO.

$\Rightarrow$ Power signal:

$\Rightarrow$ A signal which maintain constant amplitude over infinite time is a power signal.

e.g. $y(t) = A \cdot u(t)$.




$$P_{av} = \lim_{T \to \infty} \frac{1}{2T} \int_0^T A^2 \cdot dt$$

$$P_{av} = \lim_{T \to \infty} \frac{1}{2T} \times A^2 \cdot T.$$

$$= \lim_{T \to \infty} \frac{A^2}{2}.$$

$$\therefore \boxed{P_{av} = \frac{A^2}{2}.}$$

Now, $P_{av} = \lim_{T \to \infty} \frac{1}{2T} \cdot E.$

$$\Rightarrow E = \lim_{T \to \infty} 2T \cdot (P_{av}).$$

$$E = \lim_{T \to \infty} \cancel{2} T \cdot A^2/\cancel{2}$$

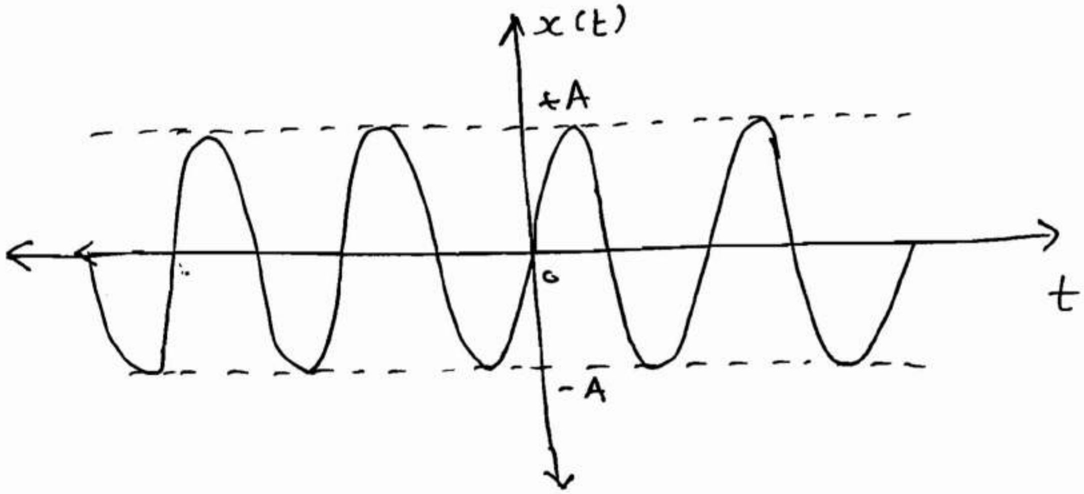
$$\boxed{E = \infty}$$

$\Rightarrow$ So, Power signal require $\infty$ energy.

$\Rightarrow$ The signal cannot be both energy and

$\Rightarrow$ All periodic signals are power signals because they maintain constant amplitude over a infinite time.

for. e.g. (i) $x(t) = A\cos(\omega_0 t + \theta)$.




$$P_{av} = \left(\frac{A}{\sqrt{2}}\right)^2 = (rms)^2 = (\text{mean square})^2.$$

$$\therefore P_{av} = \frac{A^2}{2}.$$

(ii) $x(t) = A \cdot e^{j\omega_0 t}$

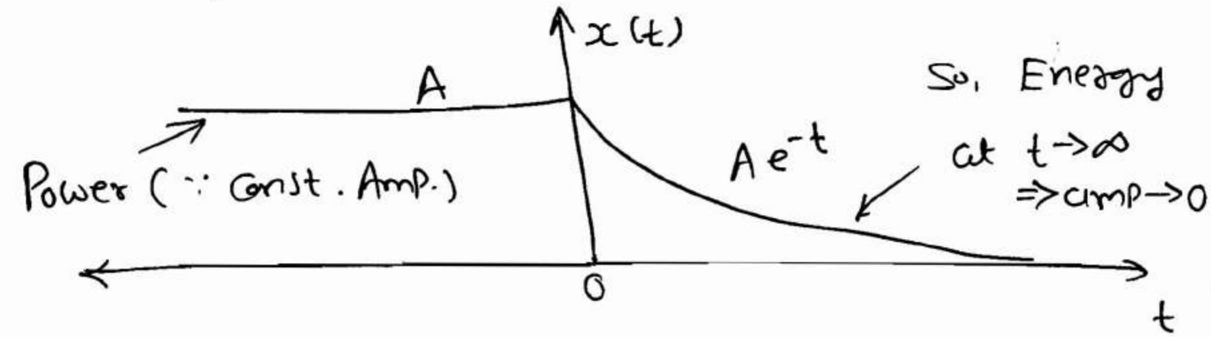
$$P_{av} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |A \cdot e^{j\omega_0 t}|^2 \cdot dt.$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} A^2 \cdot |e^{j\omega_0 t}|^2 \cdot dt.$$

$$= \lim_{T \to \infty} \frac{1}{\cancel{2T}} \cdot A^2 \cdot \cancel{2T}$$

$$P_{av} = A^2.$$

⇒ What is Nature of signal below:




$$\Rightarrow P_{av} = \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt.$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[ \int_{-T}^{0} A^2 \cdot dt + \int_{0}^{T} A^2 \cdot |e^{-2t}|^2 \cdot dt \right]$$

0

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{0} A^2 \cdot dt$$

case-(i): if power signal then E signal has zero power so power + 0 = power

case-(ii) if E signal then P has infinite E.
so, $E = \infty$ means at $t \to \infty \Rightarrow E \to \infty$

$$P_{av.} = \frac{A^2}{2}$$

So, Power + Energy = Power.

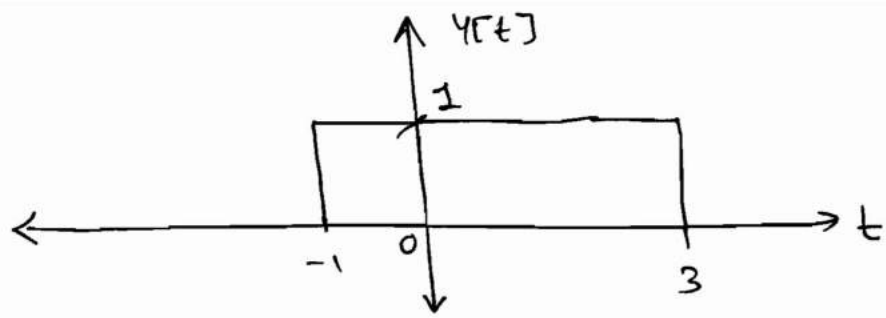
$\boxed{Q}$ If $x[t] = \delta(t+1) - \delta(t-3)$. Find the energy in $y[t] = \int_{-\infty}^{t} x(\tau) d\tau$.

$\underline{\underline{Sol^n}}$:

$$y(t) = \int_{-\infty}^{t} (\delta(t+1) - \delta(t-3)) d\tau.$$

$$= u[t+1] - u[t-3].$$

$\Rightarrow$



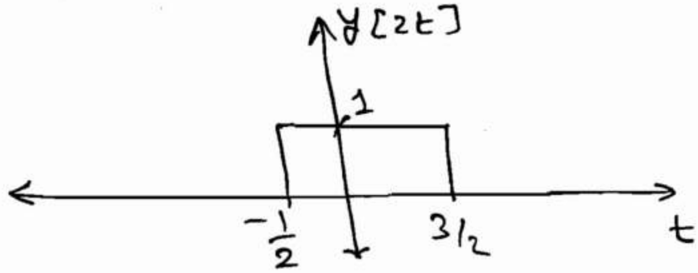

$$\Rightarrow \quad E_{y(t)} = \int_{-1}^{3} (1)^2 \cdot dt = 4$$

$$\therefore \quad \boxed{E_{y(t)} = 4.} \quad — ⊛$$

$\Rightarrow$ All finite duration signal of finite amplitude are energy signals.

$\Rightarrow$ for the above example find E for $y[2t]$.

$\Rightarrow$




$$\Rightarrow \quad E_{y(2t)} = \int_{-1/2}^{3/2} (1)^2 \cdot dt = \frac{4}{2} = 2.$$

$$\therefore \quad \boxed{E_{y(2t)} = 2} \quad — ⊛$$

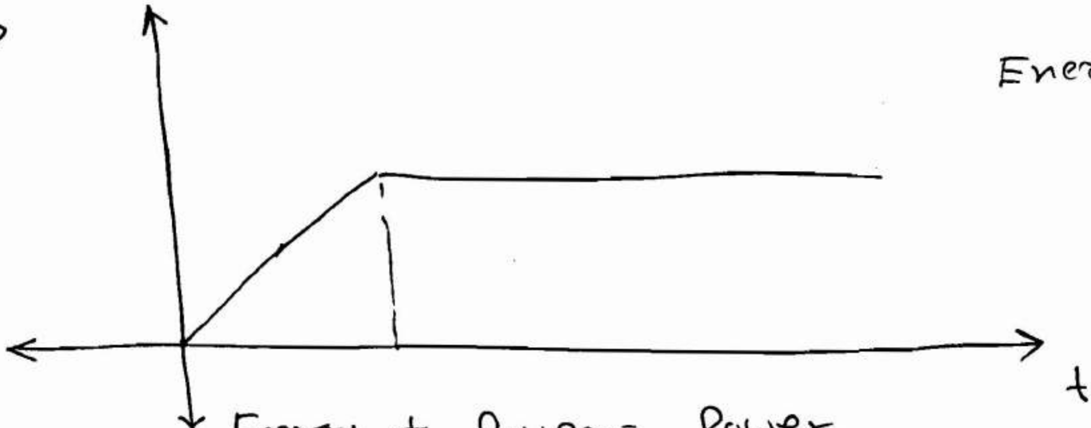
$$\Rightarrow \quad \boxed{E\ y(\alpha t) = \frac{E_{y(t)}}{|\alpha|}.} \qquad \text{Imp} \ldotp ☆$$

$$\Rightarrow \quad \boxed{A\, y(t) \Longrightarrow \underline{\text{Energy}}\ A^2 E_{y(t)}.} \qquad \text{Imp.}$$

* $x(t) = r(t) - r(t-1)$.

For power see constant amp.
Energy very by $t \to \pm\infty \Rightarrow Amp = 0$.

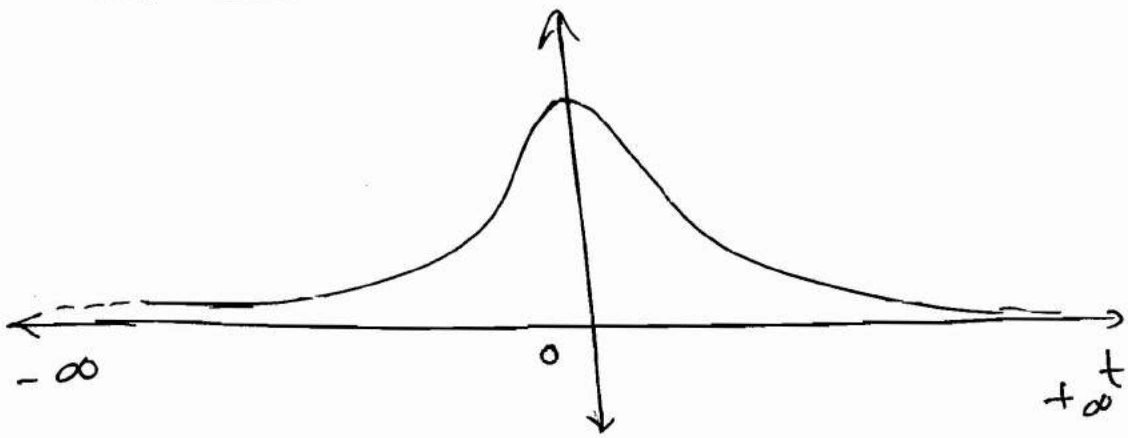
$\Rightarrow$




Energy + Power = Power.

* $x(t) = e^{-\pi t^2}$

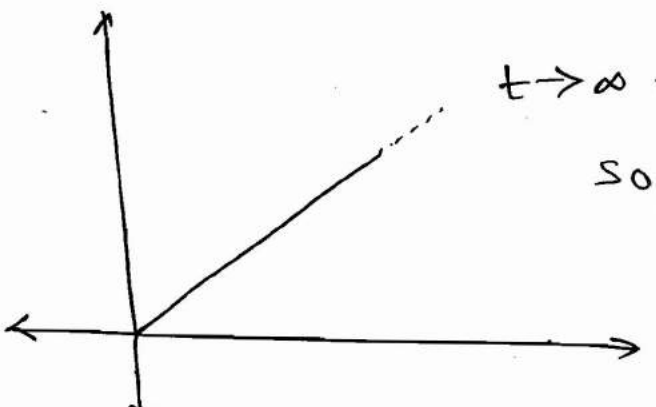
$\Rightarrow$




Energy signal

* $x(t) = t u(t)$.

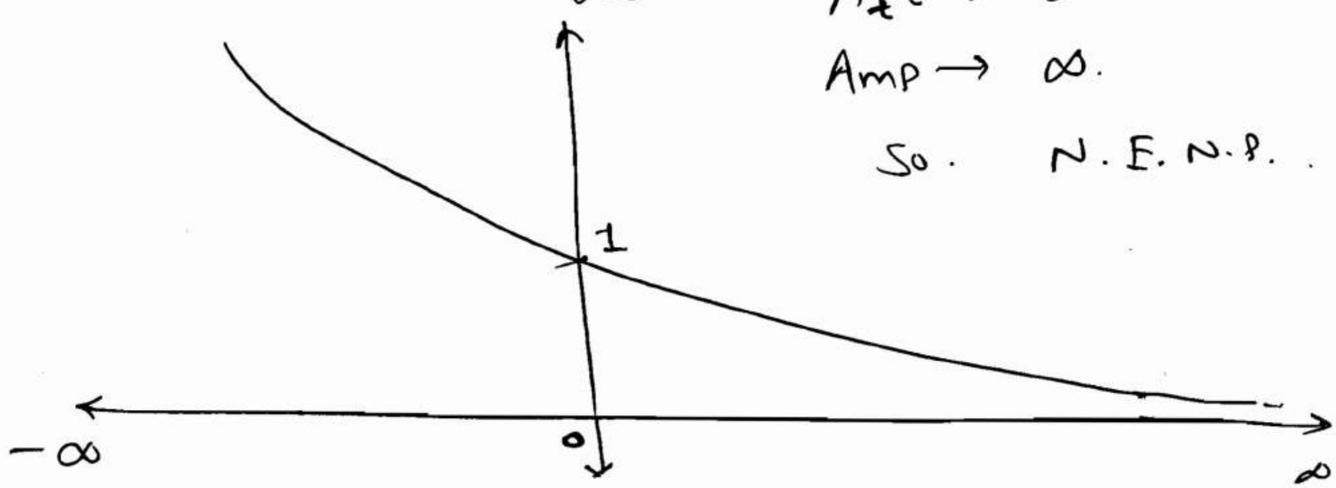
$\Rightarrow$



$t \to \infty \Rightarrow Amp \to \infty$

So, NENP.

* $\underline{y(t) = e^{-3t}}$.




As $t \to -\infty$

Amp $\to \infty$.

So. N.E.N.P.

To make Energy signal,

amp at $\left.\begin{matrix} t \to -\infty \\ t \to +\infty \end{matrix}\right\}$ & must be zero.

Note:

=> $\text{Cont}^n$ Impulse function is N.E.N.P. signal,

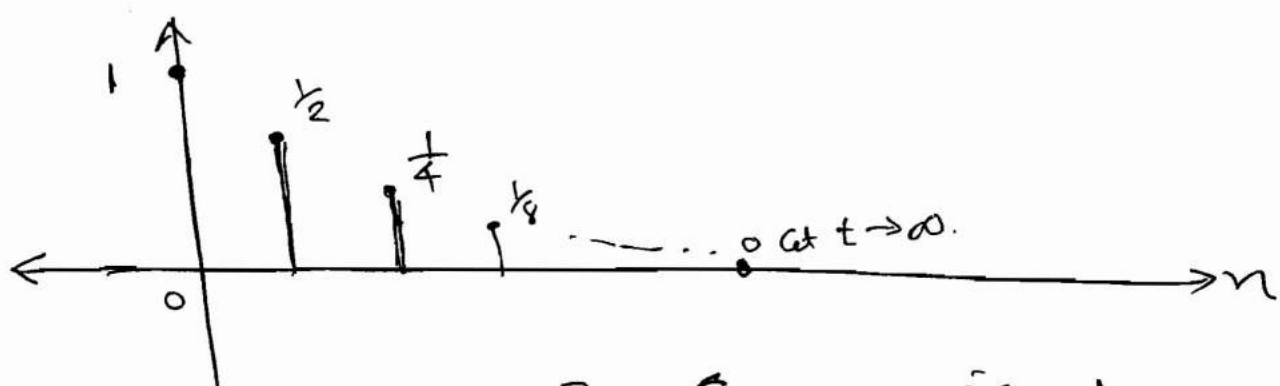
=> Discrete impulse is Energy signal.

* Discrete Signal:

=> $x[n] = \alpha^n \cdot u[n]$.

(i) For $\underline{|\alpha| < 1}$. $\longrightarrow$ Energy.

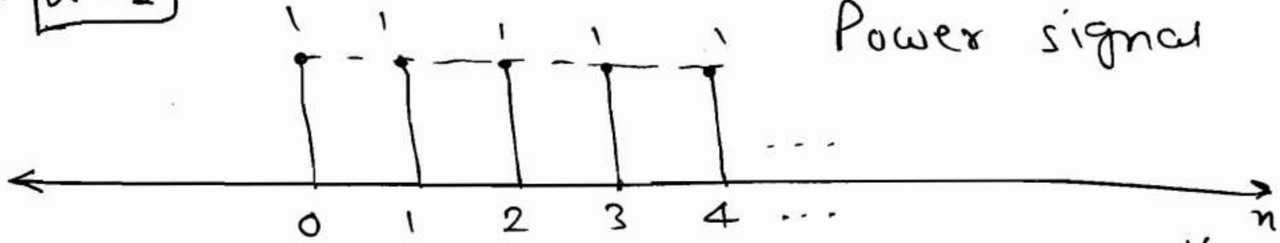
$x[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$.



(ii) $\underline{|\alpha| = 1}$ $\longrightarrow$ Power

$x[n] = u[n]$

* $\boxed{\alpha = 1}$



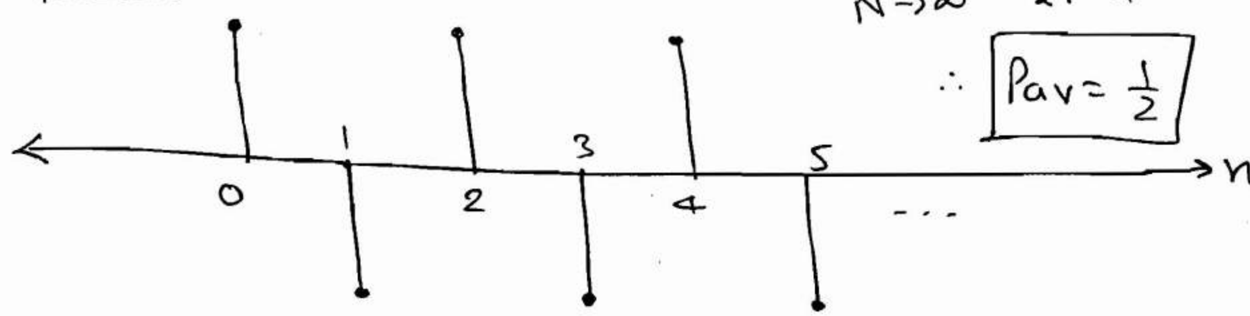

Power signal

$$P_{av} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

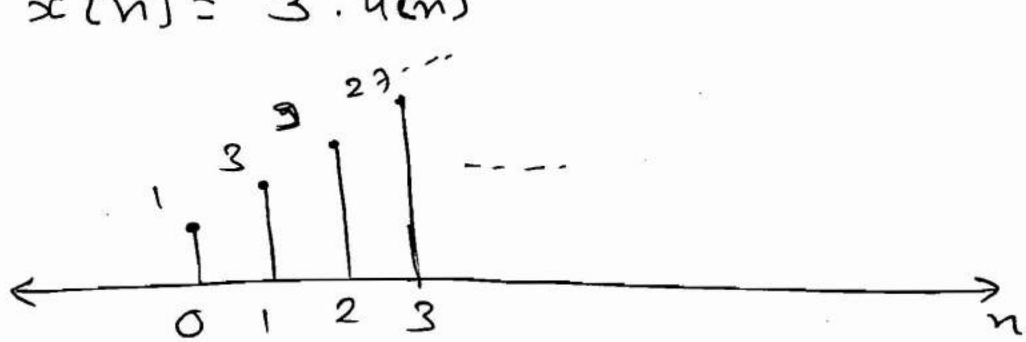
$$\therefore \boxed{P_{av} = \frac{1}{2}}$$

* $\boxed{\alpha = -1}$




(iii) $\underline{|\alpha| > 1}$ $\longrightarrow$ NENP.

$x[n] = 3^n . u(n)$




As $n \to \infty$

amp $\to \infty$.

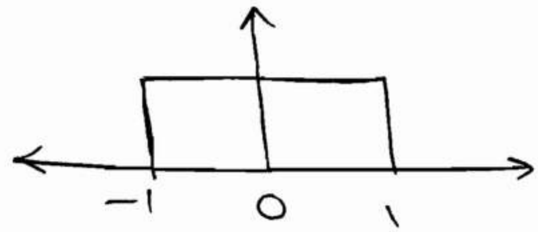
So, N.E.N.P.

## [2] Even & Odd signals.

### ① Even signals:

$\Rightarrow$ $\boxed{x(t) = x(-t).}$

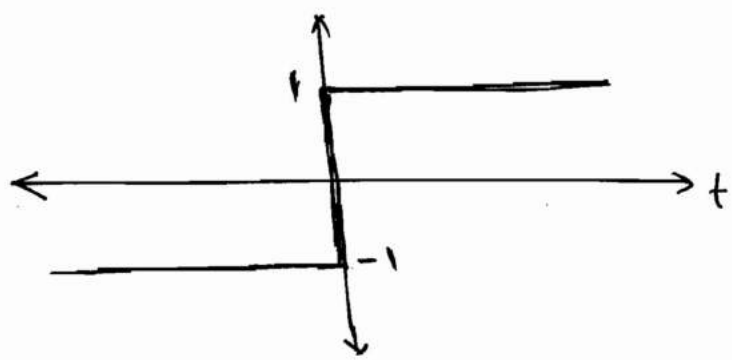
for e.g.




### ② Odd signals:

$\Rightarrow$ $\boxed{x(t) = -x(-t).}$

for. eg. signum fn.




$\Rightarrow$ For Conjugate signals,

$$x(t) = a(t) + j\,b(t).$$

Even conjugate : $\boxed{x(t) = x^{*}(-t).}$

e.g.: $x(t) = t^2 \cdot e^{j\omega t}$

$$x(-t) = t^2 \cdot e^{-j\omega t}$$

→ Odd Conjugate:

$$\boxed{x(t) = -x^*(-t).}$$

for. e.g.: $x(t) = t \cdot e^{j\omega t}$.

$\Rightarrow \quad x(t) = x_e(t) + x_o(t).$ —— (I).

Replace 't' by '-t'

$\therefore \; x(-t) = x_e(-t) + x_o(-t).$

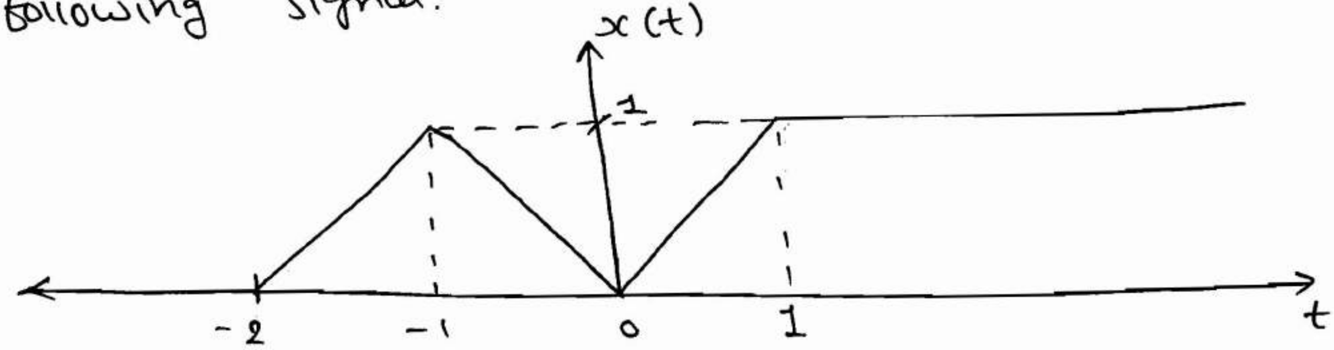
$\Rightarrow \; x(-t) = x_e(t) - x_o(-t)$ —— (II).

⇒ from eqn (I) & (II).

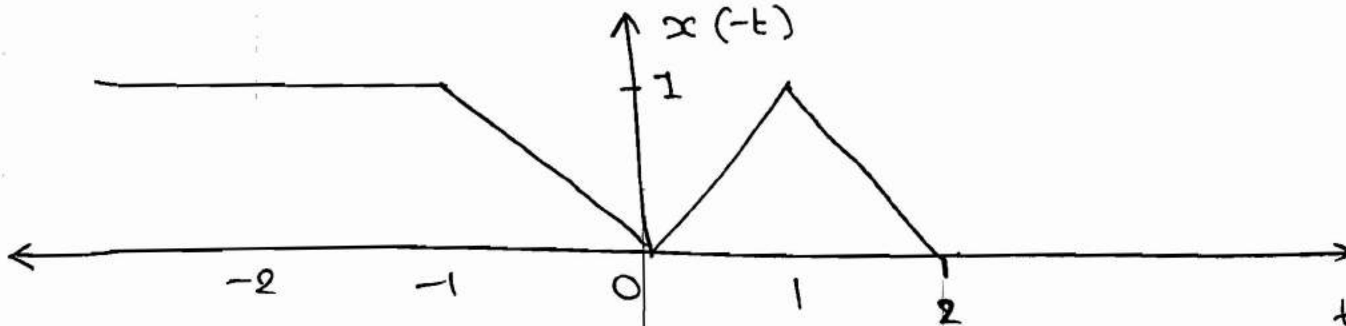
$$\Rightarrow \boxed{\begin{aligned} x_e(t) &= \frac{x(t) + x(-t)}{2} \\ x_o(t) &= \frac{x(t) - x(-t)}{2} \end{aligned}}$$

Q Find the odd and even signal for the following signal.

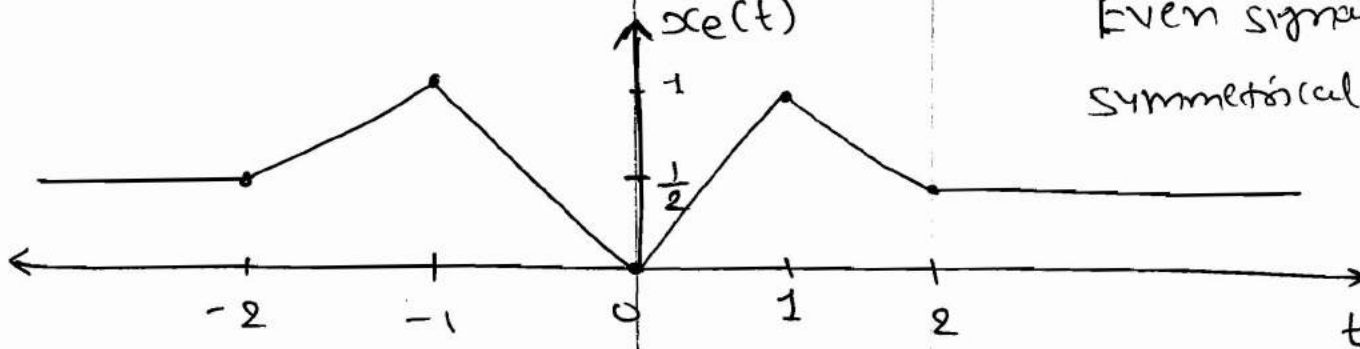
⇒

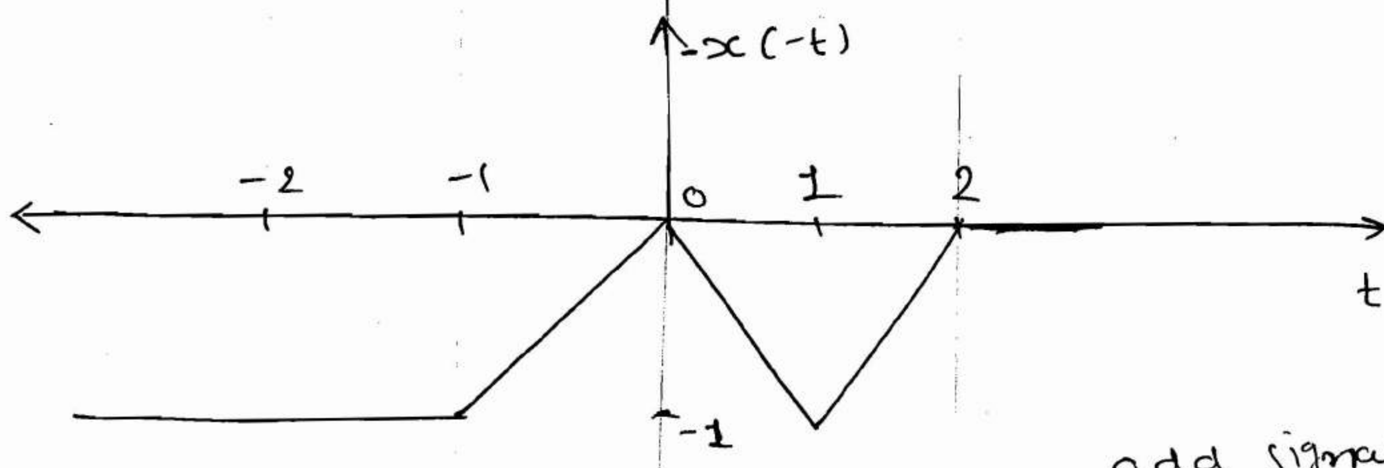



Sol$^n$:

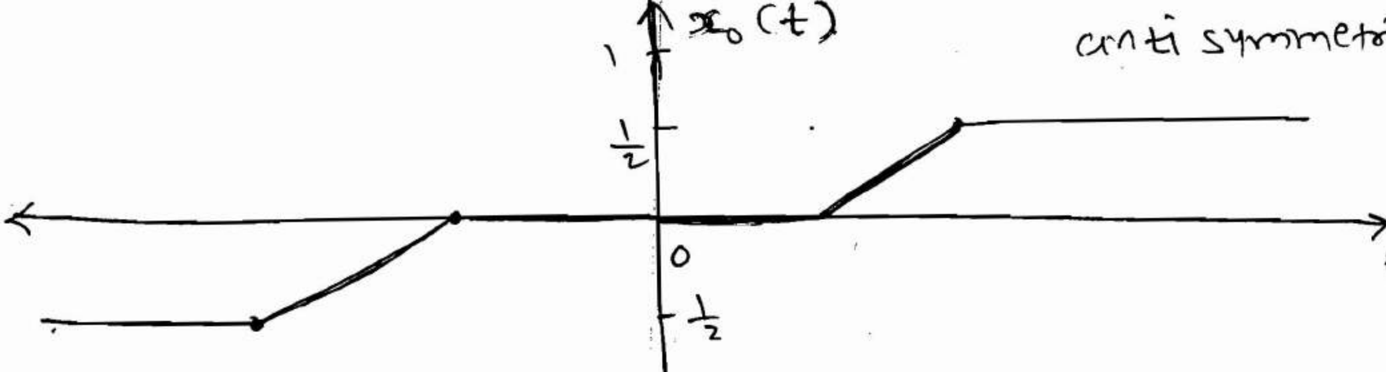



Even signal symmetrical.




odd signal anti symmetrical



$\Rightarrow$ Area under even signal:

$$\int_{-a}^{a} x_e(t)\,dt = 2\int_{0}^{a} x_e(t).dt.$$

$\Rightarrow$ Area under odd signal = zero.

$$\Rightarrow \boxed{\begin{array}{ll} e+e = O & e \cdot e = e \\ o+o = O & o \cdot o = O \\ e+o = \text{neno} & e \cdot O = o \\ \multicolumn{2}{l}{\Rightarrow \frac{d}{dt}(e) = o \ \ \& \ \ \frac{d}{dt}(o) = e.} \\ \multicolumn{2}{l}{\Rightarrow \int (e)dt = o \ \ \& \ \ \int (o)dt = e.} \end{array}}$$

$\Rightarrow E_{total} = E_{x(t)} + \cancel{[illegible]} = E_{even} + E_{odd}.$

$$E_{x(t)} = \int_{-\infty}^{+\infty} \left[ x_e(t) + x_o(t) \right]^2 dt.$$

$$= \int_{-\infty}^{\infty} x_e^2(t).\,dt + \int_{-\infty}^{\infty} x_o^2(t).dt + 2\int_{-\infty}^{\infty} \cancelto{0}{x_e(t).\,x_o(t)dt}$$

$$E_{total} = E_{even} + E_{odd}.$$

$\boxed{Q}$ The Conjugate Antisymmetric part of

$$x[n] = \{ \underset{\uparrow \atop -1}{1+j2},\ \underset{\uparrow \atop 0}{3},\ \underset{\uparrow \atop 1}{j4} \}.$$

Soln:

$$x_{oc}[n] = \frac{x[n] - x^*[-n]}{2}$$

$$x_{oc}[-1] = \frac{x[-1] - x^*[1]}{2}$$

$$= \frac{1+j2 - \{-j4\}}{2}$$

$$x_{oc}[-1] = \frac{1+j6}{2}$$

similarly,
$$x_{oc}[0] = \frac{x[0] - x^*[0]}{2} = \frac{3-3}{2} = 0.$$

$$x_{oc}[1] = \frac{x[1] - x^*[-1]}{2} = \frac{j4 - 1 + j2}{2}$$

$$x_{oc}[1] = \frac{-1+j6}{2}.$$

$$\Rightarrow x_{oc}[n] = \left\{ \frac{1+j6}{2},\ 0,\ \frac{-1+j6}{2} \right\}.$$

[3] Periodic & Non-Periodic signal.

$\Rightarrow$ Harmonics : $n\omega_0$

$$x(t) = x(t+T)$$

$T \rightarrow$ Least period of $x(t)$.

* Steps for finding time period of $x_1(t) + x_2(t) + x_3(t) + \ldots$

① $T_1, T_2, T_3, \ldots$

② $T_1/T_2,\ T_1/T_3,\ T_1/T_4, \ldots$

③ If the ratios of second step is rational no. then the overall signal is periodic.

④ L.C.M. of Denominators of $2^{nd}$ step.

⑤ $T = (\text{L.C.M.})\, T_1$.

$\boxed{Q}$ $y_1(t) = \cos 50\pi t + \sin 60\pi t$.

Soln: ① $\omega_1 = 50\pi$

$\therefore \dfrac{2\pi}{T_1} = 50\pi$

$T_1 = \dfrac{1}{25}$

$\omega_2 = 60\pi$

$\dfrac{2\pi}{T_2} = 60\pi$

$T_2 = \dfrac{1}{30}$.

② $\dfrac{T_1}{T_2} = \dfrac{30}{25} = 6/5$

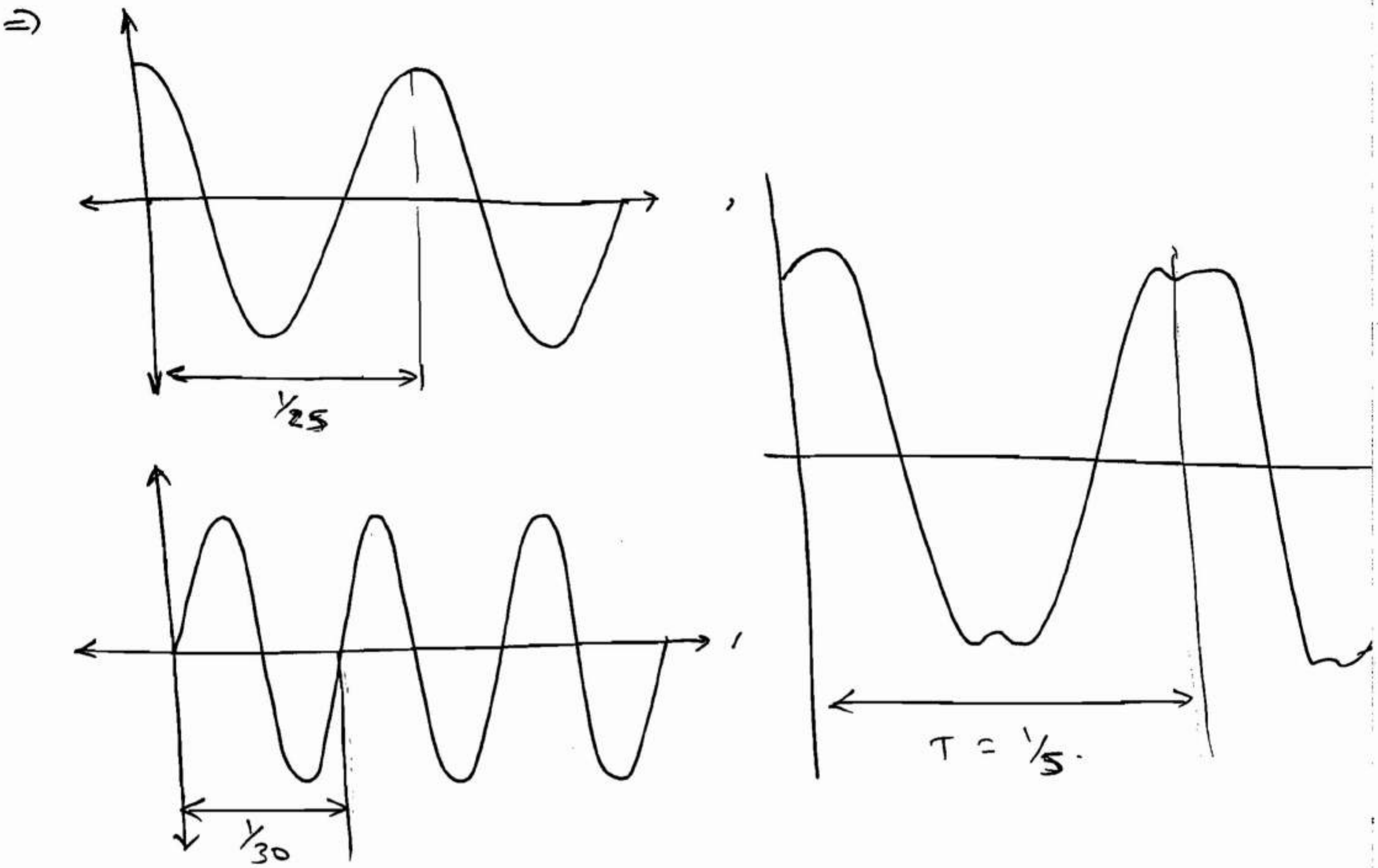
③ valid ✓

④ L.C.M. = 5.

⑤ $T = (\text{L.C.M.}) \times T_1$

$= 5 \times \dfrac{1}{25}$

⇒




* Alternative.

⇒ G.C.D. (Greatest common factor).

$50\pi \Rightarrow 5\,(10\pi)$.

$60\pi \Rightarrow 6\,(10\pi)$.

$\therefore \quad \omega_0 = 10\pi$

$$\frac{2\pi}{T} = 10\pi \Rightarrow \boxed{T = \frac{1}{5}}.$$

③ $y(t) = e^{j\frac{5\pi t}{6}} + e^{j\frac{\pi t}{3}}$.

Soln:

$$y(t) = e^{j\frac{5\pi t}{6}} + e^{j\frac{2\pi t}{6}}$$

$$\therefore \quad \mathrm{GCD}\left(\frac{5\pi}{6}, \frac{2\pi}{6}\right).$$

$$= \frac{\pi}{6} = \omega_0 \Rightarrow \frac{2\pi}{T} = \frac{\pi}{6} \Rightarrow \boxed{T=12}$$

④ $y(t) = \sin\left(\frac{2\pi t}{3}\right) \cdot \cos\left(\frac{4\pi t}{5}\right)$.

Solⁿ:

$y(t) = \frac{1}{2} \cdot 2\sin\left(\frac{2\pi t}{3}\right) \cdot \cos\left(\frac{4\pi t}{5}\right)$.

$y(t) = \frac{1}{2}\sin\left(\frac{22\pi t}{15}\right) - \frac{1}{2}\sin\left(\frac{2\pi}{15}\right)$.

$\therefore \quad GCD\left(\frac{22\pi}{15}, \frac{2\pi}{15}\right)$.

$= \frac{2\pi}{15} = \omega_0$

$\Rightarrow \frac{2\pi}{T} = \frac{2\pi}{15}$

$\Rightarrow \boxed{T = 15}$

⑤ $y(t) = \underset{\substack{\downarrow \\ T_1 = 1.08}}{y_1(t)} + \underset{\substack{\downarrow \\ T_2 = 3.6}}{y_2(t)} + \underset{\substack{\downarrow \\ T_3 = 2.205}}{y_3(t)}$.

⑥ $x(t) = \cos 13\pi t + \sin 17t$.

Solⁿ:

$T_1 = \frac{2}{13}, \quad T_2 = \frac{2\pi}{17}$.

So, $\frac{T_1}{T_2} = \underset{\substack{\uparrow \\ \text{irrational}}}{\frac{17}{13\pi}}$

So, Non-periodic.

⑦ $y(t) = j e^{j10t}$

Solⁿ:

$y(t) = e^{j\frac{\pi}{2}} \, e^{j10t} = e^{j\left(\frac{\pi}{2} + 10t\right)}$

$\Rightarrow$ Additional phase angle of $90^\circ$

$$\Rightarrow \quad j = e^{j\pi/2}$$

$\Rightarrow$ It never changes periodicity.

$$\omega_0 = 10 \Rightarrow T = \frac{2\pi}{\omega_0} = \pi/5.$$

Proof:

$$y\left(t + \frac{\pi}{5}\right) = j \cdot e^{j\left(10t + \frac{\pi}{5}\right)}.$$

$$= j \cdot e^{j10t} \cdot e^{j2\pi}$$

$$= j \cdot e^{j10t} \cdot (\cos 2\pi + j\sin 2\pi)$$

$$= j \cdot e^{j10t}$$

$$\therefore y\left(t + \frac{\pi}{5}\right) = y(t)$$

(7) $x(t) = e^{-(2+j3)t}$

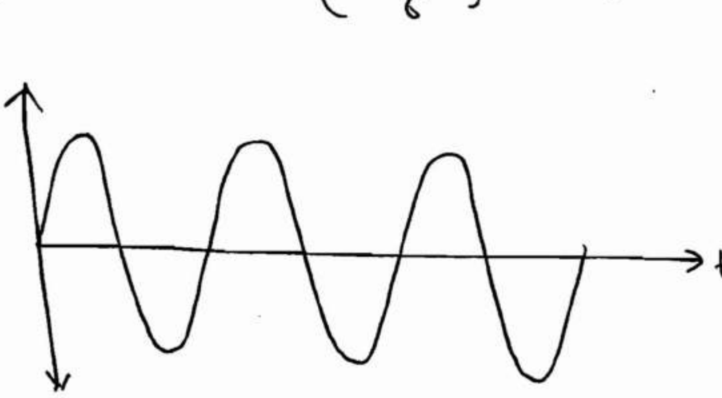
Soln:

$$x(t) = e^{-2t} \cdot e^{j3t}.$$

Non-periodic     periodic.

(8) $y(t) = \sin\left(\frac{\pi t}{6}\right) \cdot u(t).$

Soln:




$\Rightarrow$ Non-Periodic

⑨ $x(t) = \mathcal{E}v\{\cos 3\pi t \cdot u(t)\}$.

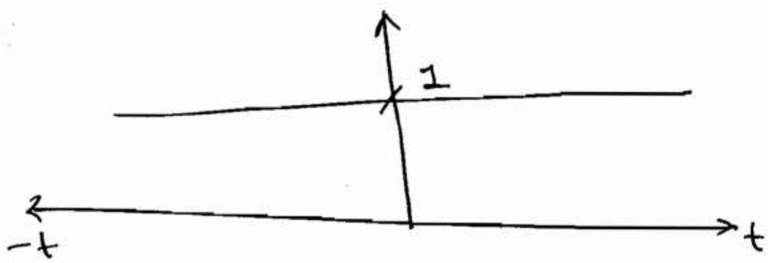
Sol$^n$:

$$x_e(t) = \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi(-t) \cdot u(-t)}{2}$$

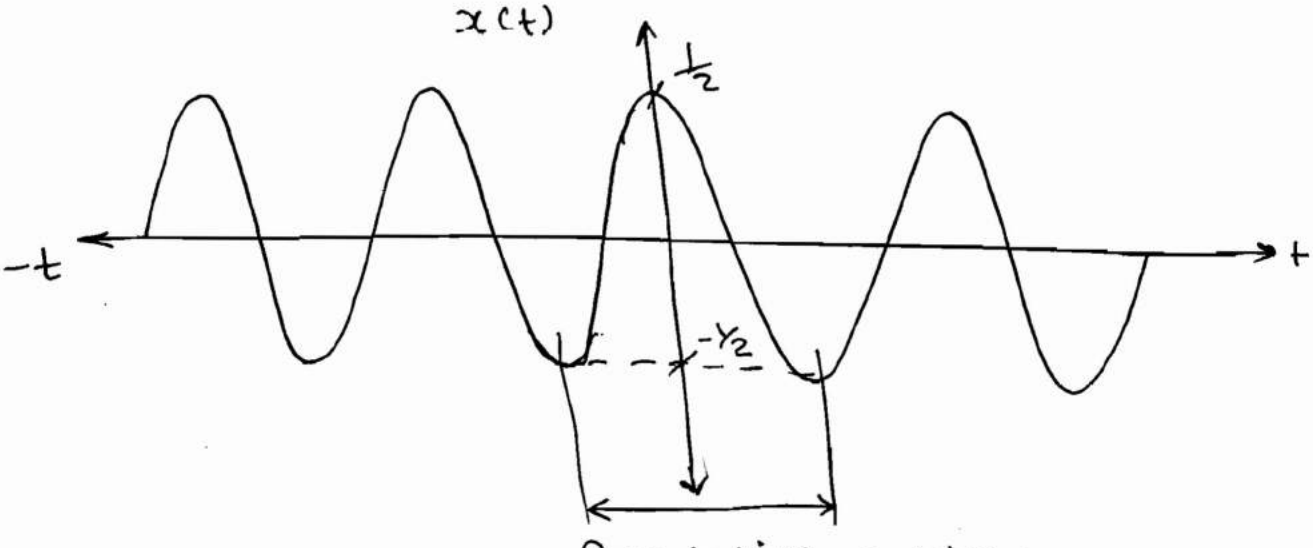
$$= \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi t \cdot u(-t)}{2}$$

$$\therefore \quad x(t) = \frac{\cos 3\pi t}{2}\,[u(t) + u(-t)].$$

$u(t) + u(-t) \Rightarrow$




$\Rightarrow$



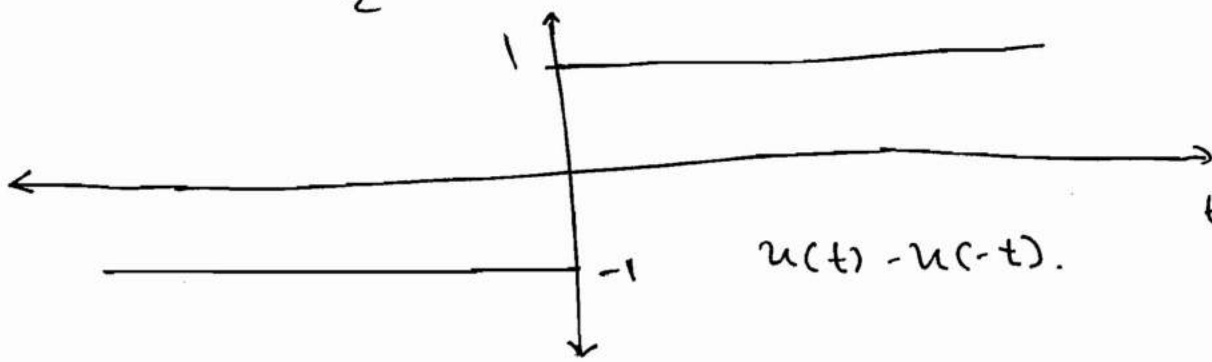

$$\Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 3\pi$$

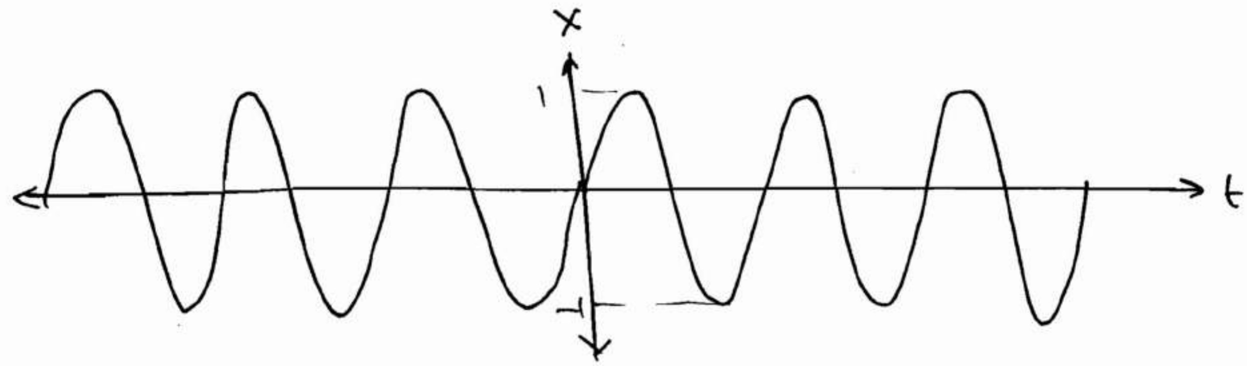
$$\Rightarrow \quad \boxed{T = \frac{2}{3}}$$

$*$ $x(t) = \mathcal{E}v\{\sin 3\pi t \cdot u(t)\}$.

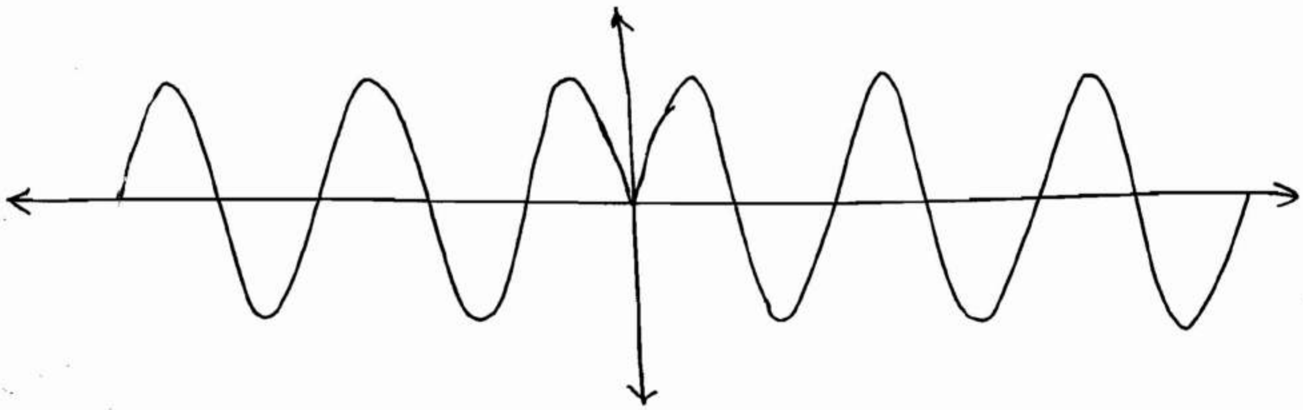
Sol$^n$: $x(t) = \sin 3\pi t \cdot u(t) + \sin 3\pi(-t) \cdot u(-t)$

$\Rightarrow x(t) = \frac{\sin 3\pi t}{2}\,[u(t) - u(-t)].$



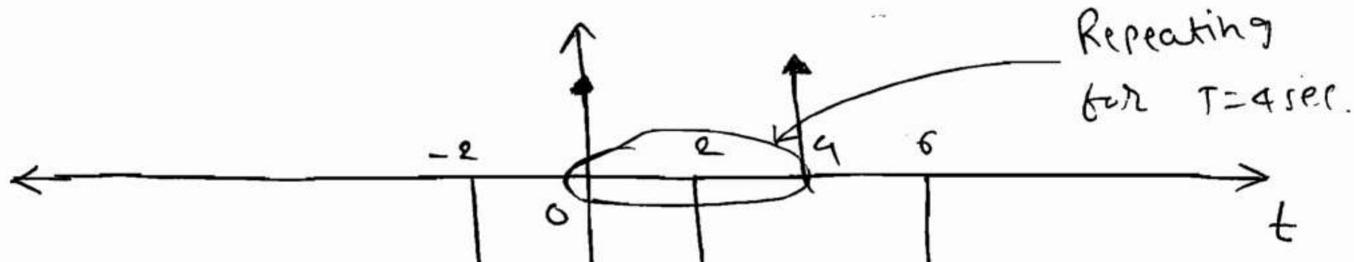

||



So, Non-periodic.

(11) $$y(t) = \sum_{k=-\infty}^{\infty} (-1)^k \cdot \delta(t-2k).$$

Sol$^n$: $y(t) = \cdots + \delta(t+2) + \delta(t) - \delta(t-2) + \delta(t-4) + \cdots$



★ $x[n] = \sin\left[\frac{3\pi n}{5}\right]$.

⇒ Continuous sinusoids and complex sinusoids are periodic for every value of $\omega_0$ whereas the equivalent discrete terms are periodic if $\frac{\omega_0}{2\pi}$ is a Rational Number $\left(\frac{m}{N}\right)$.

---

⇒ To make N as the time period signal, m no. of full cycles continuous signal is repeated.

$$\Rightarrow \quad x[n] = x[n+N].$$

$$x[n] = \sin\omega_0 n.$$

$$\Rightarrow x[n+N] = \sin\omega_0(n+N).$$

$$= \sin.\omega_0 n.\underbrace{\cos\omega_0 N}_{1} + \cos\omega_0 n.\underbrace{\sin\omega_0 N}_{0}.$$

$$\Rightarrow \cos\omega_0 N = \cos 2\pi m = 1. \quad \text{for } m = 0, \pm1, \pm2, \ldots$$

$$\therefore \omega_0 N = 2\pi m.$$

$$\therefore \boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}.$$

$\rightarrow$ here, $\omega_0 = \frac{3\pi}{5}$.

$$\therefore \frac{\omega_0}{2\pi} = \frac{3}{10} = \frac{m}{N}$$

$\therefore$ Time period of Discrete signal $\boxed{N=10}$

(2) $x[n] = \cos\left[\frac{n}{6} + \frac{\pi}{4}\right]$.

Sol^n: here, $\omega_0 = \frac{1}{6}$.

$\therefore \frac{\omega_0}{2\pi} = \frac{1}{12\pi}$, so irrational

$\therefore$ Non-periodic.

(3) $x[n] = \sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{4}\right)$.

Sol^n: here, $\omega_{01} = \frac{\pi}{3}$, $\omega_{02} = \frac{\pi}{4}$.

$\therefore N_1 = \cancel{\tfrac{1}{8}}\ 6$, $N_2 = 8$.

$\therefore \boxed{N = \text{LCM}(N_1, N_2)}$

* * *

$\boxed{\begin{array}{l} t \rightarrow \text{LCM} \\ f \rightarrow \text{GCD}. \end{array}}$

$\Rightarrow N = \text{LCM}(6, 8)$.

$\boxed{N = 24}$.

$$\begin{array}{r|l} 2 & 6, 8 \\ \hline & 3, 4 \end{array}$$

So, LCM = $2 \times 3 \times 4$

(4) $x(t) = 2\cos(150\pi t + 45^\circ)$. , $f_s = 200\text{ Hz}$

Sol$^n$: convert cont$^n$ to discrete by

put $t = nT_s$.

$\therefore x[nT_s] = 2\cos(150\pi n \cdot T_s + 45^\circ)$.

$= 2\cos(150 \cdot \pi \cdot n \cdot \frac{1}{f_s} + 45^\circ)$.

$\therefore x[nT_s] = 2\cos(150 \cdot \pi \cdot n \times \frac{1}{200} + 45^\circ)$.

$\Rightarrow \quad \omega_0 = \frac{150\pi}{200}$.

$\therefore \cancel{2\pi} \quad \frac{\omega_0}{2\pi} = \frac{15}{20} \times \cancel{\pi} \times \frac{1}{2\cancel{\pi}} = 3/8$.

So, $\boxed{N = 8}$

(5) $x[n] = (j)^{n/2}$.

Sol$^n$: $x[n] = e^{j\frac{\pi}{2} \times \frac{n}{2}}$. $\quad (\because j = e^{j\frac{\pi}{2}})$.

$\therefore x[n] = e^{j\frac{n\pi}{4}}$.

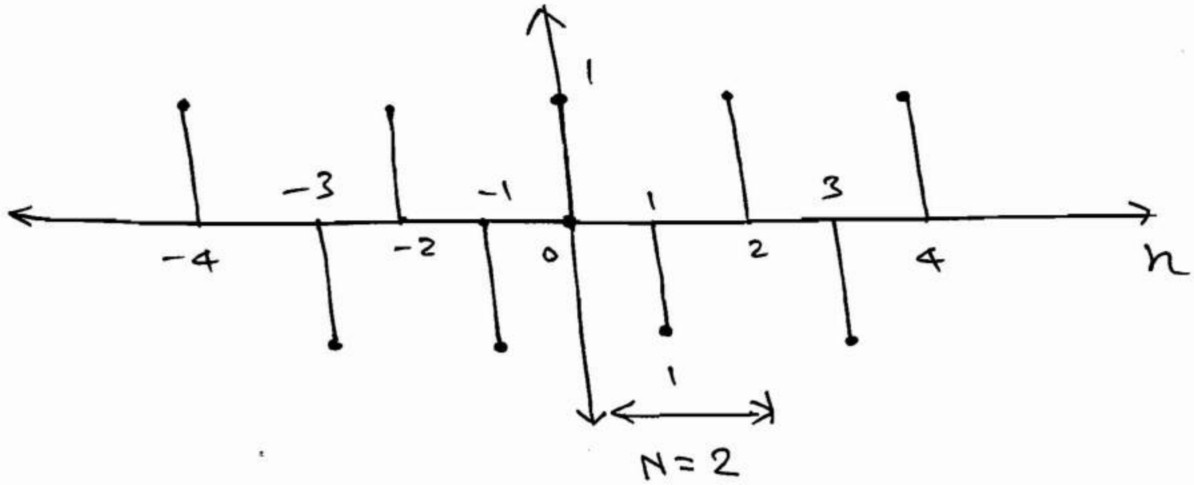
$\therefore \quad \omega_0 = \frac{\pi}{4}$.

$\therefore \frac{\omega_0}{2\pi} = \frac{\pi}{4} \times \frac{1}{2\pi} = \frac{1}{8}$.

So, $\boxed{N = 8.}$

(6) $x[n] = (-1)^{n}$

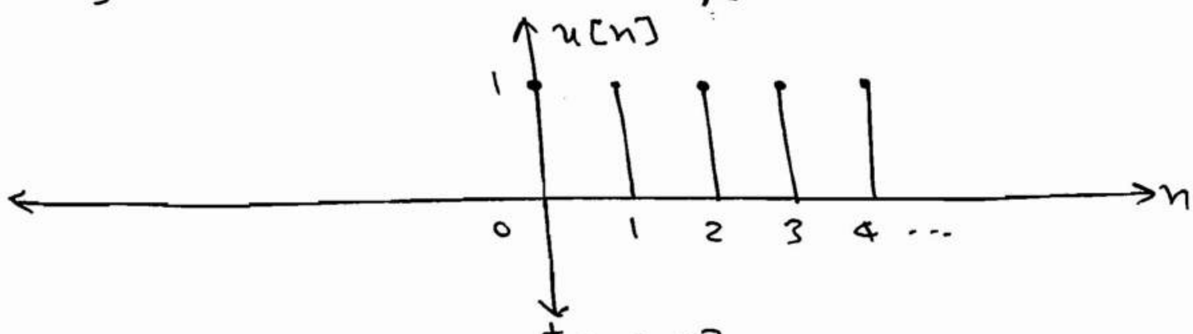
Sol$^n$:

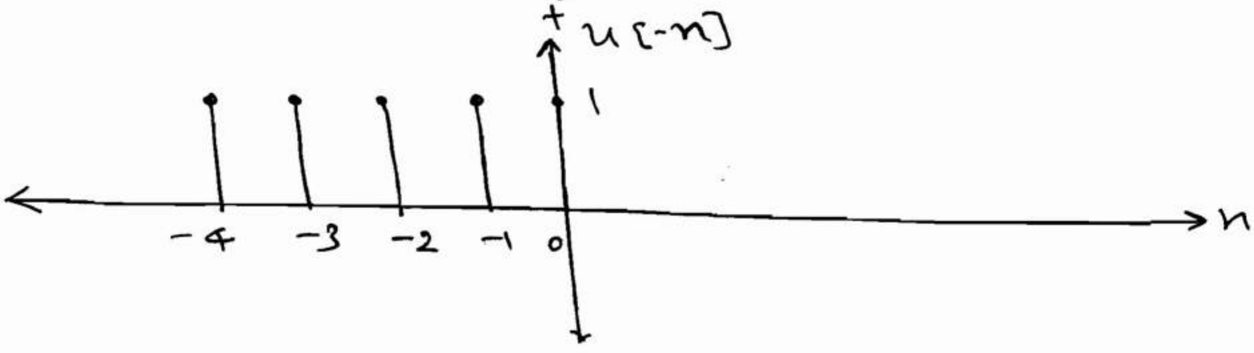


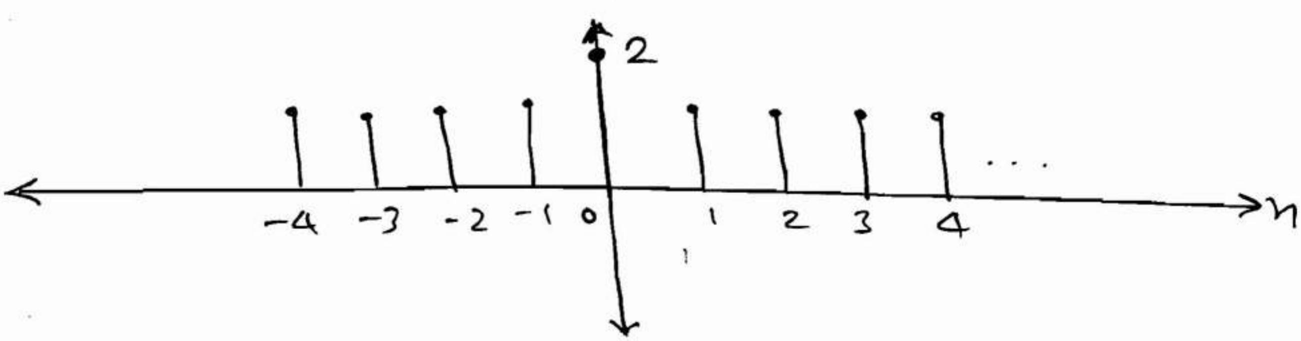

so, $\boxed{N=2}$

(7) $x[n] = u[n] + u[-n]$

Sol$^n$:



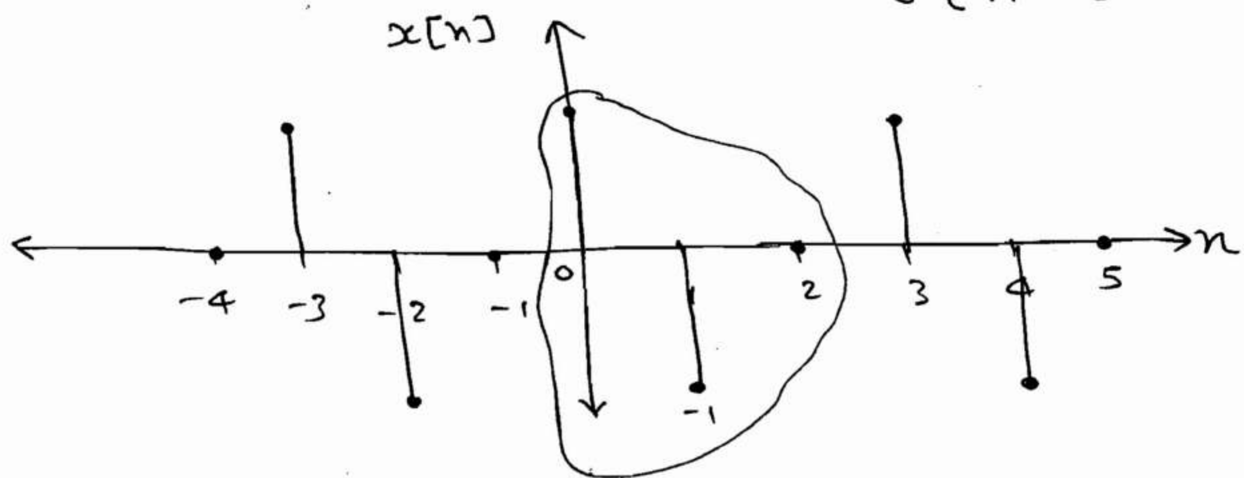




so, non - Periodic.

[8] $x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-3k] - \delta[n-1-3k].$

Sol$^n$: $x[n] = \ldots + \delta[n+3] - \delta[n+2] + \delta[n] - \delta[n-1] + \delta[n-3] - \delta[n-4] + \ldots$




$\Downarrow$

$\boxed{N=3}$

[4] Causal & Non-Causal signals:

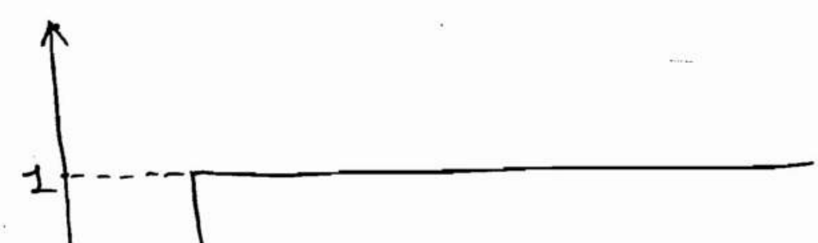
Sol$^n$: Causal: Defined for +ve time only.

e.g. $\Rightarrow t > 0$ (or) $n \geq 0$.

$\Rightarrow x(t) = 0 \ ; \ t < 0$

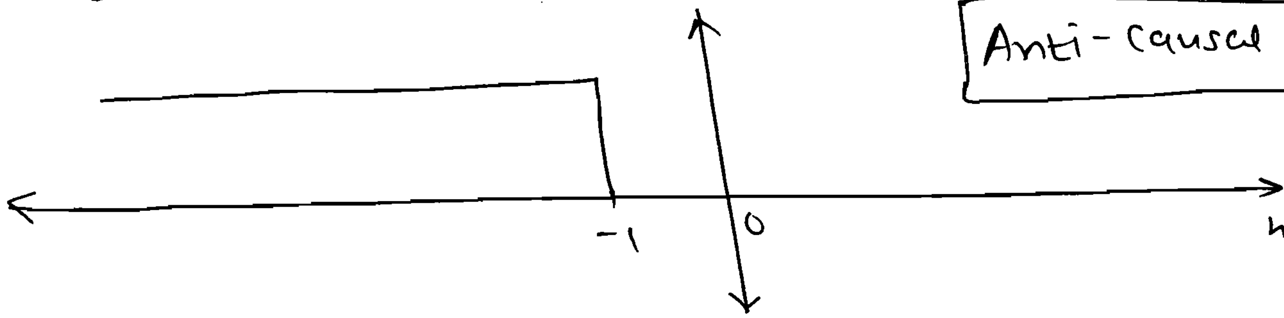
$x[n] = 0 \ ; \ n < 0.$

e.g. ① $x(t) = u(t-1)$.



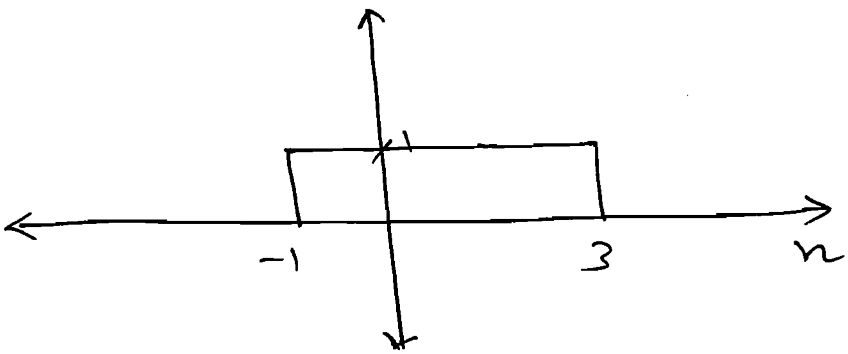

$\boxed{\text{Causal.}}$

② $u[-t-1] = u[-(t+1)]$.




$\boxed{\text{Anti-causal}}$

③

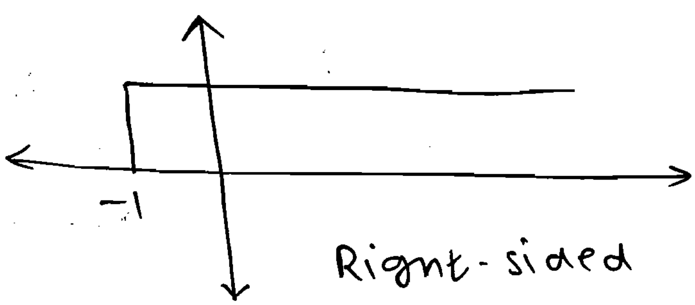



$\boxed{\text{Non-Causal}}$

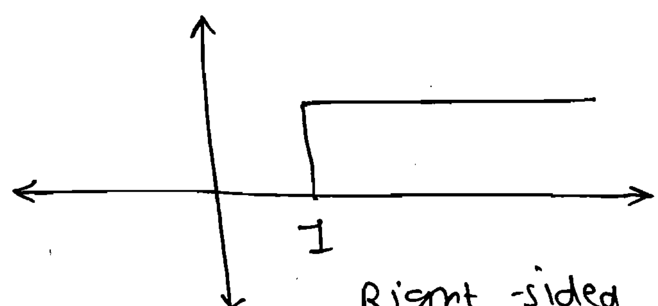
**Note:**

⇒ All Causal signals are Right sided signal, but all right sided signal may (or) may not be Causal signal.

⇒ Right-sided: $(t \geq t_0)$

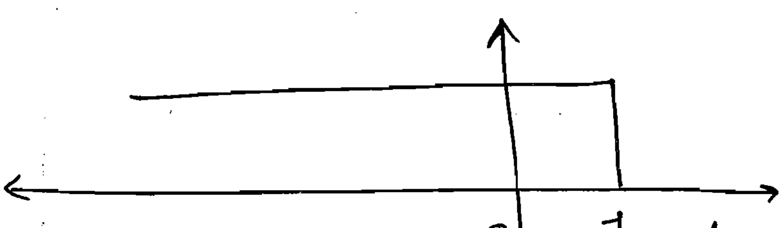


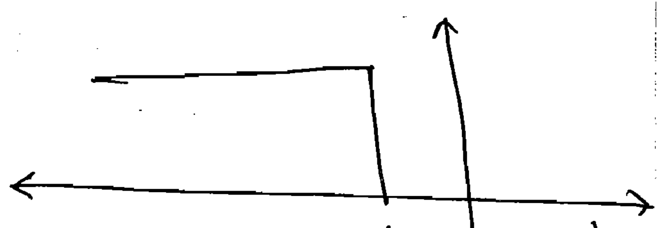

Right-sided
Non-Causal




Right-sided
Causal.

⇒ Left-sided: $(t \leq t_0)$.





→ Periodic and Random signals are Power signal.

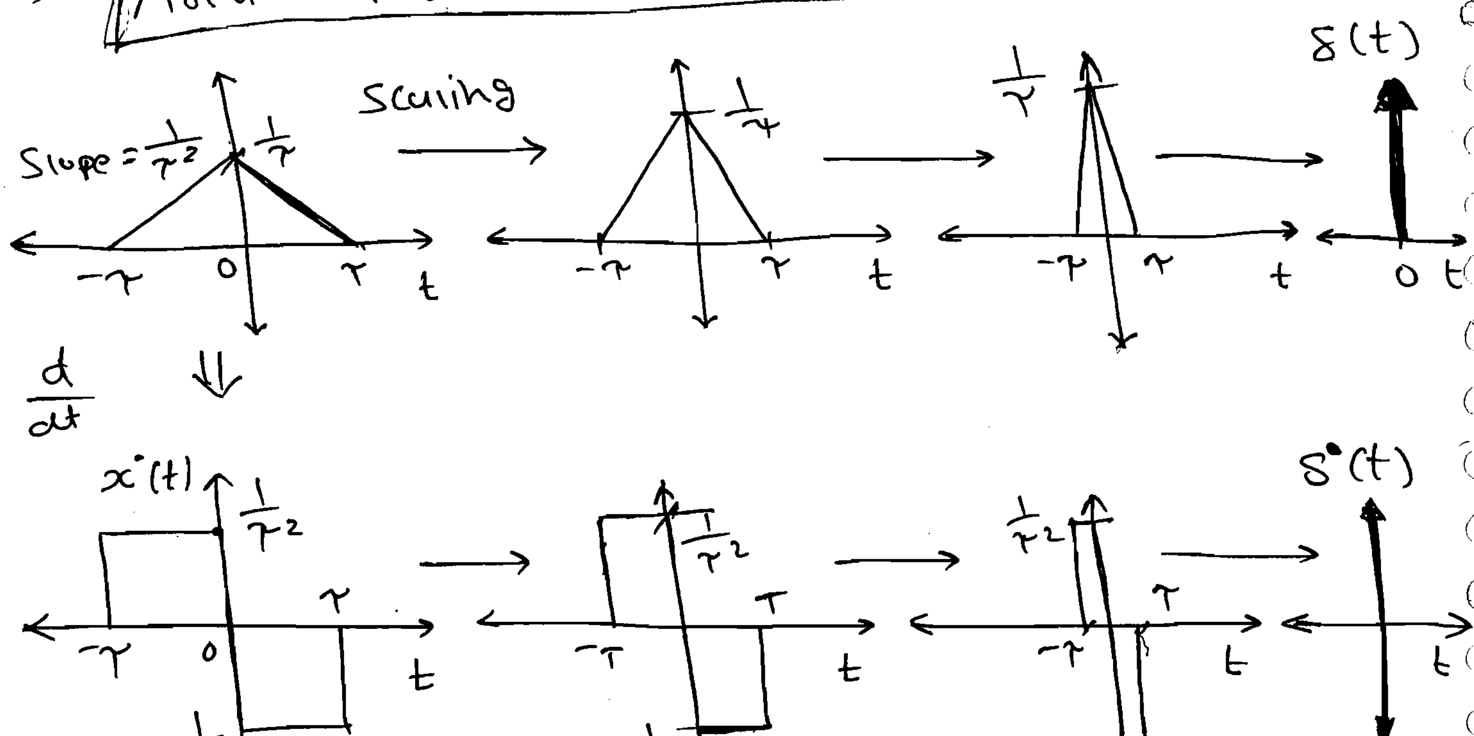
→ But Aperiodic and Deterministic signals may (or) may not be energy signal.

e.g. $u(t)$ → is non-periodic and it is power-signal.

## * Doublet:

⇒ By approximating a unit area triangular function we can generate unit impulse fn. In a similar manner by approximating derivative of triangular we can produce doublet. ($\dot{\delta}(t)$).

⇒ $\boxed{\text{Area under Doublet is zero.}}$



⇒ Sifting Property:

$$\Rightarrow \int_{t_1}^{t_2} x(t) \cdot \delta(t-t_0)\, dt = x(t_0) \quad ; \quad t_1 \le t_0 \le t_2.$$

$$\boxed{\Rightarrow \int_{t_1}^{t_2} x(t) \cdot \delta^{(n)}(t-t_0)\, dt = (-1)^n \left. \frac{d^n x(t)}{dt^n} \right|_{t=t_0}}$$

⇒ Derivative of Doublet is Triplet and we should consider parabolic curve instead of triangular curve since triplet is double derivative of impulse. Double derivative of Parabolic curve is unit step function.

**Q** $\displaystyle\int_0^{\infty} e^{-2t^2} \cdot \delta'(t-10)\, dt.$

**Soln:** $t_0 = 10$, $0 < 10 < \infty$.

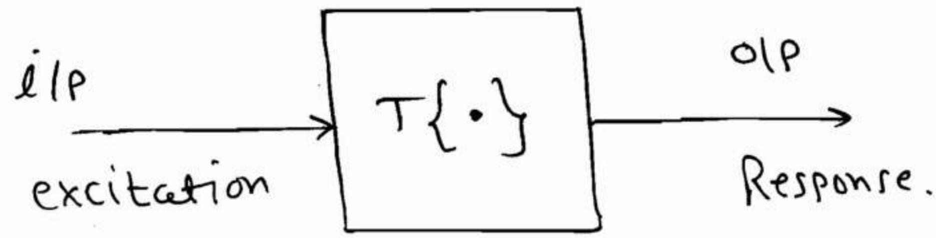
so,
$$= (-1)^n \cdot \left. \frac{d^n}{dt} x(t) \right|_{t=t_0}$$

$$= (-1)^1 \left. \frac{d}{dt} \cdot e^{-2t^2} \right|_{t=10}$$

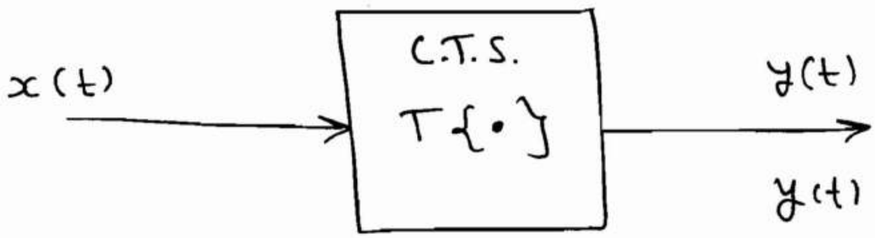
$$= - e^{-2t^2} \cdot (-4t) \Big|_{t=10}$$

# ☆ Systems :-

⇒




⇒




$y(t) = T\{x(t)\}.$

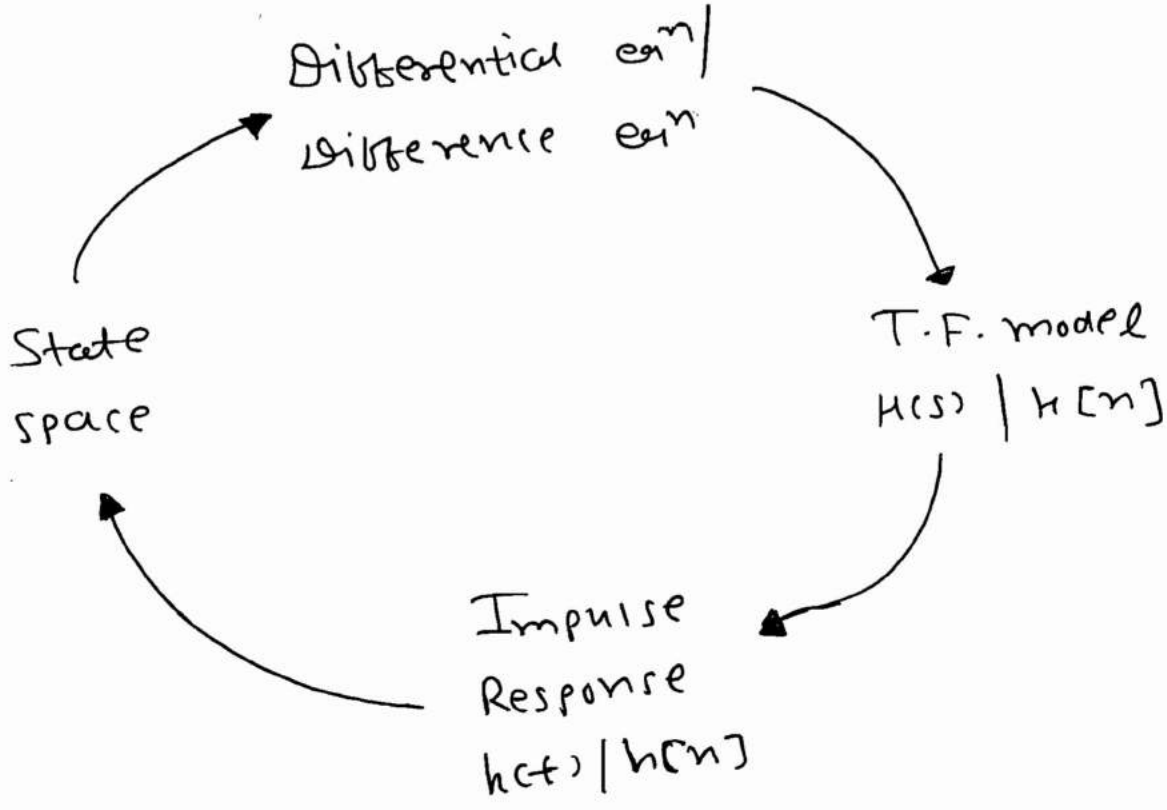
e.g. $y(t) = \ln x(t)$

$= x^2(t).$

⇒ Designing systems ⇒ Synthesis.

⇒ ~~An~~ Getting o/p ⇒ Analysis.

⇒ –



⇒ Linear, Stable, Invertible → Considering Amp.

⇒ T.I., static, Causal ⇒ Time.

## [1] Linear & Non-linear System:

⇒ Linear ⇒ Supperpossion = Additivity + Homoginity.

① **Additivity:**

$$x_1(t) \xrightarrow{T} y_1(t)$$

$$x_2(t) \xrightarrow{T} y_2(t).$$

$$\Rightarrow \quad x_1(t) + x_2(t) \xrightarrow{T} y_1(t) + y_2(t).$$

② **Scaling (Homoginity):**

$$\Rightarrow \quad \alpha x(t) \longrightarrow \alpha y(t).$$

($\alpha$ = need not be real; can be Imaginary).

e.g. $y(t) = x(t) \cdot x(t-3)$.

$$x_1(t) \longrightarrow y_1(t) = x_1(t) \cdot x_1(t-3).$$

$$x_2(t) \longrightarrow y_2(t) = x_2(t) \cdot x_2(t-3).$$

$$y(t) = (x_1(t) + x_2(t)) \longrightarrow (x_1(t) + x_2(t)).$$

$\boxed{Q}$ - 1  $y(t) = x(t) \cdot \cos 5t$.

<u>Sol^n:</u>

$$x_1(t) \Rightarrow y_1(t) = x_1(t) \cdot \cos 5t.$$

$$x_2(t) \Rightarrow y_2(t) = x_2(t) \cdot \cos 5t.$$

$$\Rightarrow x_1(t) + x_2(t) \Rightarrow y(t) = \left(x_1(t) + x_2(t)\right) \cos 5t$$

$$= x_1(t) \cdot \cos 5t + x_2(t) \cdot \cos 5t$$

$$y(t) = y_1(t) + y_2(t)$$

$\Rightarrow$ Linear.

<u>NOTE:</u>

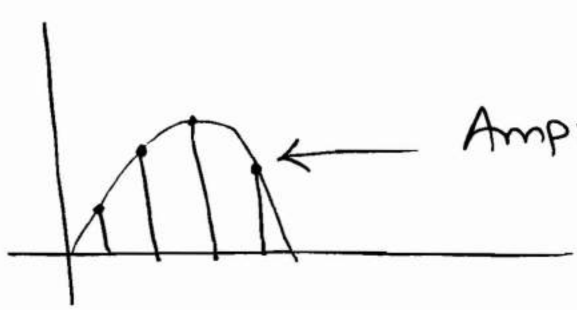
$\Rightarrow$ Unknown signal product make the system non-linear.

2  $y(t) = \int_{-\infty}^{t} x(\tau) \cdot d\tau$.

$\Rightarrow$ Linear.

3  $y(t) = \text{Sample}\{x(t)\}$.

$\Rightarrow$ System operation is sampling.



Amplitude is not changing so, Linear.

$\Rightarrow$ Sampling is a linear operation, but

③ $y(t) = \sin\{x(t)\}$.

$\downarrow$

Non-linear

**Soln:** $y_1(t) = \sin\{x_1(t)\}$ , $y_2(t) = \sin\{x_2(t)\}$.

$y(t) \Rightarrow \sin\{x_1(t) + x_2(t)\}$

$\neq \sin\{x_1(t)\} + \sin\{x_2(t)\}$.

**Note:** When o/p is trignomatric function System is always Non-Linear. ✓

④ $y(t) = |m(t)|$

**Soln:** Non-linear $\Rightarrow$ All +ve values becomes -ve values

o/p due to $-5x(t) = |-5x(t)| = 5|x(t)|$.

$\alpha y(t) = -5|x(t)| \neq 5|x(t)|$

So, Non-linear.

**Note:** Modulas of i/p makes the system Non-linear. because all $\bar{+}$ve values becomes +ve value. ✓

⑤ $y(t) = 3x(t) + 5$.

**Soln:** Non-linear.

o/p due to $\alpha y(t) = \alpha(3x(t) + 5)$.

o/p due to $\alpha x(t)$

$$= 3\alpha x(t) + 5.$$

So, Non-linear.

Note: Addition of Constant makes the System Non-linear.

→ This system is also called as "Incremental Linear" since o/p is increasing because of constant term.

[6] $y[n] = 2^{x[n]}$.

Sol$^n$: $y_1[n] = 2^{x_1[n]}$, $y_2[n] = 2^{x_2[n]}$.

∴ o/p due to $x_1[n] + x_2[n]$

$$= 2^{x_1[n]} \cdot 2^{x_2[n]}$$

$$\neq 2^{x_1[n]} + 2^{x_2[n]}.$$

So, Non-linear.

Note: Exponential terms makes the system Non-Linear.

[7] $y[n] = \text{sgn}[x[n]]$.

Sol$^n$: $y[n] = 1; \quad x[n] > 0.$

$$= -1 \; ; \quad x[n] < 0.$$

$\Rightarrow$ For signam we are always maintaing Const. amp of $\pm 1$. So it is non-linear.

<u>Note:</u> When o/p signal is input dependent then it is always Non-Linear.

[8] $y[n] = x^*[n]$.

<u>Sol$^n$:</u> o/p due to $(2+j3) = \{(2+j3)\ x[n]\}^*$.

$$= (2-j3) \cdot x[n]$$

$$\alpha\, x[n] = (2+j3) \cdot x[n] \quad \neq \text{Non-linear.}$$

[9] $y[n] = \text{Real}[x(n)]$.

<u>Sol$^n$:</u> o/p due to $2+j3 = \text{Real}\{(2+j3) \cdot x[n]\}$.

$$= 2 \cdot \text{Real}[x[n]].$$

$$(2+j3)\ \text{Real}\{x[n]\} \neq 2\,\text{Real}\,(x[n]).$$

So, Non-linear.

<u>Note:</u> $\rightarrow$ Real, Complex part and Conjugate all are Non-linear.

[10] $y[n] = \text{median}[x(n)]$.

<u>Sol$^n$:</u> $O/P_1 = \{1, 2, 3, 4, 5\} = 3.$ $\quad \overset{3+1}{4}.$

$OP_1 + OP_2 = \{1, 1, 1, 1, 2, 2, 2, 3, 4, 5\}.$

$$\frac{2+2}{2} = 2.$$

So, Non-linear.

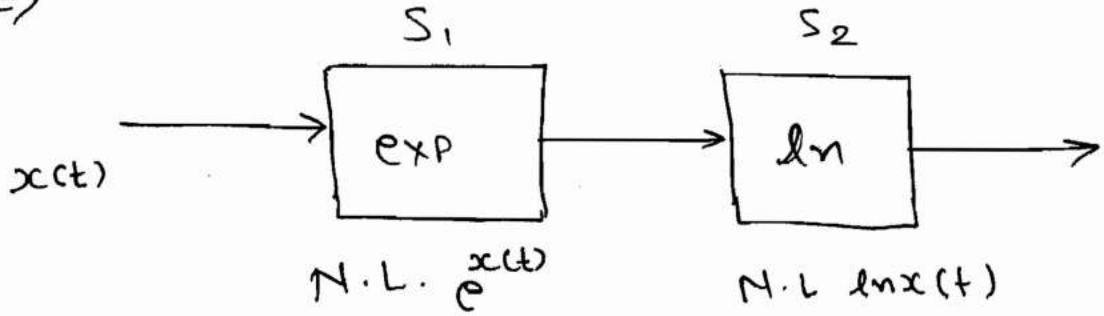
<u>Note</u>: → Quantization is Non-linear ope. &

Sampling is Linear operation.

→ Phase - plane Analysis → Non-linear.

* <u>Two - System Connected in Series.</u>

⇒




$y(t) = x(t).$

⇓

Linear.

⇒


$y(t)$

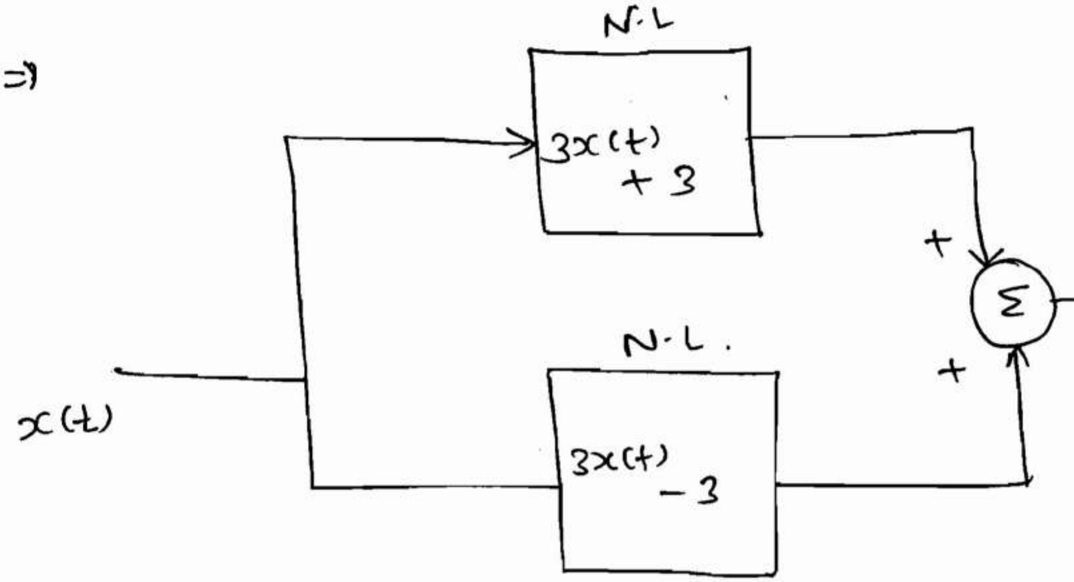
⇓

Non-linear.

⇒ when both $S_1$ & $S_2$ are opposite to each other then it becomes Linear.

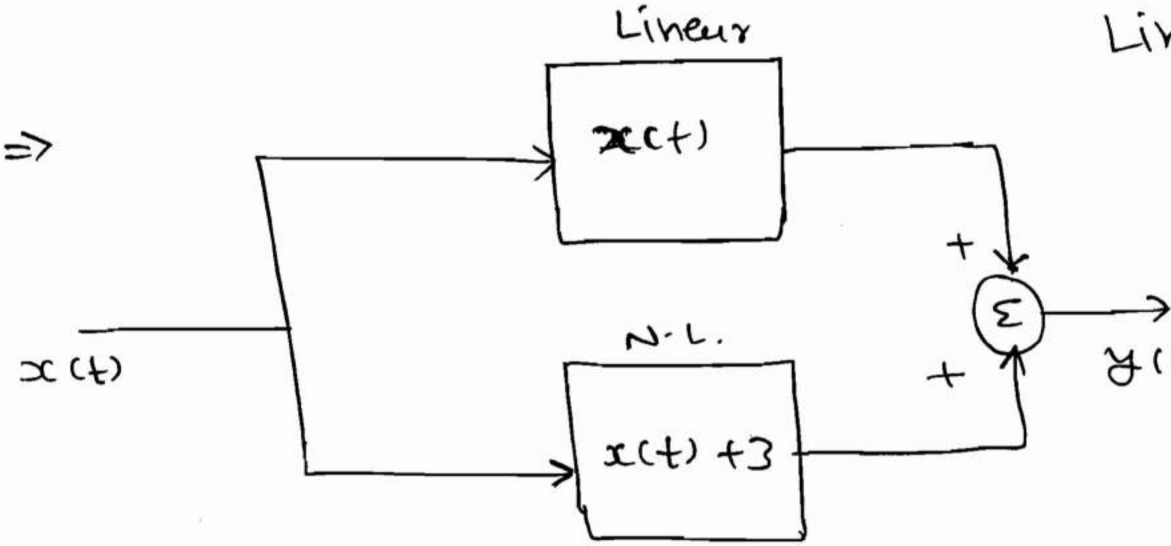
## * Two - Systems Connected in Parallel:

=>




$y(t) = 3x(t) + 3$
$\quad\;\; 3x(t) - 3$
$= 6x(t)$
$\Downarrow$
Linear.

=>



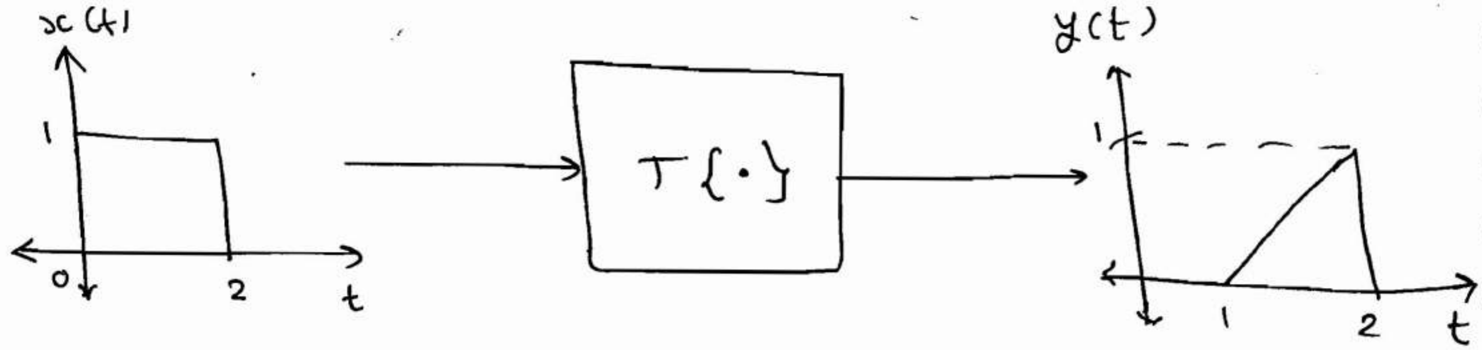

$y(t) = 2x(t) + 3$
$\Downarrow$
Non-linear.

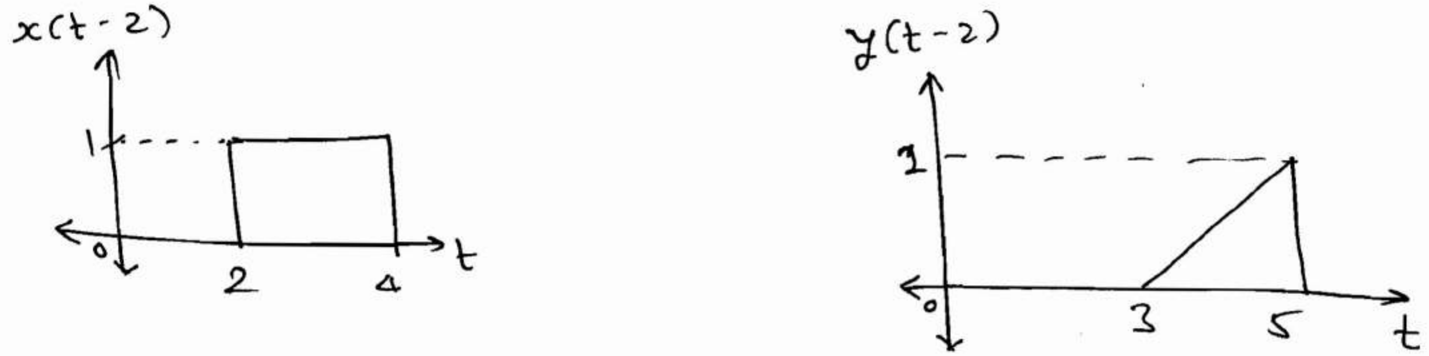
## [2] Time Invariant | Shift Invariant :-

=> A System is T.I. if i/p o/p char. do not change with time.

=> If $x(t) \longrightarrow y(t)$

$x(t - t_0) \longrightarrow y(t - t_0).$




e.g.

$\boxed{a}$ ① $y(t) = t \cdot x(t)$.

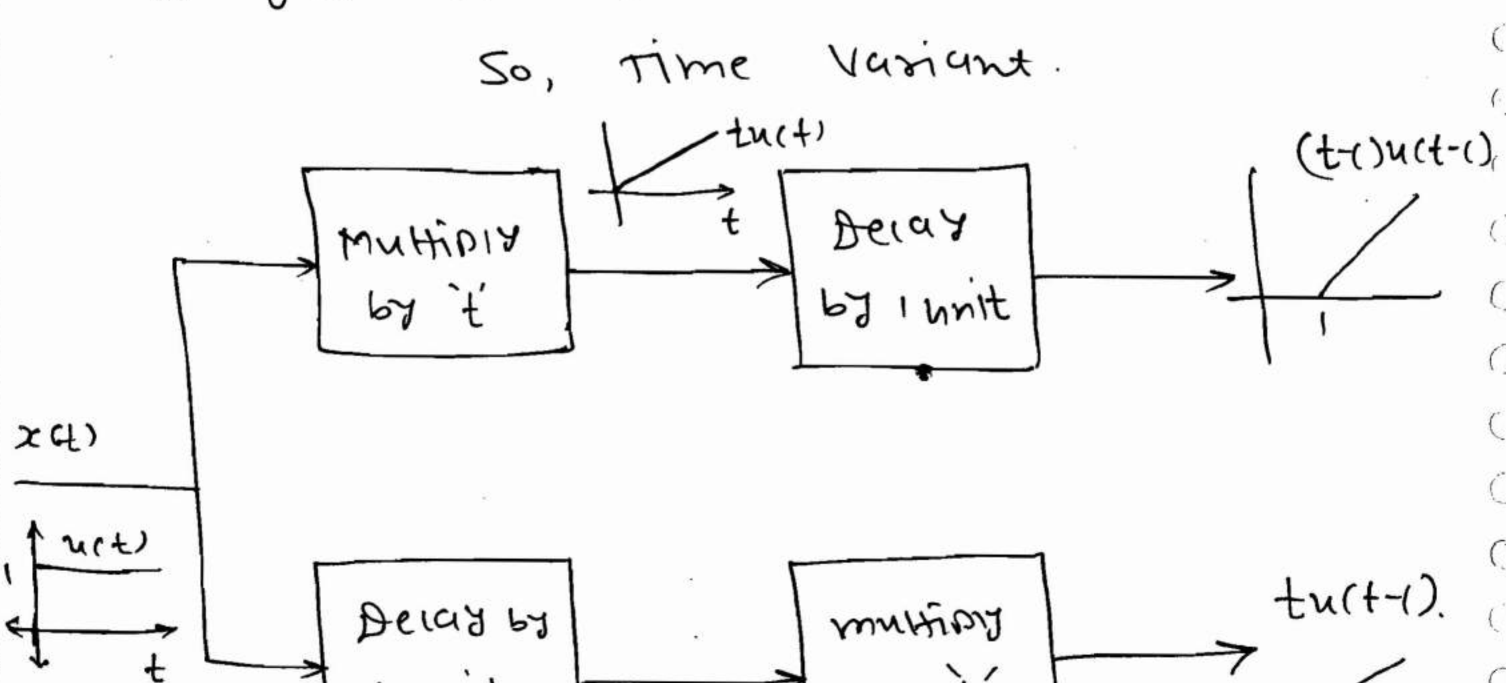
<u>Soln</u>: o/p due to delay input

$$y_1(t) = t\, x(t-t_0).$$

Delay o/p by '$t_0$' $\Rightarrow t \Rightarrow t - t_0$.

$$\therefore\ y(t-t_0) = (t-t_0)\cdot x(t-t_0) \neq y_1(t)$$

So, Time Variant.



② $y(t) = e^{-x(t)}$.

$\underline{\underline{Sol^n}}$: time Invariant

$$y(t) = e^{-x(t-t_0)}$$

$$y(t-t_0) = e^{-x(t-t_0)}$$

③ $y(t) = x^2(t)$.

$\underline{\underline{Sol^n}}$:

$$y(t) = x^2(t-t_0)$$

$$y(t-t_0) = x^2(t-t_0)$$

So, Time Invariant.

④ $y(t) = x(2t)$.

$\underline{\underline{Sol^n}}$: I/P : $y(t) = x(2t - t_0)$.

O/P : $y(t-t_0) = x(2(t-t_0)) = x(2t-2t_0)$.

So, Time Variant.

⑤ $y(t) = x(-t)$.

$\underline{\underline{Sol^n}}$:

I/P : $y(t) = x(-t-t_0)$.

O/P : $y(t-t_0) = x(-(t-t_0)) = x(-t+t_0)$.

So, Time Variant.

$\Rightarrow$ In General,

IMP

$$\boxed{x(\alpha t) = T.V.}$$

$\boxed{6}$ $y(t) = \dfrac{d\,x(t)}{dt}$.

Sol$^n$:

$$y(t - t_0) = \frac{d\,x(t-t_0)}{dt\,(t-t_0)} = \frac{d\,x(t-t_0)}{dt}$$

$\left(\because \ \dfrac{d}{dt}(t_0) = 0 \right.$ $\uparrow$ constant$\left.\right)$.

So, Time Invariant.

$\boxed{7}$ $y(t) = \begin{cases} x(t) & ; \ x(t) > 0 \\ 0 & ; \ x(t) < 0. \end{cases}$

Sol$^n$:

Amp. dependent $\rightarrow$ Non-linear.

Independent of time $\longrightarrow$ T.I.

$\boxed{8}$ $y(t) = \begin{cases} x(t) & ; \ t > 0 \\ 0 & ; \ t < 0. \end{cases}$

Sol$^n$:

Amp. Independent $\Rightarrow$ Linear

Time dependent $\Rightarrow$ T.V.

* discrete:

① $y[n] = g[n].\, x[n]$.

Sol$^n$: $y[n]$ I/P: $g[n].\, x[n-n_0]$

O/P: $g[n-n_0].\, x[n-n_0]$

So, T.V.

② $y[n] = 3\,x[n]$.

$3x[n-n_0]$ )

③ $\frac{dy(t)}{dt} + t \cdot y(t) = x^2(t).$ $\frac{dy}{dx}$ $y \to D.V$ ; $x \to I.V$

Sol$^n$: $t \cdot y(t) \to$ T.V. ; $x^2(t) \to$ N.L.

Linear
→ First degree of differentiation &
→ Co-efficient must be constant or I.V. (here I.V. is $t$)

④ $\frac{d^2y(t)}{dt} + 4y(t) = 2x(t).$

Sol$^n$: $\frac{d^2y(t)}{dt} \to$ L ; $4y(t) \to$ T.I.

So, Linear time Invariant (L.T.I.).

⇒ Because of co-efficient are fixed ⇒ T.I.

⇒ In any differential eq$^n$ if all the co-efficient are fixed only with linear elements that is LTI system.

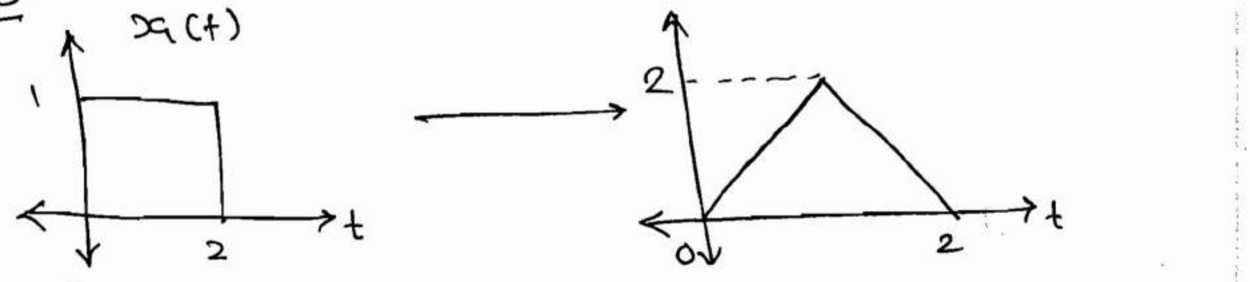
i) I/P O/P depending on time → V

ii) Time scaling. → V

iii) Modulator (signal multiply by sine or cos). → V

Page-⑰

P: 1.4.5.



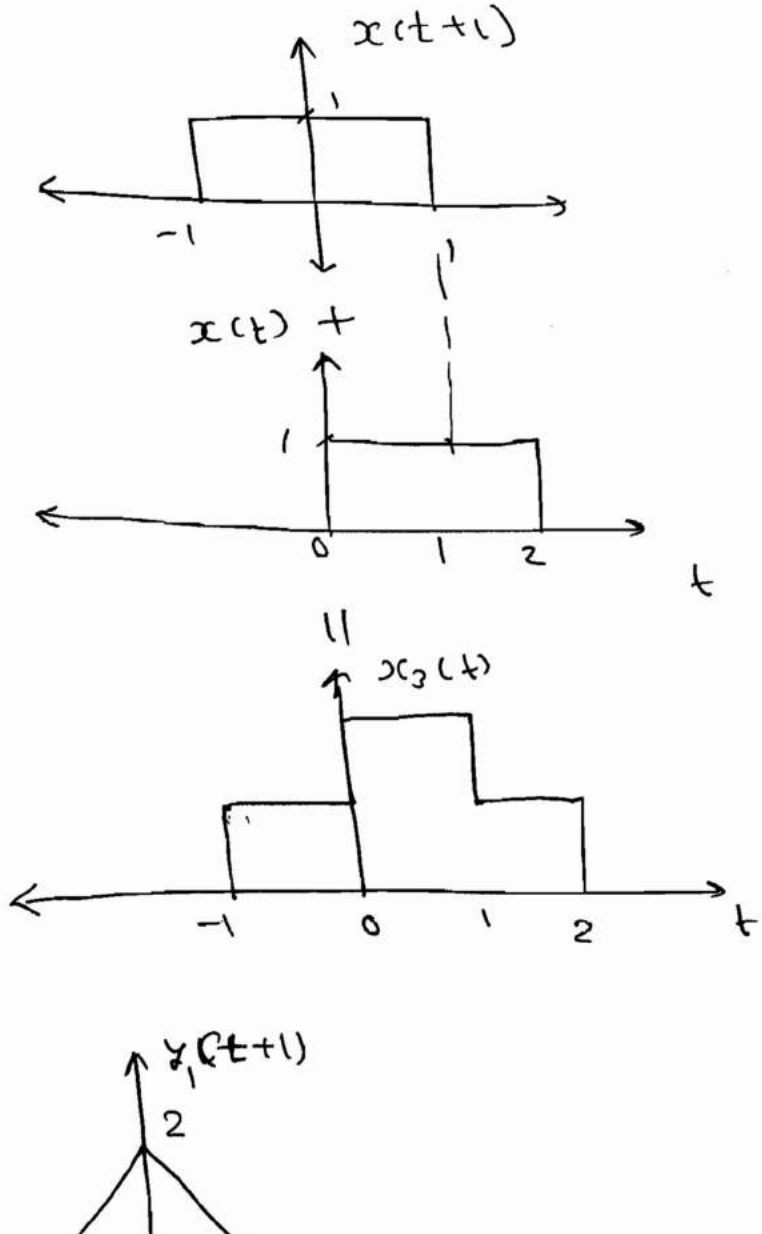

⇒

$$\Rightarrow \quad x_2(t) = x_1(t) - x_1(t-2).$$

$$\Big\downarrow \text{ LTI (given).}$$

$$\therefore \quad x_2(t) = y_1(t) - y_1(t-2).$$

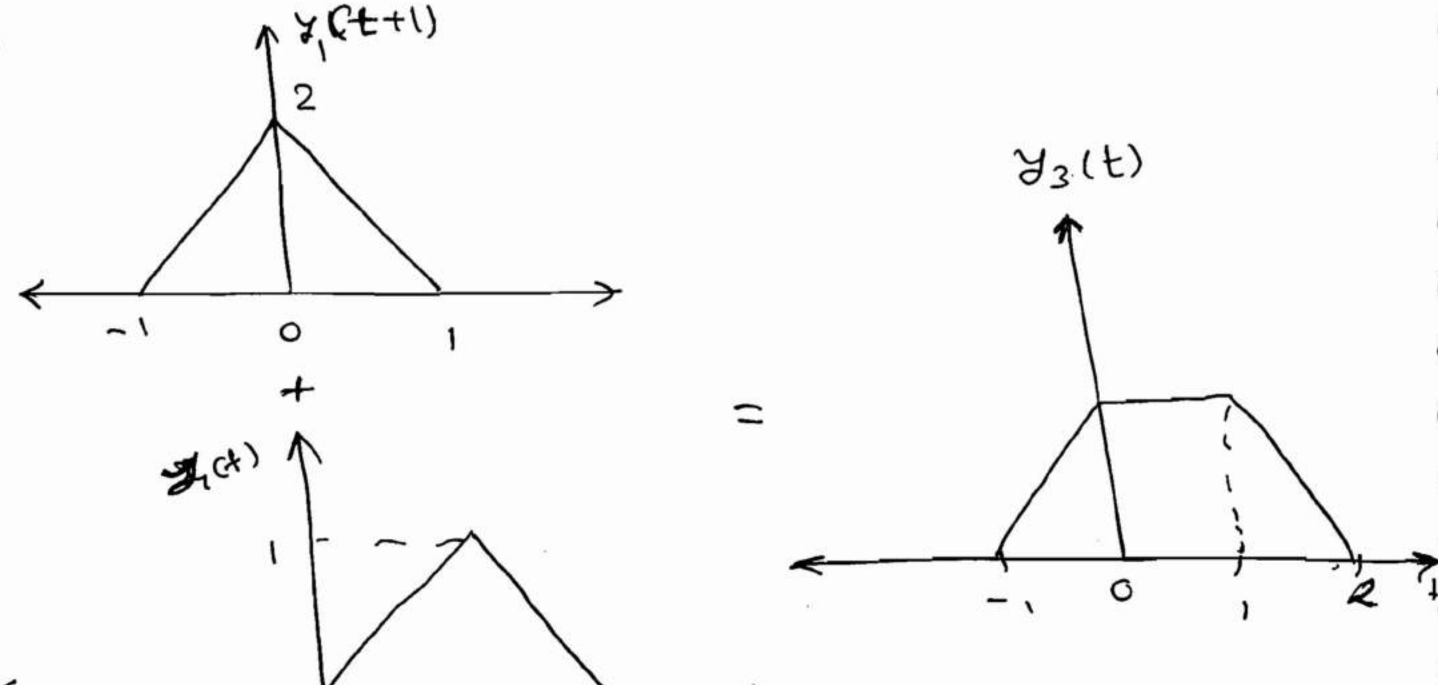
$\Rightarrow$




$x_3(t) = x_1(t+1) + x_1(t).$

So, $y_3(t) = y_1(t+1) + y_1(t).$

$\Rightarrow$



★ $x(t) \rightarrow y(t)$. ⇒ LTI.

Initial cond$^n$ $y(0) = 0$.

⇒ For a LTI system Same relation is Valid for i/p and o/p.

[3] Causal and Non-Causal System.

⇒ A system is Causal system if the present o/p depends only on the present i/p and past values of the i/p but not on future values i.e. Causal systems are non-anticipative.

eg.: (i) $y(t) = (3t+1)\, x(t)$.

Causal.

(ii) $y(t) = x(t) \cdot \cos\omega_0 t$

causal.

Q

① $y(t) = \sin\{x(t)\}$.

Sol$^n$:

Causal.

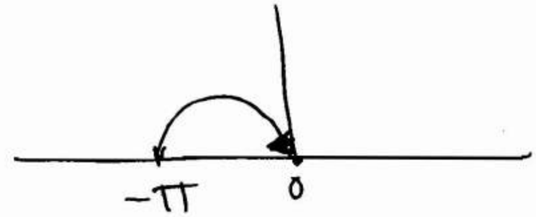
② $y(t) = x\{\sin(t)\}$.

Sol$^n$: $\sin t = 0$ for $t = \pm m\pi$, $m = 0, 1, 2, \ldots$

$y(-\pi) = x\{\sin(-\pi)\}$

$\Rightarrow \quad y(-\pi) = x(0).$

o/p is expecting future value.




So, Non- Causal.

③ $$y(t) = \int_{-\infty}^{2t} x(\tau) \cdot d\tau.$$

$\underline{\underline{\text{Sol}^n}}$: $$y(1) = \int_{-\infty}^{2} x(\tau) \cdot dt.$$

So, Non - Causal.

④ $$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau.$$

$\underline{\underline{\text{Sol}^n}}$: $$y(1) = \int_{-\infty}^{1} x(\tau) \cdot d\tau. \Rightarrow \text{Causal.}$$

⑤ $$y[n] = 2x[n]. + 3u[n+1].$$

$\underline{\underline{\text{Sol}^n}}$: Causal

$\hookrightarrow$ Not affecting causality.

⑥ $$y[n] = \sum_{k=n_0}^{n} x[k]$$ . "$n_0$" is finite.

$\underline{\underline{\text{Sol}^n}}$: $\underline{\text{Case}} - \underline{\text{I}}$: $n_0 > n$. $\Rightarrow$ Non Causal (NC).

$2 > 1$

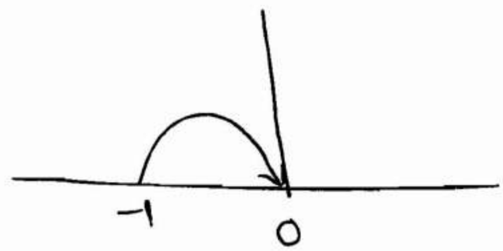
Case - II: $n_0 \le n$.

$$y(2) = \sum_{K=1}^{2} x[K] = x[1] + x[2]. \Rightarrow (c)$$

$\Rightarrow$ So, Conditionally Causal.

$\Rightarrow$ for $n_0 = 0$.

$$\therefore \; y[n] = \sum_{K=0}^{n} x[K].$$




$$\therefore \; y[-1] = \sum_{K=0}^{-1} x[K] = x[-1] + x[0].$$

So, Non - Causal.

[7] $y[n] = \sum_{K=-\infty}^{n} x[K].$

Sol$^n$: It is accumulator.

$$y[1] = \sum_{K=-\infty}^{1} x(K] = \cdots + x[-2] + x[-1] + x[0] + x[1].$$

So, Causal.

[8] $y[n] = \dfrac{1}{2M+1} \sum_{K=-M}^{+M} x[n-K].$

Sol$^n$: It is Moving avg. system.

Let., $M = 1$

$$\therefore \; y[n] = \frac{1}{3} \sum x[n-K].$$

$\therefore\ y[n] = \frac{1}{3}\left[ x[n+1] + x[n] + x[n-1] \right]$.

So, Non - Causal system.

<u>Note</u>: Non - Causal systems Can not be design when the independent Variable is time. ( t or n ).

[9] $y^{\cdot}(t+4) + 2(t) = x(t+2)$.

Soln: Put $t+4 = \tau$.

$\therefore\ y^{\cdot}(\tau) + 2(\tau-4) = x(\tau-2)$.

So, Causal system.

$\Rightarrow$ Here, Present time is $t+4$.

[4] Static | Memory less &

Dynamic | with memory :

$\Rightarrow$ Static system is that in which o/p at perticular instant is depend only on input at that instant only.

eg: ① $y(t) = e^{-(t+3)} \cdot x(t)$.

Static.

[Q] ① $y(t) = 2x(t) + 3$

(2) $y[n] = g[n+3] \cdot x[n]$

Static.

(3) $y[n] = x[3n]$.

Soln: $y[1] = x[3] \Rightarrow$ Dynamic.

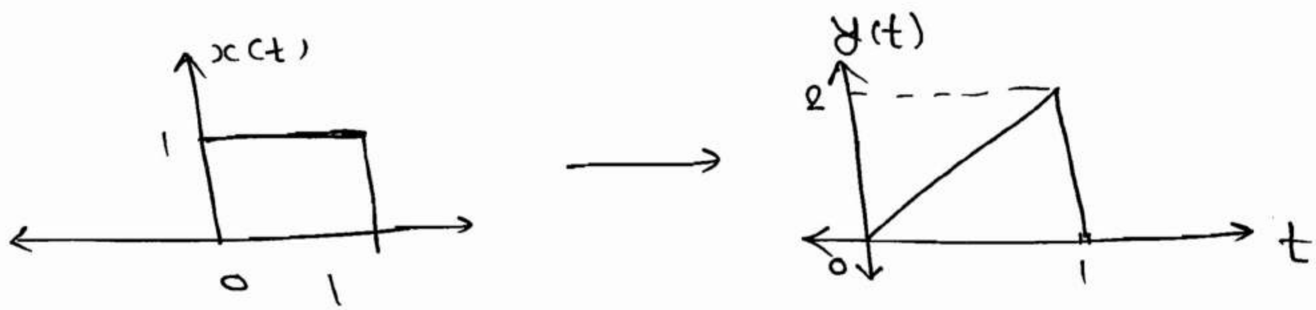
(4) $y(t) = \frac{d}{dt}(x(t))$

Soln: differential terms is because of system is energy storing elements

So, System is alway dynamic.

$\Rightarrow$ All Static systems are Causal system.

(a)

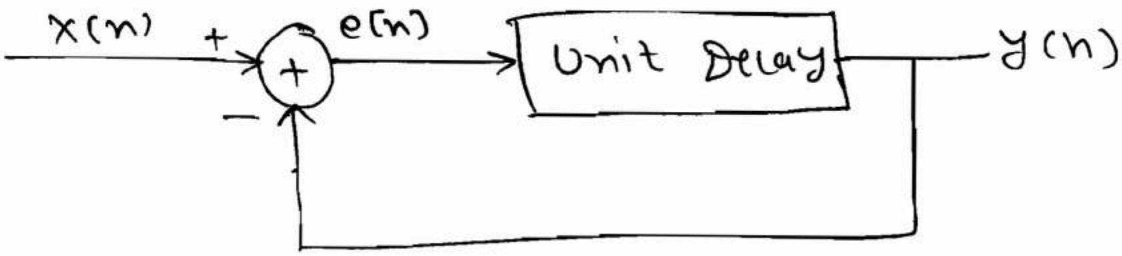



Soln: Before applying an input, O/P should not start $\Rightarrow$ Causal.

$\Rightarrow$ I/P is starting at '0' and O/P is also starting at '0'. So, causal system.

$$y(t) = 2\int_{-\infty}^{t} x(\tau)\, d\tau.$$

Q Consider the F/B system shown in figure, assume that $y[n]=0$ for $n<0$.




⇒ Find $y(n)$ when the input is

(i) $x(n) = \delta[n]$.

(ii) $x(n) = u[n]$.

Solⁿ:

$$e[n] = x[n] - y[n].$$

$$\therefore \quad y[n] = e[n-1].$$

$$\therefore \quad y[n] = x[n-1] - y[n-1].$$

⇒ $n=0, \quad y[0] = x[-1] - y[-1] = 0-0=0.$

① $x[n] = \delta[n]$.

$\rightarrow y[0] = 0-0=0.$

$\therefore \quad y[1] = x[0] - y[0].$

$\rightarrow y[1] = 1-0 = 1.$

$\rightarrow y[2] = x[1] - y[1]$

$= 0-1=-1.$

$$\therefore y[n] = [0, 1, -1, +1, -1, \dots].$$

$\Rightarrow \dfrac{z^{-1}}{1+z^{-1}} \Rightarrow$ IZT $\Rightarrow (-1)^{n-1} \cdot u(n-1)$.

② $x[n] = u[n]$.

O/P $y[n] = \{0, 1, 0, 1, 0, \ldots\}$.

**Note:**

$\Rightarrow$ Present o/p depends only on present i/p and past input then it is Finite Impulse Response [FIR].

$\Rightarrow$ If it also depends on past o/p it is IIR (Infinite impulse Response) filter. FB is there $\Rightarrow$ Recursive.

## [5] Stable & Unstable Systems:

$\Rightarrow$ It is a magnitude concept.

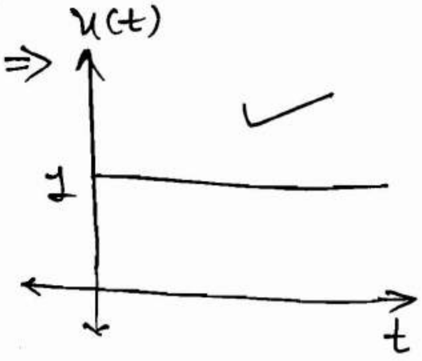
$\Rightarrow$ Boundedness $\Rightarrow$ Bounded amplitude.

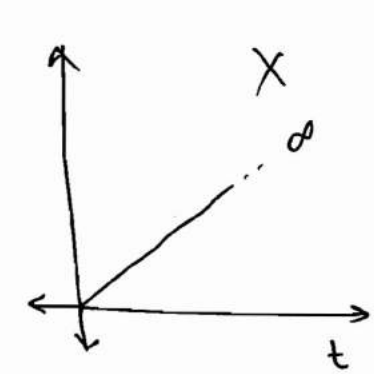
$\Rightarrow$ B.I.B.O.

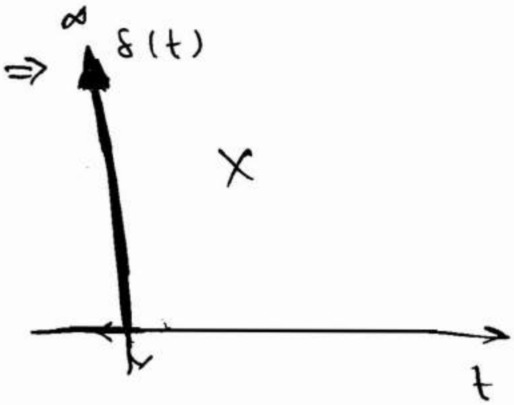
$\Rightarrow$

$$\underset{\text{B.I.}}{x(t)} \longrightarrow \boxed{T\{\cdot\}} \longrightarrow \underset{\text{B.O.}}{y(t)}.$$

$$\left.\begin{array}{l} \text{If } |x(t)| \le M_x < \infty \text{ then} \\ |y(t)| \le M_y < \infty. \end{array}\right\} \longrightarrow \text{Stable.}$$

⇒ 

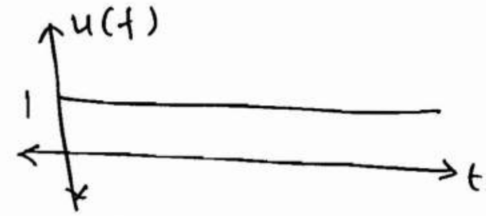

⇒ 


⇒ 


⇒ Sine & Cos ⇒ Stable.

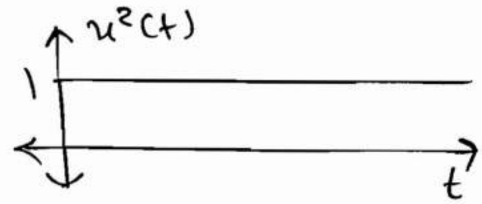
Q

① $y(t) = x^2(t)$.




Sol$^n$: If $x(t) = u(t)$

$\therefore y(t) = u^2(t) = u(t)$.




Stable

* Alternative

→ Magnitude Concept.

→ $y(t) = |x(t)|^2$

$= (\text{finite})^2 = \text{finite} \Rightarrow$ Stable.

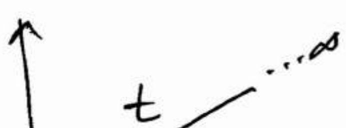
② $y(t) = x(2t)$.

Sol$^n$. Stable.

③ $y(t) = t \cdot x(t)$.

Sol$^n$: if $x(t) = u(t)$

$\Rightarrow y(t) = t \cdot u(t)$




So, Unstable.

$\boxed{Q}$ (4) $y(t) = \frac{d}{dt} . x(t)$.

$\underline{\underline{Sol^n:}}$ let, $x(t) = u(t)$.

$y(t) = \frac{d}{dt} \cdot u(t) = \delta(t)$. (Unbounded).

So, Unstable.

(5) $y(t) = x(t). \cos\omega_c t$.

$\underline{\underline{Sol^n:}}$ $|y(t)| = |x(t). \cos\omega_c t|$

$= |x(t)| \cdot \cos\omega_c t \rightarrow$ max value +1 & -1.

$=$ finite $=$ stable.

(6) $y(t) = \int_{-\infty}^{t} |x(\tau). \cos\omega_c t| \, dt$.

$\underline{\underline{Sol^n:}}$ $y(t) = \int_{-\infty}^{t} (\text{finite}) \, . dt$.

$= \infty$

$\Rightarrow$ Unstable.

(7) $y[n] = 2x[n] + 3$.

$\underline{\underline{Sol^n:}}$ Stable $\quad \hookrightarrow$ finite + finite = finite.

$\Rightarrow$ for discrete we have $\delta[n]$, $u[n]$.

(8) $y[n] = e^{x[n]}$.

$|x[n]|$

⑨ $y[n] = \sum_{n=n_0}^{\infty} x[k]$ "$n_0$ is finite".

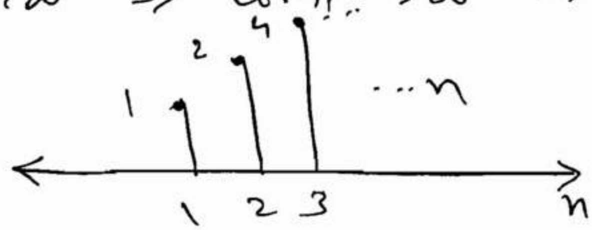
$\underline{\underline{Sol^n}}$: Let, $x[k] = 1$.

$$\therefore y[n] = \sum_{n=n_0}^{n} (1).$$

$$= 1 + 1 + 1 + \dots n \text{ terms}.$$

$$= n.$$

as $n \to \infty \Rightarrow amp. \to \infty \Rightarrow$ Unstable




⑩ $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k].$

$\underline{\underline{Sol^n}}$: $x[k] = 1$.

$$y[n] = \sum_{k=n-n_0}^{n+n_0} (1).$$

Put, $k - n + n_0 = m$.

$$\therefore y[n] = \sum_{m=0}^{m=2n_0} (1).$$

$$\therefore y[n] = (2n_0 + 1)$$

$\Downarrow$

## [6] Invertible & Inverse System :-

$\Rightarrow$ A system is an Invertible if different inputs (different amplitudes) leads to different outputs i.e. two different i/ps for a given system should not produce same o/p.

$$x(t) \longrightarrow \boxed{T\{\cdot\}} \xrightarrow{y(t)} \boxed{T^{-1}\{\cdot\}} \longrightarrow x(t)$$

$$\underline{\underline{T \cdot T^{-1} = 1}}$$

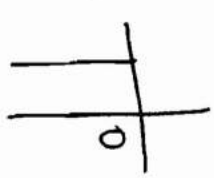
e.gs:

① $y(t) = x^2(t)$.

Sol^n:

i/p $\longrightarrow$ o/p

$u(t) \longrightarrow u(t)$.



$-u(t) \longrightarrow u(t)$.

So, Non- Invertible.

② $y(t) = x(t-4)$.

Sol^n:

$\delta(t) \longrightarrow \delta(t-4)$.

$-\delta(t) \longrightarrow -\delta(t-4)$

$u(t) \longrightarrow +u(t-4)$.

$-u(t) \longrightarrow -u(t-4)$

So, Invertible & Inverse is $y(t+4)$.

③ $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau.$

$\underline{\underline{Sol^n}}$:

i/p ⟶ o/p.

$\delta(t) \longrightarrow u(t).$

$-\delta(t) \longrightarrow -u(t).$

$u(t) \longrightarrow t\,u(t).$

$-u(t) \longrightarrow -t\,u(t).$

So, Invertible & Inverse is $\frac{dy(t)}{dt}$.

*

③ $y(t) = \frac{d}{dt}(x(t)).$

$\underline{\underline{Sol^n}}$:

i/p ⟶ o/p

$2 \longrightarrow 0$

$3 \longrightarrow 0$

$4 \longrightarrow 0$

So, Non-Invertible.

④ $y[n] = n \cdot x[n].$

$\underline{\underline{Sol^n}}$:

i/p ⟶ o/p.

$\delta[n] \longrightarrow n \cdot \delta[n]$
$= 0 \cdot \delta[n] = 0$ $(\because n_0 = 0)$. property of $\delta[n]$).

$-\delta[n] \longrightarrow -n \cdot \delta[n]$
$= 0 \cdot \delta[n] = 0.$

So, Non-Invertible.

⑤ $y[n] = x[n] \cdot x[n-3].$

$\underline{\underline{Sol^n}}$:

i/p ⟶ o/p

So, Non- Invertebile.

[6] $$y[n] = \sum_{k=-\infty}^{n} x[k].$$

Sol$^n$: Cont. $\quad$ Discrete

$$\sum \Rightarrow \int$$

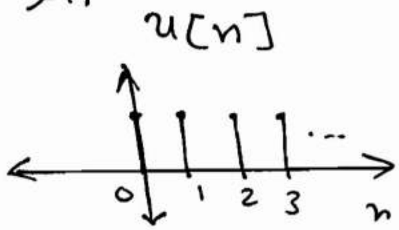
$$i/p \longrightarrow o/p$$

$$\delta[n] \longrightarrow u[n].$$

$$-\delta[n] \longrightarrow -u[n].$$

So, Invertible.

$\Rightarrow$ i/p




$$\longrightarrow \sum_{k=0}^{n} (1) = (n+1)u[n].$$

So, Invertible.

Inverse is $y[n] - y[n-1]$.

$$= \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k].$$

$$= x[n] + \cancel{\sum_{k=-\infty}^{n-1} x[k]} - \cancel{\sum_{k=-\infty}^{n-1}} x[k]$$

$$= x[n].$$

(7) $$y[n] = x[n] \cdot \sin\left[\frac{5\pi n}{6}\right].$$

Sol$^n$: $$i/p \longrightarrow o/p$$

$$\delta[n] \longrightarrow \delta[n] \cdot \sin\left[\frac{5\pi n}{6}\right].$$

$$\rightarrow \quad -\delta[n] \longrightarrow -\delta[n] \cdot \sin\left(\frac{5\pi n}{6}\right)$$

$$= 0.$$

So, Non- Invertible.