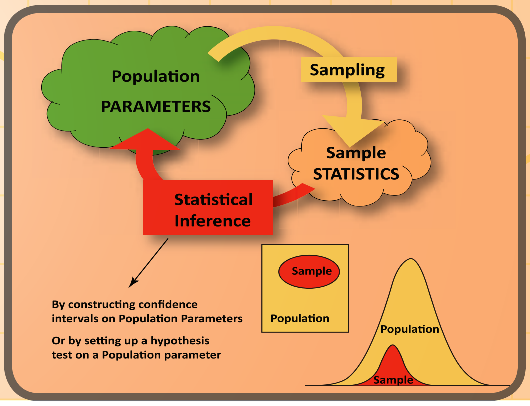
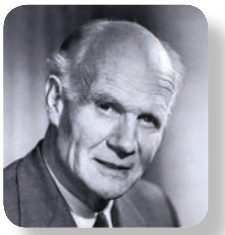
CHAPTER

1 TESTS OF SIGNIFICANCE – BASIC CONCEPTS AND LARGE SAMPLE TESTS




Jerzy Neyman

Jerzy Neyman (1894-1981) was born into a Polish family in Russia. He is one of the Principal architects of Modern Statistics. He developed the idea of confidence interval estimation during 1937. He had also contributed to other branches of Statistics, which include Design of Experiments, Theory of Sampling and Contagious Distributions. He established the Department of Statistics in University of California at Berkeley, which is one of the preeminent centres for statistical research worldwide.


Egon Sharpe Pearson

Egon Sharpe Pearson (1885-1980) was the son of Prof. Karl Pearson. He was the Editor of *Biometrika*, which is still one of the premier journals in Statistics. He was instrumental in publishing the two volumes of *Biometrika Tables for Statisticians*, which has been a significant contribution to the world of Statistical Data Analysis till the invention of modern computing facilities.

Neyman and Pearson worked together about a decade from 1928 to 1938 and developed the theory of testing statistical hypotheses. *Neyman-Pearson Fundamental Lemma* is a milestone work, which forms the basis for the present theory of testing statistical hypotheses. In spite of severe criticisms for their theory, in those days, by the leading authorities especially Prof.R.A.Fisher, their theory survived and is currently in use.

"Statistics is the servant to all sciences" – *Jerzy Neyman*

LEARNING OBJECTIVES

The students will be able to

- ❖ understand the purpose of hypothesis testing;
- ❖ define parameter and statistic;
- ❖ understand sampling distribution of statistic;
- ❖ define standard error;
- ❖ understand different types of hypotheses;
- ❖ determine type I and type II errors in hypotheses testing problems;
- ❖ understand level of significance, critical region and critical values;
- ❖ categorize one-sided and two-sided tests;
- ❖ understand the procedure for tests of hypotheses based on large samples; and
- ❖ solve the problems of testing hypotheses concerning mean(s) and proportion(s) based on large samples.

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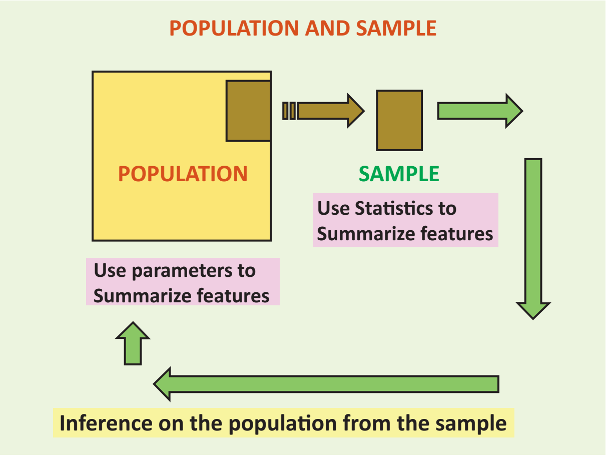
Introduction

In XI Standard classes, we concentrated on collection, presentation and analysis of data along with calculation of various measures of central tendency and measures of dispersion. These kinds of describing the data are popularly known as **descriptive statistics**. Now, we need to understand another dimension of statistical data analysis, which is called **inferential statistics**. Various concepts and methods related to this dimension will be discussed in the first four Chapters of this volume. Inferential Statistics may be described as follows from the statistical point of view:

One of the main objectives of any scientific investigation or any survey is to find out the unknown facts or characteristics of the population under consideration. It is practically not feasible to examine the entire population, since it will increase the time and cost involved. But one may examine a part of it, called **sample**. On the basis of this limited information, one can make decisions or draw inferences on the unknown facts or characteristics of the population.



Thus, inferential statistics refers to a collection of statistical methods in which random samples are used to draw valid inferences or to make decisions in terms of probabilistic statements about the population under study.

Before going to study in detail about Inferential Statistics, we need to understand some of the important terms and definitions related to this topic.

1.1 PARAMETER AND STATISTIC

A **population**, as described in Section 2.4 in XI Standard text book, is a collection of units/objects/numbers under study, whose elements can be considered as the values of a random variable, say, X. As mentioned in Section 9.3 in XI Standard text book, there will be a probability distribution associated with X.

Parameter: Generally, **parameter** is a quantitative characteristic, which indexes/identifies the respective distribution. In many cases, statistical quantitative characteristics calculated based on all the units in the population are the respective parameters. For example, population mean, population standard deviation, population proportion are parameters for some distributions.

Recall: The unknown constants which appear in the *probability density function or probability mass function* of the random variable X, are also called **parameters** of the corresponding distribution/population.

The parameters are commonly denoted by Greek letters. In Statistical Inference, some or all the parameters of a population are assumed to be unknown.

Random sample: Any set of reliazations $(X_1, X_2, \ldots, X_n)$ made on X under independent and identical conditions is called a **random sample**.

Statistic: Any statistical quantity calculated on the basis of the random sample is called a **statistic**. The sample mean, sample standard deviation, sample proportion *etc*., are called **statistics** (plural form of *statistic*).They will be denoted by Roman letters.

Let $(x_1, x_2, \ldots, x_n)$ be an observed value of $(X_1, X_2, \ldots, X_n)$. The collection of $(x_1, x_2, \ldots, x_n)$ is known as *sample space*, which will be denoted by '**S**'.

Note 1:

A set of n sample observations can be made on X, say, $x_1, x_2, \ldots, x_n$ for making inferences on the unknown parameters. It is to be noted that these n values may vary from sample to sample. Thus, these values can be considered as the realizations of the random variables $X_1, X_2, \ldots, X_n$, which are assumed to be independent and have the same distribution as that of X. These are also called independently and identically distributed (*iid*) random variables.

NOTE

The *statistic* itself is a random variable and has a probability distribution.

Note 2:

In Statistical Inference, the sample standard deviation is defined as $S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$, where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. It may be noted that the divisor is $n - 1$ instead of n.

Note 3:

The statistic itself is a random variable, until the numerical values of $X_1, X_2, \ldots, X_n$ are observed, and hence it has a probability distribution.

Notations to denote various population parameters and their corresponding sample statistics are listed in Table 1.1. The notations will be used in the first four chapters of this book with the same meaning for the sake of uniformity.

Table 1.1 Notations for Parameters and Statistics

Statistical measure	Parameter	Statistic	Value of the Statistic for a given sample
Mean	μ	$\bar{X}$	$\bar{x}$
Standard deviation	σ	S	s
Proportion	P	p	p_0

1.2 SAMPLING DISTRIBUTION

The probability distribution of a statistic is called **sampling distribution** of the statistic. In other words, it is the probability distribution of possible values of the statistic, whose values are computed from possible random samples of same size.

The following example will help to understand this concept.

Example 1.1

Suppose that a population consists of 4 elements such as 4, 8, 12 and 16. These may be considered as the values of a random variable, say, X. Let a random sample of size 2 be drawn from this population under *sampling with replacement* scheme. Then, the possible number of samples is 4^2.

It is to be noted that, if we take samples of size n each from a finite population of size N, then the number of samples will be N^n under with replacement scheme and ^NC_n samples under without replacement scheme.

In each of the 4^2 samples, the sample elements x_1 and x_2 can be considered as the values of the two *iid* random variables X_1 and X_2. The possible samples, which could be drawn from the above population and their respective means are presented in Table 1.2.

Table 1.2 Possible Samples and their Means

Sample Number	Sample elements (x_1, x_2)	Sample Mean $\bar{x}$
1	4,4	4
2	4,8	6
3	4,12	8
4	4,16	10
5	8,4	6
6	8,8	8
7	8,12	10
8	8,16	12
9	12,4	8
10	12,8	10
11	12,12	12
12	12,16	14
13	16,4	10
14	16,8	12
15	16,12	14
16	16,16	16

The set of pairs (x_1, x_2) listed in column 2 constitute the sample space of samples of size 2 each.

Hence, the sample space is:

S = {(4,4), (4,8), (4,12), (4,16), (8,4), (8,8), (8,12), (8,16), (12,4), (12,8), (12,12), (12,16), (16,4), (16,8), (16,12), (16,16)}

The sampling distribution of $\overline{X}$, the sample mean, is determined and is presented in Table 1.3.

Table 1.3 Sampling Distribution of Sample Mean

Sample mean: $\bar{x}$	4	6	8	10	12	14	16	Total
Probability: $P(\bar{X}=\bar{x})$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	1

Note 4: The sample obtained under sampling *with replacement* from a finite population satisfies the conditions for a random sample as described earlier.

Note 5: If the sample values are selected under *without replacement scheme*, independence property of $X_1, X_2, \ldots X_n$ will be violated. Hence it will not be a random sample.

Note 6: When the sample size is greater than or equal to 30, in most of the text books, the sample is termed as a **large sample**. Also, the sample of size less than 30 is termed as **small sample**. However, in practice, there is no rigidity in this number *i.e.*, 30, and that depends on the nature of the population and the sample.

Note 7: The learners may recall from XI Standard Textbook that some of the probability distributions possess the additive property. For example, if $X_1, X_2, \ldots, X_n$ are *iid* $N(\mu, \sigma^2)$ random variables, then the probability distributions of $X_1 + X_2 + \ldots + X_n$ and $\bar{X}$ are respectively the $N(n\mu, n\sigma^2)$ and $N(\mu, \sigma^2/n)$. These two distributions, in statistical inference point of view, can be considered respectively as the sampling distributions of the sample total and sample mean of a random sample drawn from the $N(\mu, \sigma^2)$ distribution. The notation $N(\mu, \sigma^2)$ refers to the normal distribution having mean μ and variance σ^2.

1.3 STANDARD ERROR

The standard deviation of the sampling distribution of a statistic is defined as the **standard error** of the statistic, which is abbreviated as *SE*.

For example, the standard deviation of the sampling distribution of the sample mean, $\bar{x}$, is known as the standard error of the sample mean, or $SE(\bar{X})$.

If the random variables $X_1, X_2, \ldots, X_n$ are independent and have the same distribution with mean μ *and variance* σ^2, then variance of $\bar{X}$ becomes as

$$V(\bar{X}) = V\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} V(X_i) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

Thus, $SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}$.

Also, note that mean of $\bar{X} = E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\mu = \frac{n\mu}{n} = \mu$

Example 1.2

Calculate the standard error of $\bar{X}$ for the sampling distribution obtained in *Example 1*.

Solution:

Here, the population is {4, 8, 12, 16}.

Population size $(N) = 4$, Sample size $(n) = 2$

Population mean $(\mu) = (4 + 8 + 12 + 16)/4 = 40/4 = 10$

The population variance is calculated as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2$$

$$= \frac{1}{4}\left[(4-10)^2 + (8-10)^2 + (12-10)^2 + (16-10)^2\right] = \frac{1}{4}\left[36+4+4+36\right] = 20.$$

Hence, $SE(\bar{X}) = \sqrt{\frac{\sigma^2}{n}} = \sqrt{\frac{20}{2}} = \sqrt{10}$

This can also be verified from the sampling distribution of $\bar{X}$ (*see* Table 1.3)

$$V(\bar{X}) = \sum(\bar{x} - \mu)^2 P(\bar{X} = \bar{x})$$

where the summation is taken over all values of $\bar{x}$

Thus, $V(\bar{X}) = (4-10)^2\frac{1}{16} + (6-10)^2\frac{2}{16} + (8-10)^2\frac{3}{16} + (10-10)^2\frac{4}{16}$

$$+(12-10)^2\frac{3}{16} + (14-10)^2\frac{2}{16} + (16-10)^2\frac{1}{16}$$

$$= \frac{1}{16}(36+32+12+0+12+32+36) = 10$$

Hence, the standard deviation of the sampling distribution of $\bar{X}$ is = $\sqrt{10}$.

Standard Errors of some of the frequently referred statistics are listed in Table 1.4.

Table 1.4 Statistics and their Standard Errors

Statistic	Standard error
Sample proportion: p	$\sqrt{\frac{PQ}{n}}$, where P is the population proportion and $Q = 1 - P$.
Difference between the means $\bar{X}$ and $\bar{Y}$ of two independent samples: $(\bar{X} - \bar{Y})$	$\sqrt{\frac{\sigma_X^2}{m} + \frac{\sigma_Y^2}{n}}$ where m and n are the sizes of samples drawn from the populations whose variances are σ_X^2 and σ_Y^2 respectively.
	$\sigma\sqrt{\frac{1}{m} + \frac{1}{n}}$, where σ^2 is the common variance of the populations.
Difference between the proportions p_X and p_Y of two independent samples: $(p_X - p_Y)$	$\sqrt{\frac{P_X Q_X}{m} + \frac{P_Y Q_Y}{n}}$, where m and n are sizes of the samples drawn from the populations whose proportions are respectively P_X and P_Y; $Q_X = 1 - P_X$, $Q_Y = 1 - P_Y$.
	$\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}$, where $\hat{p} = \frac{mp_X + np_Y}{m+n}$, $\hat{q} = 1 - \hat{p}$, m and n are sample sizes, when P_X and P_Y are unknown.

1.4 NULL HYPOTHESIS AND ALTERNATIVE HYPOTHESIS

In many practical studies, as mentioned earlier, it is necessary to make decisions about a population or its unknown characteristics on the basis of sample observations. For example, in bio-medical studies, we may be investigating a particular theory that the recently developed medicine is much better than the conventional medicine in curing a disease. For this purpose, we propose a statement on the population or the theory. Such statements are called hypotheses.

Thus, a **hypothesis** can be defined as a statement on the population or the values of the unknown parameters associated with the respective probability distribution. All the hypotheses should be tested for their validity using statistical concepts and a representative sample drawn from the study population. '*Hypotheses*' is the plural form of '*hypothesis*'.

A **statistical test** is a procedure governed by certain determined/derived rules, which lead to take a decision about the null hypothesis for its rejection or otherwise on the basis of sample values. This process is called **statistical hypotheses testing**.

The statistical hypotheses testing plays an important role, among others, in various fields including industry, biological sciences, behavioral sciences and Economics. In each hypotheses testing problem, we will often find as there are two hypotheses to choose between *viz*., null hypothesis and alternative hypothesis.

Null Hypothesis:

A hypothesis which is to be actually tested ***for possible rejection*** based on a random sample is termed as **null hypothesis**, which will be denoted by H_0.

YOU WILL KNOW

(i) Generally, it is a hypothesis of no difference in the case of comparison.

(ii) Assigning a value to the unknown parameter in the case of single sample problems

(iii) Suggesting a suitable model to the given environment in the case of model construction.

(iv) The given two attributes are independent in the case of *Chi*-square test for independence of attributes.

Alternative Hypothesis:

A statement about the population, which contradicts the null hypothesis, depending upon the situation, is called **alternative hypothesis**, which will be denoted by H_1.

For example, if we test whether the population mean has a specified value μ_0, then the null hypothesis would be expressed as:

$$H_0: \mu = \mu_0$$

The alternative hypothesis may be formulated suitably as anyone of the following:

(i) $H_1: \mu \neq \mu_0$

(ii) $H_1: \mu > \mu_0$

(iii) $H_1: \mu < \mu_0$

The alternative hypothesis in (i) is known as two-sided alternative and the alternative hypothesis in (ii) is known as one-sided (right) alternative and (iii) is known as one-sided (left) alternative.

1.5 ERRORS IN STATISTICAL HYPOTHESES TESTING

A **statistical decision** in a hypotheses testing problem is either of rejecting or not rejecting H_0 based on a given random sample. Statistical decisions are governed by certain rules, developed by applying a statistical theory, which are known as **decision rules**. The decision rule leading to rejection of H_0 is called as **rejection rule**.

The null hypothesis may be either true or false, in reality. Under this circumstance, there will arise four possible situations in each hypotheses testing or decision making problem as displayed in Table 1.5.

Table 1.5 Decision Table

	H_0 is true	H_0 is false
Reject H_0	Type I error	Correct decision
Do not Reject H_0	Correct decision	Type II error

It must be recognized that the final decision of rejecting H_0 or not rejecting H_0 may be incorrect. The error committed by rejecting H_0, when H_0 is really true, is called **type I error**. The error committed by not rejecting H_0, when H_0 is false, is called **type II error**.

Example 1.3

A soft drink manufacturing company makes a new kind of soft drink. Daily sales of the new soft drink, in a city, is assumed to be distributed with mean sales of ₹40,000 and standard deviation of ₹2,500 per day. The Advertising Manager of the company considers placing advertisements in local TV Channels. He does this on 10 random days and tests to see whether or not sales has increased. Formulate suitable null and alternative hypotheses. What would be type I and type II errors?

Solution:

The Advertising Manager is testing whether or not sales increased more than ₹40,000. Let μ be the average amount of sales, if the advertisement does appear.

The null and alternative hypotheses can be framed based on the given information as follows:

Null hypothesis: H_0: $\mu = 40000$

i.e., The mean sales due to the advertisement is not significantly different from ₹40,000.

Alternative hypothesis: H_1: $\mu > 40000$

i.e., Increase in the mean sales due to the advertisement is significant.

(i) If type I error occurs, then it will be concluded as the advertisement has improved sales. But, really it is not.

(ii) If type II error occurs, then it will be concluded that the advertisement has not improved the sales. But, really, the advertisement has improved the sales.

The following may be the penalties due to the occurrence of these errors:

If type I error occurs, then the company may spend towards advertisement. It may increase the expenditure of the company. On the other hand, if type II error occurs, then the company will not spend towards advertisement. It may not improve the sales of the company.

1.6 LEVEL OF SIGNIFICANCE, CRITICAL REGION AND CRITICAL VALUE(S)

In a given hypotheses testing problem, the *maximum probability* with which we would be willing to tolerate the occurrence of type I error is called **level of significance** of the test. This probability is usually denoted by 'α'. Level of significance is specified before samples are drawn to test the hypothesis.

The level of significance normally chosen in every hypotheses testing problem is 0.05 (5%) or 0.01 (1%). If, for example, the level of significance is chosen as 5%, then it means that among the 100 decisions of rejecting the null hypothesis based on 100 random samples, maximum of 5 of among them would be wrong. It is emphasized that the 100 random samples are drawn under identical and independent conditions. That is, the null hypothesis H_0 is rejected wrongly based on 5% samples when H_0 is actually true. We are about 95% confident that we made the right decision of rejecting H_0.

Critical region in a hypotheses testing problem is a subset of the sample space whose elements lead to rejection of H_0. Hence, its elements have the dimension as that of the sample size, say, $n(n > 1)$. That is,

$$\text{Critical Region} = \left\{ \underset{\sim}{x} = (x_1, x_2, ..., x_n) \,|\, H_0 \text{ is rejected} \right\}.$$

A subset of the sample space whose elements does not lead to rejection of H_0 may be termed as **acceptance region**, which is the complement of the critical region. Thus,

$$\mathbf{S} = \{\text{Critical Region}\} \cup \{\text{Acceptance Region}\}.$$

Test statistic, a function of statistic(s) and the known value(s) of the underlying parameter(s), is used to make decision on H_0. Consider a hypotheses testing problem, which uses a **test statistic** $t(\underset{\sim}{X})$ and a constant c for deciding on H_0. Suppose that H_0 is rejected, when $t(\underset{\sim}{x}) > c$. It is to be noted here that $t(\underset{\sim}{X})$ is a scalar and is of dimension one. Its sampling distribution is a univariate probability distribution. The values of $t(\underset{\sim}{X})$ satisfying the condition $t(\underset{\sim}{x}) > c$ will identify the samples in the sample space, which lead to rejection of H_0. It does not mean that $\{t \,|\, t(\underset{\sim}{x}) > c\}$ is the corresponding critical region. The value 'c', distinguishing the elements of the critical region and the acceptance region, is referred to as **critical value**. There may be one or many critical values for a hypotheses testing problem. The critical values are determined from the sampling distribution of the respective test statistic under H_0.

Example 1.4

Suppose an electrical equipment manufacturing industry receives screws in lots, as raw materials. The production engineer decides to reject a lot when the number of defective screws is one or more in a randomly selected sample of size 2.

$$\text{Define } X_i = \begin{cases} 1, & \text{if } i^{th} \text{ screw is defective} \\ 0, & \text{if } i^{th} \text{ screw is not defective} \end{cases}, \qquad i = 1, 2$$

Then, X_1 and X_2 are *iid* random variables and they have the *Bernoulli* (P) distribution.

Let $H_0 : P = \frac{1}{3}$ and $H_1 : P = \frac{2}{3}$

The sample space is $\mathbb{S} = \{(0,0),(0,1),(1,0),(1,1)\}$

If $T(X_1, X_2)$ represents the number of defective screws, in each random sample, then the statistic $T(X_1, X_2) = X_1 + X_2$ is a random variable distributed according to the *Binomial* (2, P) distribution. The possible values of $T(X_1, X_2)$ are 0, 1 and 2. The values of $T(X_1, X_2)$ which lead to rejection of H_0 constitute the set {1,2}.

But, the critical region is defined by the elements of $\mathbb{S}$ corresponding to $T(X_1,X_2) = 1$ or 2. Thus, the critical region is {(0,1), (1,0), (1,1)} whose dimension is 2.

Note 8: When the sampling distribution is continuous, the set of values of $t(\underset{\sim}{X})$ corresponding to the rejection rule will be an interval or union of intervals depending on the alternative hypothesis. It is empahazized that **these intervals identify the elements of critical region, but they do not constitute the critical region**.

When the sampling distribution of the test statistic Z is a normal distribution, the critical values for testing H_0 against the possible alternative hypothesis at two different levels of significance, say 5% and 1% are displayed in Table 1.6.

Table 1.6 Critical values of the Z statistic

Alternative hypothesis	Level of Significance (α)	
	0.05 or 5%	0.01 or 1%
One- sided (right)	$z_\alpha = z_{0.05} = 1.645$	$z_\alpha = z_{0.01} = 2.33$
One- sided (left)	$-z_\alpha = -z_{0.05} = -1.645$	$-z_\alpha = -z_{0.01} = -2.33$
Two-sided	$z_{\alpha/2} = z_{0.025} = 1.96$	$z_{\alpha/2} = z_{0.005} = 2.58$

1.7 ONE-TAILED AND TWO-TAILED TESTS

In some hypotheses testing problem, elements of the critical region may be identified by a rejection rule of the type $t(\underset{\sim}{X}) \geq c$. In this case, P($t(\underset{\sim}{X}) \geq c$) will be the area, which falls at the right end (Figure1.1) under the curve representing the sampling distribution of $t(\underset{\sim}{X})$. The statistical test defined by this kind of critical region is called **right-tailed test**.

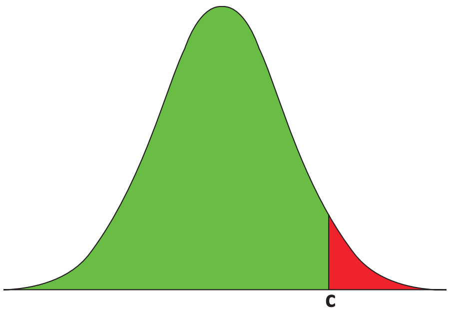


Figure 1.1. Right-tailed Test

On the other hand, suppose that the rejection rule $t(\underset{\sim}{X}) \leq c$ determines the elements of the critical region. Then, P($t(\underset{\sim}{X}) \leq c$) will be the area, which falls at the left end (Figure.1.2) under the curve representing the sampling distribution of $t(\underset{\sim}{X})$. The statistical test defined by this kind of critical region is called **left -tailed test**.

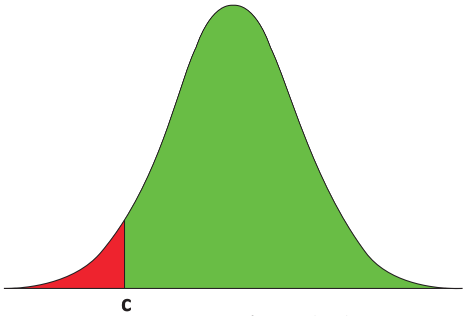


Figure 1.2 Left-tailed Test

The above two tests are commonly known as **one-tailed tests**.

Note 9: It should be noted that the sampling distribution of $t(\underset{\sim}{X})$ need not be with symmetric shape always. Sometimes, it may be positively or negatively skewed.

Example 1.5

Suppose a pizza restaurant claims its average pizza delivery time is 30 minutes. But you believe that the restaurant takes more than 30 minutes. Now, the null and the alternate hypotheses can be formulated as

$H_0 : \mu = 30$ minutes and $H_1 : \mu > 30$ minutes

Suppose that the decision is taken based on the delivery times of 4 randomly chosen pizza deliveries of the restaurant. Let X_1, X_2, X_3, and X_4 represent the delivery times of the such four occasions. Also, let H_0 be rejected, when the sample mean exceeds 31. Then, the critical region is

$$\text{Critical Region} = \left\{(x_1, x_2, x_3, x_4) \,|\, \bar{x} = \frac{x_1 + x_2 + x_3 + x_4}{4} > 31\right\}$$

In this case, $P(\bar{X} > 31)$ will be the area, which fall at the right end under the curve representing the sampling distribution of $\bar{X}$. Hence, this test can be categorized as a right-tailed test.

Suppose that H_0 is rejected, when either $t(\underset{\sim}{X}) \leq a$ or $t(\underset{\sim}{X}) \geq b$ holds. In this case, P($t(\underset{\sim}{X}) \leq a$) and P($t(\underset{\sim}{X}) \geq b$) will be the areas, which fall respectively at left and right ends under the curve representing the sampling distribution of $t(\underset{\sim}{X})$ (Figure 1.3). The statistical test defined with this kind of rejection rule is known as **two-tailed test**.

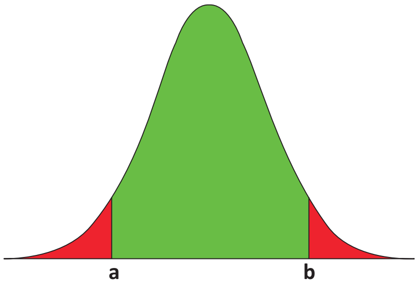


Figure 1.3 Two-tailed Test