Chapter 12

Pythagoras Theorems

Exercise 12.1

1. Lengths of sides of triangles are given below. Determine which of them are right triangles. In case of a right triangles, write the length of its hypotenuse:

(i) 3 cm, 8cm, 6cm

(ii) 13cm, 12cm, 5cm

(iii) 1.4cm, 4.8cm, 5cm

Solution:

We use the Pythagoras theorem to check whether the triangles are right triangles.

We have $h^2 = b^2 + a^2$ [Pythagoras theorem]

Where h is the hypotenuse, b is the base and a is the altitude.

(i) Given sides are 3cm, 8cm and 6cm

$b^2 + a^2 = 3^2 + 6^2 = 9 + 36 = 45$

$h^2 = 8^2 = 64$

here $45 \neq 64$

Hence the given triangle is not a right triangle.

(ii) Given sides are 13cm, 12cm and 5cm

$b^2 + a^2 = 12^2 + 5^2 = 144 + 25 = 169$

$h^2 = 13^2 = 169$

here = $b^2 + a^2 = h^2$

Hence the given triangle is a right triangle.

Length of the hypotenuse is 13 cm.

(ii) Given sides are 1.4cm, 4.8cm and 5cm

$b^2 + a^2 = 1.4^2 + 4.8^2 = 1.96 + 23.04 = 25$

$h^2 = 5^2 = 25$

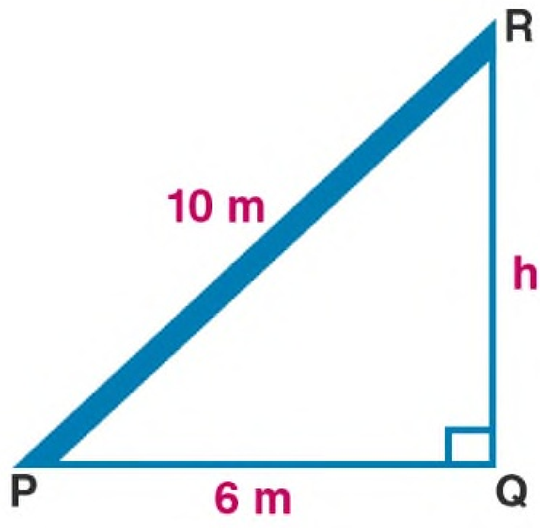
here $b^2 + a^2 = h^2$

Hence the given triangle is a right triangle.

Length of the hypotenuse is 5cm.

2. Foot of a 10m long ladder leaning against a vertical well is 6m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

Solution:



Let PR be the ladder and QR be the vertical wall.

Length of the ladder PR = 10m

PQ = 6m

Let height of the wall, QR = h

According to Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$10^2 = 6^2 + QR^2$

$100 = 36 + QR^2$

$QR^2 = 100 - 36$

$QR^2 = 64$

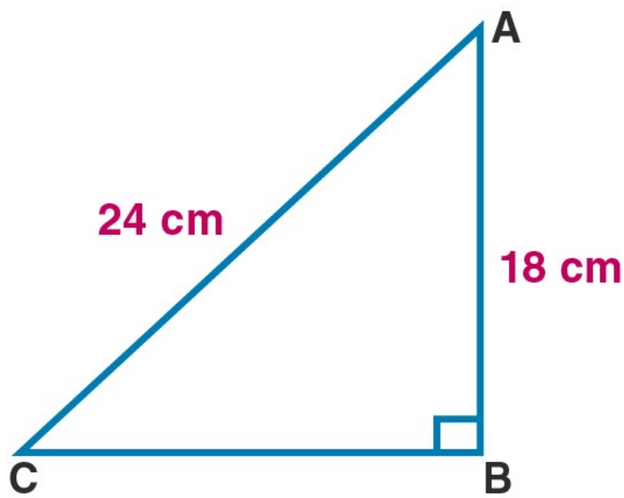
Taking square root on both sides,

QR = 8

Hence the height of the wall where the top of the ladder reaches is 8m.

3. A guy attached a wire 24m long to a vertical pole of height 18m and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be tight ?

Solution:



Let AC be the wire and AB be the height of the pole.

AC = 24cm

AB = 18cm

According to Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$24^2 = 18^2 + BC^2$

576 = 324 + BC^2

$BC^2 = 576 - 324$

$BC^2 = 252$

Taking square root on both sides,

$BC = \sqrt{252}$

$= \sqrt{4 \times 9 \times 7}$

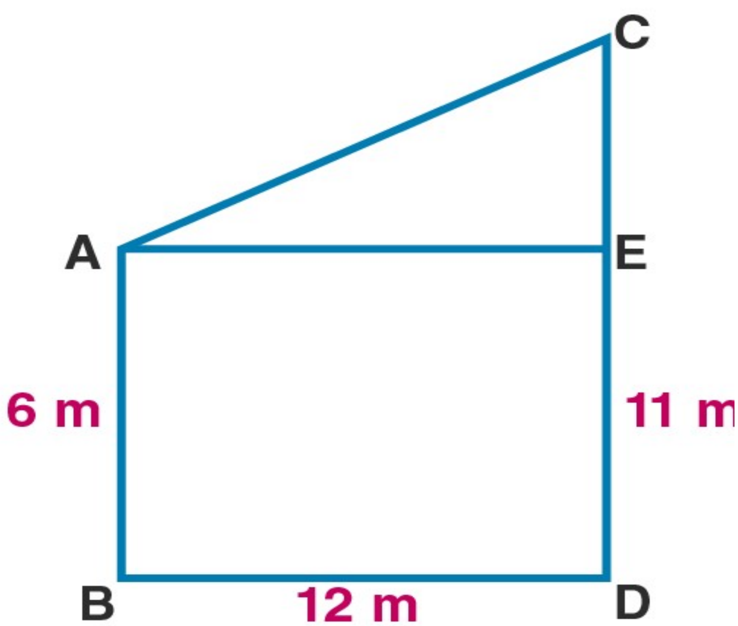
$= 2 \times \sqrt[3]{7}$

$= 6\sqrt{7}$ cm

Hence the distance is $6\sqrt{7}$ cm.

4. Two poles of heights 6m and 11m stand on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

Solution:



Let AB and CD be the poles which are 12m apart.

AB = 6m

CD = 11m

BD = 12m

Draw AE BD

CE = 11 – 6 = 5m

AE = 12m

According to Pythagoras theorem,

$AC^2 = AE^2 + CE^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

Taking square root on both sides

AC = 13

Hence the distance between their tops is 13m.

5. In a right-angles triangle, if hypotenuse is 20 cm and the ratio of the other two sides is 4:3, find the sides.

Solution:

Given hypotenuse, h = 20cm

Ratio of other two sides, a:b = 4 : 3

Let altitude of the triangle be 4x and base be 3x.

According to Pythagoras theorem,

$h^2 = b^2 + a^2$

$20^2 = (3x)^2 + (4x)^2$

$400 = 9x^2 + 16x^2$

$25x^2 = 400$

$x^2 = \frac{400}{25}$

$x^2 = 16$

Taking square root on both sides

$x = 4$

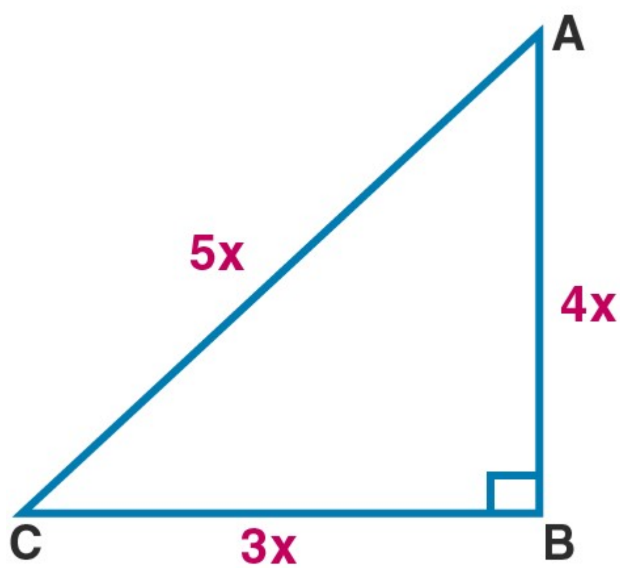
so base, $b = 3x = 3 \times 4 = 12$

altitude, $a = 4x = 4 \times 4 = 16$

Hence the other sides are 12cm and 16cm

6. If the sides of a triangle are in the ratio 3:4:5, prove that it is right-angles triangle.

Solution:



Given the sides are in the ratio 3:4:5.

Let ABC be the given triangle.

Let the sides be $3x, 4x$ and hypotenuse be $5x$.

According to Pythagoras theorem,

$AC^2 = BC^2 + AB^2$

$BC^2 + AB^2 = (3x)^2 + (4x)^2$

$= 9x^2 + 16x^2$

$= 25x^2$

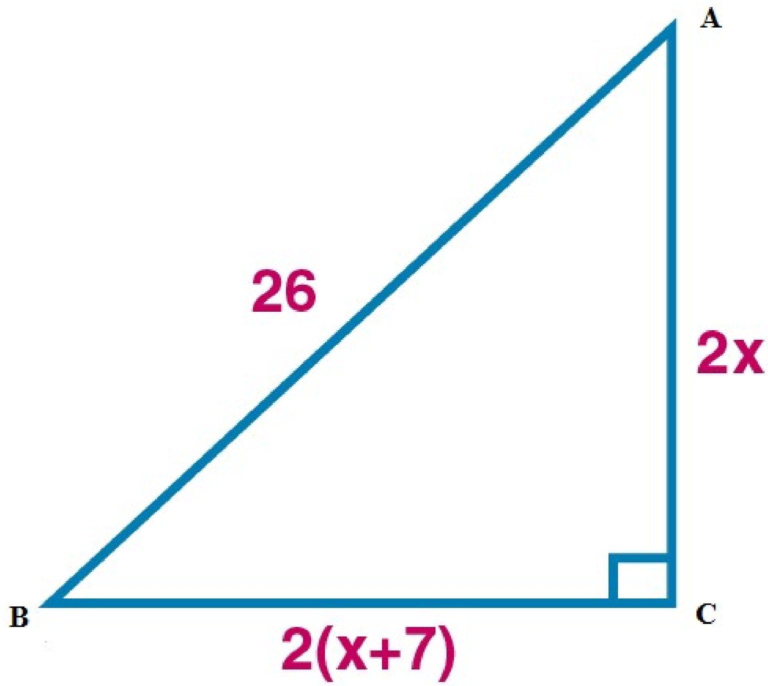
$AC^2 = (5x)^2 = 25x^2$

$AC^2 = BC^2 + AB^2$

Hence ABC is a right angles triangle.

7. For gooing to a city B from city A, there is route via city C such that AC⊥ CB, AC = $2x$km and CB = $2(x + 7)$km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of highway.

Solution:



Given AC = $2x\ km$

CB = $2(x+7)km$

AB = 26

Given AC CB.

According to Pythagoras theorem,

$AB^2 = CB^2 + AC^2$

$26^2 = (2(x+7))^2 + (2x)^2$

$676 = 4(x^2 + 14x + 149) + 4x^2$

$4x^2 + 56\text{x} + 196 + 4x^2 = 676$

$8x^2 + 56x + 196 = 676$

$8x^2 + 56x + 196 - 676 = 0$

$8x^2 + 56x - 480 = 0$

$x^2 + 7x - 60 = 0$

$(x - 5)(x + 12) = 0$

$(x - 5) = 0$ or $(x + 12) = 0$

$x = 5$ or $x = -12$

Length cannot be negative. So $x = 5$

$BC = 2(x + 7) = 2(5 + 7) = 2 \times 12 = 24km$

$AC = 2x = 2 \times 5 = 10km$

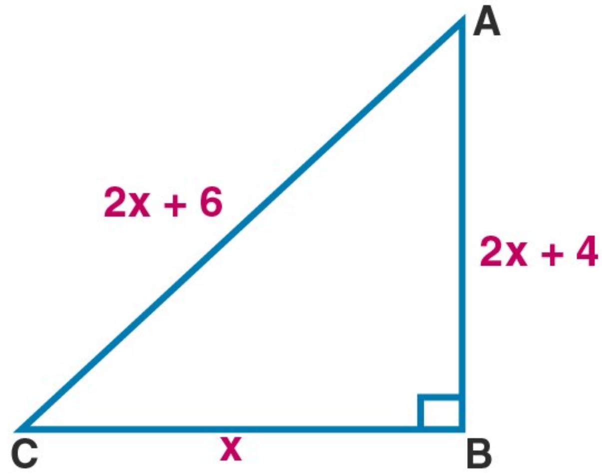
Total distance = AC + BC = 10 + 24 = 34 km

Distance saved = 34 – 26 = 8 km

Hence the distance saved is 8km.

8. The hypotenuse of right triangle is 6m more than twice the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Solution:



Let the shortest side be x.

Then hypotenuse = $2x + 6$

Third side = $2x + 6 - 2 = 2x + 4$

According to Pythagoras theorem,

$AB^2 = CB^2 + AC^2$

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 24x + 36 = x^2 + 4x^2 + 16x + 16$

$x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) = 0$

$x - 10 = 0$ or $x + 2 = 0$

$x = 10$ or $x = -2$

x cannot be negative.

So, shortest side is 10m.

Hypotenuse = $2x + 6$

= $2 \times 10 + 6$

= 20 + 6

= 26 m

Third side = $2x + 4$

= $2 \times 10 + 4$

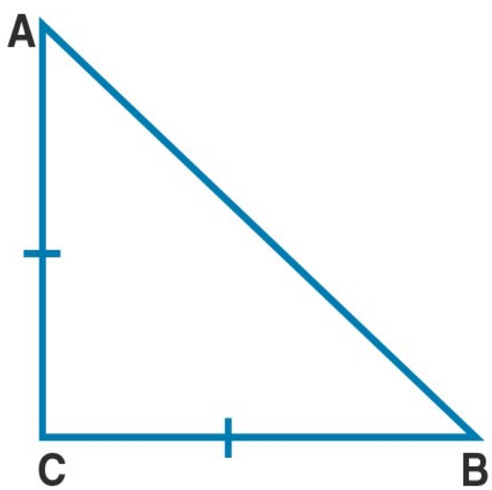
= 20 + 4

= 24 m

Hence the shortest side, hypotenuse and third side of the triangle are 10m, 26m, and 24m respectively.

9. ABC is an isosceles triangle right angles at C. Prove that $AB^2 = 2AC^2$.

Solution:



Let ABC be the isoscles right angled triangle.

C = 90°

AC = BC [isosceles triangle]

According to Pythagoras theorem,

$AB^2 = BC^2 + AC^2$

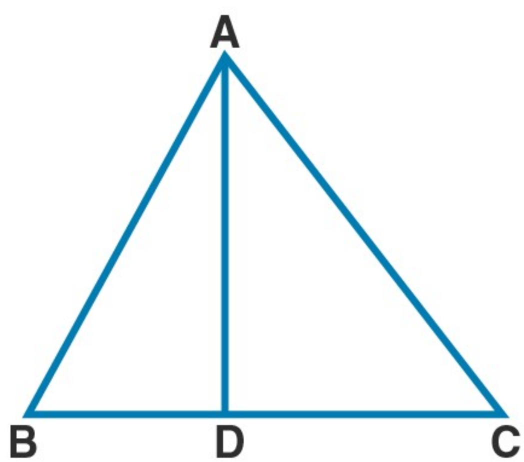
$AB^2 = AC^2 + AC^2 \quad [\because AC = BC]$

$AB^2 = 2AC^2$

Hence proved.

10. In a triangle ABC, AD is perpendicular to BC. Prove that $\boldsymbol{AB^2 + CD^2 = AC^2 + BD^2}$.

Solution:



Given AD BC.

So ADB and ADC are right triangles.

In ADB.

$AB^2 = AD^2 + BD^2$ [Pythagoras theorem]

$AD^2 = AB^2 - BD^2$ ….(i)

In ADC,

$AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$AD^2 = AC^2 - CD^2$ ….(ii)

Comparing (i) and (ii)

$AB^2 - BD^2 = AC^2 - CD^2$

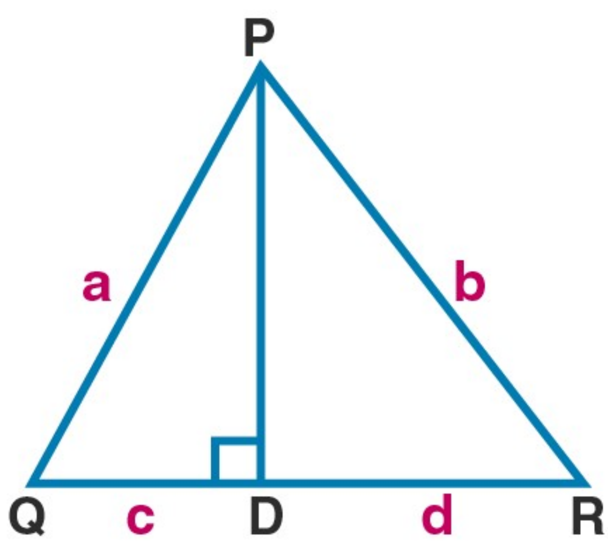
$AB^2 + CD^2 = AC^2 + BD^2$

Hence proved.

11. In $\Delta PQR, PD \perp QR$, such that D lies on QR. If PQ = a, PR= b, QD = c and DR = d,

prove that (a+b)(a-b) = (c+d)(c-d).

Solution:



Given PQ = a, PR = b, QD = c and DR = d.

PD QR.

So PDQ and PDR are right triangles.

In PDQ,

$PQ^2 = PD^2 + QD^2 [Pythagoras\ theorem]$

$PD^2 = PQ^2 - QD^2$

$PD^2 = a^2 - c^2$ ….(i) $[\because PQ = a \text{ and } QD = c]$

In PDR,

$PR^2 = PD^2 + DR^2$ [*Pythagoras theorem*]

$PD^2 = PR^2 - DR^2$

$PD^2 = b^2 - d^2$ …(ii) $[\because PR = b \text{ and } DR = d]$

Comparing (i) and (ii)

$a^2 - c^2 = b^2 - d^2$

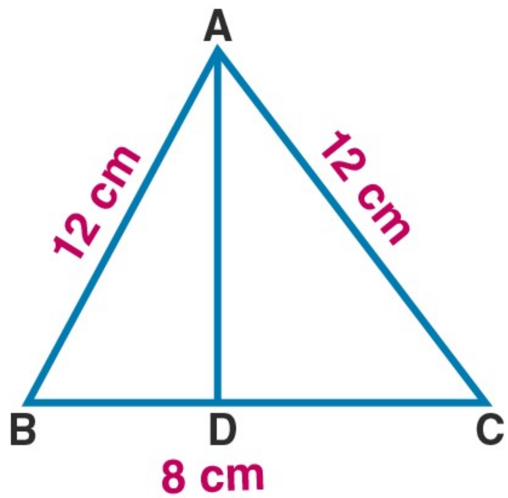
$a^2 - b^2 = c^2 - d^2$

$(a + b)(a - b) = (c + d)(c - d)$

Hence proved.

12. ABC is an isosceles triangle with AB = AC = 12cm and BC =8cm. Find the altitude on BC and Hence, calculate its area.

Solution:



Let AD be the altitude of ABC.

Given AB = AC = 12cm

BC = 8cm

The altitude to the base of an isosceles triangle bisects the base.

So BD = DC

BD = $\frac{8}{2}$ = 4cm

DC = 4cm

ADC is a right triangle.

$AB^2 = BD^2 + AD^2$ [Pythagoras theorem]

$AD^2 = 12^2 - 4^2$

$AD^2 = 144 - 16$

$AD^2 = 128$

Taking square root on both sides,

AD = $\sqrt{128} = \sqrt{2 \times 64} = 8\sqrt{2}$ cm

Area of ABC = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 8 \times \sqrt[8]{2}$

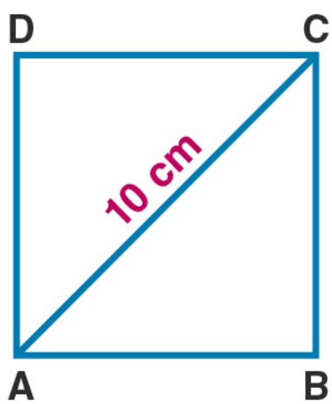
$= 4 \times \sqrt[8]{2}$

$= 32\sqrt{2}\ cm^2$

Hecne the area of triangle is $32\sqrt{2}\ cm^2$.

13. Find the area and the perimeter of a square whose diagonal is 10cm long.

Solution:



Given length of the diagonal of the square is 10cm.

AC = 10

Let AB = BC = x [sides of square are equal in measure]

B = 90° [all angles of a square are 90°]

ABC is a right triangle.

$AC^2 = AB^2 + BC^2$

$10^2 = x^2 + x^2$

$100 = 2x^2$

$x^2 = \frac{100}{2}$

$x^2 = 50$

$x = \sqrt{50}$

$x = \sqrt{(25 \times 2)}$

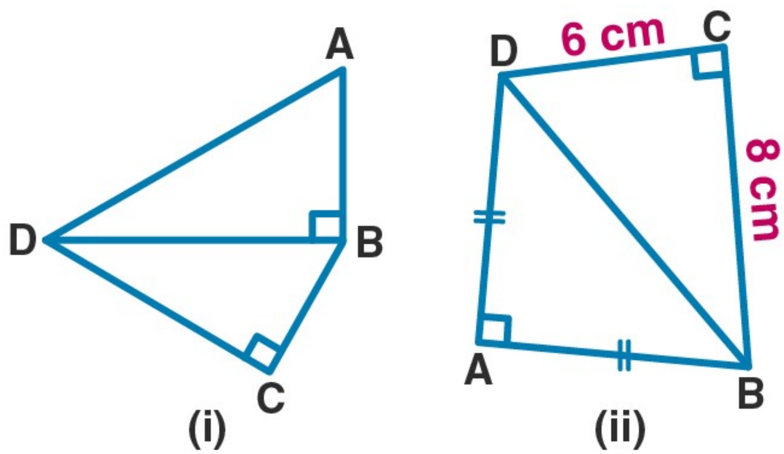
$x = 4 \times 5\sqrt{2}$

$= 20\sqrt{2}$ cm

Hence area and perimeter of the square are $50cm^2$ and $20\sqrt{2}$ cm.

14. (a) In fig. (i) given below, ABCD is a quadrilateral in which AD = 13cm, DC = 12cm, BC =3cm, $\angle ABD = \angle BCD = 90^{\circ}$. Calculate the length of AB.

(b) In fig. (ii) given below, ABCD is a quadrilateral in which AB =Ad, $\angle A = 90^{\circ} = \angle C$, BC = 8cm and CD = 6cm. Find AB and calculate the area of ΔABD.



Solution:

(i) Given AD = 13cm, DC = 12m

BC = 3m

ABD = BCD = 90°

BCD is a right triangle.

$BD^2 = BC^2 + DC^2$ [Pythagoras theorem]

$BD^2 = 3^2 + 12^2$

$BD^2 = 9 + 144$

$BD^2 = 153$

ABD is a right triangle.

$AD^2 = AB^2 + BD^2$ [Pythagoras theorem]

$13^2 = AB^2 + 153$

$169 = AB^2 + 153$

$AB^2 = 169 - 153$

$AB^2 = 16$

Talking square root on both sides,

AB = 4cm

Hence the length of AB is 4cm.

(ii) Given AB = AD, A = 90°= C, BC = 8 cm and DC = 6cm

BCD is a right triangle.

$BD^2 = BC^2 + DC^2$ [Pythagoras theorem]

$BD^2 = 8^2 + 6^2$

$BD^2 = 64 + 36$

$BD^2 = 100$

Taking square root on both sides,

BD = 10 cm

ABD is a right triangle.

$BD^2 = AB^2 + AD^2$ [Pythagoras theorem]

$10^2 = 2AB^2$ [$\because AB = CD$]

100 = $2AB^2$

$AB^2 = \frac{100}{2}$

$AB^2 = 50$

Taking square root on both sides,

AB = $\sqrt{50}$

AB = $\sqrt{(2 \times 25)}$

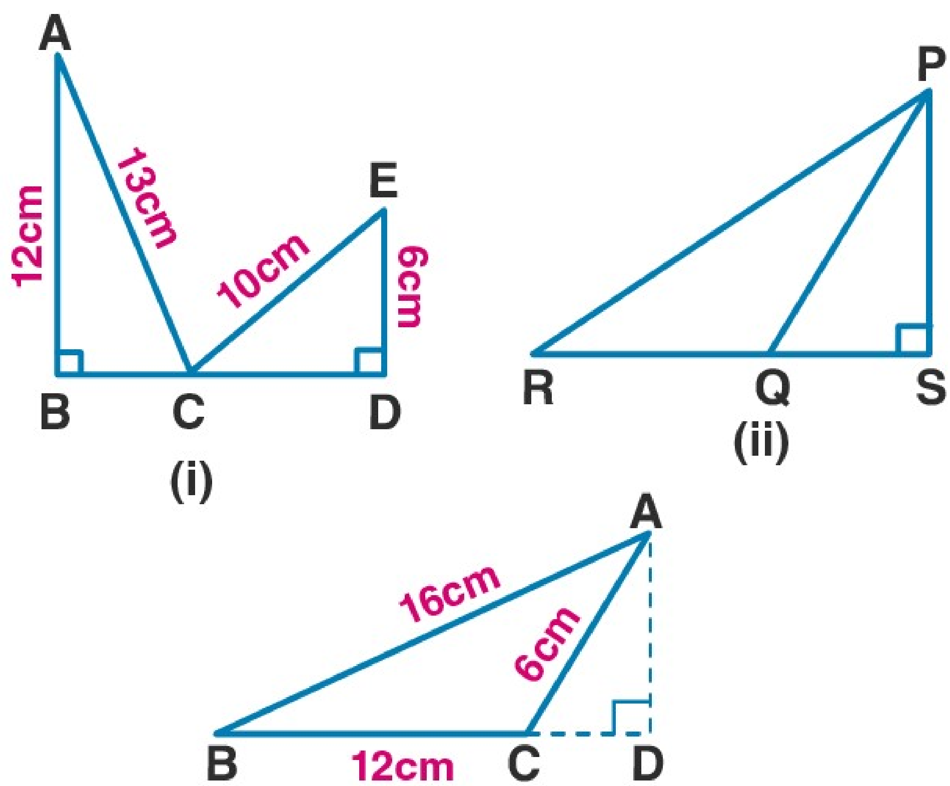
AB = $5\sqrt{2}$ cm

Hence the length of AB is $5\sqrt{2}$ cm.

15. (a) In figure (i) given below, AB = 12cm, AC = 13cm, CD = 10cm and DE = 6cm. Calculate the length of BD.

(b) In figure (ii) given below, $\angle PSR = 90^\circ$, PQ = 10 cm,

(c) In figure (iii) given below, $\angle D = 90^\circ$, AB = 16cm, BC = 12cm and CA = 6cm, Find CD.



(iii)

Solution :

(a) Given AB = 12cm, AC =13cm, CE = 10 cm and DE = 6cm

ABC is a right triangle.

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$13^2 = 12^2 + BC^2$

$BC^2 = 13^2 - 12^2$

$BC^2 = 169 - 144$

$BC^2 = 25$

Taking square root on both sides,

BC = 5 cm

CDE is a right triangle.

$CE^2 = CD^2 + DE^2$ [Pythagoras theorem]

$10^2 = CD^2 + 6^2$

100 = $CD^2 + 36$

$CD^2 = 100 - 36$

$CD^2 = 64$

Taking square root on both sides,

CD = 8cm

BD = BC + CD

BD = 5 + 8

BD = 13 cm

Hence the length of BD is 13 cm.

(b) Given PSR = 90°, PQ = 10cm, QS = 6cm and RQ = 9cm

PSQ is a right triangle.

$PQ^2 = PS^2 + QS^2$ [Pythagoras theorem]

$10^2 = PS^2 + 6^2$

$100 = PS^2 + 36$

$PS^2 = 100 - 36$

$PS^2 = 64$

Taking square root on both sides,

PS = 8cm

PSR is a right triangle.

RS = RQ + QS

RS = 9 + 6

RS = 15 cm

$PR^2 = PS^2 + RS^2$ [Pythagoras theorem]

$PR^2 = 8^2 + 15^2$

$PR^2 = 64 + 225$

$PR^2 = 289$

Taking square root on both sides,

PR = 17 cm

Hence the length of PR is 17 cm.

(c) D = $90^{\circ}, AB = 16cm, BC = 12cm\ and\ CA = 6cm$

ADC is a right triangle.

$AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$6^2 = AD^2 + CD^2 \ldots\ldots(i)$

ABD is a right triangle.

$AB^2 = AD^2 + BD^2$ [Pythagoras theorem]

$16^2 = AD^2 + (BC + CD)^2$

$16^2 = AD^2 + (12 + CD)^2$

$256 = AD^2 + 144 + 24CD + CD^2$

$256 - 144 = AD^2 + CD^2 + 24CD$

$AD^2 + CD^2 = 112 - 24CD$

$6^2 = 112 - 24CD$ [from (i)]

36 = $112 - 24CD$

$24CD = 112 - 36$

$24CD = 76$

CD = $\frac{76}{24} = \frac{19}{6}$

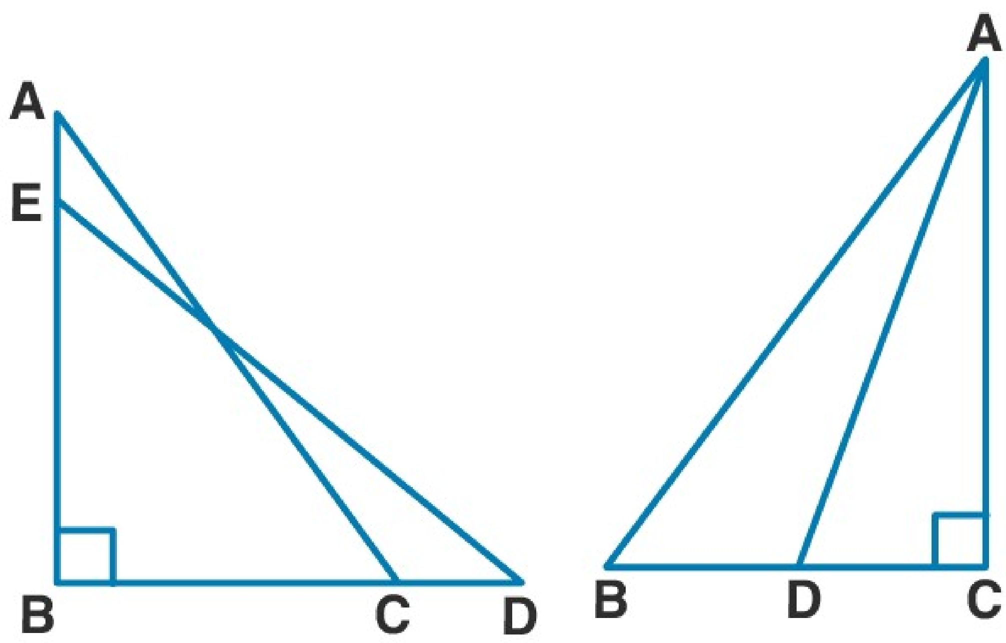
∴ CD = $3\frac{1}{6}$

Hence the length of CD is $3\frac{1}{6}$ cm

16. (a) In figure (i) given below, BC = 5cm,

$\angle B = 90^\circ$, AB = 5AE, CD = 2AE and AC = ED. Calculate the lengths of EA, CD, AB and AC.

(b) In the figure (ii) given below, ABC is a right triangle right angles at C. If D is mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$.



(i) (ii)

Solution:

(a) Given BC = 5cm,

B = $90^\circ, AB = 5AE,$

CD = 2AE and AC = ED

ABC is a right triangle.

$AC^2 = AB^2 + BC^2$ ….(i) [Pythagoras theorem]

BED is a right triangle.

$ED^2 = BE^2 + BD^2$ [Pythagoras theorem]

$AC^2 = BE^2 + BD^2$ ….(ii) [$\because AC = ED$]

Comparing (i) and (ii)

$AB^2 + BC^2 = BE^2 + BD^2$

$(5AE)^2 + 5^2 = (4AE)^2 + (BC + CD)^2$ [$\because BE = AB - AE = 5AE - AE = 4AE$]

$(5AE)^2 + 25 = (4AE)^2 + (5 + 2AE)^2$ …(iii)

[$\because BC = 5,\ CD = 2AE$]

Let AE = x. So (iii) becomes,

$(5x)^2 + 25 = (4x)^2 + (5 + 2x)^2$

$25x^2 + 25 = 16x^2 + 25 + 20x + 4x^2$

$25x^2 = 20x + 20x$

$\mathbf{5}x^2 = 20x$

$x = \frac{20}{5} = 4$

AE = 4cm

CD = 2AE = $2 \times 4 = 8cm$

AB = 5AE

AB = $5 \times 4 = 20cm$

ABC is a right triangle.

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$AC^2 = 20^2 + 5^2$

$AC^2 = 400 + 25$

$AC^2 = 425$

Taking square root on both sides,

AC = $\sqrt{425}$ = $\sqrt{(25 \times 17)}$

AC = $5\sqrt{17}$ cm

Hence EA = 4cm, CD = 8cm, AB = 20cm and AC = $\sqrt[5]{17}$ cm.

(b) Given D is the midpoint of BC.

DC = $\frac{1}{2}$BC

ABC is a right triangle.

$AB^2 = AC^2 + BC^2$ …(i) [Pythagoras theorem]

ADC is a right triangle.

$AD^2 = AC^2 + DC^2$ …(ii) [Pythagoras theorem]

$AC^2 = AD^2 - DC^2$

$AC^2 = AD^2 - \left(\frac{1}{2}\, BC\right)^2$ $[\because DC = \frac{1}{2}\, BC\ ]$

$AC^2 = AD^2 - \frac{1}{4}BC^2$

$4AC^2 = 4AD^2 - BC^2$

$AC^2 + 3AC^2 = 4AD^2 - BC^2$

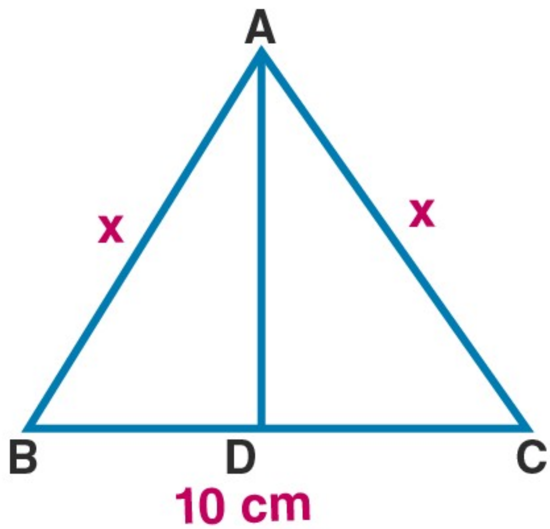
$AC^2 + BC^2 = 4AD^2 - 3AC^2$

$AB^2 = 4AD^2 - 3AC^2$ [from(i)]

Hence proved.

17. In ΔABC, $AB = AC = x, BC = 10cm$ and the area of ΔABC is $60cm^2$. Find x.

Solution:



Given AB = AC = x

So ABC in an isosceles triangle.

AD BC

The altitude to the base of an isosceles triangle bisects the base.

BD = DC = $\frac{10}{2}$ = 5 cm

Given area = 60 cm^2

$\frac{1}{2} \times base \times height = \frac{1}{2} \times 10 \times AD = 60$

AD = 60 $\times \frac{2}{10}$

AD = $\frac{60}{5}$

AD = 12cm

ADC is a right triangle.

$AC^2 = AD^2 + DC^2$

$x^2 = 12^2 + 5^2$

$x^2 = 144 + 25$

$x^2 = 169$

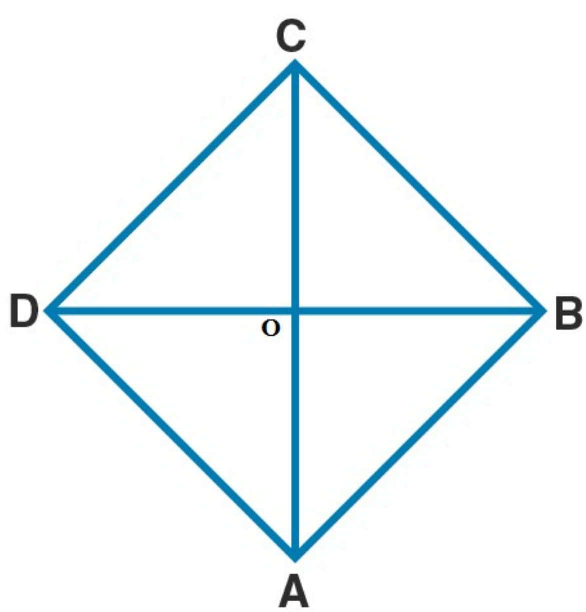
Taking square root on both sides

$x = 13cm$

Hence the value of x is 13 cm.

18. In a rhombus, if diagonals are 30cm and 40cm, find its perimeter.

Solution:



Let ABCD be the rhombus.

Given AC = 30cm

BD = 40 cm

Diagonals of a rhombus are perpendicular bisectors of each other.

OB = $\frac{1}{2} BD = \frac{1}{2} \times 40 = 20cm$

OC = $\frac{1}{2} AC = \frac{1}{2} \times 30 = 15cm$

OCB is a right triangle.

$BC^2 = OC^2 + OB^2$ [Pythagoras theorem]

$BC^2 = 15^2 + 20^2$

$BC^2 = 225 + 400$

$BC^2 = 625$

Taking square root on both sides

BC = 25cm

So side of a rhombus, a = 25cm

Perimeter = 4a = 4×25 =100 cm

Hence the perimeter of the rhombus is 100 cm.

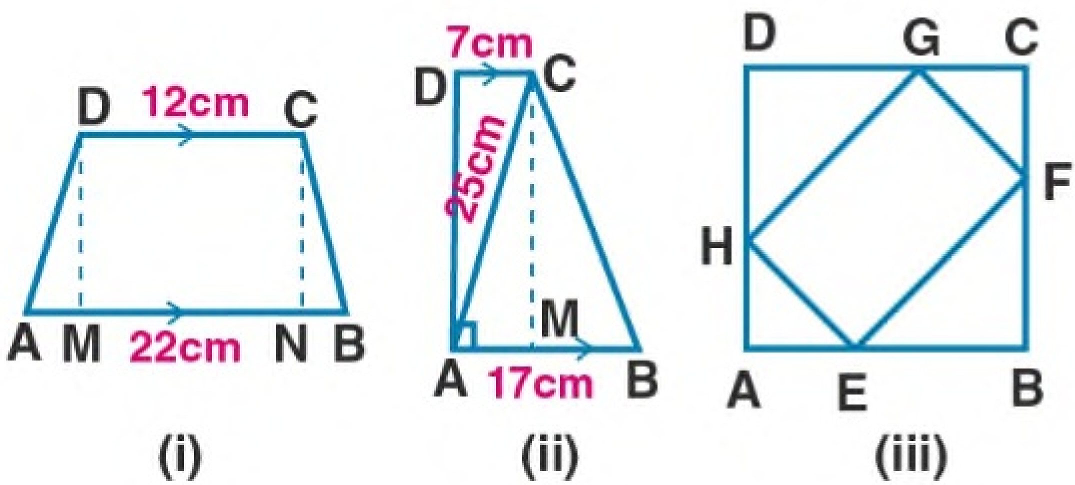
19. (a) In figure (i) given below, AB || DC, BC = AD = 13 cm. AB = 22 cm and DC = 12cm. Calculate the height of the trapezium ABCD.

(b) In figure (ii) given below, AB || DC, $\angle A = 90^{\circ}$, DC = 7cm, AB = 17cm and AC = 25cm. Calculate BC.

(c) In figure (iii) given below, ABCD is a square of side 7cm. If AE = FC = CG = HA = 3cm,

(i) Prove that EFGH is a rectangle.

(ii) find the area and perimeter of EFGH.



Solution:

(i) Given AB || DC, BC = AD = 13cm.

AB = 22cm and DC = 12cm

Here DC = 12

MN = 12cm

AM = BN

AB = AM + MN + BN

22 = AM + 12 + AM $[\because AM = BN]$

2AM = 22 – 12 = 10

AM = $\frac{10}{2}$

AM = 5cm

AMD is a right triangle.

$AD^2 = AM^2 + DM^2$ [Pythagoras theorem]

$13^2 = 5^2 + DM^2$

$DM^2 = 13^2 - 5^2$

$DM^2 =$ 169 -25

$DM^2 =$ 144

Taking square root on both sides,

DM = 12cm

Hence the height of the trapezium is 12 cm.

(b) Given AB || Dc, A = 90°, DC = 7cm, AB = 17cm and AC = 25cm.

ADC is a right triangle.

$\boldsymbol{AC^2 = AD^2 + DC^2}$ **[Pythagoras theorem]**

$25^2 = AD^2 + 7^2$

$AD^2 = 25^2 - 7^2$

$AD^2 = 625 - 49$

$AD^2 = 576$

Taking square root on both sides

AD = 24cm

CM = 24 cm [$\because ABCD$]

DC = 7cm

AM = 7 cm

BM = AB – AM

BM = 17 – 7 = 10 cm

BMC is a right triangle.

$BC^2 = BM^2 + CM^2$

$BC^2 = 10^2 + 24^2$

$BC^2 =$ 100 + 576

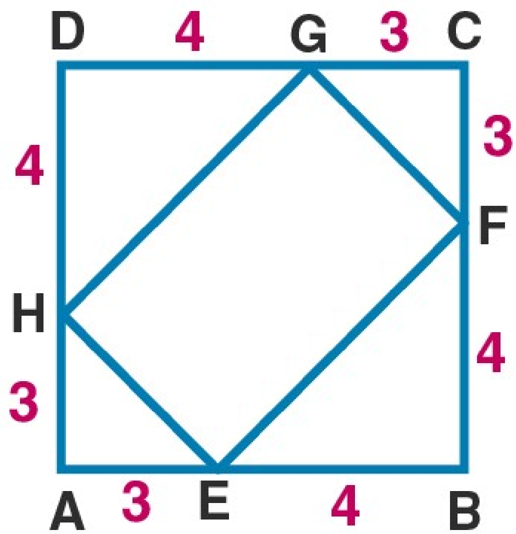
$BC^2 =$ 676

Taking square root on both sides

BC = 26 cm

Hence length of BC is 26 cm.

(c) (i) Proof:



Given ABCD is a square of side 7cm.

So AB = BC = CD = AD = 7cm

Also given AE = FC = CG = HA = 3cm

BE = AB – AE = 7 – 3 = 4cm

BF = BC – FC = 7 – 3 = 4 cm

GD = CD – CG = 7 – 3 = 4 cm

DH – AD – HA = 7 – 3 = 4 cm

A = 90° [Each angle of a square equals 90°]

AHE is a right triangle.

$HE^2 = AE^2 + AH^2$ [Pythagoras theorem]

$HE^2 = 3^2 + 3^2$

$HE^2 =$ 9 + 9 = 18

HE = $\sqrt{9 \times 2} = 3\sqrt{2}$ cm

Similarly GF = $3\sqrt{2}$ cm

EBF is a right triangle.

$EF^2 = BE^2 + BF^2$ [Pythagoras theorem]

$EF^2 = 4^2 + 4^2$

$EF^2 = 16 + 16 = 32$

Taking square root on both sides

EF = $\sqrt{16 \times 2}$ = $4\sqrt{2}$ cm

Similarly HG = $4\sqrt{2}$ cm

Now join EG

In EFG

$EG^2 = EF^2 + GF^2$

$EG^2 = (4\sqrt{2})^2 + (3\sqrt{2})^2$

$EG^2 =$ 32 + 18 = 50

$EG = \sqrt{50}$ = $5\sqrt{2}$ cm….(i)

Join HF.

Also $HF^2 = EH^2 + HG^2$

= $(3\sqrt{2})^2$ + $(4\sqrt{2})^2$

= 18 + 32

= 50

HF = $\sqrt{50}$

= $5\sqrt{2}$ cm…..(ii)

From (i) and (ii)

EG = HF

Diagonals of the quadrilateral are congruent.. So EFGH is a rectangle.

Hence proved.

(ii) Area of rectangle EFGH = length $\times$ $breadth$

= HE $\times$ EF

= $3\sqrt{2} \times 4\sqrt{2}$

= 24 cm^2

Perimeter of rectangle EFGH = 2(length +breadth)

= 2 $\times$ $\left(4\sqrt{2} + 3\sqrt{2}\right)$

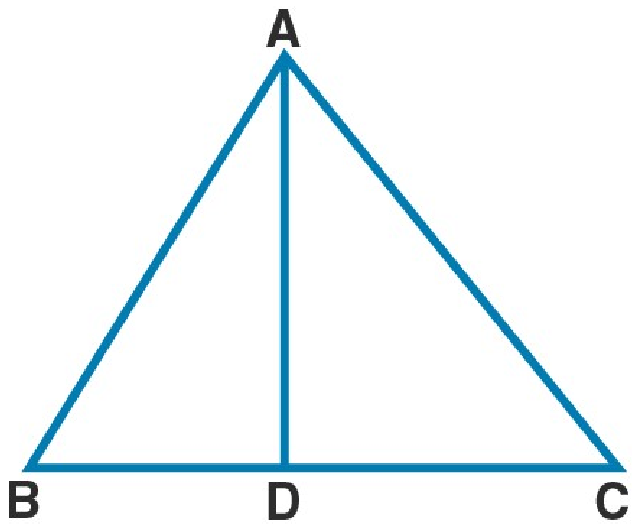
= 2 $\times$ $7\sqrt{2}$

= $14\sqrt{2}$ cm

Hence area of the rectangle is $24cm^2$ and perimeter is $14\sqrt{2}$ cm.

20. AD is perpendicular to the side BC of an equilateral ΔABC. Prove that $4AD^2 = 3AB^2$.

Solution:



Given ADBC

D = 90°

Proof :

Since ABC is an equilateral triangle,

AB = AC = BC

ABD is a right triangle.

According to Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

BD = $\frac{1}{2}$ BC

$AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$

$AB^2 = AD^2 + \left(\frac{1}{2}AB\right)^2$ [$\because BC = AB$]

$AB^2 = AD^2 + \frac{1}{4}AB^2$

$AB^2 = \frac{(4AD^2 + AB^2)}{4}$

$4AB^2 = 4AD^2 + AB^2$

$4AD^2 = 4AB^2 - AB^2$

$4AD^2 = 3AB^2$

Hence proved.

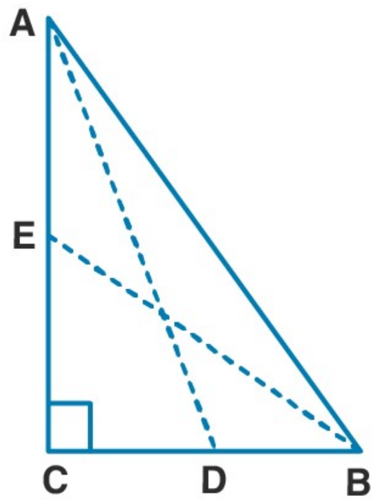
21. In figure (i) given below, D and E are mid-points of the sids BC and CA respectively of a ΔABC, right angles at C.

Prove that :

(i) $4AD^2 = 4AC^2 + BC^2$

(ii) $4BE^2 = 4BC^2 + AC^2$

(iii) $4(AD^2 + BE^2) = 5AB^2$



Solution:

Proof :

(i) C = 90°

So ACD is a right triangle.

$AD^2 = AC^2 + CD^2$ [Pythagoras theorem]

Multiply both sides by 4, we get

$4AD^2 = 4AC^2 + 4CD^2$

$4AD^2 = 4AC^2 + 4BD^2$

$[\because D \text{ is the midpoint of } BC, CD = BD = \frac{1}{2}BC]$

$4AD^2 = 4AC^2 + (2BD)^2$

$4AD^2 = 4AC^2 + BC^2$ ……(i) $[\because BC = 2BD]$

Hence proved.

(ii) BCE is right triangle.

$BE^2 = BC^2 + CE^2$ [Pythagoras theorem]

Multiply both sides by 4, we get

$4BE^2 = 4BC^2 + 4CE^2$

$4BE^2 = 4BC^2 + (2\,CE)^2$

$4BE^2 = 4BC^2 + AC^2$ …. (ii)

$\left[\because E \text{ is the midpoint of } AC, AE = CE = \frac{1}{2}\,AC\right]$

Hence proved.

(iii) Adding (i) and (ii)

$4AD^2 + 4BE^2 = 4AC^2 + BC^2 + 4BC^2 + AC^2$

$4AD^2 + 4BE^2 = 5AC^2 + 5BC^2$

$4(AD^2 + BE^2) = 5(AC^2 + BC^2)$

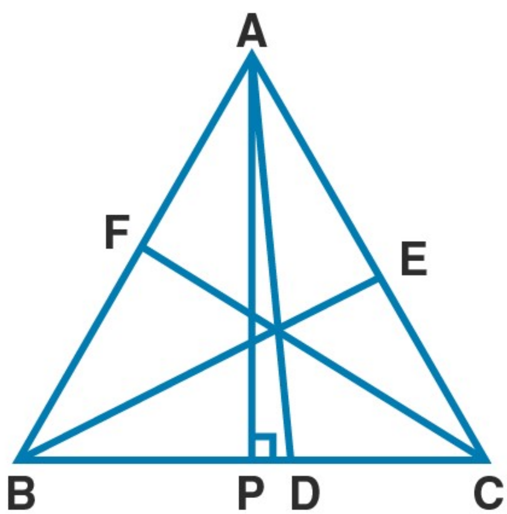
$4(AD^2 + BE^2) = 5(AB^2)$

$[\because ABC\ is\ a\ right\ triangle, AB^2 = AC^2 + BC^2]$

Hence proved.

22. If AD, BE and CF are medians of ABC, prove that $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$.

Solution:



Construction :

Draw APBC

Proof :

APB is a right triangle.

$AB^2 = AP^2 + BP^2$ [Pythagoras theorem]

$AB^2 = AP^2 + (BD - PD)^2$

$AB^2 = AP^2 + BD^2 + PD^2 - 2BD \times PD$

$AB^2 = (AP^2 + PD^2) + BD^2 - 2BD \times PD$

$AB^2 = AD^2 + \left(\frac{1}{2} BC\right)^2 - 2 \times \left(\frac{1}{2} BC\right) \times PD$

$\left[\because AP^2 + PD^2 = AD^2 \text{ and } BD = \frac{1}{2} BC\right]$

$AB^2 = AD^2 + \frac{1}{4} BC^2 - BC \times PD \ldots\ldots (i)$

APC is a right triangle.

$AC^2 = AP^2 + PC^2$ [Pythagoras theorem]

$AC^2 = AP^2 + (PD^2 + DC^2)$

$AC^2 = AP^2 + PD^2 + DC^2 + 2 \times PD \times DC$

$AC^2 = (AP^2 + PD^2) + \left(\frac{1}{2} BC\right)^2 + 2 \times PD \times \left(\frac{1}{2} BC\right)$

$\left[DC = \frac{1}{2} BC\right]$

$AC^2 = AD^2 + \frac{1}{4} BC^2 + PD \times DC$ …(ii)

[In APD, $AP^2 + PD^2 = AD^2$]

Adding (i) and (ii), we get

$AB^2 + AC^2 = 2AD^2 + \frac{1}{2} BC^2$….(iii)

Draw perpendicular from B and C to AC and AB respectively.

Similarly we get,

$BC^2 + CA^2 = 2CF^2 + \frac{1}{2} AB^2$ ………(iv)

$AB^2 + BC^2 = 2BE^2 + \frac{1}{2} AC^2$ ………(v)

Adding (iii), (iv) and (v) we get

$2(AB^2 + BC^2 + CA^2) = 2(AD^2 + BE^2 + CF^2) + \frac{1}{2}(BC^2 + AB^2 + AC^2)$

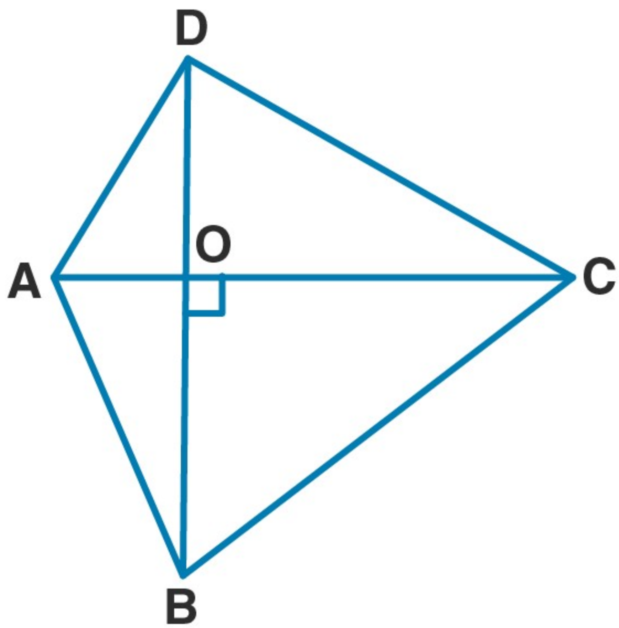
$2(AB^2 + BC^2 + CA^2) = 2(AB^2 + BC^2 + CA^2) - \frac{1}{2}(AB^2 + BC^2 + CA^2)$

$2(AD^2 + BE^2 + CF^2) = \left(\frac{3}{2}\right) \times (AB^2 + BC^2 + CA^2)$

$4(AD^2 + BE^2 + CF^2) = 3(AB^2 + BC^2 + CA^2)$

Hence proved.

23. (a) In fig. (i) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that $\boldsymbol{AB^2 + CD^2 = AD^2 + BC^2}$.



Solution:

Given diagonals of quadrilateral ABCD, AC and Bd intersect at O at right angles.

Proof :

AOB is a right triangle.

$AB^2 = OB^2 + OA^2$ …..(i) [Pythagoras theorem]

COD is a right triangle.

$CD^2 = OC^2 + OD^2$ ….(ii) [Pythagoras theorem]

Adding (i) and (ii), we get

$AB^2 + CD^2 = OB^2 + OA^2 + OC^2 + OD^2$

$AB^2 + CD^2 = (OA^2 + OD^2) + (OC^2 + OB^2)..(iii)$

AOD is a right triangle.

$AD^2 = OA^2 + OD^2$ ..(iv) [Pythagoras theorem]

BOC is a right triangle.

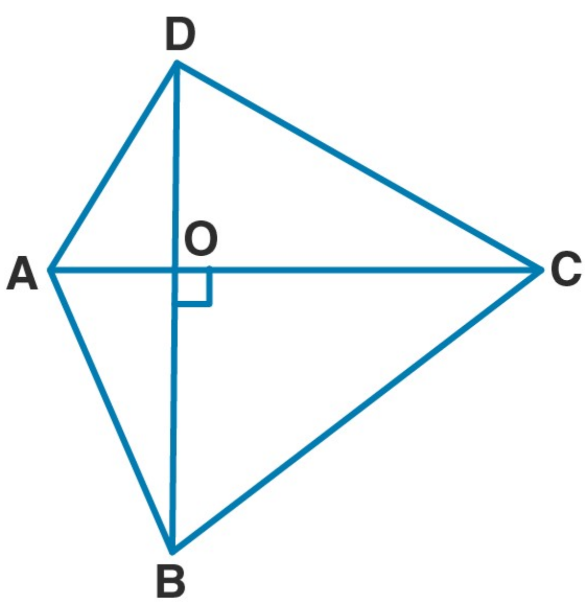
$BC^2 = OC^2 + OB^2$ ..(v) [Pythagoras theorem]

Substitute (iv) and (v) in (iii), we get

$AB^2 + CD^2 = AD^2 + BC^2$

Hence proved.

23. (a) In fig. (i) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that $\boldsymbol{AB^2 + CD^2 = AD^2 + BC^2}$.



Solution:

Given diagonals of quadrilateral ABCD, AC and BD intersect O at right angles.

Proof :

Given diagonals of quadrilateral ABCD, AC and BD intersect at O at right angles.

Proof :

AOB is a right triangle.

$AB^2 = OB^2 + OA^2$ ….. (i) [Pythagoras theorem]

COD is a right triangle.

$CD^2 = OC^2 + OD^2$ ….. (i) [Pythagoras theorem]

Adding (i) and (ii), we get

$AB^2 + CD^2 = OB^2 + OA^2 + OC^2 + OD^2$

$AB^2 + CD^2 = (OA^2 + OD^2) + (OC^2 + OB^2)$…(iii)

AOD is a right triangle.

$AD^2 = OA^2 + OD^2$ ..(iv) [Pythagoras theorem]

BOC is a right triangle.

$BC^2 = OC^2 + OB^2$ ..(v) [Pythagoras theorem]

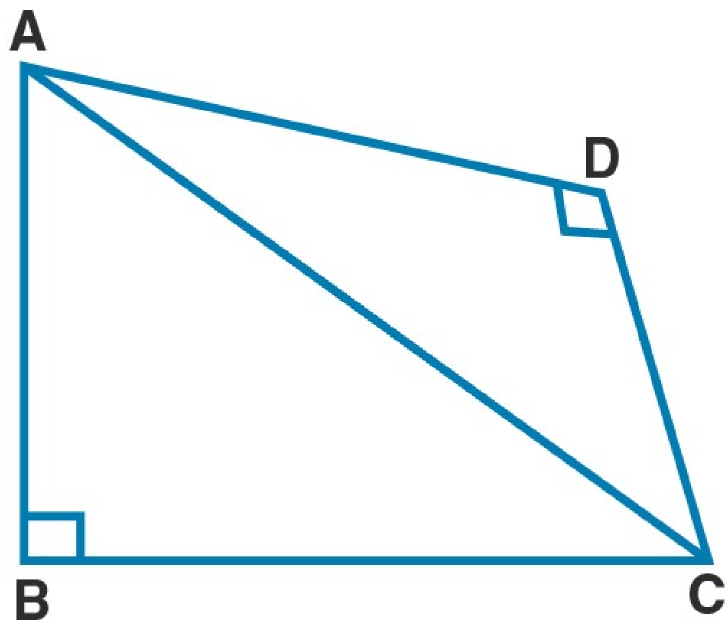
Substitute (iv) and (v) in (iii), we get

$AB^2 + CD^2 = AD^2 + BC^2$

Hence proved.

24. In a quadrilateral ABCD, B = 90° = D. Prove that $2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$.

Solution:



Given B = D = 90°

So, ABC and ADC are right triangles.

In ABC,

$AC^2 = AB^2 + BC^2$ ...(i) [Pythagoras theorem]

In ADC,

$AC^2 = AD^2 + DC^2$ ...(ii) [Pythagoras theorem]

Adding (i) and (ii)

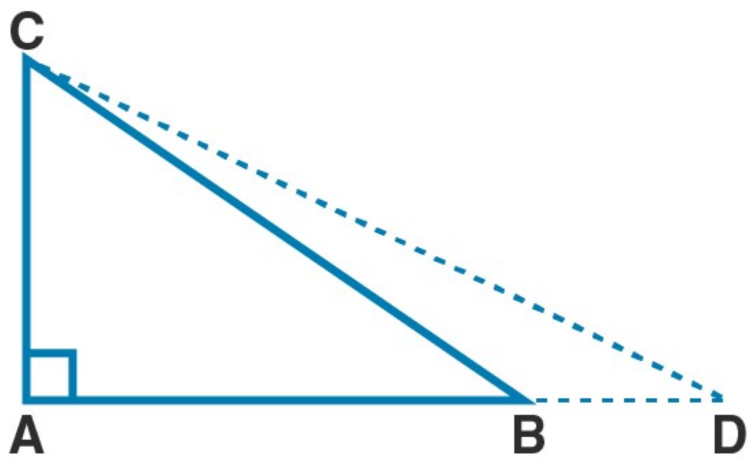
$2AC^2 = AB^2 + BC^2 + AD^2 + DC^2$

$2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$

Hence proved.

25. In a ΔABC, A= 90°, $CA = AB$ and D is a point on AB produced. Prove that : $DC^2 - BD^2 = 2AB \times AD$.

Solution:



Given A = 90°

CA = AB

Proof :

in ACD,

$DC^2 = CA^2 + AD^2$ $[Pythagoras\ theorem]$

$DC^2 = CA^2 + (AB + BD)^2$

$DC^2 = CA^2 + AB^2 + BD^2 + 2AB \times BD$

$DC^2 - BD^2 = CA^2 + AB^2 + 2AB \times BD$

$DC^2 - BD^2 = AB^2 + AB^2 + 2AB \times BD \quad [\because CA = AB]$

$DC^2 - BD^2 = 2AB^2 + 2AB \times BD$

$DC^2 - BD^2 = 2AB\ (AB + BD)$

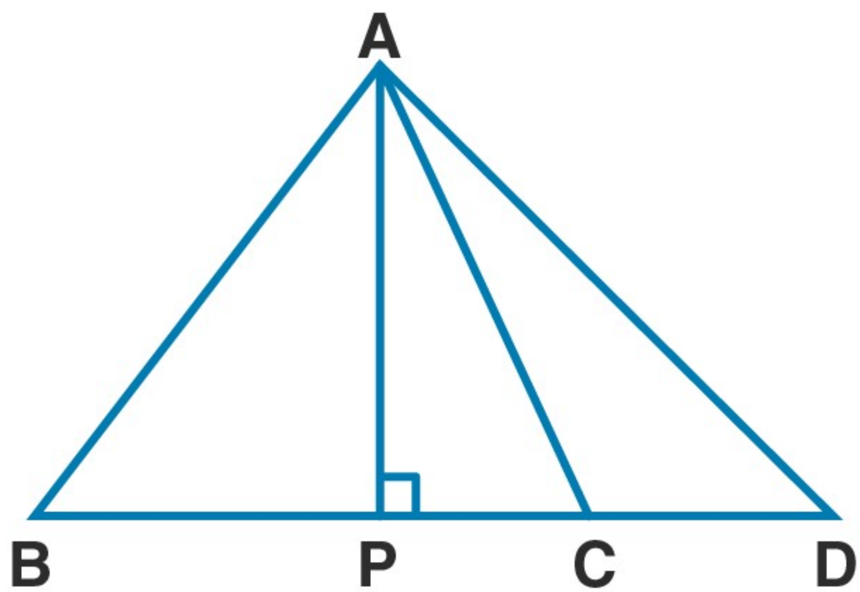
$DC^2 - BD^2 = 2AB \times AD[A - B - D]$

Hence proved.

26. In an isosceles triangle ABC, AB = AC and D is a point on BC produced.

Prove that $AD^2 = AC^2 + BD.CD.$

Solution:



Given ABC is an isosceles triangle.

AB = BC

Construction : Draw AP BC

Proof :

APD is a right triangle.

$AD^2 = AP^2 + PD^2$ [Pythagoras theorem]

$AD^2 = AP^2 + (PC + CD)^2$ [PD = PC + CD]

$AD^2 = AP^2 + PC^2 + CD^2 + 2PC \times CD$ ….(i)

APC is a right triangle.

$AC^2 = AP^2 + PC^2$ …(ii) [Pythagoras theorem]

Substitute (ii) in (i)

$AD^2 = AC^2 + CD^2 + 2PC \times CD$ .... (iii)

Since ABC is an isoscels triangle,

PC = $\frac{1}{2}$ BC

[The altitude to the base of an isosceles triangle bisects the base]

$AD^2 = AC^2 + CD^2 + 2 \times \frac{1}{2} BC \times CD$

$AD^2 = AC^2 + CD^2 + BC \times CD$

$AD^2 = AC^2 + CD(CD + BC)$

$AD^2 = AC^2 + CD \times BD$ $[CD + BC = BD]$

$AD^2 = AC^2 + BD \times CD$

Hence proved.

Chapter Test

1. a) In fig. (i) given below, AD $\perp BC, AB = 25cm,$ AC= 17 cm and AD = 15 cm. Find the length of BC.

b) In figure (ii) given below, BAC = 90°, ADC = 90°, AD = 6cm, CD = 8cm and BC = 26 cm.

Find : (i) AC

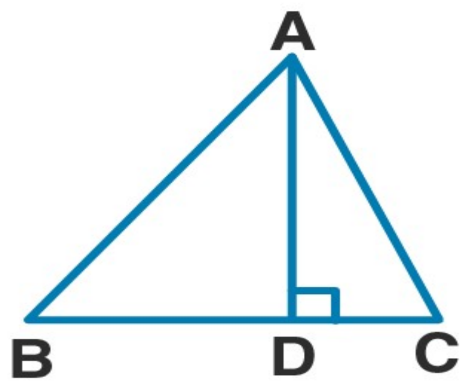
(ii) AB

(iii) area of the shaded region.

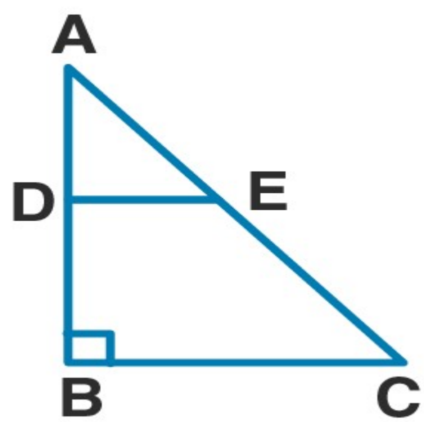
(c) In figure (iii) given below, triangle ABC is right angles at B. Given that AB = 9 cm, AC = 15cm and D, E are mid-points of the sides AB and AC respectively, calculate

(i) the length of BC

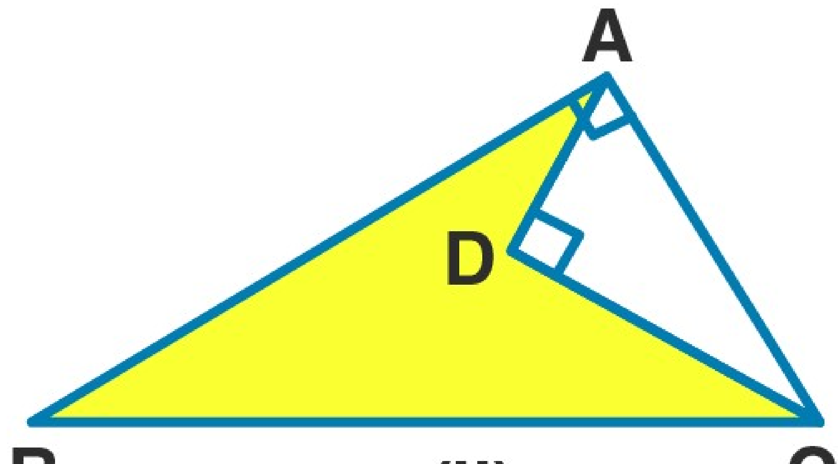
(ii) the area of ΔADE.



(i)



(iii)



(ii)

Solution:

(a) Given AD⊥BC,AB=25cm, AC= 17 cm and AD = 15 cm

ADC is a right triangle.

$AC^2 = AD^2 + DC^2$ [Pythagoras theorem]

$17^2 = 15^2 + DC^2$

289 = 225 + DC^2

$DC^2 = 289 - 225$

$DC^2 = 64$

Taking square root on both sides,

DC = 8cm

ADB is a right triangle.

$AB^2 = AD^2 + BD^2$ [Pythagoras theorem]

$25^2 = 15^2 + BD^2$

625 = 225 + BD^2

$BD^2 = 625 - 225 = 400$

Taking square root on both sides,

BD = 20 cm

BC = BD + Dc

= 20 + 8

= 28 cn

Hence the length of BC is 28 cm.

(b) Given BAC = 90°, ADC = 90°, AD = 6cm, CD = 8cm and BC = 26 cm.

(i) ADC is a right triangle.

$AC^2 = AD^2 + DC^2$ [Pythagoras theorem]

$AC^2 = 6^2 + 8^2$

$AC^2 = 36 + 64$

$AC^2 = 100$

Taking square root on both sides,

AC = 10 cm

Hence length of AC is 10 cm.

(ii) ABC is a right triangle.

$BC^2 = AC^2 + AB^2$ [Pythagoras theorem]

$26^2 = 10^2 + AB^2$

$AB^2 = 26^2 - 10^2$

$AB^2 = 676 - 100$

$AB^2 = 576$

Taking square root on both sides,

AB = 24 cm

Hence length of AB is 24 cm.

(iii) Area of ABC = $\frac{1}{2} \times AB \times AC$

= $\frac{1}{2} \times 24 \times 10$

=120 cm^2

Area of ADC = $\frac{1}{2} \times AD \times DC$

= $\frac{1}{2} \times 6 \times 8$

=24 cm^2

Area of shaded region = area of ABC – area of ADC

= 120 -24

= 96 cm^2

Hence the area of shaded region is 96 cm^2.

(c) Given B = 90°

AB = 9 cm, AC = 15 cm.

D, E are mid- points of the sides AB and AC respectively.

(i) ABC is a right triangle.

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$15^2 = 9^2 + BC^2$

225 = 81 + BC^2

BC^2 = 225 - 81

$BC^2 = 144$

Taking square root on both sides,

BC = 12 cm

Hence the length of BC is 12 cm.

(ii) AD = $\frac{1}{2}$ AB [D is the midpoint of AB]

AD = $\frac{1}{2} \times 9 = \frac{9}{2}$

AE = $\frac{1}{2} AC$ [E is the midpoint of AC]

AE = $\frac{1}{2} \times 15 = \frac{15}{2}$

ADE is a right triangle.

$AE^2 = AD^2 + DE^2$ [Pythagoras theorem]

$\left(\frac{15}{2}\right)^2 = \left(\frac{9}{2}\right)^2 + DE^2$

$DE^2 = \left(\frac{15}{2}\right)^2 - \left(\frac{9}{2}\right)^2$

$DE^2 = \frac{225}{4} - \frac{81}{4}$

$DE^2 = \frac{144}{4}$

Taking square root on both sides,

DE = $\frac{12}{2} =$ 6 cm.

Area of ADE = $\frac{1}{2} \times DE \times AD$

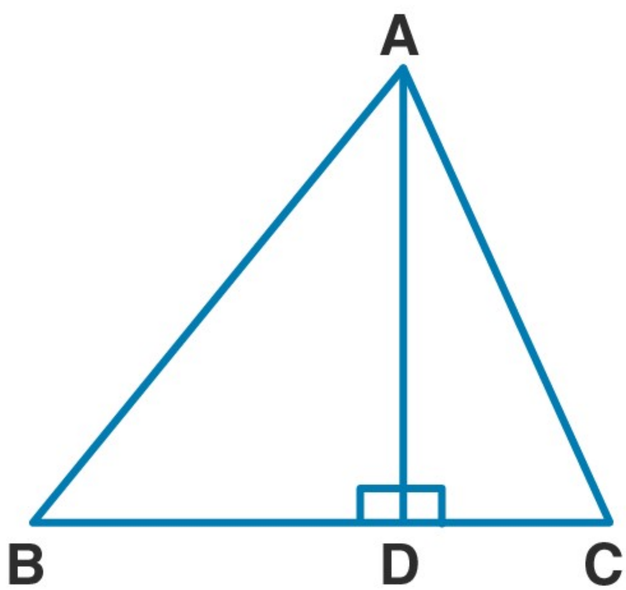
$= \frac{1}{2} \times 6 \times \frac{9}{2}$

=13.5 cm^2

Hence the area of the ADE is 13.5 cm^2.

2. If in $\Delta ABC, AB > AC$ and AD BC, prove that $AB^2 - AC^2 = BD^2 - CD^2$

Solution:



Given AD BC, $AB > AC$

So ADB and ADC are right triangles.

Proof :

In ADB,

$AB^2 = AD^2 + BD^2$ [Pythagoras theorem]

$AD^2 = AB^2 - BD^2$ ..(i)

In ADC,

$AC^2 = AD^2 + CD^2$ [Pythagoras theorem]

$AD^2 = AC^2 - CD^2$ ..(ii)

Equation (i) and (ii)

$AB^2 - BD^2 = AC^2 - CD^2$

$AB^2 - AC^2 = BD^2 - CD^2$

Hence proved.

3. In a right angles triangle ABC, right angles at C, P and Q are the points on the sides CA and CB respectively which divide these sides in the ratio 2:1.

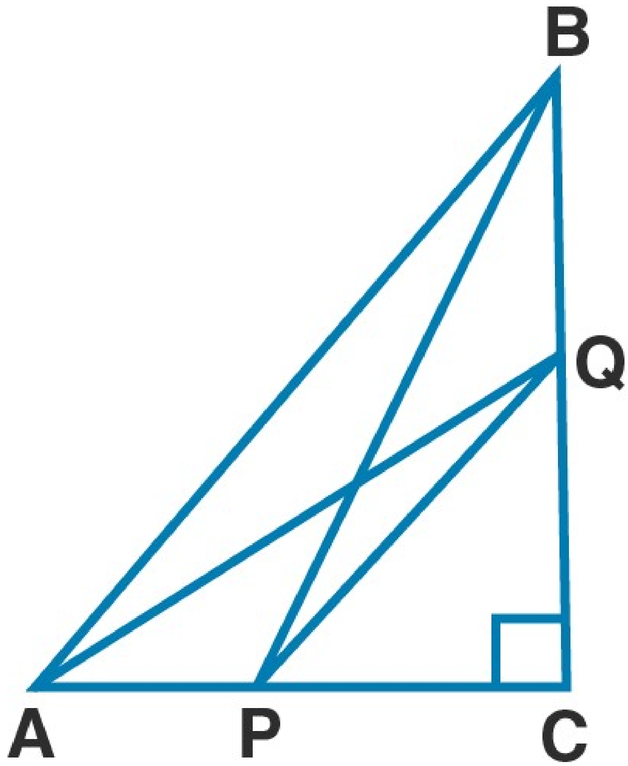
Prove that

(i) $9\boldsymbol{AQ^2} = \mathbf{9}\boldsymbol{AC^2} + \mathbf{4}\boldsymbol{BC^2}$

(ii) $9\boldsymbol{BP^2} = \mathbf{9}\boldsymbol{BC^2} + \mathbf{4}\boldsymbol{AC^2}$

(iii) $9(\boldsymbol{AQ^2} + \boldsymbol{BP^2}) = \mathbf{13}\boldsymbol{AB^2}$

Solution:



Construction :

Join AQ and BP.

Given C = 90°

Proof :

(i) In ACQ,

$AQ^2 = AC^2 + CQ^2$ [Pythagoras theorem]

Multiplying both sides by 9, we get

$9AQ^2 = 9AC^2 + 9CQ^2$…(i)

Given BQ : CQ = 1 : 2

$\frac{CQ}{BC} = \frac{CQ}{(BQ+CQ)}$

$\frac{CQ}{BC} = \frac{2}{3}$

3CQ = 2BC ….(ii)

Substitute (ii) in (i)

$9AQ^2 = 9AC^2 + (2BC)^2$

$9AQ^2 = 9AC^2 + 4BC^2$ ….(iii)

Hence proved.

(ii) In BPC,

$BP^2 = BC^2 + CP^2$ [Pythagoras theorem]

Multiplying both sides by 9, we get

$9BP^2 = 9BC^2 + 9CP^2$

$9BP^2 = 9BC^2 + (3CP)^2$ ….(iv)

Given AP : PC = 1 : 2

$\frac{CP}{AC} = \frac{CP}{AP} + PC$

$\frac{CP}{AC} = \frac{2}{3}$

$3CP = 2AC$ …..(v)

Substitute (v) in (iv)

$9BP^2 = 9BC^2 + (2AC)^2$

$9BP^2 = 9BC^2 + 4AC^2$ ….(vi)

Hence proved.

(iii) Adding (iii) and (vi), we get

$9AQ^2 = 9BP^2 + 9AC^2 + 4BC^2$ $+9BC^2 + 4AC^2$

9$(AQ^2 + BP^2)$ = 13 $(AC^2 + BC^2)$ ….(vii)

In ABC,

$AB^2 = AC^2 + BC^2$ …..(viii)

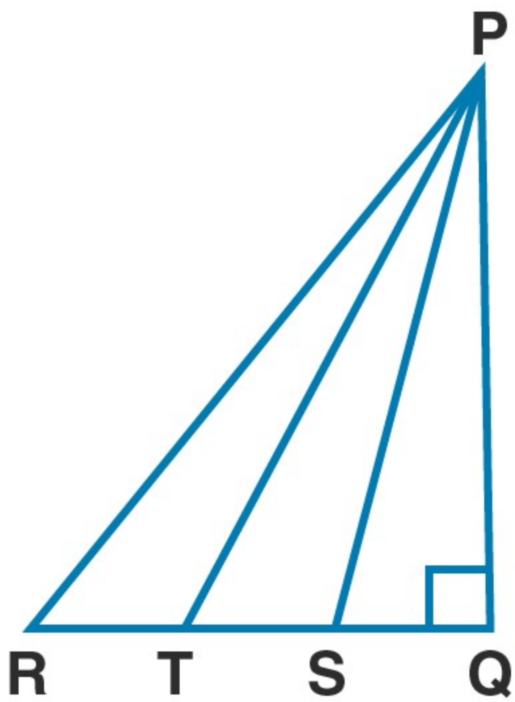
Substitute (viii) in (viii), we get

9$(AQ^2 + BP^2)$ = 13AB^2

Hence proved.

4. In the given figure, ΔPQR is right angles at Q and points S and T trisect side QR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.

Solution:



Given Q = 90°

S and T are points on RQ such that these points trisect it.

So RT = TS = SQ

To prove : $8PT^2 = 3PR^2 + 5PS^2$

Proof :

Let RT = TS = SQ = x

In PRQ,

$PR^2 = RQ^2 + PQ^2$ [Pythagoras theorem]

$PR^2 = (3x)^2 + PQ^2$

$PR^2 = 9x^2 + PQ^2$

Multiply above equation by 3

$3PR^2 = 27x^2 + 3PQ^2$ ……(i)

Similarly in PTs,

$PT^2 = TQ^2 + PQ^2$ [Pythagoras theorem]

$PT^2 = (2x)^2 + PQ^2$

$PT^2 = 4x^2 + PQ^2$

Multiply above equation by 8

$8PT^2 = 32x^2 + 8PQ^2$ …..(ii)

Similarly in PSQ,

$PS^2 = SQ^2 + PQ^2$ [Pythagoras theorem]

$PS^2 = x^2 + PQ^2$

Multiply above equation by 5

$5PS^2 = 5x^2 + 5PQ^2$ ….(iii)

Add (i) and (iii), we get

$3PR^2 + 5PS^2 = 27x^2 + 3PQ^2 + 5x^2 + 5PQ^2$

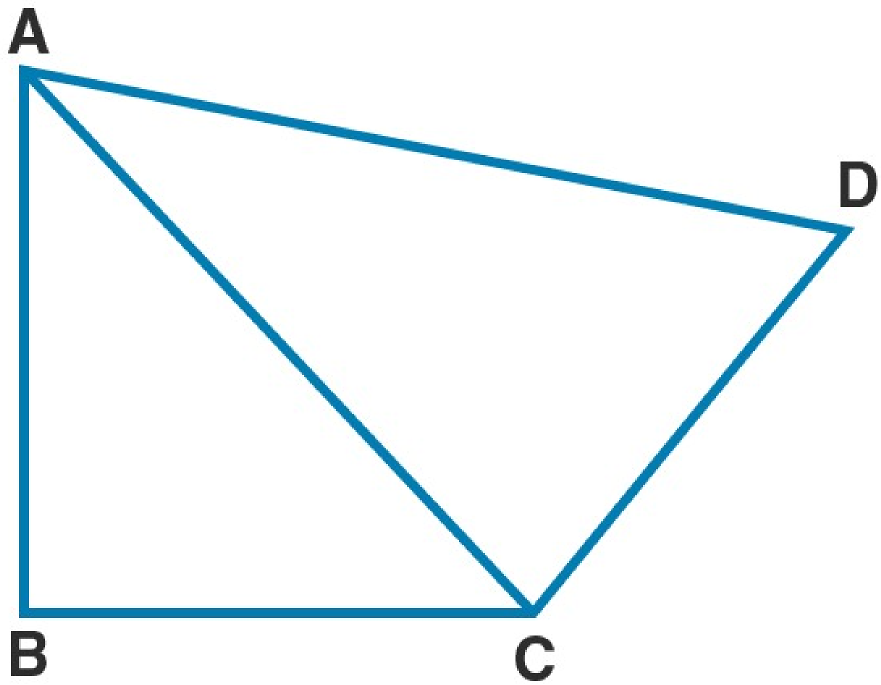
$3PR^2 + 5PS^2 = 32x^2 + 8PQ^2$

$3PR^2 + 5PS^2 = 8PT^2$ [from (ii)]

$8PT^2 = 3PR^2 + 5PS^2$

Hence proved.

5. In a quadrilateral ABCD, B = 90°, if $AD^2 = AB^2 + BC^2 + CD^2$
To prove : ACD = 90°
Solution:



Given : B = 90° in quadrilateral ABCD

$AD^2 = AB^2 + BC^2 + CD^2$

To prove : ACD = 90°

Proof :

In ABC,

$AC^2 = AB^2 + BC^2$ ..(i) [Pythagoras theorem]

Given

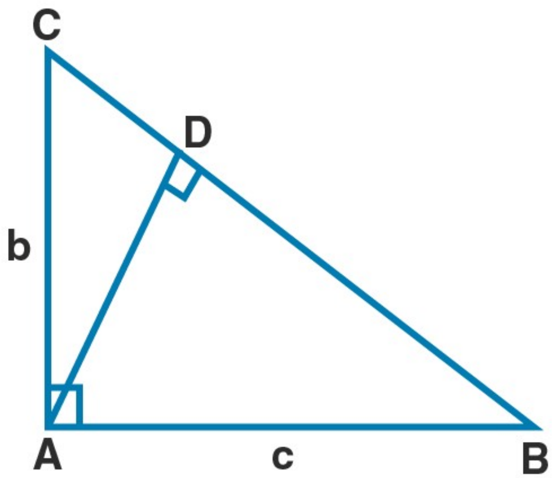
$AD^2 = AB^2 + BC^2 + CD^2$

$AD^2 = AC^2 + CD^2$ [from (i)]

In ACD, ACD = 90° [Converse ofpyhagoras theorem]

Hence proved.

6. In the given figure, find the length of AD in terms of b and c.



Solution:

Given : A = 90°

AB = c

AC = b

ADB = 90°

In ABC,

$BC^2 = AC^2 + AB^2$ [Pythagoras theorem]

$BC^2 = b^2 + c^2$

$Bc = \sqrt{b^2 + c^2}$ ….(i)

Area of ABC = $\frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times BC \dots (iii)$

Also, Area of ABC = $\frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times \sqrt{b^2 + c^2} \times AD$ …(iii)

Equating (ii) and (iii)

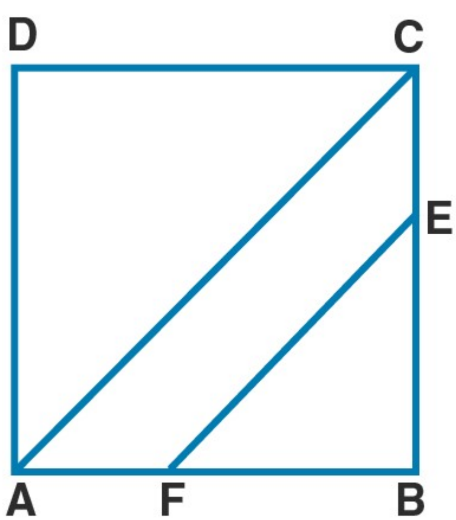
$\frac{1}{2} \times$ bc = $\frac{1}{2} \times (b^2 + c^2) \times AD$

AD = $\frac{bc}{(\sqrt{b^2+c^2})}$

Hence AD is $\frac{bc}{(\sqrt{b^2+c^2})}$.

7. ABCD is a square, F is mid-point of AB and BE is one-third of BC. If area of ΔFBE is 108 cm^2, find the length of AC.

Solution:



Let x be each side of the square ABCD.

FB = $\frac{1}{2}AB$ [$\because F$ *is the midpoint of* AB]

FB = $\frac{1}{2}x$ …..(i)

BE = $\left(\frac{1}{3}\right)BC$

BE = $\left(\frac{1}{3}\right)x$ …..(ii)

AC = $\sqrt{2}$ × side [Diagonal of a square]

AC = $\sqrt{2\,x}$

Area of FBE = $\frac{1}{2}\ FB \times BE$

108 = $\frac{1}{2} \times \frac{1}{2}\ x\ \times \left(\frac{1}{3}\right)x$ [given area of FBE = $108cm^2$]

108 = $\left(\frac{1}{12}\right)x^2$

$x^2 = 108 \times 12$

$x^2 = 1296$

Taking squares root on both sides.

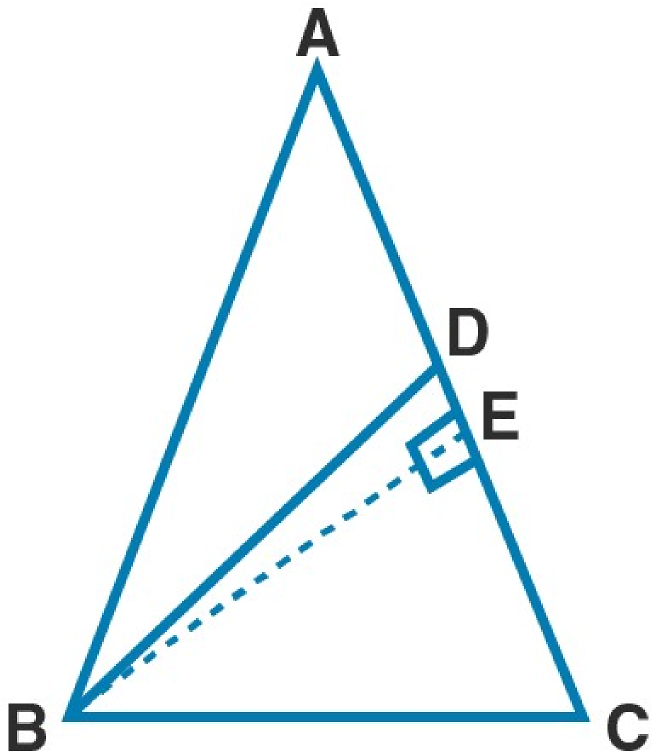
$x = 36$

AC = $\sqrt{2} \times 36 = 36\sqrt{2}$

Hence length of AC is $36\sqrt{2}$ cm.

8. In a triangles ABC, AB = AC and D is a point on side AC such that $BC^2 = AC \times CD, prove\ that\ BD = BC$.

Solution:



Given : In ABC, AB = AC

D is a point on sides AC such that $BC^2 = AC \times CD$

To prove : BD = BC

Construction : Draw BEAC

Proof :

In BCE,

$BC^2 = BE^2 + EC^2$ [Pythagoras theorem]

$BC^2 = BE^2 + (AC - AE)^2$

$BC^2 = BE^2 + AC^2 + AE^2 - 2AC \times AE$

$BC^2 = BE^2 + AE^2 + AC^2 - 2AC \times AE$ …(i)

In ABC,

$AB^2 = BE^2 + AE^2 .. (ii)$

Substitute (ii) in (i)

$BC^2 = AB^2 + AC^2 - 2AC \times AE$

$BC^2 = AC^2 + AC^2 - 2AC \times AE$ $[\because AB = AC]$

$BC^2 = 2AC^2 - 2AC \times AE$

$BC^2 = 2AC (AC - AE)$

$BC^2 = 2AC \times EC$

Given $BC^2 = AC \times CD$

$2AC \times EC = AC \times CD$

2EC = CD ….(ii)

E is the midpoint of CD.

EC = DE …(iii)

In BED and BEC,

EC = DE [from (iii)]

BE = BE [common side]

BED = BEC

BED BEC [BY SAS congruency rule]

BD = BD [c.p.c.t.]

Hence proved.