WITH GRAPH PAPER

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Mathematics

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| 65/2/C | ① ● ③ ④ |

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## Section - C

Q: 20

$$(1+x^2)\frac{dy}{dx} = e^{m\tan^{-1}x} - y$$

$$(1+x^2)\frac{dy}{dx} + y = e^{m\tan^{-1}x}$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{m\tan^{-1}x}}{1+x^2}$$

comparing it with

$$\frac{dy}{dx} + Py = Q \rightarrow \text{linear differential equation}$$

$$P = \frac{1}{1+x^2}$$

$$I.F = e^{\int P dx}$$

$$= e^{\int \frac{1}{1+x^2} dx}$$

$$I.F = e^{\tan^{-1}x}$$

$$\left\{ \int \frac{1}{x^2+1} dx = \tan^{-1}x \right\}$$

So ~~dif~~ equation becomes

$$y\ I.F = \int Q \times I.F\, dx + c$$

$$y \times e^{\tan^{-1}x} = \int \frac{e^{m\tan^{-1}x}}{1+x^2} \times e^{\tan^{-1}x} dx + c$$

$$y \times e^{\tan^{-1}x} = \int \frac{e^{(m+1)\tan^{-1}x}}{1+x^2} dx + c$$

put $\tan^{-1}x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$y e^{\tan^{-1}x} = \int e^{(m+1)t} \times dt + c$$

$$y e^{\tan^{-1}x} = \frac{e^{(m+1)t}}{(m+1)} + c$$

$$\cancel{y\,to}\ y e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + c$$

when $x = 0$ $y = 1$

$$ye^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{(m+1)} + c$$

when $x=0$ $y=1$

$$1 \times e^{\tan^{-1}0} = \frac{e^{(m+1)\tan^{-1}0}}{(m+1)} + c$$

$$1 \times e^{0} = \frac{e^{(m+1)(0)}}{m+1} + c \qquad e^0 = 1$$

$$1 = \frac{1}{m+1} + c$$

$$1 - \frac{1}{m+1} = c$$

$$\frac{m+\cancel{1}-\cancel{1}}{m+1} = c$$

$$c = \frac{m}{m+1}$$

So equation is

$$\left[ye^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{(1+m)} + \frac{m}{m+1}\right]$$

Q: 21.

$$f(x) = \sin^2 x - \cos x \qquad x \in [0, \pi]$$

$$f'(x) = 2\sin x \cos x - (-\sin x)$$

$$f'(x) = 2\sin x \cos x + \sin x$$

put $f'(x) = 0$

then $(2\sin x \cos x + \sin x) = 0$

$$\sin x (2\cos x + 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{-1}{2}$$

$$x = 0, \pi \quad \text{or} \quad x = \frac{2\pi}{3}$$

both the extreme value are automatically included.

So $f(0) = [\sin(0)]^2 - \cos 0$

$= 0 - 1$

$= -1$

$f(\pi) = \sin^2\pi - \cos\pi$

$= 0 - (-1)$

$= 1$

$f\left(\frac{2\pi}{3}\right) = \sin^2\frac{2\pi}{3} - \cos\frac{2\pi}{3}$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{2} = \frac{10}{8} = \frac{5}{4}$

$f(0) = -1$

$f(\pi) = 1$

$f\left(\frac{2\pi}{3}\right) = \frac{5}{4}$

So $\boxed{\text{Absolute maxima is } \frac{5}{4} \text{ at } x = \frac{2\pi}{3}}$

$\boxed{\text{Absolute minima is } -1 \text{ at } x = 0}$

Q: 22:

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 4\hat{j} + 2\hat{k} + \mu(2\hat{i} - \hat{j} + 3\hat{k})$

These line are coplaner if they are parallel or they are intersecting

But these lines are not parallel.

So they are coplaner if shortest distance between them is zero:

$a_1 = (1, 1, 1)$ $\quad (a_2 - a_1) = (-1, 3, 1)$

$a_2 = (0, 4, 2)$

$b_1 = \langle 1, -1, 1 \rangle$

$b_2 = \langle 2, -1, 3 \rangle$

Shortest distance $= \dfrac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$

Let us find $|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} -1 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

(expanding by 1st row)

$$= -1(-3+1) - 3(3-2) + 1(-1+2)$$

$$= -1(-2) - 3(1) + 1$$

$$= 2 + 1 - 3$$

$$= 0$$

So shortest distance between the lines is zero

So they are coplaner.

Let $\langle a,b,c\rangle$ be the direction ratio of normal to the plane.

So the dot product of direction ratio of normal to plane and the direction ratio of line is 0.

So $a - b + c = 0$

$2a - b + 3c = 0$

| $a$ | $b$ | | $c$ |
|---|---|---|---|
| $-1$ | $1$ | $1$ | $-1$ |
| $-1$ | $3$ | $2$ | $-1$ |

$$\frac{a}{-3+1} = \frac{b}{2-3} = \frac{c}{-1+2}$$

So $\langle a,b,c\rangle = \langle -2,-1,1\rangle$

So passing point of line is also the passing point of plane

So passing point $= (1,1,1)$

So equation becomes

$$-2(x-1)-1(y-1)+1(z-1)=0$$

$$-2x+2-y+\cancel{1}+z-\cancel{1}=0$$

$$\boxed{-2x-y+z+2=0}$$

$\downarrow$

equation of plane.

Q:23 $f: W \to W$

$$f(n) = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even} \end{cases}$$

prove that it is one - one :

Let us suppose that $n_1, n_2 \in W$

$n_1 \neq n_2$ but $f(n_1) = f(n_2)$

(i) 1st case : if $n_1$ & $n_2$ are odd

$$f(n_1) = f(n_2)$$

$$n_1 - \cancel{1} = n_2 - \cancel{1}$$

$n_1 = n_2$ which is contradiction.

(ii) 2nd case : $n_1, n_2$ are even

$$f(n_1) = f(n_2)$$

$$n_1 + \cancel{1} = n_2 + \cancel{1}$$

$$n_1 = n_2$$

which is contradiction.

(iii) 3rd case if $n_1$ is even & $n_2$ is odd

$$f(n_1) = f(n_2)$$

$$n_1 + 1 = n_2 - 1$$

$$\underbrace{n_1}_{\text{even}} + 2 = \underbrace{n_2}_{\text{odd}}$$

if we add 2 in an even no. then we get an even no. but $n_2$ is odd which is again contradiction

So from these 3 point we see that

$f(n_1) = f(n_2)$ only if $n_1 = n_2$

$\Rightarrow$ $f$ is one-one.

To prove that $f$ is onto:

Let $y \in W$
and $y$ is even
then $y+1$ is odd and it belongs in whole no.
$\cancel{f(x_1 -}\ f(y+1) = y+1-1$
$= y$
So for every $y \in W$ which is even there exist a preimage $y+1$ which is odd and $(y+1) \in W$

Let $y \in W$
$y$ is odd
then $y-1$ is even and $y-1 \in W$
$f(y-1) = y-1+1$
$= y$
So for every $y \in W$ which is odd there exist a pre image $y-1 \in W$ which is even.
So for all $y \in W$ there exist pre image in whole no.
So $f$ is onto.

$f$ is one-one and onto So it is invertible

To find $f^{-1}(x)$

$y = x - 1$ if $x$ is odd

$y$ here is even

$y + 1 = x$

So $x = y + 1$ if $y$ is even

$f^{-1}(x) = x + 1$ if $x$ is even

$y = x + 1$ if $x$ is even

$y$ is odd here

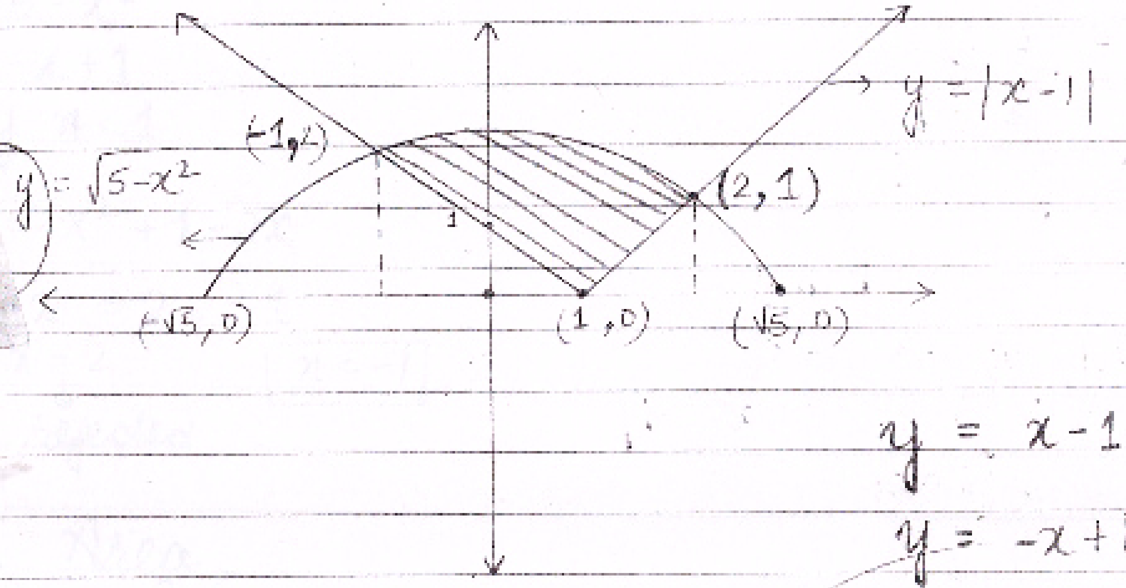
$y - 1 = x$

So $f^{-1}(x) = x - 1$ if $x$ is odd

So $f^{-1}(x) = \begin{cases} x-1 & \text{if } x \text{ is odd} \\ x+1 & \text{if } x \text{ is even} \end{cases}$

$\boxed{x \in W}$ So $\boxed{f^{-1} = f}$

24°




$y = x-1$ if $x \geq 1$

$y = -x+1$ if $x < 1$

intersection point :

$y = \sqrt{5-x^2}$

$y = x-1$

So $\sqrt{5-x^2} = (x-1)$

$5-x^2 = x^2+1-2x$

$2x^2-2x-4=0$

$\cancel{2}x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

So $x=2$ or $x=-1$

$x=2$ rejected as $x \geq 1$

$y = \sqrt{5-x^2}$

$y = -x+1$

$y \quad x<1$

$5 - x^2 = x^2 + 1 - 2x$

So $x = 2, -1$

$x = 2$ $\downarrow$ rejected $\quad \boxed{x=-1}$

Required Area

$$(A) = \int_{-1}^{2} \sqrt{5-x^2}\,dx - \left[\int_{-1}^{1} (-x+1)\,dx + \int_{1}^{2} (x-1)\,dx\right]$$

$$A = \left.\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right|_{-1}^{2} - \left[\left.\cancel{-}\frac{x^2}{2} + x\right|_{-1}^{1} + \left.\frac{x^2}{2} - x\right|_{1}^{2}\right]$$

$$A = 1\times 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} - \left(\frac{-1\times 2}{2} + \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right) - \left\{\frac{-1}{2}[1-1] + 2 + \frac{1}{2}[4-1] - 1\right\}$$

$$A = 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - \left[0 + 2 + \frac{3}{2} - 1\right]$$

$$A = 2 + \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right] - \frac{5}{2}$$

$$A = \frac{5}{2} \times \frac{\pi}{2} + 2 - \frac{5}{2}$$

$$\boxed{A = \frac{5\pi}{4} - \frac{1}{2} \text{ sq units}}$$

$$\left\{\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right.$$
$$= \sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}\frac{2}{\sqrt{5}}$$
$$\left.= \frac{\pi}{2}\right\}$$

$\theta$1) dx

$x\Big|_1^2$

2

$[4-1]-1\}$

**25.** Ist 6 positive integers = (1, 2, 3, 4, 5, 6)

Random Variable = X = larger of 2 numbers.

Sample Space S = { (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (3,1) (4,1) (5,1) (6,1)
(2,3) (2,4) (2,5) (2,6) (3,2) (4,2) (5,2)
(6,2) (3,4) (3,5) (3,6) (4,3) (5,3) (6,3)
(4,5) (4,6), (5,4), (6,4) (5,6) (6,5) }

| X | P(X) | $x_i p_i$ | $x_i^2 p_i$ |
|---|---|---|---|
| 2 | $\frac{2}{30} = \frac{1}{15}$ | $\frac{2}{15}$ | $\frac{4}{15}$ |
| 3 | $\frac{4}{30} = \frac{2}{15}$ | $\frac{6}{15}$ | $\frac{18}{15}$ |
| 4 | $\frac{6}{30} = \frac{3}{15}$ | $\frac{12}{15}$ | $\frac{48}{15}$ |
| 5 | $\frac{8}{30} = \frac{4}{15}$ | $\frac{20}{15}$ | $\frac{100}{15}$ |
| 6 | $\frac{10}{30} = \frac{5}{15}$ | $\frac{30}{15}$ | $\frac{180}{15}$ |

$\Sigma x_i p_i = \frac{70}{15}$ $\Sigma x_i^2 p_i \; \frac{350}{15}$

Mean $= \Sigma x_i p_i^2 = \frac{14}{3} = 4.66$

Variance $= \Sigma x_i^2 p_i - (\mu)^2$

$= \frac{350}{15} - \frac{70}{15} \times \frac{70}{15}$

Rough

2+6

$\begin{array}{r} 336 \\ \times 5 \\ \hline 180 \end{array}$

$\begin{array}{r} 22 \\ 48 \\ \hline 70 \end{array}$

200
150
350

$$\sigma^2 = \frac{70}{15}\left[5 - \frac{70}{15}\right]$$

$$= \frac{70}{15}\left[\frac{75-70}{15}\right]$$

$$= \frac{\cancel{70}^{14} \times \cancel{5}}{{}_{3}\cancel{15} \; \cancel{15}\,{}_{3}}$$

$$= \frac{14}{9}$$

$$\sigma^2 = 1 \cdot 55$$

$$\boxed{\text{Variance} = 1 \cdot 55}$$

## Section - B

Q:19

$$x^x + x^y + y^x = a^b$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ P & Q & R \end{array}$$

$$\frac{d}{dx}\left[P + Q + R\right] = 0$$

$$\left[\frac{dP}{dx} + \frac{dQ}{dx} + \frac{dR}{dx} = 0\right]$$

$$P = x^x$$

$$\log P = x \log x$$

$$\frac{1}{P}\frac{dP}{dx} = x\left[\frac{1}{x}\right] + \log x$$

$$\frac{dP}{dx} = x^x(1 + \log x) \quad -(1)$$

$$Q = x^y$$

$$\log Q = y \log x$$

$$\frac{1}{Q}\frac{dQ}{dx} = \left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$$

$$\frac{dQ}{dx} = x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] \quad -(2)$$

$$\cancel{Q = y^x}$$

$$R = y^x$$

$$\log R = x \log y$$

$$\frac{1}{R}\frac{dR}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$$

$$\frac{dR}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \quad -③$$

adding ① & ② & ③ we get

$$x^x + x^x \log x + x^{y-1} y + x^y \log x \frac{dy}{dx} + y^{x-1} x \frac{dy}{dx} + y^x \log y = 0$$

$$\frac{dy}{dx} = \frac{-\left[x^x(1+\log x) + x^{y-1} y + y^x \log y\right]}{x^y \log x + y^{x-1} x}$$

Q:18 $y = e^{ax} \cos bx$

$$\frac{dy}{dx} = e^{ax}(-\sin bx)b + \cos bx \times e^{ax} \times a$$

$$\frac{dy}{dx} = -be^{ax}\sin bx + ay \quad \Rightarrow \left[\frac{-1}{b}\left(\frac{dy}{dx} - ay\right) = e^{ax}\sin bx \quad ①\right]$$

$$\frac{d^2y}{dx^2} = -b\left[e^{ax}\cos bx \times b + \sin bx \times e^{ax} \times a\right] + a\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -b\left[by - \frac{a}{b}\left(\frac{dy}{dx} - ay\right)\right] + a\frac{dy}{dx} \quad \text{from } ①$$

$$\frac{d^2y}{dx^2} = -b^2y + a\frac{dy}{dx} - a^2y + a\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} + (a^2+b^2)y - 2a\frac{dy}{dx} = 0$$

hence proved.

26.

$Z = 5x+2y$
↓
objective function to be made maximum and minimum

$x - 2y \leq 2$

$3x+2y \leq 12$

$-3x+2y \leq 3$

$x \geq 0 \quad , y \geq 0$

corresponding equations

$x - 2y = 2$

put (0, 0) in it

$\cancel{0, 0}$ $0 \leq 2$

which is true so the region is towards origin

$3x + 2y \leq 12$ , $\Rightarrow$ $3x + 2y = 12$

put (0, 0)

$0 \leq 12$ which is true

So region towards origin

at A (-2,

$-3x + 2y = 3$

put (0, 0)

at B

$0 \leq 3$

at C

which is true

So region is towards origin

at

$x \geq 0$ $y \geq 0$. So it means Ist quadrant.

intersection point of

$$-3x + 2y = 3$$
$$3x + 2y = 12$$
$$4y = 15$$
$$y = \frac{15}{4}$$

$$-3x + 2 \times \frac{15}{4_2} = 3$$
$$-3x = 3 - \frac{15}{2}$$
$$-x = 1 - \frac{5}{2}$$
$$-x = -\frac{3}{2}$$
$$x = \frac{3}{2}$$

intersection point of

$$x - 2y = 2$$
$$3x + 2y = 12$$
$$4x = 14$$
$$x = \frac{7}{2}$$

$$\frac{7}{2} - 2y = 2$$
$$\frac{7}{2} - 2 = 2y$$
$$\frac{3}{2} = 2y$$
$$y = \frac{3}{4}$$

$Z = 5x + 2y$

at A $(2,0)$ $\quad Z = 10$

at B $(0,0)$ $\quad Z = 0$

at C $\left(0, \frac{3}{2}\right)$ $\quad Z = 3$

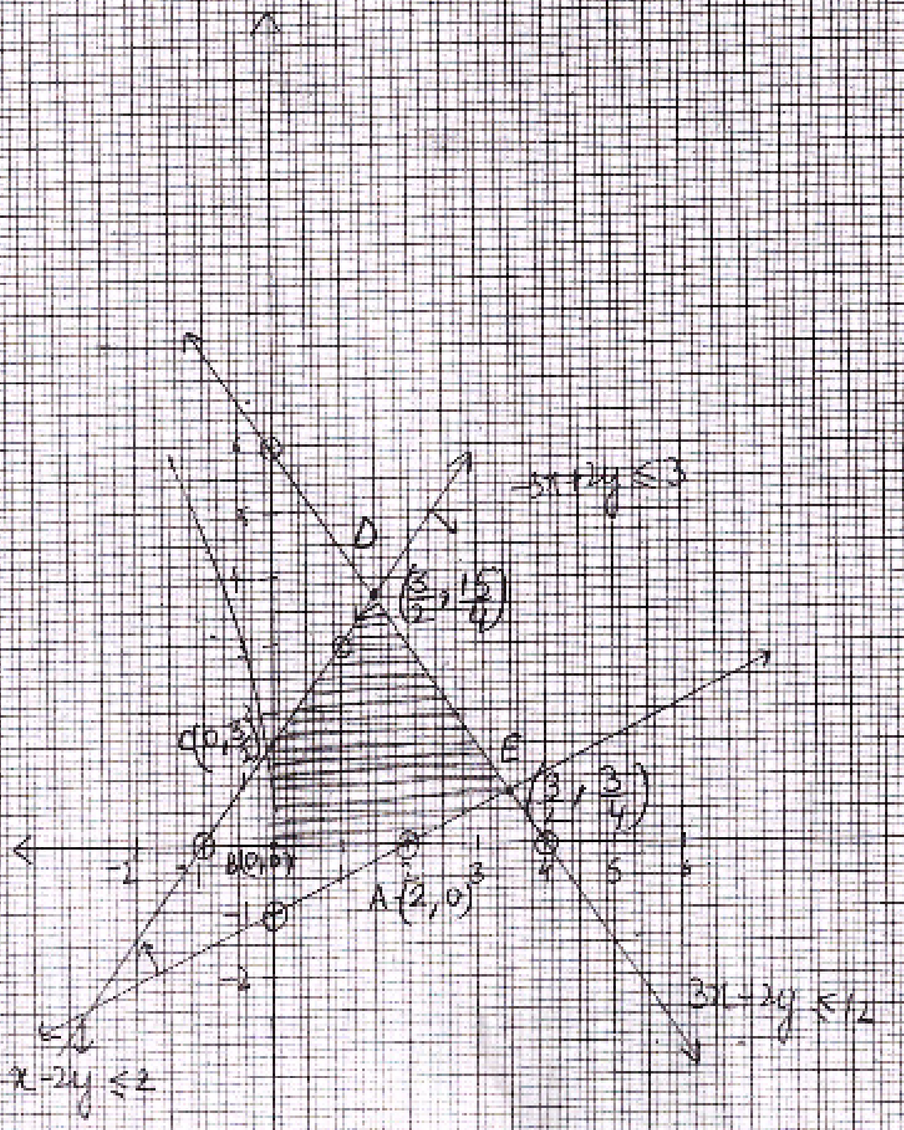
at D $\left(\frac{3}{2}, \frac{15}{4}\right)$ $\quad Z = \frac{15}{2} + \frac{15}{2} = 15$

at E $\left(\frac{3}{2}, \frac{3}{4}\right)$

$$Z = \frac{15}{2} + \frac{3}{2}$$
$$Z = 9$$

**Q: 26**

10 small box = 1 unit



So Z is maximum

at $D\left(\frac{3}{2}, \frac{15}{4}\right)$

$Z = 15$

Z is minimum at (0, 0)

$\boxed{Z = 0}$

170

$$A = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{l} \begin{array}{ccc} \text{Jans} & \text{Mats} & \text{Toys} \end{array} \\ \begin{bmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{bmatrix}_{3\times 3} \end{array} \begin{array}{l} \\ \rightarrow \text{school X} \\ \rightarrow \text{school Y} \\ \rightarrow \text{school Z} \end{array}$$

$$B = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \rightarrow \text{cost of Jans} \\ \rightarrow \text{cost of Mats} \\ \rightarrow \text{cost of toys} \end{array}$$

$$AB = \begin{bmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$X = AB = \begin{bmatrix} 750 + 1200 + 3500 \\ 1000 + 1500 + 2750 \\ 875 + 2000 + 3750 \end{bmatrix}$$

$$X = \begin{bmatrix} 5450 \\ 5250 \\ 6625 \end{bmatrix} \begin{array}{l} \rightarrow \text{fund collected by school } X \\ \rightarrow \text{fund collected by school } Y \\ \rightarrow \text{fund collected by school } Z \end{array}$$

fund by X = Rs. 5450

fund by Y = Rs 5250

fund by Z = Rs 6625

Total fund = Rs 17325

They are helping victims and hence

the value of helping other is generated.

16. $I = \int \frac{x+3}{(x+5)^3} e^x dx$

$I = \int \frac{x+5}{(x+5)^3} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$

$I = \int \frac{1}{(x+5)^2} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$

integrating this by parts $\frac{1}{(x+5)^2}$ is taken as Ist function

$I = \frac{1}{(x+5)^2} e^x - \int \frac{d}{dx} \frac{1}{(x+5)^2} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$

$I = \frac{1}{(x+5)^2} e^x + \cancel{\int \frac{2}{(x+5)^3} e^x dx} - \cancel{\int \frac{2}{(x+5)^3} e^x dx}$

So $I = \dfrac{e^x}{(x+5)^2} + c$

15°

$x = a\sin 2t\,(1+\cos 2t)$

$\dfrac{dx}{dt} = a\left[(\sin 2t)(-\sin 2t)(2) + (1+\cos 2t)\cos 2t \times 2\right]$

$\dfrac{dx}{dt} = 2a\left[\cos^2 2t + \cos 2t - \sin^2 2t\right]$

$\dfrac{dx}{dt} = 2a\left[\cos 4t + \cos 2t\right]$ $\quad\left[\cos^2 x - \sin^2 x = \cos 2x\right]$

$y = b\cos 2t(1 - \cos 2t)$

$y = b\cos 2t - b\cos^2 2t$

$\dfrac{dy}{dt} = -b\sin 2t \times 2 + b \times 2\cos 2t\,\sin 2t \times 2$

$\dfrac{dy}{dt} = 2b\left[-\sin 2t + \sin 4t\right]$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{2b}{2a}\left[\frac{\sin 4t - \sin 2t}{\cos 2t + \cos 4t}\right].$$

at $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{b}{a}\left[\frac{\sin \pi - \sin \frac{\pi}{2}}{\cos \frac{\pi}{2} + \cos \pi}\right]$$

$$= \frac{b}{a}\left[\frac{0 - 1}{0 - 1}\right]$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \frac{b}{a}$$

142

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x}\, dx \quad \text{—} (1)$$

$$\left[ \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx \right]$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x}\, dx \quad \text{—} (2)$$

adding (1) & (2) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{1}{(\sin x + \cos x)}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}$$

$$\left\{ \begin{array}{l} \text{put } \sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\ \text{and } \cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \end{array} \right.$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}\, dx$$

put $\tan\frac{x}{2} = t$

$$\sec^2\frac{x}{2} \times \frac{1}{2}\, dx = dt$$

$x \to 0 \quad t \to 0$

when $x \to \frac{\pi}{4} \quad t \to 1$

$$\cancel{2}I = \cancel{2}\int_0^1 \frac{dt}{2t + 1 - t^2}$$

$$I = \int_0^1 \frac{dt}{2t + 1 + 1 - 1 - t^2} \Rightarrow I = \int_0^1 \frac{dt}{2 - (t-1)^2}$$

$$I = \int_0^1 \frac{dt}{(\sqrt{2})^2 - (t-1)^2} \qquad \int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right|$$

$$I = \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|_0^1$$

$$I = \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+\cancel{1}-\cancel{1}}{\sqrt{2}-\cancel{1}+\cancel{1}}\right| - \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

$$I = \frac{1}{2\sqrt{2}} \log 1 - \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

$$= 0 - \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

$$I = \frac{-1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|$$

13.

$$\Delta = \begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 3x+7 & 3x+7 & 3x+7 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$$\Delta = 3x+7 \begin{vmatrix} 1 & 1 & 1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$C_1 \to C_1 - C_2 \qquad C_2 \to C_2 - C_3$

$$\Delta = (3x+7) \begin{vmatrix} 0 & 0 & 1 \\ 7 & -3 & x+2 \\ -3 & -4 & x+6 \end{vmatrix}$$

expanding by $R_1$

$$\Delta = (3x+7)\left[-28 - 9\right]$$

$$\Delta = 0$$

$$\Rightarrow 3x+7 = 0$$

$$\boxed{x = \frac{-7}{3}}$$

12.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 9 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

So $A^2 - 4A - 5I = O$

Hence proved.

$A^2 - 4A - 5I = 0$

pre-multiplying by $A^{-1}$

$A^{-1}AA - 4A^{-1}A - 5A^{-1}I = 0$

$A^{-1}A = I$

$IA - 4I - 5A^{-1} = 0$

$IA = A$

$A - 4I - 5A^{-1} = 0$

$A - 4I = 5A^{-1}$

$$A^{-1} = \frac{A - 4I}{5}$$

$$A^{-1} = \frac{1}{5}\left[\,A - 4I\,\right]$$

$$5A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$5A^{-1} = \begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix}$$

So $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

11.

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$

$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$(1-x) = \cos(2\sin^{-1}x)$

$\sin^{-1}x = \theta$

$x = \sin\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = 1 - 2x^2$

$\Rightarrow$ $1 - x = \cos 2\theta$

$1 - x = 1 - 2x^2$

$\Rightarrow 2x^2 - x = 0$

$x[2x - 1] = 0$

$x = 0$ or $x = \frac{1}{2}$

put $x = \frac{1}{2}$ in equation

$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{2}$

$= \frac{\pi}{6} - 2 \times \frac{\pi}{6}$

$\neq \frac{\pi}{2}$

So $x \neq \frac{1}{2}$

So $\boxed{x = 0}$

10.

passing point of line $= (4, 2, 2)$

since $\vec{b}$ is $\parallel$ to line

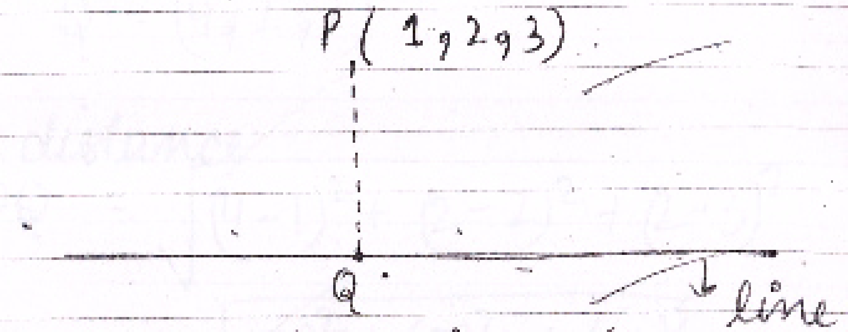
So direction ratio of line $= \langle 2, 3, 6 \rangle$

So equation of line

$$\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-2}{6} = \lambda$$

general point on line

$(2\lambda+4, 3\lambda+2, 6\lambda+2)$




Q be the foot of $\perp$ar

PQ is $\perp$ar to line

So dot product of direction ratio of line and PQ is 0

direction ratio of PQ $= \langle 2\lambda+3, 3\lambda, 6\lambda-1 \rangle$

So according to question:

$$2.(2\lambda+3)+(3\lambda)3+6(6\lambda-1)=0$$

$$4\lambda+\cancel{6}+9\lambda+36\lambda-\cancel{6}=0$$

$$\boxed{\lambda=0}$$

So point $Q=(4,2,2)$

So ⊥ar distance

$$PQ=\sqrt{(4-1)^2+(2-2)^2+(2-3)^2}$$

$$=\sqrt{(3)^2+(0)^2+(-1)^2}$$

$$=\sqrt{9+1}$$

$$\boxed{\text{length of ⊥ar} = \sqrt{10}\text{ unit}}$$

9.

~~AB~~

these A, B, C, D are coplaner

so $\vec{AB}\cdot(\vec{BC}\times\vec{CD}) = 0$

triple product is 0.

$\vec{AB} = 1\hat{i} + (x-1)\hat{j} + 4\hat{k}$

$\vec{BC} = 0\hat{i} + (1-x)\hat{j} - 7\hat{k}$

$\vec{CD} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{AB}\cdot(\vec{BC}\times\vec{CD}) = \begin{vmatrix} 1 & x-1 & 4 \\ 0 & 1-x & -7 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

expanding by $R_1$

$1(1-x+21) - (x-1)14 + 4(2(x-1)) = 0$

$22 - x - 14x + 14 + 8x - 8 = 0$

$-7x = -28$

$\boxed{x = 4}$

8.

p (probability of success) $= \frac{1}{2}$
i.e that head comes

q (probability of failure) $= \frac{1}{2}$
i.e that tail comes

Let the coin be tossed n times

this event follow the conditions of bernoulli trial

X be the random variable = no of heads

$P(X \geq 1) = P(1) + \cdots + P(n)$

$P(X \geq 1) = 1 - P(0)$

$= 1 - {}^{n}C_0 \times \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n$

$P(X \geq 1)$ should be more than 80%.

$P(X \geq 1) \geq \frac{8}{10}$

$\frac{80}{100} < 1 - \left(\frac{1}{2}\right)^n$

$$1-\left(\frac{1}{2}\right)^n \geq \frac{8}{10}$$

$$\left(1-\frac{8}{10}\right) \geq \left(\frac{1}{2}\right)^n$$

$$\frac{1\cancel{2}}{5\cancel{10}} \geq \left(\frac{1}{2}\right)^n$$

$$5 \leq 2^n$$

$$n = 3$$

follow the condition

So the coin should be tossed at least 3 times.

7. $$I = \int \frac{x^2}{x^4+x^2-2}\,dx$$

$$I = \int \frac{x^2}{(x^4+2x^2-x^2-2)}\,dx$$

$$I = \int \frac{x^2}{x^2(x^2+2)-1(x^2+2)}\,dx$$

$$I = \int \frac{x^2}{(x^2-1)(x^2+2)}\,dx$$

put $x^2 = t$

then $\dfrac{t}{(t-1)(t+2)} = \dfrac{A}{t-1} + \dfrac{B}{t+2}$

$$t = A(t+2) + B(t-1)$$

put $t = 1$

$1 = A \times 3$

$\boxed{A = \frac{1}{3}}$

put $t = -2$

$-2 = -3B$

$\boxed{B = \frac{2}{3}}$

$$I = \int \frac{1}{3}\frac{dx}{(t-1)} + \frac{2}{3}\frac{dx}{(t+2)}$$

$$I = \int \frac{1}{3}\frac{dx}{(x^2-1)} + \frac{2}{3}\int \frac{dx}{(x^2+2)}$$

$$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$$

$$I = \frac{1}{3} \times \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + \frac{2}{3} \times \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$I = \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

## Section - A

Q: 6 equation of plane

$$6x - 3y + 2z - 4 = 0$$

distance $= \dfrac{|6\times2 - 3\times5 + 2(-3) - 4|}{\sqrt{36+9+4}}$

$= \dfrac{|12 - 15 - 6 - 4|}{\sqrt{49}}$

$= \dfrac{|12 - 15 - 10|}{7}$

$\boxed{\text{distance} = \frac{13}{7} \text{ units}}$

Q: 5

$\vec{a} = \hat{i} - \hat{j}$ $\quad |\vec{a}| = \sqrt{2}$

$\vec{b} = \hat{j} - \hat{k}$ $\quad |\vec{b}| = \sqrt{2}$

0902

Fictitious Roll No.
(To be entered by Board)

4474874

अपना अनुक्रमांक इस उत्तर-पुस्तिका पर न लिखें
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अतिरिक्त उत्तर-पुस्तिका (ओं) की संख्या
Supplementary Answer-Book(s) No.

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$(\hat{i}-\hat{j})\cdot(\hat{j}-\hat{k}) = \sqrt{2}\times\sqrt{2}\cdot\cos\theta$$

$$-\frac{1}{2} = \cos\theta$$

$$\boxed{\theta = \frac{2\pi}{3}}$$

angle between vectors $= \frac{2\pi}{3}$

4.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$\vec{a}\times\vec{b} = \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$$

$$\vec{a}\times\vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-17)^2+(13)^2+(7)^2}$$
$$= \sqrt{289+169+49}$$
$$= \sqrt{507}$$
$$= \sqrt{3\times 169}$$
$$|\vec{a}\times\vec{b}| = 13\sqrt{3}$$

Q: 3.

$$x\log x\frac{dy}{dx} + y = 2\log x$$
$$\frac{dy}{dx} + \frac{y}{x\log x} = 2\log x$$

compare it with $\frac{dy}{dx}+Py=Q$

$$\text{I.F} = e^{\int P\,dx}$$
$$= e^{\int \frac{1}{x\log x}dx}$$

put $\log x = t$

$\frac{1}{x}dx = dt$

$$I.F = e^{\int \frac{dt}{t}}$$

$$= e^{\log|t|}$$

$$= t$$

$$\boxed{I.F = \log x}$$

20. general equation of family of lines passing throug origin

$$y = mx \qquad m = \frac{y}{x}$$

$$\frac{dy}{dx} = m$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\boxed{x\frac{dy}{dx} - y = 0}$$

Q. 11.

$a_{12} = e^{2\times 1\times x} \sin 2x$

$\boxed{a_{12} = e^{2x} \sin 2x}$

Poonam 01074009/A

Evaluation done as per CBSE Marking scheme.