Cubes and Cube Roots

- A number is said to be a **perfect cube** if each of its prime factors appears in group of three.
- Prime factorization method can be used to check whether a number is a perfect cube or not.

  For example, 5832 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3
  Here, each of the prime factors occurs in groups of three. Hence, 5832 is a perfect cube.
- We can use the method of prime factorization to find the smallest number with which the given number must be multiplied with or divided by to obtain a perfect cube.

  Example: The smallest number that has to be multiplied with 91476 to make a perfect cube is 1078.
  Prime factorization of 91476

  2 | 91476
  2 | 45738
  3 | 22869
  3 | 7623
  3 | 2541
  7 | 847
  11 | 121
  11 | 11
  1

  $91476 = \underline{3\times3\times3}\times\underline{2\times2}\times\underline{11\times11}\times\underline{7}$

  Here, only prime factor 3 appears in a group of three. The prime factors 2, 11 and 7 appear twice, twice and once respectively.
  Thus,we have to multiply 91476 with 2 × 11 × 7 × 7, which
  gives $\underline{3\times3\times3}\times\underline{2\times2\times2}\times\underline{11\times11\times11}\times\underline{7\times7\times7}$, which is a perfect cube.
  2 × 11 × 7 × 7 = 1078
  Therefore, 1078 is the smallest number that should be multiplied with 91476 to obtain a perfect cube.

  Example: The smallest number that has to be divided by 91476 to make it a perfect cube is 3388.
  Prime factorization of 91476

$2 \mid 91476$
$2 \mid 45738$
$3 \mid 22869$
$3 \mid 7623$
$3 \mid 2541$
$7 \mid 847$
$11 \mid 121$
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$91476 = \underline{3 \times 3 \times 3} \times \underline{2 \times 2} \times \underline{11 \times 11} \times \underline{7}$

Here, only prime factor 3 appears in a group of three. The prime factors 2, 11 and 7 appear twice, twice and once respectively.
Since each group of 2, 11, and 7 is incomplete, we can remove these groups and still end up with a perfect cube. Therefore, we have to divide 91476 by 2 × 2 × 11 × 11 × 7, which gives us 3 × 3 × 3, which is a perfect cube.
2 × 2 × 11 × 11 × 7 = 3388
Therefore, 3388 is the smallest number that divides 91476 and gives a perfect cube.

- Cube root is the inverse operation of finding a cube. The symbol $\sqrt[3]{\ }$ denotes cube-root.

  Example: $\sqrt[3]{64} = 4$ since 4 × 4 × 4 = 64

- The cube root of a perfect cube can be found by prime factorization method.

  Example: Cube root of 287496 is 66.
  Prime factorization of 287496
  $2 \mid 287496$
  $2 \mid 143748$
  $2 \mid 71874$
  $3 \mid 35937$
  $3 \mid 11979$
  $3 \mid 3993$
  $11 \mid 1331$
  $11 \mid 121$
  $11 \mid 11$
  1

  Thus, the number 287496 can be expressed as a product of its prime factors as

$$287496 = \underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{11\times11\times11} = 2^3\times3^3\times11^3 = (2\times3\times11)^3$$

$$\therefore\ \sqrt[3]{287496} = 2\times3\times11 = 66$$

- The units digit of a number and that of its cube exhibits a particular relationship. This is true for ever number and its cube.
- If a number has any of the digits 0, 1, 4, 5, 6, and 9 at its units place, then its cube also ends with the same digit.

  Example: Cube of 4 is 64.

  Cube of 11 is 1331.

- If a number ends with 2, then its cube ends with 8, and vice-versa.

  Example:

  Cube of 12 is 1728.

- If a number ends with 3, then its cube ends with 7, and vice-versa.

  Example:

  Cube of 3 is 27.

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