4 PERCENTAGE

PERCENT

The word "percent" is derived from the latin phrase "per centum", which means "per hundred".

A percentage is a fraction with denominator hundred.

It is denoted by the symbol %.

Numerator of the fraction is called the rate percent.

Consider the following example :

Mohan scores 125 in Paper-I and 115 in Paper II out of 200 marks in each. Then as a total in both he scored $125 + 115 = 240$ and total full marks $= 200 + 200 = 400$.

∴   out of 400 marks he scored 240

∴   out of 1 mark he scored $\dfrac{240}{400}$

∴   out of 100 marks he scored $\dfrac{240}{400} \times 100 = 60$

Hence we say that he scored 240 out of 400 or he scored 60% of 400

Value of Percentage :

Value of percentage always depends on the quantity to which it refers.

Consider the following example :

"65% of the students in the class are boys". From the context, it is understood that boys form 65% of the total number of students in the class. To know the value of 65%, the value of the total number of student should be known.

If the total number of students is 200, then,

the number of boys $= \dfrac{200 \times 65}{100} = 130$

It can also be written as $(200) \times (0.65) = 130$

To Express the Fraction Equivalent to % :

Express the fraction with the denominator 100, then the numerator is the answer.

Suppose we have to convert the fraction $\dfrac{3}{4}$ into its equivalent percentage then

$\dfrac{3}{4} = \dfrac{3 \times 25}{4 \times 25} = \dfrac{75}{100} = 75\%$

Also, $\dfrac{3}{5} = \dfrac{3 \times 20}{5 \times 20} = \dfrac{60}{100} = 60\%$

We can also convert $\dfrac{3}{4}$ directly into percentage by multiplying $\dfrac{3}{4}$ with 100.

$\dfrac{3}{4}$ in percent $= \dfrac{3}{4} \times 100 = 75\%$

Example 1: Express the fraction $\dfrac{11}{12}$ into the percent.

Solution : $\dfrac{11}{12} = \dfrac{\dfrac{11}{12} \times 100}{100} = \dfrac{91\dfrac{2}{3}}{100} = 91\dfrac{2}{3}\%$

To Express % Equivalent to Fraction :

$$a\% = \dfrac{a}{100}$$

Example 2: Express $45\dfrac{5}{6}\%$ into fraction.

Solution : $45\dfrac{5}{6}\% = \dfrac{45\dfrac{5}{6}}{100} = \dfrac{275}{6 \times 100} = \dfrac{11}{24}$

Fractional Equivalents of %	
$1\% = \dfrac{1}{100}$	$33\dfrac{1}{3}\% = \dfrac{1}{3}$
$2\% = \dfrac{1}{50}$	$40\% = \dfrac{2}{5}$
$4\% = \dfrac{1}{25}$	$50\% = \dfrac{1}{2}$
$5\% = \dfrac{1}{20}$	$66\dfrac{2}{3}\% = \dfrac{2}{3}$
$6\dfrac{1}{4}\% = \dfrac{1}{16}$	$60\% = \dfrac{3}{5}$
$10\% = \dfrac{1}{10}$	$75\% = \dfrac{3}{4}$
$11\dfrac{1}{3}\% = \dfrac{17}{150}$	$80\% = \dfrac{4}{5}$
$12\dfrac{1}{2}\% = \dfrac{1}{8}$	$96\% = \dfrac{24}{25}$
$16\% = \dfrac{4}{25}$	$100\% = 1$

$$16\frac{2}{3}\% = \frac{1}{6} \qquad\qquad 115\% = \frac{23}{20}$$

$$20\% = \frac{1}{5} \qquad\qquad 133\frac{1}{3}\% = \frac{4}{3}$$

$$25\% = \frac{1}{4}$$

$$\text{Increase \%} = \frac{\text{Increase value}}{\text{Original value}} \times 100$$

Example 3: Rent of the house is increased from ₹ 7000 to ₹ 7700. Express the increase in price as a percentage of the original rent.

Solution : Increase value = ₹ 7700 – ₹ 7000 = ₹ 700

$$\text{Increase \%} = \frac{\text{Increase value}}{\text{Original value}} \times 100 = \frac{700}{7000} \times 100$$
$$= 10$$

∴ Percentage rise = 10 %

Decrease Percent

$$\text{Decrease \%} = \frac{\text{Decrease value}}{\text{Original value}} \times 100$$

Example 4: The cost of a bike last year was ₹ 19000. Its cost this year is ₹17000. Find the percent decrease in its cost.

Solution : % decrease = $\dfrac{19000 - 17000}{19000} \times 100$

$$= \frac{2000}{19000} \times 100 = 10.5\%$$

∴ Percent decrease = 10.5 %

Example 5: A positive number is divided by 5 instead of being multiplied by 5. By what percent is the result of the required correct value ?

Solution : Let the number be 1, then the correct answer = 5

The incorrect answer that was obtained = $\dfrac{1}{5}$

∴ The required % = $\dfrac{1}{5 \times 5} \times 100 = 4\%$

Remember

✧ If A is x % of C and B is y % of C, then A is $\dfrac{x}{y} \times 100$ % of B.

✧ If two numbers are x% and y% more than a third number respectively, then the first number is $\left(\dfrac{100+x}{100+y} \times 100\right)\%$ of the second and the second is $\left(\dfrac{100+y}{100+x} \times 100\right)\%$ of the first.

✧ If two numbers are x% and y% less than a third number respectively, then the first number is $\left(\dfrac{100-x}{100-y} \times 100\right)\%$ of the second and the second is $\left(\dfrac{100-y}{100-x} \times 100\right)\%$ of the first.

Example 6: Two numbers are 30% and 60% more than a third number respectively. What percent is the first number of the second?

Solution : Given x = 30, y = 60

$$\text{First number} = \left(\frac{100+x}{100+y} \times 100\right)\% \text{ of the second}$$

∴ First number = $\left(\dfrac{100+30}{100+60} \times 100\right)\%$ of the second

$$= 81.25\% \text{ of the second number.}$$

Example 7: Two numbers are 32% and 20% less than a third number respectively. What percent is the first number of the second ?

Solution : Given x = 32, y = 20

∴ First number = $\left(\dfrac{100-x}{100-y} \times 100\right)\%$ of the second

$$= \left(\frac{100-32}{100-20} \times 100\right)\% \text{ of the second}$$
$$= 85\% \text{ of the second.}$$

Remember

✧ x % of a quantity is taken by the first, y % of the remaining is taken by the second and z % of the remaining is taken by third person. Now, if A is left in the fund, then the initial amount

$$= \frac{A \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)} \text{ in the begining.}$$

✧ x % of a quantity is added. Again, y % of the increased quantity is added. Again z % of the increased quantity is added. Now it becomes A, then the initial amount

$$= \frac{A \times 100 \times 100 \times 100}{(100+x)(100+y)(100+z)}$$

Example 8: 3.5 % income is taken as tax and 12.5 % of the remaining is saved. This leaves ₹ 4,053 to spend. What is the income?

Solution : By direct method,

$$\text{Income} = \frac{4053 \times 100 \times 100}{(100-3.5)(100-12.5)} = ₹4800.$$

Remember

❖ If the price of a commodity increases by r %, then reduction in consumption, so as not to increase the expenditure is

$$\left(\frac{r}{100+r}\times100\right)\%.$$

If the price of a commodity decreases by r %, then the increase in consumption so as not to decrease the expenditure is

$$\left(\frac{r}{100-r}\times100\right)\%$$

Example 9: If the price of coal be raised by 20%, then find by how much a householder must reduce his consumption of this commodity so as not to increase his expenditure ?

Solution : Reduction in consumption

$$=\left(\frac{20}{100+20}\times100\right)\%$$

$$=\left(\frac{20}{120}\times100\right)\%=16.67\%$$

POPULATION FORMULA

• If the original population of a town is P, and the annual increase is r %, then the population after n years

$$=P\left(1+\frac{r}{100}\right)^{n}$$

and population n years ago $=\dfrac{P}{\left(1+\dfrac{r}{100}\right)^{n}}$

• If the annual decrease be r %, then the population after n years

$$=P\left(1-\frac{r}{100}\right)^{n}$$

and population n years ago

$$=\frac{P}{\left(1-\dfrac{r}{100}\right)^{n}}$$

Example 10 : The population of a certain town increased at a certain rate percent per annum. Now it is 456976. Four years ago, it was 390625. What will it be 2 years later?

Solution : Suppose the population increases at r % per annum.

Then, $390625\left(1+\dfrac{r}{100}\right)^{4}=456976$

$\therefore\ \left(1+\dfrac{r}{100}\right)^{2}=\sqrt{\dfrac{456976}{390625}}=\dfrac{676}{625}$

Population 2 years later

$$=456976\left(1+\frac{r}{100}\right)^{2}$$

$$=456976\times\frac{676}{625}=494265\,(\text{approx})$$

Example 11: The population of a city increases at the rate of 4% per annum. There is an additional annual increase of 1 % in the population due to the influx of job seekers. Find the % increase in the population after 2 years.

Solution : The net annual increase = 5%.

Let the initial population be 100.

Then, population after 2 years

$=100\times1.05\times1.05$

$=110.25$

Therefore, % increase in population

$=(110.25-100)=10.25\%$

Remember

❖ If a number A is increased successively by x% followed by y% and then z%, then the final value of A will be

$$A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

In case a given value decreases by a percentage then we will use negative sign before that.

Example 12: The population of a town is 1,44,000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years ?

Solution : Given, P = 1,44,000

$x=5\%$

$y=10\%$

$z=15\%$

$\therefore$ Populater after 3 years

$$=1,44,000\left(1+\frac{5}{100}\right)\left(1-\frac{10}{100}\right)\left(1+\frac{15}{100}\right)$$

$$=\frac{1,44000\times105\times90\times115}{100\times100\times100}=156492$$

FIRST INCREASE AND THEN DECREASE

• If the value is first increased by x % and then decreased by y % then there is $\left(x-y-\dfrac{xy}{100}\right)\%$ increase or decrease, according to the + ve or – ve sign respectively.

• If the value of a number is increased by x% and then decreased by x%, then there is only net decrease which is equal to $\left(\dfrac{x}{10}\right)^{2}$

Example 13: A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease percent.

Solution : % change $= 10 - 10 - \dfrac{10 \times 10}{100} = -1\%$

i.e., 1% decrease.

Alternate Method :

% change $= \left(\dfrac{10}{10}\right)^2 = 1\%$

Remember

✧ Average percentage rate of change over a period

$= \dfrac{(\text{New Value} - \text{Old Value})}{\text{Old Value}} \times \dfrac{100}{n}\%$

[where n = period]

✧ The percentage error $= \dfrac{\text{The Error}}{\text{True Value}} \times 100\%$

Successive Increase or Decrease

If the value is increased successively by $x\%$ and $y\%$ then the final increase is given by

$\left(x + y + \dfrac{xy}{100}\right)\%$

If the value is decreased successively by $x\%$ and $y\%$ then the final decrease is given by

$\left(-x - y + \dfrac{xy}{100}\right)\%$

Example 14: The price of a car is decreased by 10 % and 20 % in two successive years. What percent of price of a car is decreased after two years?

Solution : Put $x = -10$ and $y = -20$, then

$-10 - 20 + \dfrac{(-10)(-20)}{100} = -28\%$

∴ The price of the car decreases by 28%.

STUDENT AND MARKS

The percentage of passing marks in an examination is $x\%$. If a candidate who scores y marks fails by z marks, then the maximum marks

$M = \dfrac{100(y + z)}{x}$

A candidate scoring $x\%$ in an examination fails by a marks, while another candidate who scores $y\%$ marks gets b marks more than the minimum required passing marks. Then the maximum marks

$M = \dfrac{100(a + b)}{y - x}$.

In an examination $x\%$ and $y\%$ students respectively fail in two different subjects while $z\%$ students fail in both

subjects. then the % of student who pass in both the subjects will be $\{100 - (x + y - z)\}\%$

Example 15: Vishal requires 40% to pass. If he gets 185 marks, and fails by 15 marks, what was the maximum he could have got?

Solution : If Vishal had 15 more marks, he could have scored 40% marks.

Now, 15 marks more than 185 is $185 + 15 = 200$

Let the maximum marks be x, then

40% of $x = 200$

$\Rightarrow \quad \dfrac{40}{100} \times x = 200$

$\Rightarrow \quad x = \dfrac{200 \times 100}{40} = 500$

Thus, maximum marks $= 500$

Alternate method :

Maximum marks $= \dfrac{100(185 + 15)}{40} = \dfrac{100 \times 200}{40} = 500$.

Example 16: A candidate scores 15% and fails by 30 marks, while another candidate who scores 40% marks, gets 20 marks more than the minimum required marks to pass the examination. Find the maximum marks of the examination.

Solution : Alternate method :

Maximum marks $= \dfrac{100(30 + 20)}{40 - 15} = 200$

Example 17: In an examination 42% students failed in science and 52% failed in mathematics. If 17% failed in both the subjects. Find the percentage of those who passed in both the subjects.

Solution : Given, $x = 42$, $y = 52$, $z = 17$

∴ % of those who passed in both the subjects

$= 100 - (42 + 52 - 17) = 23\%$

TWO–DIMENSIONAL FIGURE AND AREA

• If the sides of a triangle, square, rectangle, rhombus or radius of a circle are increased by $a\%$, its area is increased by

$\dfrac{a(a + 200)}{100}\%$

• If the sides of a triangle, square, rectangle, rhombus or radius of a circle decreased by a % them its area is decreased by

$\dfrac{a(200 - a)}{100}\%$

Example 18: If the radius of a circle is increased by 10 %, what is the percentage increase in its area?

Solution : Radius is increased by 10%. so, area is increased by

$\dfrac{10(10 + 200)}{100} = 21\%$

Example 19: If the length and width of a rectangular garden were each increased by 20%, then what would be the per cent increase in the area of the garden?

Solution : Let the original length and width of the garden be x and y units, respectively.

Then, the original area $= x \times y = xy$ square units

New area $= 1.2 x \times 1.2 y = 1.44xy$ square units

% increase in area $= \dfrac{(1.44xy - xy)}{xy} \times 100 = 44\%$

Alternate method :

% increase in area $= \dfrac{20(20 + 200)}{100} = 44\%$

Remember

✧ If A's income is $r\%$ more than that of B, then B's income is less than that of A by

$$\left(\dfrac{r}{100+r} \times 100 \right)\%$$

✧ If A's income is $r\%$ less than that of B, then B's income is more than that of A by

$$\left(\dfrac{r}{100-r} \times 100 \right)\%$$

✧ If the both sides of rectangle are changed by $x\%$ and $y\%$ respectively, then % effect on area $= x + y + \dfrac{xy}{100}$ (+/− according to increase or decrease)

Example 20: If A's salary is 50 % more than B's, then by what percent B's salary is less than A's salary?

Solution : By direct method,

B's salary is less than A's salary by

$$\left(\dfrac{50}{100+50} \times 100 \right)\%$$

$$= \dfrac{50}{150} \times 100 \% = 33.33 \%.$$

Example 21: Ravi's weight is 25% that of Meena's and 40% that of Tara's. What percentage of Tara's weight is Meena's weight?

Solution : Let Meena's weight be x kg and Tara's weight be y kg.

Then Ravi's weight $= 25\%$ of Meena's weight

$$= \dfrac{25}{100} \times x \qquad \dots (i)$$

Also, Ravi's weight $= 40\%$ of Tara's weight

$$= \dfrac{40}{100} \times y \qquad \dots (ii)$$

From (i) and (ii), we get

$$\dfrac{25}{100} \times x = \dfrac{40}{100} \times y$$

$$\Rightarrow \qquad 25x = 40y$$

$$\Rightarrow \qquad 5x = 8y \Rightarrow x = \dfrac{8}{5}y$$

Meena's weight as the percentage of Tara's weight

$$= \dfrac{x}{y} \times 100 = \dfrac{\frac{8}{5}y}{y} \times 100 = \dfrac{8}{5} \times 100 = 160$$

Hence, Meena's weight is 160% of Tara's weight.

Depreciation Formula

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then :

1. Value of the machine after n years $= P\left(1 - \dfrac{R}{100} \right)^n$

2. Value of the machine n years ago $= \dfrac{P}{\left(1 - \dfrac{R}{100} \right)^n}$.

Example 22: The value of a machine depreciates at the rate of 10% per annum. If its present value is ₹1,62,000, what will be its worth after 2 years ? What was the value of the machine 2 years ago ?

Solution : Value of the machine after 2 years

$$₹\left[162000 \times \left(1 - \dfrac{10}{100} \right)^2 \right] = ₹\left(162000 \times \dfrac{9}{10} \times \dfrac{9}{10} \right)$$

$$= ₹ 131220.$$

Value of the machine 2 years ago

$$= ₹\left[\dfrac{162000}{\left(1 - \dfrac{10}{100} \right)^2} \right] = ₹\left(162000 \times \dfrac{10}{9} \times \dfrac{10}{9} \right) = ₹ 200000.$$

EXERCISE

1. If two numbers are respectively 20% and 50% of a third number, what is the percentage of the first number to the second?
   (a) 10   (b) 20
   (c) 30   (d) 40

2. When the price of a radio was reduced by 20%, its sale increased by 80%. What was the net effect on the sale ?
   (a) 44% increase   (b) 44% decrease
   (c) 66% increase   (d) 75% increase

3. The digit at unit place of a two-digit number is increased by 100% and the digit at ten places of the same number is increased by 50%. The new number thus formed is 19 more than the original number. What is the original number ?
   (a) 22   (b) 63
   (c) 24   (d) None of these

4. Groundnut oil is now being sold at ₹ 27 per kg. During last month its cost was ₹ 24 per kg. Find by how much % a family should reduce its consumption, so as to keep the expenditure same.
   (a) $11\frac{1}{9}\%$ (b) $11\frac{1}{11}\%$
   (c) $11\frac{9}{10}\%$ (d) $9\frac{1}{10}\%$

5. 10% of the inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.
   (a) 5000   (b) 6000   (c) 7000   (d) 8000

6. When the price of a pressure cooker was increased by 15%, the sale of pressure cookers decreased by 15%. What was the net effect on the sales ?
   (a) 15% decrease   (b) no effect
   (c) 2.25% increase   (d) 2.25% decrease

7. A = 10% of x, B = 10% of y, C = 10% of x + 10% of y. On the basis of the above equalities, what is true in the following?
   (a) A is equal to B
   (b) A is greater than B
   (c) B is greater than A
   (d) Relation cannot be established between A and B

8. The ratio of salary of a worker in July to that in June was $2\frac{1}{2} : 2\frac{1}{4}$, by what % the salary of July more than salary of June. Also find by what %, salary of June was less than that of July.
   (a) $11\frac{1}{9}\%$ and 10%   (b) 10% and $11\frac{1}{9}\%$
   (c) Both 10%   (d) Both $11\frac{1}{9}\%$

9. By reduction of 20% in the price of oranges, one can purchase 5 oranges more for ` 2.50. Find the reduced price of the oranges per dozen. Find also the original price.
   (a) 120 paise, 140 paise   (b) ₹ 0.8, ₹ 1.5
   (c) ₹ 1.0, ₹ 1.5   (d) ₹ 1.2., ₹ 1.5

10. A speaks truth in 75% and B in 80% cases. In what percentage of cases are they likely to contradict each other when narrating the same incident ?
    (a) 35   (b) 30
    (c) 25   (d) 20

11. If the price of sugar is increased by 7%, then by how much percent should a housewife reduce her consumption of sugar, to have no extra expenditure ?
    (a) 7 over 107%   (b) 107 over 100%
    (c) 100 over 107%   (d) 7%

12. Lucknow bound Shatabdi Express has a capacity of 500 seats of which 10% are in the Executive class and the rest chair cars. During one journey, the train was booked to 85% of its capacity. If Executive class was booked to 96% of its capacity, then how many chair car seats were empty during that journey?
    (a) 78   (b) 73
    (c) 72   (d) None of these

13. A sum of ₹ 4558 is divided among A, B and C such that A receives 20% more than C, and C receives 25% less than B. What is A's share in the amount ?
    (a) ₹ 1548   (b) ₹ 1720
    (c) ₹ 1290   (d) ₹ 1345

14. In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is $\frac{2}{3}$ of the number of students of 8 years age which is 48. What is the total number of students in the school?
    (a) 72   (b) 80
    (c) 120   (d) None of these

15. From the salary of an officer, 10% is deducted as house rent, 20% of the rest, he spends on conveyance, 20% of the rest he pays as income tax and 10% of the balance, he spends on clothes. Then , he is left with ₹. 15,552. Find his total salary.
    (a) ₹ 25,000   (b) ₹ 30,000
    (c) ₹ 35,000   (d) ₹ 40,000

16. If the radius of a circle is diminished by 10%, the area is diminished by
    (a) 36%   (b) 20%
    (c) 19%   (d) 10%

17. In an election between two candidates, 75% of the voters cast their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of total valid votes. Find the total number of votes enrolled in that election.
    (a) 16080
    (b) 16800
    (c) 18600
    (d) 16008

18. In an election between two candidates, the candidate who gets 30% of the votes polled is defeated by 15,000 votes. What is the number of votes polled by the winning candidate?
    (a) 11,250
    (b) 15,000
    (c) 26,250
    (d) 37,500

19. In an examination, 65% students passed in Civics and 60% in Histroy, 40% passed in both of these subjects. If 90 students failed in History and Civics both, then what is the total number of students?
    (a) 600
    (b) 650
    (c) 700
    (d) 750

20. 40% of the people read newspaper X, 50% read newspaper Y and 10% read both the papers. What percentage of the people read neither newspaper?
    (a) 10%
    (b) 15%
    (c) 20%
    (d) 25%

21. In a shipment of 120 machine parts, 5 per cent were defective. In an another shipment of 80 machine parts, 10 per cent were also defective. For the two shipments combined, what per cent of the machine parts were defective?
    (a) 6.5 %
    (b) 7.0%
    (c) 7.5%
    (d) 8.0%

22. The sum of two numbers is $\dfrac{28}{25}$ of the first number. The second number is what percent of the first?
    (a) 12%
    (b) 14%
    (c) 16%
    (d) 18%

23. A screw driver and a hammer currently have the same price. If the price of a screw driver rises by 5% and the price of hammer goes up by 3%, then how much more will it cost to buy 3 screw drivers and 3 hammers ?
    (a) 3%
    (b) 4%
    (c) 5%
    (d) 8%

24. In measuring the side of a square, an error of 5% in excess is made. The error % in the calculated area is
    (a) $10\dfrac{1}{4}\%$
    (b) $10\dfrac{3}{4}\%$
    (c) $1\dfrac{3}{4}\%$
    (d) 25%

25. If A's salary is 25% higher than B's salary, then how much per cent is B's salary lower than A's ?
    (a) $16\dfrac{1}{3}\%$
    (b) 20%
    (c) 25%
    (d) $33\dfrac{1}{3}\%$

26. The length of a rectangular plot is increased by 25%. To keep its area unchanged, the width of the plot should be
    (a) kept unchanged
    (b) increased by 25%
    (c) increased by 20%
    (d) reduced by 20%

27. When the cost of petroleum increases by 40%, a man reduces his annual consumption by 20%. Find the percentage change in his annual expenditure on petroleum.
    (a) 20%
    (b) 16%
    (c) 12%
    (d) 40%

28. A reduction of 20% in the price of an apple enable a man to buy 10 apple more for ₹54. The reduced price of apple per dozen is
    (a) ₹4.32
    (b) ₹12.96
    (c) ₹10.80
    (d) ₹14.40

29. After three successive equal percentage rise in the salary the sum of 100 rupees turned into 133 rupees and 10 paise. Find the percentage rise in the salary.
    (a) 13%
    (b) 10%
    (c) 15%
    (d) 14%

30. In an examination in which full mark were 500, A got 10% less than B. B got 25% more than C. C got 20% less than D. If a got 360 marks what % of full mark was obtained by D.
    (a) 90%
    (b) 80%
    (c) 50%
    (d) 60%

31. In an examination 80% of student passed in English 85% in mathematics and 75% in both English and mathematics. If 40 student failed in both the subject find total number of students.
    (a) 350
    (b) 400
    (c) 450
    (d) 600

32. The length of a rectangle is increased by 15% and breadth decreased by 15%. Then the area of the new rectangle is
    (a) unchanged
    (b) increased by 2.25%
    (c) decreased by 2.25%
    (d) increased by 15%

33. If 50% of $(x - y) = 40\%$ of $(x + y)$, then what per cent of x is y?
    (a) $10\dfrac{1}{9}\%$
    (b) $11\dfrac{1}{9}\%$
    (c) $13\dfrac{1}{9}\%$
    (d) $21\dfrac{1}{9}\%$

34. If the height of a cone is increased by 50%, then what is the percentage increase in the volume of the cone?
    (a) $\dfrac{100}{3}\%$
    (b) 40 %
    (c) 50 %
    (d) $\dfrac{200}{3}\%$

35. 38L of milk was poured into a tub and the tub was found to be 5% empty. To completely fill the tub, what amount of additional milk must be poured?
    (a) 1 ℓ
    (b) 2 ℓ
    (c) 3 ℓ
    (d) 4 ℓ

36. Water contains $14\frac{2}{7}\%$ of hydrogen and the rest is oxygen. In 350 g of water, oxygen will be
   (a) 300g   (b) 250g
   (c) 200g   (d) None of these

37. To an examination, a candidate needs 40% marks. All questions carry equal marks. A candidate just passed by getting 10 answers correct by attempting 15 of the total questions. How many questions are there in the examination?
   (a) 25   (b) 30
   (c) 40   (d) 45

38. What is the number whose 20% is 30% of 40?
   (a) 90   (b) 80
   (c) 60   (d) 50

39. An employee is required to contribute 10% of his payment to General Provident Fund. If he gets ₹13500 as net pay in a month, then what is the monthly General Provident Fund contribution (assuming no other deductions)?
   (a) ₹ 1215   (b) ₹ 1350
   (c) ₹ 1500   (d) ₹ 1650

40. A man losses 20% of his money. After spending 25% of the remaining, he has ₹480 left. What is the amount of money he originally had?
   (a) ₹ 600   (b) ₹ 720
   (c) ₹ 800   (d) ₹ 840

41. The radius of the base of a right circular cone is increased by 15% keeping the height fixed. The volume of the cone will be increased by
   (a) 30%   (b) 31%
   (c) 32.25%   (d) 34.75%

42. The price of an article is ₹ 25. After two successive cuts by the same percentage, the price becomes ₹20.25. If each time the cut was $x\%$, then
   (a) $x = 9$ (b) $x = 10$
   (c) $x = 11$ (d) $x = 11.5$

43. What is 5% of 50% of 500?
   (a) 12.5   (b) 25
   (c) 1.25   (d) 6.25

44. X, Y and Z had taken a dinner together. The cost of the meal of Z was 20% more than that of Y and the cost of the meal of X was 5/6 as much as the cost of the meal of Z. If Y paid ₹100, then what was the total amount that all the three of them had paid?
   (a) ₹285   (b) ₹300
   (c) ₹355   (d) None of these

45. A person's salary has increased from ₹7200 to ₹ 8100. What is the percentage increase in his salary?
   (a) 25%   (b) 18%
   (c) $16\frac{2}{3}\%$ (d) $12\frac{1}{2}\%$

46. A person sold an article from ₹ 3600 and got a profit of 20%. Had he sold the article for ₹ 3150, how much profit would he have got?
   (a) 4%   (b) 5%
   (c) 6%   (d) 10%

47. A water pipe is cut into two pieces. The longer piece is 70% of the length of the pipe. By how much percentage is the longer piece longer than the shorter peice?
   (CDS)
   (a) 140%   (b) $\frac{400}{3}\%$
   (c) 40%   (d) None of these

48. On a 20% discount sale, an article costs 596. What was the original price of the article?   **(CDS)**
   (a) ₹ 720   (b) ₹ 735
   (c) ₹ 745   (d) ₹ 775

49. A person could save 10% of his income. But 2 years later, when his income increased by 20%, he could save the same amount only as before. By how much percentage has his expenditure increased?   **(CDS)**
   (a) $22\frac{2}{9}\%$ (b) $23\frac{1}{3}\%$
   (c) $24\frac{2}{9}\%$ (d) $25\frac{2}{9}\%$

50. The salary of a person is increased by 10% of his original salary. But he received the same amount even after increment. What is the percentage of his salary he did not receive?   **(CDS)**
   (a) 11%   (b) 10%
   (c) 100/11%   (d) 90/11%

51. The value of a single discount on some amount which is equivalent to a series of discounts of 10%, 20% and 40% on the same amount, is equal to   **(CDS)**
   (a) 43.2%   (b) 50%
   (c) 56.8%   (d) 70%

52. ABCD is a square. If the sides AB and CD are increased by 30%, sides BC and AD are increased by 20%, then the area of the resulting rectangle exceeds the area of the square by   **(CDS)**
   (a) 50%   (b) 52%
   (c) 54%   (d) 56%

HINTS & SOLUTIONS

1. (d) Let the third number be 100. Then, the first and second numbers will be 20 and 50, respectively.

   Required % $= \dfrac{20}{50} \times 100 = 40\%$

2. (a) Let the original price be x and sale be of y units.
   Then, the revenue collected initially $= x \times y$
   Now, new price $= 0.8\,x$, new sale $= 1.8\,y$
   Then, new revenue collected $= 1.44\,xy$

   % increase in revenue $= \dfrac{0.44xy}{xy} \times 100$

   $= 44\%$ increase

3. (d) Working with options, we have

	Original number	New number	Difference
(a)	22	34	12
(b)	63	96	33
(c)	24	38	14

   Obviously, (d) is the correct option.

4. (a) % change in rate $= \dfrac{27-24}{24} \times 100 = \dfrac{100}{8}\%$
   For fixed expenditure, % change in consumption

   $= \dfrac{\% \text{ change in rate}}{100 + \% \text{ change in rate}} \times 100$

   $= \dfrac{100/8}{100\left[1+\dfrac{1}{8}\right]} \times 100 = \dfrac{100}{9}\% = 11\dfrac{1}{9}\%$

5. (b) Let the total number of original inhabitants be x. Then,
   $(100-25)\%$ of $(100-10)\%$ of $x = 4050$

   $\Rightarrow \left(\dfrac{75}{100} \times \dfrac{90}{100} \times x\right) = 4050 \Rightarrow \dfrac{27}{40}x = 4050$

   $\Rightarrow x = \left(\dfrac{4050 \times 40}{27}\right) = 6000.$

   $\therefore$ Number of original inhabitants $= 6000$

6. (d) Net effect on sale $= -\dfrac{(\text{common } \% \text{ change})^2}{100}$

   $= \dfrac{-(15)^2}{100} = 2.25\%$ decrease

7. (d) The given information gives no indication regarding the comparison of x and y.

8. (a) Let the salary of July be ₹ $\dfrac{5}{2}x$

   and the salary of June be ₹ $\dfrac{9}{4}x$.

   Required percentages

   $= \dfrac{\dfrac{5}{2}x - \dfrac{9}{4}x}{\dfrac{9}{4}x} \times 100$ and $\dfrac{\dfrac{5}{2}x - \dfrac{9}{4}x}{\dfrac{5}{2}x} \times 100$

   $= \dfrac{100}{9}\%$ and $\dfrac{100}{10}\% = 11\dfrac{1}{9}\%$ and 10%

9. (d) Let original price be ₹ x per orange. Then,
   Reduced rate $= (1-0.2)x = ₹\, 0.8\,x$

   $\therefore \dfrac{2.50}{0.8x} - \dfrac{2.50}{x} = 5$

   $\Rightarrow \dfrac{25}{8x} - \dfrac{2.5}{x} = 5 \Rightarrow x = \dfrac{1}{8}$

   $\therefore$ Original price of oranges per dozen $\dfrac{1}{8} \times 12 = ₹\, 1.5$

   and Reduced price $= ₹\,(0.8)(1.5) = ₹\, 1.2$

10. (a) Let the truth spoken by A and B be p_1 and p_2

    respectively, i.e., $p_1 = \dfrac{3}{4}$ and $p_2 = \dfrac{4}{5}$

    They will contradict each other only when one speaks truth and the other is lying.

    i.e., $\dfrac{3}{4} \times \dfrac{1}{5} + \dfrac{4}{5} \times \dfrac{1}{4} = \dfrac{3}{20} + \dfrac{4}{20} = \dfrac{7}{20} = \dfrac{35}{100}$ i.e., 35%

11. (a) % reduction in consumption

    $= \dfrac{\% \text{ change in price}}{100 + \% \text{ change in price}} \times 100$

    $= \dfrac{7}{100+7}\% = \dfrac{7}{107}\%$

12. (b) Seats in executive class $= 50$
    Seats for chair car $= 450$
    Booked seats in total $= 425$
    Booked in executive class $= 48$
    Therefore, seats booked in chair class $= (425-48)$
    $= 377$
    Empty seats for chair class $= 450 - 377 = 73$

13. (a) Let B get ₹ x. Then C gets = 75% of $x = \dfrac{3x}{4}$

and A gets = 120% of $\dfrac{3x}{4} = \dfrac{120}{100} \times \dfrac{3x}{4} = \dfrac{9x}{10}$

Now, $\dfrac{9x}{10} + \dfrac{3x}{4} + x = 4558$

$\Rightarrow \dfrac{53x}{20} = 4558 \Rightarrow x = \dfrac{4558 \times 20}{53} = 1720$

Hence, A's share = $\dfrac{9x}{10} = ₹ \dfrac{9 \times 1720}{10} = ₹ 1548$

14. (d) Let the number of students be x. Then,
Total number of students of 8 years and above 8 years
$= (100 - 20)\%$ of $x = 80\%$ of x.

$\therefore \quad 80\%$ of $x = 48 + 2/3$ of $48 \Rightarrow \dfrac{80}{100} x = 80 \Rightarrow x = 100.$

15. (b) Let the total salary be ₹. x.
Then, $(100 - 10)\%$ of $(100 - 20)\%$ of $(100 - 20)\%$ of $(100 - 10)\%$ of $x = 15552$

$\Rightarrow \left(\dfrac{90}{100} \times \dfrac{80}{100} \times \dfrac{80}{100} \times \dfrac{90}{100} \times x \right) = 15552$

$\Rightarrow x = \left(\dfrac{15552 \times 10000}{64 \times 81} \right) = 30,000$

16. (c) If the radius is diminised by r %, then

Area is diminished by $\left(2r - \dfrac{r^2}{100} \right) \%$

$= 2 \times 10 - \dfrac{10^2}{100} = 19\%$

17. (b) Let the total number of votes enrolled be x. Then,
Number of votes cast = 75% of x. Valid votes = 98% of (75% of x).
$\therefore \quad 75\%$ of $[98\%$ of $(75\%$ of $x)] = 9261$

$\Rightarrow \left(\dfrac{75}{100} \times \dfrac{98}{100} \times \dfrac{75}{100} \times x \right) = 9261$

$\Rightarrow x = \left(\dfrac{9261 \times 100 \times 100 \times 100}{75 \times 98 \times 75} \right) = 16800$

18. (c) Let the total number of votes be x
$\therefore \quad$ votes polled by winning candidate
$= (100 - 30)\%$ of $x = 70\%$ of x
Now, 70% of x – 30% of $x = 15,000$
$\Rightarrow \quad 40\%$ of $x = 15,000$

$\Rightarrow \quad x = \dfrac{15000 \times 100}{40} = 37,500$

$\therefore \quad$ number of votes polled by winning candidate
$= 70\%$ of 37500

$= \dfrac{70 \times 37500}{100} = 26,250$

19. (a) Let the total number of students be x.
Number of students passed in one or both is given by :
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 65\%$ of $x + 60\%$ of $x - 40\%$ of x

$= \left(\dfrac{65}{100} x + \dfrac{60}{100} x - \dfrac{40}{100} x \right) = \dfrac{85}{100} x = \dfrac{17}{20} x.$

Failed in both $= \left(x - \dfrac{17}{20} x \right) = \dfrac{3x}{20}.$

$\therefore \dfrac{3x}{20} = 90 \Rightarrow x = \left(\dfrac{90 \times 20}{3} \right) = 600$

20. (c) $n(A) = 40, n(B) = 50, n(A \cap B) = 10.$
$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 40 + 50 - 10 = 80.$
$\therefore \quad$ Percentage reading either or both newspapers
$= 80\%.$
Hence, percentage reading neither newspaper
$= (100 - 80)\% = 20\%$

21. (b) Total no. of machine parts in both the shipments
$= (120 + 80) = 200$
Total defective machine parts in both the shipments
$= 120 \times 5\% + 80 \times 10\% = 6 + 8 = 14$

Therefore, required % $= \dfrac{14}{200} \times 100 = 7\%$

22. (a) Let the numbers be x and y. Then,

$x + y = \dfrac{28}{25} x \Rightarrow y = \dfrac{28}{25} x - x \Rightarrow y = \dfrac{3}{25} x$

$\Rightarrow \dfrac{y}{x} = \left(\dfrac{3}{25} \times 100 \right) \% = 12\%$

23. (b) Let the original price of a screw driver and a hammer be ₹ 100 each.
Then, price of 3 screw drivers and 3 hammers = ₹ 600
Now, after increase of 5%, the price of 3 screw drivers = ₹ 315
And after 3% increase the price of 3 hammers = ₹ 309
Increased price of 3 hammers and 3 screw drivers
$= ₹ 624$

Therefore, % increase in price $= \dfrac{24}{600} \times 100 = 4\%$

24. (a) If side is increased by a%, area increased by

$\left(2a + \dfrac{a^2}{100} \right) \%$

$= 2 \times 5 + \dfrac{5^2}{100} = 10 \dfrac{1}{4} \%$

25. (b) Let B's salary be ₹ 100,

then A's salary = ₹ 125

$$\% \text{ lesser} = \frac{125-100}{125} \times 100 = \frac{25}{125} \times 100$$

$$= \frac{1}{5} \times 100 = 20\%$$

Short cut method:

B's salary is lower than A's salary by

$$\left(\frac{25}{100+25} \times 100\right)\% = 20\%$$

26. (d) Let the original length and breadth be both 10 cm each.
Then original area = 100 cm^2
New length = $10 \times 1.25 = 12.5$ cm
Let new breadth be x. Then, $12.5x = 100$

$$\Rightarrow \quad x = \frac{100}{12.5} = 8 \text{cm}$$

Hence, % reduction in breadth = $\frac{2}{10} \times 100 = 20\%$

27. (c) **First expenditure:** Suppose 100 litres of petroleum at 100 units of money per litre, then total expenditure = 100×100 units of money = 10000 units of money.
Second expenditure: Now 80 litres of petroleum at 140 units of money per litre, total expenditure = 80×140 units of money = 11200 units.

$\Rightarrow$ Expenditure increases by

$$\frac{11200-10000}{10000} \times 100 = 12\%$$

Short-cut: $\text{Exp}_1 = PX$, $\text{Exp}_2 = 1.4P(0.8X) = 1.12PX$.
$\Rightarrow$ Directly we see, answer = 12%.

28. (b) Let the original price of apple be ₹ x /dozen.

New price ₹ = $\frac{4x}{5}$ /dozen.

$$= \frac{54}{\frac{4x}{5}} - \frac{54}{x} = \frac{10}{12}$$

$$\Rightarrow \quad 54\left(\frac{5}{4x} - \frac{1}{x}\right) = \frac{5}{6}$$

$$\Rightarrow \quad 54\left(\frac{1}{4x}\right) = \frac{5}{6}$$

$$\Rightarrow \quad 4x = \frac{54 \times 6}{5}$$

$$\Rightarrow \quad \frac{4x}{5} = 12.96$$

29. (b) Let rise in salray be x%

$$100\left(1 + \frac{x}{100}\right)\left(1 + \frac{x}{100}\right)\left(1 + \frac{x}{100}\right) = 133.1$$

$$\left(1 + \frac{x}{100}\right)^3 = \frac{133.1}{100} = 1.331$$

$$1 + \frac{x}{100} = 1.1$$

$$\frac{x}{100} = 0.1 \Rightarrow x = 10$$

Rise in salary is 10%

30. (b) $360 = x - \frac{x \times 10}{100} = \frac{9x}{10} \Rightarrow x = 400$

when x is mark obtained by B mark obtained by C = y

$$400 = y + y \times \frac{25}{100} = \frac{5y}{4}$$

$$y = 320$$

mark obtained by D = z

$$320 = z - z \times \frac{20}{100} = \frac{4z}{5}$$

$$z = 400$$

% of mark obtained of D = $\frac{400}{500} \times 100 = 80\%$

31. (b) Let total no. of student = x
Let A and B represent the sets of students who passed in English and mathematics respectively

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 80\% \text{ of } x + 85\% \text{ of } x - 75\% \text{ of } x$$

$$= \frac{80}{100}x + \frac{85}{100}x - \frac{75x}{100} = \frac{90x}{100} = \frac{9x}{10}$$

∴ Students failed in both subjects

$$= x - \frac{9x}{10} = \frac{x}{10}$$

So, $\frac{x}{10} = 40$ x = 400

32. (c) $A = l \times b$

$$A' = \left(l + \frac{15}{100}l\right)\left(b - \frac{15}{100}b\right) = 1.15\,l \times 0.85\,b$$

$$A' = 0.9775\,A$$

$$\% \text{ change} = \frac{A - 0.9775A}{A} \times 100 = 2.25\%$$

33. (b) Given that, 50% of $(x-y) = 40\%$ of $(x+y)$

$\Rightarrow \quad \dfrac{50}{100} \times (x-y) = \dfrac{40}{100} \times (x+y)$

$\Rightarrow \quad 5x - 5y = 4x + 4y$

$\Rightarrow \quad x = 9y \qquad \qquad ...(i)$
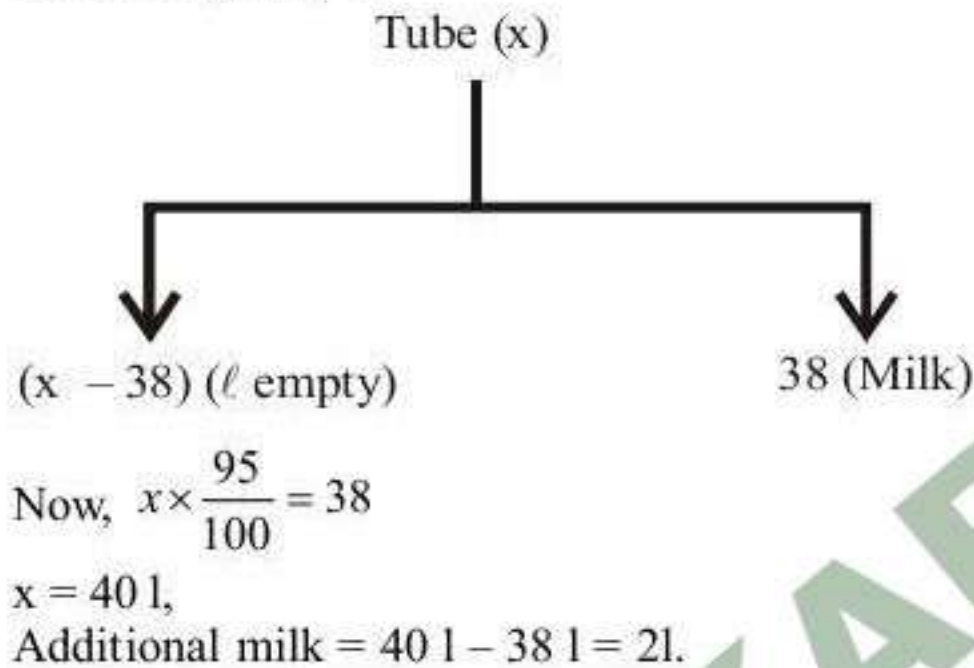
Let $\quad r \% $ of $x = y$

$\Rightarrow \quad \dfrac{r}{100} \times 9y = y \qquad$ [from Eq. (i)]

$\therefore \quad r = \dfrac{100}{9} = 11\dfrac{1}{9}\%$

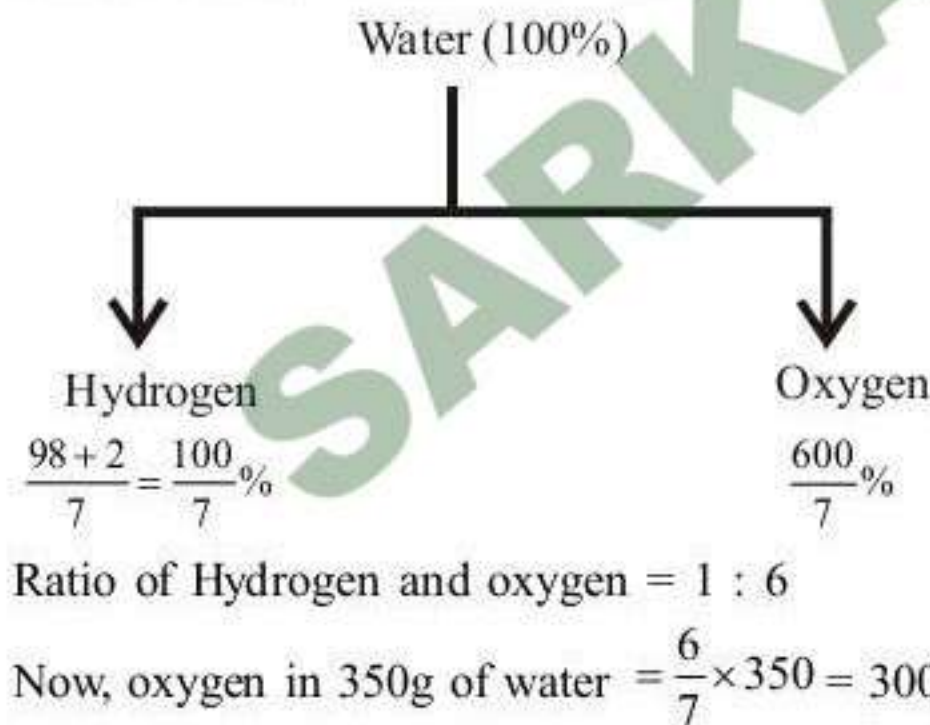34. (c) Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Here radius is constant, then volume is directly proportional to height. Now height increase 50%. So percentage increase in volume is 50%.

35. (b) Let tub capacity x l.

Tube (x)

$(x - 38)$ (ℓ empty)   38 (Milk)

Now, $x \times \dfrac{95}{100} = 38$

$x = 40$ l,

Additional milk $= 40$ l $- 38$ l $= 2$l.

36. (a) Water (100%)

Hydrogen
$\dfrac{98+2}{7} = \dfrac{100}{7}\%$

Oxygen
$\dfrac{600}{7}\%$

Ratio of Hydrogen and oxygen $= 1 : 6$

Now, oxygen in 350g of water $= \dfrac{6}{7} \times 350 = 300$g

37. (a) Let the number of questions in examination $= x$

By given condition, 40% of $x = 10$

$\Rightarrow \quad \dfrac{x \times 40}{100} = 10$

$\therefore \quad = \dfrac{1000}{40} = 25$

38. (c) Let the number be x

According to question 20% of $x = 30\%$ of 40

$\Rightarrow \quad \dfrac{x \times 20}{100} = \dfrac{40 \times 30}{100}$

$\Rightarrow \quad x = \dfrac{40 \times 30}{20} = 60$

39. (c) Let the net pay of employee $= x$

After contributing $10\% = x \times \dfrac{90}{100} = \dfrac{9x}{10}$

According to question $\dfrac{9x}{10} = 13500$

$\because \quad x = \dfrac{13500 \times 10}{9} = 15000$

$\because$ General provident fund $= 10\%$ of basic pay

$= \dfrac{10 \times 15000}{100} = 1500$

40. (c) Let man has originally ₹ x

After 20% loss $= \dfrac{x \times 80}{100} = \dfrac{8x}{10}$

After spending $25\% = \dfrac{8x}{10} \times \dfrac{75}{100} = \dfrac{8x}{10} \times \dfrac{3}{4}$

According to the question,

$\dfrac{8x}{10} \times \dfrac{3}{4} = 480$

$\Rightarrow \quad 8x \times 3 = 480 \times 4 \times 10$

$\therefore \quad x = \dfrac{480 \times 4 \times 10}{8 \times 3} = 800$

41. (c) $x + y + \dfrac{xy}{100}$

$= 15 + 15 + \dfrac{15 \times 15}{100}$

$= 30 + \dfrac{225}{100}$

$= 32.25\%$

42. (b) According to the question,

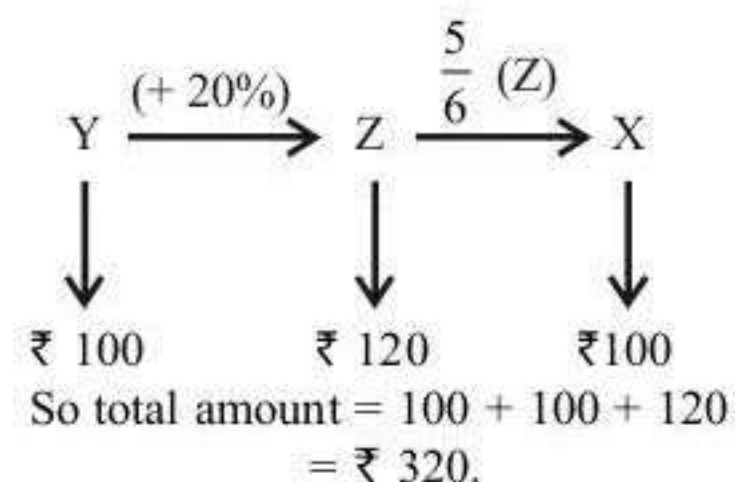$\Rightarrow \quad 25 \times \left(\dfrac{100-x}{100}\right)\left(\dfrac{100-x}{100}\right) = 20.25$

$\Rightarrow \quad (100-x)^2 = \dfrac{202500}{25} \Rightarrow (100-x)^2 = 8100$

$\Rightarrow \quad 100 - x = 90$

$\therefore \quad x = 10$

43 (a) 5% of 50% of $500 = \dfrac{5}{100} \times \dfrac{50}{100} \times 500 = 12.5$

44. (d) According to question.

$Y \xrightarrow{(+ 20\%)} Z \xrightarrow{\frac{5}{6} (Z)} X$

₹ 100   ₹ 120   ₹100

So total amount $= 100 + 100 + 120$

$= ₹ 320.$

45. (d) Percentage increase in salary

$$= \frac{8100 - 7200}{7200} \times 100$$

$$= \frac{900}{7200} \times 100 = 12.5\% = 12\frac{1}{2}\%$$

46. (b) Let the cost price of the article be ₹ x

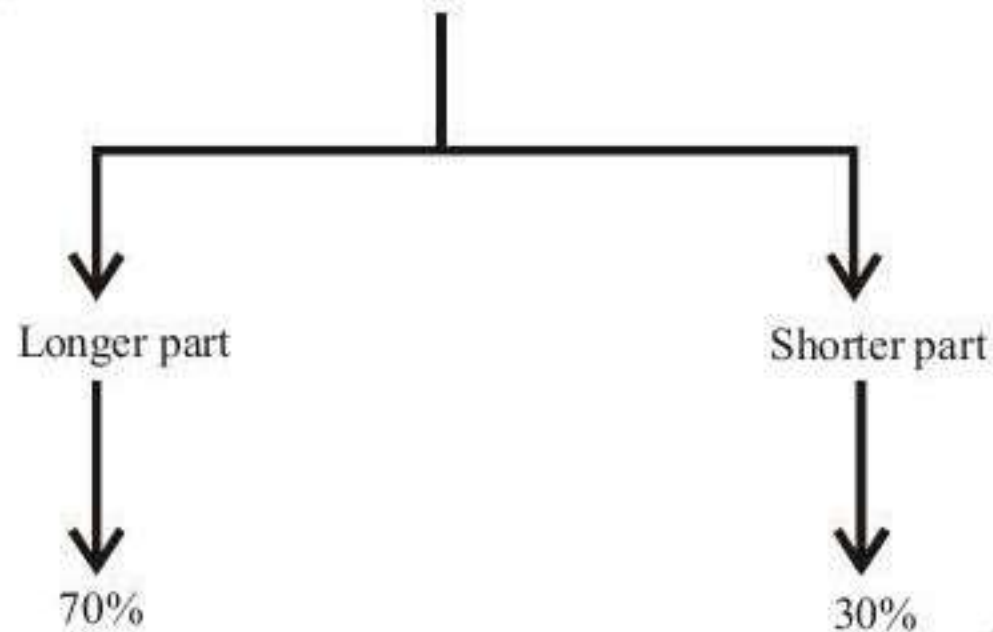After 20% profit $\Rightarrow$ $\frac{120x}{100} = 3600$

$x = 3000$

Now, profit percentage, when the article is sold for ₹3150

$$= \frac{3150 - 3000}{3000} \times 100 - \frac{150}{3000} \times 100 = 5\%$$

47. (b)

Pipe

Longer part   Shorter part

70%   30%

Percentage of longer pipe as compare to shorter pipe

$$= \frac{70 - 30}{30} \times 100 = \frac{40}{30} \times 100 = \frac{400}{3}\%$$

48. (c) Let the original price of article be ₹ x
Now, After 20% discount article costs ₹ 596

So, $596 = \frac{80}{100} \times x$ $\Rightarrow$ $x = \frac{596 \times 100}{80} = 745$

∴ Original price = ₹ 745

49. (a) Let income be ₹ 100

Expenditure amount $= 100 \times \frac{90}{100} = ₹ 90$

Now, income increased by 20% $= 100 \times \frac{120}{100} = ₹ 120$

Expenditure amount $= (120 - 10) = ₹ 110$
Increase in expenditure $= 110 - 90 = ₹ 20$

Increase in % of expenditure $= \frac{20}{90} \times 100$

$$= \frac{200}{9} = 22\frac{2}{9}\%$$

50. (c) Let original salary of a person = x ₹
Increase in salary = 10% of x
Then New Salary = x + 10% of x

$$x + \frac{10}{100} \times x = \frac{11}{10}x$$

But he received same amount even after increment.
Let y be the percentage salary he did not receive.

then $\frac{11}{10}x - \frac{11}{10}x \times \frac{y}{100} = x$

$\Rightarrow \frac{11}{10}x\left[1 - \frac{y}{100}\right] = x$

$\Rightarrow 1 - \frac{y}{100} = \frac{10}{11}$

$\Rightarrow \frac{y}{100} = 1 - \frac{10}{11} = \frac{1}{11}$

$\Rightarrow y = \frac{100}{11}\%$

51. (c) Discount equivalent to 10% and 20%
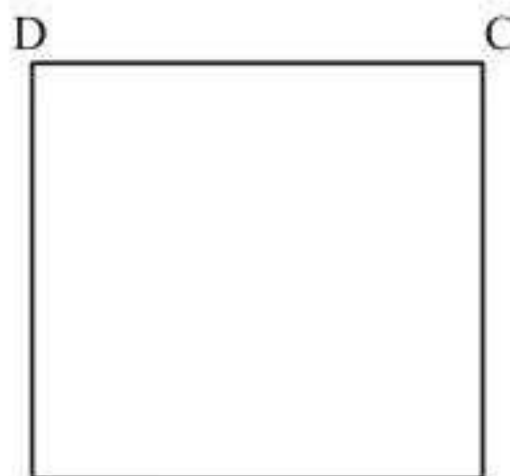
$$= 10 + 20 - \frac{10 \times 20}{100} = 28\%$$

Discount equivalent to 28% and 40%

$$= 28 + 40 - \frac{28 \times 40}{100}$$

$$= 68 - 11.2$$

$$= 56.8\%$$

52. (d)

D   C

A ← a → B

Let initially the side of square be 'a'.
Area of square = a^2
Area of reactangle = 1.3 a × 1.2 a
Percentage increase = 1.56 a^2 in Area

$$= \frac{(1.56a^2 - a^2)}{a^2} \times 100$$

$$= (1.56 - 1) \, 100 = 56\%$$