Applications of Definite Integration

EXERCISE 7.1 [PAGE 157]

Exercise 7.1 | Q 1.1 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: $y = x^4$, $x = 1$, $x = 5$

Solution:

Let A be the required area.

Consider the equation $y = x^4$.

$\therefore A = \int_1^5 y \cdot dx$

$= \int_1^5 x^4 \cdot dx$

$= \left[\frac{x^5}{5}\right]_1^5$

$= \frac{1}{5}\left[x^5\right]_1^5$

$= \frac{1}{5}\left[(5)^5 - (1)^5\right]$

$= \frac{1}{5}(3125 - 1)$

$\therefore A = \frac{3124}{5}$ sq . units.

Exercise 7.1 | Q 1.2 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: $y = \sqrt{6x + 4}, x = 0, x = 2$

Solution:

Let A be the required area.

Consider the equation y = $\sqrt{6x+4}$.

$\therefore$ A = $\int_0^2 y \cdot dx$

= $\int_0^2 \sqrt{6x+4} \cdot dx$

= $\int_0^2 (6x+4)^{\frac{1}{2}}$*dx

= $\left[\frac{(6x+4)^{\frac{1}{2}}}{\frac{3}{2} \times 6}\right]$

= $\frac{1}{9}\left[(6x+4)^{\frac{3}{2}}\right]$

= $\frac{1}{9}\left[(6 \times 2+4)^{\frac{3}{2}} - (6 \times 0+4)^{\frac{3}{2}}\right]$

= $\frac{1}{9}\left[(16)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right]$

= $\frac{1}{9}\left[(4^2)^{\frac{3}{2}} - (2^2)^{\frac{3}{2}}\right]$

= $\frac{1}{9}\left[(4)^2 - (2)^3\right]$

= $\frac{1}{9}(64-8)$

$\therefore$ A = $\frac{56}{9}$ sq.units.

Exercise 7.1 | Q 1.3 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: y = $\sqrt{16-x^2}$, x = 0, x = 4

Solution:

Let A be the required area.

Consider the equation y = $\sqrt{16-x^2}$.

$\therefore$ A = $\int_0^4 y \cdot dx$

= $\int_0^4 \sqrt{16-x^2} \cdot dx$

= $\int_0^4 \sqrt{(4)^2-(x)^2} \cdot dx$

= $\left[\frac{x}{2}\sqrt{(4)^2-x^2}+\frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

= $\left[\frac{4}{2}\sqrt{16-(4)^2}+\frac{16}{2}\sin^{-1}\left(\frac{4}{4}\right)\right]-\left[\frac{0}{2}\sqrt{16-(0)^2}+\frac{16}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]$

= $[2(0) + 8\sin^{-1}(1)] - [0 + 0]$

= $8 \times \frac{\pi}{2}$

$\therefore$ A = 4π q. units.

Exercise 7.1 | Q 1.4 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: 2y = 5x + 7, x = 2, x = 8

Solution: Let A be the required area.

Consider the equation 2y = 5x + 7

i.e. y = $\frac{5x+7}{2}$

$\therefore$ A = $\int_2^8 y \cdot dx$

$$= \int_2^8 \frac{5x+7}{2} \cdot dx$$

$$= \frac{1}{2} \int_2^8 (5x+7) \cdot dx$$

$$= \frac{1}{2} \left[\frac{5x^2}{2} + 7x \right]_2^8$$

$$= \frac{1}{2} \left[\left(\frac{5 \times 8^2}{2} + 7 \times 8 \right) - \left(\frac{5 \times 2^2}{2} + 7 \times 2 \right) \right]$$

$$= \frac{1}{2} [(160 + 56) - (10 + 14)]$$

$$= \frac{1}{2} (216 - 24)$$

$$= \frac{1}{2} \times 192$$

∴ A = 96 sq. units.

Exercise 7.1 | Q 1.5 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: 2y + x = 8, x = 2, x = 4

Solution:

Let A be the required area.

Consider the equation 2y + x = 8

i.e., y = $\frac{8-x}{2}$

∴ A = $\int_2^4 y \cdot dx$

$$= \int_2^4 \frac{8-x}{2} \cdot dx$$

$$= \frac{1}{2} \int_2^4 (8-x) \cdot dx$$

$$= \frac{1}{2}\left[8x - \frac{x^2}{2}\right]_2^4$$

$$= \frac{1}{2}\left[\left(8 \times 4 - \frac{4^2}{2}\right) - \left(8 \times 2 - \frac{2^2}{2}\right)\right]$$

$$= \frac{1}{2}(32 - 8) - (16 - 2)\Big]$$

$$= \frac{1}{2}(24 - 14)$$

$$= \frac{1}{2} \times 10$$

∴ A = 5 sq. units.

Exercise 7.1 | Q 1.6 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: $y = x^2 + 1$, $x = 0$, $x = 3$

Solution: Let A be the required area.

Consider the equation $y = x^2 + 1$.

$$\therefore A = \int_0^3 y \cdot dx$$

$$= \int_0^3 (x^2 + 1) \cdot dx$$

$$= \left[\frac{x^3}{3} + x\right]_0^3$$

$$= \left(\frac{3^3}{3} + 3\right) - (0)$$

= (9 + 3)

∴ A = 12 sq. units.

Exercise 7.1 | Q 1.7 | Page 157

Find the area of the region bounded by the following curves, the X-axis and the given lines: $y = 2 - x^2$, $x = -1$, $x = 1$

Solution: Let A be the required area.

Consider the equation $y = 2 - x^2$.

$$\therefore A = \int_{-1}^{1} y \cdot dx$$

$$= \int_{-4}^{1} (2 - x^2) \cdot dx$$

$$= \left[2x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \left[2 \times 1 - \frac{1^3}{3}\right] - \left[2 \times (-1) - \frac{(-1)^3}{3}\right]$$

$$= \left(2 - \frac{1}{3}\right) - \left(-2 + \frac{1}{3}\right)$$

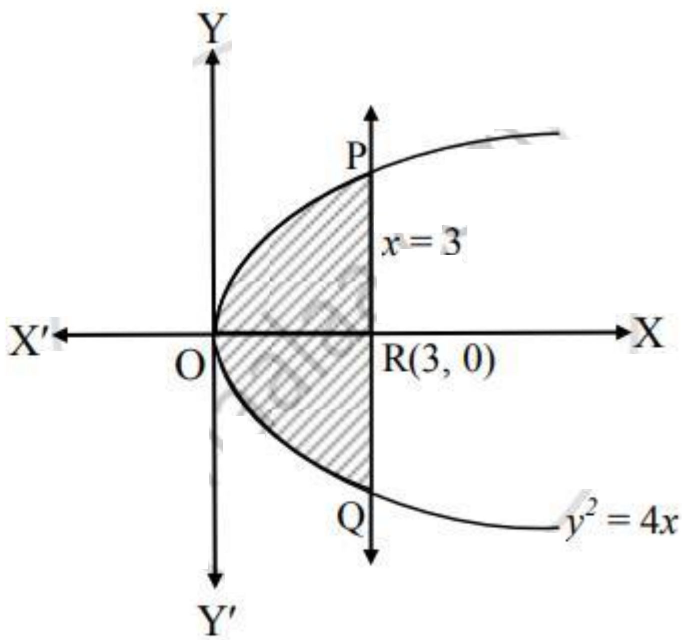
$$= \frac{5}{3} - \left(\frac{-5}{3}\right)$$

$$\therefore A = \frac{10}{3} \text{ sq. units.}$$

Exercise 7.1 | Q 2 | Page 157

Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$.

Solution:



GIven equation of the parabola is $y^2 = 4x$

$\therefore y = 2\sqrt{x}$ ...[$\because$ In first quadrant, $y > 0$]

and equation of the line is $x = 3$

$\therefore$ Required = area of the region OQRPO

= 2(area of the region ORPO

$$= 2\int_0^3 y \cdot dx$$

$$= 2\int_0^3 2\sqrt{x} \cdot dx$$

$$= 4\int_0^3 \sqrt{x} \cdot dx$$

$$= 4\int_0^3 x^{\frac{1}{2}} \cdot dx$$

$$= 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3$$

$$= 4 \times \frac{2}{3}\left[(3)^{\frac{3}{2}} - 0\right]$$

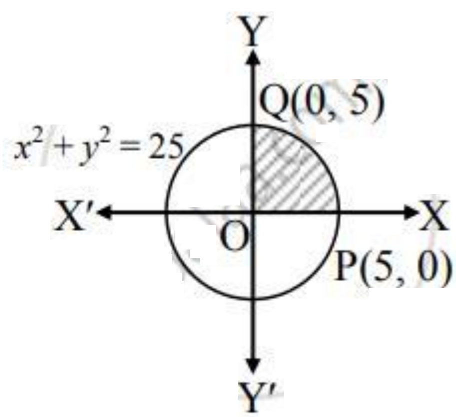
$$= \frac{8}{3}\left(3\sqrt{3}\right)$$

$\therefore$ Required area = $8\sqrt{3}$sq. units.

Exercise 7.1 | Q 3 | Page 157

Find the area of circle $x^2 + y^2 = 25$.

Solution:



By the symmetry of the circle, required area of the circle is 4 times the area of the region OPQO.

For the region OPQO, the limits of integration are x = 0 and x = 5.

Given equation of the circle is

$x^2 + y^2 = 25$

$\therefore y^2 = 25 – x^2$

$\therefore$ y = $\pm \sqrt{25 - x^2}$

$\therefore$ y = $\sqrt{25 - x^2}$ ...[$\because$ In first quadrant , y > 0]

$\therefore$ Required area = 4 (area of the region OPQO)

$$= 4 \times \int_0^5 y \cdot dx$$

$$= 4 \times \int_0^5 \sqrt{25 - x^2} \cdot dx$$

$$= 4 \int_0^5 \sqrt{(5)^2 - x^2} \cdot dx$$

$$= 4\left[\frac{x}{2}\sqrt{(5)^2 - x^2} + \frac{(5)^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= 4\left\{\left[\frac{5}{2}\sqrt{25 - (5)^2} + \frac{25}{2}\sin^{-1}\left(\frac{5}{5}\right)\right] - \left[\frac{0}{2}\sqrt{25 - (0)^2} + \frac{25}{2}\sin^{-1}\left(\frac{0}{5}\right)\right]\right\}$$

$$= 4\left\{\left[\frac{5}{2}(0) + \frac{25}{2}\sin^{-1}(1)\right] - [0 + 0]\right\}$$

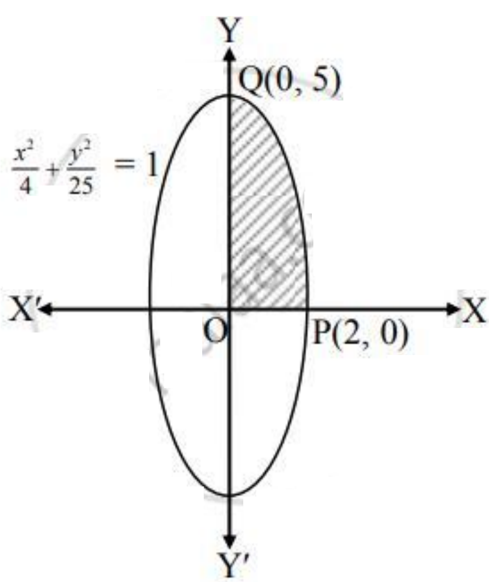
$$= 4\left(\frac{25}{2} \times \frac{\pi}{2}\right)$$

= 25π sq. units.

Exercise 7.1 | Q 4 | Page 157

Find the area of ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

Solution:



By the symmetry of the ellipse, required area of the ellipse is 4 times the area of the region OPQO.

For the region OPQO, the limits of integration are x = 0 and x = 2.

Given equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{25} = 1$

$\therefore \frac{y^2}{25} = 1 - \frac{x^2}{4}$

$\therefore y^2 = 25\left(1 - \frac{x^2}{4}\right)$

$= \frac{25}{4}(4 - x^2)$

$\therefore y = \pm\frac{5}{2}\sqrt{4 - x^2}$

$\therefore y = \frac{5}{2}\sqrt{4 - x^2}$ ...[$\because$ In first quadrant, y . 0]

$\therefore$ Required area = 4(area of the region OPQO)

$= 4\int_0^2 y \cdot dx$

$= 4\int_0^2 \frac{5}{2}\sqrt{4 - x^2} \cdot dx$

$= \frac{4 \times 5}{2}\int_0^2 \sqrt{(2)^2 - x^2} \cdot dx$

$= 10\left[\frac{x}{2}\sqrt{(2)^2 - x^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= 10\left\{\left[\frac{2}{2}\sqrt{(2)^2 - (2)^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{2}{2}\right)\right] - \left[\frac{0}{2}\sqrt{(2)^2 - (0)^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]\right\}$

$= 10\{[0 + 2\sin^{-1}(1)] - [0 + 0]\}$

$= 10\left(2 \times \frac{\pi}{2}\right)$

$= 10\pi$ sq. units.

MISCELLANEOUS EXERCISE 7 [PAGES 157 - 158]

Miscellaneous Exercise 7 | Q 1.1 | Page 157

Choose the correct alternative :

Area of the region bounded by the curve $x^2 = y$, the X-axis and the lines x = 1 and x = 3 is _______.

1. **26/3sq. units**
2. 3/26sq. units
3. 26 sq. units
4. 3 sq. units

Solution:

$$\text{Required area} = \int_1^2 y \cdot dx$$

$$= \int_1^3 x^2 \cdot dx$$

$$= \left[\frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{3}(27 - 1)$$

$$= \frac{26}{3} \text{ sq. units.}$$

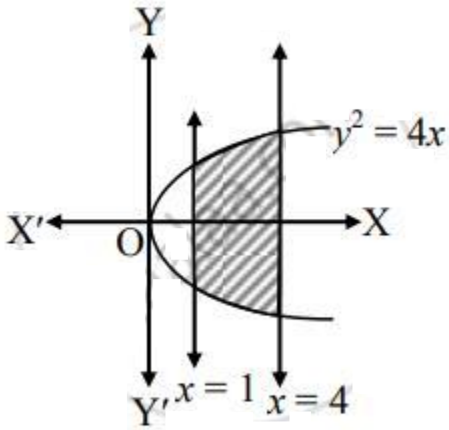
Miscellaneous Exercise 7 | Q 1.2 | Page 157

Choose the correct alternative :

The area of the region bounded by $y^2 = 4x$, the X-axis and the lines x = 1 and x = 4 is _______.

1. 28 sq. units
2. 3 sq. units
3. **56/3 sq. units**
4. 63/7 sq. units

Solution:



Required area = $2\int_1^4 y \cdot dx$

$= 2\int_1^4 2\sqrt{x} \cdot dx$

$= 4\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \dfrac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$

$= \dfrac{8}{3}(8 - 1)$

$= \dfrac{56}{3}$ sq.units.

Miscellaneous Exercise 7 | Q 1.3 | Page 157

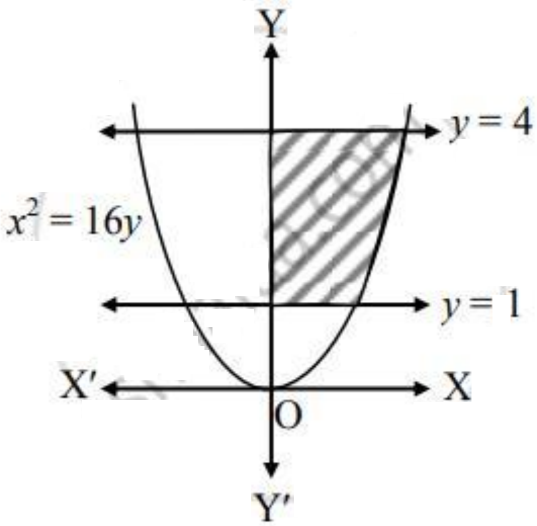
Choose the correct alternative :

Area of the region bounded by $x^2 = 16y$, $y = 1$ and $y = 4$ and the Y-axis, lying in the first quadrant is _______.

1. 53 sq. units
2. 3/56 sq. units

3. **56/3 sq. units**
4. 63/7 sq. units

Solution:



$$\text{Required area} = \int_1^4 x \cdot dy$$

$$= \int_1^4 4\sqrt{y} \cdot dy$$

$$= 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= \frac{8}{3}(8-1)$$

$$= \frac{56}{3}\text{sq.units.}$$

Miscellaneous Exercise 7 | Q 1.4 | Page 157

Choose the correct alternative :

Area of the region bounded by $y = x^4$, x = 1, x = 5 and the X-axis is _____.

1. 3142/5 sq.unts
2. **3124/5 sq.unts**
3. 3142/3 sq.unts
4. 3124/3 sq.unts

Solution: Let A be the required area.

Consider the equation $y = x^4$.

$$\therefore A = \int_1^5 y \cdot dx$$

$$= \int_1^5 x^4 \cdot dx$$

$$= \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{1}{5}\left[x^5\right]_1^5$$

$$= \frac{1}{5}\left[(5)^5 - (1)^5\right]$$

$$= \frac{1}{5}(3125 - 1)$$

$$\therefore A = \frac{3124}{5} \text{sq . units.}$$

Miscellaneous Exercise 7 | Q 1.5 | Page 157

Choose the correct alternative :

Using definite integration, area of circle $x^2 + y^2 = 25$ is _______.

1. 5π sq. units
2. **4π sq. units**
3. 25π sq. units
4. 25 sq. units

Solution: Let A be the required area.

Consider the equation y = $\sqrt{16 - x^2}$.

$\therefore$ A = $\int_0^4 y \cdot dx$

= $\int_0^4 \sqrt{16 - x^2} \cdot dx$

= $\int_0^4 \sqrt{(4)^2 - (x)^2} \cdot dx$

= $\left[\frac{x}{2}\sqrt{(4)^2 - x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

= $\left[\frac{4}{2}\sqrt{16 - (4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{4}{4}\right)\right] - \left[\frac{0}{2}\sqrt{16 - (0)^2} + \frac{16}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]$

= $[2(0) + 8\sin^{-1}(1)] - [0 + 0]$

= $8 \times \frac{\pi}{2}$

$\therefore$ A = **4π q. units**.

Miscellaneous Exercise 7 | Q 2.1 | Page 158

Fill in the blank :

Area of the region bounded by $y = x^4$, $x = 1$, $x = 5$ and the X-axis is _______.

Solution: Let A be the required area.

Consider the equation $y = x^4$.

$\therefore$ A = $\int_1^5 y \cdot dx$

$$= \int_1^5 x^4 \cdot dx$$

$$= \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{1}{5}\left[x^5\right]_1^5$$

$$= \frac{1}{5}\left[(5)^5 - (1)^5\right]$$

$$= \frac{1}{5}(3125 - 1)$$

$$\therefore \text{A} = \frac{3124}{5} \text{sq . units.}$$

Miscellaneous Exercise 7 | Q 2.2 | Page 158

Using definite integration, area of the circle $x^2 + y^2 = 49$ is _______.

Solution: Area of the circle $x^2 + y^2 = r^2$ is πr^2 sq. units.

Here, $r^2 = 49$

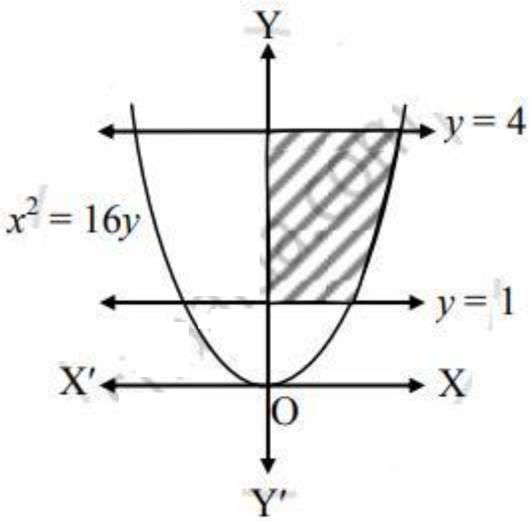
∴ Required area = **49π sq. units**.

Miscellaneous Exercise 7 | Q 2.3 | Page 158

Fill in the blank :

Area of the region bounded by $x^2 = 16y$, $y = 1$, $y = 4$ and the Y-axis, lying in the first quadrant is _______.

Solution:



Required area $= \int_1^4 x \cdot dy$

$$= \int_1^4 4\sqrt{y} \cdot dy$$

$$= 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= \frac{8}{3}(8 - 1)$$

$$= \frac{56}{3}\text{sq.units.}$$

Miscellaneous Exercise 7 | Q 2.4 | Page 158

Fill in the blank :

The area of the region bounded by the curve $x^2 = y$, the X-axis and the lines x = 3 and x = 9 is _______.

Solution:

Required area = $\int_3^9 y \cdot dx$

$= \int_3^9 x^2 \cdot dx$

$= \left[\frac{x^3}{3}\right]_3^9$

$= \frac{1}{3}\left(9^3 - 3^3\right)$

$= \frac{1}{3}(729 - 27)$

$= \frac{702}{3}$

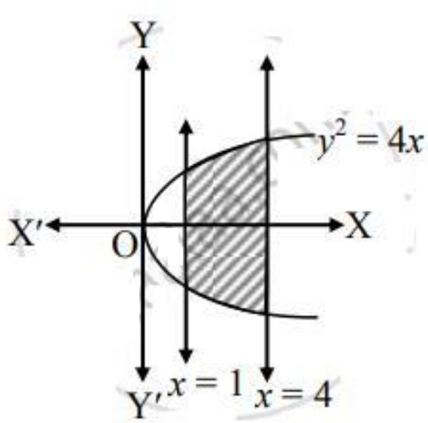
= **234 sq. units**.

Miscellaneous Exercise 7 | Q 2.5 | Page 158

Fill in the blank :

The area of the region bounded by $y^2 = 4x$, the X-axis and the lines x = 1 and x = 4 is _______.

Solution:



Required area = $2\int_1^4 y \cdot dx$

$= 2\int_1^4 2\sqrt{x} \cdot dx$

$= 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \frac{8}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$

$= \frac{8}{3}(8 - 1)$

$= \frac{56}{3}\text{sq.units.}$

Miscellaneous Exercise 7 | Q 3.1 | Page 158

State whether the following is True or False :

The area bounded by the curve x = g (y), Y-axis and bounded between the lines y = c and y = d is given by

$$\int_{c}^{d} x \cdot dy = \int_{y=c}^{y=d} g(y) \cdot dy$$

1. **True**
2. False

Solution: The area bounded by the curve x = g (y), Y-axis and bounded between the lines y = c and y = d is given by

$\int_{c}^{d} x \cdot dy = \int_{y=c}^{y=d} g(y) \cdot dy$ **True.**

Miscellaneous Exercise 7 | Q 3.2 | Page 158

State whether the following is True or False :

The area bounded by the two cures y = f(x), y = g (x) and X-axis is $\left|\int_a^b f(x) \cdot dx - \int_b^a g(x) \cdot dx\right|$.

1. True
2. **False**

Solution:

The area bounded by two curves y = f(x), y = g (x) and X-axis is $\left|\int_a^b f(x) \cdot dx - \int_a^b g(x) \cdot dx\right|$ **False**.

Miscellaneous Exercise 7 | Q 3.3 | Page 158

State whether the following is True or False :

The area bounded by the curve y = f(x), X-axis and lines x = a and x = b is $\left|\int_a^b f(x) \cdot dx\right|$.

1. **True**
2. False

Solution:

The area bounded by the curve y = f(x), X-axis and lines x = a and x = b is $\left|\int_a^b f(x) \cdot dx\right|$ **True**.

Miscellaneous Exercise 7 | Q 3.4 | Page 158

State whether the following is True or False :

If the curve, under consideration, is below the X-axis, then the area bounded by curve, X-axis and lines x = a, x = b is positive.

1. **True**
2. False

Solution: If the curve, under consideration, is below the X-axis, then the area bounded by curve, X-axis and lines x = a, x = b is positive **True**.

Miscellaneous Exercise 7 | Q 3.5 | Page 158

State whether the following is True or False :

The area of the portion lying above the X-axis is positive

1. **True**
2. False

Solution: The area of the portion lying above the X-axis is positive **True**.

Miscellaneous Exercise 7 | Q 4.1 | Page 158

Solve the following :

Find the area of the region bounded by the curve $xy = c^2$, the X-axis, and the lines x = c, x = 2c.

Solution: Given equation of the curve is $xy = c^2$

$$\therefore y = \frac{c^2}{x}$$

$$\therefore \text{Required area} = \int_c^{2c} y \cdot dx$$

$$= \int_c^{2c} \frac{c^2}{x} \cdot dx$$

$$= c^2 \int_c^{2c} \left(\frac{1}{x}\right) \cdot dx$$

$= c^2[\log x]_c^{2c}$

= c2(log 2c – log c)

$= c^2 \log\left(\frac{2c}{c}\right)$

$= c^2 \log 2$ sq.units.

Miscellaneous Exercise 7 | Q 4.2 | Page 158

Solve the following :

Find the area between the parabolas $y^2 = 7x$ and $x^2 = 7y$.

Solution: Given equations of the parabolas are $y^2 = 7x$ ...(i)

and $x^2 = 7y$

$\therefore y = \frac{x^2}{7}$ ...(ii)

From (i), we get

$y = \sqrt{7x}$ ...(iii) [$\because$ In first quadrant, y > 0]

Find the points of intersection of $y^2 = 7x$ and $x^2 = 7y$.

Subbstituting (ii) in (i) we get

$\left(\frac{x^2}{7}\right)^2 = 7x$

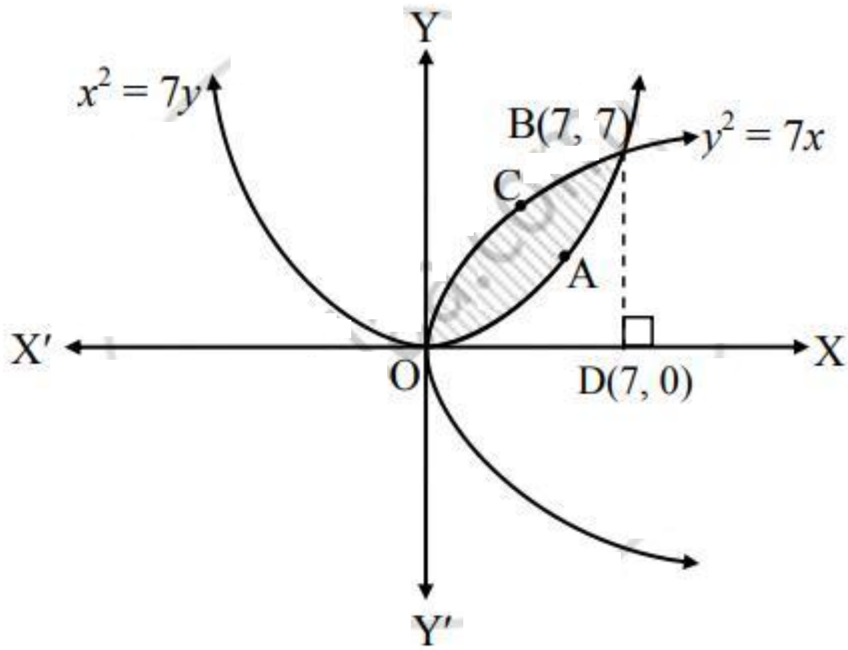
$\therefore x^4 = 343x$

$\therefore x^4 - 34x = 0$

$\therefore x(x^3 - 343) = 0$

$\therefore x = 0$ or $x^3 = 343 = 7^3$

$\therefore x = 0$ or $x = 7$



When x = 0, y = 0 and when x = 7, y = 7

∴ The points of intersection of y2 = 7x and x2 = 7y are O(0, 0) and B(7, 7).

Draw BD ⊥ OX

Required area = area of the region OABCO

= area of the region ODBCO – area of the region ODBAO

= area under the parabola $y^2 = 7x$ – area under the parabola $x^2 = 7y$

$= \int_0^7 \sqrt{7x} \cdot dx - \int_0^7 \frac{x^2}{7} \cdot dx$ ...[from (iii) and (ii)]

$= \sqrt{7} \int_0^7 x^{\frac{1}{2}} \cdot dx - \frac{1}{7} \int_0^7 x^2 \cdot dx$

$= \sqrt{7} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^7 - \frac{1}{7} \left[\frac{x^3}{3} \right]_0^7$

$= \frac{2\sqrt{7}}{3} [97)^{\frac{3}{2}} - 0\Big] - \frac{1}{21} \left[(7)^3 - 0 \right]$

$$= \frac{2\sqrt{7}}{3}\left(7\sqrt{7}\right) - \frac{1}{21}(343)$$

$$= \frac{98}{3} - \frac{49}{3}$$

$$= \frac{49}{3} \text{ sq. units.}$$

Miscellaneous Exercise 7 | Q 4.3 | Page 158

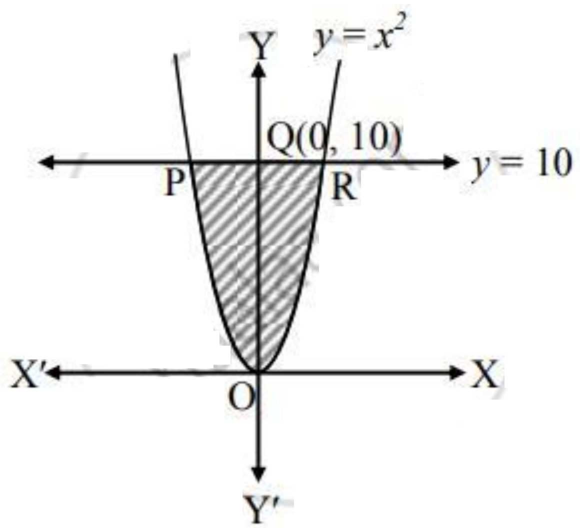
Solve the following :

Find the area of the region bounded by the curve $y = x^2$ and the line $y = 10$.

Solution: Given equation of the curve is

$y = x^2$

$\therefore x = \sqrt{y}$ ...[$\because$ In first quadrant, $x > 0$]



Required area = area of the region ORQPO

= 2 (area of the region ORQO)

$$= 2\int_0^{10} x \cdot dy$$

$$= 2\int_0^{10} y^{\frac{1}{2}} \cdot dy$$

$$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{10}$$

$$= \frac{4}{3}\left[(10)^{\frac{3}{2}} - 0\right]$$

$$= \frac{4}{3}\left(10\sqrt{10}\right)$$

$$= \frac{40\sqrt{10}}{3}\text{sq.units.}$$

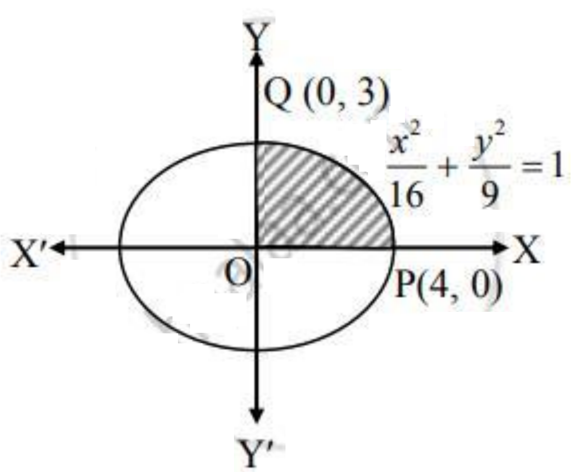
Miscellaneous Exercise 7 | Q 4.4 | Page 158

Solve the following :

Find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1.$

Solution: By the symmetry of the ellipse, required area of the ellipse is 4 times the area of the region OPQO.

For the region OPQO, the limits of integration are x = 0 and x = 4.



Given equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{9}$ = 1

$\therefore \frac{y^2}{9}=1-\frac{x^2}{16}$

$\therefore$ y^2 = $9\left(1-\frac{x^2}{16}\right)$

= $\frac{9}{16}\left(16-x^2\right)$

$\therefore$ y = $\pm\frac{3}{4}\sqrt{16-x^2}$

$\therefore$ y = $\frac{3}{4}\sqrt{16-x^2}$ ...[$\because$ In first quadrant, y > 0]

$\therefore$ Required area = 4(area of the region OPQO)

= $4\int_0^4 y\cdot dx$

= $4\int_0^4 \frac{3}{4}\sqrt{16-x^2}\cdot dx$

= $3\int_0^4 \sqrt{(4)^2-x^2}\cdot dx$

= $3\left[\frac{x}{2}\sqrt{(4)^2-x^2}+\frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

= $3\left\{\left[\frac{4}{2}\sqrt{(4)^2-(4)^2}+\frac{(4)^2}{2}\sin^{-1}\left(\frac{4}{4}\right)\right]-\left[\frac{0}{2}\sqrt{(4)^2-(0)^2}+\frac{(4)^2}{2}\sin^{-1}\left(\frac{0}{4}\right)\right]\right\}$

= 3{[0 + 8sin–1 (1)] – [0 + 0]}

= $3\left(8\times\frac{\pi}{2}\right)$

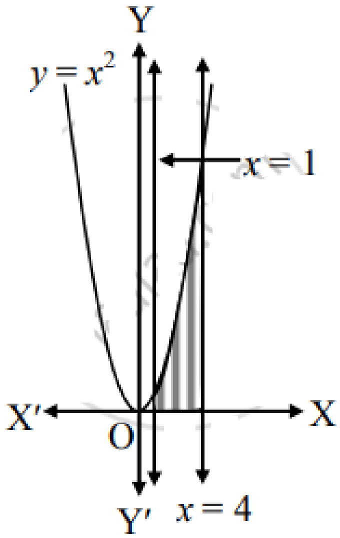
= 12π sq. units.

Miscellaneous Exercise 7 | Q 4.5 | Page 158

Solve the following :

Find the area of the region bounded by $y = x^2$, the X-axis and $x = 1$, $x = 4$.

Solution:



Required area = $\int_1^4 y \cdot dx$

$= \int_1^4 x^2 \cdot dx$

$= \left[\frac{x^3}{3}\right]_1^4$

$= \frac{1}{3}\left(4^3 - 1^3\right)$

$= \frac{1}{3}(64 - 1)$

$= \frac{1}{3}(63)$

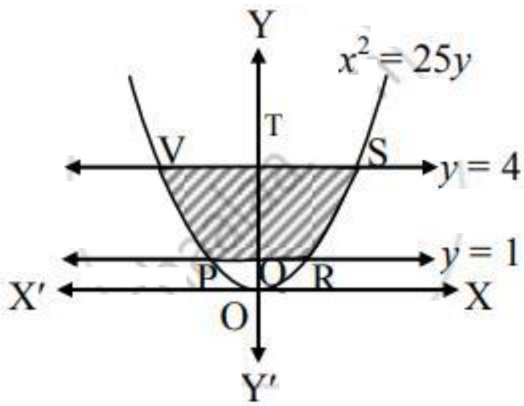
= 21 sq. units.

Miscellaneous Exercise 7 | Q 4.6 | Page 158

Solve the following :

Find the area of the region bounded by the curve $x^2 = 25y$, $y = 1$, $y = 4$ and the Y-axis.

Solution:



Given equation of the curve is $x^2 = 25y$

$\therefore 5\sqrt{y}$ ...[$\because$ In first quadrant, x > 0]

Required area = area of the region PRSVP

= 2(area of the region QRSTQ)

$$= 2\int_1^4 x \cdot dy$$

$$= 2\int_1^4 5\sqrt{y} \cdot dy$$

$$= 10\int_1^4 y^{\frac{1}{2}} \cdot dy$$

$$= 10\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$= \frac{20}{3}\left[(4)\left(\frac{3}{2}\right) - (1)^{\frac{3}{2}}\right]$

$= \frac{20}{3}(8 - 1)$

$= \frac{20}{3}(7)$

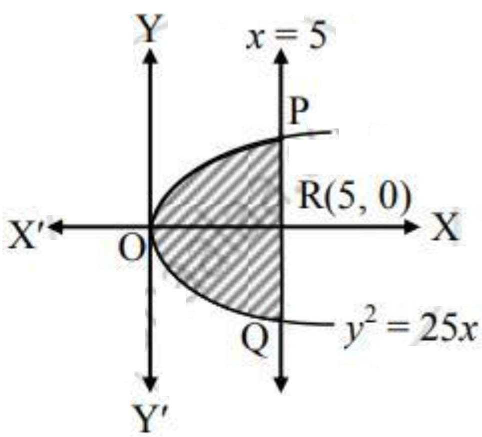
$= \frac{140}{3}$ sq. units.

Miscellaneous Exercise 7 | Q 4.7 | Page 158

Solve the following :

Find the area of the region bounded by the parabola $y^2 = 25x$ and the line $x = 5$.

Solution:



Given equation of the parabola is $y^2 = 25x$

$\therefore y = 5\sqrt{x}$ ...[∵ IIn first quadrant, y > 0]

Requred areaa = area of the region OQRPO

= 2(area of the region ORPO)

$= 2\int_0^5 y \cdot dx$

$$= 2\int_0^5 5\sqrt{x}\cdot dx$$

$$= 10\int_0^5 x^{\frac{1}{2}}\cdot dx$$

$$= 10\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^5$$

$$= \frac{20}{3}\left[(5)^{\frac{3}{2}} - 0\right]$$

$$= \frac{20}{3}\left(5\sqrt{(5)}\right.$$

$$= \frac{100\sqrt{5}}{3}\text{sq.units.}$$