Number Systems

MATHMATICAL REASONING

1. If $2^{x-3}.3^{2x-8} = 36,$ then the value of x is_____.

(a) 2 (b) 5
(c) 3 (d) 1

2. If $N = \dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}},$ then N equals ____.

(a) 1 (b) $2\sqrt{2}-1$
(c) $\dfrac{\sqrt{5}}{2}$ (d) $\dfrac{2}{\sqrt{\sqrt{5}+1}}$

3. Express the mixed recurring decimal 4.32 in the form $\dfrac{p}{q}$.

(a) $\dfrac{389}{90}$ (b) $\dfrac{329}{90}$
(c) $\dfrac{29}{90}$ (d) $\dfrac{233}{990}$

4. If $x = (7+4\sqrt{3})$, then $\left(x+\dfrac{1}{x}\right) =$ _____.

(a) $8\sqrt{3}$ (b) 14
(c) 49 (d) 48

5. $\dfrac{7\sqrt{3}}{(\sqrt{10}+\sqrt{3})} - \dfrac{2\sqrt{5}}{(\sqrt{6}+\sqrt{5})} - \dfrac{3\sqrt{2}}{(\sqrt{15}+3\sqrt{2})} =$ _____.

(a) 1 (b) 2
(c) $\dfrac{1}{2}$ (d) 3

6. The rationalising factor of $\sqrt[5]{a^2b^3c^4}$ is ______.

(a) $\sqrt[5]{a^3b^2c}$ (b) $\sqrt[4]{a^3b^2c}$
(c) $\sqrt[3]{a^3b^2c}$ (d) $\sqrt{a^3b^2c}$

7. Rational number $\dfrac{-18}{5}$ lies between consecutive integers ____.

(a) – 2 and – 3 (b) – 3 and – 4
(c) – 4 and – 5 (d) – 5 and – 6

8. An irrational number between $\dfrac{1}{7}$ and $\dfrac{2}{7}$ is _____.

(a) $\dfrac{1}{2}\left(\dfrac{1}{7}+\dfrac{2}{7}\right)$ (b) $\left(\dfrac{1}{7}\times\dfrac{2}{7}\right)$
(c) $\sqrt{\dfrac{1}{7}\times\dfrac{2}{7}}$ (d) None of these

9. The denominator of $\dfrac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} + \dfrac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}$ is ______.

(a) a^2 (b) b^2
(c) a^2-b^2 (d) $\dfrac{4a^2-2b^2}{b^2}$

10. The ascending order of the surds $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4},$ is ____.

(a) $\sqrt[9]{4}, \sqrt[6]{3}, \sqrt[3]{2}$ (b) $\sqrt[9]{4}, \sqrt[3]{2}, \sqrt[6]{3}$
(c) $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ (d) $\sqrt[6]{3}, \sqrt[9]{4}, \sqrt[3]{2}$

11. The greater number among $\sqrt{17}-\sqrt{12}$ and $\sqrt{11}-\sqrt{6}$ is_____.

(a) $\sqrt{17}-\sqrt{12}$ (b) $\sqrt{11}-\sqrt{6}$
(c) Both are equal (d) Can't be compared

12. The value of x, if $5^{x-3}.3^{2x-8} = 225$ is_____.

(a) 3 (b) 4
(c) 2 (d) 5

13. The value of $\dfrac{1}{3}$ of 15^{27} is ____.

(a) 5^{27} (b) 15^{9}
(c) 5×15^{26} (d) 5×3^{9}

14. If $x = 2-\sqrt{3}$ then the value of $x^2+\dfrac{1}{x^2}$ and $x^2-\dfrac{1}{x^2}$ respectively, are____.

(a) $14, 8\sqrt{3}$ (b) $-14, -8\sqrt{3}$
(c) $14, -8\sqrt{3}$ (d) $-14, 8\sqrt{3}$

15. Which of the following statements is INCORRECT?

(a) Every integer is a rational number.
(b) Every natural number is an integer.
(c)Every natural number is a real number.
(d) Every real number is a rational number.

16. If $\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}=(a-b\sqrt{3})$, find the values of a and b.

(a) $a=1, b=2$ (b) $a=2, b=1$

(c) $a=2, b=3$ (d) $a=3, b=2$

17. Which of the following numbers has the terminating decimal representation?

(a) $\frac{5}{12}$ (b) $\frac{8}{35}$

(c) $\frac{7}{24}$ (d) $\frac{13}{80}$

18. The number $x=1.242424....$ can be expressed in the form $x=\frac{p}{q}$, where p and q q are positive integers having no common factors. Then p + q equals ____.

(a) 72 (b) 74

(c) 41 (d) 53

19. If x and y are positive real numbers, then which of the following is CORRECT?

(a) $x>y \Rightarrow -x>-y$ (b) $x>y \Rightarrow -x<-y$

(c) $x>y \Rightarrow \frac{1}{x}>\frac{1}{y}$ (d) $x>y \Rightarrow \frac{1}{x}<\frac{-1}{y}$

20. If $x=1-\sqrt{2}$, then find the value of $\left(x-\frac{1}{x}\right)^2$.

(a) 2 (b) 3

(c) 4 (d) 5

ACHIEVERS SECTION (HOTS)

21. The value of

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$ is ____.

(a) 0 (b) 1

(c) 2 (d) 4

22. Which of the following statements is INCORRECT?

(a) If 'a' is a rational number and 'b' is irrational, then a + b is irrational.

(b) The product of a non-zero rational number with an irrational number is always irrational.

(c) Addition of any two irrational numbers can be rational.

(d) Division of any two integers is an integer.

23. Fill in the blanks.

(i) The decimal form of an irrational number is neither **P** nor **Q**.

(ii) There are **R** rational numbers between any two consecutive integers.

(iii) Every rational number is **S**.

	P	Q	R	S
A	non-repeating	terminating	zero	real
B	repeating	terminating	infinite	real
C	non-repeating	non-terminating	zero	integer
D	repeating	terminating	infinite	integer

24. Read the statements carefully.

Statement-1: Every point on the number line represent a unique real number.

Statement-2: Irrational numbers cannot be represented on a number line.

Which of the following options hold?

(a) Both Statement-1 and Statement-2 are true.

(b) Statement-1 is true but Statement-2 is false.

(c) Statement-1 is false but Statement-2 is true.

(d) Both Statement-1 and Statement-2 are false.

25. Match the following,

Column – I	Column – II
(a) If $\frac{3}{x+8}=\frac{4}{6-x}$, then x is ______.	(i) 3
(b) if, $\frac{2^{x-1}.4^{2x+1}}{8^{x-1}}=64$,	(ii) -2
(c) if $4^x-4^{x-1}=24$	(iii) -2
(d) $4^{x+1}=256$, then x is	(iv) 1

(a) (a) → (i); (b) → (ii); (c) → (iii); (d) → (iv)

(b) (a) → (iii); (b) → (iv); (c) → (i); (d) → (ii)

(c) (a) → (i); (b) → (iv); (c) → (ii); (d) → (iii)

(d) (a) → (iii); (b) → (iv); (c) → (iii); (d) → (i)

HINTS & EXPLANATIONS

1. (b): We have $2^{x-3}.3^{2x-8}=36$

$\Rightarrow 2^x 2^{-3}.3^{2x}3^{-8}=36$

$\Rightarrow 2^x.\frac{1}{2^3}.3^x.3^x.\frac{1}{3^8}=36$

$\Rightarrow (2\times3\times3)^x=36\times8\times6561$

$\Rightarrow (18)^x = (18)^5$

On comparing. We get $x = 5$

2. (a) :

3. (a) : Let $x = 4.3\overline{2} = 4.3222...$ ...(i)

$\Rightarrow 10x = 43.222...$ ...(ii)

Subtracting (i) form (ii), we get

$9x = 38.9 \Rightarrow x = \frac{389}{90}$

4. (b) : We have, $x = 7 + 4\sqrt{3}$

Now, $\frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{7-4\sqrt{3}}{49-48} = 7 - 4\sqrt{3}$

$\therefore x + \frac{1}{x} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$

5. (a) : $\frac{7\sqrt{3}}{(\sqrt{10}+\sqrt{3})} - \frac{2\sqrt{5}}{(\sqrt{6}+\sqrt{5})} - \frac{3\sqrt{2}}{(\sqrt{15}+3\sqrt{2})}$

$= \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3} - \frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{6-5}$

$- \frac{3\sqrt{2}(\sqrt{5}-3\sqrt{2})}{15-18}$

$= \frac{7\sqrt{30}-21}{7} - \frac{2\sqrt{30}-10}{1} - \frac{3\sqrt{30}-18}{-3}$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6 = 1$

6. (a) : We have $\sqrt[5]{a^2b^3c^4}$

Since, $\sqrt[5]{a^2b^3c^4} \times \sqrt[5]{a^3b^2c} = \sqrt[5]{a^2a^3b^3b^2c^4c}$

$\therefore$ The rationalising factor is $\sqrt[5]{a^3b^2c}$.

7. (b) : We have, $\frac{-18}{5}$

On expressing it in decimal form, we get -3.6

$\therefore$ The number lies between -3 and -4

8. (c) : An irrational number between $\frac{1}{7}$ and $\frac{2}{7}$ is

$\sqrt{\frac{1}{7} \times \frac{2}{7}}$

9. (b) : We have, $\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} + \frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}$

$= \frac{(a+\sqrt{a^2-b^2})^2 + (a-\sqrt{a^2-b^2})^2}{a^2 - (\sqrt{a^2-b^2})^2}$

$a^2 + a^2 - b^2 + 2a\sqrt{a^2-b^2} + a^2 + a^2$

$= \frac{-b^2 - 2a\sqrt{a^2-b^2}}{a^2 - a^2 + b^2}$

$= \frac{4a^2 - 2b^2}{b^2}$

$\therefore$ The denomination of the given expression is b^2

10. (a) : LCM of 3, 6, 9 = 18

$\therefore \sqrt[3]{2} = (2)^{\frac{1}{3}\times\frac{6}{6}} = (2^6)^{\frac{1}{18}} = (64)^{\frac{1}{18}} = \sqrt[18]{64}$

$\sqrt[6]{3} = (3)^{\frac{1}{6}\times\frac{3}{3}} = (3^3)^{\frac{1}{18}} = (27)^{\frac{1}{18}} = \sqrt[18]{27}$

$\sqrt[9]{4} = (4)^{\frac{1}{9}\times\frac{2}{2}} = (4^2)^{\frac{1}{18}} = (16)^{\frac{1}{18}} = \sqrt[18]{16}$

So, the order is, $\sqrt[18]{27} < \sqrt[18]{27} < \sqrt[18]{64}$

$\Rightarrow \sqrt[9]{4} < \sqrt[18]{27} < \sqrt[18]{64}$

11. (b) : $\sqrt{11} - \sqrt{6} = 3.3 - 2.4 = 0.9$

and $\sqrt{17} - \sqrt{12} = 4.1 - 3.4 = 0.7$

So, $\sqrt{11} - \sqrt{16}$ is greater.

12. (d) : We have, $5^{x-3}.3^{2x-8} = 225$

$\Rightarrow \frac{5^x}{5^3}.\frac{3^{2x}}{3^8} = (15)^2$

$\Rightarrow 5^x.3^{2x} = 5^2 \times 3^2 \times 5^3 \times 3^8$

$\Rightarrow 5^x.3^{2x} = 5^5 \times 3^{10}$

On comparing, we get $x = 5$

13. (c) : We have, $\frac{1}{3} \times (15)^{27} = \frac{15 \times (15)^{26}}{3}$

$= 5 \times (15)^{26}$

14. (c) : We have, $x = 2 - \sqrt{3}$

$\therefore \frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$

$\therefore \left(x - \frac{1}{x}\right) = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

Also, $\left(x - \frac{1}{x}\right) = 2 - \sqrt{3} - 2 - \sqrt{3} = -2\sqrt{3}$

Now, $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow (4)^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow x^2 + \frac{1}{x^2} = 14$

and $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 4 \times (-2\sqrt{3}) = -8\sqrt{3}$

15. (d) : Every rational number is a rest number.

16. (b) : we have, $a - b\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{3-1}$

$= \frac{3+1-2\sqrt{3}}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$

On comparing, we get $a = 2, b = 1$

17. (d) : $\frac{13}{80} = 0.1625$

18. (b) : We have,

$x = 1.2424.....$

$\Rightarrow 100x = 124.2424.......$ ...(i)

Subtracting (i) from (ii), we get ...(ii)

$100x - x = 123$

$\Rightarrow 99x = 123 \Rightarrow x = \frac{123}{99} = \frac{41}{33}$

$\therefore p = 41, q = 33$

and $p + q = 41 + 33 = 74$

19. (b) : We have, x and y are positive real numbers.

$\therefore x > y \Rightarrow -x < -y$ and $x > y \Rightarrow \frac{1}{x} < \frac{1}{y}$

20. (c) : We have, $x = 1 - \sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2} = -(1+\sqrt{2})$

Now, $\left(x - \frac{1}{x}\right)^2 = (1-\sqrt{2}+1+\sqrt{2})^2 = (2)^3 = 4$

21. (c) : We have,

$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}}$

$+ \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}}$

On rationalizing each of the above number separately, we get

$\frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4} + \frac{\sqrt{4}-\sqrt{5}}{4-5}$

$+ \frac{\sqrt{5}-\sqrt{6}}{5-6} + \frac{\sqrt{6}-\sqrt{7}}{6-7} + \frac{\sqrt{7}-\sqrt{8}}{7-8} + \frac{\sqrt{8}-\sqrt{9}}{8-9}$

$= -(1-\sqrt{2}) - (\sqrt{2}-\sqrt{3}) - (\sqrt{3}-\sqrt{4})$

$-(\sqrt{4}-\sqrt{5}) - (\sqrt{5}-\sqrt{6}) - (\sqrt{6}-\sqrt{7}) - (\sqrt{7}-\sqrt{8})$

$-(\sqrt{8}-\sqrt{9})$

$= -1 + \sqrt{9} = -1 + 3 = 2$

22. (d) : Division of any two integers is always not an integer.

23. (b) :

24. (b) : Statement -1 is true and statement -2 is false as irrational numbers can be represented on number line.

25. (d) : we have. $\frac{3}{x+8} = \frac{4}{6-x}$

$\Rightarrow 18 - 3x = 4x + 32 \Rightarrow 7x = -14 \Rightarrow x = -2$

(b) We have, $\frac{2^{x-1}.4^{2x+1}}{8^{x-1}} = 64$

$\Rightarrow \frac{2^x 2^{-1}.4^{2x}.4}{8^x 8^{-1}} = 64$

$\Rightarrow \frac{1}{4^x}.4^{2x} = 64 \Rightarrow 4^x \Rightarrow \frac{64}{16} = 4$

On comparing, we get $x = 1$

(c) we have, $4^{x-1} = 24$

$\Rightarrow 4^x\left(1 - \frac{1}{4}\right) = 24 \Rightarrow (2^2)^x = \frac{24 \times 4}{3} = 32$

$\Rightarrow 2^{2x} = 2^5$

On comparing. We get $2x = 5 \Rightarrow x = \frac{5}{2}$

Now, $(2x)^x = (5)^{5/2}$

(d) We have, $4^{x+1} = 256$

$\Rightarrow \quad 4^{x+1} = (4)^4$

On comparing. We get $x + 1 = 4 \Rightarrow x = 3$