12

Introduction to Three Dimensional Geometry

Short Answer Type Questions

Q. 1 Locate the following points

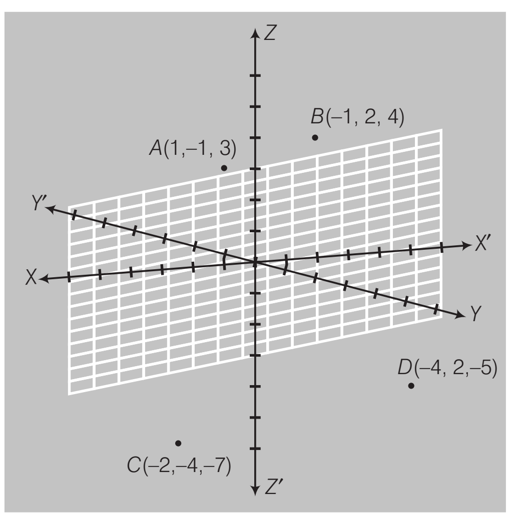
(i) (1, – 1,3) (ii) (– 1, 2, 4)

(iii) (– 2,– 4, – 7) (iv) (– 4, 2,– 5)

Sol. Given, coordinates are

(i) (1,–1,3) **(ii)** *B* (–1, 2, 4)

(iii) *C* (– 2, – 4,– 7) **(iv)** *D* (– 4, 2,– 5)



X-increment = *Y*-increment = *Z*-increment = 1

Q. 2 Name the octant in which each of the following points lies.

(i) (1, 2, 3,) (ii) (4,– 2, 3)
(iii) (4,– 2, – 5) (iv) (4, 2,– 5)
(v) (– 4, 2, 5) (iv) (– 3, – 1, 6)
(vii) (2,– 4, – 7) (viii) (– 4, 2, – 5).

Sol. **(i)** Point (1, 2 ,3) lies in first quadrant. **(ii)** (4, – 2, 3) in fourth octant.
(iii) (4, – 2, – 5) in eight octant. **(iv)** (4, 2, – 5) in fifth octant.
(v) (– 4, 2, 5) in second octant. **(vi)** (– 3, – 1, 6) in third octant.
(vii) (2,– 4, – 7) in eight octant. **(viii)** (– 4, 2, – 5) in sixth octant.

Q. 3 If *A*, *B*, *C* be the feet of perpendiculars from a point *P* on the *X*, *Y* and *Z*-axes respectively, then find the coordinates of *A*, *B* and *C* in each of the following where the point *P* is

(i) A (3, 4, 2) (ii) *B* (– 5, 3, 7)
(iii) *C* (4, – 3, – 5)

Sol. The coordinates of *A*, *B* and *C* are the following

(i) *A* (3, 0, 0), *B* (0,4, 0), *C* (0,0, 2)
(ii) *A* (– 5, 0, 0), *B* (0, 3, 0), *C* (0,0, 7)
(iii) *A* (4, 0, 0), *B* (0, – 3, 0), *C* (0,0,– 5)

Q. 4 If *A*, *B*, and *C* be the feet of perpendiculars from a point *P* on the *XY*, *YZ* and *ZX*-planes respectively, then find the coordinates of *A*, *B* and *C* in each of the following where the point *P* is

(i) (3, 4, 5) (ii) (– 5, 3, 7)
(iii) (4, – 3, – 5)

Sol. We know that, on *XY*-plane $z = 0$, on *YZ*-plane, $x = 0$ and on *ZX*-plane, $y = 0$. Thus, the coordinate of *A*, *B* and *C* are following

(i) *A* (3, 4, 0), *B* (0, 4, 5), *C* (3, 0, 5)
(ii) *A* (– 5, 3, 0), *B* (0, 3, 7), *C* (– 5, 0, 7)
(iii) *A* (4,– 3, 0), *B* (0, – 3, – 5), *C* (4, 0,– 5)

Q. 5 How far apart are the points (2, 0, 0) and (–3, 0, 0)?

Thinking Process

Distance between two points (x_1, y_1, z_1) *and* (x_2, y_2, z_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

Sol. Given points, *A* (2, 0, 0) and *B* (– 3, 0, 0)

$$AB = \sqrt{(2 + 3)^2 + 0^2 + 0^2} = 5$$

Q. 6 Find the distance from the origin to(6, 6, 7).

Sol. Distance from origin to the point (6, 6,7)

$$= \sqrt{(0-6)^2 + (0-6)^2 + (0-7)^2} \qquad [\because d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}]$$

$$= \sqrt{36 + 36 + 49}$$

$$= \sqrt{121} = 11$$

Q. 7 Show that, if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1 - x^2 - y^2})$ is at a distance 1 unit form the origin.

Sol. Given that, $x^2 + y^2 = 1$

$\therefore$ Distance of the point $(x, y, \sqrt{1 - x^2 - y^2})$ from origin is given as

$$d = \left| \sqrt{x^2 + y^2 + (\sqrt{1 - x^2 - y^2})^2} \right|$$

$$= \left| \sqrt{x^2 + y^2 + 1 - x^2 - y^2} \right| = 1$$

Hence proved.

Q. 8 Show that the point A (1, – 1, 3), B (2, – 4, 5) and C (5, – 13, 11) are collinear.

Thinking Process

If the three points A, B, and C are collinear, then AB + BC = AC.

Sol. Given points, A (1, – 1, 3), B (2, – 4, 5) and C (5, – 13, 11).

$$AB = \sqrt{(1-2)^2 + (-1+4)^2 + (3-5)^2}$$

$$= \sqrt{1 + 9 + 4} = \sqrt{14}$$

$$BC = \sqrt{(2-5)^2 + (-4+13)^2 + (5-11)^2}$$

$$= \sqrt{9 + 81 + 36} = \sqrt{126}$$

$$AC = \sqrt{(1-5)^2 + (-1+13)^2 + (3-11)^2}$$

$$= \sqrt{16 + 144 + 64} = \sqrt{224}$$

$\because \quad AB + BC = AC$

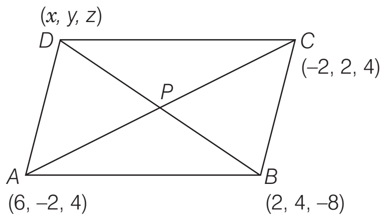
$\Rightarrow \quad \sqrt{14} + \sqrt{126} = \sqrt{224}$

$\Rightarrow \quad \sqrt{14} + 3\sqrt{14} = 4\sqrt{14}$

So, the points A, B and C are collinear.

Q. 9 **Three consecutive vertices of a parallelogram *ABCD* are *A* (6, – 2, 4), *B* (2, 4, –8) and *C* (– 2, 2, 4). Find the coordinates of the fourth vertex.**

Sol. Let the coordinates of the fourth vertices $D(x, y, z)$.



Mid-points of diagonal *AC*,

$$x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2},\ z = \frac{z_1 + z_2}{2}$$

and $$x = \frac{6 - 2}{2} = 2,\ y = \frac{-2 + 2}{2} = 0,\ z = \frac{4 + 4}{2} = 4$$

Since, the mid-point of *AC* are (2, 0, 4).

Now, mid-point of *BD*, $2 = \frac{x + 2}{2} \Rightarrow x = 2$

$\Rightarrow$ $$0 = \frac{y + 4}{2} \Rightarrow y = -4$$

$\Rightarrow$ $$4 = \frac{z - 8}{2} \Rightarrow z = 16$$

So, the coordinates of fourth vertex *D* is (2, – 4, 16).

Q. 10 **Show that the ΔABC with vertices *A* (0, 4, 1), *B* (2, 3, – 1) and *C* (4, 5, 0) is right angled.**

Thinking Process

In a right angled triangle sum of the square of two sides is equal to square of third side.

Sol. Given that, the vertices of the ΔABC are *A* (0, 4, 1), *B* (2, 3, – 1) and *C* (4, 5, 0).

Now, $$AB = \sqrt{(0 - 2)^2 + (4 - 3)^3 + (1 + 1)^2}$$

$$= \sqrt{4 + 1 + 4} = 3$$

$$BC = \sqrt{(2 - 4)^2 + (3 - 5)^2 + (-1 - 0)^2}$$

$$= \sqrt{4 + 4 + 1} = 3$$

$$AC = \sqrt{(0 - 4)^2 + (4 - 5)^2 + (1 - 0)^2}$$

$$= \sqrt{16 + 1 + 1} = \sqrt{18}$$

$\because$ $$AC^2 = AB^2 + BC^2$$

$\Rightarrow$ $$18 = 9 + 9$$

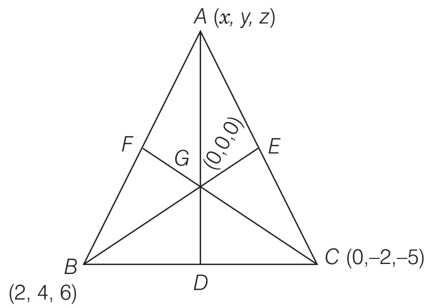
Hence, vertices ΔABC is a right angled triangle.

Q. 11 **Find the third vertex of triangle whose centroid is origin and two vertices are (2, 4, 6) and (0, – 2, 5).**

Thinking Process

The vertices of the ΔABC are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, then the coordinates of the centroid G are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Sol. Let third vertex of ΔABC *i.e.,* is $A(x, y, z)$.



Given that, the coordinate of centroid G are (0, 0, 0).

$$\because \quad 0 = \frac{x+2+0}{3} \Rightarrow x = -2$$

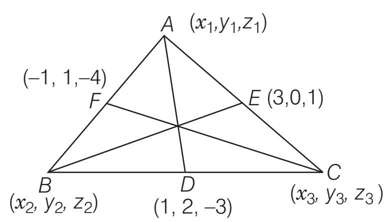
$$0 = \frac{y+4-2}{3} \Rightarrow y = -2$$

$$0 = \frac{z+6-5}{2} \Rightarrow z = -1$$

Hence, the third vertex of triangle is (– 2, – 2, –1).

Q. 12 **Find the centroid of a triangle, the mid-point of whose sides are *D* (1, 2, – 3), *E* (3, 0, 1) and *F* (– 1, 1, – 4).**

Sol. Given that, mid-points of sides are *D* (1, 2, – 3), *E* (3, 0, 1) and *F* (– 1, 1, – 4).



Let the vertices of the ΔABC are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_3)$ and $C(x_3, y_3, z_3)$.

Then, mid-point of *BC* are (1, 2, – 3).

$$\therefore \quad 1 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 2 \quad \text{...(i)}$$

$$2 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 4 \quad \text{...(ii)}$$

$$-3 = \frac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = -6 \quad \text{...(iii)}$$

Similarly for the sides AB and AC,

$$\Rightarrow \qquad -1 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = -2 \qquad \text{...(iv)}$$

$$\Rightarrow \qquad 1 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 2 \qquad \text{...(v)}$$

$$\Rightarrow \qquad -4 = \frac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = -8 \qquad \text{...(vi)}$$

$$\Rightarrow \qquad 3 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 6 \qquad \text{...(vii)}$$

$$\Rightarrow \qquad 0 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 0 \qquad \text{...(viii)}$$

$$\Rightarrow \qquad 1 = \frac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = 2 . \qquad \text{...(ix)}$$

On adding Eqs. (i) and (iv), we get

$$x_1 + 2x_2 + x_3 = 0 \qquad \text{...(x)}$$

On adding Eqs. (ii) and (v), we get

$$y_1 + 2y_2 + y_3 = 6 \qquad \text{...(xi)}$$

On adding Eqs. (iii) and (vi), we get

$$z_1 + 2z_2 + z_3 = -14 \qquad \text{...(xii)}$$

From Eqs. (vii) and (x),

$$2x_2 = -6 \Rightarrow x_2 = -3$$

If $x_2 = -3$, then $x_3 = 5$

If $x_3 = 5$, then $x_1 = 1, \quad x_2 = -3, x_3 = 5$

From Eqs. (xi) and (viii),

$$2y_2 = 6 \Rightarrow y_2 = 3$$

If $y_2 = 3$, then $y_1 = -1$ If $y_1 = -1$, then $y_3 = 1, \quad y_2 = 3, y_3 = 1$

From Eqs. (xii) and (ix),

$$2z_2 = -16 \Rightarrow z_2 = -8$$

$$z_2 = -8, \text{ then } z_1 = 0$$

$$z_1 = 0, \text{ then } z_3 = 2$$

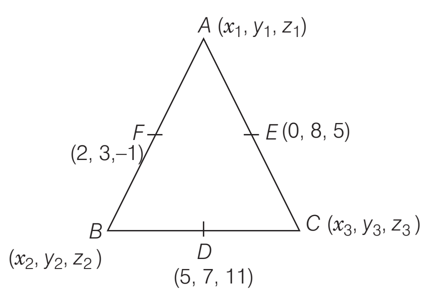
$$z_1 = 0, z_2 = -8, z_3 = 2$$

So, the points are A (1 – 1,0), B (– 3, 3,– 8) and C (5, 1, 2).

$\therefore$ Centroid of the triangle $= G\left(\frac{1-3+5}{3}, \frac{-1+3+1}{3}, \frac{0-8+2}{3}\right)$ *i.e.*, G (1, 1,– 2)

Q. 13 The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1). Find its vertices.

Sol. Let vertices of the ΔABC are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$, then the mid-point of BC (5, 7, 11).



$$5 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 10 \quad \text{...(i)}$$

$$7 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 14 \quad \text{...(ii)}$$

$$11 = \frac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = 22 \quad \text{...(iii)}$$

Similarly for the sides *AB* and *AC*,

$$2 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 4 \quad \text{...(iv)}$$

$$3 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 6 \quad \text{...(v)}$$

$$-1 = \frac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = -2 \quad \text{...(vi)}$$

$$0 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 0 \quad \text{...(vii)}$$

$$8 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 16 \quad \text{...(viii)}$$

$$5 = \frac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = 10 \quad \text{...(ix)}$$

From Eqs. (i) and (iv),

$$x_1 + 2x_2 + x_3 = 14 \quad \text{...(x)}$$

From Eqs. (ii) and (v),

$$y_1 + 2y_2 + y_3 = 20 \quad \text{...(xi)}$$

From Eqs. (iii) and (vi),

$$z_1 + 2z_2 + z_3 = 20 \quad \text{...(xii)}$$

From Eqs. (vii) and (x),

$$2x_2 = 14 \Rightarrow x_2 = 7$$
$$x_2 = 7, \text{ then } x_3 = 10 - 7 = 3$$
$$x_3 = 3, \text{ then } x_1 = -3$$
$$x_1 = -3, x_2 = 7, x_3 = 3$$

From Eqs. (viii) and (xi),

$$2y_2 = 4 \Rightarrow y_2 = 2$$
$$y_2 = 2, \text{ then } y_1 = 4$$
$$y_1 = 4, \text{ then } y_3 = 12$$
$$y_1 = 4, y_2 = 2, y_3 = 12$$

From Eqs. (ix) and (xii),

$$2z_2 = 10 \Rightarrow z_2 = 5$$
$$z_2 = 5, \text{ then } z_1 = -7$$
$$z_1 = -7, \text{ then } z_3 = 17$$
$$z_1 = -7, z_2 = 5, z_3 = 17$$

So, the vertices are *A* (– 3, 4, – 7), *B* (7, 2, 5) and *C* (3, 12, 17).

Q. 14 **If the vertices of a parallelogram *ABCD* are *A* (1, 2, 3), *B* (– 1, – 2, – 1) and *C* (2, 3, 2), then find the fourth vertex *D*.**

Thinking Process

The diagonal of a parallelogram have the same vertices. Use this property to solve the problem.

Sol. Let the fourth vertex of the parallelogram *ABCD* is *D* (*x*, *y*, *z*). Then, the mid-point of *AC* are

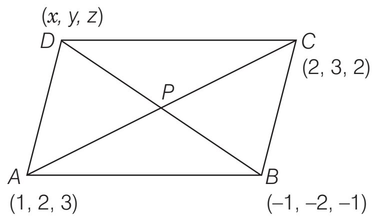
$$P\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) \text{ i.e., } P\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right).$$

Now, mid-point of *BD*,

$$\frac{3}{2} = \frac{-1 + x}{2} \Rightarrow x = 4$$

$$\frac{5}{2} = \frac{-2 + y}{2} \Rightarrow y = 7$$

$$\frac{5}{2} = \frac{-1 + z}{2} \Rightarrow z = 6$$



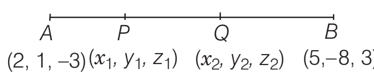
So, the coordinates of fourth vertex is (4, 7, 6).

Q. 15 Find the coordinate of the points which trisect the line segment joining the points *A* (2, 1, –3) and *B* (5, – 8, 3).

Thinking Process

If point P divided line segment joint the point $A(x_1, y_1, z_1)$ *and* $B(x_2, y_2, z_2)$ *in* $m_1 : m_2$ *internally then the coordinate of P are* $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$

Sol. Let the $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ trisect line segment *AB*.



Since, the point *P* divided line *AB* in 1 : 2 internally, then

$$x_1 = \frac{2 \times 2 + 1 \times 5}{1 + 2} = \frac{9}{3} = 3$$

$$y_1 = \frac{2 \times 1 + 1 \times (-8)}{3} = \frac{-6}{3} = -2$$

$$z_1 = \frac{2 \times (-3) + 1 \times 3}{3} = \frac{-6 + 3}{3} = \frac{-3}{3} = -1$$

Since, the point *Q* divide the line segment *AB* in 2 : 1, then

$$x_2 = \frac{1 \times 2 + 2 \times 5}{3} = 4,$$

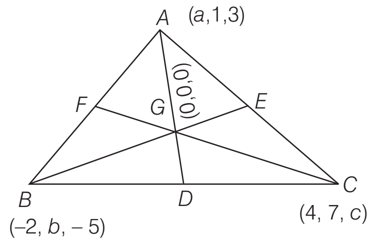
$$y_2 = \frac{1 \times 1 + (-8 \times 2)}{3} = -5$$

$$z_2 = \frac{1 \times (-3) + 2 \times 3}{3} = -1$$

So, the coordinates of *P* are (3, – 2, – 1) and the coordinates of *Q* are (4, – 5, 1).

Q. 16 **If the origin is the centroid of a ΔABC having vertices $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, then find the values of a, b, c.**

Sol. Given that origin is the centroid of the ΔABC *i.e.*, $G(0, 0, 0)$.



$$\because \quad 0 = \frac{a - 2 + 4}{3} \Rightarrow a = -2$$

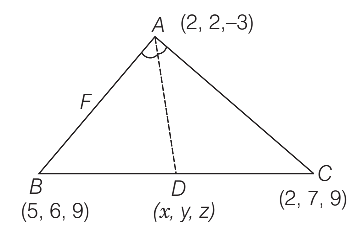
$$0 = \frac{1 + b + 7}{3} \Rightarrow b = -8$$

$$0 = \frac{3 - 5 + c}{3} \Rightarrow c = +2$$

$$\therefore \quad a = -2,\ b = -8 \text{ and } c = 2$$

Q. 17 **If $A(2, 2, -3)$, $B(5, 6, 9)$, $C(2, 7, 9)$ be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D, then find the coordinates of D.**

Sol. Let the coordinates of D are (x, y, z).



$$AB = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

$$AC = \sqrt{0 + 25 + 144} = \sqrt{169} = \sqrt{13}$$

$$\Rightarrow \quad \frac{AB}{AC} = \frac{13}{13} \Rightarrow AB = AC$$

$$\frac{BD}{DC} = \frac{1}{1} \Rightarrow BD = DC$$

Since, D is divide the line BC in two equal parts. So, D is the mid-point of BC.

$$\therefore \quad x = \frac{5 + 2}{2} = 7/2$$

$$\Rightarrow \quad y = \frac{6 + 7}{2} = 13/2$$

$$\Rightarrow \quad z = \frac{9 + 9}{2} = 9$$

So, the coordinates of D are $\left(\frac{7}{2}, \frac{13}{2}, 9\right)$.

Long Answer Type Questions

Q. 18 Show that the three points A (2, 3, 4), B (1, 2, – 3) and C (– 4, 1,– 10) are collinear and find the ratio in which C divides AB.

Sol. Given points are A (2, 3, 4), B (– 1, 2, – 3) and C (– 4, 1,– 10).

$$\therefore \quad AB = \sqrt{(2+1)^2 + (3-2)^2 + (4+3)^2}$$
$$= \sqrt{9+1+49} = \sqrt{59}$$
$$BC = \sqrt{(-1+4)^2 + (2-1)^2 + (-3+10)^2}$$
$$= \sqrt{9+1+49} = \sqrt{59}$$
$$AC = \sqrt{(2+4)^2 + (3-1)^2 + (4+10)^2}$$
$$= \sqrt{36+4+196}$$
$$= \sqrt{236} = 2\sqrt{59}$$

Now, $\quad AB + BC = \sqrt{59} + \sqrt{59} = 2\sqrt{59}$

$\because \quad AB + BC = AC$

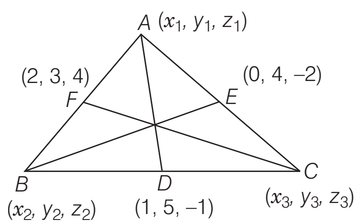
Hence, the points A, B and C are collinear.

Now, $\quad AC : BC = 2\sqrt{59} : \sqrt{59} = 2 : 1$

So, C divide AB in 2 :1 externally.

Q. 19 The mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices and also find the centroid of the triangle.

Sol. Let the vertices of ΔABC are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$.



Since, the mid-point of side BC is D (1, 5, –1).

Then, $\quad \dfrac{x_2 + x_3}{2} = 1 \Rightarrow x_2 + x_3 = 2 \quad$ ...(i)

$\dfrac{y_2 + y_3}{2} = 5 \Rightarrow y_2 + y_3 = 10 \quad$ ...(ii)

$\dfrac{z_2 + z_3}{2} = -1 \Rightarrow z_2 + z_3 = -2 \quad$ ...(iii)

Similarly, the mid-points of AB and AC are F (2, 3, 4) and E (0, 4, – 2),

$\dfrac{x_1 + x_2}{2} = 2 \Rightarrow x_1 + x_2 = 4 \quad$ ...(iv)

$\dfrac{y_1 + y_2}{2} = 3 \Rightarrow y_1 + y_2 = 6 \quad$ ...(v)

and $\quad \dfrac{z_1 + z_2}{2} = 4 \Rightarrow z_1 + z_2 = 8 \quad$ ...(vi)

Now, $\frac{x_1 + x_3}{2} = 0 \Rightarrow x_1 + x_3 = 0$ ...(vii)

$\frac{y_1 + y_3}{2} = 4 \Rightarrow y_1 + y_3 = 8$ ...(viii)

$\frac{z_1 + z_3}{2} = -2 \Rightarrow z_1 + z_3 = -4$ ...(ix)

From Eqs. (i) and (iv),

$x_1 + 2x_2 + x_3 = 6$ ...(x)

From Eqs. (ii) and (v),

$y_1 + 2y_2 + y_3 = 16$ ...(xi)

From Eqs. (iii) and (vi),

$z_1 + 2z_2 + z_3 = 6$ ...(xii)

From Eqs. (vii) and (x),

$2x_2 = 6 \Rightarrow x_2 = 3$

$x_2 = 3$, then $x_3 = -1$

$x_3 = -1$,

Then, $x_1 = 1 \Rightarrow x_1 = 1, x_2 = 3, x_2 = -1$

From Eqs. (viii) and (xi),

$2y_2 = 8 \Rightarrow y_2 = 4$

$y_2 = 4$,

Then, $y_1 = 2$

$y_1 = 2$,

Then, $y_3 = 6$,

$\Rightarrow$ $y_1 = 2, y_2 = 4, y_3 = 6$

From Eqs. (ix) and (xii),

$2z_2 = 10 \Rightarrow z_2 = 5$

$z_2 = 5$,

Then, $z_1 = 3$

$z_1 = 3$,

Then, $z_3 = -7$

$\Rightarrow$ $z_1 = 3, z_2 = 5, z_3 = -7$

So, the vertices of the triangle A (1, 2, 3), B (3, 4, 5) and C (– 1, 6, –7).

Hence, centroid of the triangle $G\left(\frac{1+3-1}{3}, \frac{2+4+6}{3}, \frac{3+5-7}{3}\right)$ *i.e.*, G (1, 4, 1/3).

Q. 20 Prove that the points (0, –1, – 7),(2, 1, – 9) and (6, 5, – 13) are collinear. Find the ratio in which the first point divides the join of the other two.

Thinking Process

First of all find the value of AB, AC and BC using distance formula i.e., $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2) + (z_1 - z_2)^2}$ *, then show that AB + BC = AC for collinearity of the points A, B and C.*

Sol. Given points are $A(0, -1, -7)$, B (2, 1, – 9) and C (6, 5, – 13)

$$AB = \sqrt{(0-2)^2 + (-1-1)^2 + (-7+9)^2} = \sqrt{4+4+4} = 2\sqrt{3}$$

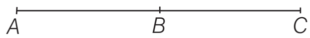
$$BC = \sqrt{(2-6)^2 + (1-5)^2 + (-9+13)^2} = \sqrt{16+16+16} = 4\sqrt{3}$$

$$AC = \sqrt{(0-6)^2 + (-1-5)^2 + (-7+13)^2} = \sqrt{36+36+36} = 6\sqrt{3}$$

$\because$ $$AB + BC = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$$

So, $$AB + BC = AC$$

Hence, the points A, B and C are collinear.



$$AB : AC = 2\sqrt{3} : 6\sqrt{3} = 1 : 3$$

So, point A divide B and C in $1 : 3$ externally.

Q. 21 What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

Sol. The coordinates of the cube which edge is 2 units, are (2, 0, 0), (2, 2, 0), (0, 2, 0), (0, 2, 2),(0, 0, 2), (2, 0, 2), (0, 0, 0) and (2, 2, 2).

Objective Type Questions

Q. 22 The distance of point $P(3, 4, 5)$ from the YZ-plane is

(a) 3 units (b) 4 units

(c) 5 units (d) 550

Sol. *(a)* Given, point is P (3, 4, 5).

Distance of P from YZ-plane, [$\because$ YZ-plane, $x = 0$]

$$d = \sqrt{(0-3)^2 + (4-4)^2 + (5-5)^2} = 3$$

Q. 23 What is the length of foot of perpendicular drawn from the point P (3, 4, 5) on Y-axis?

(a) $\sqrt{41}$ (b) $\sqrt{34}$

(c) 5 (d) None of these

Sol. *(b)* We know that, on the Y-axis, $x = 0$ and $z = 0$.

$\therefore$ Point A (0, 4, 0),

$$PA = \sqrt{(0-3)^2 + (4-4)^2 + (0-5)^2}$$

$$= \sqrt{9 + 0 + 25} = \sqrt{34}$$

Q. 24 Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(a) $\sqrt{50}$ (b) 3

(c) 4 (d) 5

Sol. *(a)* Given, points P (3, 4, 5) and O (0, 0, 0),

$$PO = \sqrt{(0-3)^2 + (0-4)^2 + (0-5)^2}$$

$$= \sqrt{9 + 16 + 25} = \sqrt{50}$$

Q. 25 If the distance between the points $(a, 0, 1)$ and $(0, 1, 2)$ is $\sqrt{27}$, then the value of a is

(a) 5 (b) ± 5
(c) -5 (d) None of these

Sol. *(b)* Given, the points are $A(a, 0, 1)$ and $B(0, 1, 2)$.

$\therefore \quad AB = \sqrt{(a-0)^2 + (0-1)^2 + (1-2)^2}$

$\Rightarrow \quad \sqrt{27} = \sqrt{a^2 + 1 + 1}$

$\Rightarrow \quad 27 = a^2 + 2$

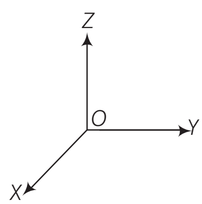
$\Rightarrow \quad a^2 = 25$

$\Rightarrow \quad a = \pm 5$

Q. 26 X-axis is the intersection of two planes

(a) XY and XZ (b) YZ and ZX
(c) XY and YZ (d) None of these

Sol. *(a)* We know that, on the XY and XZ-planes, the line of intersection is X-axis.



Q. 27 Equation of Y-axis is considered as

(a) $x = 0, y = 0$ (b) $y = 0, z = 0$
(c) $z = 0, x = 0$ (d) None of these

Sol. *(c)* On the Y-axis, $x = 0$ and $z = 0$.

Q. 28 The point $(-2, -3, -4)$ lies in the

(a) first octant (b) seventh octant
(c) second octant (d) eight octant

Sol. *(b)* The point $(-2, -3, -4)$ lies in seventh octant.

Q. 29 A plane is parallel to YZ-plane, so it is perpendicular to

(a) X-axis (b) Y-axis
(c) Z-axis (d) None of these

Sol. *(a)* A plane is parallel to YZ-plane, so it is perpendicular to X-axis.

Q. 30 The locus of a point for which $y = 0$ and $z = 0$, is

(a) equation of X-axis
(b) equation of Y-axis
(c) equation at Z-axis
(d) None of these

Sol. *(a)* We know that, equation on the X-axis, $y = 0$, $z = 0$.
So, the locus of the point is equation of X-axis.

Q. 31 The locus of a point for which $x = 0$ is

(a) XY-plane
(b) YZ-plane
(c) ZX-plane
(d) None of these

Sol. *(b)* On the YZ-plane, $x = 0$, hence the locus of the point is YZ-plane.

Q. 32 If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is

(a) $2\sqrt{3}$
(b) $3\sqrt{2}$
(c) $\sqrt{2}$
(d) $\sqrt{3}$

Sol. *(a)* Given points of the parallelopiped are A (5, 8, 10) and B (3, 6, 8).

$$\therefore \quad AB = \sqrt{(5-3)^2 + (6-8)^2 + (10-8)^2}$$
$$= \sqrt{4+4+4} = 2\sqrt{3}$$

Q. 33 L is the foot of the perpendicular drawn from a point P (3, 4, 5) on the XY-plane. The coordinates of point L are

(a) (3, 0, 0)
(b) (0, 4, 5)
(c) (3, 0, 5)
(d) None of these

Sol. *(d)* We know that, on the XY-plane $z = 0$.
Hence, the coordinates of the points L are (3, 4, 0).

Q. 34 L is the foot of the perpendicular drawn from a point (3, 4, 5) on X-axis. The coordinates of L are

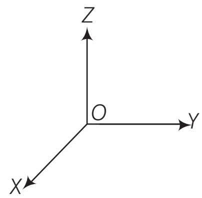
(a) (3, 0, 0)
(b) (0, 4, 0)
(c) (0, 0, 5)
(d) None of these

Sol. *(a)* On the X-axis, $y = 0$ and $z = 0$
Hence, the required coordinates are (3, 0, 0).

Fillers

Q. 35 The three axes *OX*, *OY* and *OZ* determine ....

Sol. The three axes *OX*, *OY* and *OZ* determine three coordinates planes.



Q. 36 The three planes determine a rectangular parallelopiped which has .... of rectangular faces.

Sol. Three points

Q. 37 The coordinates of a point are the perpendicular distance from the .... on the respectives axes.

Sol. Given points

Q. 38 The three coordinate planes divide the space into .... parts.

Sol. Eight parts

Q. 39 If a point *P* lies in *YZ*-plane, then the coordinates of a point on *YZ*-plane is of the form ....

Sol. We know that, on *YZ*-plane, $x = 0$. So, the coordinates of the required point is $(0, y, z)$.

Q. 40 The equation of *YZ*-plane is ....

Sol. The equation of *YZ*-plane is $x = 0$.

Q. 41 If the point *P* lies on *Z*-axis, then coordinates of *P* are of the form ....

Sol. On the *Z*-axis, $x = 0$ and $y = 0$. So, the required coordinates are $(0, 0, z)$.

Q. 42 The equation of *Z*-axis, are ....

Sol. The equation of *Z*-axis, $x = 0$ and $y = 0$.

Q. 43 A line is parallel to *XY*-plane if all the points on the line have equal ....

Sol. *z*-coordinates.

Q. 44 A line is parallel to X-axis, if all the points on the line have equal ....

Sol. y and z-coordinates.

Q. 45 $x = a$ represent a plane parallel to ....

Sol. $x = a$ represent a plane parallel to YZ-plane.

Q. 46 The plane parallel to YZ-plane is perpendicular to ....

Sol. The plane parallel to YZ-plane is perpendicular to X-axis.

Q. 47 The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 10, 13 and 8 units are ....

Sol. Given dimensions are $a = 10$, $b = 13$ and $c = 8$.

$$\therefore \quad \text{Required length} = \sqrt{a^2 + b^2 + c^2}$$
$$= \sqrt{100 + 169 + 64} = \sqrt{333}$$

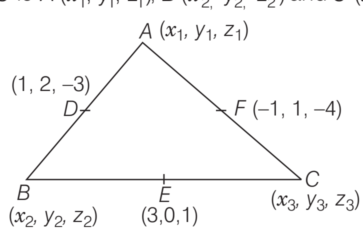
Q. 48 If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then a ....

Sol. Given points are $(a, 2, 1)$ and $(1, -1, 1)$.

$$\therefore \quad \sqrt{(a-1)^2 + (2+1)^2 + (1-1)^2} = 5$$
$$\Rightarrow \quad (a-1)^2 + 9 + 0 = 25$$
$$\Rightarrow \quad a^2 - 2a + 1 + 9 = 25$$
$$\Rightarrow \quad a^2 - 2a - 15 = 0$$
$$\Rightarrow \quad a^2 - 5a + 3a - 15 = 0$$
$$\Rightarrow \quad a(a-5) + 3(a-5) = 0$$
$$\Rightarrow \quad (a-5)(a+3) = 0$$
$$\Rightarrow \quad a - 5 = 0 \text{ or } a + 3 = 0$$
$$\therefore \quad a = +5 \text{ or } -3$$

Q. 49 If the mid-points of the sides of a triangle AB, BC and CA are $D\ (1, 2, -3)$, $E\ (3, 0, 1)$ and $F\ (-1, 1, -4)$, then the centroid of the ΔABC is ....

Sol. Let the vertices of ΔABC is $A\ (x_1, y_1, z_1)$, $B\ (x_2, y_2, z_2)$ and $C\ (x_3, y_3, z_3)$.



Since, D is the mid-point of AB, then

$$\frac{x_1 + x_2}{2} = 1 \Rightarrow x_1 + x_2 = 2 \quad \text{...(i)}$$

$$\frac{y_1 + y_2}{2} = 2 \Rightarrow y_1 + y_2 = 4 \quad \text{...(ii)}$$

$$\frac{z_1 + z_2}{2} = -3 \Rightarrow z_1 + z_2 = -6 \quad \text{...(iii)}$$

Similarly, E and F are the mid-points of sides BC and AC, respectively.

$$\frac{x_2 + x_3}{2} = 3 \Rightarrow x_2 + x_3 = 6 \quad \text{...(iv)}$$

$$\frac{y_2 + y_3}{2} = 0 \Rightarrow y_2 + y_3 = 0 \quad \text{...(v)}$$

$$\frac{z_2 + z_3}{2} = 1 \Rightarrow z_2 + z_3 = 2 \quad \text{...(vi)}$$

$$\frac{x_1 + x_3}{2} = -1 \Rightarrow x_1 + x_3 = -2 \quad \text{...(vii)}$$

$$\frac{y_1 + y_3}{2} = 1 \Rightarrow y_1 + y_3 = 2 \quad \text{...(viii)}$$

$$\frac{z_1 + z_3}{2} = -4 \Rightarrow z_1 + z_3 = -8 \quad \text{...(ix)}$$

From Eqs. (i) and (iv),

$$x_1 + 2x_2 + x_3 = 8 \quad \text{...(x)}$$

From Eqs. (ii) and (v),

$$y_1 + 2y_2 + y_3 = 4 \quad \text{...(xi)}$$

From Eqs. (iii) and (vi),

$$z_1 + 2z_2 + z_3 = -4 \quad \text{...(xii)}$$

From Eqs. (vii) and (x),

$$2x_2 = 10 \Rightarrow x_2 = 5$$

$\Rightarrow$ $\quad x_2 = 5$, then $x_3 = 1$

If $x_3 = 1$, then $x_1 = -3$

$\because$ $\quad x_1 = -3,\ x_2 = 5,\ x_3 = 1$

From Eqs. (viii) and (xi),

$$2y_2 = 2 \Rightarrow y_2 = 1$$

If $\quad y_2 = 1$, then $y_3 = -1$

If $\quad y_3 = -1$, then $y_1 = 3$

$\because$ $\quad y_1 = 3,\ y_2 = 1,\ y_3 = -1$

From Eqs. (ix) and (xii),

$$2z_2 = 4 \Rightarrow z_2 = 2$$

If $\quad z_2 = 2$, then $z_3 = 0$

If $\quad z_3 = 0$, then $z_1 = -8$

$\because$ $\quad z_1 = -8, z_2 = 2,\ z_3 = 0$

So, the vertices of ΔABC are $A\ (-3, 3, -8)$, $B\ (5, 1, 2)$ and $C\ (1, -1, 0)$.

Hence, coordinates of centroid of ΔABC, $G\left(\frac{-3+5+1}{3}, \frac{3+1-1}{3}, \frac{-8+2+0}{3}\right)$

i.e., $\quad G\ (1, 1, -2)$.

Q. 50 Match each item given under the Column I to its correct answer given under Column II.

	Column I		Column II
(i)	In -XY-plane	(a)	Ist octant
(ii)	Point (2, 3, 4) lies in the	(b)	YZ-plane
(iii)	Locus of the points having X coordinate 0 is	(c)	z-coordinate is zero
(iv)	A line is parallel to X-axis if and only	(d)	Z-axis
(v)	If $X = 0$, $y = 0$ taken together will represent the	(e)	plane parallel to XY-plane
(vi)	$z = c$ represent the plane	(f)	if all the points on the line have equal y and z-coordinates
(vii)	Planes $X = a, Y = b$ represent the line	(f)	from the point on the respective
(viii)	Coordinates of a point are the distances from the origin to the feet of perpendiculars	(h)	parallel to Z-axis
(ix)	A ball is the solid region in the space enclosed by a	(i)	disc
(x)	Region in the plane enclosed by a circle is known as a	(j)	sphere

Sol. **(i)** In XY-plane, z-coordinates is zero.

(ii) The point (2, 3, 4) lies in 1st octant .

(iii) Locus of the points having x-coordinate is zero is YZ-plane.

(iv) A line is parallel to X-axis if and only if all the points on the line have equal y and z -coordinates.

(v) $x = 0$, $y = 0$ represent Z-axis.

(vi) $z = c$ represent the plane parallel to XY-plane.

(vii) The planes $x = a$, $y = b$ represent the line parallel to Z-axis.

(viii) Coordinates of a point are the distances from the origin to the feet of perpendicular from the point on the respective.

(ix) A ball is the solid region in the space enclosed by a sphere.

(x) The region in the plane enclosed by a circle is known as a disc.