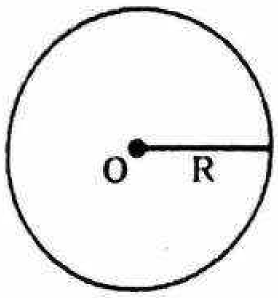
CHAPTER 14

CIRCLES (CHORDS AND TANGENTS)

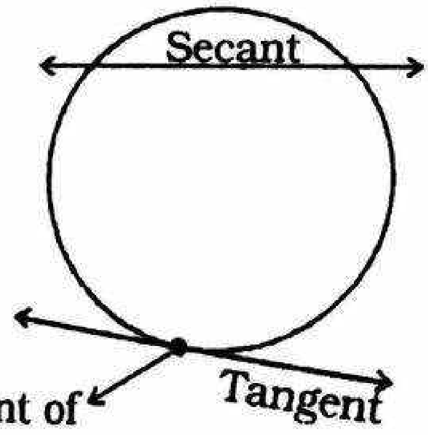
A circle is a set of points on a plane which lie at a fixed distance from a fixed point.



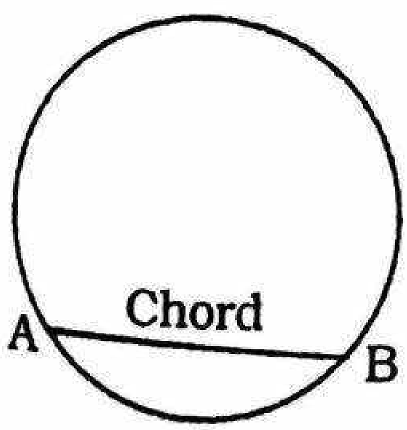
Fixed-point (O) is called "Centre" and R = OP = Radius = Fixed distance

TANGENT:- A line meeting a circle in only one point is called a tangent.

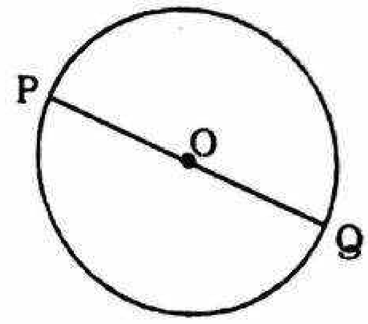
SECANT:- A line which intersects a circle in two distinct points is called a "Secant".



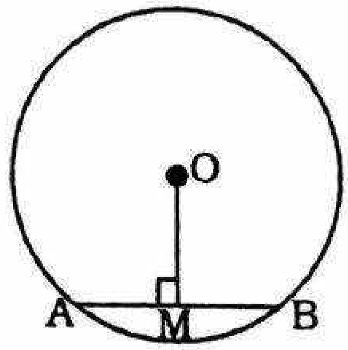
CHORD:- A line segment whose end-points lie on the circle.



DIAMETER:- A chord which passes through the centre is called the diameter of the circle.

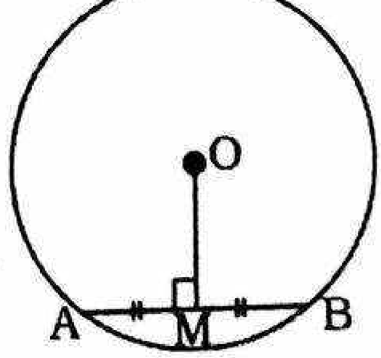


2. The perpendicular from the centre of a circle to a chor d bisects the chord.



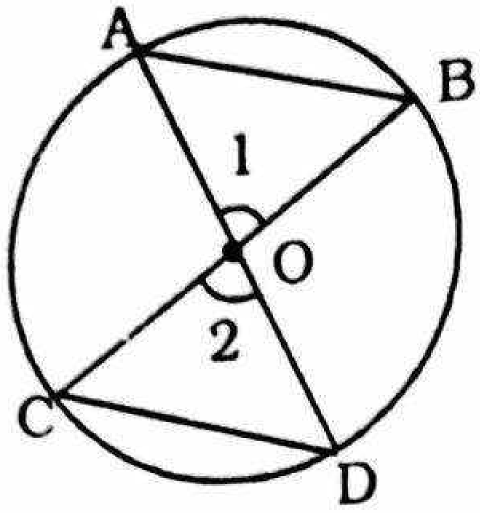
i.e. if OM $\perp$ AB, then AM = BM

3. **Converse of the above theorem:-** The line joining the centre of a circle to the midpoint of a chord is perpedicular to the chord.



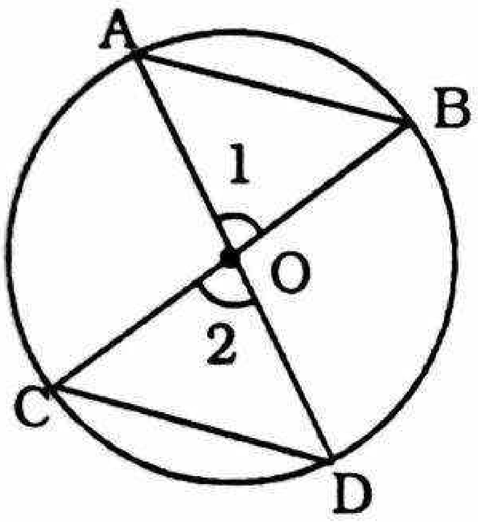
i.e. AM = MB, then OM $\perp$ AB.

4. Equal chords of a circle subtend equal angles at the centre.



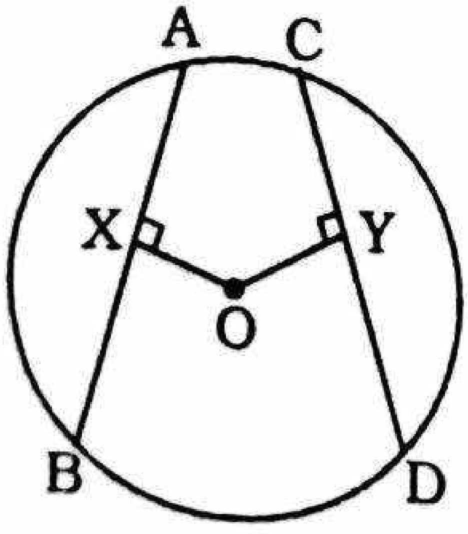
i.e. if AB = CD, then Ð1 = Ð2.

5. (Converse of the above theorem) angles subtened by two chords at the centre of a circle are equal then the chords are equal.



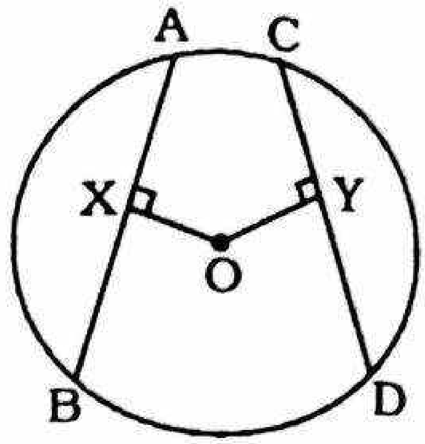
i.e. if Ð1 = Ð2, then AB = CD.

6. Equal chords of a circle are equidistance from the centre.



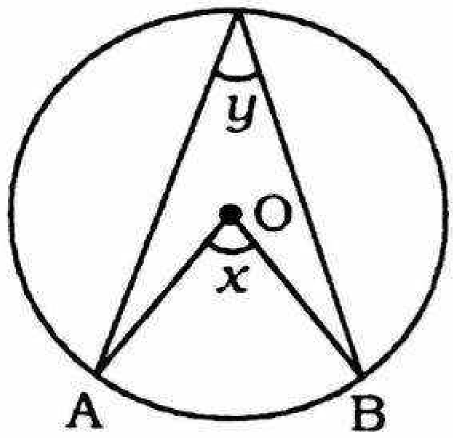
i.e. if AB = CD, OX ⊥ AB and OY ⊥ CD, then OX = OY.

7. (Converse of the above theorem) chords equidistant from the centre of the circle are equal.



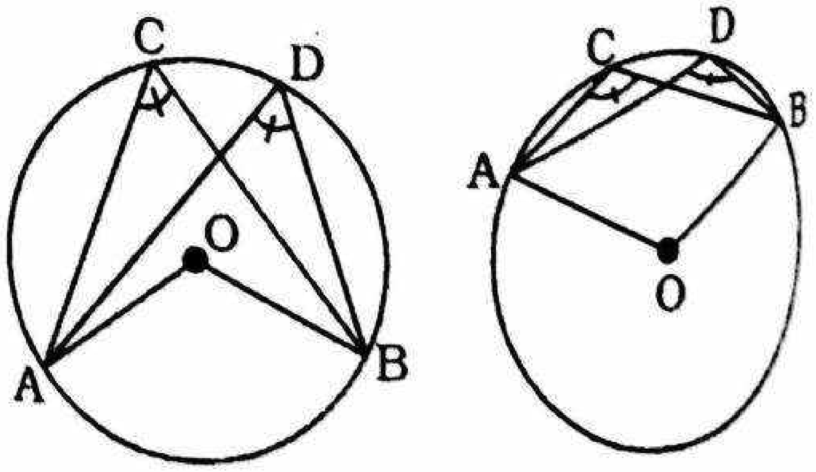
i.e, If OX ⊥ AB, OY ⊥ CD and OX = OY then AB = CD.

8. **(Degreee Measure Thoerem):-** The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.



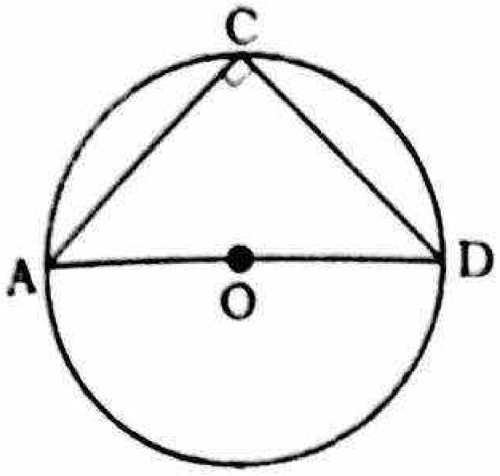
i.e Ðx at the centre and Ðy at the circumference made by the same arc AB, then $\boxed{\angle x = 2\angle y}$

9. Angles in the same segment of a circle are equal.



i.e. ÐACB = ÐADB
(angles in same arc) or
(angles in same segment)

10. The angles in a semi circle is a right angle.

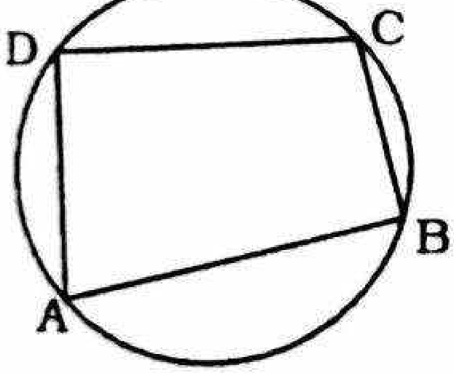


i.e ÐACB = 90°.

11. (Converse of the above thoerem)- The circle, drawn with hyptenuse of a right triangle as diameter, passes through its opposite vertex.

12. If ÐAPB = ÐAQB, and if P, Q are on the same side of AB, then A, B, Q, Pareconylic i.e. lie on the same circle.

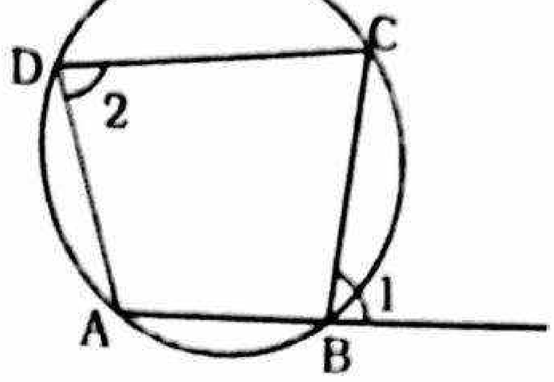
13. The sum of the either pair of the oppo site angles of a cyclic quadrilateral is 180°.



i.e. ÐA + ÐC = ÐB + ÐD = 180°

14. (Converse of the above theorem)- If the two angles of a pair of oppo site angles of a quadrilateral are supplementary,then the quadrilateral is 'cyclic'.

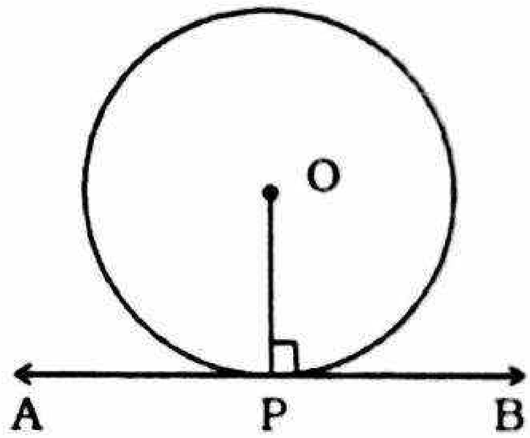
15. If a side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.



i.e. Ð1 = Ð2.

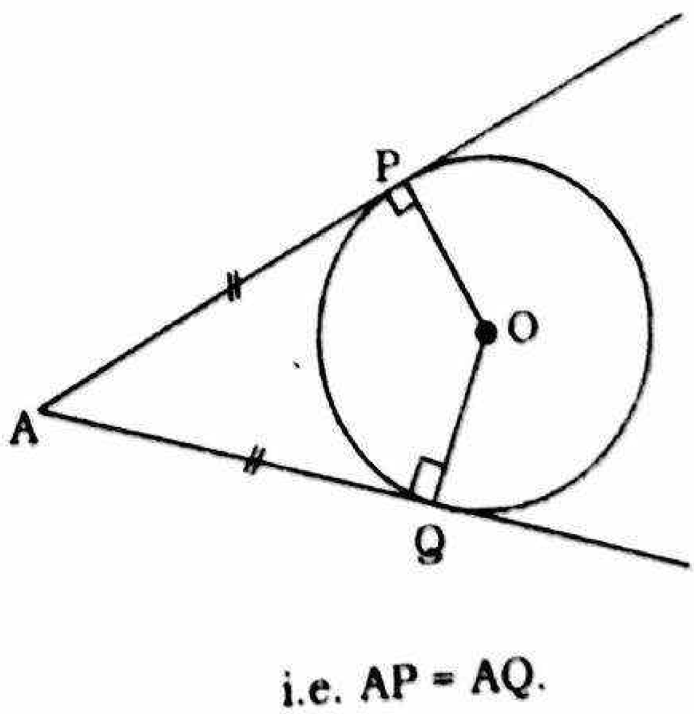
THEOREM ON TANGENTS:-

16. A tangent at any point of a circle is perpendicular to the radius through the point of contact.



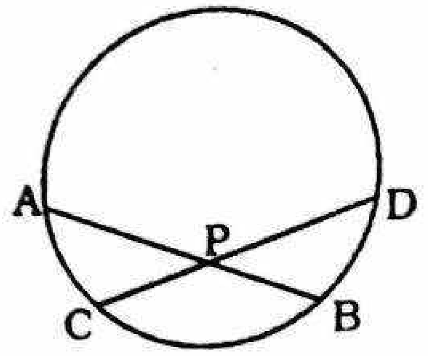
i.e. If AB is a tangent at P, then OP ⊥ AB. (converse of this theorem is also true)

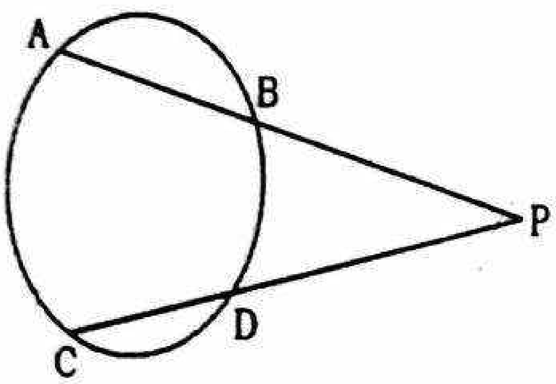
17. The lengths of two tangents, drawn from an external point to a circle, are equal.



i.e. AP = AQ.

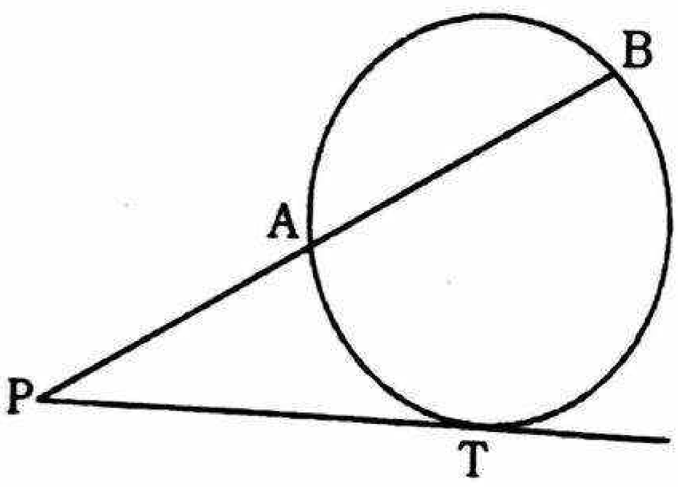
18. If two chords AB and CD intersectinternally or externally at a point P, then



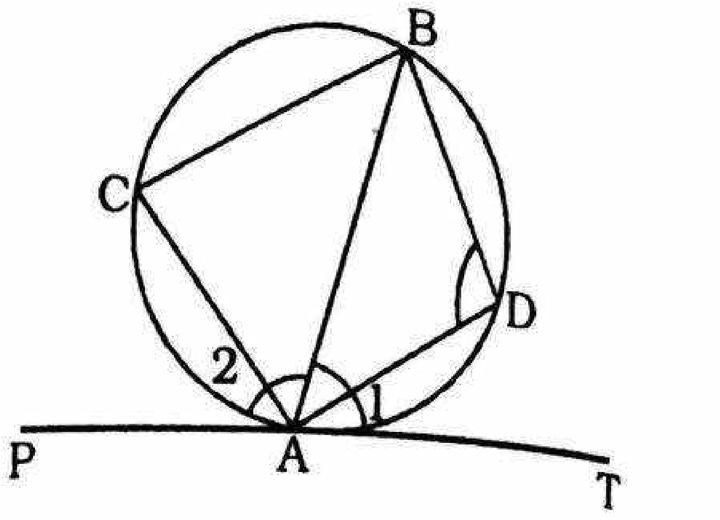


PA × PB = PC × PD

19. If PAB is a secant which intersects the circle at A and B and PT be a tangent at T, then $PT^2 = PA \times PB$



20. **<u>ALTERNATIV SEGMENT THEOREM</u>**:- If from the point of contact of a tangent, a chord s drawn, then the angles which the chord makes with the tangent line areequal respectively to the angles formed in the corresponding alternate segments.

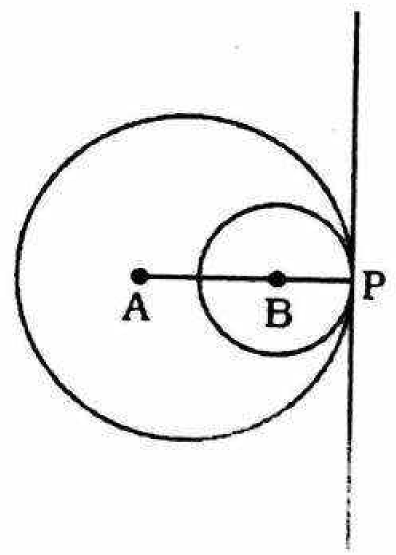


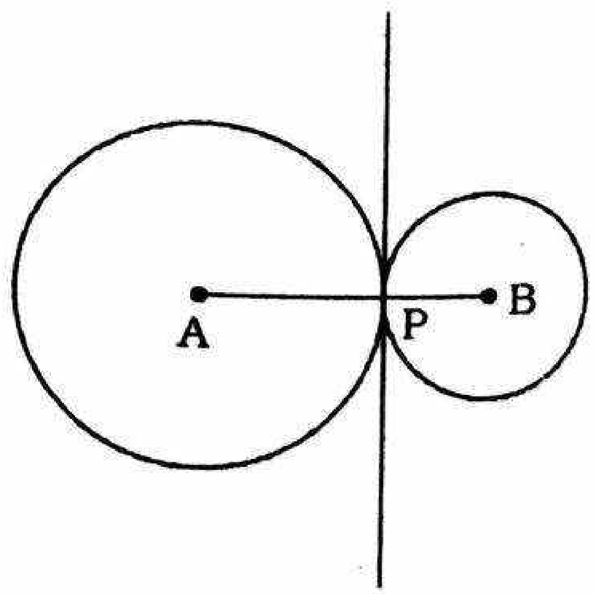
i.e. ÐBAT = ÐBCA = Ð1

and ÐBAP = ÐBDA = Ð2

(The converse of this theorem is also true)

21. If two circles touch each other internally or externally the point of contact lies on the line joining their centres.





i.e. A, B ad P are collinear.

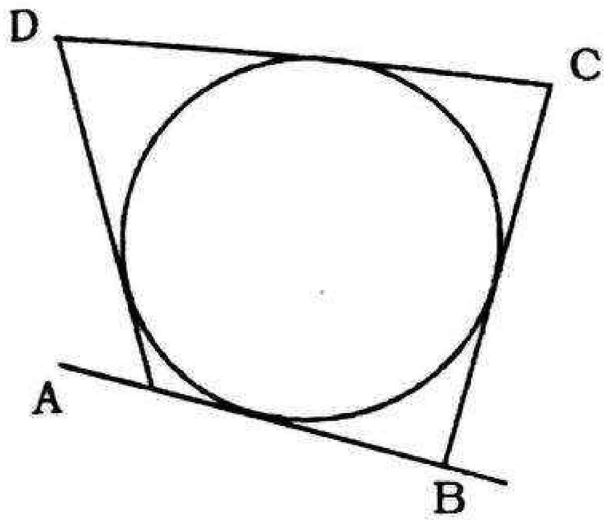
Distance between thier centres (d)

(i)When touch internallyd = AP – BP

(ii)When touch externallyd = AP + BP

Some useful results:-

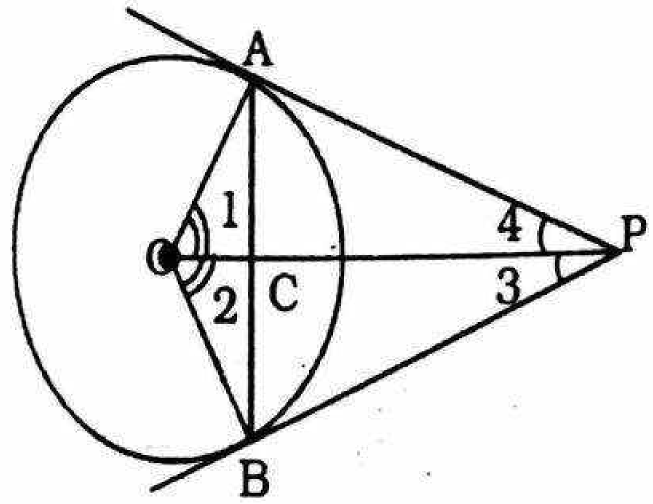
1. Two circles are congruent if and only if they have equal radii.
2. Of any two chords of a circle, the one which is greater is nearer to the centre.
3. Angle in a major segment of a circle is acute and angle in a minor segment is obtuse.
4. If a circle touches all the four sides of a quadrilateral then the sum of opposite pair of sides are equal.



i.e. AB + CD = AD + BC

5. If two tangent PA & PB are drawn from an external point P, then

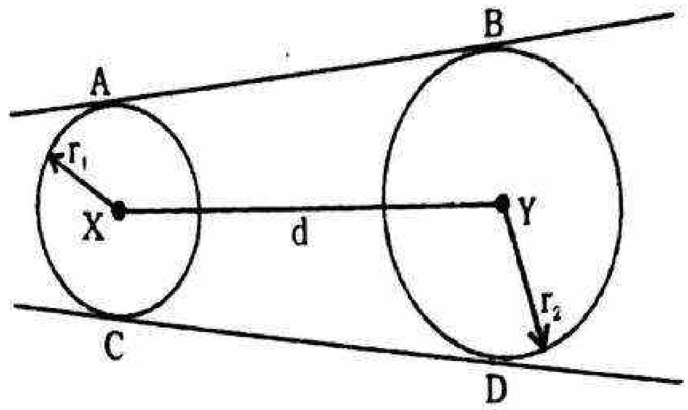
$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$	OP $\perp$ AB and AC = BC



6. For two circles with centres X and Y and radii r_1 and r_2. AB and CD are two direct common tangents (DCT), then the length of DCT.

$= \sqrt{d^2 - (r_1 - r_2)^2}$

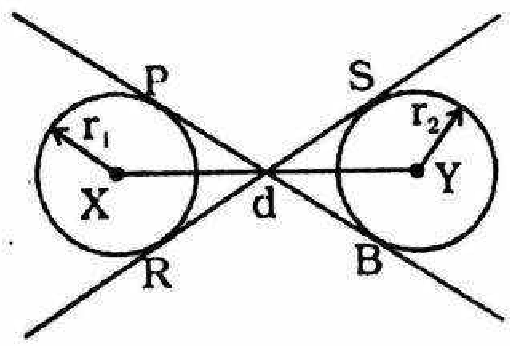
where d = distance between centres (X and Y)



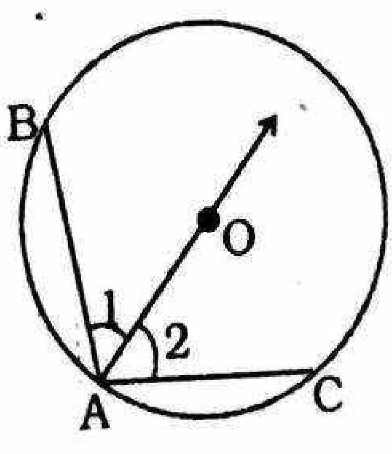
7. For the two circles with centres X and Y and radii r_1 & r_2. PQ and RS are twotransverse common tangents (TCT), then the length of TCT.

$= \sqrt{d^2 - (r_1 + r_2)^2}$

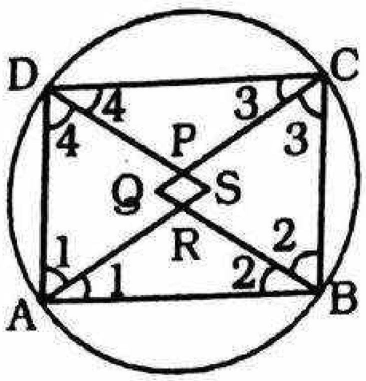
d = XY



8. If two chords AB and AC of a circle are equal, then the bisector of ĐBAC passes through the centre O of the circle. Đ1 = Đ2

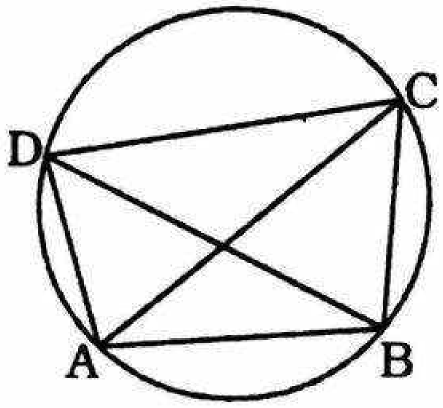


9. The equailateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



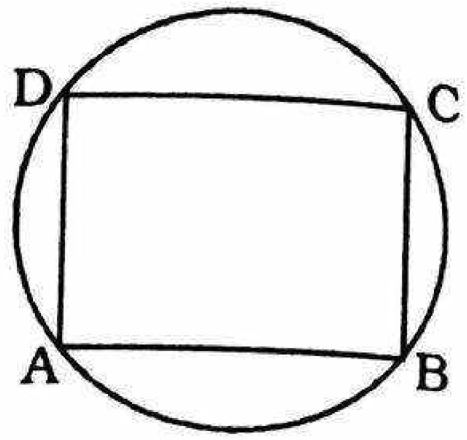
i.e. If ABCD is a cylic quadrilateral, then □ PQRS is also cyclic.

10. If A cyclic trapezium is isosceles and its diagonals are equal



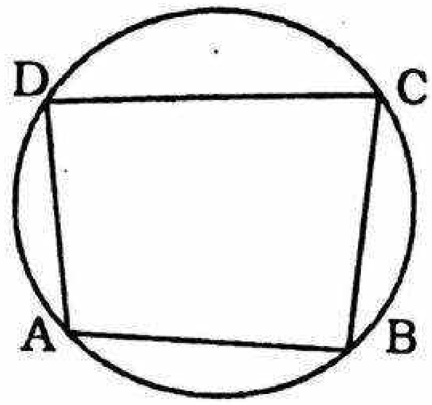
i.e. If ABCD is a cyclic trapezium s.t. AB | | DC, then AD = BC and AC = BD.

11. If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.



i.e. If AD = BC, then AB | | CD.

12. An isosceles trapezium is always cyclic.



i.e. If AB | | DC and AD = BC. Then, ABCD is a cyclic trapezium.

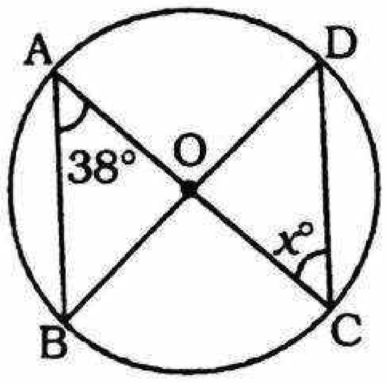
QUESTIONS
LEVEL - I

1. Through any given set of four points A, B, C, D it is possible to draw:-
(a) atmost one circle
(b) exactly one circle
(c) exactly two circles
(d) exactly three circles

2. The number of common tangents that can be drawn to two given circles is at the most :-
(a) one (b) two
(c) three (d) four

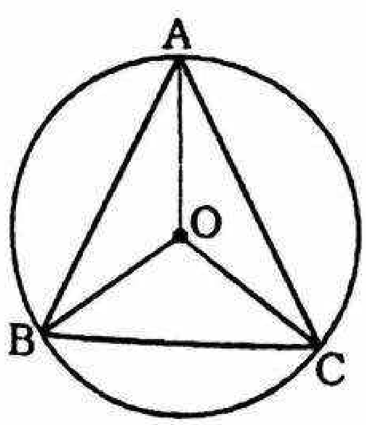
3. The radius of a circle is 13 cm and the length of one of its chords is 10 cm. Find the distance of chord from the centre.
(a) 8 cm (b) 10 cm
(c) 9 cm (d) 12 cm

4. In the given figure O is the centre of the circle. If ∠BAC = 38°, then ∠OCD is



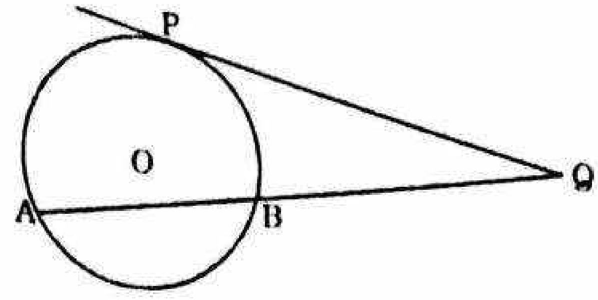
(a) 76° (b) 52°
(c) 38° (d) 19°

5. In the given figure, O is the centre of the circle. If ∠OBC = 20°, the ∠BAC:-



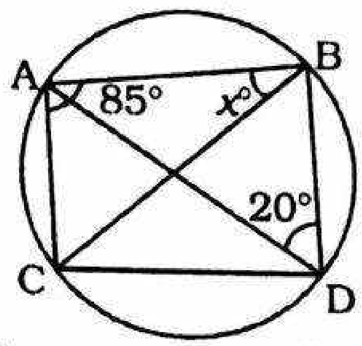
(a) 80° (b) 70°
(c) 100° (d) 140°

6. In the given figure PQ = 12 cm, BQ = 8cm, then the length of chord AB:-



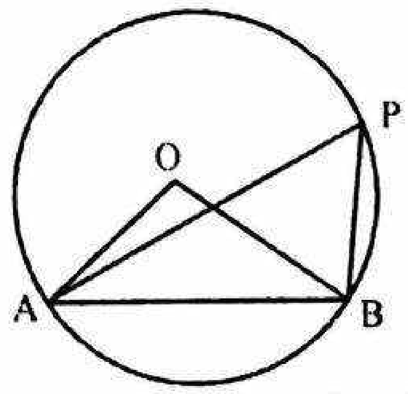
(a) 10 cm (b) $4\sqrt{5}$ cm
(c) 4 cm (d) 18 cm

7. The value of x will be:-



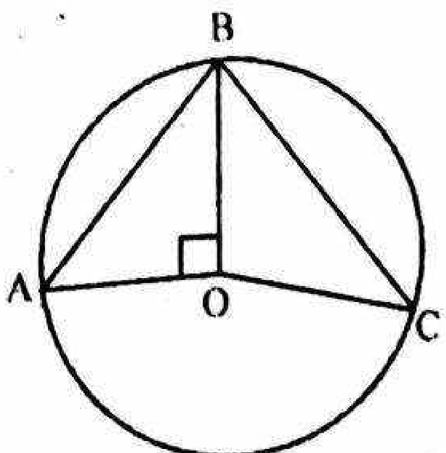
(a) 70° (b) 90°
(c) 60° (d) 75°

8. In the given figure, O is the centre of the circle and ∠AOB = 90°, then ∠APB will be:-



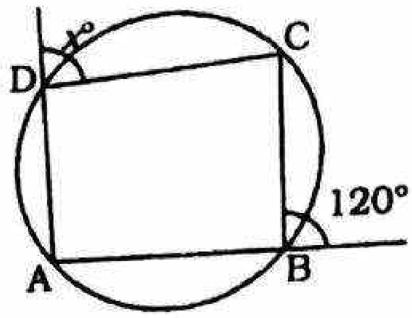
(a) 90° (b) 60°
(c) 45° (d) 30°

9. In the given figure, O is the centre ∠AOB = 90°, ∠BOC = 110°, then ∠ABC is



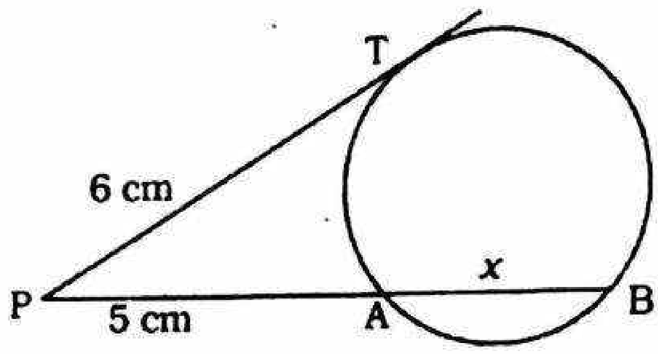
(a) 75° (b) 60°
(c) 80° (d) 70°

10. ABCD is a cyclic quadrilateral, then the of x will be



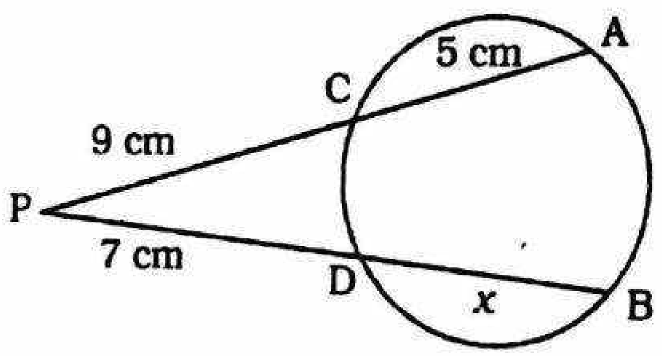
(a) 50° (b) 60°
(c) 120° (d) 70°

11. The value of x :-



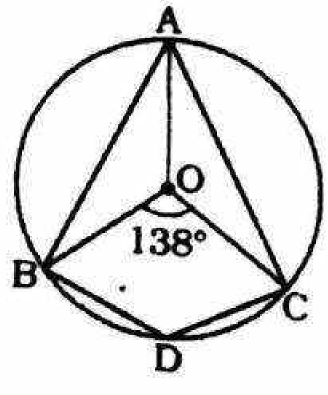
(a) 2.2 cm (b) 1.6 cm
(c) 3 cm (d) 2.6 cm

12. The value of x :-



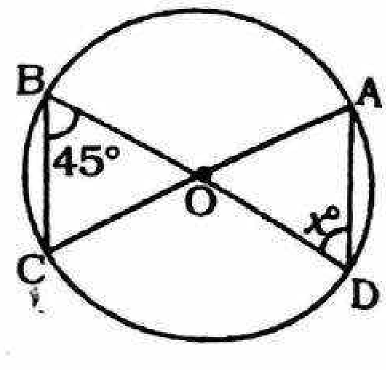
(a) 10 cm (b) 9 cm
(c) 7.5 cm (d) 11 cm

13. In the given figure ABCD is a cyclic quadrilateral and O is the centre of the circle. If ÐBOC = 138°, then ÐBDC will be



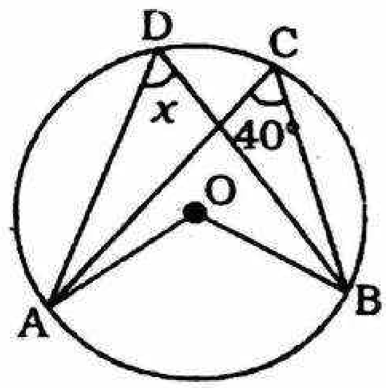
(a) 112° (b) 111°
(c) 109° (d) None of these

14. In the given figure, O is the centre of the circle, then the value of x will be.



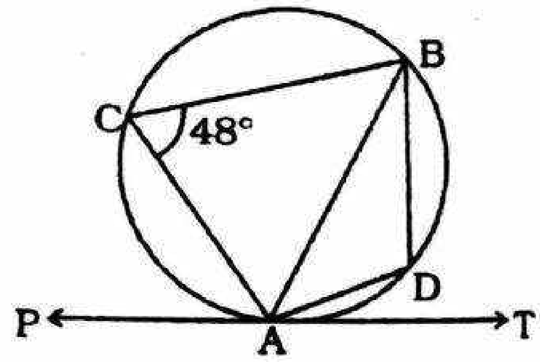
(a) 40° (b) 90°
(c) 45° (d) 30°

15. If O is centre of the circle, then x is equal to



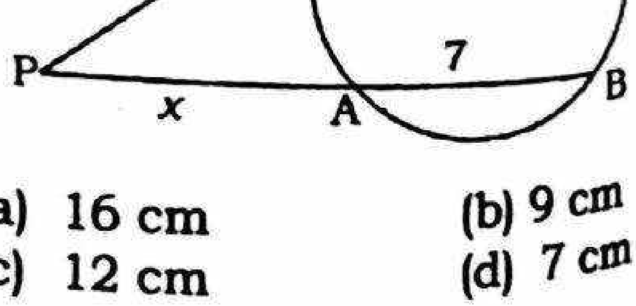
(a) 40° (b) 45°
(c) 39° (d) 35°

16. In the given figure, ÐADB:-



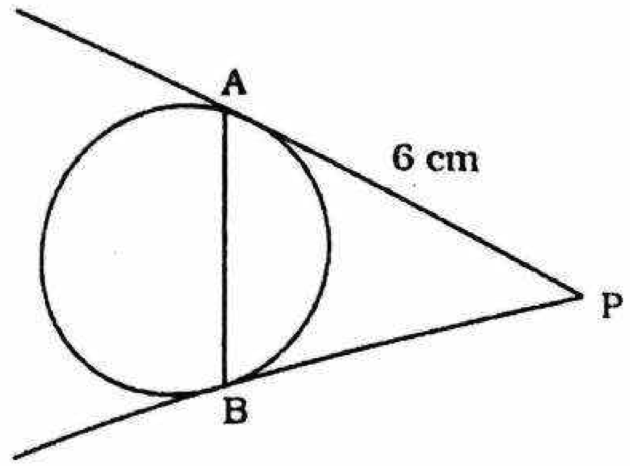
(a) 144° (b) 132°
(c) 48° (d) 96°

17. Find the value of x in the given figure:



(a) 16 cm (b) 9 cm
(c) 12 cm (d) 7 cm

18. In the given figure, PA and PB are tangents from a point P to a circle such that PA = 6 cm and ĐAPB = 60°. What is the length of the chord AB?



(a) 12 cm (b) 8 cm
(c) 9 cm (d) 6 cm

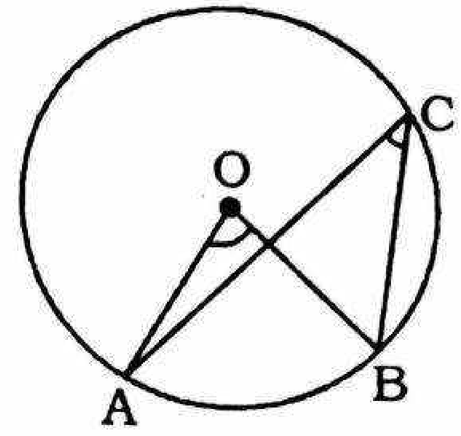
19. ABC is a right angled triangle AB = 3 cm,BC = 5 cm and AC = 4 cm, then the inradius of the circle is
(a) 1 cm (b) 1.25 cm
(c) 1.5 cm
(d) None of these

20. The number of common tangents that can be drawn to two given circles is at the most
(a) 1 (b) 2
(c) 3 (d) 4

21. Two circle of radii 12 cm and 7 cm touch each other internally. Find the distance between their centres.
(a) 6 cm (b) 13 cm
(c) 9 cm (d) 5 cm

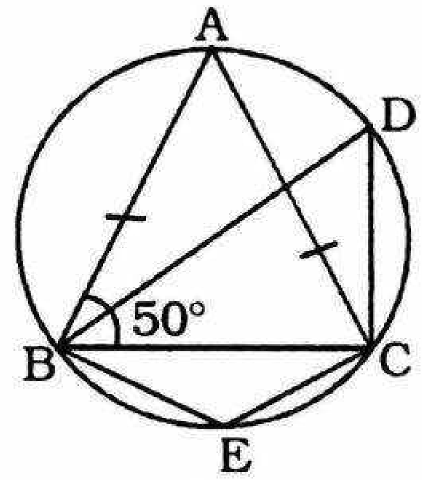
22. Three circles touch each other externally.The distance between their cen tres is 5 cm, 6 cm and 7 cm. Find radii of the circles:-
(a) 2 cm, 3 cm, 4 cm
(b) 3 cm, 4 cm, 1 cm
(d) 1 cm, 2 cm, 4 cm
(d) None of these

23. In the given figure, O is the centre of the circle and ĐACB = 30°. Find ĐAOB.



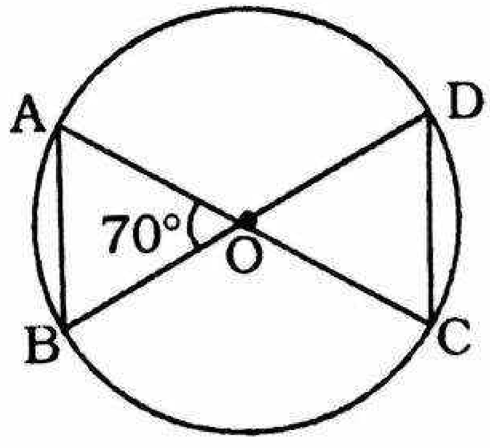
(a) 30° (b) 90°
(c) 60° (d) 50°

24. In the given figure, AB = AC and ĐABC = 50°, ĐBDC:-



(a) 60° (b) 80°
(c) 100° (d) 90°

25. In the given figure, O is the centre of the circle. ĐAOB = 70°, find ĐOCD.



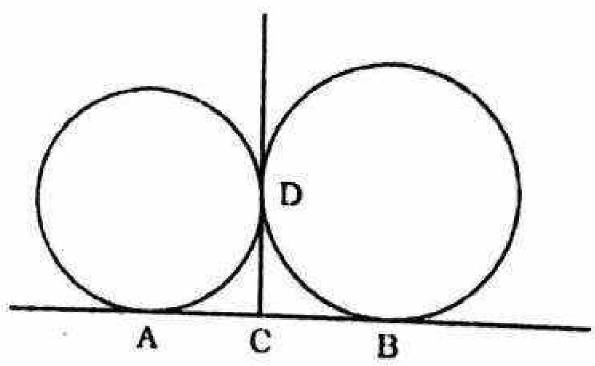
(a) 70° (b) 55°
(c) 65° (d) 110°

26. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is
(a) rhombus (b) square
(c) rectangle
(d) None of these

27. The quadrilateral formed by angle bisec tors of cyclic quadrilateral is a
(a) rectangle
(b) square
(c) parallelogram
(d) cyclic quadrilateral

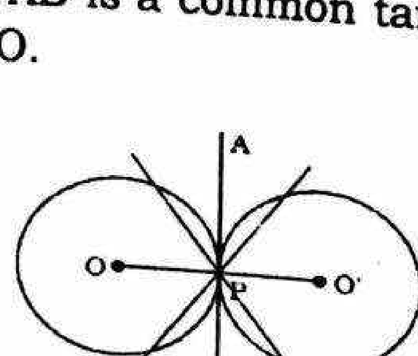
28. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is
(a) 4 cm (b) 6 cm
(c) 8 cm (d) 10 cm

29. In the given figure, AB and CD are two common tangents to the two touching circle. If CD = 7 cm, then AB is equal to



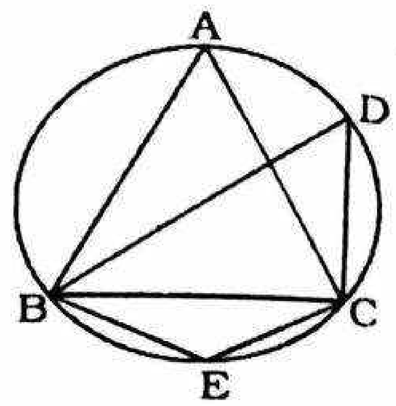
(a) 14 cm (b) 10.5 cm
(c) 12 cm
(d) None of these

30. O and O' are the centres of two circles which touch each other externally at P. If AB is a common tangent. Find ÐAPO.



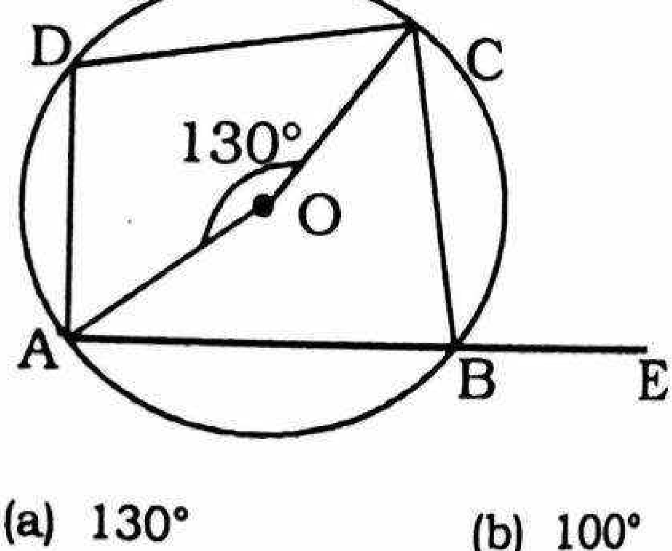
(a) 90° (b) 120°
(c) 60°
(d) data insufficient

31. In the given figure, ΔABC is an equilateral triangle. Find ÐBEC.



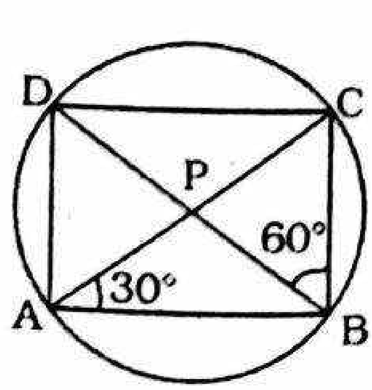
(a) 60° (b) 120°
(c) 80° (d) 90°

32. In the given figure, ÐAOC = 130°. Find ÐCBE, where O is the centre.



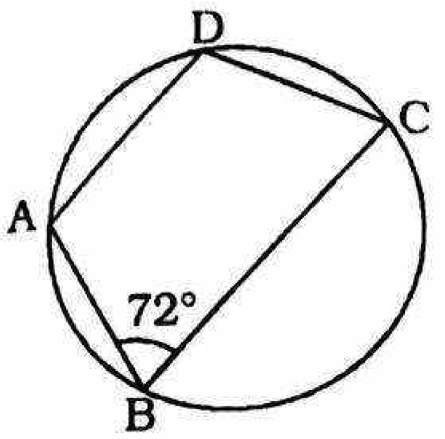
(a) 130° (b) 100°
(c) 115° (d) 105°

33. In the given figure, ABCD is a cyclic quadrilateral and diagonals bisect each other at P. If ÐDBC = 60°, and ÐBAC = 30° then ÐBCD is



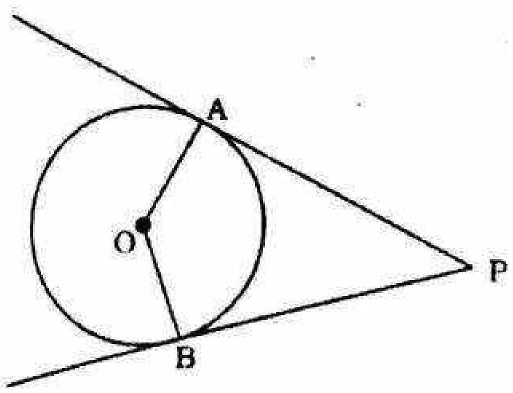
(a) 90° (b) 60°
(c) 80°
(d) None of these

34. In the given figure, AD | | BC, if ÐABC = 72°, then ÐBCD = ?



(a) 108° (b) 36°
(c) 90° (d) 72°

35. In the given figure, O is the centre of the circle. PA and PB are tangents if ÐAOB :ÐAPB = 5 : 1, then ÐAPB



(a) 150° (b) 30°
(c) 60° (d) 90°

36. R and r are the radius of two circles (R>r). If the distance between the centre of the two circles be d, then length of commmon tangent of two circles is :

(a) $\sqrt{r^2-d^2}$ (b) $\sqrt{d^2-(R-r)^2}$

(c) $\sqrt{(R-r)^2-d^2}$ (d) $\sqrt{R^2-d^2}$

37. Tow circles of radii 8cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. Then length of QP is equal to :
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 8 cm

38. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is :
(a) r_1r_2 (b) $2r_1r_2$
(c) $3r_1r_2$ (d) $4r_1r_2$

39. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then AP : AQ is :
(a) 8 : 5 (b) 5 : 8
(c) 3 : 4 (d) 4 : 5

40. The radius of a circle is 6 cm. An external point is at a distance of 10 cm from the centre. Then the length of the tangent drawn to the circle from the external point upto the point of contact is :
(a) 8 cm (b) 10 cm
(c) 6 cm (d) 12 cm

41. A triangle is inscribed in a circle and the diameter of the circle is its one side. Then the triangle will be :
(a) right-angled
(b) obtuse-angled
(c) equilateral (d) a square

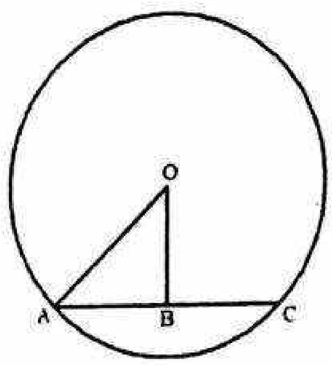
42. Two circles of radii 4 cm and 9 cm respectivley touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of square with one side PQ, is :
(a) 97 sq.cm (b) 194 sq.cm
(c) 72 sq.cm (d) 144 sq.cm

43. The length of the chord of a circle is 8 cm and perpendicular distance between centre and the chord is 3 cm. Then the radius of the circle is equal to :
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

44. The radius of a circle is 13 cm and XY is a chord which is at a distance of 12 cm from the centre. The length of the chord is :
(a) 15 cm (b) 12 cm
(c) 10 cm (d) 20 cm

45. SR is a direct common tangent to the circles of radii 8 cm and 3 cm respectively, their centres being 13 cm apart. If the points S and R are the respective points of contact, then the length of SR is :
(a) 12 cm (b) 11 cm
(c) 17 cm (d) 10 cm

46. In the following figure, if OA = 10 and AC = 16, then OB must be :



(a) 5 (b) 6
(c) 3 (d) 4

47. One chord of a circle is known to be 10.1 cm. The radius of this circle must be :
(a) 5 cm
(b) greater than 5 cm
(c) greater than or equal to 5 cm
(d) less than 5 cm

48. The length of two chords AB and AC of a circle are 8 cm and 6 cm and $\angle BAC = 90°$, then the radius of circle is :
(a) 25 cm (b) 20 cm
(c) 30 cm (d) 5 cm

49. If a chord of length 16 cm is at a distance of 15 cm from the centre of the circle, then the length of the chord of the same circle which is at distance of 8 cm from the centre is equa to :
(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

50. PR is tangent to circle, with centre O and radius 4 cm, at point Q. If $\angle POR = 90°$, OR = 5 cm and $OP = \frac{20}{3}$ cm, then, in cm, the length of PR is :
(a) 3 (b) $\frac{16}{3}$
(c) $\frac{23}{3}$ (d) $\frac{25}{3}$

51. Circumcentre of ΔABC is O. If $\angle BAC = 85°$, $\angle BCA = 80°$, then $\angle AOC$ is :
(a) 80° (b) 30°
(c) 60° (d) 75°

52. If O is the circumcentre of ΔABC and $\angle OBC = 35°$, then the $\angle BAC$ is equal to :
(a) 55° (b) 110°
(c) 70° (d) 35°

53. If I is the incentre of ΔABC and $\angle BIC = 135°$, then the ΔABC is :
(a) Acute angled (b) equilateral
(c) right angled
(d) obtuse angled

54. If S is the circumcentre of ΔABC and $\angle A = 50°$, then the value of $\angle BCS$ is :
(a) 20° (b) 40°
(c) 60° (d) 80°

55. The distance between the centres of two equal circles, each of radius 3 cm, is 10 cm. The length of a transverse common tangent is :
(a) 8 cm (b) 10 cm
(c) 4 cm (d) 6 cm

56. A unique circle can always be drawn through x number of given non-collinear points, then x must be :
(a) 2 (b) 3
(c) 4 (d) 1

57. The length of radius of a circumcircle of a triangle having sides 3cm, 4cm and 5cm is :
(a) 2 cm (b) 2.5 cm
(c) 3 cm (d) 1.5 cm

1.

2.

3.

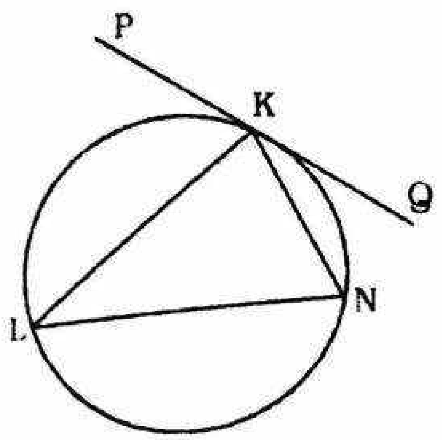
4.

LEVEL - II

1. AB and CD are two parallel chords of a circle such that AB = 6cm and CD=8cm. If the chords lie on the same side of centre O and radius 5cm the distance between AB and CD is :

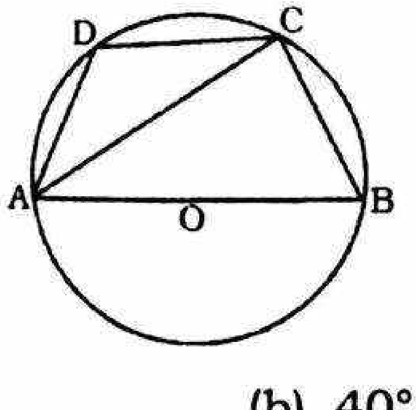
(a) 2 cm (b) 1 cm
(c) 2.5 cm (d) 3 cm

2. In the given figure PQK is a tangent and LN is the diameter of the circles. If $\angle KLN = 30^0$ then $\angle PKL$ will be :



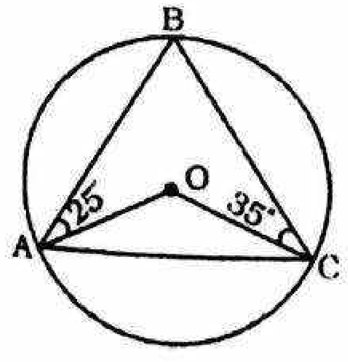
(a) 30° (b) 50°
(c) 60° (d) 70°

3. In the given figure $\angle ADC = 120^0$ and AOB is the diameter of the circle, then $\angle BAC$:



(a) 30° (b) 40°
(c) 50° (d) 60°

4. $\angle OAB = 25°, \angle OCB = 35°$ then $\angle AOC$ will be :

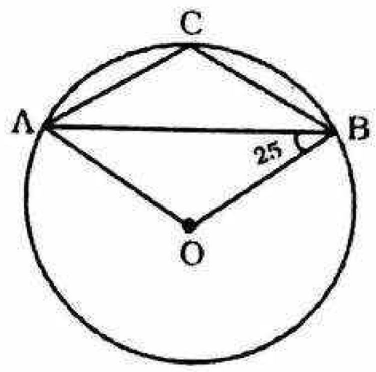


(a) 60° (b) 80°
(c) 100° (d) 120°

5. AB and CD are two parallel chords of a circle such that AB =10cm and CD = 24cm, If the chords are on the opposite sides of the centre and the distance between them is 17cm, then the radius of the circle is :

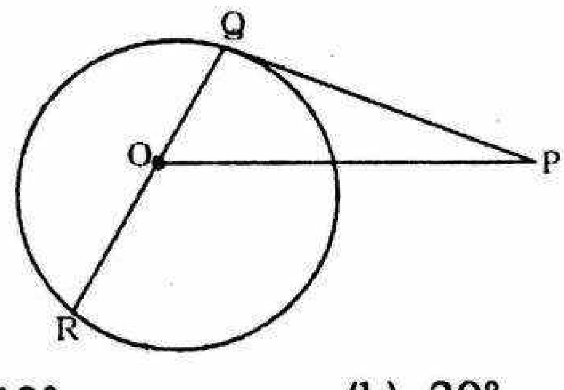
(a) 8 cm (b) 15 cm
(c) 11 cm (d) 13 cm

6. In the given figure, O is the centre of the circle then $\angle ACB$ will be :



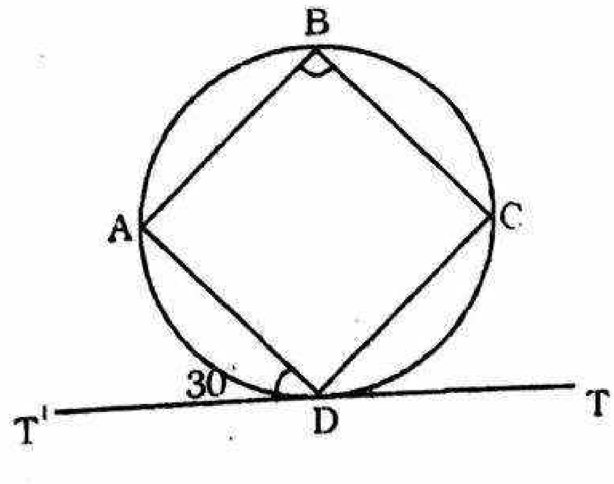
(a) 105° (b) 230°
(c) 115° (d) 100°

7. In the given figure, ROQ is the diameter of the circle. If $\angle POR = 120^0$ then $\angle QPO$ will be:



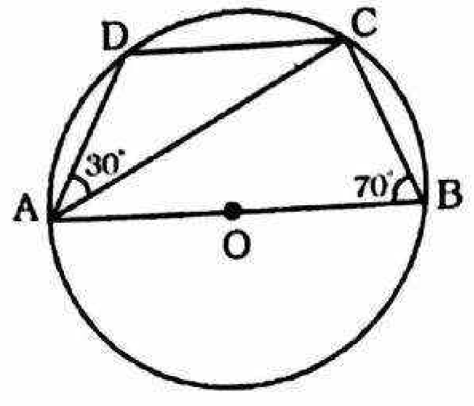
(a) 40° (b) 30°
(c) 60° (d) 50°

8. In the given figure $\angle ABC = 55^0$, the $\angle CDT$ is:



(b) 20°
(c) 15°
(d) 30°
(c) 25°

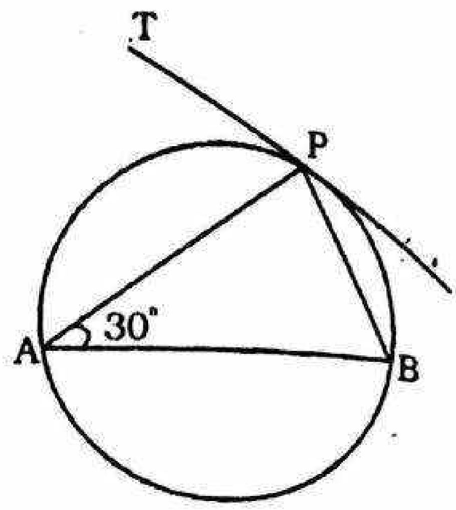
9. In the given figure if AB is the diameter of the circle, then $\angle ACD$ will be :



(a) 40° (b) 50°
(c) 35° (d) 90°

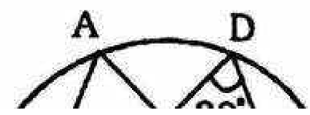
10. $\angle QSR$ is :-

12. In the given figure AB is the diameter of the circle and $\angle PAB = 30^0$. Find $\angle TPA$



(a) 30^0 (b) 60^0
(c) 50^0 (d) 70^0

13. In the following figure, find the value of x



(a) 40^0 (b) 80^0
(c) 20^0 (d) 30^0

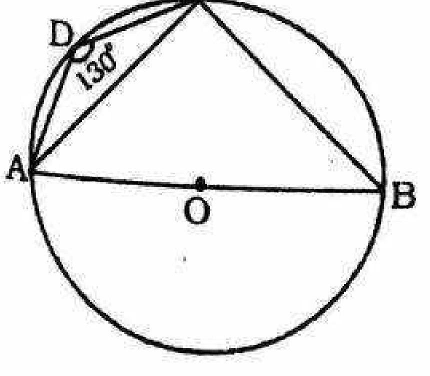
16. Two circles of radius 37cm and 20cm intersect each other at A and B. O and O' are the centres of the circles. If the length of AB is 24cm, then OO':-
(a) 50cm (b) 51cm
(c) 40cm (d) 57cm

17. Two circles of radius 4cm and 6cm touch each other internally. Find the longest chord of the bigger circle which is outside of the smaller circle?
(a) $8\sqrt{2}$cm (b) $4\sqrt{2}$cm
(c) $6\sqrt{2}$cm (d) $3\sqrt{2}$cm

18. In a circle of radius 17cm two parallel chords are present on the opposite side of the diameter. If the distance between them is 23cm and the length of one chord is 16cm then the length of other chord is:-
(a) 15 cm (b) 20cm
(c) 18 cm (d) 30cm

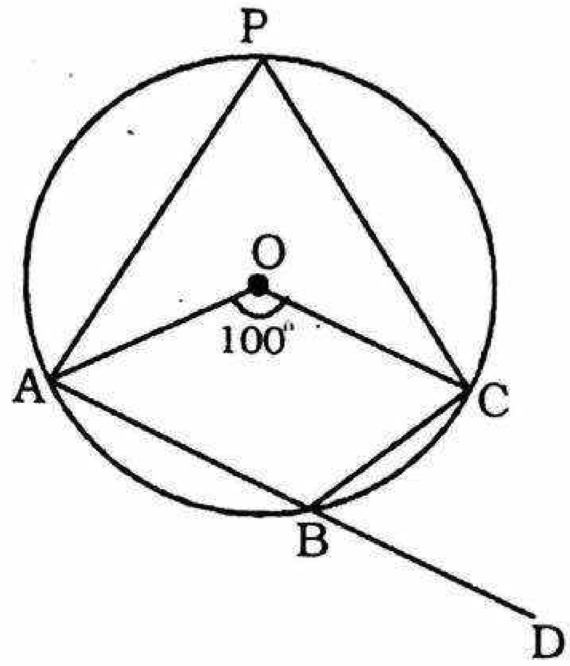
19. AB is a chord of the circle (centre O). P is a point on the circle such that $OP \perp AB$ and OP intersect AB at point M. If AB = 8cm, MP = 2cm then radius (r):-
(a) 7 cm (b) 5cm
(c) 6 cm (d) 4cm

20. In the given figure, ABCD is a cyclic quadrilaleral whose side AB is a diameter of the circle through A,B and C. If $\angle ADC = 130^0$ find $\angle CAB$.



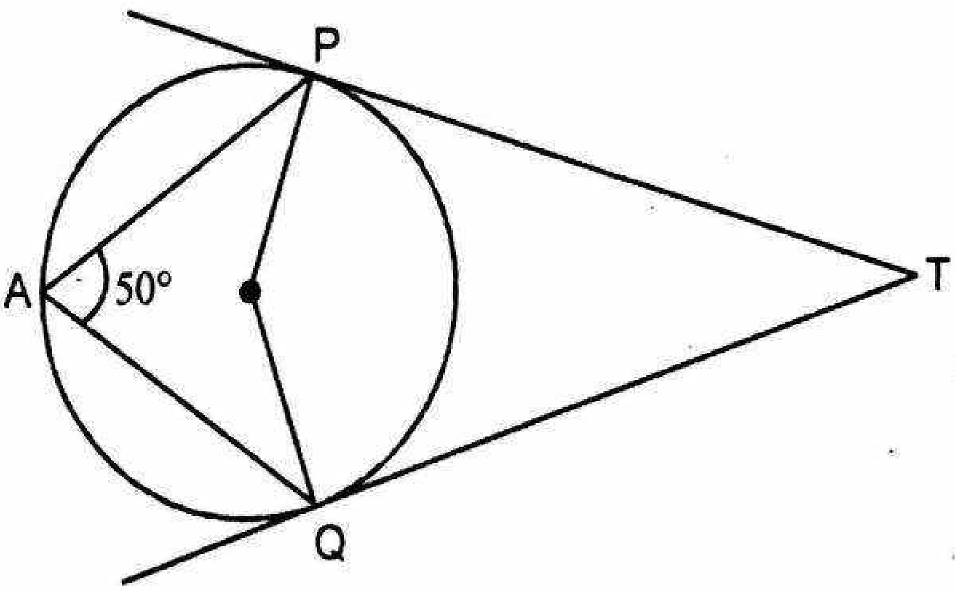
(a) 40^0 (b) 50^0
(c) 30^0 (d) 130^0

21. In the given figure, O is the centre of the circle find $\angle CBD$.



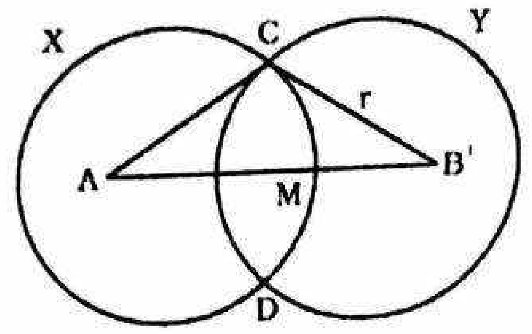
(a) 140^0 (b) 50^0
(c) 40^0 (d) 130^0

22. In the given figure, TP and TQ are tangents to the circle. If $\angle PAQ = 50^0$, what is $\angle PTQ$?



(a) 80^0 (b) 70^0
(c) 100^0 (d) 90^0

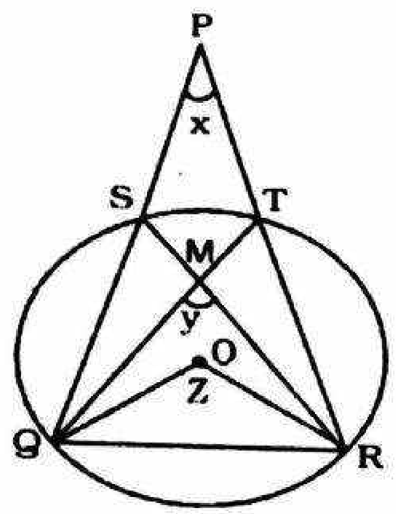
23. Two circles X and Y with centres A and B intersect at C and D. If Area of circle X is 4 time area of circle Y, then AB = ?



(a) 5 r (b) $\sqrt{5}$ r

(c) 3 r (d) $\frac{\sqrt{5}}{2}$ r

24. In the given figure, O is the centre of the circle. Then $\angle x + \angle y$ is equal to-

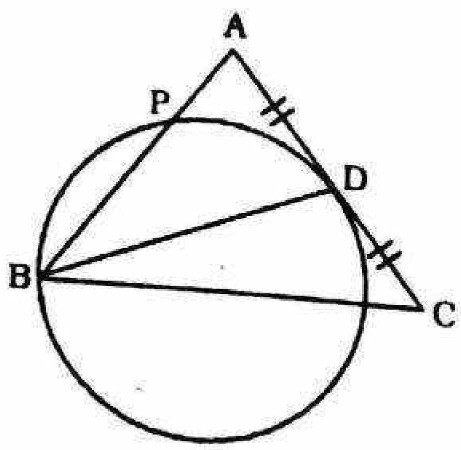


(a) 2 Z (b) $\frac{Z}{2}$

(c) Z

(d) None of these

25. In the figure, ABC is a triangle in which AB = AC. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, Find the value of AB :-

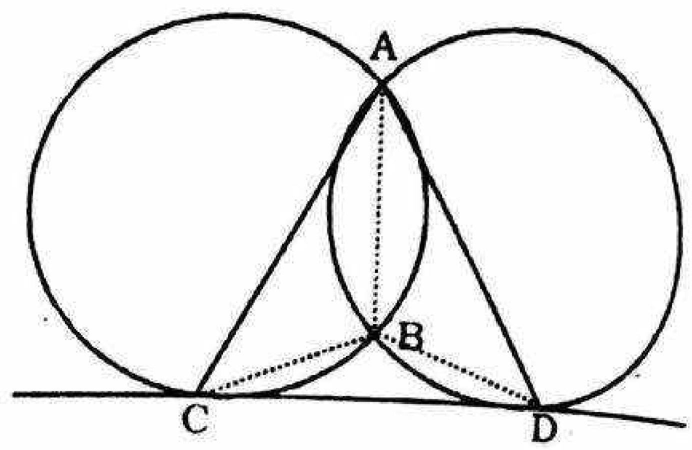


(a) 2AP (b) 3AP

(c) 4AP

(d) None of these

26. In the given figure, CD is a direct common tangent to two circles inntersecting each other at A and B, then $\angle CAD + \angle CBD$ =?

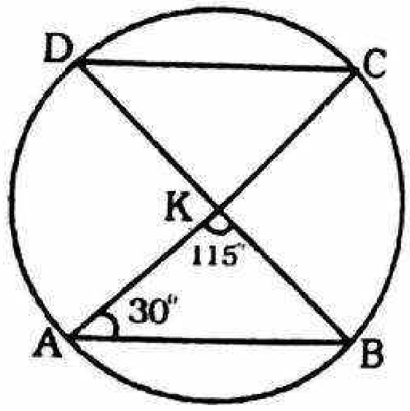


(a) 120° (b) 90°

(c) 360°

(d) None of these

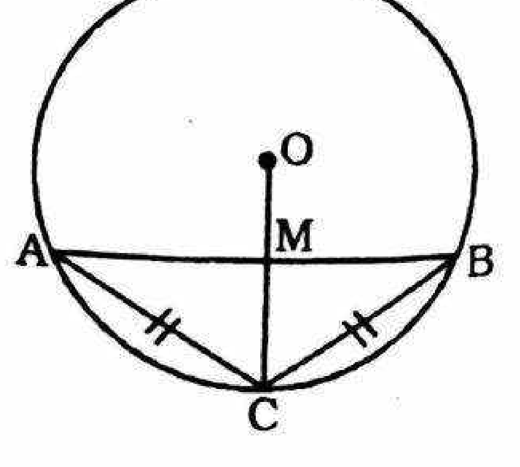
27. In the given figure, $\angle CAB = 30^0$ and $\angle AKB = 115^0$ find $\angle KCD$:-



(a) 65^0 (b) 35^0

(c) 40^0 (d) 72^0

28. In the given figure, the chords AC and BC are equal. The radius OC intersect AB at M then AM : BM :-

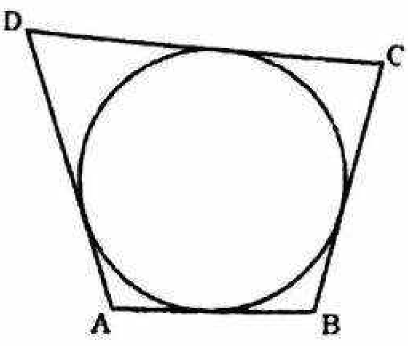


(a) 1:1
(b) $\sqrt{2}:3$
(c) $3:\sqrt{2}$
(d) None of these

29. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :-
(a) $\sqrt{3}:2$
(b) $\sqrt{3}:1$
(c) $\sqrt{5}:1$
(d) None of these

30. If AB is a chord of a circle, P and Q are two points on the circle diffrentfrom A and B, then :-
(a) the sum of the angles subtended by AB at P and Q is equal to four right angle.
(b) the sum of the angles subtended by AB at P and Q is always equal to two right angles.
(c) the angles subtended by AB at P and Q are either equal or supplementary.
(d) the angles subtended at P and Q by AB are always equal.

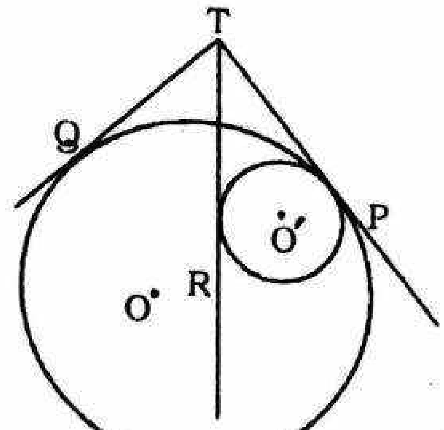
31. A circle touches a quadrilateral ABCD. Find the true statement :-



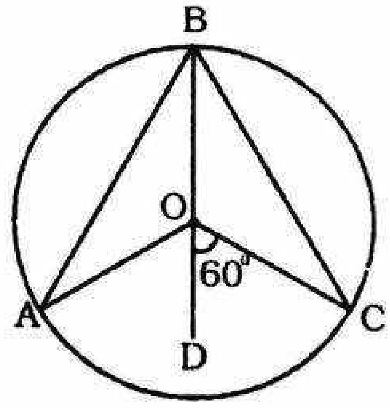
(a) BD = AC
(b) AB + BC = CD + AD
(c) AB + BC = AC
(d) AB + CD = BC + AD

32. In the given figure, Tangents TQ and TP are drawn to the larger circle centre O and tangents TP and TR are drawn to the smaller circle (centre O'). Find TQ : TR :-

(a) 1 : 1
(b) 5 : 4
(c) 8 : 7
(d) 7 : 8



33. 'O' is the centre of the circle, line segment BOD is the angle bisector of $\angle AOC$, $\angle COD = 60^0$. Find $\angle ABC$:-



(a) 120^0
(b) 60^0
(c) 30^0
(d) 90^0

34. If O is the centre of the circle and PA and PB are two tangents drawn from a point P on the circumference of the circle. If $\angle APB = 68^0$ the $\angle POA = ?$
(a) 68^0
(b) 34^0
(c) 56^0
(d) 90^0

35. In a cicle, AB is the diameter of the circle, and CD is a chord such that CD||AB .P is any point on the circle such that $\angle BPC = 48^0$, then $\angle BCD = ?$
(a) 48^0
(b) 42^0
(c) 24^0
(d) 96^0

36. AB and CD are two chords of a circle intersect at a point P. If $\angle APC = 80^0$ and then $\angle BCD = ?$

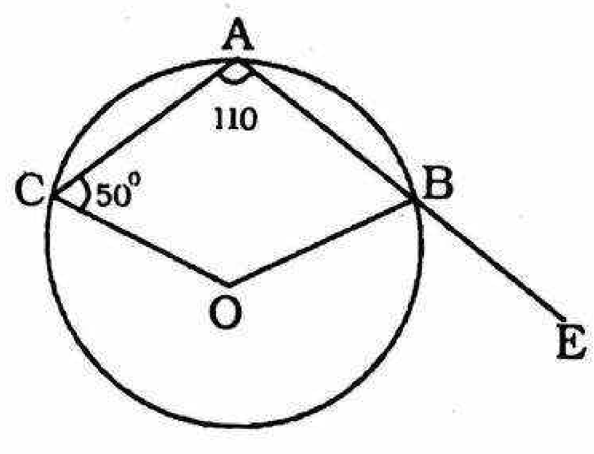
(a) 30^0 (b) 80^0
(c) 100^0 (d) 50^0

37. ABCD is a cylic quadrilateral. Side AB and DC when produced meet at P and side AD and BC when Produced when Produced meet at Q. If $\angle APD = 40^0, \angle ADC = 85^0$, then $\angle AQB$ is equal to :-
(a) 30^0 (b) 40^0
(c) 50^0 (d) 55^0

38. AB is the diameter of the circle with centre O. DC is a chord such that DC | | AB. If $\angle BAC = 20^0$, then $\angle ADC$ is equal to :-
(a) 100^0 (b) 90^0
(c) 110^0 (d) 120^0

39. In question 38 find $\angle COD$?
(a) 50^0 (b) 100^0
(c) 25^0 (d) 90^0

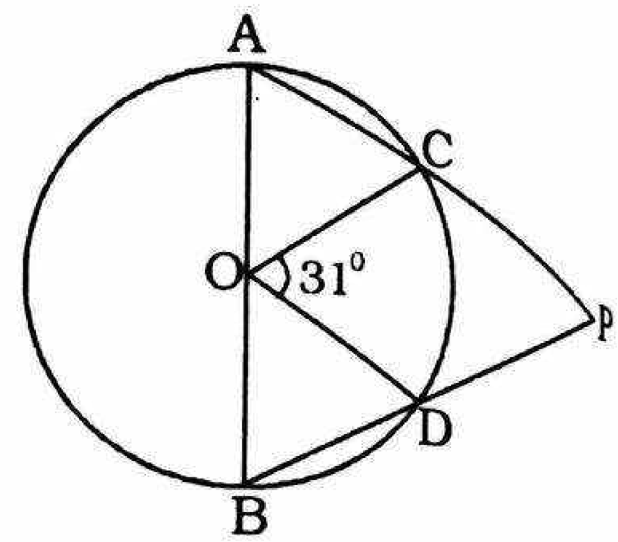
40. Find $\angle OBE$?



(a) 120^0 (b) 100^0
(c) 115^0
(d) None of these

41. AB is the diameter of a circle whose center is O and CD is a chord in the circle and CD=$\frac{1}{2}$ABAC and BD on producing meet at P. Find $\angle APB$?
(a) 30^0 (b) 40^0
(c) 50^0 (d) 60^0

42. In the given figure, AB is the diameter of the circle and O is the centre, Find $\angle APB$?

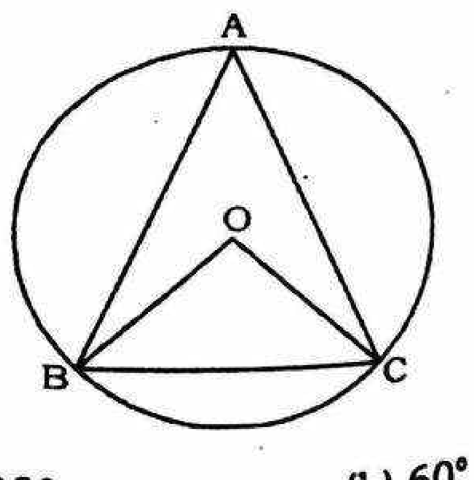


(a) 149^0 (b) 74.5^0
(c) 62^0
(d) None of these

43. O is the circum centre of the traingle ABC with circumradius 13 cm. Let BC = 24 cm and OD is perpendicular to BC. Then the length of OD is :
(a) 7 cm (b) 3 cm
(c) 4 cm (d) 5 cm

44. A, B, C are three points on a circle. The tangent at A meets. BC produced at T, $\angle BTA = 40°$ and $\angle CAT = 44°$ The angle subtended by BC at the centre of the circle is :
(a) 84° (b) 92°
(c) 96° (d) 104°

45. BC is the chord of a circle with centre O. A is a point on major arc BC as shown in the above figure. What is the value of $\angle BAC + \angle OBC$?



(a) 120° (b) 60°
(c) 90° (d) 180°

46. AB and CD are two parallel chords drawn on two opposite sides of their parallel diameter such that AB = 6 cm, CD = 8 cm. If the radius of the circle is 5 cm, the distance between the chords, in cm, is :

(a) 2 (b) 7
(c) 5 (d) 3

47. A chord AB of length $3\sqrt{2}$ unit subtends a right angle at the centre O of a circle. Area of the sector AOB (in sq. units) is :

(a) $\frac{9}{4}n$ (b) 5n
(c) 9n (d) $\frac{9}{2}n$

48. AB and BC are two chords of a circle with centre O. If P and Q are the mid-points of AB and BC respectively, then the quadrilateral OQBP must be:

(a) a rhombus (b) concyclic
(c) a rectangle (d) a square

49. If the area of the circle in the figure is 36 sq. cm, and ABCD is a square, then the area of Δ ACD, in sq. cm, is :

(a) 12n (b) $\frac{36}{n}$
(c) 12 (d) 18

50. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If $\angle$ AOP = 60°, then $\angle$ APB is :

(a) 120° (b) 90°
(c) 60° (d) 30°

51. If the length of a chord of a circle, which makes an angle 45° with the tangent drawn at one end point of the chord, is 6cm, then the radius of the circle is :

(a) $6\sqrt{2}$ cm (b) 5 cm
(c) $3\sqrt{2}$ cm (d) 6 cm

52. Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord?

(a) 5 (b) $5\sqrt{3}$
(c) $10\sqrt{3}$ (d) $\frac{5\sqrt{3}}{2}$

53. PA and PB are two tangents drawn from and external point P to a circle with centre O where the points A and B are the points of contact. The quadrilateral OAPB must be:

(a) a rectangle (b) a rhombus
(c) a square (d) concyclic

54. The radius of two concentric circles are 9 cm and 15 cm. If the chord of the greater circle be a tangent to the smaller circle, then the length of that chord is :

(a) 24 cm (b) 12 cm
(c) 30 cm (d) 18 cm

55. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to :

(a) 30° (b) 45°
(c) 60° (d) 90°

56. The ratio of the areas of the circumcircle and the incircle of an equilateral traingle is:

(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 3 : 2

57. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the centre of a circle. The distance between them is 1 cm. The radius of the circle is :

(a) 5 cm (b) 4 cm
(c) 3 cm (d) 2 cm

58. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of the common chord is :

(a) $2\sqrt{2}$ cm (b) $4\sqrt{3}$ cm
(c) $2\sqrt{3}$ (d) 8 cm

59. ABCD is a cyclic parallelogram. The angle $\angle B$ is equal to :
(a) 30° (b) 60°
(c) 45° (d) 90°

60. From four corners of a square sheet of side 4 cm, four pieces, each in the shape of arc of a circle with radius 2 cm, are cut out. The area of the remaining portion is :
(a) (8 - n)sq.cm.
(b) (16 - 4n)sq.cm.
(c) (16- 8n)sq.cm.
(d) (4 - 2n) sq.cm.

61. If a chord of a circle of radius 5 cm is a tnagent to a circle of radius 3 cm, both the circles being concentric, then the length of the chord is :
(a) 10 cm (b) 12.5 cm
(c) 8 cm (d) 7 cm

62. Two circles touch each other externally at point A and PQ is a direct common tangent which touches at P and Q respectively. Then $\angle PAQ$ =
(a) 45° (b) 90°
(c) 80° (d) 100°

63. AB is a chord to a circle and PAT is the tangent to the circle at A. If $\angle BAT$ = 75° and $\angle BAC$ = 45°, C being a point on the circle, then $\angle ABC$ is equal to :
(a) 40° (b) 45°
(c) 60° (d) 70°

64. O is the centre of a circle and arc ABC subtends an angle of 130° at O. AB is extended to P. Then $\angle PBC$ is :
(a) 75° (b) 70°
(c) 65° (d) 80°

65. The circumcentre of a triangle ABC is O. If $\angle BAC$ = 85° and $\angle BCA$ = 75°, then the value of $\angle OAC$ is:
(a) 40° (b) 60°
(c) 70° (d) 90°

66. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle, in cm^2, is :
(a) 450 (b) 308
(c) 154 (d) 77

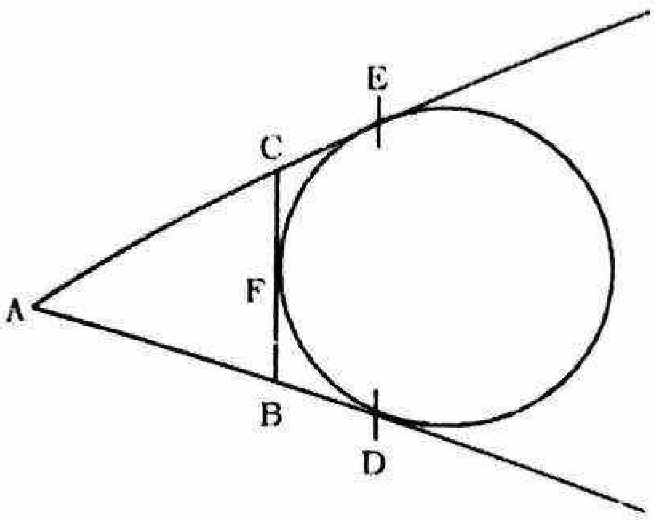
67. If the radii of two circles be 6 cm and 3 cm and the length of the transverse common tangent be 8 cm, then the distance between the two centres is:
(a) $\sqrt{154}$ cm (b) $\sqrt{140}$ cm
(c) $\sqrt{145}$ cm (d) $\sqrt{135}$ cm

68. Two parallel chords are drawn in a circle of diameter 30 cm. The length of one chord is 24 cm and the distance between the two chords in 21 cm. The length of the other chord is:
(a) 10 cm (b) 18 cm
(c) 12 cm (d) 16 cm

69. If two equal circles whose centres are O and O', intersect each other at the points A and B, OO'= 12 cm and AB = 16cm, then the radius of the circles is :
(a) 10 cm (b) 8 cm
(c) 12 cm (d) 14 cm

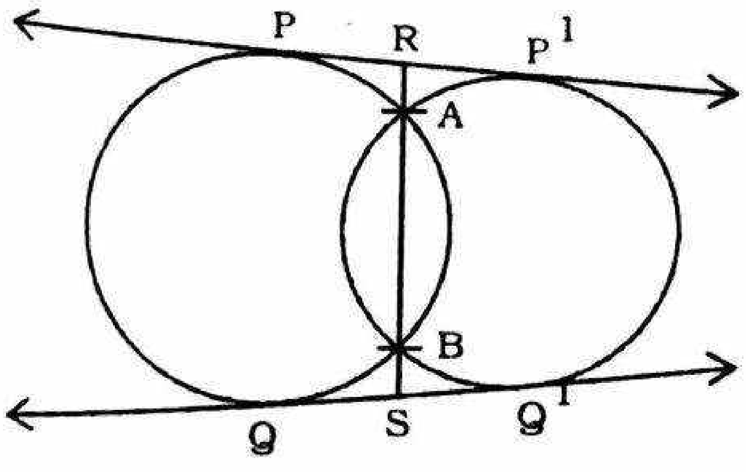
LEVEL - III

1. In the given figure, AD, AE and BC are tangents, then:-



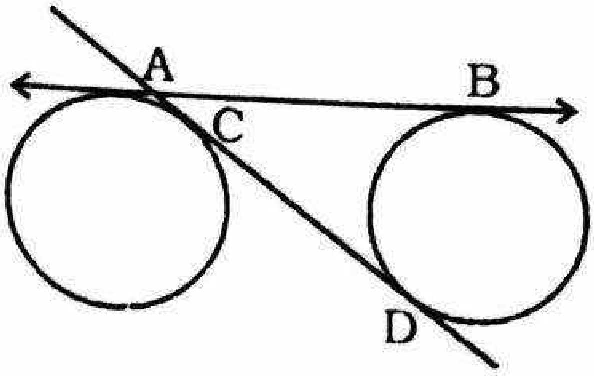
(a) AD = AB + BC + CA
(b) 2AD = AB + BC + CA
(c) 3AD = AB + BC + CA
(d) 4AD = AB + BC + CA

2. PP^1 and QQ^1 are two direct common tengents to two circles intersecting at points A and B. The common chord on produced intersects PP^1 in R and QQ^1 in S. Which of the following is true ?



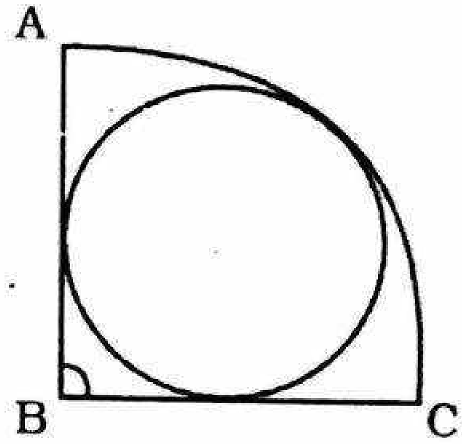
(a) $RA^2 + BS^2 = AB^2$
(b) $RS^2 = PP^{12} + AB^2$
(c) $RS^2 + PP'^2 = QQ'^2$
(d) $RS^2 = BS^2 + PP^{12}$

3. If two equal circles of radius 5cm have two common tangent AB and CD which touch the circle on A,C, and B,D respectively and if CD=24cm, find the length of AB.



(a) 27cm (b) 25cm
(c) 26 cm (d) 30cm

4. If ABC is a Quarter Circle and a circle is inscribld in it and if AB=1cm, find radius of smaller circle.



(a) $\sqrt{2} - 1$ (b) $\frac{\sqrt{2} - 1}{2}$

(c) $\frac{\sqrt{2} + 1}{2}$ (d) $1 - 2\sqrt{2}$

5. Find the lenght of the commaon chord of two circles of radius 15cm and 20cm if their centres are 25cm apart ?
(a) 12cm (b) 20cm
(c) 18cm (d) 24cm

6. AB and AC are two chords of a circle such that AB=AC=6cm. If radius of the circle is 5cm, then BC is:-
(a) 4.8cm (b) 9.6cm
(c) 2.4cm (d) 8.4cm

7. '2a' and '2b' are the lenght of two chords which intersect at right angle. If the distance between the centre of the circle and the intersecting point of the chords is 'C' then the radius of the circle is:-

(a) $\frac{\sqrt{a^2b^2c^2}}{2}$

(b) $\sqrt{a^2+b^2+c^2}$

(c) $\frac{\sqrt{a^2+b^2+c^2}}{2}$

(d) None of these

8. AB and CD are two chords of a circle which intersect at right angle at E. If AE = 2cm, EB = 6cm, ED = 3cm, then radius (r) is equal to:-

(a) $\frac{\sqrt{65}}{2}$ (b) $\sqrt{65}$

(c) $2\sqrt{65}$

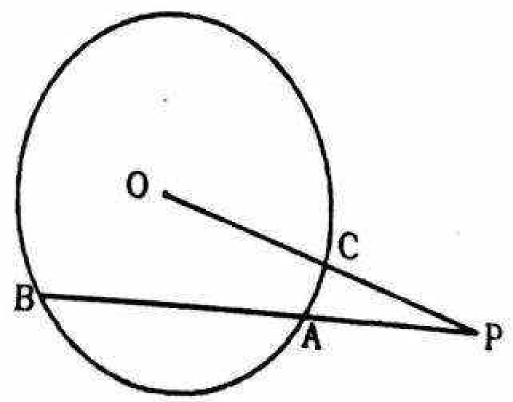
(d) None of these

9. AB is a chord of a circle (centre O) and DOC is a line segment originating from a point D on the circle and intersecting AB on producing at C such that BC=OD. If $\angle BCD = 20°$, then $\angle AOD$:-

(a) 30° (b) 40°

(c) 100° (d) 60°

10. In the given figure, O is the centre of the circle. If BA = 7cm, OP = 13cm & AP = 9cm then radius (r):-



(a) 7cm (b) 5cm

(c) 4cm (d) 6cm

11. Two tangents PA and PB are drawn to the circle(centre O) from a point P. CD is another tangenton the circle which intersects PA and PB at C and D respectively. If $\angle APB = 34°$ then $\angle COD$:-

(a) 146° (b) 68°

(c) 73°

(d) None of these

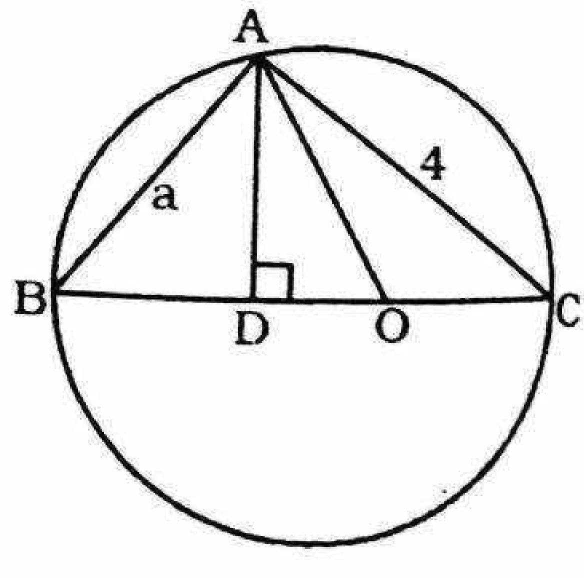
12. Two tangents PA and PB are drawn from from apoint P to the circle. If the radius of the circle is 5 cm and AB=6cm and O is the centre of the circle. OP cuts AB at Cand OC=4cm, then OP:-(a) $\frac{25}{4}$ cm

(b) 25cm

(c) 13 cm

(d) None of these

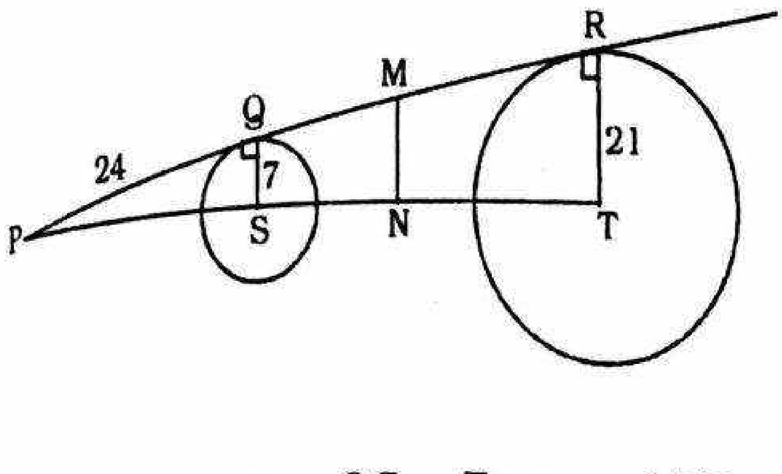
13. If i n the given figure, AB=a, AC=4cm, while O is the centre of the circle and D is a point between O and B such that $AD \perp BC$. Find the length of OD.



(a) $\frac{4-a}{4}$ (b) $\frac{16-a^2}{2\sqrt{a^2+16}}$

(c) $\frac{4a-16}{16a-a^2}$ (d) $\frac{2\sqrt{a^2-16}}{16+a^2}$

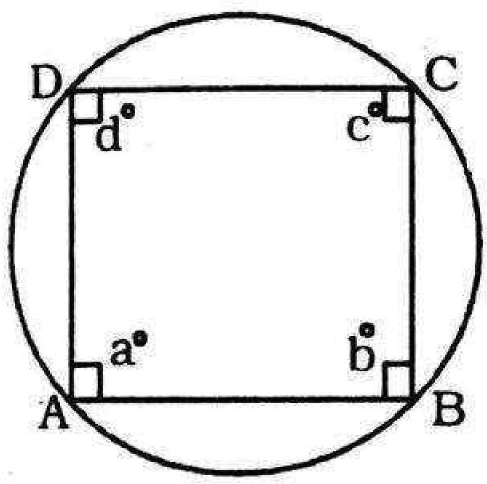
14. In the given figure, PQ = 24cm. M is the mid-point of QR.



Also, $MN \perp PR$, QS = 7cm and TR = 21cm, then SN = ?

(a) 50 cm (b) 12.5cm
(c) 31 cm (d) 25 cm

15. In the given figure, AB | | CD if a,b,c and d are integers, what is the number of possible value of (a+b-cd)?



(a) 179 (b) 89
(c) 357 (d) 358

16. Three equal circle of unit radius touch each other. Then , the area of the circle circumscribing the three circle is :-

(a) $6\pi(2+\sqrt{3})^2$ (b) $\frac{\pi}{6}(2+\sqrt{3})^2$

(c) $\frac{\pi}{3}(2+\sqrt{3})^2$ (d) $3\pi(2+\sqrt{3})^2$

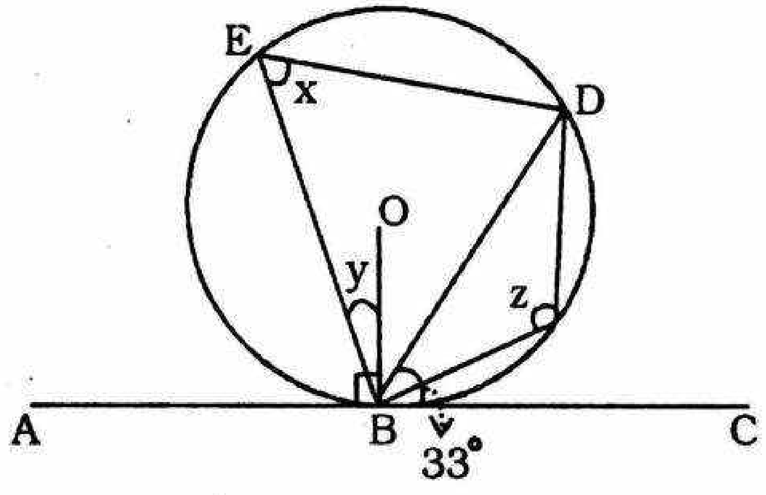
17. In ΔABC, AB=4cm, BC=3.4cm and AC=2.2cm. Three circles are drawn with centre A, B and C in such a way that each circle touches the other two. Then the diameter of the bigger circle is :

(a) 5.2 cm (b) 2.6 cm
(c) 2.8 cm
(d) None of these

18. The angle bisectors of angle A, B and C of a ΔABC intersect the circumference of the circum circle at X, Y and Z respectively. If $\angle A = 50°$, $\angle CZY = 42°$, then $\angle BYZ$ is equal to :-

(a) 46° (b) 42°
(c) 23° (d) 21°

19. In the given figure, chord BE = BD, $\angle CBD = 33°$, & $OB \perp AC$ then $x+y+z$ is equal to



(a) 230° (b) 237°
(c) 337°
(d) None of these

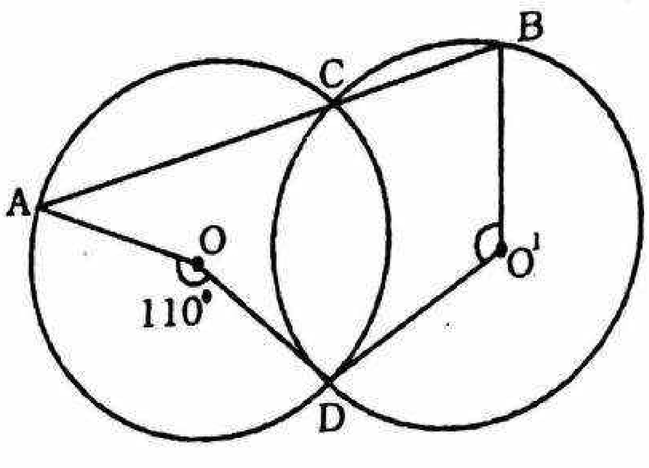
20. ABC and MNC are two secants of a circle whose centre is O. AN is the diameter of the circle if $\angle BAN = 38°$ and $\angle ACM = 20°$ then $\angle MBN$:-

(a) 38° (b) 42°
(c) 28° (d) 32°

21. PT is a tangent of a circle at T and AB is a chord. IF AB =18cm and PT = 2AP then find PT ?

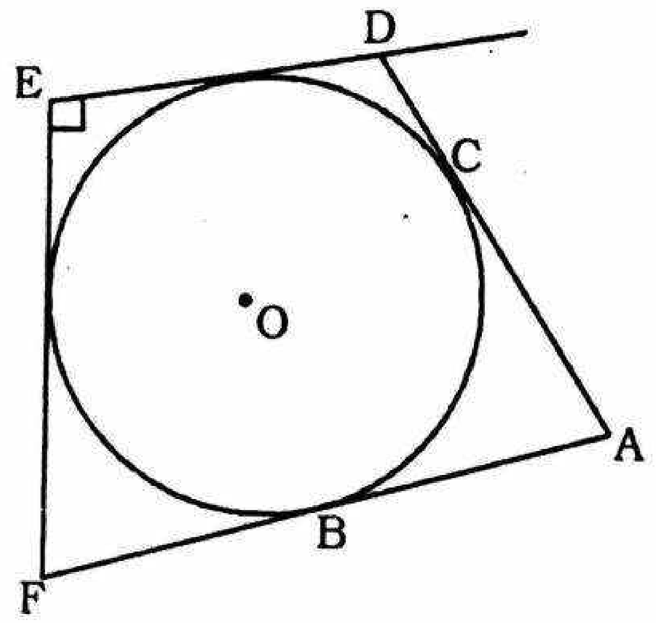
(a) 12cm (b) 18cm
(c) 6 cm (d) 9cm

22. Find $\angle BO'D$?



(a) 220° (b) 110°
(c) 55° (d) 70°

23. In the given figure, AB = 27, AD = 38cm, ED = 24cm and $\angle E = 90°$, then radius of the circle is equal to :-



(a) 11 cm (b) 15 cm
(c) 13 cm (d) 17 cm

24. Two circles having radius 'a'cm and 'b'cm touch each other externally. another circle whose radius is 'c'cm, touches both the circles and also their common tangent. Then which statement will be true :-

(a) $\sqrt{a} + \sqrt{b} = \sqrt{c}$

(b) $\sqrt{a} = \sqrt{b} + \sqrt{c}$

(c) $\sqrt{a}b + \sqrt{b}c = \sqrt{ac}$

(d) $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$

25. In a DABC, I and O are the in-centre and circum-centre respectively. The line AI is produced to a point D on the circumcircle. If $\angle BOD = Z$, $\angle BID = Y$ and $\angle ABC = X$, then $\frac{x+z}{3y}$ is eqval to :-

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{3}$ (d) 1

26. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the circle respectviely at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32°$, $\angle RTS$ = ?

(a) 32° (b) 74°
(c) 106° (d) 64°

27. O and C are respectively the orthocentre and circumcentre of an acute-angled triangle PQR. The points P and O are joined and produced to meet the side QR at S. If $\angle QCR = 130°$, then $\angle RPS$ =

(a) 30° (b) 65°
(c) 100° (d) 60°

28. Two chords AB and CD of circle whose centre is O, meet at the point P and $\angle AOC = 50°$, $\angle BOD = 40°$. Then the value of $\angle BPD$ is :

(a) 60° (b) 40°
(c) 45° (d) 75°

29.

30.

Ad

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29. The tangents are drawn at the extremities of a diameter AB of a circle with centre P. If a tangent to the circle at the point C intersects the other two tangents at Q and R, then the measure of the ∠QPR is :

(a) 45° (b) 60°
(c) 90° (d) 180°

30. Two circles of radii 9 cm and 2 cm respectively have centres X and Y and $\overline{XY}$ = 17cm. Circle of radius r cm with centre Z touches two given circles externally. If ∠XZY = 90°, find r :

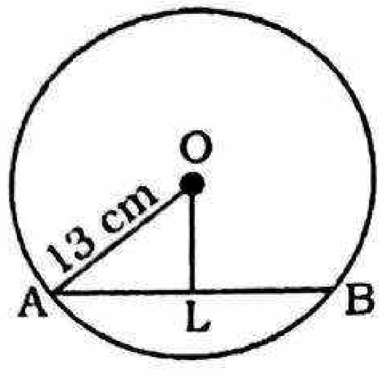
(a) 18 cm (b) 3 cm
(c) 12 cm (d) 6 cm

31. A circle (with centre at O) is touching two intersecting lines AX and BY. The two points of contact A and B subtend an angle of 65° at any point C on the circumference of the circle. If P is the point of intersection of the two lines, then the measure of ∠APO is :

(a) 25° (b) 65°
(c) 90° (d) 40°

SOLUTIONS (LEVEL -I)

1. (a)
2. (b)
3. (d)



The perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ AL = LB = $\frac{1}{2}$AB = 5 cm

$\because$ $OA^2 = OL^2 + AL^2$

$\Rightarrow 13^2 = OL^2 + 5^2$

$\Rightarrow OL^2 = 13^2 - 5^2$

$\Rightarrow$ OL = 12 cm

4.(c) $\angle ODC = \angle BAC = 38°$(Angle made by same arc BC)
and OC = OD = Radius

$\therefore$ $\angle OCD = \angle ODC = 38°$

5.(b) $\because$ OB = OC $\Rightarrow \angle OCB = \angle OBC = 20°$

$\therefore$ $\angle BOC = 180° - (20 + 20) = 140°$

$\therefore$ $\angle BAC = \frac{1}{2}\angle BOC = 70°$

6. (a) $PQ^2 = BQ \times AQ$

$\Rightarrow (12)^2 = AQ \times 8 \Rightarrow AQ = 18$ cm

$\therefore$ AB = AQ – BQ = 18 – 8 = 10 cm

7. (d) $\angle ACB = \angle ADB = 20°$
(made by same arc AB)

$\therefore$ in ΔACB, $\angle x° = 180° - 85° - 20° = 75°$

8.(c) $\angle APB = \frac{1}{2} \times \angle AOB$

$\frac{1}{2} \times \angle AOB = \frac{1}{2} \times 90°$

$= 45°$

9(c) $\angle AOC = 360° - (90° + 110°) = 160°$

$\therefore$ $\angle ABC = \frac{1}{2}\angle AOC = 80°$

10.(b) $\angle ADC$ = Opposite exterior angle = 120°

$\therefore$ $x° = 180° - 120° = 60°$

11.(a) $PT^2 = PA \times PB \Rightarrow 36 = 5(5 + x)$

$\Rightarrow 5 + x = \frac{36}{5} = 7.2 \Rightarrow x = 2.2$ cm

12.(d) PA × PC = PB × PD

$\Rightarrow 14 \times 9 = (7 + x) \times 7$

$\Rightarrow 18 = 7 + x \Rightarrow x = 11$ m

13.(b) $\angle BAC = \frac{1}{2} \times 138° = 69°$

$\therefore$ $\angle BDC = 180° - 69° = 111°$

14.(c) In ΔOBC

OB = OC $\therefore$ $\angle B = \angle C = 45°$

$\therefore$ $\angle D = \angle C$
($\because$ made by same arc AB)

$\therefore$ $\angle D = x° = 45°$

15.(a) $x = 40°$ ($\because$ made by same arc AB)

16.(b) $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ $\angle C + \angle D = 180°$

$\Rightarrow \angle D = 180° - 48° = 132°$

17.(b) $(PT)^2 = PA \times PB \Rightarrow 144 = x \times (7 + x)$

$\Rightarrow x^2 + 7x - 144 = 0$

$\Rightarrow (x + 16)(x - 9) = 0 \Rightarrow x = 9$ or -16

-16 cannot be the length, hence this value is discarded thus, x = 9 cm.

18.(d) PA = PB

$\therefore$ $\angle PAB = \angle PBA$

Also, $\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow \angle PAB + \angle PBA = 120°$

$\therefore \angle PAB = \angle PBA = 60°$

i.e. ΔPAB is an equilateral triangle.

$\therefore AB = 6$ cm

19.(a) A = Area of $\Delta ABC = \frac{1}{2} \times 3 \times 4$

$= 6 \text{ cm}^2$

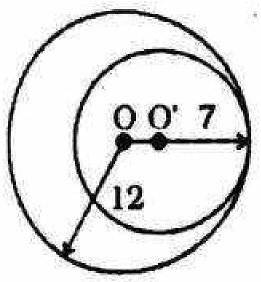
S = Semiperimeter of ΔABC =

$\frac{3+5+4}{2} = 6$ cm

$\therefore$ inradius $= \frac{A}{S} = \frac{6}{6} = 1$ cm

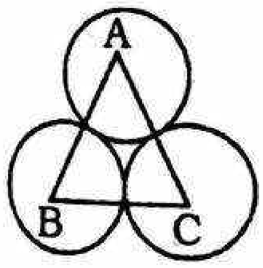
20.(b)

21.(d)



$\therefore OO' = 12 - 7 = 5$ cm

22.(a)



AB = 5 cm = $x + y$

BC = 6 cm = $y + z$

AC = 7 cm = $z + x$

$\therefore 2(x + y + z) = 5 + 6 + 7 = 18$

$\Rightarrow x + y + z = 9$

$\Rightarrow 5 + z = 9 \Rightarrow z = 4$ cm

$\therefore x = 7 - z = 3$ cm and $y = 6 - z = 2$ cm

$\therefore x = 3$ cm, $y = 2$ cm, $z = 4$ cm

23.(c) $\angle AOB = 2\angle ACB = 2 \times 30° = 60°$

24.(b) $AB = AC \Rightarrow \angle ACB = \angle ABC = 50°$

$\therefore \angle BAC = 180° - (50 + 50) = 80°$

$\therefore \angle BDC = \angle BAC = 80°$

(angle by same arc BC)

25.(b) $\angle DOC = \angle AOB = 70°$

$\because$ OD = OC = radius

$\therefore \angle OCD = \angle ODC = \frac{1}{2}(180° - 70°)$

$= 55°$

26.(c) It is necessarily a rectangle.

27.(d)

28.(c) Length of transverse tangent

$= \sqrt{d^2 - (R_1 + R_2)^2}$

here d = 10 cm, $R_1 = R_2 = 3$ cm

$\therefore$ length $= \sqrt{(10)^2 - (6)^2} = 8$ cm

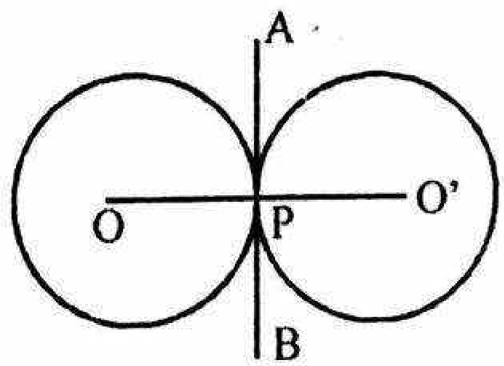
29.(a) CD = 7cm

$\therefore$ AC = 7 cm and BC = 7 cm

($\because$ AC = CD and BC = CD. Two tangents from the same point are always equal)

$\therefore$ AB = 7 + 7 = 14 cm

30.(a) Tangent is always perpendicular to the radius.



31.(b) $\angle BAC = 60°$

$\therefore \angle BEC = 180° - 160° = 120°$

32.(c) $\angle CBA = \frac{1}{2}\angle AOC = 65°$

$\therefore \angle CBE = 180° - 65° = 115°$

33.(a) $\angle BDC = \angle BAC = 30°$

$\therefore \angle BCD + \angle BDC + \angle DBC = 180°$

$\therefore \angle BCD = 180° - (30° + 60°)$

$= 90°$

34.(d) $\angle D = 180° - 72° = 108°$

$\therefore \angle BCD = 180° - 108°$

$= 72°$ ($\because AD || BC$)

35.(b) $\angle OAP = \angle OBP = 90°$

In □AOBP, $\angle O + \angle P = 180°$

$\Rightarrow \angle AOB + \angle APB = 180°$

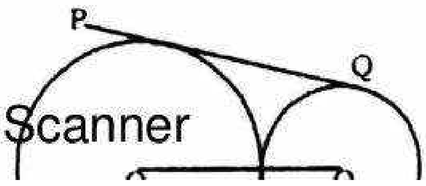
$\therefore \angle APB = \frac{1}{6} \times 180° = 30°$

[$\angle AOB : \angle APB = 5 : 1$]

36.(b) Length of common tangent

$= \sqrt{d^2 - (R-r)^2}$

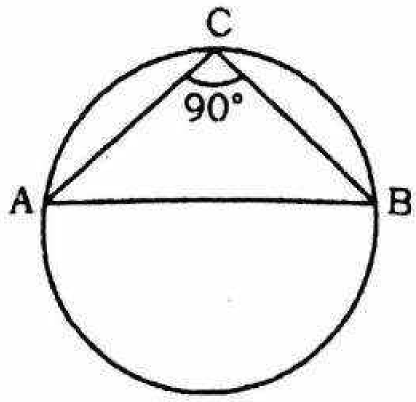
37.(d)



$= \sqrt{10^2 - 6^2}$

$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

41.(a)



AB = diameter of circle.

Angle of a semi-circle is a right angle.

i.e $\angle ACB = 90°$

$\therefore$ ABC is a right angled triangle.

42.(d)

XM = MY

OM = 12 cm

OX = 13 cm

$\therefore XM = \sqrt{OX^2 + CM^2} = \sqrt{13^2 - 12^2}$

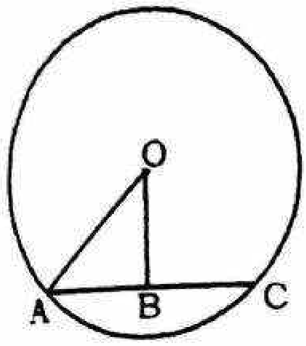
$\therefore$ XY = 2XM = 10 cm

45.(a)

$SR = \sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$

$\sqrt{(13)^2 - (5)^2} = \sqrt{18 \times 8} = 12 \text{ cm}$

46.(b)



AB = BC = 8

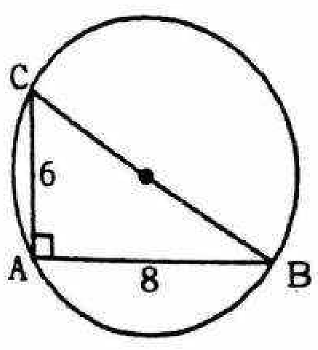
OA = 10

$\therefore OB = \sqrt{OA^2 - AB^2}$

$= \sqrt{10^2 - 8^2} = \sqrt{36} = 6$

47.(b) The largest chord of circle is its diameter.

48.(d)



$\angle BAC = 90°$

$\therefore$ BC is the diameter of the circle.

$\therefore BC = \sqrt{AB^2 - AC^2} = \sqrt{8^2 + 6^2}$

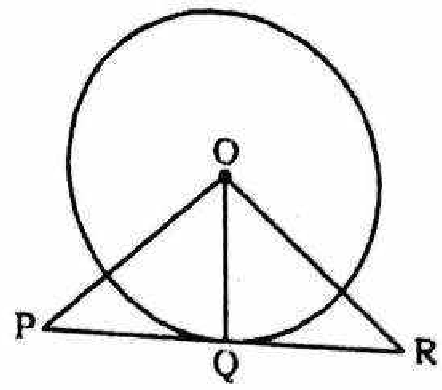
$= \sqrt{64 + 36} = \sqrt{100} = 10 \text{ cm}$

$\therefore$ Radius of the circle = 5cm

49.(c) The chord nearer to the centre is larger.

$\therefore \frac{15}{8} = \frac{x}{16} \Rightarrow x = \frac{15 \times 16}{8} = 30 \text{ cm}$

50.(d)



$OQ \perp PR$

$\therefore$ From Δ OPQ,

$PQ = \sqrt{OP^2 - OQ^2}$

$= \sqrt{\left(\frac{20}{3}\right)^2 - 4^2} = \sqrt{\frac{400}{9} - 16}$

$= \sqrt{\frac{400 - 144}{9}} = \sqrt{\frac{256}{9}} = \frac{16}{3} \text{ cm}$

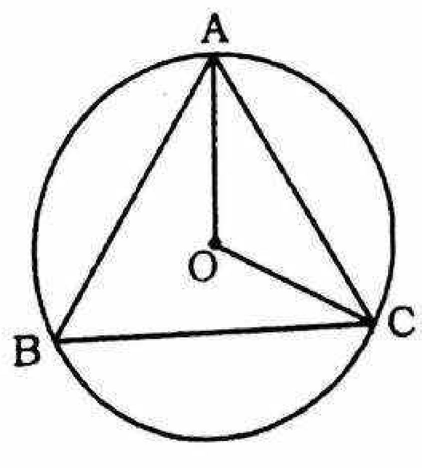
From Δ OQR,

$QR = \sqrt{OR^2 - OQ^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$

$= \sqrt{9} = 3 \text{ cm}$

$\therefore PR = PQ + QR = \frac{16}{3} + 3 = \frac{25}{3} \text{ cm}$

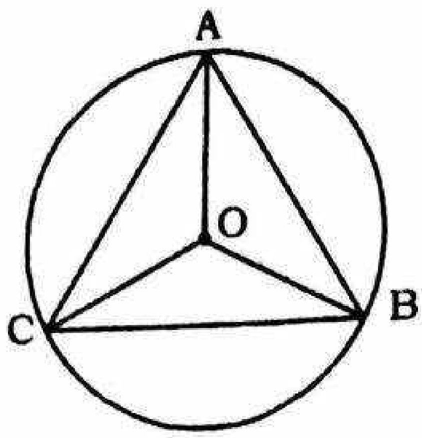
51.(b)



$\angle BAC = 180° - 80° = 15°$

$\Rightarrow \angle AOC = 2\angle ABC = 2 \times 15 = 30°$

52.(a)



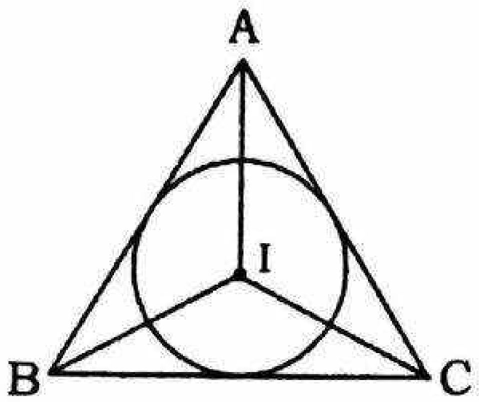
OB = OC = radius

$\therefore$ $\angle OBC = \angle OCB = 35^\circ$

$\angle BOC = 180^\circ - 35^\circ - 35^\circ = 110^\circ$

$\therefore$ $\angle BAC = \frac{1}{2} \times \angle BOC = 55^\circ$

53.(c)



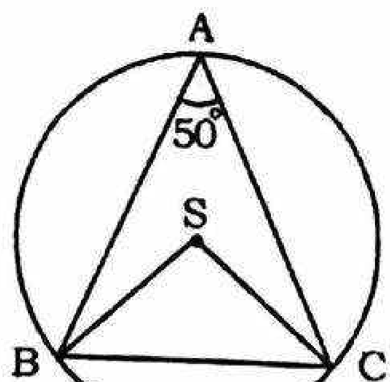
$\angle BIC = 135^\circ$

$\therefore \frac{1}{2}(\angle B + \angle C) = 45^\circ$

$\Rightarrow$ $\angle B + \angle C = 90^\circ$

$\therefore$ $\angle A = 90^\circ$

54.(b)



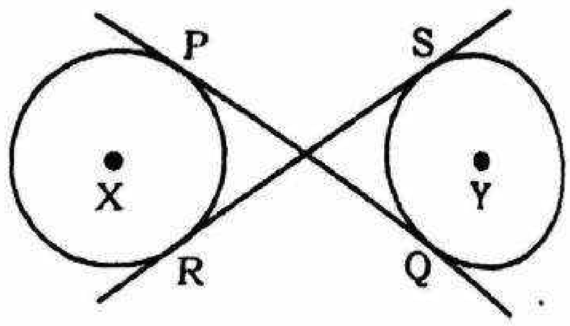
$\angle BAC = 50^\circ$

$\therefore$ $\angle BSC = 100^\circ$

BS = SC = radius

$\therefore$ $\angle BCS = \frac{1}{2}(180 - 100) = 40^\circ$

55.(a)



Transverse common tangent

$= \sqrt{(\text{Distance between centres}) - (r_1 + r_2)^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{16 \times 4} = 8cm$

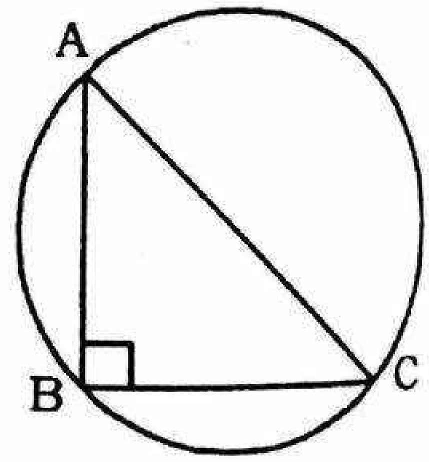
56.(b) One and only circle can pass through three non-collinear points

57.(b) $3^2 + 4^2 = 5^2$

ΔABC is a right angled triangle.

$\angle B = 90^\circ$ = angle at the circumference

$\therefore$ Diameter of circle = 5 cm



$\therefore$ Circum-radius = 2.5 cm

LEVEL - II

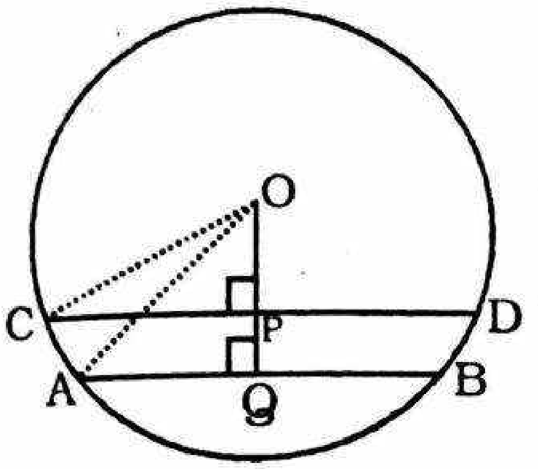
1.(B) in ΔOPC

$OC^2 = OP^2 + CP^2$

$\Rightarrow 5^2 = OP^2 + \left(\frac{8}{2}\right)^2 \Rightarrow OP^2 = 5^2 - 4^2$

$\Rightarrow OP^2 = 9 \Rightarrow OP = 3cm$

in ΔOQA



$OA^2 = OQ^2 + AQ^2$

$\Rightarrow 5^2 = OQ^2 + \left(\frac{6}{2}\right)^2 \Rightarrow OQ^2 = 5^2 - 3^2$

$\Rightarrow OQ = 4cm$

$\therefore$ distance between chords AB and CD=

PQ = OQ - OP = 4-3 = 1cm

2.(c) $\angle LKN = 90^0$ (angle in semicircle)

$\therefore \angle LNK = 180^0 - (90 + 30) = 60^0$

$\therefore \angle PKL = \angle LNK = 60^0$

(angle in alternate sigment)

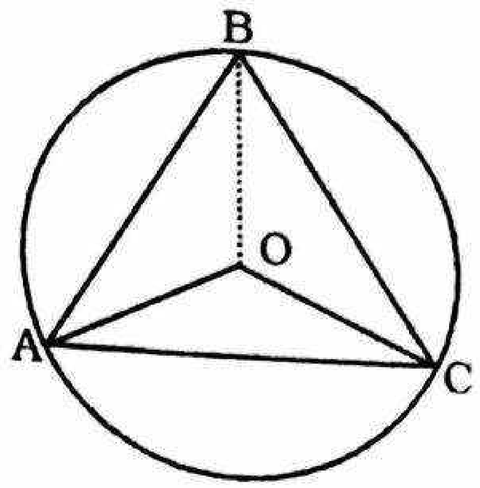
3.(a) $\angle ABC = 180^0 - \angle ADC = 180^0 - 120$
$= 60°$

and $\angle ACB = 90^0$ (angle in semicircle)

$\therefore \angle BAC = 180^0 - (90^0 + 60^0) = 30^0$

4.(d) in ΔAOB,

AO = OB (radius)



$\therefore \angle OBA = \angle OAB = 25^0$

similarly in $\therefore \Delta OBC$,

$\therefore \angle OBC = \angle OCB = 35^0$

$\therefore \angle ABC = 25 + 35 = 60^0$

$\therefore \angle AOC = 2 \times \angle ABC$

$= 2 \times 60^0$

$= 120^0$

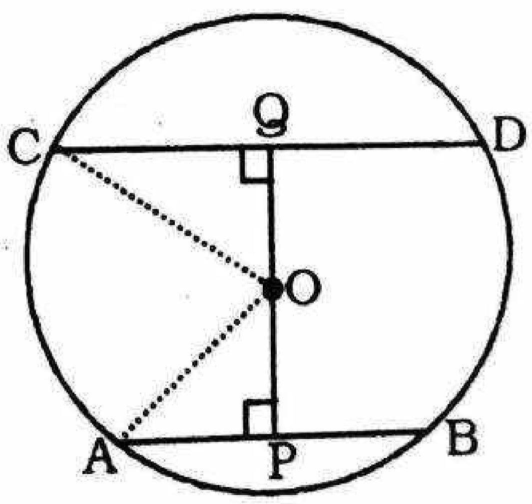
5.(d) Let OP = x cm

$\therefore$ OQ=(17-x) cm and radius =r cm

$\therefore$ in ΔOQC,

$(OC)^2 = (OQ)^2 + (QC)^2 \Rightarrow r^2$

$= (17 - x)^2 + (12)^2$ ....(i)



and in ΔOAP,

$(OA)^2 = (OP)^2 + (AP)^2 \Rightarrow r^2$

$= x^2 + (5)^2$ ....(ii)

$\therefore (17 - x)^2 + (12)^2 = x^2 + 5^2$

$\Rightarrow 289 - 34x + x^2 + 144 = x^2 + 25$

$\Rightarrow 34x = 408 \Rightarrow x = 12cm$

$\therefore$ from (ii) $r^2 = (12)^2 + (5)^2 = (13)^2$

$\Rightarrow r = 13 cm$

6. (C) OA = OB = radius

$\Rightarrow \angle OAB = \angle OBA = 25^0$

$\therefore \angle ACB = 180^0 - 50^0 = 130^0$

$\therefore$ major $\angle AOB = 360^0 - 130^0 = 230^0$

$\Rightarrow \angle AOB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 230 = 115^0$

7.(b) $\angle QOP = 180^0 - 120^0 = 60^0$

and $\angle PQO = 90^0$

$\therefore \angle QPO = 180^0 - (90^0 + 60^0) = 30^0$

8.(c) $\angle ADC = 180^0 - 55 = 125^0$

$\therefore \angle CDT = 180^0 - (125 + 30^0) = 25^0$

9.(a) in cylic □ABCD,

$\angle ADC + \angle ABC = 180^0$

$\Rightarrow \angle ADC = 180^0 - 70^0 = 110^0$

now in ΔADC,

$\Delta ACD = 180^0 - (30^0 + 110^0) = 40^0$

10.(d) $\angle PQO = \angle PRO = 90^0$

($\because$ PQ and PR are tangents)

$\therefore$ in □PQOR

$\angle ROQ = 360^0 - (90^0 + 90^0 + 60^0) = 120^0$

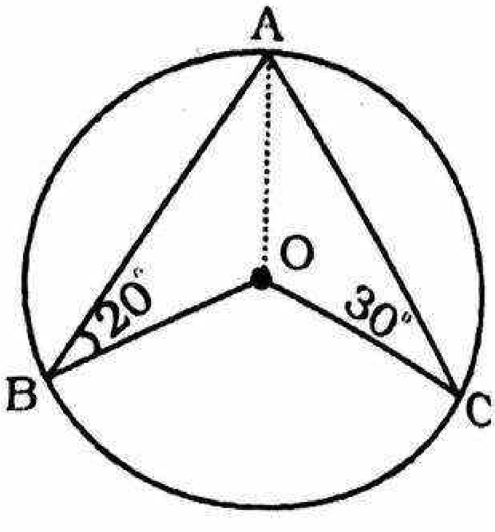
$\therefore \angle QSR = \frac{1}{2} \angle QOR = \frac{1}{2} \times 120^0 = 60^0$

11.(d) in ΔAOB

OA=OB=radius

$\therefore \angle OAB = \angle OBA = 20^0$

similarly in ΔAOC,



$\angle OAC = \angle OCA = 30^0$

$\therefore \angle BAC = 20^0 + 30^0 = 50^0$

$\therefore \angle BOC = 2 \times \angle BAC \Rightarrow \angle x = 100^0$

12.(b) $\angle APB = 90^0$

(angle in a semicircle = 90^0)

$\therefore \angle PBA = 180^0 - (90^0 + 30^0) = 60^0$

$\therefore \angle TPA = \angle PBA = 60^0$

(by alternate segment theorem)

13.(c) $\angle BAC = \angle BDC = 30^0$

($\because$ made by same are BC)

in ΔABC, $\angle x = 180^0 - (100+30^0) = 50^0$

14.(c) It will always be possible to divide a circle into 360 equal parts, because the sum of angle that can be subtended at the centre = 360^0

15.(d) ABCD is a cyclic quadrilateral. There fore

$\angle DCB = 180^0 - \angle A = 180^0 - 60^0 = 120^0$

$\therefore \angle BCQ = 180^0 - 120^0 = 60^0$

$\angle ABC = 80$;

$\therefore \angle CBQ = 180^0 - 80^0 = 100^0$

in ΔBCQ,

$\angle Q = 180^0 - (100 + 60^0) = 20^0$

16.(d) AB=24cm

$\therefore$ AM=MB=12cm

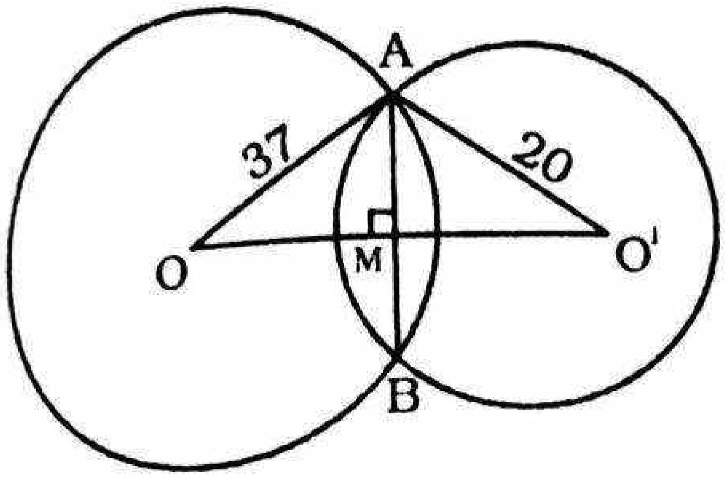
in ΔAMO,

$(OM)^2 = (AO)^2 - (AM)^2$

$(OM)^2=(37)^2-(12)^2$

$\Rightarrow OM = 35cm$

in $\Delta AMO'$;



$(O^1M)^2 = (20)^2 - (12)^2 \Rightarrow O'M = 16$

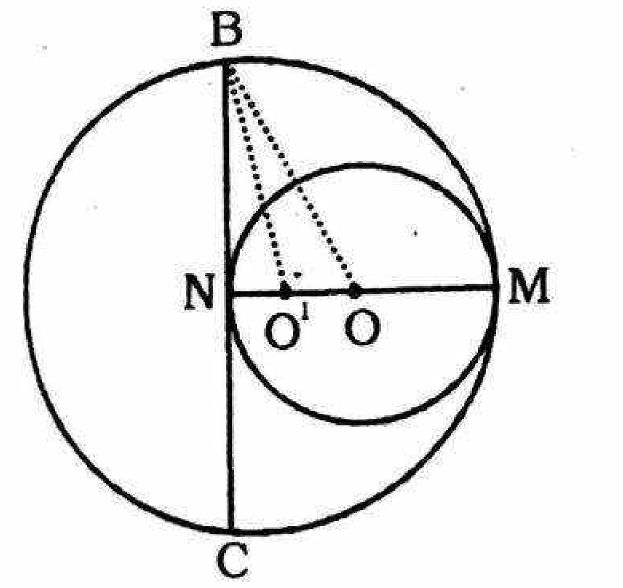
$\therefore OO^1 = OM + O'M = 35 + 16$

$= 51cm$

17.(a) OM = 4cm = radius of smaller circle and O'M = 6cm = radius of bigger circle

$\therefore$ O'N = 6-4 = 2cm

in Δ O'NB,



$(O'B)^2=(O'N)^2+(BN)^2$

$\Rightarrow (BN)^2 = 36- 4 = 32$

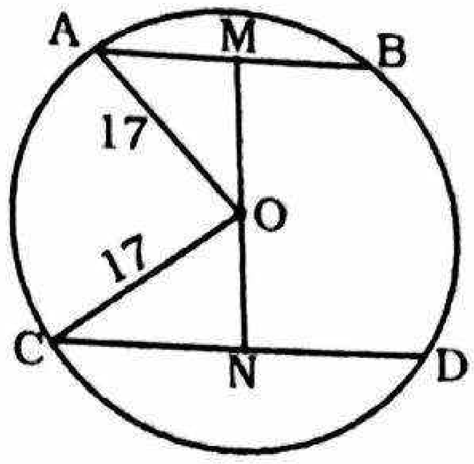
$\Rightarrow BN = 4\sqrt{2}$

$\therefore NC = BN = 4\sqrt{2}$

$\therefore BC = 4\sqrt{2} + 4\sqrt{2} = 8\sqrt{2}cm$

18.(d) MN=23cm

$AM = MB = \frac{16}{2} = 8cm$



$\therefore$ in ΔAMO,

$(OM)^2 = (17)^2 - (8)^2$

$\Rightarrow OM = 15cm$

$\therefore ON = 23 - 15 = 8cm$

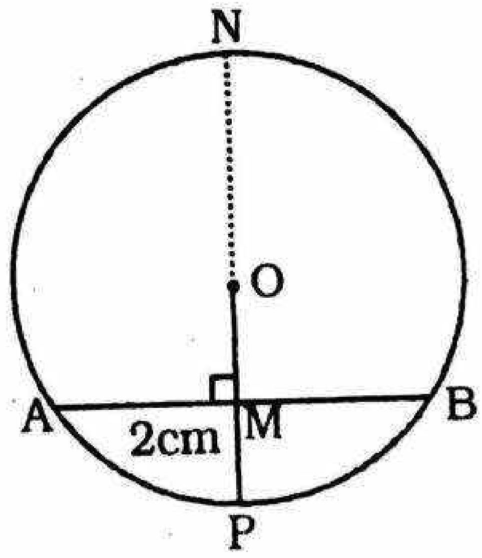
In ΔONC,

$(CN)^2 = (17)^2 - (8)^2 \Rightarrow CN = 15cm$

$\therefore CD = 2CN = 30cm$

19.(b) AB = 8cm

$\therefore$ AM = MB = 4cm



$AM \times MB = PM \times MN$

$\Rightarrow 4 \times 4 = 2 \times (2r - 2)$

$\Rightarrow 4 = r - 1 \Rightarrow r = 5cm$

20.(a) since ABCD is a cyclic quadrilateral

$\therefore \angle ADC + \angle ABC = 180^0$

$\Rightarrow \angle ABC = 180^0 - 130^0 = 50^0$

also, $\angle ACB = 90^0$

$\therefore$ in ΔABC,

$\angle CAB = 180^0 - (90^0 + 50^0) = 40^0$

21.(b) $\angle AOC = 2\angle APC$

$\Rightarrow \angle APC = 50^0$

Also, ABCP is a cyclic quadrilateral

$\therefore \angle ABC + \angle APC = 180^0$

$\Rightarrow \angle ABC = 180^0 - 50^0 = 130^0$

$\therefore \angle CBD = 180^0 - 130^0 = 50^0$

22.(a) $\angle TPQ = \angle PAQ = 50^0$

($\angle$s in the alternate segment)

$\because TP = TQ \Rightarrow \angle TQP = \angle TPQ = 50^0$

$\because \angle PTQ = 180^0 - 50^0 - 50^0 = 80^0$

23.(b) $\angle ACB = 90^0$ [angle at the point of intersection to the centres of the circles.]

BC = r

AC = 2r (as area of x=4 area of y)

$\therefore AB = \sqrt{r^2 + 4r^2} = \sqrt{5}r$

24.(c) $\angle QSR = \angle QTR = \frac{Z}{2}$

$\therefore \angle PSR = \angle PTQ = 180^0 - \frac{Z}{2}$

Also, $\angle SMT = y$

$\therefore$ In quadrilateral PSMT

$180^0 - \frac{z}{2} + 180^0 - \frac{z}{2} + x + y = 360^0$

$\Rightarrow x + y = z$

25.(c) AB = AC and AD = CD

$\therefore$ AB = 2AD

Now, since AD is a tangent

$\therefore AD^2 = AP \times AB$

$\Rightarrow \left(\frac{AB}{2}\right)^2 = AP \times AB \Rightarrow AB = 4AP$

26.(d) $\angle CAB = \angle BCD$

and $\angle DAB = \angle BDC$

(alternate segment theorem)

$\therefore \angle CAD = \angle CAB + \angle DAB$

$= \angle BCD + \angle BDC$

$\therefore \angle CAD + \angle CBD = \angle BCD + \angle BDC + \angle CBD = 180^0$

27.(b) $\angle ABK = 180^0 - (115 + 30) = 35^0$

$\therefore \angle KCD = \angle ABK = 35^0$

($\angle$s made by same arc AD)

28.(a) $\angle AOC = \angle BOC$

($\because$ AC = BC)

$\therefore$ OC is the perpendicular bisector of AB

$\therefore$ AM = BM

IInd -Method

In ΔAOM and ΔBOM

OM = OM (common)

$\angle AOM = \angle BOM$

($\because$ AC = BC)

OA = OB = radius

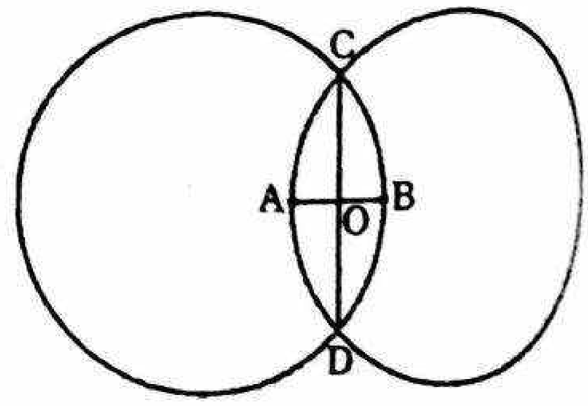
$\therefore \Delta AOM = \Delta BOM$

$\therefore AM \cong BM \Rightarrow AM : BM = 1:1$

29.(B) AB = r (say)

then AC = BC = r, also

$\therefore OA = OB = \frac{r}{2}$



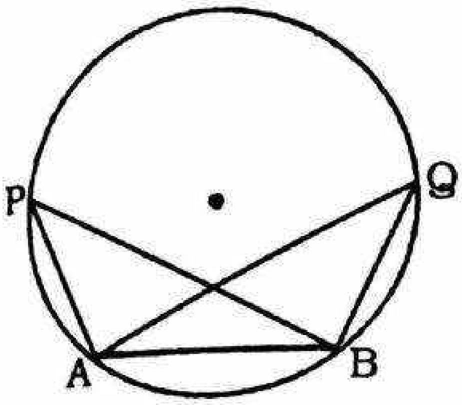
$\therefore OC = \sqrt{(AC)^2 - (OA)^2} = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{3}}{2}r$

$\therefore CD = 2CO = \sqrt{3}r$

$\therefore \frac{CD}{AC} = \frac{\sqrt{3}.r}{r} = \frac{\sqrt{3}}{1}$

30.(c) $\angle APB = \angle AQB$

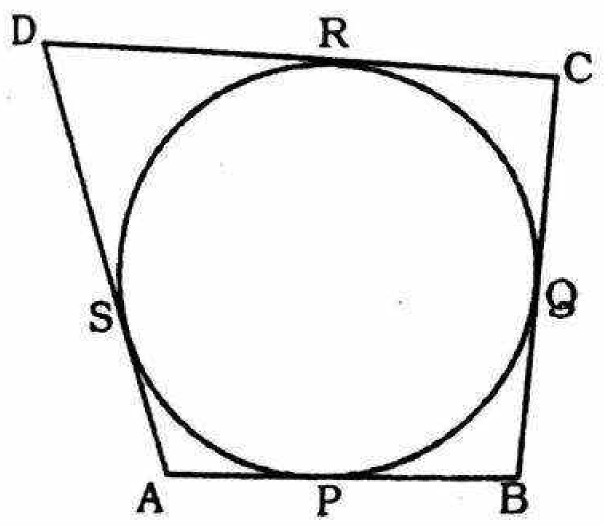
when $\angle APB = \angle AQB = 90^0$



then they are supplementary also they are supplementry, when they are in diffrent segments.

31.(d) AP = AS, BP = BQ, CQ = CR and DR = DS

$\therefore$ AB = AP + BP = AS + BQ



CD = CR + DR = CQ + DS

$\therefore$ AB + CD = (AS + DS) + (BQ + CQ) = BC = AD

32.(a) TQ = TP and TP = TR

$\therefore$ TQ = TP = TR

$\Rightarrow TQ : TR = 1 : 1$

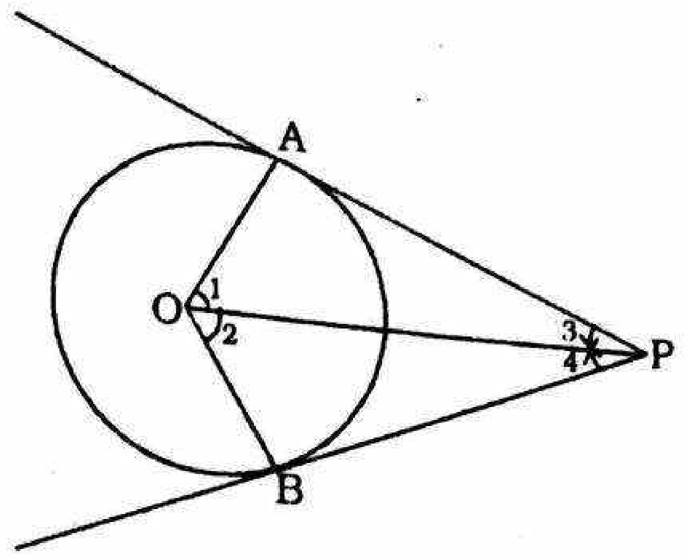
33.(b) $\angle AOC = 2 \times 60^0 = 120^0$

$\angle ABC = \frac{120}{2} = 60^0$

34.(c) $\angle APB = \angle 3 + \angle 4 = 68^0$

in ΔAOP and ΔBOP

PA = PB



OP = OP (common)

and OA=OB= radius

$\therefore AOP \cong \Delta BOP$

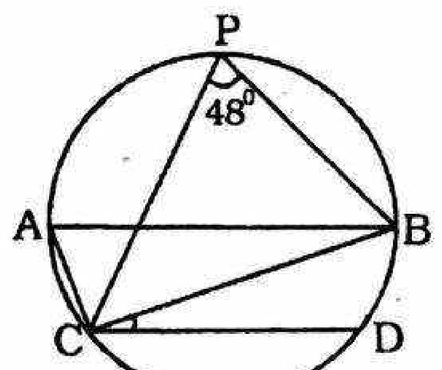
$\therefore \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$\therefore \angle 3 = \frac{68}{2} = 34^0$

$\therefore \angle 1 = \angle POA = 180^0 - (90^0 + 34^0) = 56^0$

35.(b) $\angle BAC = \angle BPC = 48^0$ (by same are BC) and $\angle ACB = 90^0$

($\because$ AB diameter)

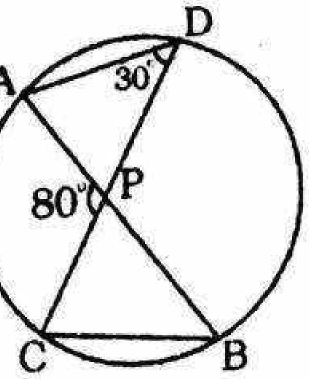


$\therefore \angle ABC = 180^0 - (90^0 + 48^0) = 42^0$

$\therefore \angle BCD = \angle ABC = 42^0$

($\because AB \parallel CD$)

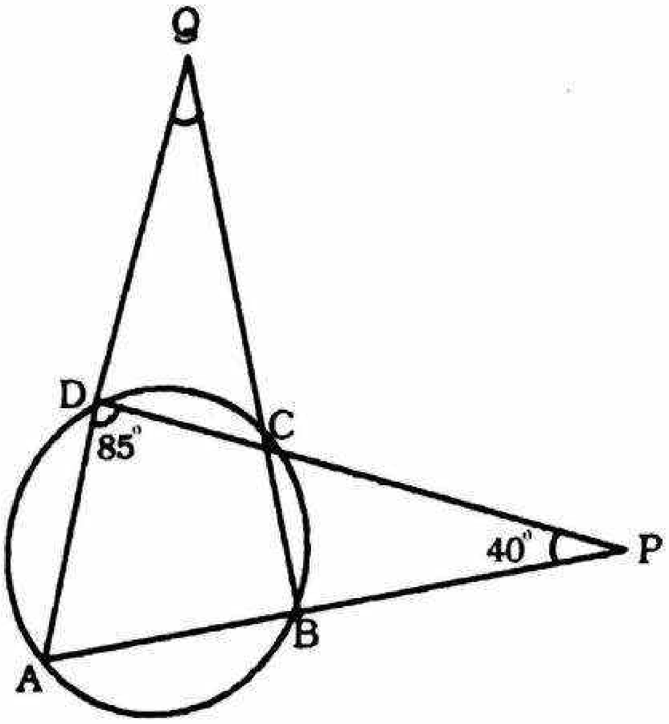
36. (d) $\angle APD = 180^0 - 80^0 = 100^0$



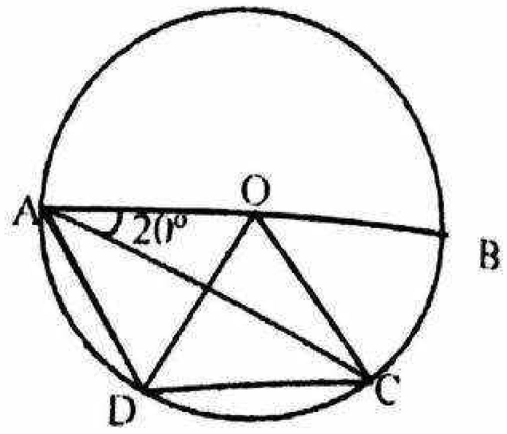
$\therefore \angle PAD = 180^0 - (100^0 + 30^0) = 50^0$

$\therefore \angle BCD = \angle BAD = 50^0$ [$\angle$s by same are BD]

37.(a) $\angle ABC = 180^0 - 85^0 = 95^0$



In ΔADP,

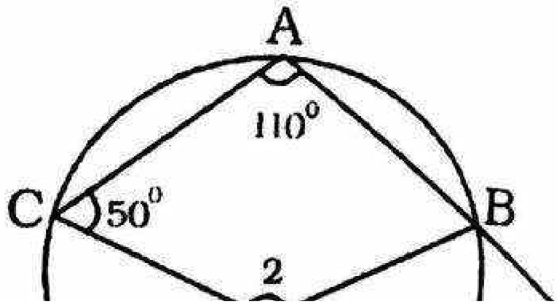


$\therefore \angle DAC = 180^0 - (110^0 - 20^0) = 50^0$

$\therefore \angle COD = 2 \times \angle DAC = 2 \times 50^0 = 100^0$

40. (A) $\angle 1 = 2\angle A = 220^0$

$\therefore \angle 2 = 360^0 - 220^0 = 140^0$



$\angle ACB = 90^0$ (angle in semicircle)

$\therefore \angle BCP = 90^0$

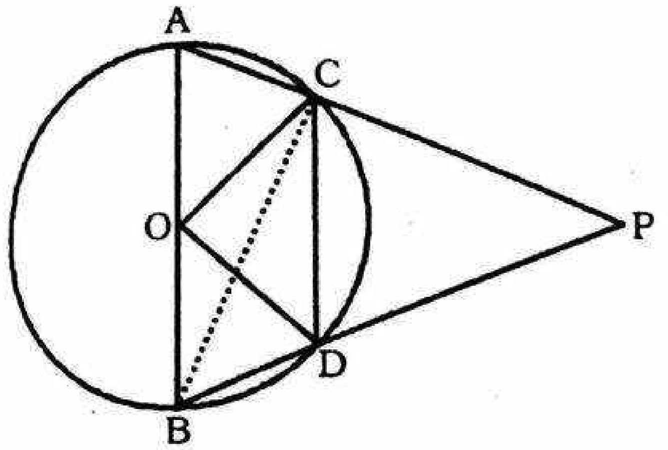
$\angle CBD = \frac{1}{2}\angle COD = 30^0$

(made by same are CD)

$\therefore$ in ΔBCP,

$\angle APB = 180^0 - 90^0 - 30^0 = 60^0$

42. (B) $\angle ACB = 90^0 \Rightarrow \angle BCP = 90^0$

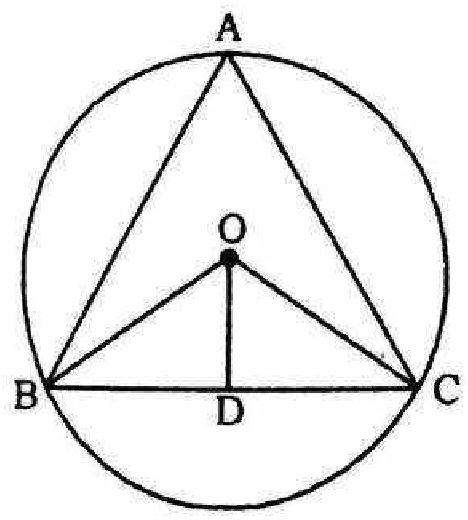


$\angle CBP = \frac{1}{2}\angle COD = 15.5^0$

$\therefore$ in ΔBPC,

$\angle APB = 180^0 - 90^0 - 15.5^0 = 74.5^0$

43. (d)



$BD = \frac{BC}{2} = 12$ cm

OB = 13 cm

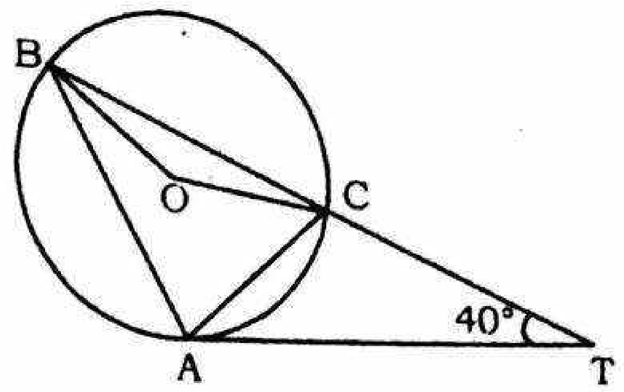
From ΔOBD,

$= OD = \sqrt{OB^2 - BD^2}$

$\sqrt{13^2 - 12^2} = \sqrt{169 - 144}$

$= \sqrt{25} = 5$ cm

44. (d)



$\angle CAT = 44°$

$\angle BTA = 40°$

$\angle ACT = 180° - 44° - 40° = 96°$

$\angle CAT = \angle CBA = 44°$

$\angle BCA = 180° - 96° = 84$

$\therefore$ BAC = 180° - 84° - 44° = 52°

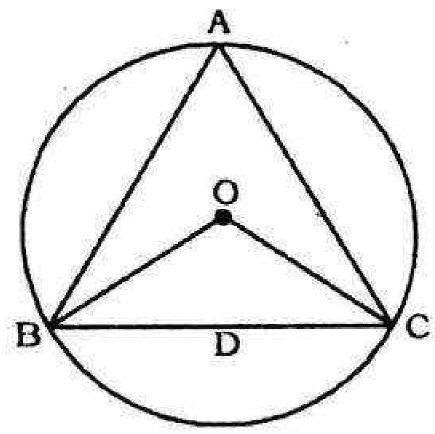
$\therefore$ Angle subtended by BC at centre

$= 2 \times 52° = 104°$

45. (c) $\angle BOC = 2\angle BAC$

OB = OC

$\therefore \angle OBC = \angle OCB$

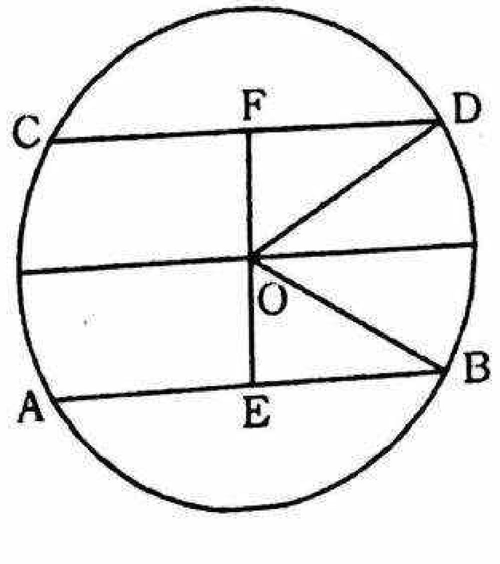


$\therefore \angle OBC = 90° - \frac{\angle BOC}{2}$

$= 90° - \angle BAC$

$\therefore \angle BAC + \angle OBC$
$= 90°$

46. (b)



$OE \perp AB$

$\therefore$ $BE = AE = 3$ cm

and, $OF \perp CD$

$\therefore$ $FD = CF = 4$ cm

From Δ OBE,

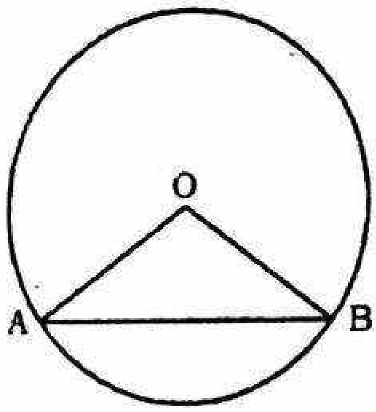
$OF = \sqrt{5^2 - 4^2} = 3$ cm

$\therefore$ $EF = OE + OF$

$= 4 + 3$

$= 7$ cm

47.(a)



From Δ OAB,

$\angle AOB = 90°$

$OA^2 + OB^2 = AB^2$

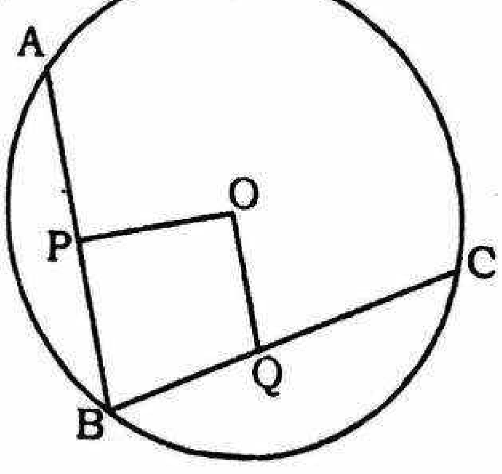
$\Rightarrow 2r^2 = \left(3\sqrt{2}\right)^3 = 18$

$\Rightarrow r^2 = 9 \Rightarrow r = 3$ units

$\therefore$ Area of the sector AOB

$= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times 9 = \frac{9\pi}{4}$ sq.units

48.(b)



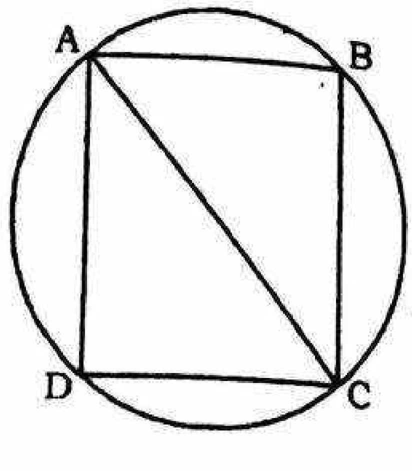
$\angle OPB = \angle PQB = 90°$

$\therefore \angle OPB + \angle OQB = 180°$

and, $\angle PBQ + \angle POQ = 180°$

hence, OQBP must be concylic

49.(b)



$\pi r_2 = 36 \Rightarrow r^2 = \frac{36}{\pi}$

$r = \frac{6}{\sqrt{\pi}} cm$

$\therefore$ AC = Diameter = $\frac{12}{\sqrt{\pi}} cm$

= Diagonal of square

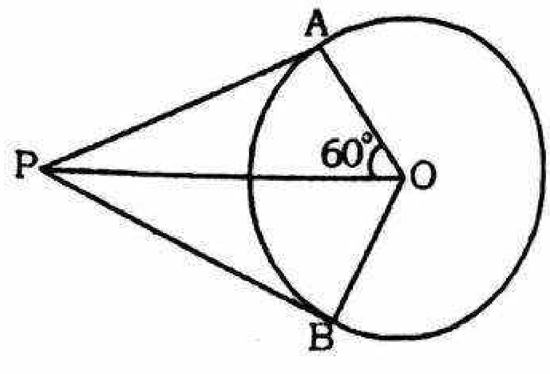
$\therefore$ Side of square = $r = \frac{1}{\sqrt{2}} \times$ Diagonal

$\frac{1}{\sqrt{2}} \times \frac{12}{\sqrt{\pi}} = \frac{6\sqrt{2}}{\sqrt{\pi}}$ cm

$\therefore$ Area of Δ ACD =

$\frac{1}{2} \times \frac{6\sqrt{2}}{\sqrt{\pi}} \times \frac{6\sqrt{2}}{\sqrt{\pi}} = \frac{36}{\pi}$ sq.cm

50.(c)



In right Δs OAP and OPB,

AP = PB, OA = OB = radius

OP = OP

$\therefore$ $\Delta OAP = \Delta OPB$

$\therefore$ $\angle AOP = \angle POB$

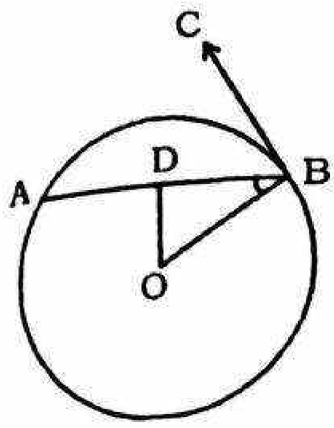
and $\angle APO = \angle OPB$

From Δ AOP,

$\angle APO = 180° - 90° - 60° = 30°$

$\therefore$ $\angle APB = 2 \times 30 = 60°$

51.(c)



$\angle ABC = 45°$

$\Rightarrow \angle ABO = 45°$ ($\because \angle OBC = 90°$)

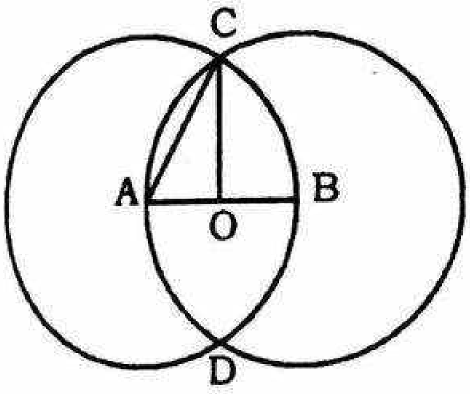
BD = 3 cm

$\therefore$ OBD

$$\cos 45° = \frac{3}{OB} = \frac{1}{\sqrt{2}} = \frac{3}{OB}$$

$\Rightarrow OB = 3\sqrt{2}$ cm

52.(b)



$$AO = OB = \frac{5}{2}$$

AC = 5

$$\therefore = \sqrt{5^2 - \left(\frac{5}{2}\right)^2} = \sqrt{25 - \frac{25}{4}}$$

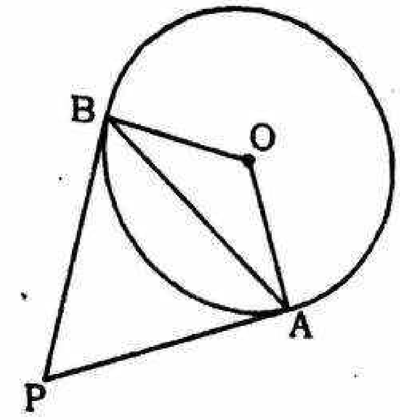
$$= \sqrt{\frac{100-25}{4}} = \sqrt{\frac{75}{4}} = \frac{5\sqrt{3}}{2}$$

$$\therefore CD = 2 \times OC = 2 \times \frac{5\sqrt{3}}{2} = 5\sqrt{3} \text{ cm}$$

53.(d) $OA \perp AP$ and $OB \perp BP$

$\angle OAP = 90°$ and $\angle OBP = 90°$

$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$



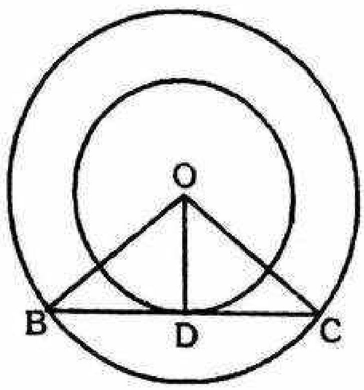
In quadrilateral OAPB,

$\angle OAP + \angle APB + \angle OBP = 360°$

$\Rightarrow \angle APB + \angle OBP = 180°$

$\therefore$ The quadrilateral will be cyclic.

54.(a)



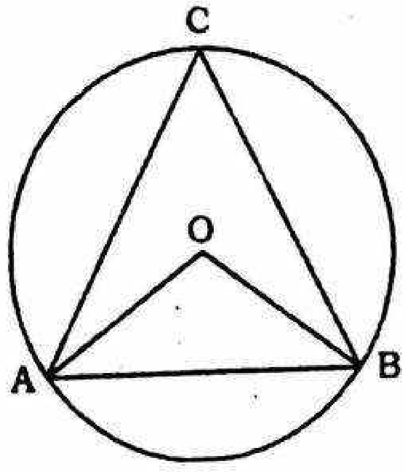
BO = OC = 15 cm.

OD = 9 cm

$\therefore BD = \sqrt{15^2 - 9^2} = \sqrt{24 \times 6} = 12$ cm

$\therefore BC = 2 \times 12 = 24$ cm.

55.(a)



AO = OB = AB

$\therefore \angle AOB = 60°$

$\therefore \angle ACB = 30°$

56.(a) For the equilateral triangle of side

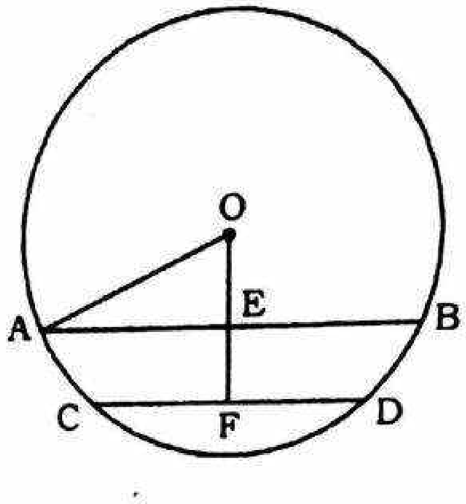
a, In radius $= \dfrac{a}{2\sqrt{3}}$

Circum-radius $= \frac{a}{\sqrt{3}}$

Required ratio

$= \pi\left(\frac{a}{\sqrt{3}}\right)^2 : \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{1}{3} : \frac{1}{12} = 4:1$

57.(a)



Let OE = x cm

$\therefore$ OF = $(x + 1)$cm

OA = OC = r cm

AE = 4 cm, CF = 3 cm

From Δ OAE,

$OA^2 = AE^2 + OE^2$

$\Rightarrow r^2 = 16 + x^2$

$\Rightarrow x^2 = r^2 - 16$

From Δ OCF,

$(x + 1)^2 = r^2 - 9$

By equation (ii) - (i)

$(x + 1)^2 - x^2 = r^2 - 9 - r^2 + 16$

$\Rightarrow 2x + 1 = 7$

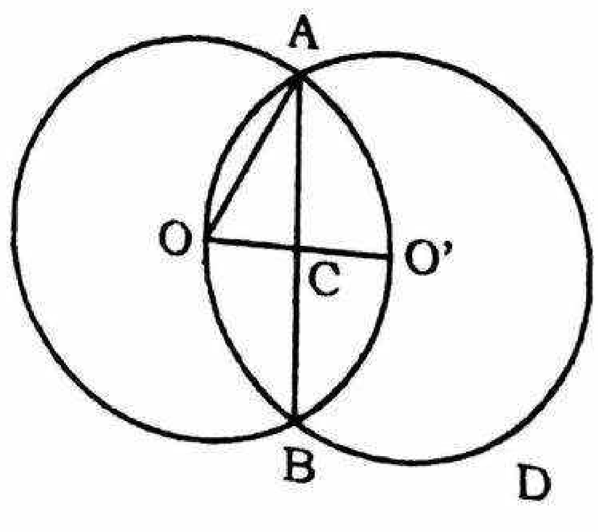
$\Rightarrow x = 3$ cm

$\therefore$ From equation (i),

$9 = r^2 - 16 \Rightarrow r^2 = 25$

$\Rightarrow r = 5$

58.(b)



OC = 2 cm

OA = 4 cm

$\therefore AC = \sqrt{4^2 - 2^2} = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$

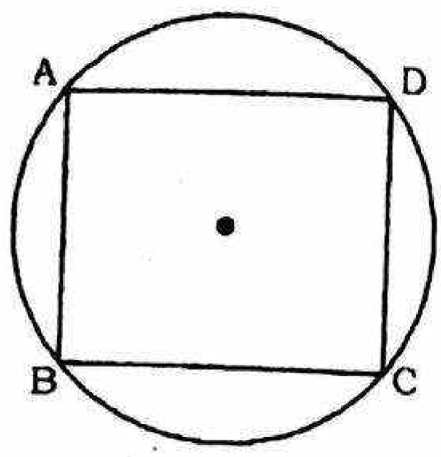
$\therefore AB = 4\sqrt{3}$ cm

59.(d) ABCD is cyclic parallelogram.

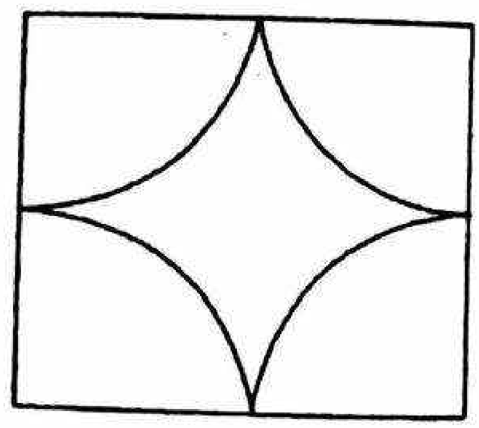
$\therefore \angle B + \angle D = 180°$

$\Rightarrow 2\angle B = 180°$

$\Rightarrow \angle B = 90°$



60.(b)



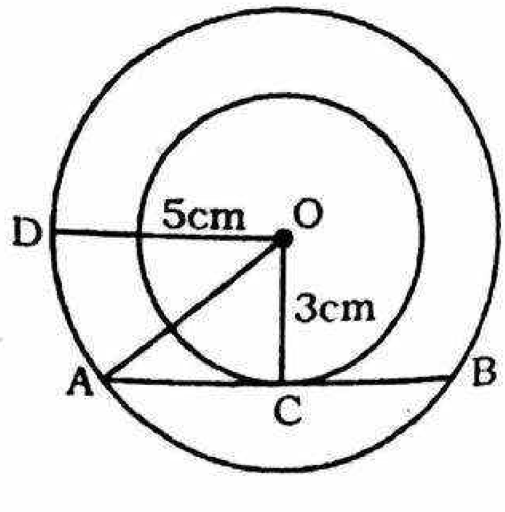
Area of sectors = $\pi r^2 = 4\pi$ sq.cm.

Area of square = $4 \times 4 = 16$ cm.

Area of the remaining portion =

$(16 - 4\pi)$sq.cm.

61.(c)

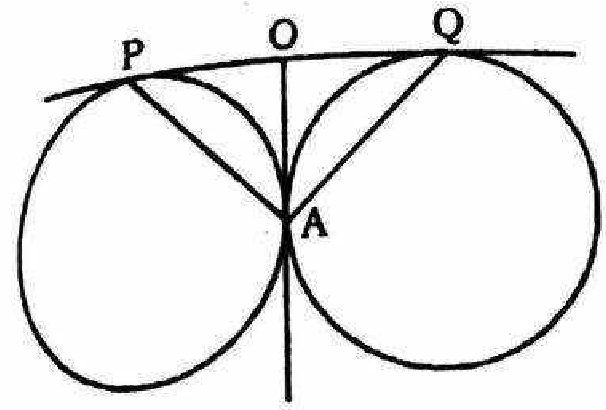


$AC = \sqrt{AC^2 - OC^2} = \sqrt{5^2 - 3^2}$

$= \sqrt{25-9} = \sqrt{16} = 4$ cm

$\therefore AB = 2 \times 4 = 8$ cm

62.(b)



OA = OP

and OA = OQ

$\therefore$ OA = OP = OQ

Let $\angle OPA = \alpha$

$\angle OQA = \beta$

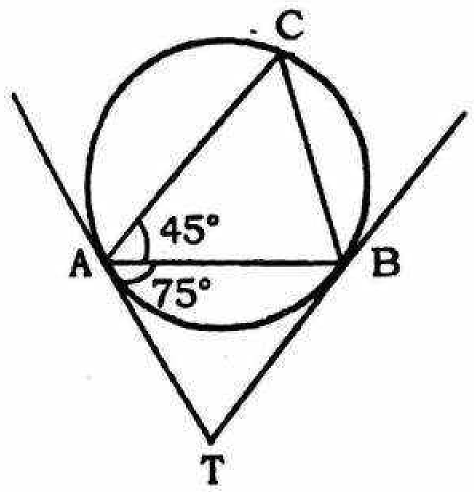
Now in ΔPAQ,

$\beta + \alpha + (\alpha + \beta) = 180°$

$\Rightarrow \alpha + \beta = 90°$

$\therefore$ PAQ = 90°

63.(c)



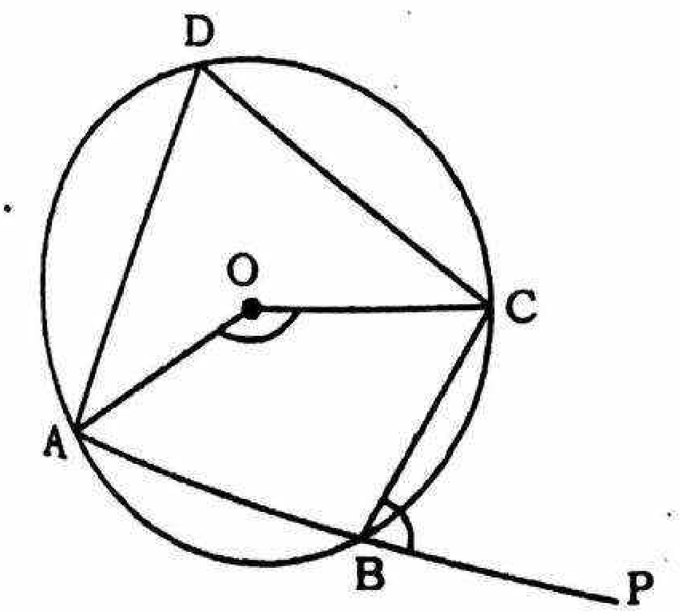
$\angle ACB = \angle BAT = 75°$

(angles in the alternate segment)

In ΔABC,

$\angle ABC = 180° - 45° - 75° = 60°$

64.(c)



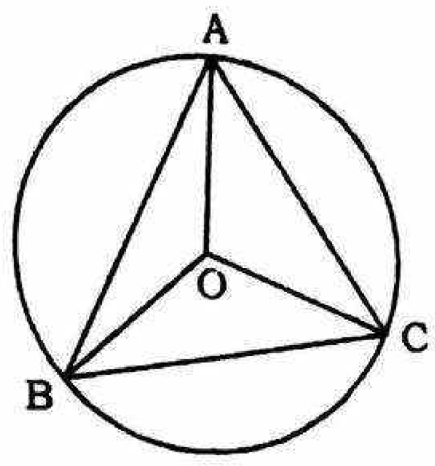
$\angle AOC = 130°$

$\angle ADC = \frac{1}{2} \times 130° = 65°$

$\angle PBC = \angle ADC = 65°$

(exterior angle is equal to the opposite interior angle)

65.(c)



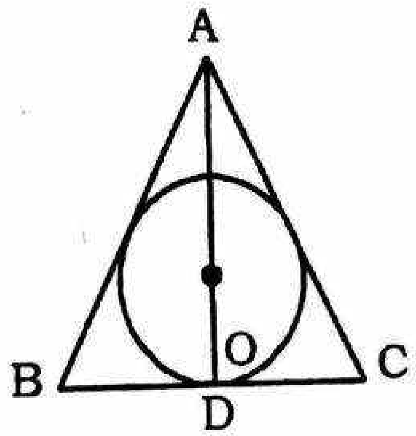
$\angle ABC = 180° - 85° - 75°$

$= 20°$

$\therefore \angle AOC = 2 \times 20°$

$= 40°$

66.(c)



$BD = DC = 7\sqrt{3}$ cm

$AD = \sqrt{AB^2 - BD^2} = \sqrt{(14\sqrt{3})^2 - (7\sqrt{3})^2}$

$= \sqrt{(14\sqrt{3} + 7\sqrt{3})(14\sqrt{3} - 7\sqrt{3})}$

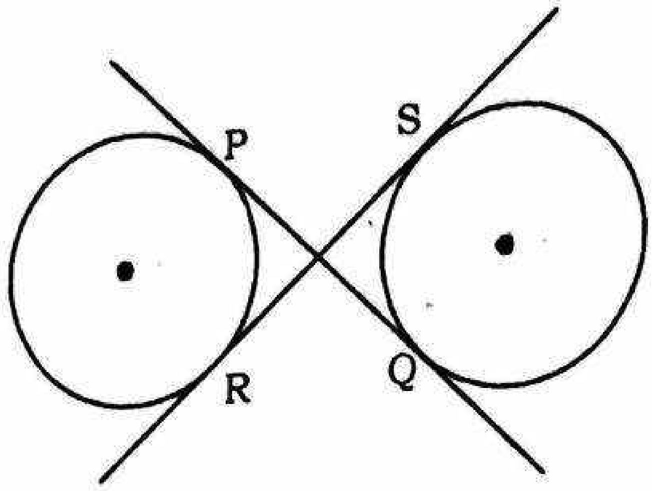
$= \sqrt{21\sqrt{3} \times 7\sqrt{3}} = 21$ cm

$\therefore OD = \frac{1}{3} \times 21 = 7$ cm

$\therefore$ Area of circle $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7 = 154$ sq.cm

67. (c)



Lenth to transverse tangent

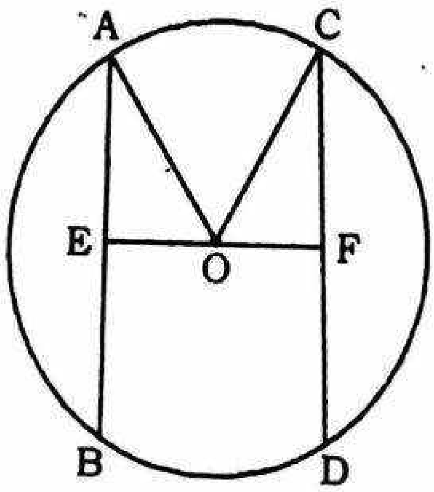
$=\sqrt{XY^2-(r_1+r_2)^2}$

$\Rightarrow 8=\sqrt{XY^2-9^2}$

$\Rightarrow XY^2=64+81=145$

$\Rightarrow XY=\sqrt{145}$

68.(b)



AB = 24 cm

AE = EB = 12 cm

$OE = \sqrt{15^2-12^2}$

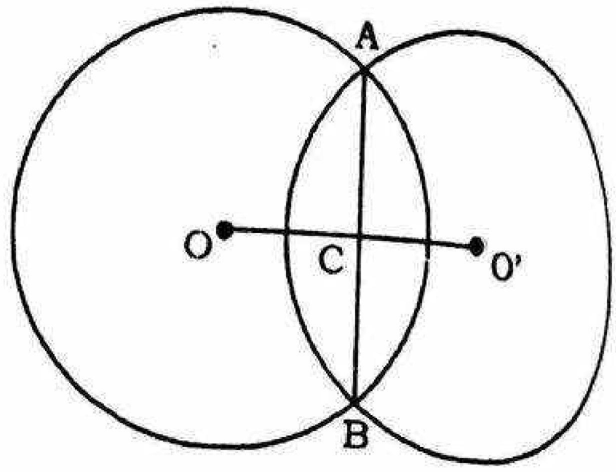
$= \sqrt{225-144}=\sqrt{81}=9$ cm

$\therefore$ OF = 21 - 9 = 12 cm

$\therefore$ $CF = \sqrt{15^2-12^2}=9$ cm

$\therefore CD = 2\times 9 = 18$ cm

69. (a)



AB = 16, AC = BC = 8cm

OC = CO' = 6 cm

$\therefore OA = \sqrt{OC^2+CA^2}$

$= \sqrt{6^2+8^2}=\sqrt{36+64}$

$= \sqrt{100}$ = 10 cm

LEVEL - III

1.(b) $\because$ Tangents drawn from any external point are of same length

$AD = AE$, $BD = BF$ and $CE = CF$

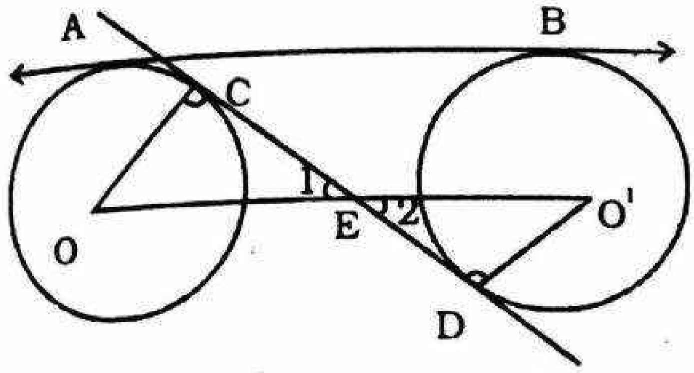
$\therefore$ $AD = AB + BD = AB + BF$

and $AD = AE = AC + CE = AC + CF$

$\therefore$ $2AD = AB + AC + BF + CF = AB + BC + CA$

2.(b)

3.(c)



$OC = O^1D = 5cm$ (radius)

$CD = 24cm$

& $OD=OE=r=$ radius

$BM=1cm$

$\therefore$ $OB=(1-r)cm$

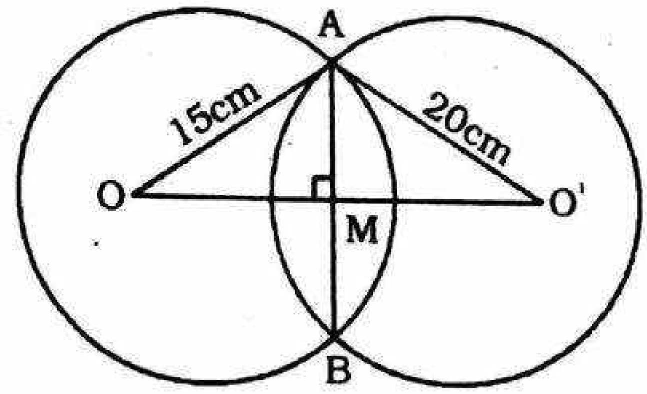
$\because$ ODBE is a square

$\therefore$ $OB=\sqrt{2}r$

$\therefore$ $\sqrt{2}r = 1 - r \Rightarrow r(\sqrt{2}+1) = 1$

$$\Rightarrow r = \frac{1}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = (\sqrt{2}-1)cm$$

5.(d)



$OO^1=25cm$

$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times OA \times BM$

$= \frac{1}{2} \times 5 \times x = \frac{5x}{2}$ cm

$ON \perp AB$

$\therefore \quad AN = BN = \frac{6}{2} = 3cm$

in ΔANO,

$ON = \sqrt{(5)^2 - (3)^2} = 4cm$

$\therefore$ again Area of ΔAOB =

$\frac{1}{2} \times AB \times ON = \frac{1}{2} \times 6 \times 4 = 12$cm

canner

$\therefore \angle NOM = 90^0$

it mean DMON is a ractangle

$\therefore \quad OM = DN = x$ (let)

and $ON = DM = y$ (let)

Let radius = OQ = r cm

in Δ OMQ, $r^2 = x^2 + b^2$ ____(i)

in Δ ONA, $r^2 = y^2 + a^2$ ____(ii)

(i)+(ii) $\quad 2r^2 = a^2 + b^2 + (x^2 + y^2)$

$= a^2 + b^2 + c^2$

$\Rightarrow r = \sqrt{\frac{a^2 + b^2 c^2}{2}}$

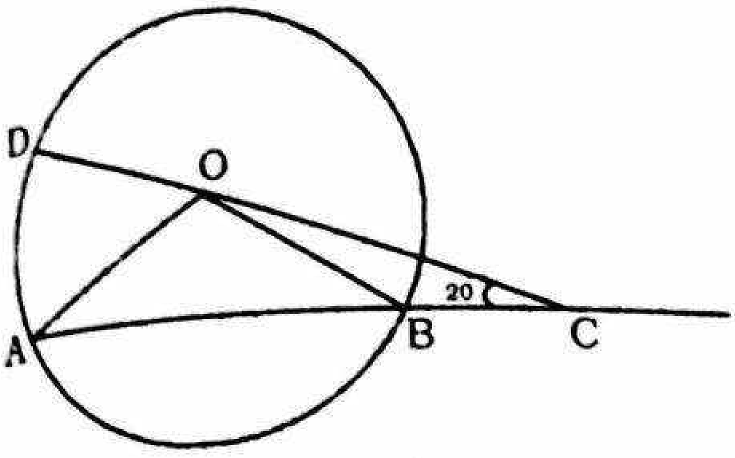
8.(a) $\quad AE \times EB = DE \times CE$

$\Rightarrow CE = \frac{2 \times 6}{3} = 4$cm

Let O is the ...

9.(d) BC = OD (given)

$\therefore$ BC = OD = OB = radius



in ΔBOC, BC = OB

$\therefore \angle BOC = \angle OCB = 20^0$

$\therefore \angle ABO = 20^0 + 20^0 = 40^0$

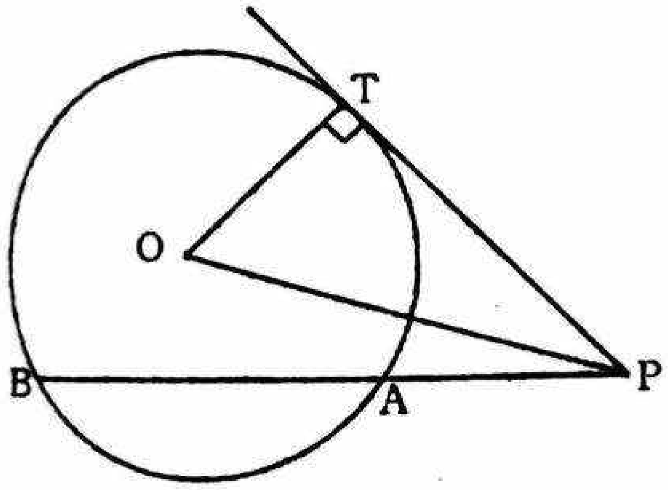
In ΔOAB, AO = OB

$\therefore \angle OAB = \angle ABO = 40^0$

$\therefore \angle AOB = 180^0 - (40^0 + 40^0) = 100^0$

$\therefore \angle AOD = 180^0 - (100^0 + 20^0) = 60^0$

10.(b) Draw a tangent (PT) from P-



$\therefore PT^2 = PA \times PB$

$\Rightarrow PT^2 = 9 \times 16 \Rightarrow PT = 12cm$

in ΔOTP, $\angle T = 90^0$

$\therefore (OT)^2 = (13)^2 - (12)^2 = 25$

$\Rightarrow OT = r = 5cm$

11.(c) in $\square AOBP$ $(\because \angle B = \angle A = 90^0)$

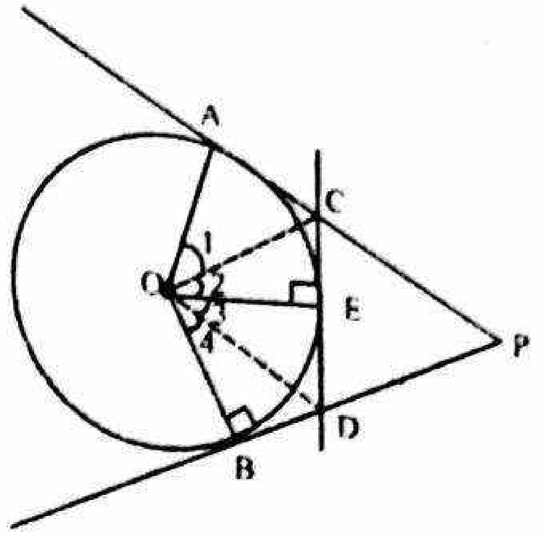
$\therefore \angle AOB = 180^0 - 34 = 146^0$

in ΔOAC and ΔOEC

OC = OC (common)

OA = OE = radius

and CA = CE



$\therefore \Delta OAC \cong \Delta OEC$

$\therefore \angle AOC = \angle COE \Rightarrow \angle 1 = \angle 2$

Similarly $\Delta OBD \cong \Delta OED$

$\therefore \angle 3 = \angle 4$

$\angle AOB = 180° - 34° = 146°$

In ΔAOB,

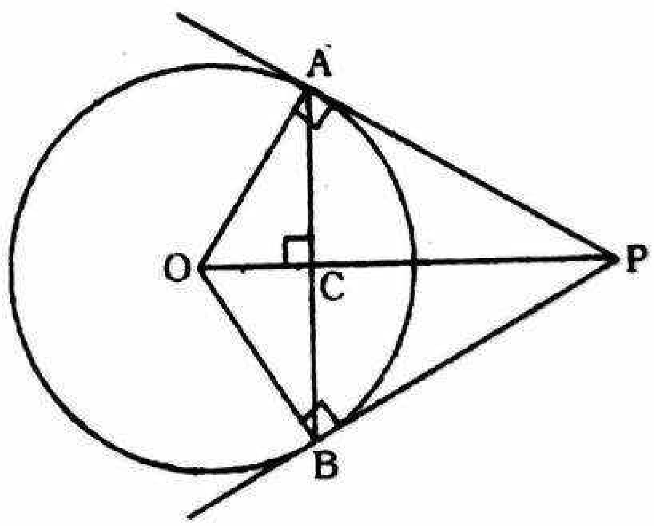
$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 146^0$

$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 = 146^0$

$\Rightarrow \angle 2 + \angle 3 = 73^0$

$\Rightarrow \angle COD = 73^0$

12.(a)



AB = 6cm

$\therefore AC = BC = \frac{6}{2} = 3cm$

in ΔOAP and ΔOCA

$\angle OAP = \angle OCA = 90^0$

$\angle AOP = \angle AOC$

$\therefore \Delta OAP \sim \Delta OCA$

$\therefore \frac{PO}{AO} = \frac{OA}{OC} \Rightarrow OP = \frac{OA^2}{OC} = \frac{(5)^2}{4}$

$\Rightarrow OP = \frac{25}{4}$ cm

13.(b) $BC = 2(OB) = \sqrt{a^2+4^2}$

$= \sqrt{a^2+16}$

$(\because \angle A = 90^0)$

$\because \Delta ABD \sim \Delta CBA$

$\therefore \frac{BD}{AB} = \frac{AB}{BC} \Rightarrow BD.BC = a^2$

$\Rightarrow BD = \frac{a^2}{BC} = \frac{a^2}{\sqrt{a^2+16}}$

$\therefore OD = OB - BD = \frac{\sqrt{a^2+16}}{2} - \frac{a^2}{\sqrt{a^2+16}} = \frac{16-a^2}{2\sqrt{a^2+16}}$

14.(d) $\Delta PQS \cong \Delta PMN \cong \Delta PRT$

$\therefore$ N is the mid-point of ST

Also in ΔPQS, $PS^2 = (24)^2 + (7)^2$
$= 625$

$\Rightarrow$ PS = 25cm As $\Delta PQS \cong \Delta PRT$

$\Rightarrow \frac{QS}{RT} = \frac{PQ}{PR} = \frac{PS}{PT} = \frac{7}{21} = \frac{1}{3}$

$\therefore PR = 3 \times PQ = 72cm$

and $PT = 3 \times PS = 75cm$

$\therefore$ ST = PT– PS = 50cm

$\therefore$ SN = 25cm

15.(a) If a pair of sides of a cyclic quadrilateral are parallel, it become an isosceles trapezium.

Here, a + c = b + d = 180⁰ (cyclic quadrilateral) a = b and c = d (Isosceles trapezium)

$\therefore$ a + b - c - d = (a + b + c + d) - 2(c + d)
= 360⁰ - 4c ($\because$ c=d)

Since, no angle of the quadrilateral ABCD is reflex i,e, $\rangle$ 180⁰

$\therefore$ C can take any value 1 to 179

$\therefore$ a + b - c -d cm take one value for each value of C, i,e,179 values.

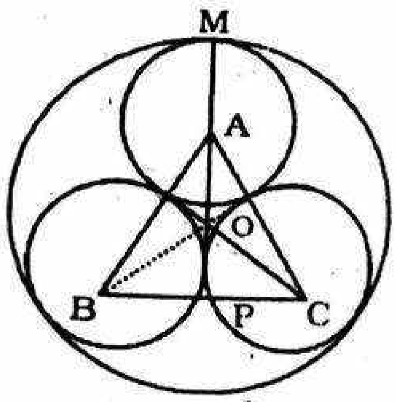
16.(c) AB = BC = AC = 2cm
($\because$ radius of ach circle=1cm)

$\therefore AP = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$cm

Let O is the centriod, then

$OA = \frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}}$ cm

$\therefore OM = OA + AM = \frac{2}{\sqrt{3}} + 1 = \frac{2+\sqrt{3}}{\sqrt{3}}$ cm



OM is the radius of the larger circle.

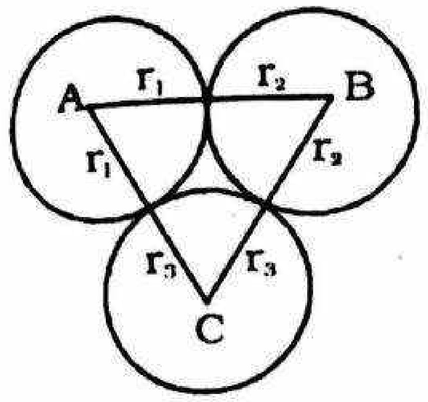
$\therefore$ Area of the circumscribing circle= πR^2

$= \pi \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)^2$

$= \frac{\pi}{3}(2+\sqrt{3})^2$

17.(a) $\therefore r_1 + r_2 = 4$

$r_2 + r_3 = 3.4$ $r_1 + r_3 = 2.2$



$\Rightarrow 2(r_1 + r_2 + r_3) = 4 + 3.4 + 2.2 = 9.6$

$\Rightarrow r_1 + r_2 + r_3 = 4.8$

$\therefore r_1 = 1.4cm,\ r_2 = 2.6cm,$ and

$r_3 = 0.8cm,$

$\therefore$ diameters

$d_1 = 2r_1 = 2.8cm$

$d_2 = 2r_2 = 5.2cm$

$d_3 = 2r_3 = 1.6cm$

$\therefore$ d_2 is bigger circle & d_2 =5.2cm

18.(c) $\angle$CBY= $\angle$ CZY=42⁰

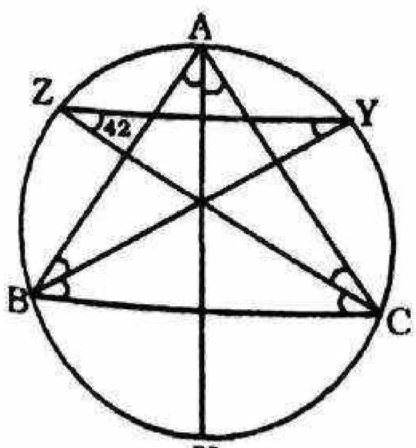
($\angle$S by same arc YC)

$\therefore$ $\angle$ABC = 42 + 42 = 84

$\therefore$ $\angle$C = 180⁰ - (84 + 50⁰) = 46⁰

$\therefore$ $\angle BCZ = \frac{\angle C}{2} = 23^0$

$\therefore$ $\angle$BYZ = $\angle$BCZ = 23⁰ ($\angle$S by same are BZ)



19.(b) $\angle x = \angle CBD = 33^0$ (by alternate segment theorem)

$\therefore \angle Z = 180^0 - \angle x = 147^0$ ($\because$ BPDE is cyclic quadrilateral) and $\angle$BDE = $\angle$BED= $x = 33^0$

{$\because$ BE=BD}

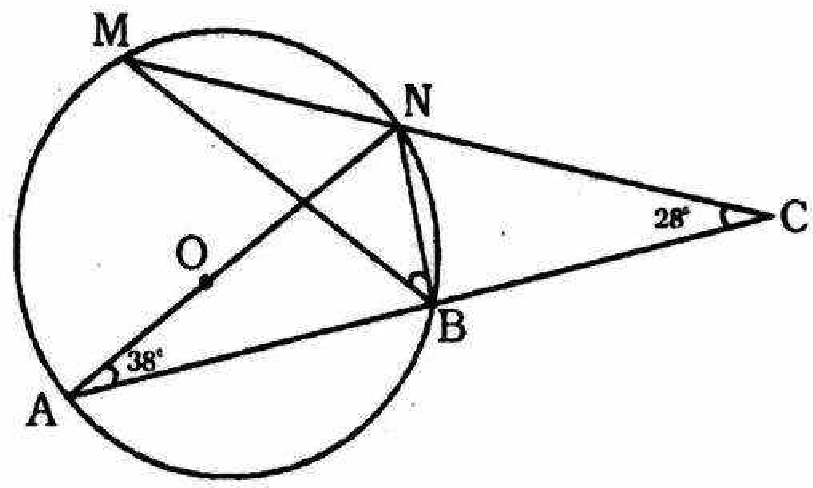
$\therefore$ $\angle$ABE= $\angle$BDE= 33⁰

{by alternate segment theorem}

$\therefore$ $\angle$y=90⁰-33⁰=57⁰

$\therefore x + y + z = 33 + 57 + 147 = 237^0$

20.(d)



$\angle ABN = 90^0$ { angle in semicircle}

$\therefore \angle CBN = 90^0$

$\therefore$ in ΔCBN,

$\angle BNC = 180^0 - 90^0 - 20^0 = 70^0$

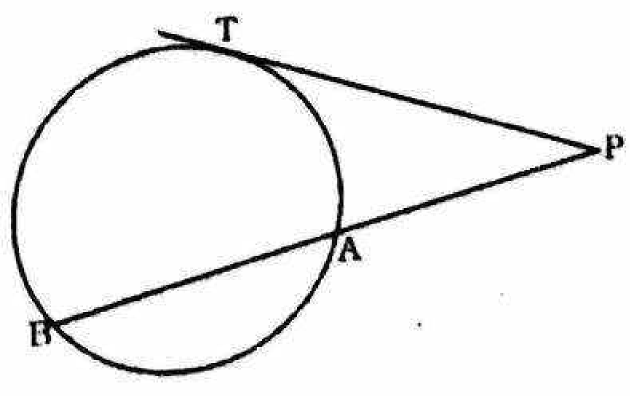
$\therefore$ ĐMNB = 180° – 70° = 110°

and $\angle BMN = \angle BAN = 38^0$

$\therefore$ in ΔMBN,

$\angle MBN = 180^0 - 38^0 - 110^0 = 32^0$

21.(a)



$(PT)^2 = PA \times PB$

$\Rightarrow (2AP)^2 = PA \times PB$

$\Rightarrow 4AP^2 = AP \times BP \Rightarrow 4AP = BP$

$\Rightarrow 4AP = (18 + AP) \Rightarrow 3AP = 18$

$\Rightarrow AP = 6cm$

$\therefore PT = 2AP = 12cm$

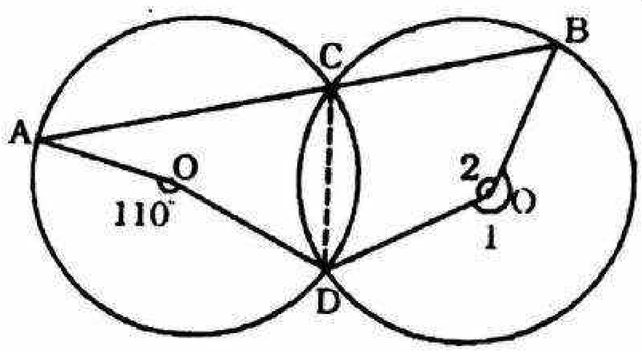
22.(b) $\angle ACD = \frac{1}{2}\angle AOD = 55^0$

$\therefore \angle BCD = 180^0 - 55^0$

$= 125^0$

$\therefore \angle 1 = 2\angle BCD = 250^0$

$\therefore \angle 2 = \angle BO'D = 360^0 - 250^0 = 110^0$



23.(c) $AB = 27cm$

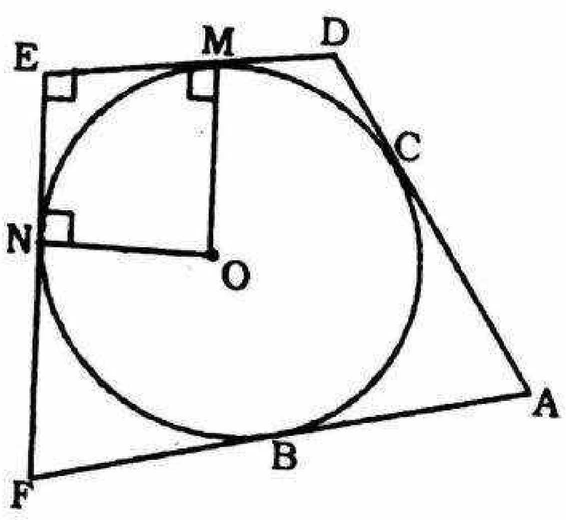
$\therefore AC = 27cm$

$\therefore CD = 38 - 27 = 11cm$

$\therefore DM = CD = 11cm$

$\therefore EM = 24 - 11 = 13cm$

$\therefore EN = EM = 13cm$



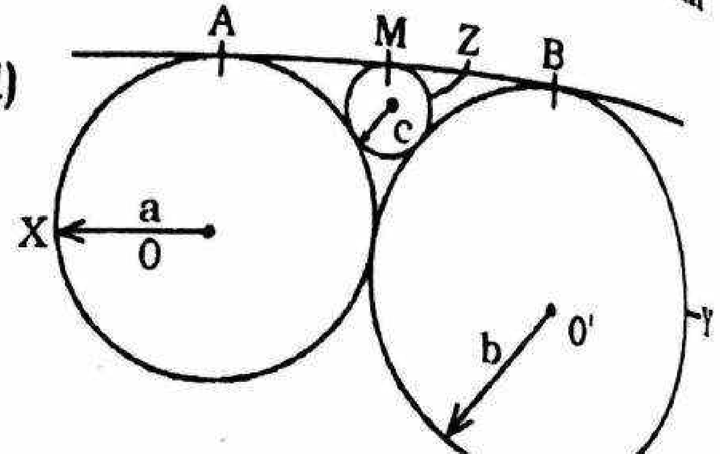
$OM \perp ED$ and $ON \perp EF$

$\therefore ONEM$ is a square

$(\because EM = EN)$

$\therefore OM = ON =$ radius = 13cm

24.(d)



AB is a common tangent of X and Y

$\therefore AB = 2\sqrt{ab}$

Similarly $AM = 2\sqrt{ac}$ and

$BM = 2\sqrt{bc}$

$\therefore 2\sqrt{ab} = 2\sqrt{ac} + \sqrt{bc}$

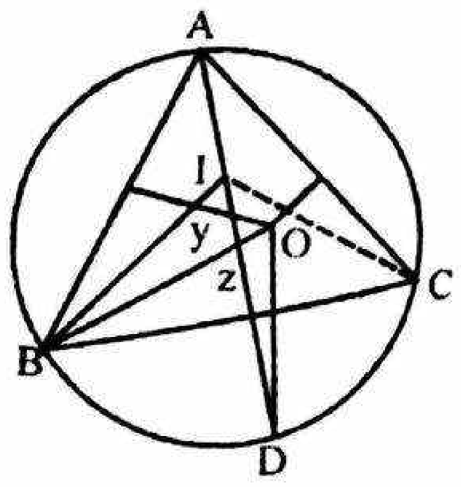
$[\because AB = AM + BM]$

$\Rightarrow \sqrt{ab} = \sqrt{ac} + \sqrt{bc}$

on diciding both sides by $\sqrt{abc}$

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{a}} \Rightarrow \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$$

25.(a)



$\because$ ÐBOD = Z

$\therefore$ ĐABD $= \frac{z}{2}$

[$\because$ angle made by are ar at the circumferences is half of the angle made by the arc at the centre.

ĐABI = ĐIBC = $\frac{x}{2}$ [$\because$ I is the incentre]

Now, in DABI,

ĐBID = y (exterior angle)

$\therefore$ y = ĐBAI + ĐABI = $\frac{z}{2} + \frac{x}{2} = \frac{x+z}{2}$

$\Rightarrow x + z = 2y$

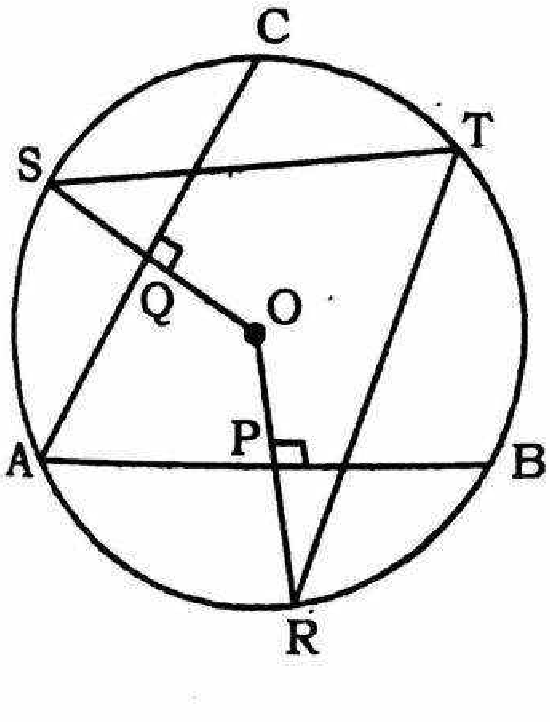
$\therefore \frac{x+z}{3y} = \frac{2y}{3y} = \frac{2}{3}$

In ΔPBC,

$\angle PBD = \angle BCP = \angle CBP$

$= 20° + 25° = 45°$

26.(b)



$\angle OQA = \angle OPA = 90°$

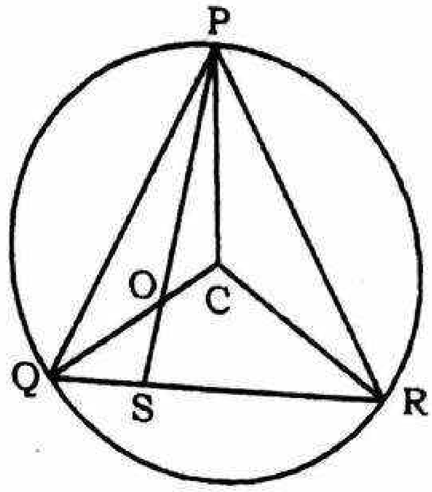
$\angle QOP + \angle QAP = 180°$

$\Rightarrow \angle QOP = 180° - 32° = 148°$

$\angle QOP = \angle SOR = 2\angle STR$

$\therefore \angle RTS = \frac{148}{2} = 74°$

27.(b)



$\angle PQS = 60°$

$\angle QCR = 130°$

$\therefore \angle QPR = \frac{1}{2} \times 130° = 65°$

$\Rightarrow \angle QRP = 180° - 60° - 65° = 55°$

$\therefore$ In $\Delta QCR = \Delta CRQ = 25°$

$\therefore \angle PQC = \angle QPC = 35°$

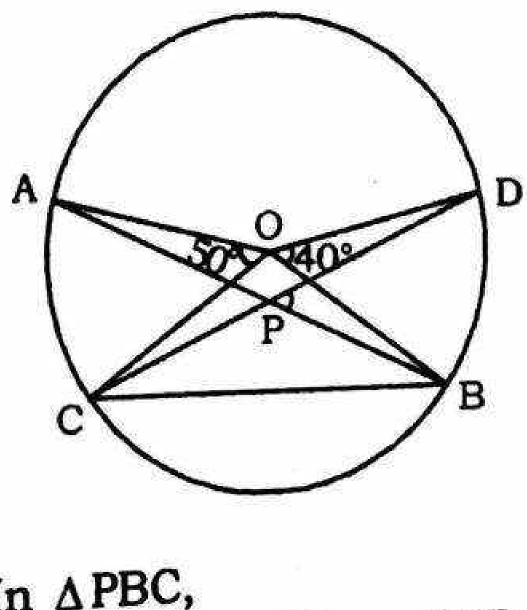
$\angle CPR = 30°$

$\therefore \angle RPS = 35°$

28.(c) Join BC

$\therefore \angle BCP = \frac{1}{2}\angle BOD = 20°$

and $\angle CBP = \frac{1}{2}\angle AOC = 25°$

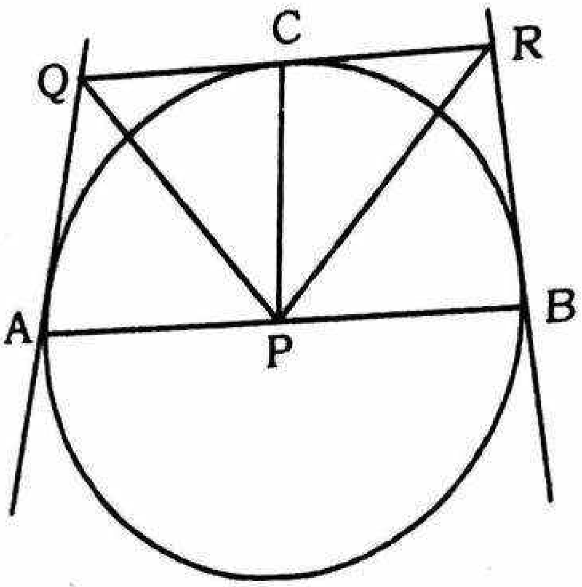


In ΔPBC,

$\angle PBD = \angle BCP = \angle CBP$

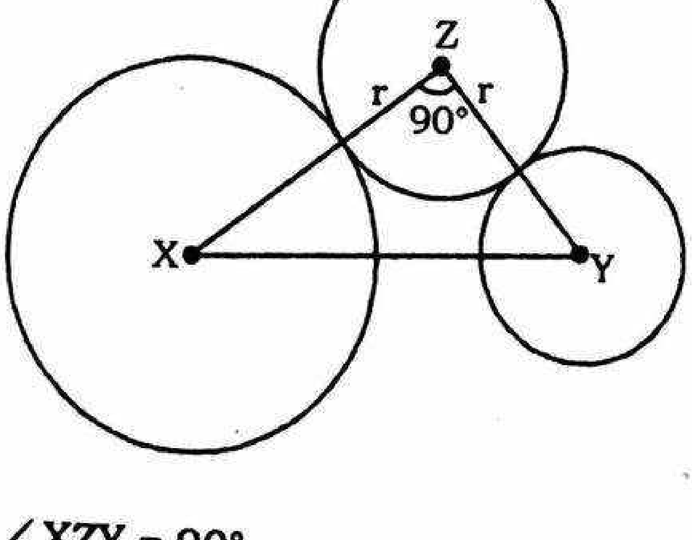
$= 20° + 25° = 45°$

29.(c)



In ΔPCR and ΔRBP,
PC = PB (radii)
RC = RB
PR is common.
$\therefore$ $\Delta PCR \cong \Delta RPB$
Similarly,
$\angle CPQ \cong \angle QPA$
$\therefore \angle QPR = 90°$
Because $\angle APB = 180°$

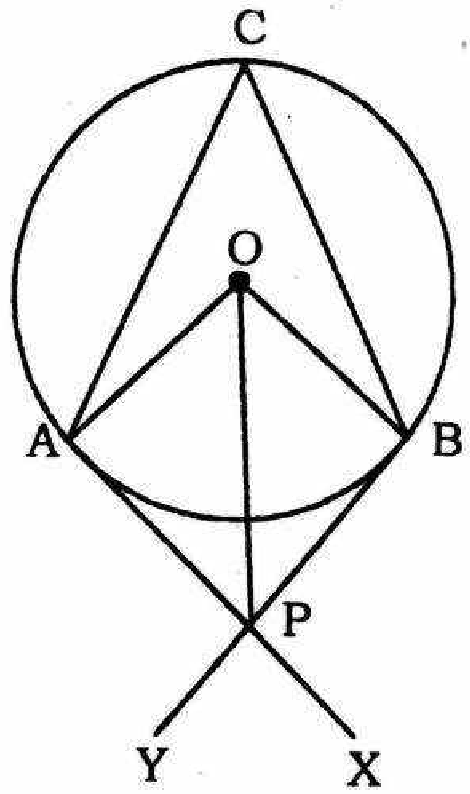
30.(c)



$\angle XZY = 90°$
XY = (9 + r)cm,
YZ = (r + 2) cm
XY = 17 cm
$\therefore XY^2 = XZ^2 + ZY^2$

$\Rightarrow 17^2 = (9 + r)^2 + (r + 2)^2$
$\Rightarrow 289 = 81 + 18r + r^2 + r^2 + 4r + 4$
$\Rightarrow 2r^2 + 22r - 204 = 0$
$\Rightarrow r^2 + 11r - 102 = 0$
$\Rightarrow r^2 + 17r - 6r - 102 = 0$
$\Rightarrow r(r + 17) (r + 17) = 0$
$\Rightarrow (r-6) (r + 17) = 0$
$\Rightarrow r = 6$ cm

31.(b) 330



$\angle ACB = 65°$
$\angle AOB = 2 \times 65° = 130°$
$\angle OAP = 90°$, $\angle AOP = 65°$
$\angle APO = 180° - 90° - 65° = 25°$