Quadratic Equations

An equation of the form $ax^2 + bx + c = 0$, where a,b,c are real numbers and $a \neq 0$, is called a quadratic equation in x.

Solution or Roots of Quadratic Equation

A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$

Application

1. Speed = $\frac{\text{Distance}}{\text{Time}}$
2. Area of figures
3. Flow rate × time = volume of water
4. Number or ages

e.g. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol. Shorter side = x cm, longer side = (x + 5) cm.
hypotenuse = 25 cm
by pythagoras Theorem
$x^2 + (x + 5)^2 = 25^2$
$x^2 + 5x - 300 = 0$
$(x + 20)(x - 15) = 0$
This gives x = 15 or x = - 20.
We reject x = - 20 and take x = 15.
Thus, length of shorter side = 15 cm.
Length of longer side = (15 + 5) cm, i.e., 20 cm.

Method of finding solution

Factorisation method

In this method $(ax^2 + bx + c)$ be expressible as the product of two linear expression, say $(px + q)$ and $(rx + s)$, where p, q, r are real numbers such that $p \neq 0$ and $r \neq 0$

Then $ax^2 + bx + c = 0 \Rightarrow (px + q)(rx + s) = 0$

$\Rightarrow (px + q) = 0$ or $(rx + s) = 0$

$\Rightarrow x = -\frac{q}{p}$ or $x = -\frac{s}{r}$

Completing the square method

$ax^2 + bx + c = 0, a \neq 0.$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS

Nature of roots

$ax^2 + bx + c = 0$, where $a \neq 0$
$D = (b^2 - 4ac)$. and the roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

Case - I
When D > 0, roots are real distinct and given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

Case - II
When D = 0, roots are real and equal and roots are given by

$$\alpha = \beta = -\frac{b}{2a}$$

Case - III
When D < 0, roots are not real.

Quadratic formula :

for $ax^2 + bx + c = 0$,
$D = b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$