

4

Chapter

Alligations

INTRODUCTION

The chapter of alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. I have often seen students having a lot of difficulty in solving questions on alligation. Please remember that all problems on alligation can be solved through the weighted average method. Hence, the student is advised to revert to the weighted average formula in case of any confusion.

The use of the techniques of this chapter for solving weighted average problems will help you in saving valuable time wherever a direct question based on the mixing of two groups is asked. Besides, in the case of questions that use the concept of the weighted average as a part of the problem, you will gain a significant edge if you are able to use the techniques illustrated here.

THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To recollect, the weighted average is used when a number of smaller groups are mixed together to form one larger group.

If the average of the measured quantity was

A_1 for group	1	containing	n_1	elements
A_2 for group	2	containing	n_2	elements
A_3 for group	3	containing	n_3	elements
A_k for group	k	containing	n_k	elements

We say that the weighted average, A_w is given by:

$$A_w = (n_1 A_1 + n_2 A_2 + n_3 A_3 + \dots + n_k A_k) / (n_1 + n_2 + n_3 \dots + n_k)$$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In the case of the situation where just two groups are being mixed, we can write this as:

$$A_w = (n_1 A_1 + n_2 A_2) / (n_1 + n_2)$$

Rewriting this equation we get: $(n_1 + n_2) A_w = n_1 A_1 + n_2 A_2$

$$n_1(A_w - A_1) = n_2(A_2 - A_w)$$

or $n_1/n_2 = (A_2 - A_w)/(A_w - A_1)$ $\propto$ The alligation equation.

The Alligation Situation

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is A_1 and the number of elements is n_1 and the average of the second group is A_2 and the number of elements is n_2 , then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

As a convenient convention, we take $A_1 < A_2$. Then, by the principal of averages, we get $A_1 < A_w < A_2$.

Illustration 1

Two varieties of rice at ` 10 per kg and ` 12 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture.

Solution $1/2 = (12 - A_w)/(A_w - 10) \propto A_w - 10 = 24 - 2A_w$

fi $3A_w = 34$ fi $A_w = 11.33$ `/kg.

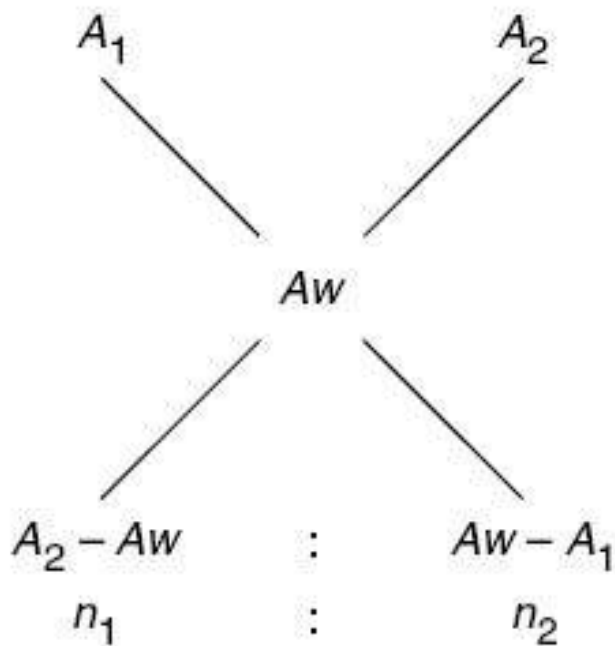
Illustration 2

On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

Solution $n_1/n_2 = (40 - 34)/(34 - 30) = 6/4 = 3/2$

Graphical Representation of Alligation

The formula illustrated above can be represented by the following cross diagram:



[Note that the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.]

As we have seen, there are five variables embedded inside the alligation equation. These being: the three averages— A_1 , A_2 and A_w and the two weights— n_1 and n_2

Based on the problem situation, one of the following cases may occur with respect to the knowns and the unknown, in the problem.

Case	Known	Unknown
I	(a) A_1, A_2, A_w	(a) $n_1 : n_2$
	(b) A_1, A_2, A_w, n_1	(b) n_2 and $n_1 : n_2$
II	A_1, A_2, n_1, n_2	A_w
III	A_1, A_w, n_1, n_2	A_2

Now, let us try to evaluate the effectiveness of the cross method for each of the three cases illustrated above:

Case 1: A_1, A_2, A_w are known; may be one of n_1 or n_2 is known.

To find: $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

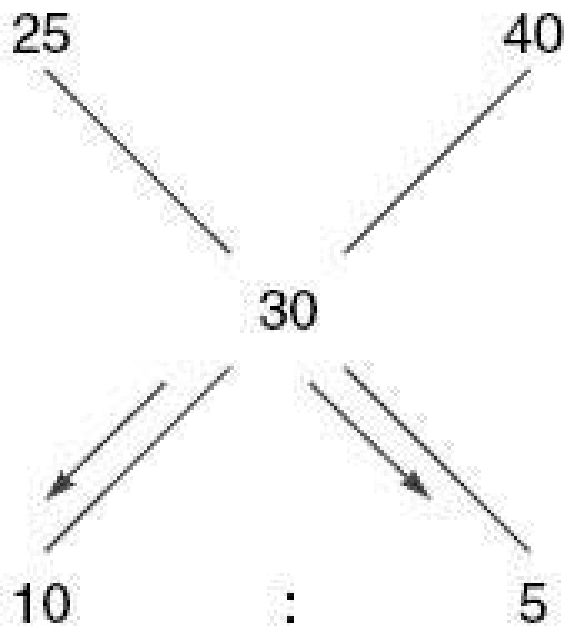
Let us illustrate through an example:

Illustration 3

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

- (a) The ratio of students in the classes
- (b) The number of students in the first class if the second class had 30 students.

Solution



- (a) Hence, solution is 2 : 1.
- (b) If the ratio is 2 : 1 and the second class has 30 students, then the first class has 60 students.

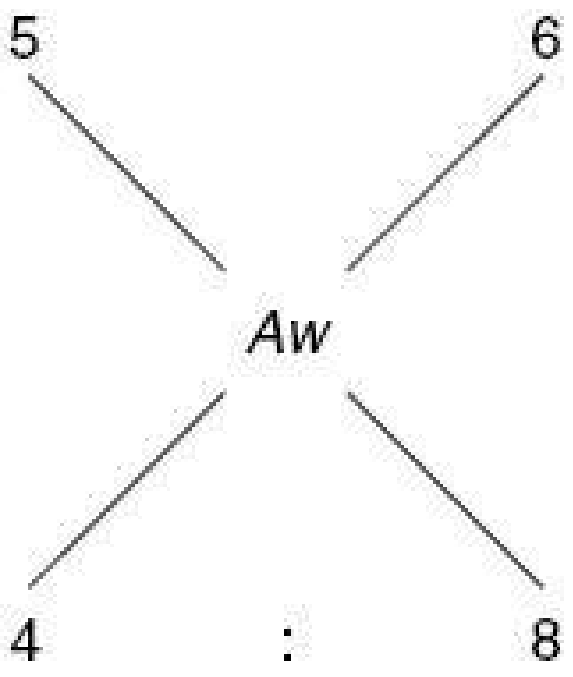
Note: The cross method becomes pretty effective in this situation when all the three averages are known and the ratio is to be found out.

Case 2: A_1, A_2, n_1 and n_2 are known, A_w is unknown.

Illustration 4

4 kg of rice at ` 5 per kg is mixed with 8 kg of rice at ` 6 per kg. Find the average price of the mixture.

Solution



$$= (6 - A_w) : (A_w - 5)$$
$$\text{fi } (6 - A_w)/(A_w - 5) = 4/8 \text{ } \therefore 12 - 2 A_w = A_w - 5$$

$$3 A_w = 17$$

$$\therefore A_w = 5.66 \text{ ₹/kg. (Answer)}$$

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice, the amount of time required in doing the same.

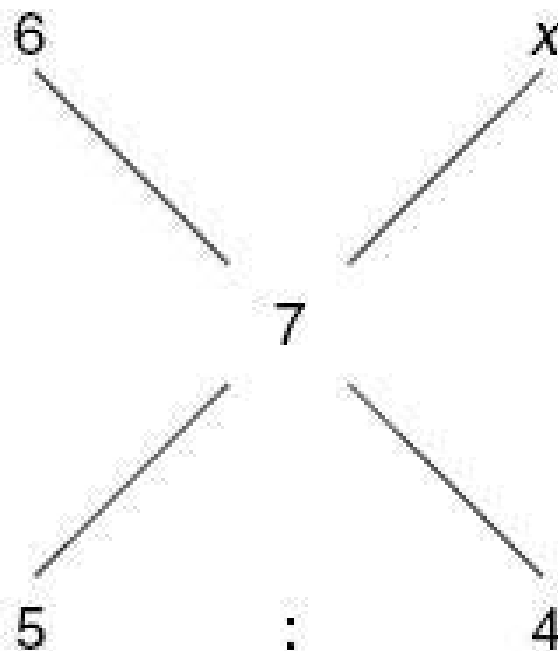
Note: The cross method becomes quite cumbersome in this case, as this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

Case 3: A_1 , A_w , n_1 and n_2 are known; A_2 is unknown.

Illustration 5

5 kg of rice at ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 per kg. Find the price of the costlier rice.

Solution Using the cross method:



$$= (x - 7) : 1$$

$$\therefore (x - 7)/1 = 5/4 \Rightarrow 4x - 28 = 5$$

$$\therefore x = ₹ 8.25.$$

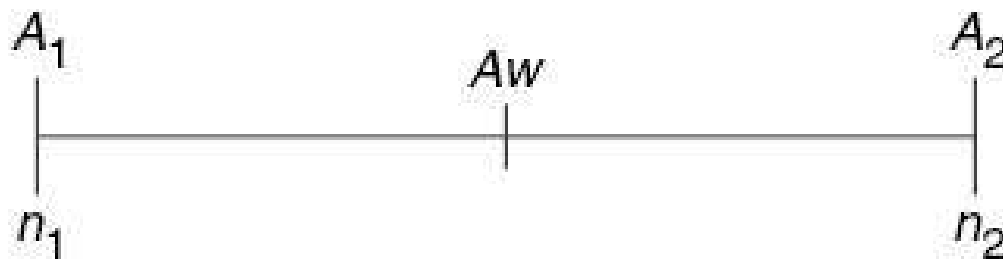
Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice the amount of time required in doing the same.

Note: The cross method becomes quite cumbersome in this case since this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

The above problems can be dealt quite effectively by using the straight line approach, which is explained below.

The Straight Line Approach

As we have seen, the cross method becomes quite cumbersome in Case 2 and Case 3. We will now proceed to modify the cross method so that the question can be solved graphically in all the three cases. Consider the following diagram, which results from closing the cross like a pair of scissors. Then the positions of A_1 , A_2 , A_w , n_1 and n_2 are as shown.



Visualise this as a fragment of the number line with points A_1 , A_w and A_2 in that order from left to right. Then,

- (a) n_2 is responsible for the distance between A_1 and A_w or n_2 corresponds to $A_w - A_1$
- (b) n_1 is responsible for the distance between A_w and A_2 . or n_1 corresponds to $A_2 - A_w$
- (c) $(n_1 + n_2)$ is responsible for the distance between A_1 and A_2 . or $(n_1 + n_2)$ corresponds to $A_2 - A_1$.

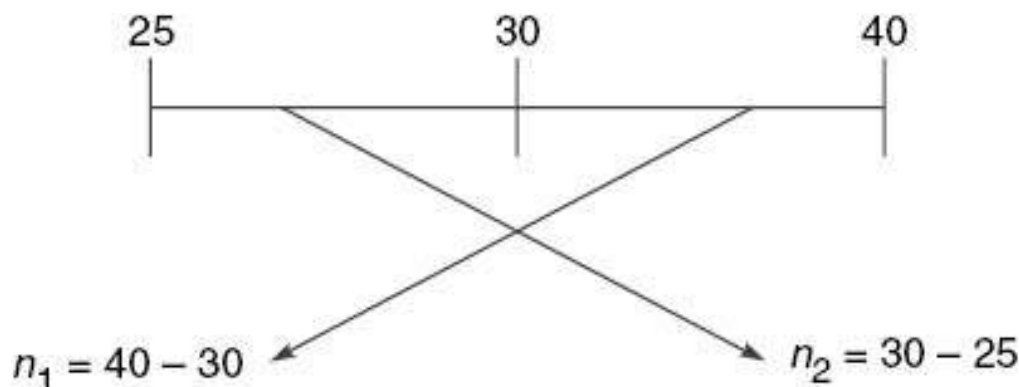
The processes for the 3 cases illustrated above can then be illustrated below:

Illustration 6

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

- (a) the ratio in which the classes were mixed.
- (b) the number of students in the first class if the second class had 30 students.

Solution



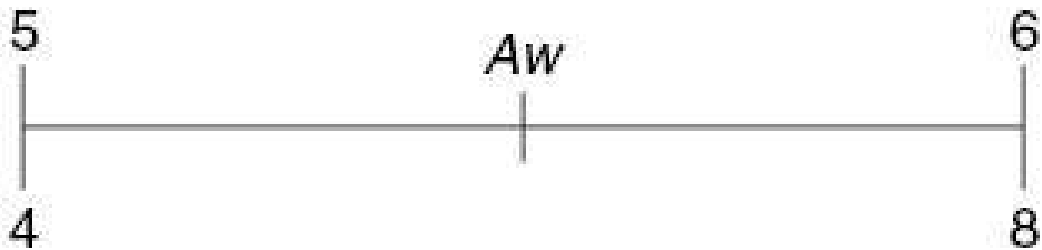
Hence, ratio is 2 : 1, and the second class has 60 students.

Case 2 A_1 , A_2 , n_1 and n_2 are known; A_w is unknown.

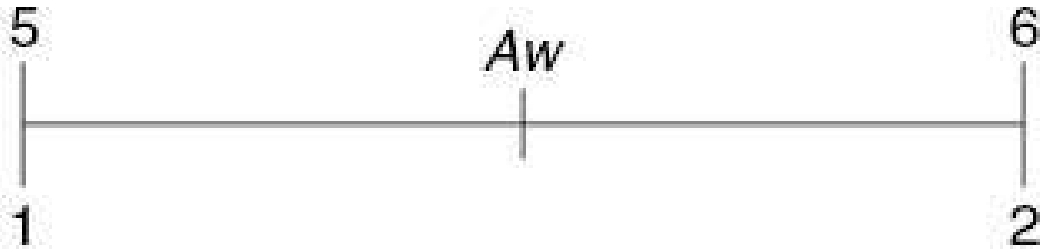
Illustration 7

4 kg of rice at ` 5 per kg is mixed with 8 kg of rice at ` 6 per kg. Find the average price of the mixture.

Solution



is the same as



Then, by unitary method:

$n_1 + n_2$ corresponds to $A_2 - A_1$

$\text{£ } 1 + 2$ corresponds to $6 - 5$

That is, 3 corresponds to 1

$\therefore n_2$ will correspond to $\frac{(A_2 - A_1) \times n_2}{(n_1 + n_2)}$

In this case $(1/3) \times 2 = 0.66$.

Hence, the required answer is 5.66.

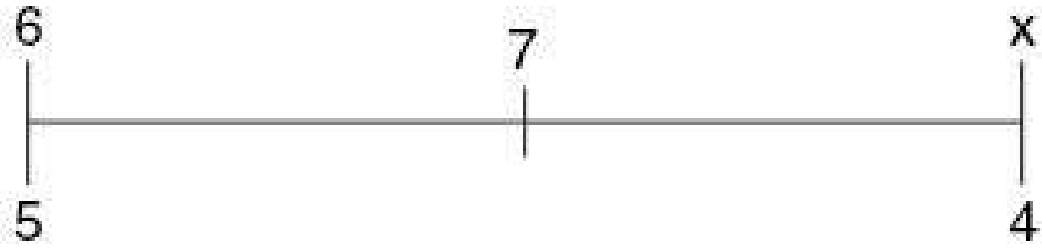
Note: In this case, the problem associated with the cross method is overcome and the solution becomes graphical.

Case 3: A_1 , Aw , n_1 and n_2 are known; A_2 is unknown.

Illustration 8

5 kg of rice at ` 6 per kg is mixed with 4 kg of rice to get a mixture costing ` 7 per kg. Find the price of the costlier rice.

Using straight line method:



4 corresponds to $7 - 6$ and 5 corresponds to $x - 7$.

The thought process should go like:

$4 \text{ £ } 1$

$$\sqrt{5} \approx 1.25$$

Hence, $x - 7 = 1.25$

and $x = 8.25$

SOME TYPICAL SITUATIONS WHERE ALLIGATIONS CAN BE USED

Given below are typical alligation situations, which students should be able to recognize. This will help them improve upon the time required in solving questions. Although in this chapter we have illustrated problems based on alligation at level 1 only, alligation is used in more complex problems where the weighted average is an intermediate step in the solution process.

The following situations should help the student identify alligation problems better as well as spot the way A_1 , A_2 , n_1 and n_2 and A_w are mentioned in a problem.

In each of the following problems the following magnitudes represent these variables:

$$A_1 = 20, A_2 = 30, n_1 = 40, n_2 = 60$$

Each of these problems will yield an answer of 26 as the value of A_w .

1. A man buys 40 kg of rice at ` 20/kg and 60 kg of rice at ` 30/kg. Find his average price. (26/kg)
2. Pradeep mixes two mixtures of milk and water. He mixes 40 litres of the first containing 20% water and 60 litres of the second containing 30% water. Find the percentage of water in the final mixture. (26%)
3. Two classes are combined to form a larger class. The first class having 40 students scored an average of 20 marks on a test while the second having 60 students scored an average of 30 marks on the same test. What was the average score of the combined class on the test. (26 marks)
4. A trader earns a profit of 20% on 40% of his goods sold, while he earns a profit of 30% on 60% of his goods sold. Find his percentage profit on the whole. (26%)
5. A car travels at 20 km/h for 40 minutes and at 30 km/h for 60 minutes. Find the average speed of the car for the journey. (26 km/hr)
6. 40% of the revenues of a school came from the junior classes while 60% of the revenues of the school came from the senior classes. If the school raises its fees by 20% for the junior classes and by 30% for the senior classes, find the percentage increase in the revenues of the school. (26%)

Some Keys to spot A_1 , A_2 and A_w and differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This isn't foolproof though, since at times the question might confuse the student by giving 3 values for quantities representing n_1 , n_2 and $n_1 + n_2$ respectively.
2. A_1 , A_2 and A_w are always rate units, while n_1 and n_2 are quantity units.
3. The denominator of the average unit corresponds to the quantity unit (i.e. unit for n_1 and n_2).
4. All percentage values represent the average values.

A Typical Problem

A typical problem related to the topic of alligation goes as follows:

4 litres of wine are drawn from a cask containing 40 litres of wine. It is replaced by water. The process is repeated 3 times

(a) What is the final quantity of wine left in the cask.

(b) What is the ratio of wine to water finally.

If we try to chart out the process, we get: Out of 40 litres of wine, 4 are drawn out.

This leaves 36 litres wine and 4 litres water. (Ratio of 9 : 1)

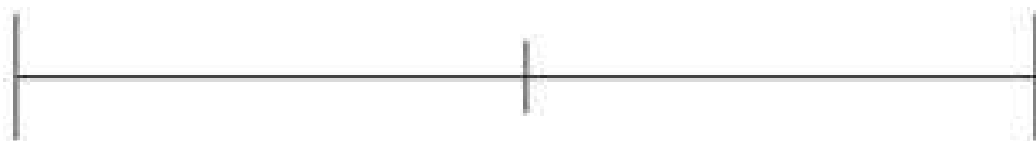
Now, when 4 litres are drawn out of this mixture, we will get 3.6 litres of wine and 0.4 litres of water (as the ratio is 9 : 1). Thus at the end of the second step we get: 32.4 litres of wine and 7.6 litres of water. Further, the process is repeated, drawing out 3.24 litres wine and 0.76 litres water leaving 29.16 litres of wine and 10.84 litres of water.

This gives the final values and the ratio required.

A closer look at the process will yield that we can get the amount of wine left by:

$$40 \times 36/40 \times 36/40 \times 36/40 = 40 \times (36/40)^3$$

$$\text{fi } 40 \times (1 - 4/40)^3$$



This yields the formula:

Wine left : Capacity $\times (1 - \text{fraction of wine withdrawn})^n$ for n operations.

Thus, you could have multiplied:

$$40 \times (0.9)^3 \text{ to get the answer}$$

That is, reduce 40 by 10% successively thrice to get the required answer.

Thus, the thought process could be:

$$40 - 10\% \text{ } \text{Æ} \text{ } 36 - 10\% \text{ } \text{Æ} \text{ } 32.4 - 10\% \text{ } \text{Æ} \text{ } 29.16$$

LEVEL OF DIFFICULTY (I)

- If 5 kg of salt costing ₹ 5/kg and 3 kg of salt costing ₹ 4/kg are mixed, find the average cost of the mixture per kilogram.
(a) ₹ 4.5 (b) ₹ 4.625
(c) ₹ 4.75 (d) ₹ 4.125
- Two types of oils having the rates of ₹ 4/kg and ₹ 5/kg respectively are mixed in order to produce a mixture having the rate of ₹ 4.60/kg. What should be the amount of the second type of oil if the amount of the first type of oil in the mixture is 40 kg?
(a) 75 kg (b) 50 kg
(c) 60 kg (d) 40 kg
- How many kilograms of sugar worth ₹ 3.60 per kg should be mixed with 8 kg of sugar worth ₹ 4.20 per kg, such that by selling the mixture at ₹ 4.40 per kg, there may be a gain of 10%?
(a) 6 kg (b) 3 kg
(c) 2 kg (d) 4 kg
- A mixture of 125 gallons of wine and water contains 20% water. How much water must be added to the mixture in order to increase the percentage of water to 25% of the new mixture?
(a) 10 gals (b) 8.5 gals
(c) 8 gals (d) 8.33 gals
- Ravi lends ₹ 3600 on simple interest to Harsh for a period of 5 years. He lends a part of the amount at 4% interest and the rest at 6% and receives ₹ 960 as the amount of interest. How much money did he lend on 4% interest rate?
(a) ₹ 2800 (b) ₹ 2100
(c) ₹ 2400 (d) ₹ 1200
- 400 students took a mock exam in Delhi. 60% of the boys and 80% of the girls cleared the cut off in the examination. If the total percentage of students qualifying is 65%, how many girls appeared in the examination?
(a) 100 (b) 120
(c) 150 (d) 300
- A man purchased a cow and a calf for ₹ 1300. He sold the calf at a profit of 20% and the cow at a profit of 25%. In this way, his total profit was $23\frac{1}{13}\%$. Find the cost price of the cow.
(a) ₹ 1100 (b) ₹ 600
(c) ₹ 500 (d) ₹ 800
- The average salary per head of all employees of a company is ₹ 600. The average salary of 120 officers is ₹ 4000. If the average salary per head of the rest of the employees is ₹ 560, find the total number of workers in the company.

- (a) 10200 (b) 10320
(c) 10500 (d) 10680
9. A dishonest milkman purchased milk at ` 10 per litre and mixed 5 litres of water in it. By selling the mixture at the rate of ` 10 per litre he earns a profit of 25%. The quantity of the amount of the mixture that he had was:
(a) 15 litres (b) 20 litres
(c) 25 litres (d) 30 litres
10. A cistern contains 50 litres of water. 5 litres of water is taken out of it and replaced by wine. The process is repeated again. Find the proportion of wine and water in the resulting mixture.
(a) 1 : 4 (b) 41 : 50
(c) 19 : 81 (d) 81 : 19
11. A container has a capacity of 20 gallons and is full of spirit. 4 gallons of spirit is drawn out and the container is again filled with water. This process is repeated 5 times. Find out how much spirit is left in the resulting mixture finally?
(a) $6\frac{257}{525}$ gallons (b) $6\frac{346}{625}$ gallons
(c) 6.5 gallons (d) 6.25 gallons
12. A vessel is full of refined oil. $\frac{1}{4}$ of the refined oil is taken out and the vessel is filled with mustard oil. If the process is repeated 4 times and 10 litres of refined oil is finally left in the vessel, what is the capacity of the vessel?
(a) 33 litres (b) $\frac{2460}{81}$ litres
(c) $\frac{2560}{81}$ litres (d) 30 litres
13. In what ratio should two qualities of coffee powder having the rates of ` 47 per kg and ` 32 per kg be mixed in order to get a mixture that would have a rate of ` 37 per kg?
(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 3 : 1
14. A thief steals four gallons of liquid soap kept in a train compartment's bathroom from a container that is full of liquid soap. He then fills it with water to avoid detection. Unable to resist the temptation he steals 4 gallons of the mixture again, and fills it with water. When the liquid soap is checked at a station it is found that the ratio of the liquid soap now left in the container to that of the water in it is 36 : 13. What was the initial amount of the liquid soap in the container if it is known that the liquid soap is neither used nor augmented by anybody else during the entire period?
(a) 7 gallons (b) 14 gallons
(c) 21 gallons (d) 28 gallons

15. In what ratio should water be mixed with soda costing ` 12 per litre so as to make a profit of 25% by selling the diluted liquid at ` 13.75 per litre?
- (a) 10 : 1 (b) 11 : 1
(c) 1 : 11 (d) 12 : 1
16. A sum of ` 36.90 is made up of 90 coins that are either 20 paise coins or 50 paise coins. Find out how many 20 paise coins are there in the total amount.
- (a) 47 (b) 43
(c) 27 (d) 63
17. A dishonest grocer professes to sell pure butter at cost price, but he mixes it with adulterated fat and thereby gains 25%. Find the percentage of adulterated fat in the mixture assuming that adulterated fat is freely available.
- (a) 20% (b) 25%
(c) 33.33% (d) 40%
18. A mixture of 70 litres of alcohol and water contains 10% of water. How much water must be added to the above mixture to make the water 12.5% of the resulting mixture?
- (a) 1 litre (b) 1.5 litre
(c) 2 litres (d) 2.5 litres
19. A mixture of 20 litres of brandy and water contains 10% water. How much water should be added to it to increase the percentage of water to 25%?
- (a) 2 litres (b) 3 litres
(c) 2.5 litres (d) 4 litres
20. A merchant purchased two qualities of pulses at the rate of ` 200 per quintal and ` 260 per quintal. In 52 quintals of the second quality, how much pulse of the first quality should be mixed so that by selling the resulting mixture at ` 300 per quintal, he gains a profit of 25%?
- (a) 100 quintals (b) 104 quintals
(c) 26 quintals (d) None of these
21. A man buys milk at ` 8.5 per litre and dilutes it with water. He sells the mixture at the same rate and thus gains 11.11%. Find the quantity of water mixed by him in every litre of milk.
- (a) 0.111 litres (b) 0.909 litres
(c) 0.1 litre (d) 0.125 litres
22. There are two mixtures of honey and water, the quantity of honey in them being 25% and 75% of the mixture. If 2 gallons of the first are mixed with three gallons of the second, what will be the ratio of honey to water in the new mixture?
- (a) 11 : 2 (b) 11 : 9
(c) 9 : 11 (d) 2 : 11
23. There are two kinds of alloys of tin and copper. The first alloy contains tin and copper such that

93.33% of it is tin. In the second alloy there is 86.66% tin. What weight of the first alloy should be mixed with some weight of the second alloy so as to make a 50 kg mass containing 90% of tin?

- (a) 15 kg
- (b) 30 kg
- (c) 20 kg
- (d) 25 kg

24. Two containers of equal capacity are full of a mixture of oil and water. In the first, the ratio of oil to water is 4 : 7 and in the second it is 7 : 11. Now both the mixtures are mixed in a bigger container. What is the resulting ratio of oil to water?

- (a) 149 : 247
- (b) 247 : 149
- (c) 143 : 241
- (d) 241 : 143

25. Two vessels contain spirit and water mixed respectively in the ratio of 1 : 3 and 3 : 5. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is 1 : 2.

- (a) 2 : 1
- (b) 3 : 1
- (c) 1 : 2
- (d) 1 : 3

26. The price of a pen and a pencil is ₹ 35. The pen was sold at a 20% profit and the pencil at a 10% loss. If in the transaction a man gains ₹ 4, how much is cost price of the pen?

- (a) ₹ 10
- (b) ₹ 25
- (c) ₹ 20
- (d) None of these

27. A person purchased a cupboard and a cot for ₹ 18,000. He sold the cupboard at a profit of 20% and the cot at a profit of 30%. If his total profit was 25.833%, find the cost price of the cupboard.

- (a) ₹ 10,500
- (b) ₹ 12,000
- (c) ₹ 7500
- (d) ₹ 10,000

28. A vessel is full of a mixture of kerosene and petrol in which there is 18% kerosene. Eight litres are drawn off and then the vessel is filled with petrol. If the kerosene is now 15%, how much does the vessel hold?

- (a) 40 litres
- (b) 32 litres
- (c) 36 litres
- (d) 48 litres

29. Two solutions of 90% and 97% purity are mixed resulting in 21 litres of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?

- (a) 15 litres
- (b) 12 litres
- (c) 9 litres
- (d) 6 litres

30. In the Singapore zoo, there are deers and there are ducks. If the heads are counted, there are 180, while the legs are 448. What will be the number of deers in the zoo?

- (a) 136
- (b) 68
- (c) 44
- (d) 22

31. A bonus of ₹ 9,85,000 was divided among 300 workers of a factory. Each male worker gets 5000 rupees and each female worker gets 2500 rupees. Find the number of male workers in the factory.

- (a) 253 (b) 47
(c) 94 (d) 206
32. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene that are present in three vessels of equal capacity in the ratios 4 : 1, 5 : 2 and 6 : 1 respectively?
(a) 166 : 22 (b) 83 : 22
(c) 83 : 44 (d) None of these
33. A mixture worth ₹ 3.25 a kg is formed by mixing two types of flour, one costing ₹ 3.10 per kg while the other ₹ 3.60 per kg. In what proportion must they have been mixed?
(a) 3 : 7 (b) 7 : 10
(c) 10 : 3 (d) 7 : 3
34. A 20 percent gain is made by selling the mixture of two types of ghee at ₹ 480 per kg. If the type costing 610 per kg was mixed with 126 kg of the other, how many kilograms of the former was mixed?
(a) 138 kg (b) 34.5 kg
(c) 69 kg (d) Cannot be determined
35. In what proportion must water be mixed with milk so as to gain 20% by selling the mixture at the cost price of the milk? (Assume that water is freely available)
(a) 1 : 4 (b) 1 : 5
(c) 1 : 6 (d) 1 : 12
36. A bartender stole champagne from a bottle that contained 50% of spirit and he replaced what he had stolen with champagne having 20% spirit. The bottle then contained only 25% spirit. How much of the bottle did he steal?
(a) 80% (b) 83.33%
(c) 85.71% (d) 88.88%
37. A bag contains a total of 105 coins of ₹ 1, 50 p and 25 p denominations. Find the total number of coins of Re 1 if there are a total of 50.5 rupees in the bag and it is known that the number of 25 paise coins are 133.33% more than the number of 1 rupee coins.
(a) 56 (b) 25
(c) 24 (d) None of these
38. A man possessing ₹ 6800, lent a part of it at 10% simple interest and the remaining at 7.5% simple interest. His total income after $3\frac{1}{2}$ years was ₹ 1904. Find the sum lent at 10% rates.
(a) ₹ 1260 (b) ₹ 1700
(c) ₹ 1360 (d) None of these
39. If a man decides to travel 80 kilometres in 8 hours partly by foot and partly on a bicycle, his speed on foot being 8 km/h and that on bicycle being 16 km/h, what distance would he travel on

foot?

- (a) 20 km (b) 30 km
(c) 48 km (d) 60 km

40. Two vessels contain a mixture of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35 litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many litres are taken from the first vessel?

- (a) 11 litres (b) 22 litres
(c) 16.5 litres (d) 17.5 litres

ANSWER KEY

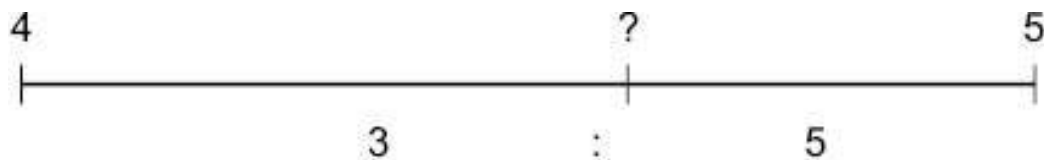
Level of Difficulty (I)

- | | | | |
|---------|---------|---------|---------|
| 1. (b) | 2. (c) | 3. (d) | 4. (d) |
| 5. (d) | 6. (a) | 7. (d) | 8. (b) |
| 9. (c) | 10. (c) | 11. (b) | 12. (c) |
| 13. (a) | 14. (d) | 15. (c) | 16. (c) |
| 17. (a) | 18. (c) | 19. (d) | 20. (c) |
| 21. (a) | 22. (b) | 23. (d) | 24. (a) |
| 25. (c) | 26. (b) | 27. (c) | 28. (d) |
| 29. (c) | 30. (c) | 31. (c) | 32. (b) |
| 33. (d) | 34. (d) | 35. (b) | 36. (b) |
| 37. (c) | 38. (c) | 39. (c) | 40. (a) |
-

Solutions and Shortcuts

Level of Difficulty (I)

1. Solving the following alligation figure:



The answer would be 4.625/kg.

2. Mixing ` 4/kg and ` 5/kg to get ` 4.6 per kg we get that the ratio of mixing is 2:3. If the first oil is 40 kg, the second would be 60 kg.
3. Since by selling at ` 4.40 we want a profit of 10%, it means that the average cost required is ` 4 per kg. Mixing sugar worth ` 3.6/kg and ` 4.2/kg to get ` 4/kg means a mixture ratio of 1:2. Thus, to 8 kg of the second variety we need to add 4 kg of the first variety to get the required cost price.
4. In 125 gallons we have 25 gallons water and 100 gallons wine. To increase the percentage of water to 25%, we need to reduce the percentage of wine to 75%. This means that 100 gallons of wine = 75% of the new mixture. Thus the total mixture = 133.33 gallons. Thus, we need to mx

$133.33 - 125 = 8.33$ gallons of water in order to make the water equivalent to 25% of the mixture.

5. Since, Ravi earns ₹ 960 in 5 years, it means that he earns an interest of $960/5 = ₹ 192$ per year. On an investment of 3600, an annual interest of 192 represents an average interest rate of 5.33%.

Then using the alligation figure below:



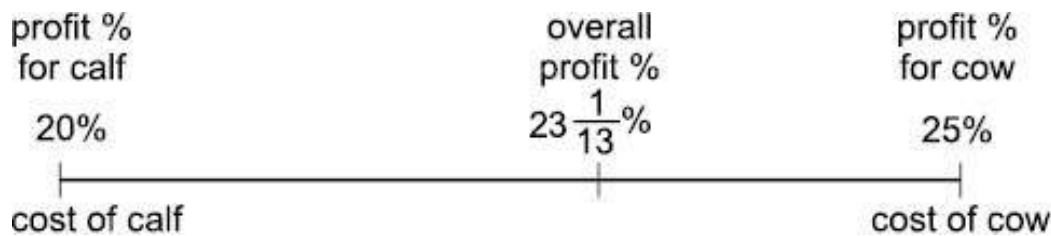
We get the ratio of investments as 1:2. Hence, he lent $1 \times 3600/3 = 1200$ at 4% per annum.

6. The ratio of boys and girls appearing for the exam can be seen to be 3:1 using the following alligation figure.



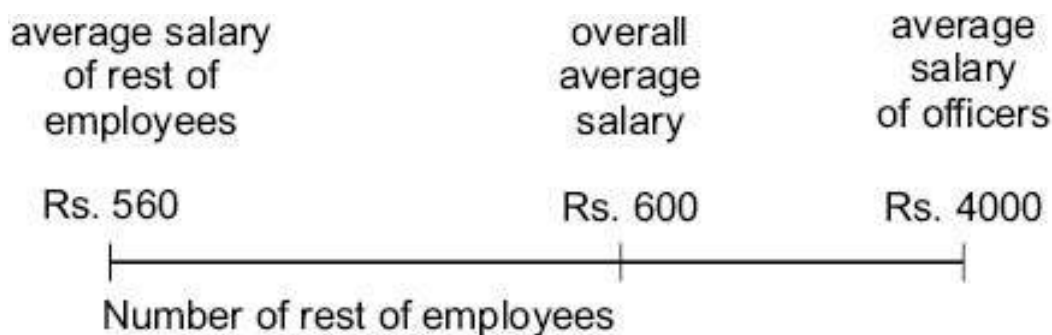
This means that out of 400 students, there must have been 100 girls who appeared in the exam.

7. The ratio of the cost of the cow and the calf would be 40:25 or 8:5 as can be seen from the following alligation figure:



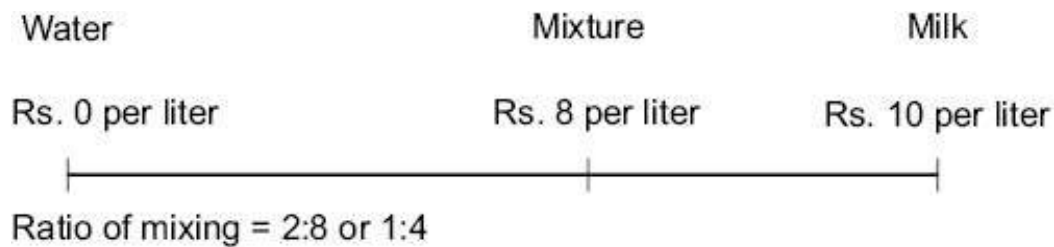
Thus, the cost of the cow would be ₹ 800.

8.



From the figure it is clear that the ratio of the number of officers to the number of other employees would be 40:3400. Since there are 120 officers, there would be $3400 \times 3 = 10200$ workers in the company. Thus the total number of employees would be $10200 + 120 = 10320$.

9. The cost price of the mixture would have been ₹ 8 per liter for him to get a profit of 25% by selling at ₹ 10 per liter. The ratio of mixing would have been 1:4 (water is to milk) as can be seen in the figure:

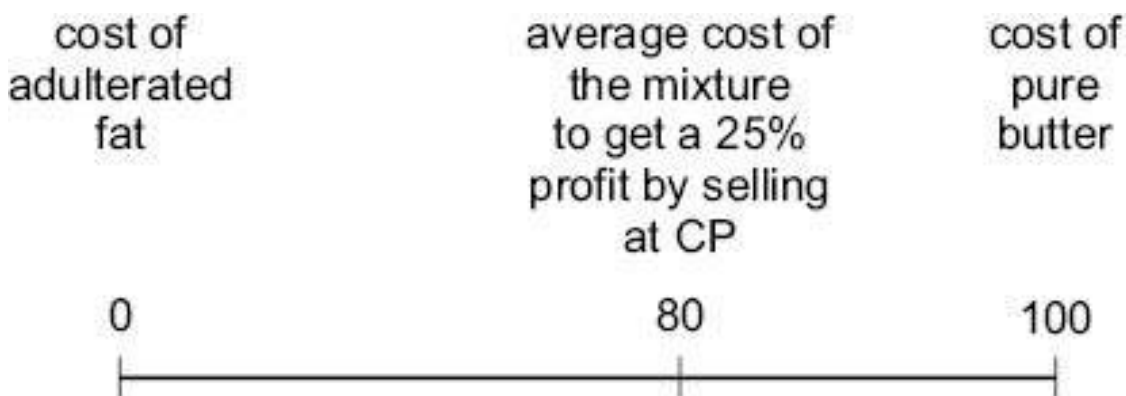


Since we are putting in 5 liters of water, the amount of milk must be 20 liters. The amount of mixture then would become 25 liters.

10. Amount of water left = $50 \times \frac{9}{10} \times \frac{9}{10} = 40.5$ liters. Hence, wine = 9.5 liters. Ratio of wine and water = 19:81. Option (c) is correct.
11. The amount of spirit left = $20 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{4096}{625} = 6 \frac{346}{625}$.
12. Let the quantity of refined oil initially be Q . Then we have $Q \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = 10 \Rightarrow Q = \frac{2560}{81}$ liters.
13. The ratio would be 1:2 as seen from the figure:



14. It can be seen from the ratio 36:13 that the proportion of liquid soap to water is $\frac{36}{49}$ after two mixings. This means that $\frac{6}{7}$ th of the liquid soap must have been allowed to remain in the container and hence $\frac{1}{7}$ th of the container's original liquid soap would have been drawn out by the thief. Since he takes out 4 gallons every time, there must have been 28 gallons in the container. (as 4 should be $\frac{1}{7}$ th of 28).
15. In order to sell at a 25% profit by selling at 13.75 the cost price should be $13.75/1.25 = 11$. Also since water is freely available, we can say that the ratio of water and soda must be 1:11.
16. The average value of a coin is 41 paise and there are only 20 paise and 50 paise coins in the sum. Hence, the ratio of the number of 20 paise coins to 50 paise coins would be $9:21 = 3:7$. Since there are a total of 90 coins, the number of 20 paise coins would be $3 \times \frac{90}{10} = 27$ coins.
- 17.

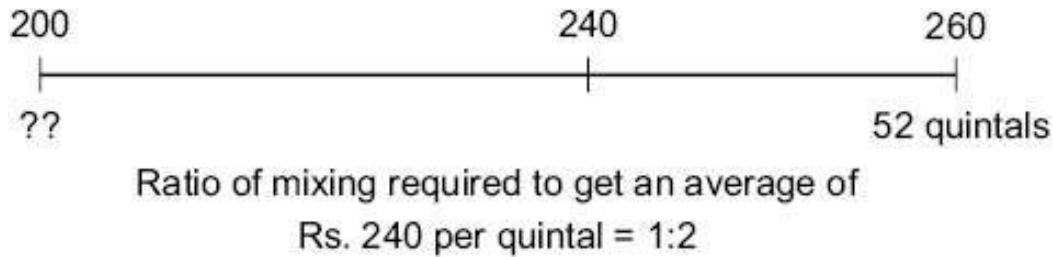


The ratio of mixing required would be 1:4 which means that the percentage of adulterated fat would be 20%.

18. Solve using options. Initially there are 7 liters of water in 70 liters of the mixture. By mixing 2 liters of water we will have 9 liters of water in 72 liters of the mixture – which is exactly 12.5%.
19. Again solve this question using options. Initially there are 2 liters of water in 20 liters of the mixture. To take the water to 25% of the mixture we would need to add 4 liters of water as that

would give us 6 liters of water in 24 liters of the mixture.

20. By selling at 300 if we need to get a profit of 25% it means that the cost price would be $300/1.25 = 240$.

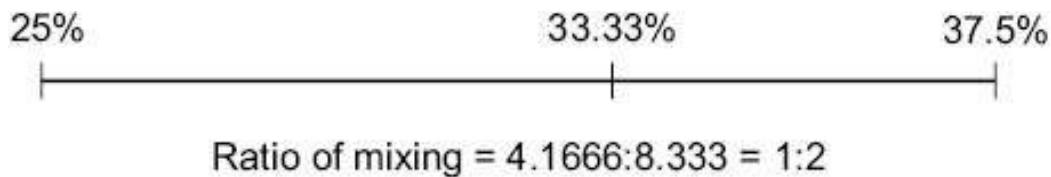


Thus, in 52 quintals of the second we need to mix 26 quintals of the first.

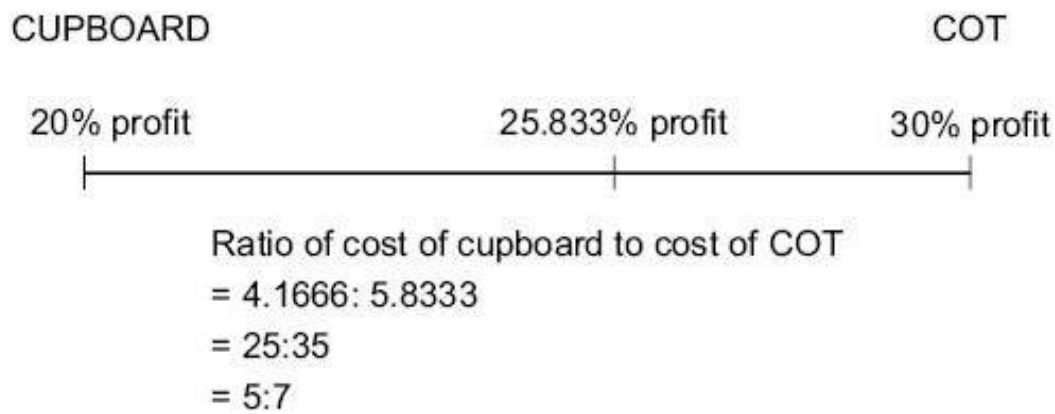
21. The requisite 11.11% profit can be got by mixing 0.111 liters of water in 1 liter of milk. In such a case the total milk quantity would be 1.111 liters and the price would be for 1 liter only. The profit would be $0.111/1 = 11.11\%$.
22. The percentage of honey in the new mixture would be:
 $(2 \times 25 + 3 \times 75)/5 = 275/5 = 55\%$. The ratio of honey to water in the new mixture would be $55:45 = 11:9$.
23. In order to mix two tin alloys containing 86.66% tin and 93.33% tin to get 90% tin, the ratio of mixing should be 1:1. Thus, each variety should be 25 kgs each.
24. Assume the capacity of the two containers is 198 liters each. When we mix 198 liters of the first and 198 liters of the second the amount of oil would be:
 $198 \times 4/11 + 198 \times 7/18 = 72 + 77 = 149$ liters.

Consequently the amount of water would be $396 - 149 = 247$ liters. Option (a) is correct.

25. The first vessel contains 25% spirit while the second vessel contains 37.5 % spirit. To get a 1:2 ratio we need 33.33% spirit in the mixture. The ratio of mixing can be seen using the following alligation figure:

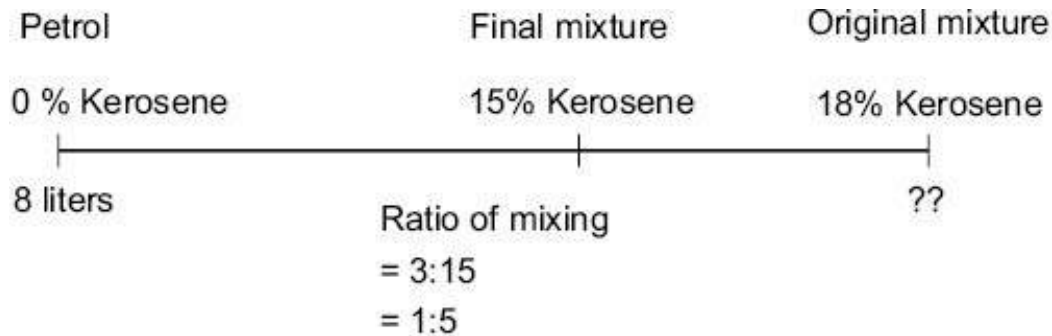


26. Solve using options as that would be the best way to tackle this question. Option (b) fits the situation perfectly as if we take the price of the pen as ` 25, the cost of the pencil would be ` 10. The profit in selling the pen would be `5 while the loss in selling the pencil would be ` 1. The total profit would be ` 4 as stipulated by the problem.
27. The following alligation visualization would help us solve the problem:



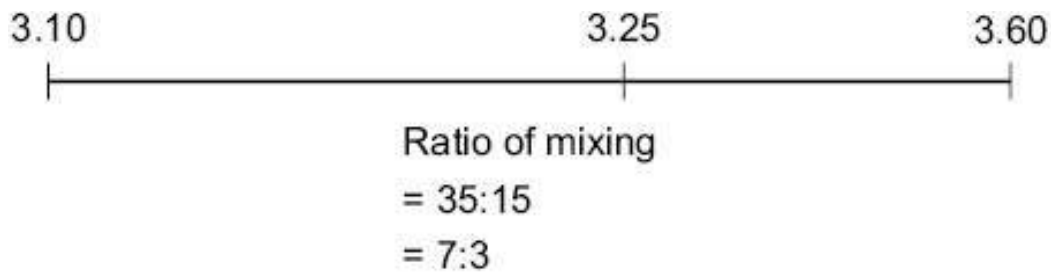
Cost of cupboard $= 5 \times 18000/12 = 7500$.

28. The following visualization would help:



From the figure we can see that the original mixture would be 40 liters and the petrol being mixed is 8 liters. Thus, the vessel capacity is 48 liters.

29. 90% and 97% mixed to form 94% means that the mixing ratio is 3:4. The first solution would be $3 \times 21/7 = 9$ liters.
30. If all the animals were ducks we would have 180 heads and 360 legs. If we reduce the number of ducks by 1 to 179 and increase the number of deers by 1 to 1, we would get an incremental 2 legs. Since, the number of legs we need to increment is 88 ($448 - 360 = 88$), we need to have 44 deers and 136 ducks.
31. Solve using options. Option (a) does not fit because $253 \times 5000 + 47 \times 2500$ is not equal to 985000.
 Similarly option (b) does not fit because $253 \times 2500 + 47 \times 5000$ is not equal to 985000.
 Option (c) fits as $94 \times 5000 + 206 \times 2500 = 985000$.
32. In order to solve this we need to assume a value for the amounts in the vessels. If we assume 35 liters as the quantities in all the three vessels we will get:
 $28 \text{ liters} + 25 \text{ liters} + 30 \text{ liters} = 83 \text{ liters of petrol and } 22 \text{ liters of kerosene in } 105 \text{ liters of the mixture.}$
 The required ratio is 83:22.
33. The required ratio would be 7:3 as seen in the following figure.



34. We cannot determine the answer to this question as we do not know the price per kg of the other type of ghee. Hence, we cannot find the ratio of mixing which would be required in order to move further in this question.
35. To gain 20% by selling at cost price, milk should comprise 83.33% of the total mixture. The ratio of mixing that achieves this is 1:5.
36. 20% spirit is mixed with 50% spirit to get 25% spirit. The ratio of mixing would be 5:1. This means he stole $\frac{5}{6}$ th of the bottle or 83.33% of the bottle.
37. $O + F + T = 95$
 $O + 0.5F + 0.25T = 50.5$
 $T = 1.333 O$.
 Solving we get: 24 coins of 25 paise each.
38. Annual interest income = $1904 / 3.5 = 544$. Interest of ` 544 on a lending of ` 6800 implies an 8% average rate of interest. This 8% is generated by mixing two loans @ 7.5% and 10% respectively. The ratio in which the two loans should be allocated would be 4:1. The amount lent at 10% would be $1 \times 6800 / 5 = 1360$.
39. Solve using options. If he travels 48 km on foot he would take 6 hours on foot. Also, in this case he would travel 32 km on bicycle @ 16kmph – which would take him 2 hours. Thus a total of 8 hours. Option (c) satisfies the conditions of the question.
40. Solving through options is the best way to tackle this question. Option (a) fits the conditions of the problem as if there are 11 liters in the first vessel, there would be 8 liters of spirit. Also it means that we would be taking 24 liters from the second vessel out of which there would be 20 liters of spirit. Thus, total spirit would be 28 out of 35 liters giving us 7 liters of water.



BLOCK REVIEW TEST

Review Test

- Rakshit bought 19 erasers for ₹ 10. He paid 20 paise more for each white eraser than for each brown eraser. What could be the price of a white eraser and how many white erasers could he have bought?
(a) 60 paise, 8 (b) 60 paise, 12
(c) 50 paise, 8 (d) 50 paise, 10
- After paying all your bills, you find that you have ₹ 7.20 in your pocket. You have equal number of 50 paise and 10 paise coins; but no other coins nor any other currency notes. How many coins do you have?
(a) 8 (b) 24
(c) 27 (d) 30
- Suresh Kumar went to the market with ₹ 100. If he buys three pens and six pencils he uses up all his money. On the other hand if he buys three pencils and six pens he would fall short by 20%. If he wants to buy equal number of pens & pencils, how many pencils can he buy?
(a) 4 (b) 5
(c) 6 (d) 7
- For the above question, what is the amount of money he would save if he were to buy 3 pens and 3 pencils?
(a) ₹ 50 (b) ₹ 25
(c) ₹ 75 (d) ₹ 40
- Abdul goes to the market to buy bananas. If he can bargain and reduce the price per dozen by ₹ 2, he can buy 3 dozen bananas instead of 2 dozen with the money he has. How much money does he have?
(a) ₹ 6 (b) ₹ 12
(c) ₹ 18 (d) ₹ 24
- Two oranges, three bananas and four apples cost ₹ 15. Three oranges, two bananas and one apple cost ₹ 10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?
(a) ₹ 10 (b) ₹ 8
(c) ₹ 15 (d) cannot be determined
- John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange would be
(a) ₹ 1 (b) ₹ 2
(c) ₹ 3 (d) ₹ 4
- Two towns A and B are 100 km apart. A school is to be built for 100 students of Town B and 30

students of Town A. The Expenditure on transport is `1.20 per km per person. If the total expenditure on transport by all 130 students is to be as small as possible, then the school should be built at

- (a) 33 km from Town A
- (b) 33 km from Town B
- (c) Town A
- (d) Town B

9. A person who has a certain amount with him goes to the market. He can buy 50 oranges or 40 mangoes. He retains 10% of the amount for taxi fare and buys 20 mangoes and of the balance he purchases oranges. Number of oranges he can purchase is

- (a) 36
- (b) 40
- (c) 15
- (d) 20

10. 72 hens costs `96.7_. Then what does each hen cost, where numbers at “_” are not visible or are written in illegible hand?

- (a) `3.43
- (b) `5.31
- (c) `5.51
- (d) `6.22

Directions for Questions 10 to 12: There are 60 students in a class. These students are divided into three groups A, B and C of 15, 20 and 25 students each. The groups A and C are combined to form group D

11. What is the average weight of the students in group D?

- (a) more than the average weight of A.
- (b) more than the average weight of C.
- (c) less than the average weight of C.
- (d) Cannot be determined.

12. If one student from Group A is shifted to group B, which of the following will be true?

- (a) The average weight of both groups increases
- (b) The average weight of both groups decreases
- (c) The average weight of the class remains the same.
- (d) Cannot be determined.

13. If all the students of the class have the same weight then which of the following is false?

- (a) The average weight of all the four groups is the same.
- (b) The total weight of A and C is twice the total weight of B.
- (c) The average weight of D is greater than the average weight of A.
- (d) The average weight of all the groups remains the same even if a number of students are shifted from one group to another.

14. The average marks of a student in ten papers are 80. If the highest and the lowest score are not

considered the average is 81. If his highest score is 92 find the lowest.

- (a) 55 (b) 60
(c) 62 (d) Cannot be determined

15. A shipping clerk has five boxes of different but unknown weights each weighing less than 100 kg. The clerk weighs the boxes in pairs. The weights obtained are 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121 kg. What is the weight of the heaviest box?

- (a) 60 kg (b) 62 kg
(c) 64 kg (d) Cannot be determined

16. The total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is ₹ 700 when there are 25 boarders and ₹ 600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?

- (a) 550 (b) 580
(c) 540 (d) 570

17. A yearly payment to a servant is ₹ 90 plus one turban. The servant leaves the job after 9 months and receives ₹ 65 and a turban, then find the price of the turban.

- (a) ₹ 10 (b) ₹ 15
(c) ₹ 7.50 (d) Cannot be determined

18. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is ₹ 20 on a standard bag and ₹ 30 on a deluxe bag. Every bag must be processed on machine A and on Machine B. The processing times per bag on the two machines are as follows:

Time required (Hours/bag)

	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

- (a) Standard 75 bags, Deluxe 80 bags
(b) Standard 100 bags, Deluxe 60 bags
(c) Standard 50 bags, Deluxe 100 bags
(d) Standard 60 bags, Deluxe 90 bags

19. Three math classes: X, Y, and Z, take an algebra test.

The average score of class X is 83.

The average score of class Y is 76.

The average score of class Z is 85.

What is the average score of classes X, Y, Z ?

- (a) 81.5 (b) 80.5
(c) 83 (d) Cannot be determined

20. Prabhat ordered 4 Arrow shirts and some additional Park Avenue shirts. The price of one Arrow shirt was twice that of one Park Avenue shirt. When the order was executed it was found that the number of the two brands had been interchanged. This increased the bill by 40%. The ratio of the number of Arrow shirts to the number of Park Avenue shirts in the original order was:

- (a) 1:3 (b) 1:4
(c) 1:2 (d) 1:5

21. Three groups of companies: Tata, Birla and Reliance announced the average of the annual profit for all years since their establishment.

The average profit of Tata is ` 75,000 lakh

The average profit of Birla is ` 64000 lakh

The average profit of Reliance is ` 73000 lakh

The average profit of all results of Tata and Birla together is ` 70000 lakh.

The average profit of all results of Birla and Reliance together is ` 69000 lakh.

Approximately what is the average profit for all the three group of companies?

- (a) ` 70800 lakh (b) ` 71086 lakh
(c) ` 70666 lakh (d) Cannot be determined

ANSWER KEY

Review Test

- | | | | |
|---------|---------|---------|---------|
| 1. (b) | 2. (b) | 3. (a) | 4. (b) |
| 5. (b) | 6. (c) | 7. (b) | 8. (d) |
| 9. (d) | 10. (c) | 11. (d) | 12. (c) |
| 13. (c) | 14. (b) | 15. (b) | 16. (a) |
| 17. (a) | 18. (a) | 19. (d) | 20. (a) |
| 21. (b) | | | |
-