JEE Advanced

Single Correct Answer Type

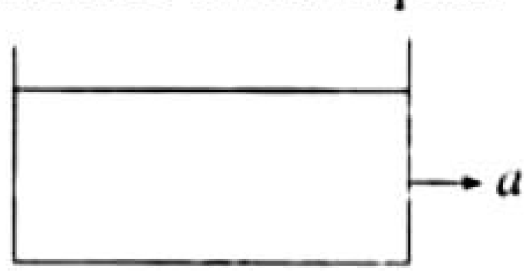
1. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 50°C. The expansion of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

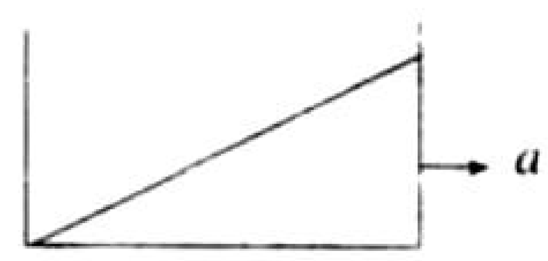
   (This problem is mixed concept of heat and fluid mechanics)

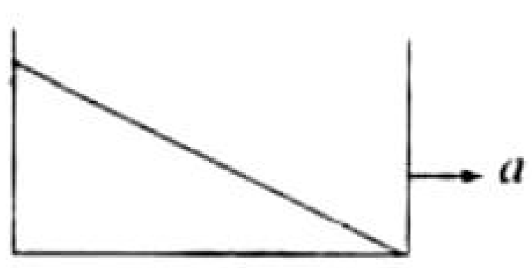
   a. $W_1 > W_2$ **b.** $W_1 = W_2$
   c. $W_1 < W_2$ **d.** none of these

   (IIT-JEE 1980)

2. A vessel containing water is given a constant acceleration 'a' towards the right along a straight horizontal path. Which of the following diagrams in the figure represents the surface of the liquid?

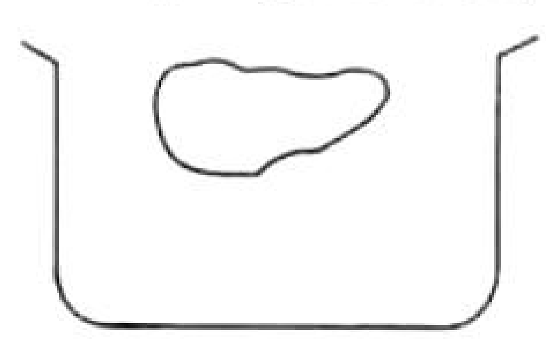
   a. 

   b. 

   c. 

   (This problem is mixed concept of heat and fluid mechanics) **(IIT-JEE 1981)**

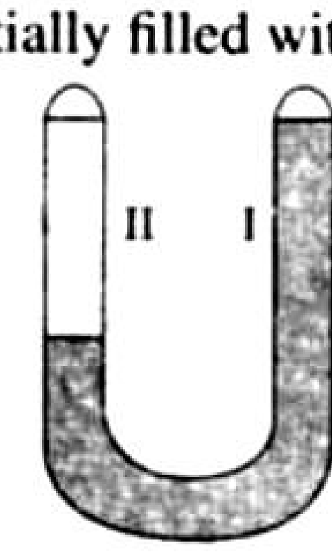
3. A body floats in a liquid contained in a beaker. The whole system as shown in the figure falls freely under gravity. The upthrust on the body is

   a. zero
   b. equal to the weight of the liquid displaced
   c. equal to the weight of the body in air
   d. equal to the weight of the immersed portion of the body

   (IIT-JEE 1982)

4. A U-tube of uniform cross section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be

   a. 1.12 **b.** 1.[illegible] **c.** 1.05 **d.** 1.0

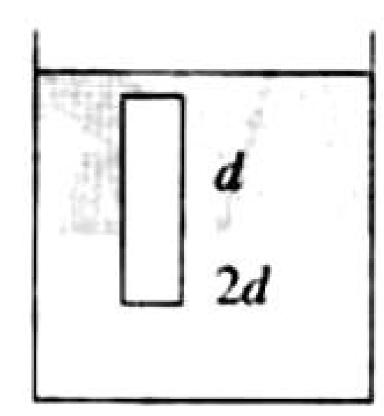
   (IIT-JEE 1983)

5. A vessel contains oil (density = 0.8 g/cm^2) over mercury (density = 13.6 g/cm^3). A homogenous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cm^3 is

   a. 3.3 **b.** 6.4 **c.** 7.2 **d.** 12.8

   (IIT-JEE 1988)

6. A homogeneous solid cylinder of length $L (L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid–liquid interface with length $L/4$ in the density liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then, density D of solid is given by

a. $\frac{5}{4}d$ b. $\frac{4}{5}d$ c. $4d$ d. $\frac{d}{5}$

(IIT-JEE 1995)

7. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to

a. $L/\sqrt{2\pi}$ b. $2\pi L$ c. L d. $1/2\pi$

(IIT-JEE 2000)

8. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as

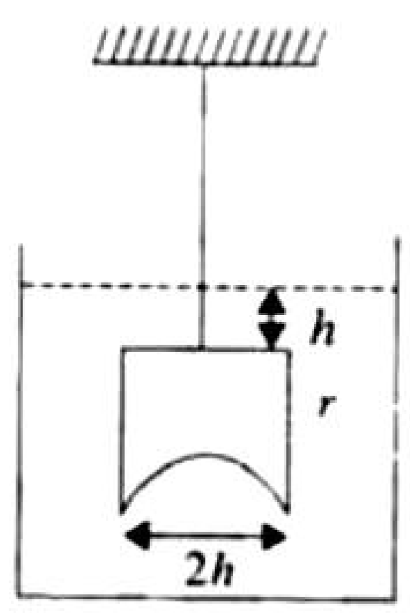
a. $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ b. $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$

c. $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ d. $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

(IIT-JEE 2001)

(This problem is mixed concept of heat and fluid mechanics)

9. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is

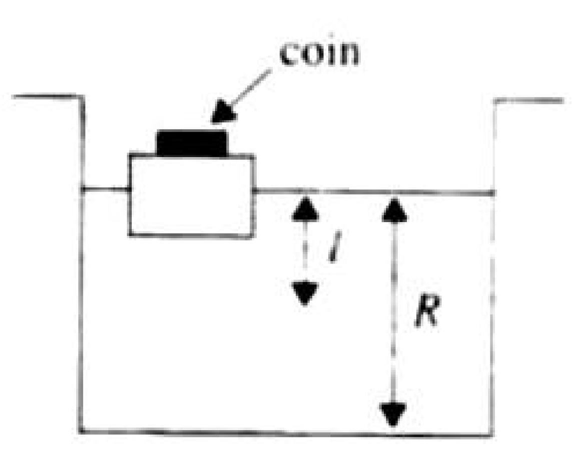


a. Mg b. $Mg - V\rho g$

c. $Mg + \pi R^2 h\rho g$ d. $\rho g(V + \pi R^2 h)$

(IIT-JEE 2001)

10. A wooden block, with a coin placed on its top, floats in water as shown in the figure. The distance l and h are shown here. After some time, the coin falls into the water. Then



a. l decreases and h increases

b. l increases and h decreases

c. both l and h increase

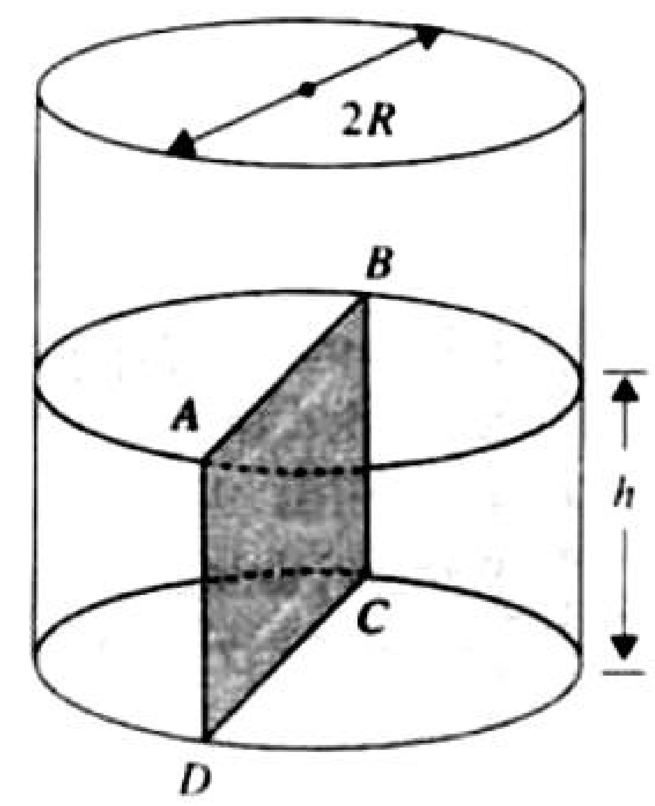
d. both l and h decrease **(IIT-JEE 2002)**

11. Water is filled in a container up to height 3 m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $a/A = 0$, then v^2 is (where v is the velocity of water coming out of the hole)

a. 50 b. 51 c. 48 d. 51.5

(IIT-JEE 2005)

12. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of the section by water on the other side of this section has magnitude



a. $|2P_0Rh + \pi R^2\rho gh - 2RT|$

b. $|2P_0Rh + R\rho gh^2 - 2RT|$

c. $|P_0\pi R^2 + R\rho gh^2 - 2RT|$

d. $|P_0\pi R^2 + R\rho gh^2 + 2RT|$ **(IIT-JEE 2007)**

13. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

a. More than half-filled if ρ_c is less than 0.5

b. More than half-filled ρ_c is more than 1.0

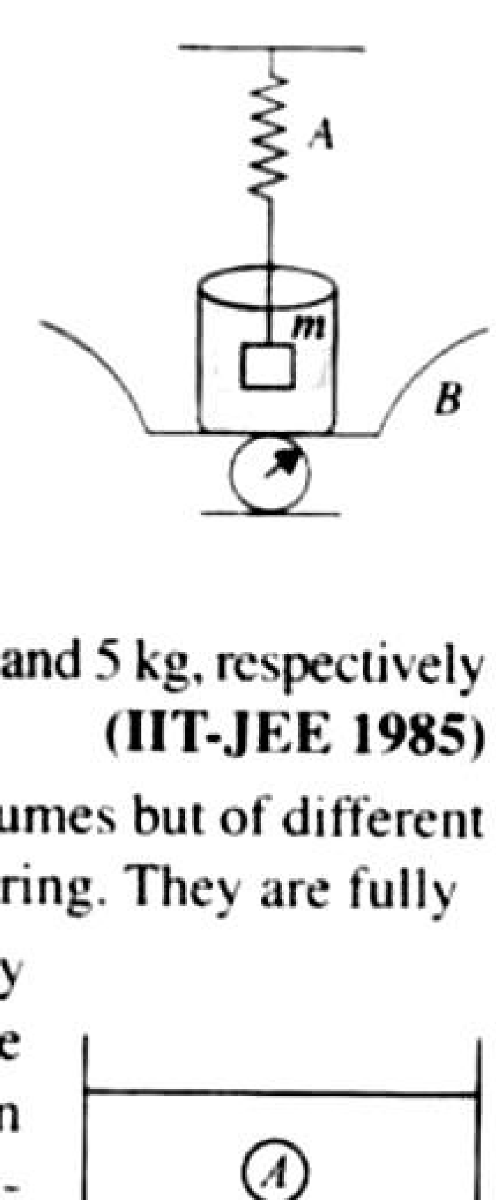
c. half-filled if ρ_c is more than 0.5

d. less than half-filled if ρ_c is less than 0.5

(IIT-JEE 2012)

Multiple Correct Answer Type

1. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hinging mass is inside the liquid in the beaker as shown in the figure. In this situation,



a. the balance A will read more than 2 kg

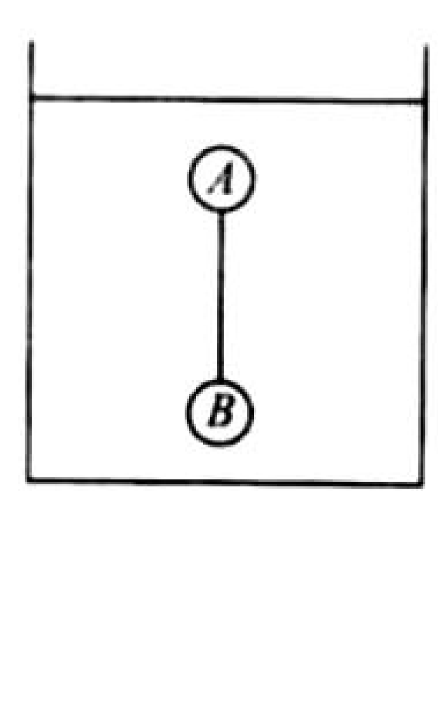
b. the balance B will read more than 5 kg

c. the balance A will read less than 2 kg and B will read more than 5 kg

d. the balance A and B will read 2 kg and 5 kg, respectively

(IIT-JEE 1985)

2. Two solid spheres and of equal volumes but of different densities and are connected by a string. They are fully immersed in a fluid of density. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



a. $d_A < d_F$ b. $d_B > d_F$

c. $d_A > d_F$ d. $d_A + d_B = 2d_F$

(IIT-JEE 2011)

3. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k.

The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

a. the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

b. the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

c. the light sphere is partially submerged.

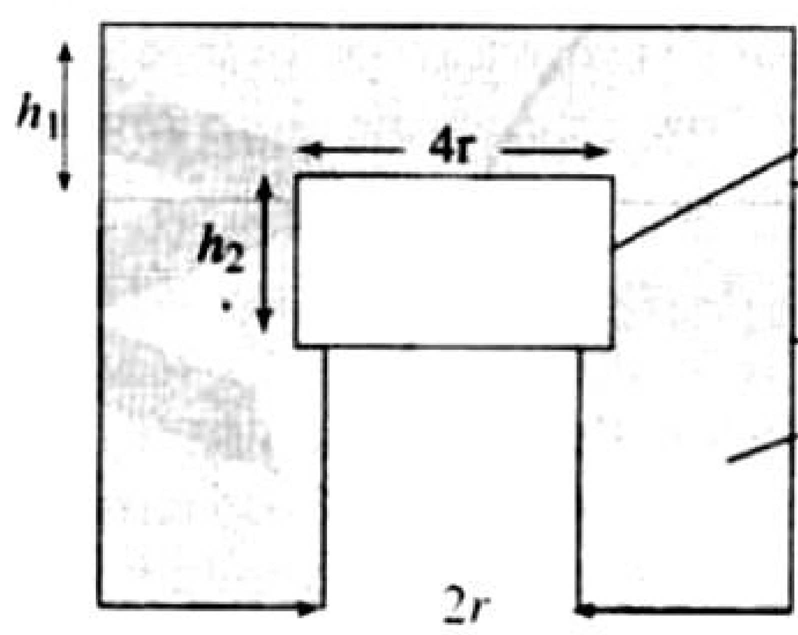
d. the light sphere is completely submerged.

(JEE Advanced 2013)

Linked Comprehension Type

For Problems 1–3

A cylindrical tank has a hole of diameter $2r$ in its bottom. The hole is covered wooden cylindrical block of diameter $4r$, height h and density $\rho/3$.



Situation 1: Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

Situation 2: The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. The height h_2 is smaller than h (height of the block) and thus the block is exposed to the atmosphere. **(IIT-JEE 2006)**

1. Find the minimum value of height h_1 (in situation 1) for which the block just starts to move up?

a. $2h/3$ b. $5h/4$ c. $5h/3$ d. $5h/2$

2. Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force.

a. $h/3$ b. $4h/9$ c. $2h/3$ d. h

3. In situation 2, if h_2 is further decreased, then

a. cylinder will not move up and remains at its original position

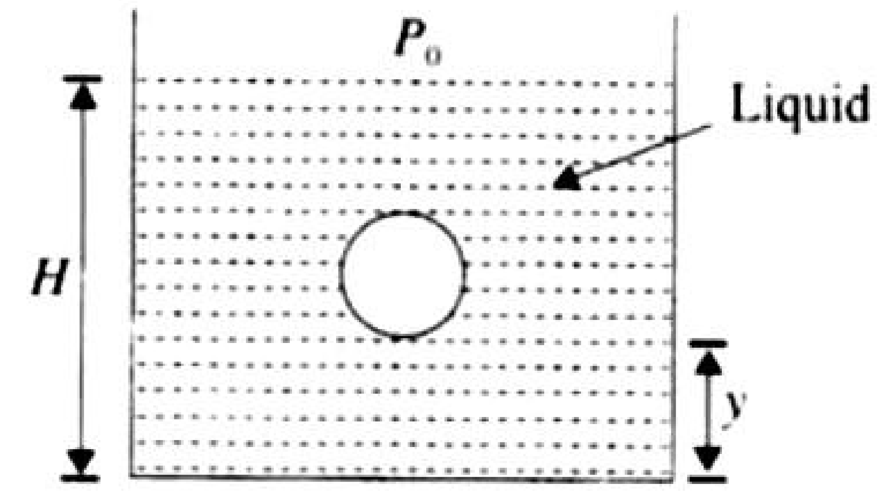
b. for $h_2 = h/3$, cylinder again starts moving up

c. for $h_2 = h/4$, cylinder again starts moving up

d. for $h_2 = h/5$, cylinder again starts moving up

For Problems 4–6

A small spherical monoatomic ideal gas bubble ($\gamma = 5/3$) is trapped inside a liquid of density ρ_l (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (neglect surface tension). **(IIT-JEE 2008)**



4. As the bubble moves upwards, beside the buoyancy force, the following forces are acting on it.

a. Only the force of gravity

b. The force due to gravity and the force due to the pressure of the liquid

c. The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

d. The force due to gravity and the force due to viscosity of the liquid

5. When the gas bubble is at height y from the bottom, its temperature is

a. $T_0\left(\frac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{2/5}$ b. $T_0\left(\frac{P_0+\rho_l g(H-y)}{P_0+\rho_l gy}\right)^{2/5}$

c. $T_0\left(\frac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{3/5}$ d. $T_0\left(\frac{P_0+\rho_l g(H-y)}{P_0+\rho_l gy}\right)^{3/5}$

6. The buoyancy force acting on the gas bubble is (assume R is the universal gas constant)

a. $\rho_l nRgT_0\frac{(P_0+\rho_l gH)^{2/5}}{(P_0+\rho_l gy)^{7/5}}$

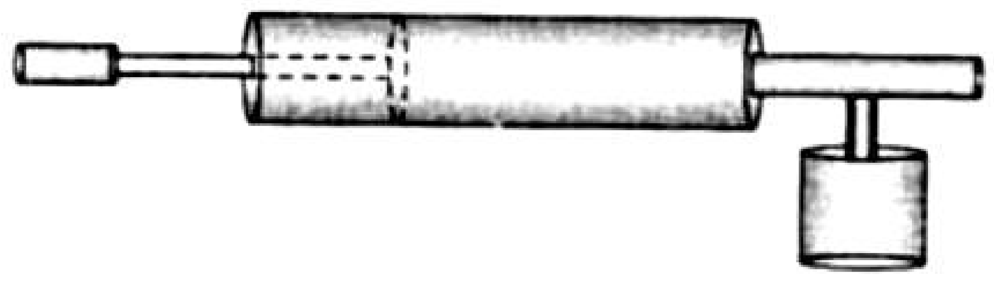
b. $\frac{\rho_l nRgT_0}{(P_0+\rho_l gH)^{2/5}[(P_0+\rho_l g(H-y)]^{3/5}}$

c. $\rho_l nRgT_0\frac{(P_0+\rho_l gH)^{3/5}}{(P_0+\rho_l gy)^{8/5}}$

d. $\frac{\rho_l nRgT_0}{(P_0+\rho_l gH)^{3/5}[(P_0+\rho_l g(H-y)]^{2/5}}$

For Problems 7–8

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



(JEE Advanced 2014)

7. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

a. 0.1 ms^{-1} **b.** 1 ms^{-1} **c.** 2 ms^{-1} **d.** 8 ms^{-1}

8. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

a. $\sqrt{\frac{\rho_a}{\rho_l}}$ **b.** $\sqrt{\rho_a\rho_l}$ **c.** $\sqrt{\frac{\rho_l}{\rho_a}}$ **d.** ρ_l

Matching Column Type

1. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

List I	List II
P. Lift is accelerating vertically up.	**1.** $d = 1.2$ m
Q. Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	**2.** $d > 1.2$ m
R. Lift is moving vertically up with constant speed.	**3.** $d < 1.2$ m
S. Lift is falling freely.	**4.** No water leaks out of the jar

Code:

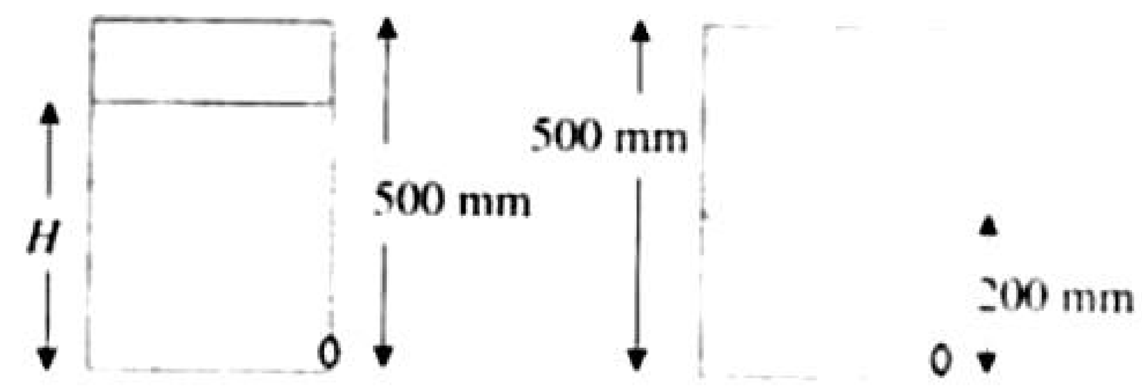
a. P-2, Q-3, R-2, S-4 **b.** P-2, Q-3, R-1, S-4

c. P-1, Q-1, R-1, S-4 **d.** P-2, Q-3, R-1, S-1

(JEE Advanced 2014)

Integer Answer Type

1. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. (Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.)



(IIT-JEE 2009)

Assertion-Reasoning Type

In each of the questions, assertion (A) is given by corresponding statement of reason (R) of the statements, mark the correct answer.

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of assertion.

c. If assertion is true but the reason is false.

d. If assertion is false but the reason is true.

1. Assertion: The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Reason: In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(IIT-JEE 2008)

Fill in the Blanks Type

1. A piece of metal floats on mercury. The coefficient of volume expansion of the metal and mercury are γ_1 and γ_2, respectively. If the temperature of both mercury and the metal is increased by an amount ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor ——.

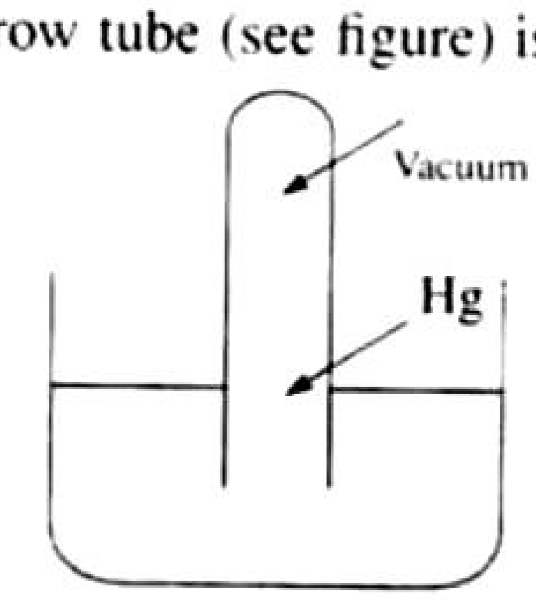
(This problem is mixed concept of heat and fluid mechanics) **(IIT-JEE 1991)**

2. A horizontal pipeline carries water in a streamline flow. At a point along the pipe, where the cross-sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2 is ____ Pa (density of water = 10^3 kg m^{-3}). **(IIT-JEE 1994)**

True/False Type

1. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, water level in the pond decreases. **(IIT-JEE 1980)**

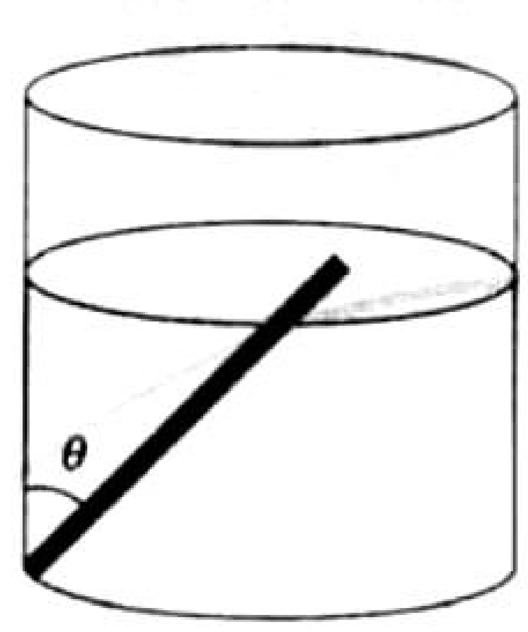
2. A barometer made of a very narrow tube (see figure) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is 0.00018 per degree centigrade and that of the tube is negligible. The temperature of mercury in the barometer is now raised by



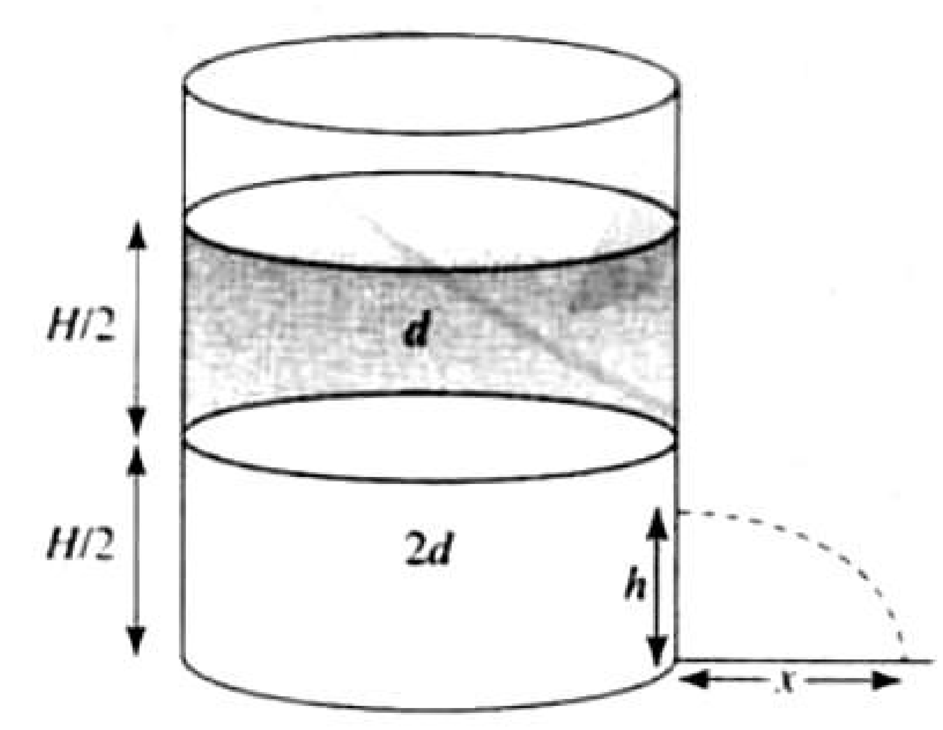
1°C, but the temperature of the atmosphere does not change. Then, the mercury height in the tube remains unchanged. **(IIT-JEE 1983)**

Subjective Type

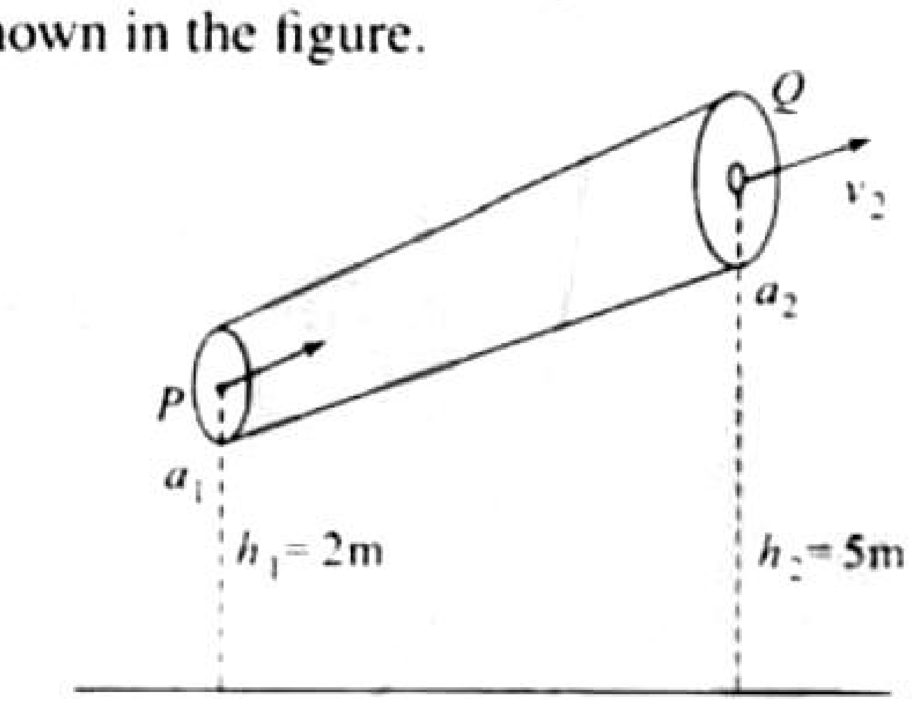
1. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? **(IIT-JEE 1979)**
2. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ. The height of the liquid in one vessel is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the two vessels are connected? **(IIT-JEE 1981)**
3. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in the figure. The tank is filled with water up to a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0°$). **(IIT-JEE 1984)**



4. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density of d_L
   a. If $d > d_L$, obtain an expansion (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.
   b. Is the motion of the ball simple harmonic?
   c. If $d = d_L$ how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large. **(IIT-JEE 1992)**
5. A container of large uniform cross-sectional area A resting on a horizontal surface holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0.



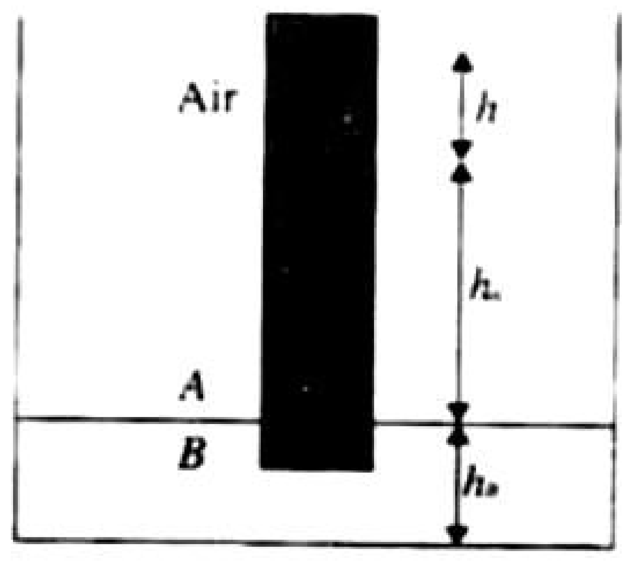
   a. A homogenous solid cylinder of length $L(L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. Determine:
      i. the density D of the solid
      ii. the total pressure at the bottom of the container.
   b. The cylinder is removed and the original arrangement is restored. A tiny hole of area $s(s << A)$ is punched on the vertical side of the container at a height $h(h < H/2)$. Determine
      i. the initial speed of efflux of the liquid at the hole.
      ii. the horizontal distance x travelled by the liquid initially, and
      iii. the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m. (Neglect the air resistance in these calculations). **(IIT-JEE 1995)**
6. A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $A/100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass M_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$, calculate (a) the acceleration of the container and (b) its velocity when 75% of the liquid has drained out. **(IIT-JEE 1997)**
7. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in a vertical plane as shown in the figure.



The areas of cross section of the tube at two points P and Q at heights of 2 m and 5 m, respectively, are 4×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from points P and Q. **(IIT-JEE 1997)**

8. A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density ρ ($>\rho$). **(IIT-JEE 1999)**
9. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of liquids A

and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder with liquid B is $h_B = 0.8$ cm.



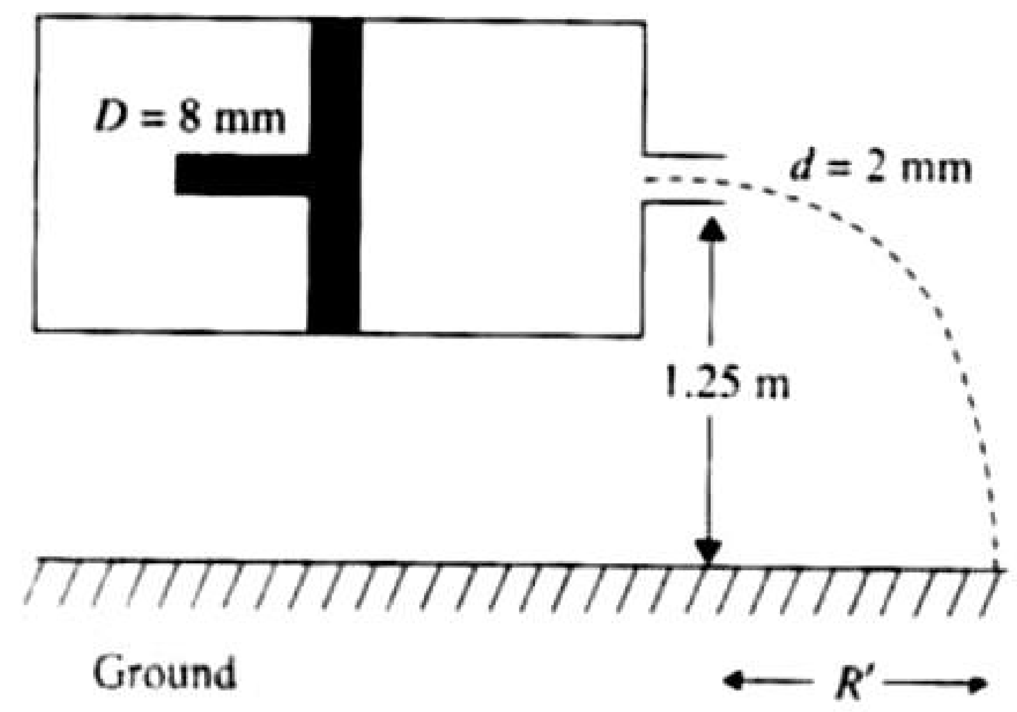
a. Find the total force exerted by liquid A on the cylinder.

b. Find h, the length of part of the cylinder in air.

c. The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately just after it is released.

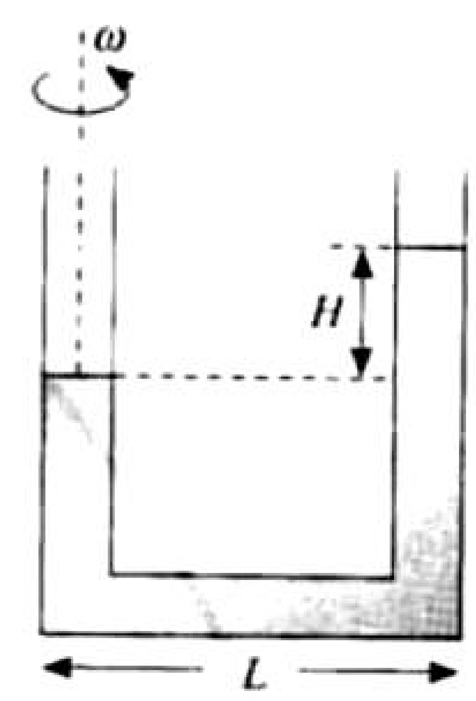
(IIT-JEE 2002)

10. Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground. Take g = 10 m/s^2. **(IIT-JEE 2004)**



11. A U tube is rotated about one of its limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube $d << L$.

(IIT-JEE 2005)



ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. c. **2.** c. **3.** a. **4.** b. **5.** c.
6. a. **7.** a. **8.** a. **9.** d. **10.** d.
11. a. **12.** b. **13.** d.

Multiple Correct Answers Type

1. b, c. **2.** a, b, d. **3.** a, d.

Linked Comprehension Type

1. c. **2.** b. **3.** a. **4.** d. **5.** b.
6. b. **7.** c. **8.** a.

Matching Column Type

1. c.

Integer Answer Type

1. (6)

Assertion–Reasoning Type

1. d.

Fill in the Blanks Type

1. $-\frac{(\gamma_1+\gamma_2)\Delta T}{1-\gamma_1\Delta T}$ **2.** 500 Pa

True/False Type

1. False. **2.** False.

Subjective Type

1. level will come down. **2.** $\frac{\rho A g}{2}(h_1-h_2)^2$

3. $\theta = 45°$

4. (a) $t_2 = \frac{d_L t_1}{d_L - d}$ (b) no (c) remain same

5. (a) (i) $\frac{5d}{4}$ (ii) $P = P_0 + \left(\frac{H}{2}+\frac{L}{4}\right)dg$

(b) (i) $\frac{\sqrt{(3H-4h)}}{2}g$ (ii) $\sqrt{(3H-4h)h}$ (iii) $t = \frac{3H}{8}; \frac{3}{4}H$

6. (a) $\frac{g}{50}$ (b) $\sqrt{\frac{gm_0}{2A\rho}}$

7. 30375 J/m^3, 25000 J/m^3

8. $\pi R^2 L(\sqrt{\rho\sigma}-\rho)$

9. (a) zero (b) 0.25 cm (c) $\frac{g}{6}$

10. 2 m **11.** $H = \frac{\omega^2 L^2}{2g}$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. c. $W_1 = mg - Vd_a g$

$$W_2 = mg - V'd_a'g$$

$$= mg - V(1 + 50\gamma_b)\frac{d_a g}{(1+50\gamma_a)}$$

$$= mg - Vd_a g\left[\frac{1+50\gamma_b}{1+50\gamma_a}\right]$$

V = volume of ball at 0°C

d_a = density of alcohol at 0°C

V' = volume of ball at 50°C

d'_a = density of alcohol at 0°C

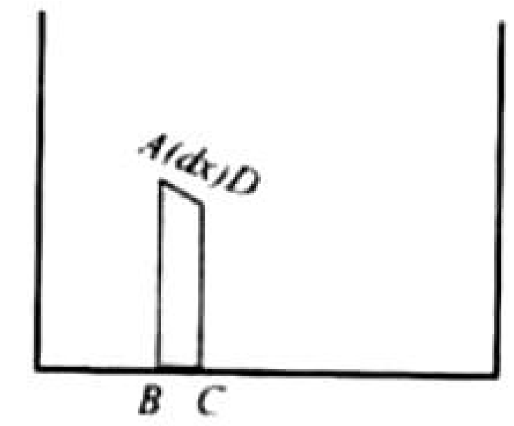
Given $\gamma_b < \gamma_a$

$\therefore \quad 1 + 50\,\gamma_b < 1 + 50\gamma_a$

$\therefore \quad \dfrac{1+50\gamma_b}{1+50\gamma_a} < 1$

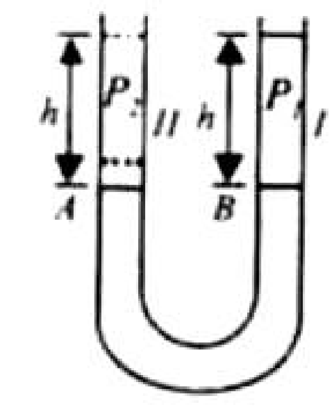
$\therefore \quad W_2 > W_1$ or $W_1 > W_2$

2. c Let us consider a small dotted segment of thickness dx for observation. Since this segment is accelerated towards right, a net force is acting in this segment towards right from the liquid towards the left of $ABCD$. According to Newton's third law, the segment $ABCD$ will also apply a force on the previous section creating a pressure on it which makes the liquid rise.



3. a. The whole system falls freely under gravity. Upthrust = weight of fluid displaced = (mass of fluid displaced) × g. For a freely falling body, g = 0.Therefore, upthrust = 0.

4. b. The level of liquid is the same in both the limbs. Pressure in limbs I at B = Pressure of limb II at A.

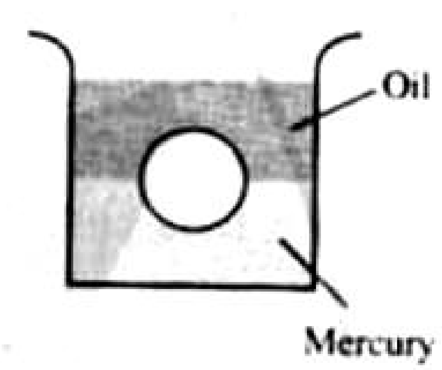


$$h\rho_1 g = h\rho_2 g$$

$$\Rightarrow \quad \rho_1 = \rho_2$$

5. c. Weight of sphere = Upthrust due to Hg + Upthrust due to oil, i.e.,

$$Vdg = \frac{V}{2}d_{Hg}g + \frac{V}{2}d_{oil} \times g$$



$$\Rightarrow \quad d = \frac{d_{Hg} + d_{oil}}{2}$$

$$= \frac{13.6 + 0.8}{2} = 7.2 \text{ g/cm}^3$$

6. a. Weight of cylinder = Upthrust due to upper liquid + upthrust due to lower liquid

$$\Rightarrow \quad D = \frac{A}{5} \times L \times g = d\left(\frac{A}{5}\right)\left[\frac{3}{4}L\right]g + 2d\left[\frac{A}{5}\right]\left[\frac{L}{4}\right] \times g$$

$\therefore \quad D = 5d/4$

7. a. Equating the rate of flow, we have

$$\sqrt{(2gy)} \times L^2 = \sqrt{(2g \times 4y)}\pi R^2$$

$$\Rightarrow \quad L^2 = 2\pi R^2 \quad \Rightarrow R = \frac{L}{\sqrt{2\pi}}$$

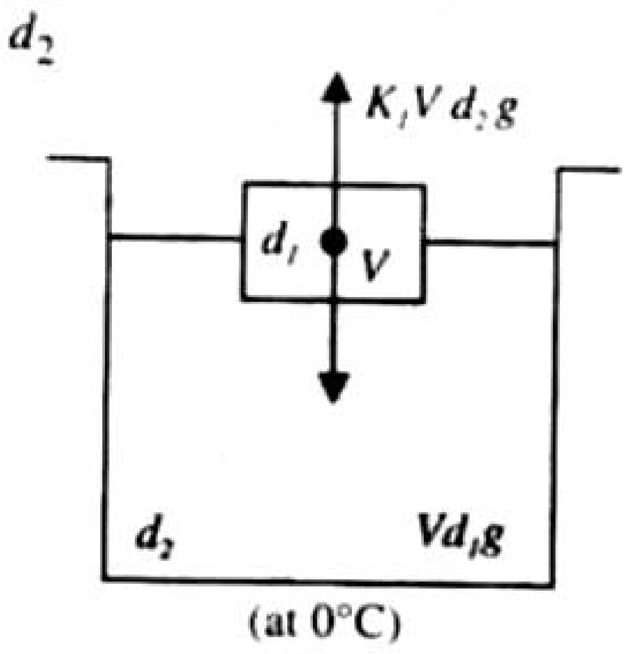
Flow = (area) × (velocity). Here, Vel = $\sqrt{2gx}$,

where x = ht from top

8. a. For equilibrium is case 1 at 0°C: Upthrust = Weight of the body

$$\Rightarrow \quad K_1 V d_2 g = V d_1 g$$

$$\Rightarrow \quad K_1 = \frac{d_1}{d_2} \qquad \text{(i)}$$

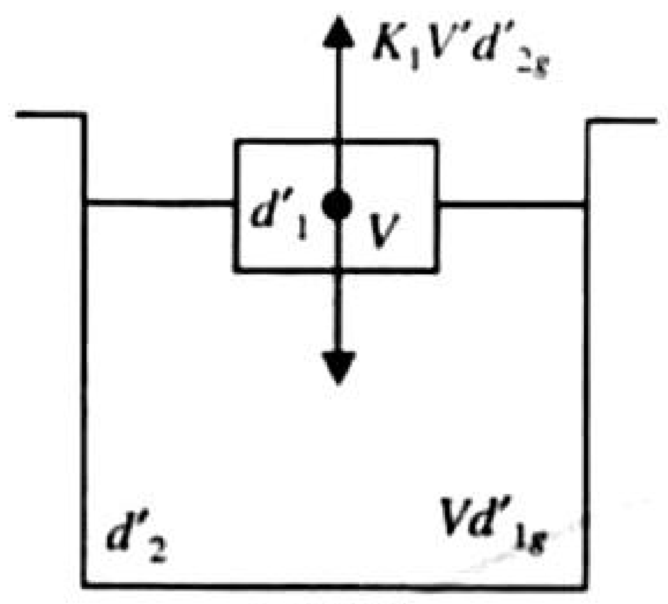


(at 0°C)

For equilibrium in case 2 at 60°C: When the temperature is increased, the density will decrease.

$\therefore \quad d'_1 = d_1(1 + \gamma_{Fe} \times 60)$ and $d'_2 = d_2(1 + \gamma_{Hg} \times 60)$

Again, upthrust = Weight of the body



(at 0°C)

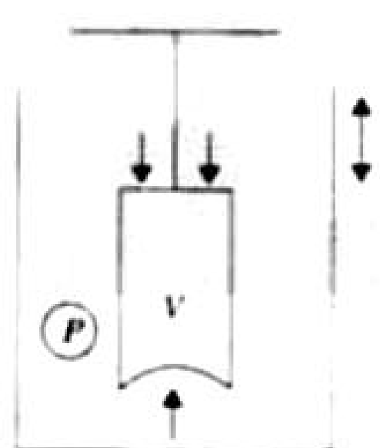
$$\therefore \quad K_2V'd'_2g = V'd'_1g$$

$$\therefore \quad K_2\left[\frac{d_2}{1+\gamma_{Hg}\times 60}\right] = \frac{d_1}{1+\gamma_{Fe}\times 60}$$

$$\therefore \quad K_2\left[\frac{1+\gamma_{Fe}\times 60}{1+\gamma_{Hg}\times 60}\right] = \frac{d_1}{d_2}$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{1+\gamma_{Fe}\times 60}{1+\gamma_{Hg}\times 60}$$

9. d. According to Archimedes' principle, upthrust = weight of the fluid displaced $F_{bottom} - F_{top}$ $= V\rho g$



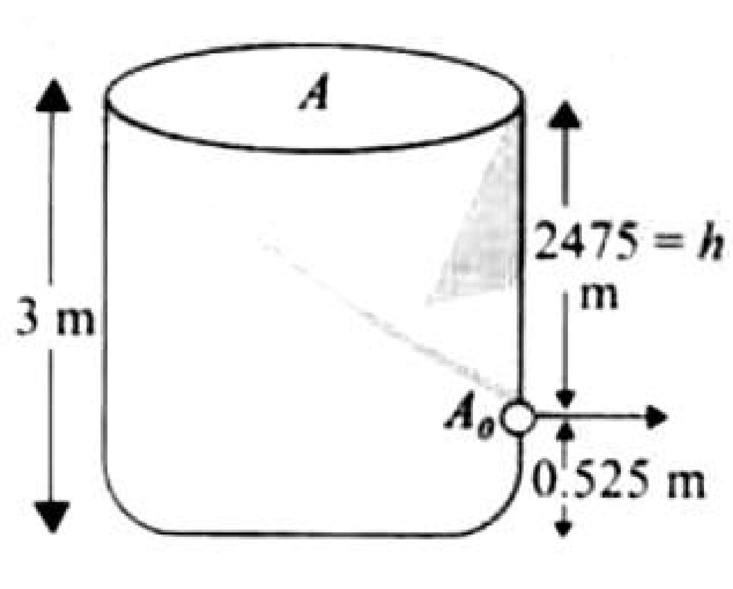
$$\therefore \quad F_{bottom} = F_{top} + V\rho g$$
$$= P_l \times A + V\rho g$$
$$= (h\rho g) \times (\pi R^2) + V\rho g$$
$$= \rho g[\pi R^2 h + V]$$

10. d. l decreases as the block moves up. H will also decrease because when the coin is in the water it will displace equal volume of water, whereas when it is on the block an equal weight of water is displaced.

11. a. The square of the velocity of flux

$$v^2 = \frac{2gh}{\sqrt{1-\left(\frac{A_0}{A}\right)^2}}$$

$$= \frac{2\times 10\times 2.475}{\sqrt{1-(0.1)^2}} = 50 \text{ m}^2/\text{s}^2$$



12. b. Force from right hand side of the liquid on left hand side liquid.

i. Due to surface tension force = $2RT$ (towards right)

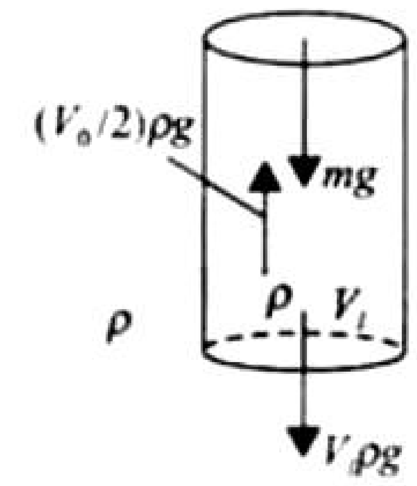
ii. Due to liquid pressure force

$$= \int_{x=0}^{x=h} (p_0 + \rho g h)(2Rx)dx$$

$$= (2p_0Rh + R\rho g h^2) \text{ (towards left)}$$

$\therefore$ Net force is $|\,2p_0Rh + R\rho g h^2 - 2RT\,|$

13. d. For flotation:



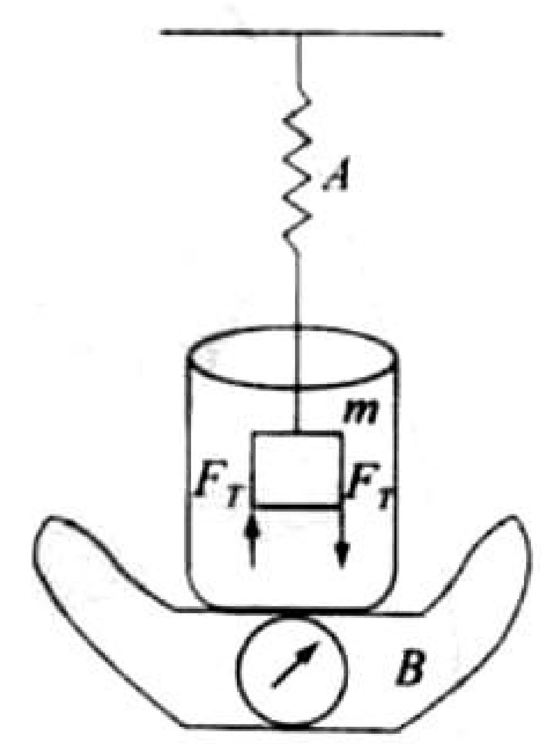
$$mg + \rho \times V_l \times g = \frac{V_0}{2} \times \rho \times g$$

$$V_l = \frac{V_0}{2} - \frac{m}{\rho}$$

So $\quad V_l < \dfrac{V_0}{2}$

Multiple Correct Answer Type

1. b, c. When the block of mass m is arranged as shown in the figure, an upthrust F_v will act on the mass which will decrease the reading on A. But according to Newton's second law, to each and every action, there is equal and opposite reaction. So F_v will act on the liquid of the beaker which will increase the reading in B.



2. a., b., d.

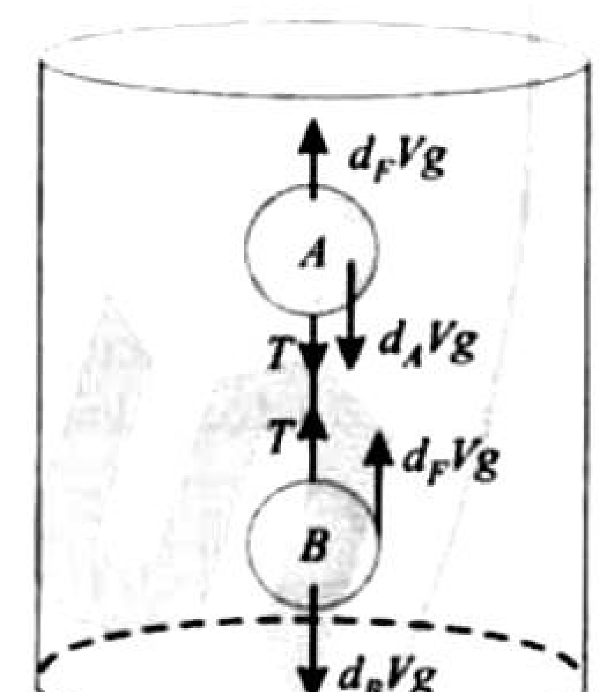
Let V be the volume of each sphere and T be the tension in the string.

For the string to be taut,

$$d_FVg > d_AVg$$
$$\Rightarrow \quad d_F > d_A$$
and $\quad d_EVg > d_FVg$
$$\Rightarrow \quad d_B > d_F$$

For equilibrium,

$$d_FVg + d_FVg + T$$
$$= T + d_AVg + d_EVg$$
or $\quad d_A + d_B = 2d_F$

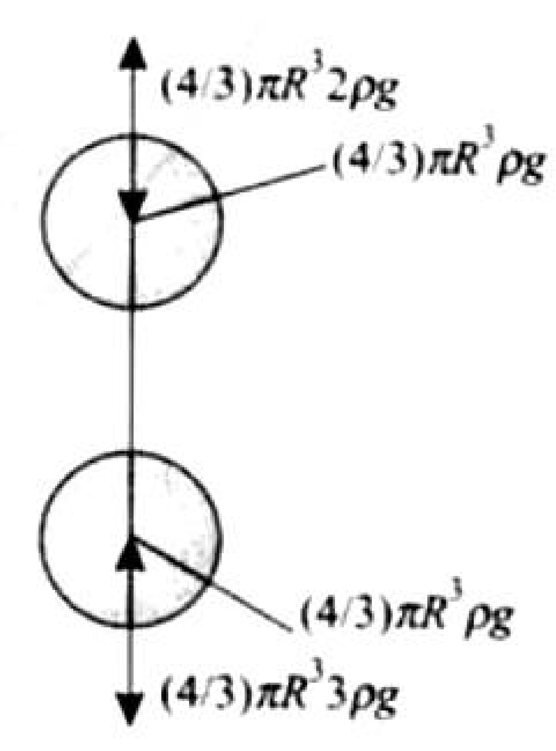


3. a, d. At equilibrium,

$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

$$T = \frac{4}{3}\pi R^3 \rho g$$

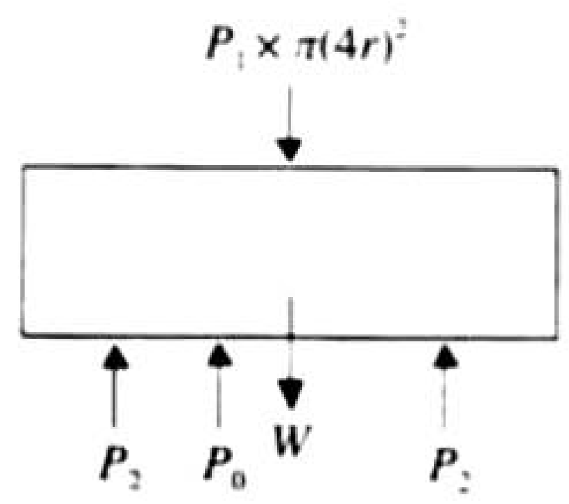
$$\therefore \quad \Delta l = \frac{4}{3k}\pi R^3 \rho g$$



For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So, both the spheres are completely submerged.

Linked Comprehension Type

1. c. Consider the equilibrium of wooden block. Forces acting in the downward direction are as follows.



a. Weight of wooden cylinder

$$= \pi(4r)^2 \times h \times \frac{\rho}{3} \times g$$

$$= \pi \times 16r^2 \frac{h\rho}{3} g$$

b. Force due to pressure (P_1) created by liquid of height h_1 above the wooden block is

$$= P_1 \times \pi(4r)^2 = [P_0 + h_1\rho g] \times \pi(4r)^2$$

$$[\rho_0 + h_1\rho g]\pi \times 16r^2$$

Force acting on the upward direction due to pressure P_0 exerted from below the wooden block and atmospheric pressure is

$$= p_2 \times \pi[(4r)^2 - (2r)^2] + P_0 \times (2r)^2$$

$$= [P_0 + (h_1 + h)\rho g] \times \pi \times 12r^2 + 4r^2 P_0$$

At the verge of rising,

$$[P_0 + (h_1 + h)\rho g]\pi \times 12r^2 + 4r^2 P_0$$

$$= \pi \times 10r^2 h \times \frac{\rho}{3} g + [P_0 + h_1\rho g] \times \pi \times 16r^2$$

$$12h_1 + 12h = \frac{16h}{3} + 16h$$

$$12h_1 - \frac{16h}{3} = 4h_1 \Rightarrow \frac{5h}{3} = h_1$$

2. b. Again considering equilibrium of wooden block.

Total downward force = Total force upwards Wt. of block + Force due to atmospheric pressure = Force due to pressure of liquid + Force due to atmospheric pressure

$$\pi(16r^2)\frac{\rho}{3} + g + P_0\pi \times 16r^2$$

$$= [h_2\rho g + p_0]\pi[16 - 4r^2] + P_0 \times 4r^2$$

$$\pi(16r^2)h\frac{\rho}{g} g = h_2\rho g \times \pi \times 12r^2$$

$$16\frac{h}{3} = 12h_2$$

$$\Rightarrow \quad h_2 = \frac{4}{9}h$$

3. a. When the height h_2 of water level is further decreased, then the upward force acting on the wooden block decreases. The total force downward remains the same. This difference will be compensated by the normal reaction by the tank wall on the wooden block. Thus, the block does not move up return to its original position.

4. d. As the bubble moves upwards, besides the buoyancy force (cause of which is pressure difference), only force of gravity and force of viscosity will act.

5. b. As there is no exchange of heat, process is adiabatic. Applying,

$$Tp^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

$$T_2 p_2^{\frac{1-\gamma}{\gamma}} = T_1 p_1^{\frac{1-\gamma}{\gamma}}$$

$$T_2 = T_1\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{1-\gamma}{\gamma}}$$

Substituting the values, we have

$$T_2 = T_0\left[\frac{p_0 + \rho_1 g(H-y)}{p_0 + \rho_1 gH}\right]^{\frac{\frac{5}{3}-1}{5/3}}$$

$$T_0 = \left[\frac{p_0 + \rho_l g(H-y)}{p_0 + \rho_l gH}\right]^{2/5}$$

6. b. Buoyancy force F = (volume of bubble) $(\rho_1)g$

$$= \left(\frac{nRT_2}{p_2}\right)\rho_l g$$

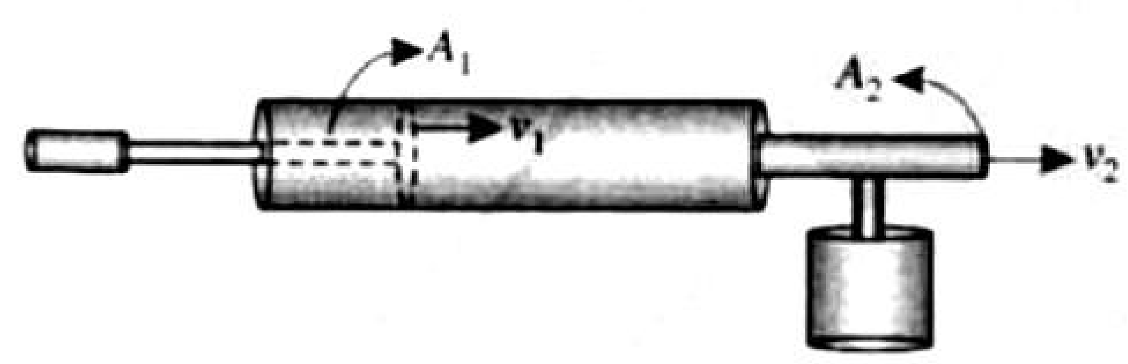
Here, $T_2 = T_0\left[\frac{p_0 + \rho_e g(H-y)}{p_0 + \rho_l gh}\right]^{2/5}$

and $\quad p_2 = p_0 + \rho_l g\,(H-y)$

Substituting the values, we get

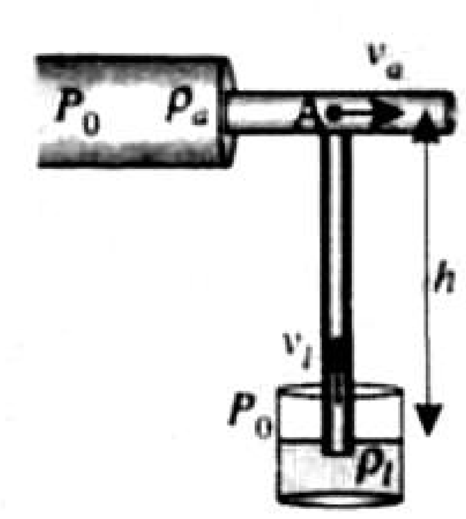
$$F = \frac{\rho_l n R_g T_0}{(p_0 + \rho_l gH)^{2/5}[p_0 + \rho_l g(H-y)]^{3/5}}$$

7. c. By continuity equation $A_1V_1 = A_2V_2$



$$\Rightarrow \quad \pi(20)^2 \times 5 = \pi(1)^2 V_2 \quad \Rightarrow \quad V_2 = 2\ \text{m/s}^2$$

8. a. Pressure at the point A in the nozzle is P.



Now, $P = P_0 - \frac{1}{2}\rho_a v_a^2$ (i) and also $P = P_0 - \frac{1}{2}\rho_l v_l^2 - \rho_l gh$ (ii)

From equations (i) and (ii)

$$\frac{1}{2}\rho_a v_a^2 = \frac{1}{2}\rho_l v_l^2 + \rho_l gh$$

$$\Rightarrow \quad v_l = \sqrt{\frac{\rho_a}{\rho_l}v_a^2 - 2gh}$$

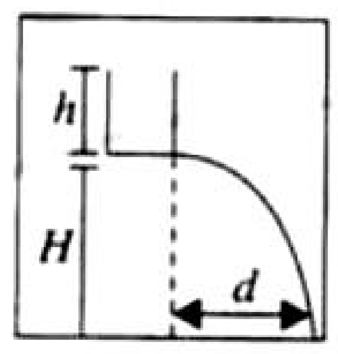
So for the given v_a and h, $v_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$

If a is area of cross-section of thin tube, then rate of flow of liquid will be equal to av_l.

Hence the rate (volume per unit time) at which the liquid is sprayed $(av_l) \propto \sqrt{\frac{\rho_a}{\rho_l}}$

Matching Column Type

1. c. When lift is moving up with acceleration a, speed of water from jar is given by, $v = \sqrt{2g_{effective}h}$.



Where $g_{effective} = (g + a)$

Hence $v = \sqrt{2(g+a)h}$ (i)

Time taken by water to reach the base of the lift is given by

$$H = \frac{1}{2}(g+a)t^2 \Rightarrow t = \sqrt{\frac{2H}{(g+a)}} \quad \text{(ii)}$$

Horizontal distance covered by water on the floor of the lift.

$d = vt$

$$= \sqrt{2(g+a)h} \times \sqrt{\frac{2H}{(g+a)}} = \sqrt{4Hh}$$

So d is independent of acceleration of the lift

$\therefore$ P, Q, R $\to$ 1.

(S) When lift falls freely, effective acceleration of the person in the lift holding the jar is zero, so no water leaks out of the jar $(v = \sqrt{2(g+a)h} = 0)$

S-4

So, P-1, Q-1, R-1, S-4

Integer Answer Type

1. (6) $P = P_0 - \rho gh = 98 \times 10^3$ N/m^2

$P_0V_0 = PV$

$10^5[A(500 - H)] = 98 \times 10^3[A(500 - 200)]$

$H = 206$ mm

Level fall = 206 − 200 = 6 mm

Assertion-Reasoning Type

1. d. From continuity equation, Av = constant

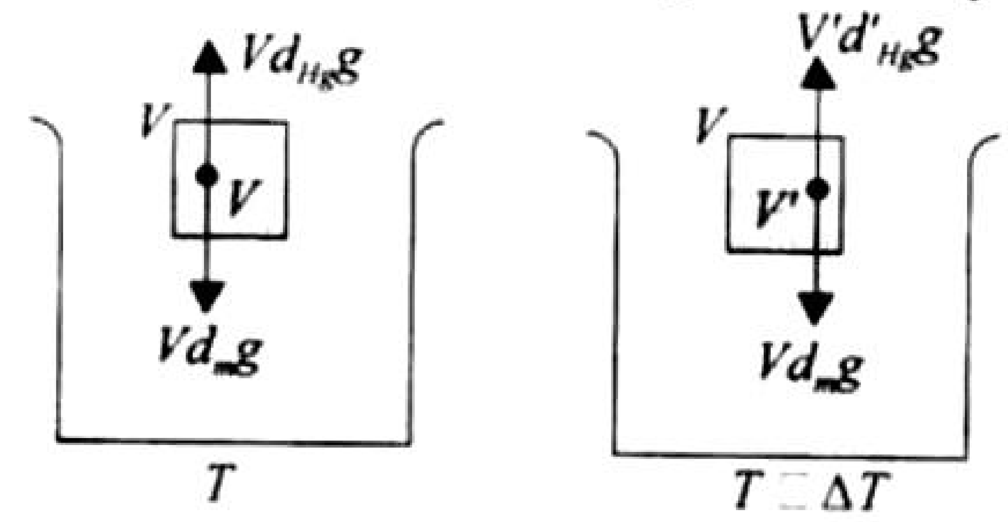
or $\quad A \propto \frac{1}{v}$

At lower heights, speed will be more. Therefore, area of cross section will be less.

Fill in the Blanks Type

1. $Vd_{Hg}g = Vd_m g$

Fraction of volume of metal submerged in mercury



$$\frac{v}{V} = \frac{d_m}{d_{Hg}} = K_1 \text{ (say)}$$

In second case $(T + \Delta T)$

$$v'd'H_g g = V'd'_m g$$

$$\Rightarrow \quad \frac{v'}{V'} = \frac{d'_m}{d'_{Hg}}$$

Fraction of volume of metal submerged in mercury = K_2 (say)

$$\therefore \quad \frac{K_2}{K_1} = \frac{d'_m \times d_{Hg}}{d'_{Hg} \times d_m} = \frac{d'_m \times d'_{Hg}(1+\gamma_2\Delta T)}{d'_{Hg} \times d'_m(1+\gamma_1\Delta T)}$$

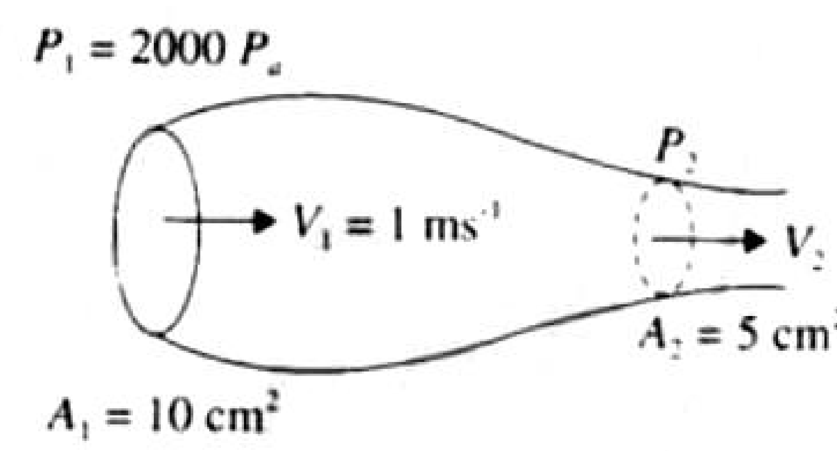
$$= \frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$$

$$\therefore \quad 1 - \frac{K_2}{K_1} = 1 - \frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$$

$$\Rightarrow \quad \frac{K_1 - K_2}{K_1} = \frac{1-\gamma_1\Delta T - 1 - \gamma_2\Delta T}{1-\gamma_1\Delta T}$$

$$\frac{K_1 - K_2}{K_1} = -\frac{(\gamma_1+\gamma_2)\Delta T}{1-\gamma_1\Delta T}$$

2. Applying the equation of continuity at section 1 and section 2, we get



$A_1V_1 = A_2V_2$

$\Rightarrow \quad 10 \times 1 = 5 \times V_2$

$\Rightarrow \quad V_2 = 2$ m/s

Applying Bernoulli's theorem, we get

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$\Rightarrow \quad 2000 + \frac{1}{2} \times 1000 \times 1^2 = P_2 + \frac{1}{2} \times 1000 \times 2^2$$

$$\Rightarrow \quad P_2 = 500 \text{ Pa}$$

Note: The conversion factor can be omitted if it gets cancelled out from the left hand side and right hand side.

True/False Type

1. False. When the man drinks some water from the pond, his weight increases and therefore the boat will sink further. The further sinking of the boat will displace the same volume of water in pond as drunk by man. Therefore, there will be no change in the level of water in the pond.

2. **False.** Pressure $P_1 = P_2 = 1$ atm $= h\rho g$. On changing the temperature, g will not change and atmosphere pressure will not change.

$\therefore \quad h \times \rho =$ constt,

When temperature is increased, the density of Hg decreases and hence h increases.

Subjective Type

1. When stones were floating with boat, they will be displacing water of volume (say V_1) whose weight should be equal to weight of stones. When the stones sink, they will displace water of volume (say V_2) whose volume is equal to the volume of stones. But since density of water is less than the density of stones.

$V_1 > V_2$ or level will fall.

2. Potential energy of liquid in cylinder 1 $U_1 = (m)g\frac{h_1}{2}$

$$\therefore \quad U_1 = (\rho \times A \times h_1)g\frac{h_1}{2} = \frac{\rho A g h_1^2}{2}$$

Similarly PE of liquid in cylinder 2 $U_2 = \frac{\rho A g h_2^2}{2}$

$\therefore$ Total PE initially $U = U_1 + U_2 = \frac{\rho A g}{2}(h_1^2 + h_2^2)$

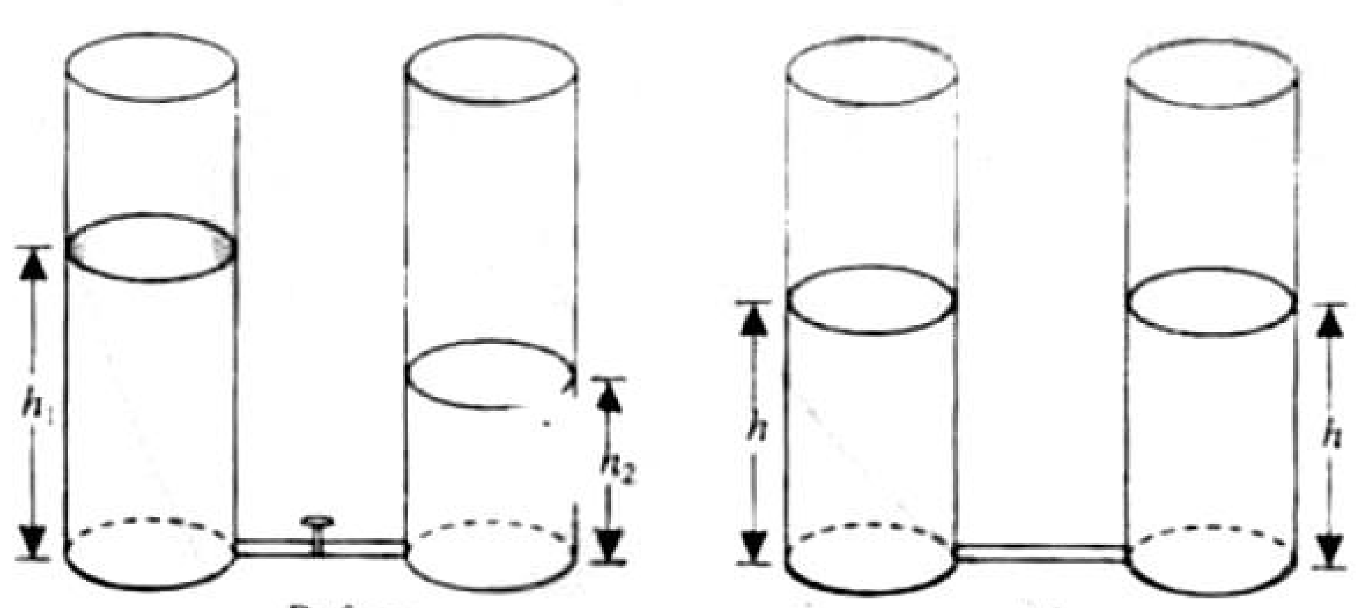
After the equalising of levels.

PE of liquid in cylinder 1 $U_1' = mg\frac{h}{2} = \frac{\rho A g}{2}h^2$

PE of liquid in cylinder 2 $U_2' = \frac{\rho A g}{2}h^2$

$\therefore$ Total PE finally $U' = U_2' + U_2' = \rho A g h^2$

Let h be the level in equilibrium. Equating the volumes, we have



Before After

$$Ah_1 + Ah_2 = 2Ah$$

$$\therefore \quad h = \left(\frac{h_1 + h_2}{2}\right)$$

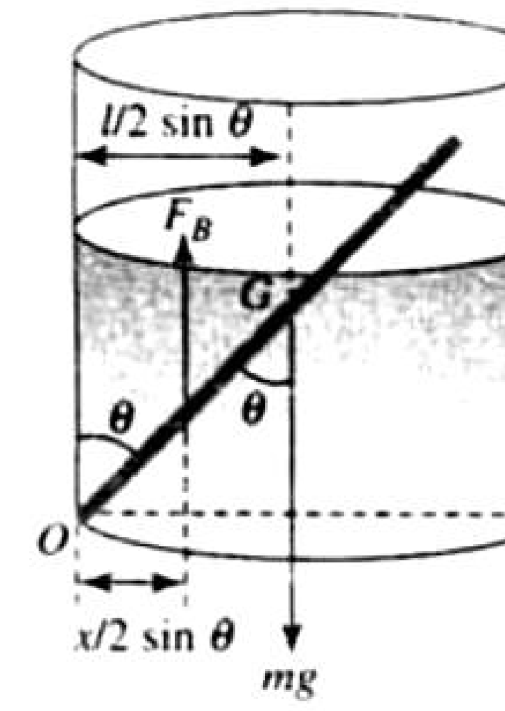
The change in P.E. $= U - U'$

$$= \rho A g\left[\frac{h_1^2}{2} + \frac{h_2^2}{2} - h^2\right]$$

$$= \rho A g\left[\frac{h_1^2}{2} + \frac{h_2^2}{2} - \left(\frac{h_1 + h_2}{2}\right)^2\right] = \frac{\rho A g}{2}(h_1 - h_2)^2$$

This change in PE is the work done by gravity.

3. a. For equilibrium



As plank is in equilibrium, $F_{net} = 0$ and $\tau_{net} = 0$

Let the length of the plank submerged inside the water is x. The upthrust force acting on the plank due to water.

$F_B = x_A \times 1000\, g$ (i)

where A is the area of cross section of the rod.

Also F_B = wt of fluid displaced.

Taking moment about 0

$$mg \times \frac{l}{2}\sin\theta = F_B\left(\frac{x}{2}\right)\sin\theta \qquad \text{(ii)}$$

From (i) and (ii)

$$mg \times \frac{l}{2}\sin\theta = (xA) \times 1000g\frac{x}{2}\sin\theta$$

$$0.5A \times 1 \times 1000 \times \frac{g}{2} = A \times 1000 \times g\frac{x^2}{2}$$

$$\therefore \quad x^2 = 0.5 \Rightarrow x = 0.707$$

From the diagram $\cos\theta = \frac{0.5}{x} = \frac{0.5}{0.707} \Rightarrow \theta = 45°$

4. a. Let the ball be dropped from a height h, using

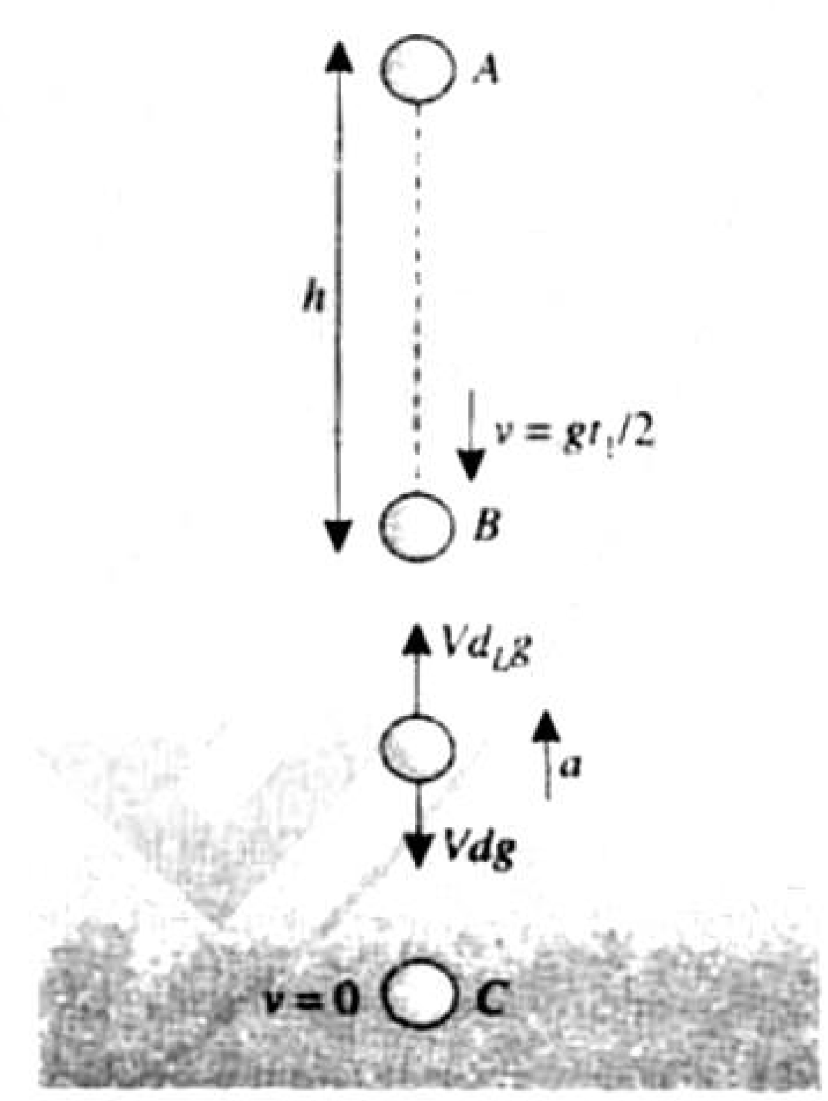
$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad h = \frac{1}{2}a\left[\frac{t_1}{2}\right]^2 = \frac{at_1^2}{8}$$

$$\Rightarrow \quad h = 0t + \frac{1}{2}at^2$$

Velocity of the ball (v) just before striking the ground, using equations $v = u + at$.

$$v = \frac{gt_1}{2} \qquad \text{(i)}$$



In the second case the ball is made to fall through the same height and then the ball strikes the surface and then the ball strikes the surface of liquid of density d_L when the ball reaches inside the liquid, it is under the influence of two force

i. Vdg, the weight of ball in downward direction

ii. Vd_Lg, the upthrust in upward direction

Note that viscous forces are absent (given) since $d_L > d$. The upward force is greater and the ball starts retarding. For motion B to C, the velocity of ball just before striking the water surface is $\frac{gt_1}{2}$.

Using $v = u + at \Rightarrow 0 = \frac{gt_1}{2} + (-a)t$

Which gives $t = \frac{gt_1}{2a}$, a is the retardation of ball inside the water.

Now, $a = \frac{F_{net}}{m} = \frac{Vd_Lg - Vdg}{Vd} = \frac{(d_L - d)g}{d}$

Again $t = \frac{gt_1}{2\left[\frac{(d_L - d)g}{d}\right]} = \frac{dt_1}{2(d_L - d)}$ (iii)

Therefore $t_2 = t_1 + 2t = t_1 + 2\left[\frac{dt_1}{2(d_L - d)}\right]$

$\Rightarrow \quad t_2 = \frac{d_L t_1}{d_L - d}$

b. Since the retardation is not proportional to displacement, the motion of the ball is not simple harmonic.

c. If $d = d_L$ then the retardation $a = 0$. Since the ball strikes the water surface with some velocity, it will continue with the same velocity in downward direction (until it is interupted by some other force).

5. a. i. Since the cylinder is in equilibrium in the liquid therefore Weight of cylinder = upthrust

F_{T_1} = Upthrust due to lower liquid

$mg = F_{T_1} + F_{T_2}$

F_{T_2} = upthrust due to upper liquid

$$\frac{A}{5} \times L \times D \times g = \frac{A}{5} \times \frac{L}{4} \times 2d \times g + \frac{A}{5} \times \frac{3L}{4} \times d \times g$$

$$\Rightarrow \quad D = \frac{2d}{4} + \frac{3d}{4} = \frac{5d}{4}$$

ii. Total pressure at the bottom of the cylinder = Atmosphere + Pressure due to liquid of density d + Pressure due to liquid of density $2d$ + Pressure due to cylinder

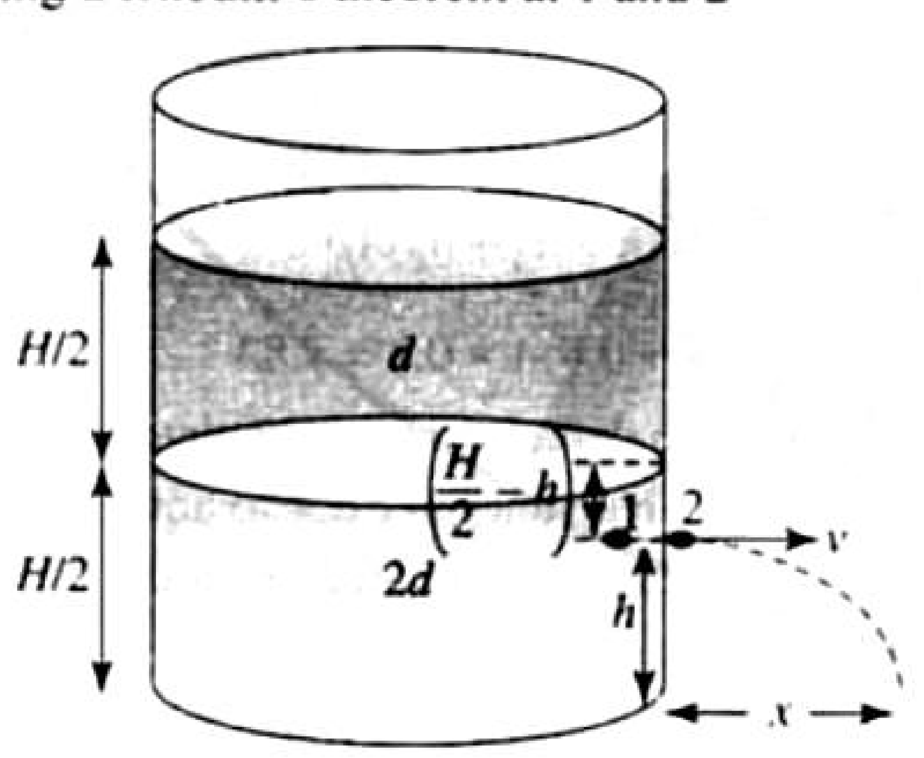
$$P = P_0 + \frac{H}{2}dg + \frac{H}{2} \times 2d \times g + \frac{\frac{A}{5} \times L \times D \times g}{A}$$

$$\left[\because P_{\text{cylinder}} = \frac{\text{weight of cylinder}}{\text{bottom area of cylinder}}\right]$$

$$P = P_0 + \frac{3H}{2}dg + \frac{5Ldg}{5 \times 4} \qquad \left[\because D = \frac{5d}{4}\right]$$

$$\Rightarrow \quad P = P_0 + \left(\frac{H}{2} + \frac{L}{4}\right)dg$$

b. Applying Bernoulli's theorem at 1 and 2



$$P_0 + \left[\frac{H}{2}dg + \left(\frac{H}{2} - h\right)2dg\right] = P_0 + \frac{1}{2}(2d)v^2$$

$$\therefore \quad \frac{Hdg}{2} + \frac{2Hdg}{2} - 2hdg = \frac{1}{2}(2d)v^2$$

$$\Rightarrow \quad v = \frac{\sqrt{(3H - 4h)}}{2}g$$

The path of the water jet after coming out of container is same as projectile motion of a particle from a height.

Now considering the motion of water jet. The water jet will fall a height h before striking the ground.

Using $s_y = u_y t + \frac{1}{2}a_y t^2$

$$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

Hence range of water jet $x = v.t$

$$x = v_x \times \sqrt{\frac{2h}{g}} = \sqrt{(3H - 4h)\frac{g}{2}} \times \sqrt{\frac{2h}{g}}$$

$$= \sqrt{(3H - 4h)h} \qquad \text{(iii)}$$

For finding the value of h for which x is maximum, we differentiate equation (iii) w.r.t t

$$\frac{dx}{dt} = \frac{1}{2}[(3H - 4h)h]^{-1/2}\{3H - 8h\}$$

Putting $\frac{dx}{dt} = 0$ for finding values h for maxima

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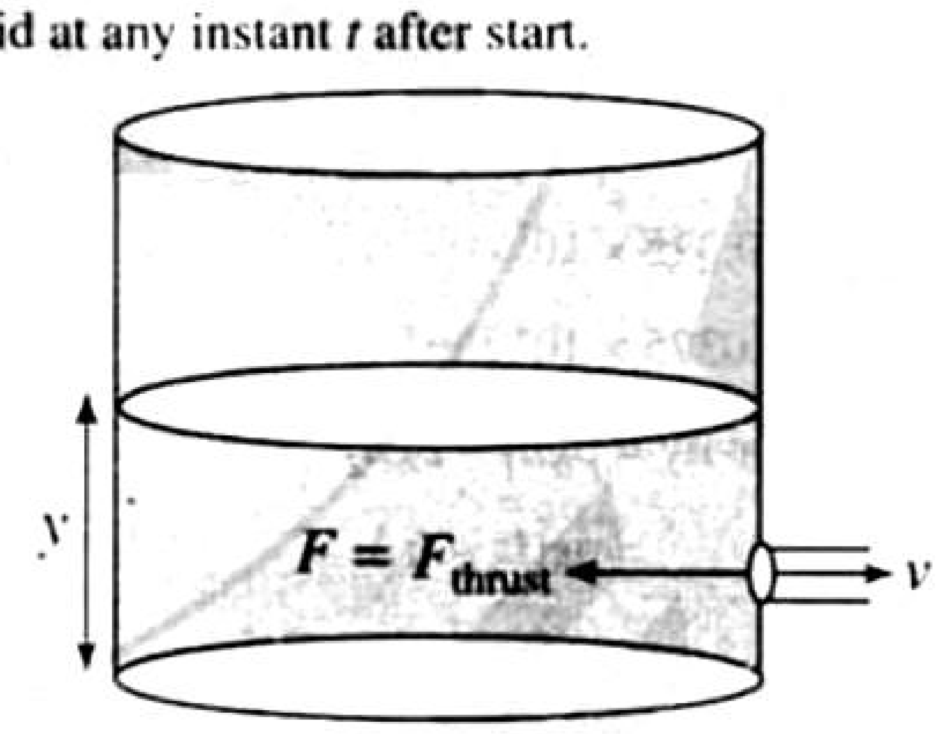
$$= \sqrt{\frac{12H}{8} \times \frac{3H}{8}} = \frac{6H}{8} = \frac{3H}{4}$$

6. Liquid is escaping from the container. Let y be the height of the liquid at any instant t after start.



Then velocity of escape is given by

$$v = \sqrt{2gy}$$

Mass of the liquid flowing in time $dt = av\rho dt$, where a = area of the hole = $A \cdot 100$.

Force is the rate of change of momentum. Hence,

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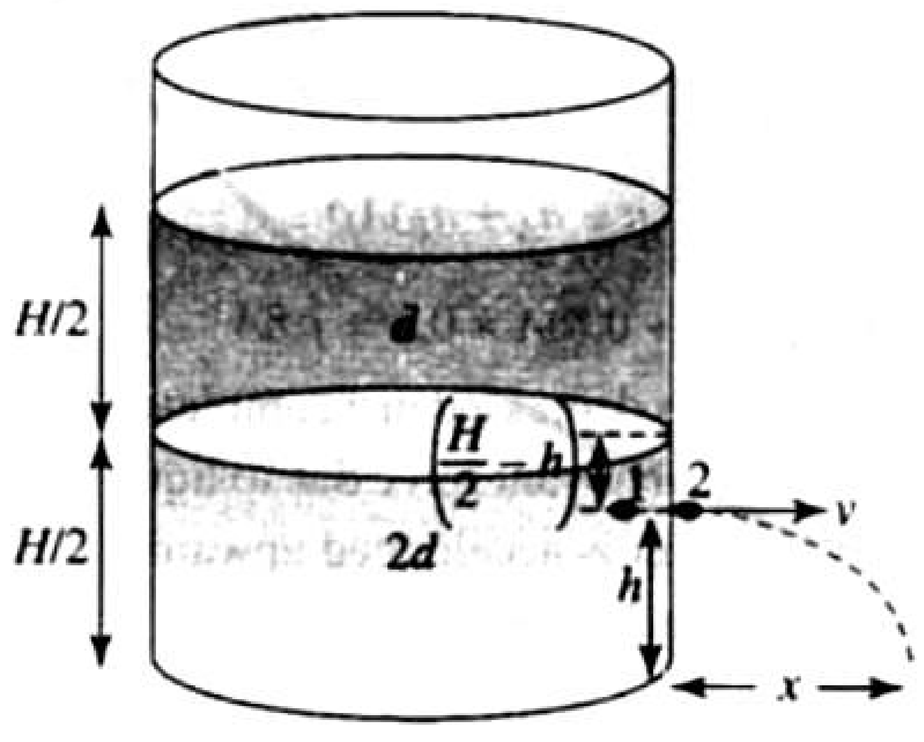

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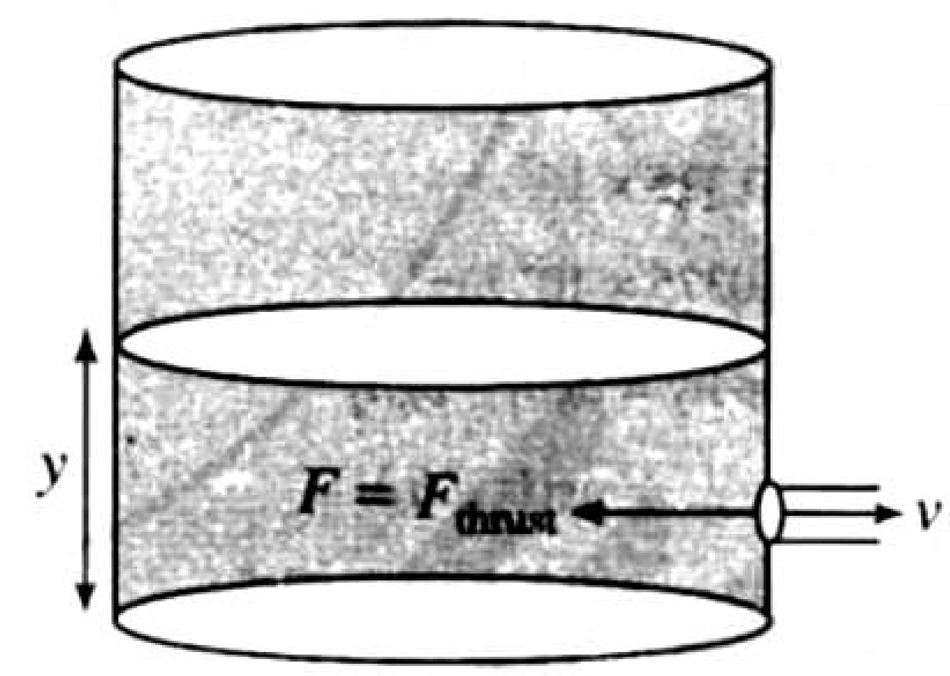

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Acceleration is given by $a = \frac{F}{m} = \frac{0.3Ag}{1.8A} = \frac{g}{6}$

$$= \frac{10}{6} = \frac{5}{3}\ \text{m/s}^2$$

10. From equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad \frac{A_1}{A_2} v_1 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right) v_1$$

or $\quad v_2 = \left(\frac{D}{d}\right)^2 v_1 = \left(\frac{8\times10^{-3}}{2\times10^{-3}}\right)^2 \times 0.25\ \text{m/s}$

$$= 4\ \text{m/s (horizontal)}$$

Vertical component of the velocity is zero.

Now, $H = \frac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2H}{g}}$$

Range is given by $R = v_2 t = v_2$

$$\sqrt{\frac{2H}{g}} = 4\times\sqrt{\frac{2\times1.25}{10}} = 2\ \text{m}$$

11. Weight of liquid of height H

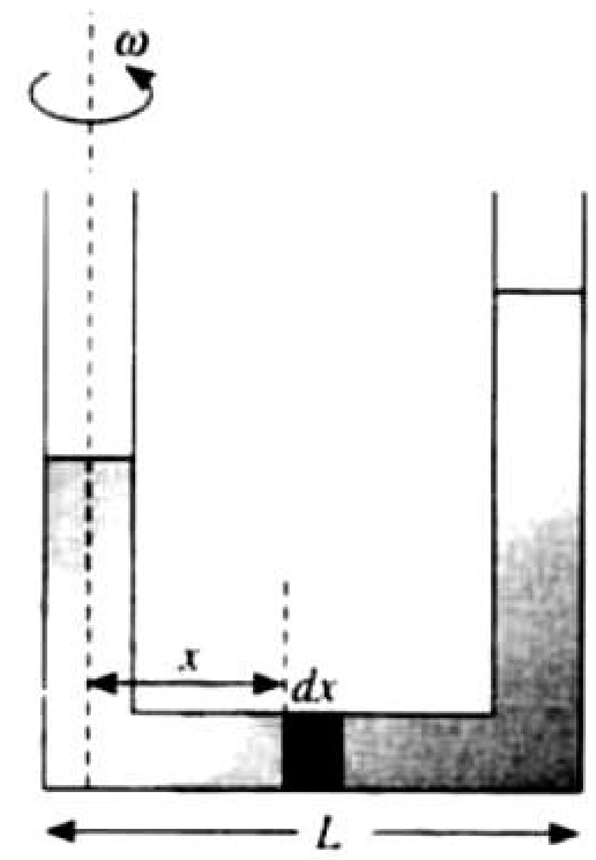
$$= \frac{\pi d^2}{4}\times H\times\rho\times g \quad \text{(i)}$$

Let us consider a mass dm situated at a distance x from A as shown in the figure.

The centripetal force required for the mass to rotate = $(dm)\,x\,\omega^2$

$\therefore$ The total centripetal force required for the mass of length L to rotate

$$= \int_0^L (dm)x\omega^2$$



Here, $dm = \rho\times\frac{\pi d^2}{4}\times dx$

$\therefore \quad$ Total centripetal force = $\int_0^L\left(\rho\times\frac{\pi d^2}{4}\times dx\right)\times x\omega^2$

$$= \rho\times\frac{\pi d^2}{4}\times\omega^2\int_0^L x\,dx = \rho\times\frac{\pi d^2}{4}\times\omega^2\times\frac{L^2}{2} \quad \text{(ii)}$$

This centripetal force is provided by the weight of liquid of height H.

From (i) and (ii)

$$\frac{\pi d^2}{4}\times H\times\rho\times g = \rho\times\frac{\pi d^2}{4}\times\frac{\omega^2\times L^2}{L}$$

$$H = \frac{\omega^2 L^2}{2g}$$