Chapter 6 Vedic Mathematics

6.1 In previous classes, we have learnt the multiplication by using "EkadhikenPoorven", "EkNeunenPoorven", "Nikhilam".In this chapter, we will again study another methods of addition, subtraction, multiplication, division, fractions, squares and square roots. If the operations of these methods are carried out by verbal explanation then calculations become very easy and fast.

6.2 Sankalan-Vyavkalnabhyam

This method is used for making calculations easy in our daily life. Use of this method is based upon wholeness of base number which is 10 or multiple of 10. In this method, deviations are taken from the whole base number for making major calculations easy.

Example 1 Find the sum $8+11+7+12+9+13$.

Solution Looking at these numbers carefully we find that 8 is 2 less than 10 and 12 is 2 more than 10. Similarly 9 is 1 less than 10 and 11 is 1 more than 10.

$$(10-2) + (10+1) + (10-3) + (10+2) + (10-1) + (10+3)$$

Arranging the numbers by expressing them in terms of whole base numbers, we have

$$(10-\not{2}) + (10+\not{2}) + (10+\not{1}) + (10-\not{1}) + (10-\not{3}) + (10+\not{3})$$

$= 20 + 20 + 20$

$= 60$

Here, -2, 2; 1, -1 and -3, 3 are the pair whose sum -2+2; 1-1; and -3+3 is zero.

Example 2 Find the sum 26+48+107+63+13+44.

Solution For making the given numbers the complete whole number we try to make in terms of 10 or multiple of 10.

26+63+48+13+107+44

By Sankalan-Vyavkalnabhyam

$= 30 - 4 + 60 + 3 + 50 - 2 + 10 + 3 + 110 - 3 + 40 + 4$

$= 30 + 60 + 10 + 50 + 110 + 40 - 4 + 3 - 2 + 3 - 3 + 4$

= 90 + 10 + 50 + 150 + 1

= 100 + 200 + 1

= 300 + 1 = 301

If we continue finding deviations and summing up the numbers in Sankalan-Vyavkalnabhyam, we can make addition easy.

6.3 Poornapoornabhyam

Make the pairs of numbers in such a way that the sum of each pair becomes a multiple of 10.

Example 3 Find the sum 27 + 58+392 + 68+ 32+ 23

Solution

= (27+23)+(58+392)+(68+32) (Try for making multiple of 10)

= 50 + 450 + 100

= (50 + 450) + 100

= 500 + 100

= 600

Example 4 Find the sum 45 + 67 + 38 + 55 + 62 + 33.

Solution Arranging terms of pairs of multiples of 10, we get

= (45 + 55) + (67 + 33) + 38 + 62

= 100 + 100 + 100

= 300

Exercise 6.1

1. Find the following sum using Sankalan-Vyavkalnabhyam and Poornapornabhyam.

(i) 282 + 718 + 796 + 524 + 804 + 376

(ii) 52 + 136 + 48 + 64

(iii) 135 + 248 + 322 + 65

6.4 Subtraction (Nikhilam Sutra)

(We do subtraction by using Nikhilam Navatah Charmam Dashatah Sutra).

If we want to subtract 362 from 1000 then use of conventional method requires many stages to follow, takes more time and chances of getting wrong answer are still high. Let us use Vedic method:

Start from right hand side and do calculations towards left. Write 9 for each 0 on left and 10 for last zero. The digit on extreme left before 0 will be reduced by 1. Doing this we have.

1000	will become	0 9 9 10
– 362		0 3 6 2
		0 6 3 8

Example 5 Subtract 1837 from 70,000.

Solution 1 less than extreme left digit (7) = 6

Now	1 less from 9	=	8
	8 less from 9	=	1
	3 less from 9	=	6
Last digit	7 less from 10	=	3
i.e.,	Remainder will be 68163.		

So, 70,000 – 1837 = 68163.

Example 6 Subtract 569 from 854.

Solution 854 – 569

Step 1 Here 4 < 9

So we take complement of 9 – 4 = 5.

Complement will be taken from 10. Therefore, complement of 5 is 5, which we write in unit place.

Step 2 Since 5 < 6 so difference of 5 and 6 is 1. Subtracting 1 from the complement 9 will give 8.

Step 3 1 less than 8 , 8 – 1 = 7. Subtracting 5 from 7 gives 2 which we write at hundreds place.

854 – 569 = 285.

6.5 Interesting Multiplication Methods

You have learnt multiplication using Nikhilam method in class VI. In this class, we will study easy methods of multiplication.

6.5.1 Multiplication of any number by 10

For example $5 \times 10 = 50$ $10 \times 10 = 100$

$68 \times 10 = 680$.

Look at these three examples and discuss with your friends 'what difference do you visualize in original numbers (5, 10, 68) when multiplied by 10'? Perhaps, you will agree that 0 appears at unit place and original number shifts to tens place and ahead.

Do and learn

1. If a number is multiplied by 100 and 1000 then what change do you observe in the product? Discuss with your friends.
2. Divide the class into two groups. One group should ask the question of multiplying the numbers by 10, 100 and 1000 and another group should answer and vice-versa. Play it like "ANTYAKSHARI".

6.5.2 Multiplication of a number by 5

You have learnt the multiplication of a number by 10. Now we will learn multiplication by 5 in a simple and interesting manner.

(i) 18×5

$= 18 \times \frac{10}{2}$ (5 being the base of 10 and can be written as $5 = \frac{10}{2}$

$= \frac{18}{2} \times 10 = 9 \times 10 \left(\frac{18}{2} = 9\right)$

$= 90$

(ii) 29×5

$= 29 \times \frac{10}{2}$

$= \frac{29}{2} \times 10$

$= 14.5 \times 10$

$= \frac{145}{10} \times 10 = 145$

$\left(5 = \frac{10}{2}\right)$

$\left(\frac{29}{2} = 14.5\right)$

$\left(14.5 = \frac{145}{10}\right)$

So, while multiplying a number by 5 the result is obtained by taking half of the number and multiplying it by 10.

Do and learn

1. Can we use the method of multiplication by 5 in the case of 50 and 500?
2. Can we multiply a number by 25 in the form of $\frac{100}{4}$ discuss in you class.

6.5.3 Multiplication of a number by 9 (Formula – EkNeunenPoorven Method)

Example 7 Multiply 6 by 9.

Solution

$$\begin{array}{r} 6 \\ \times\ 9 \\ \hline \dot{6}/9-\dot{6} \\ 5/4 \\ =54 \end{array}$$

(I) We use EkNeunenPoorven formula. So, we put the sign of Ek Neunen and write 6 on the left hand side of slash.

(ii) We subtract the ekneunen of multiplicand 6 from 9 on right hand side.

Example 8 Multiply 12 by 9.

Solution

$$\begin{array}{r} 12 \\ \times 9 \\ \hline \end{array}$$

$1\underset{\bullet}{2} / 9 - 1\underset{\bullet}{2}$

$11 / 9 - 11$

$11 / -2$ या $(\bar{2})$

$11\bar{2}$

$= 108$

(i) Here the multiple is 9 but the multiplicand is greater than 9.

(ii) Using EkNeunenPoorven put one less than 12 = 11 on the left hand side of slash

(iii) Subtract 11 from 9 (one less than 12), i.e., 9-11 on the right hand side of slash

(iv) There is 11 on the left and -2 or $\bar{2}$ on the right of slash.

(v) Removing the slash in $11\bar{2}$ and converting in normal form we obtain 108.

6.5.4 Multiplication of a number by 99

You have learnt the multiplication of a number by 9. Let us now learn the multiplication by 99. The method of multiplication by 99 is same as that of 9. So, we understand by an example using EkNeunenPoorven method.

Example 9 Solve 18 × 99.

Solution

$$\begin{array}{r} 18 \\ \times 99 \\ \hline \end{array}$$ (Symbols as usual)

$1\underset{\bullet}{8}/99-1\underset{\bullet}{8}$

17/99-17

17/82

=1782

Example 10 Solve 99 × 99.

Solution

$$\begin{array}{r} 99 \\ \times 99 \\ \hline \end{array}$$

$9\underset{\bullet}{9}/99-9\underset{\bullet}{9}$

98/99-98

98/1

[Is 99 × 99 = 981 correct? If not then could you discover the point of mistake? Yes you are correct. The base on the right hand side is 100 so, there should be two digit number but it is not so. Therefore we will write 1 as 01.]

So the solution will be 9801 and not 981.

Can you multiply a number by 999 and 9999?

Do and learn

Multiply any number by 999 and 9999 yourself and ask you teacher in case of any problem.

6.5.5 Multiplication of a number by 11

Let us learn a simple method of multiplication by 11.

Multiply 72 by 11.

$$\begin{array}{r} 72 \\ \times 11 \\ \hline 72 \\ 72\times \\ \hline =7(7+2)2 \\ =792 \end{array}$$

Let us see one more method:

72 × 11

72 × (10+1)

720+72

i.e., 7(7+2) 2

= 792

We find, in both the method, that there lies the sum of both the digits of multiplicand in between the digits of multiplicand.

Example 11 Multiply 81 by 11.

Solution

$$\begin{array}{c} 81 \\ \times 11 \\ \hline 8/(8+1)/1 \\ 891 \end{array}$$

Check if 81 × 11 = 891.

Example 12 Multiply 99 by 11.

Solution

$$\begin{array}{c} 99 \\ \times 11 \\ \hline 9/(9+9)/9 \\ 9/18/9 \\ =1089 \end{array}$$

(in the number 18, 8 will remain at tens place and 1 will be added to the number at hundreds place)

Could we apply this rule in case of three digit numbers?

Discuss and practice such problems.

Exercise 6.2

1. Subtract using Nikhilam formula:

(i) 9000 − 3768 (ii) 5872 − 2987 (iii) 4987 − 1898

$$\begin{array}{r} 9000 \\ -3768 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 5872 \\ -2987 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4987 \\ -1898 \\ \hline \\ \hline \end{array}$$

2. Multiply using appropriate formula:

(i)	87×10	(ii)	53×100	(iii)	432×1000	(iv)	64×5
(v)	72×50	(vi)	81×99	(vii)	99×999	(viii)	99×9

6.6 Fractions

You are familiar with fractions. We make fractions easy using Vedic Mathematics. Look at the following fractions carefully:

Example 13 $\frac{5}{8}, \frac{3}{8}, \frac{7}{8}, \frac{1}{8}$ Arrange in increasing order.

Solution Denominators are same but numerators are different in thee fractions.

We can write them in increasing order.

$\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$

The fraction in which the denominators are same, greater is the numerator greater would be the fraction. If numerators are same then a fraction having greater denominator would be smaller than that having smaller numerator.

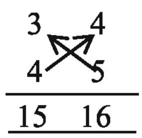
Arrange $\frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$ in increasing order.

Here, the denominator 5 is the largest number. So, the smallest fraction would be $\frac{1}{5}$ and the largest fraction would be $\frac{1}{2}$ Arranging in increasing order

$\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

Example 14 Identify the greater fraction in $\frac{3}{4}$ and $\frac{4}{5}$

Solution



(i) Write the numerators and denominators of these fractions without lines.

(ii) Cross multiplications are $3 \times 5 = 15$ and $4 \times 4 = 16$.

(iii) Fraction lying on the side of larger product will be greater.

(iv) Since $15 < 16$, so $\frac{3}{4} < \frac{4}{5}$

Example 15 Identify the order of the fractions $\frac{2}{3}$ and $\frac{6}{9}$

Solution

$$\begin{array}{cc} 2 & 6 \\ 3 & 9 \\ \hline 18 & 18 \end{array}$$

(i) Cross multiplications are 3×6 = 18 and 2×9 = 18.
(ii) Products are same, so the fractions are equal.
(iii) Hence, these are equivalent fractions.

Exercise 6.3

1. Put appropriate sign between following fractions (any one of <, =, >):

   (i) $\frac{4}{9} \square \frac{3}{9}$ (ii) $\frac{4}{5} \square \frac{4}{10}$ (iii) $\frac{3}{5} \square \frac{6}{10}$

   (iv) $\frac{5}{7} \square \frac{6}{7}$ (v) $\frac{2}{3} \square \frac{3}{2}$

2. Arrange following fractions in ascending order:

   (i) $\frac{3}{7}, \frac{4}{7}, \frac{2}{7}, \frac{5}{7}$ (ii) $\frac{3}{5}, \frac{3}{7}, \frac{3}{4}, \frac{3}{8}$

3. Arrange following fractions in descending order:

   (i) $\frac{4}{5}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ (ii) $\frac{4}{6}, \frac{4}{7}, \frac{4}{8}, \frac{4}{5}$

6.6.1 Sum of fractions

If the denominators in the fractions are same then:

Example 16 Find the sum $\frac{1}{5} + \frac{2}{5}$

Solution

$= \frac{1+2}{5}$ $\quad \frac{\text{sum of numerators}}{\text{denominator}}$

So, sum of fractions = $\frac{\text{sum of numerators}}{\text{denominator}}$

If the denominators in the fraction are not same then

Example 17 Find the sum of fractions $\frac{2}{3}$ and $\frac{4}{5}$

Solution

$\frac{2}{3} + \frac{4}{5}$

$= \frac{2\times5+3\times4}{3\times5}$

Cross multiplication values are 2×5=10 and 3x4=12
Product of denominator digits 3×5= 15

$$= \frac{10+12}{15} \quad = \frac{22}{15} \quad = 1\frac{7}{15}$$

Example 18 $\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$ Find the sum

Solution

$$\frac{1\times3\times5+2\times2\times5+4\times2\times3}{2\times3\times5}$$

Cross multiplication values are $1\times3\times5$, $2\times2\times5$ and $4\times2\times3$.

$$= \frac{15+20+24}{30}$$

Product of denominator digits $2\times3\times5 = 30$

$$= \frac{59}{30} \quad = 1\frac{29}{30}$$

When given fractions do not have equal denominators and there exist common factors:

Example 19 Solve $\frac{1}{4} + \frac{1}{10}$

Solution

$$\frac{1\times10+1\times4}{4\times10} \quad = \frac{10+4}{40} \quad = \frac{14}{40}$$

(To convert in to the simplest form, divide by same number)

$$= \frac{14 \div 2}{40 \div 2} \quad = \frac{7}{20}$$

(To be written in the simplest form)

6.6.2 Sum of mixed fractions (by Vilokanam formula and cross multiplication)

Product of mixed fractions could be evaluated easily by using Vilokanam formula and cross multiplication.

$1\frac{3}{4} + 2\frac{1}{3}$ (Split mixed fractions using Vilokanam formula)

$1\frac{3}{4} = 1+\frac{3}{4}$ and $2\frac{1}{3} = 2+\frac{1}{3}$

$$= 1+\frac{3}{4}+2+\frac{1}{3}$$

$$= (1+2)+\left(\frac{3}{4}+\frac{1}{3}\right)$$

($\frac{3}{4}+\frac{1}{3}$ By cross multiplication)

$$= 3 + \frac{3\times3+1\times4}{4\times3} \quad = 3 + \frac{9+4}{12} \quad = 3 + \frac{13}{12} \quad = 3 + 1\frac{1}{12}$$

(using vilokanam)

$$= (3+1) + \frac{1}{12} \quad = 4 + \frac{1}{12} \quad \text{or} \quad 4\frac{1}{12}$$

6.7 Subtraction of fractions

Subtraction operation of fractions is similar to the addition operation of fractions. We would use (+) sign in addition operation and (-) sign in subtraction operation.

6.7.1 Subtraction of fractions when denominators are equal

Example 20 Solve $\frac{3}{5} - \frac{1}{5}$

Solution

$$\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$$

6.7.2 Subtraction when denominators in fraction are not same and the common factors do not exist

Example 21 Solve $\frac{4}{5} - \frac{2}{3}$

Solution

$$\frac{4\times3-5\times2}{5\times3} = \frac{12-10}{15} = \frac{2}{15}$$

Example 22 Solve $\frac{1}{2}+\frac{1}{3}-\frac{1}{5}$

Solution

$$\frac{1\times3\times5+1\times2\times5-1\times2\times3}{2\times3\times5}$$ (solution is like addition of fractions)

$$= \frac{15+10-6}{30} = \frac{19}{30}$$

6.7.3 Subtraction of mixed fractions

Similar to addition operation, the subtraction of mixed fractions can be carried out using Vilokanam and cross multiplication.

Example 23 Solve $3\frac{3}{4} - 3\frac{2}{5}$

Solution

$$\left(3+\frac{3}{4}\right)-\left(3+\frac{2}{5}\right)$$

$$(3-3)+\left(\frac{3}{4}-\frac{2}{5}\right)$$

$$= 0 + \frac{3\times5-4\times2}{4\times5}$$

$$= \frac{15-8}{20} = \frac{7}{20}$$

Exercise 6.4

1. Find the sum (Vilokanam formula and cross multiplication method)

(i) $\frac{1}{9}+\frac{4}{9}$ (ii) $\frac{7}{15}+\frac{2}{15}$ (iii) $\frac{1}{2}+\frac{3}{5}$

(iv) $\frac{4}{3}+\frac{2}{5}$ (v) $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ (vi) $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}$

2. Subtract the following (Vilokanam formula and cross multiplication method)

(i) $\frac{9}{10}-\frac{3}{10}$ (ii) $\frac{19}{5}-\frac{4}{5}$ (iii) $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$

(iv) $\frac{1}{3}+\frac{1}{4}-\frac{1}{5}$ (v) $3\frac{1}{2}-1\frac{3}{4}$ (vi) $2\frac{5}{6}-2\frac{1}{6}$

6.8 Product of fractions

Product of two fractions can be evaluated very easily. In this, the product of numerators is placed at numerator and the product of denominators is placed at denominator in the resulting fraction.

Multiply $\frac{1}{2}$ and $\frac{3}{4}$

$\frac{1}{2}\times\frac{3}{4}$ $\frac{1\times3}{2\times4}=\frac{3}{8}$

6.8.1 Product of two mixed fractions (use of 'EkadhikenPurven ' formula)

If the sum of fractional parts of a mixed fraction is 1 and base and the Nikhilam digit is same then the product can be written in two parts as in the case of product of two simple numbers using "EkadhikenPoorven Formula".

Example 24 Solve $6\frac{1}{4}\times6\frac{3}{4}$

Solution (i) Sum of fractional parts $\frac{1}{4},\frac{3}{4}$ $\frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}=1$

(ii) Nikhilam digits are same = 6

(iii) Left Hand Side = First Part = Nikhilam Digit × Its Ekadhik (One more)

(iv) Right Hand Side = Second Part = Product of fractional part

$6\times(6+1)/\frac{1}{4}\times\frac{3}{4}=\frac{3}{16}$

So $6\times(6+1)+\frac{1}{4}\times\frac{3}{4}$

$6\times7+\frac{3}{16}$

$42+\frac{3}{16} \quad =42\frac{3}{16}$

Example 25 Multiply the fractions $15\frac{4}{7}\times15\frac{3}{7}$

Solution

$15\times(15+1)/\frac{4}{7}\times\frac{3}{7}$

$15\times16/\frac{12}{49}$

$240\frac{12}{49}$

6.8.2 Product of two fractions (using Vilokanam formula)

Example 26 Multiply the fractions $5\frac{1}{2}\times6$

Solution

$\left(5+\frac{1}{2}\right)\times6$ (Vilokanam Formula)

$=5\times6+\frac{1}{2}\times6$ (Solution of bracket)

$= 30+3$ (Half of 6 = 3)

$=33$

Verification of answer: $5\frac{1}{2}\times6$

$=\frac{11}{2}\times6$ $\left(5\frac{1}{2}=\frac{11}{2}\right)$

$=11\times\frac{6}{2}$

$= 11\times3$ (Half of 6 = 3)

$=33$

Example 27 Multiply the mixed fraction $7\frac{1}{2}\times8\frac{1}{2}$

Solution

$\left(7+\frac{1}{2}\right)\times\left(8+\frac{1}{2}\right)$ (By Vilokanam Formula)

$$7\times8+7\times\frac{1}{2}+8\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}$$

$$=56+3\frac{1}{2}+4+\frac{1}{4}$$

$$=56+3+4+\frac{1}{2}+\frac{1}{4}$$

$$=63+\frac{6}{8} \qquad \left(\frac{6\div2}{8\div2}=\frac{3}{4}\right)$$

$$=63\frac{3}{4}$$

Alternative Method $7\times8+\frac{1}{2}\times\frac{1}{2}+(7+8)\frac{1}{2}$

$$=56+\frac{1}{4}+15\times\frac{1}{2}$$

$$=56+7+\left(\frac{1}{4}+\frac{1}{2}\right)$$

$$=63+\frac{3}{4}$$

$$=63\frac{3}{4}$$

Exercise 6.5

Multiply the following numbers using Vilokanam formula.

(1) $\frac{1}{8}\times\frac{3}{5}$ (2) $5\frac{1}{2}\times5\frac{1}{2}$ (3) $2\frac{3}{4}\times2\frac{1}{4}$

(4) $3\frac{2}{5}\times3\frac{3}{5}$ (5) $12\frac{1}{4}\times12\frac{3}{4}$ (6) $8\frac{2}{7}\times8\frac{5}{7}$

(7) $3\frac{1}{4}\times4$ (8) $2\frac{1}{5}\times5$ (9) $3\frac{1}{2}\times4$

(10) $4\frac{1}{3}\times6$

6.9 Square Numbers

Square numbers are those for which the prime factors exist in pairs. For example: 4 is a square number because its prime factors = 2 × 2.
Here 2 is in a pair.
Is 100 a square number?
Let us find the prime factors of 100. These are 2×2×5×5. Here we have two pairs of 2 and 5. Hence both the numbers are square numbers.

Let us decide which numbers squares these are?

Prime factors of 4 = 2× 2. Here we have a pair of 2, so it is a square of 2.

Prime factors of 100 = 2 × 2 × 5 × 5 (Pairs of 2 and 5)

So, 2 × 5 = 10, i.e., it is the square of 10.

To find the square of a number we multiply the number by itself. Let us discuss some easy method to find square of the numbers.

(1) Finding the square of two/three digit numbers in which unit place digit is 5:

(i) 15×15 = 1×(1+1)/5×5 (Ekadhikenpoorven of tens digit)
= 1×2/25
= 2/25
= 225

(ii) 35×35 = 3×(3+1)/5×5 (Ekadhikenpoorven of tens digit)
= 3×4/25
= 1225

(iii) 95×95 = 9×(9+1)/5×5
= 9×10/25
= 9025

(iv) 105×105 = 10(10+1)/5×5
= 10×11/25
= 110/25 = 11025

(v) 125×125 = 12(12+1)/5×5
= 12(13)/25
= 15625

It is clear from the examples that a number having 5 at ones place is multiplied by itself or upon squaring a number there appears 25 in the last essentially. We write the product of tens digit and its next digit (Ekadhiken) before it.

Square of numbers having 5 at tens place

The numbers 51 to 59 are the only numbers having 5 at tens place.

So, 51^2 = 51×51
= 26 01

→ 1×1 = 01 (Square of unit place digit)

→ 5×5+1 = 26 (Square of tens place digit + unit place digit)

$53^2 = 53 \times 53$

28 09

- $3 \times 3 = 09$ (Square of unit place digit)
- $5 \times 5 + 3 = 28$ (Square of tens place digit + unit place digit)

$59^2 = 59 \times 59$

34 81

- $9 \times 9 = 81$
- $5 \times 5 + 9 = 34$

Square of three digits numbers having 25 in the end

$125^2 = 125 \times 125$

- $(25 \times 25 = 625)$
- $1 \times 15 = 15$ (Product of the number formed by unit and hundreds place in 125 with 1 at hundreds place)

So, $125^2 = 15625$

$325^2 = 325 \times 325$

- $(25 \times 25 = 625)$
- $3 \times 35 = 105$ (Product of the number formed by unit and hundreds place in 325 with 3 at hundredth place)

So, $325^2 = 105625$

$725^2 = 725 \times 725$

- $(25 \times 25 = 625)$
- $7 \times 75 = 525$

$= 525625$

625 always appears in the last in such cases. Product of the number formed by the digits at unit and hundredth place and the digit at hundredth place is put on the left hand side of 625.

Other methods of squaring the numbers

$11 \times 11 = 121$

- 1 (last) ← Square of digit at unit place
- 2 (middle) ← Double of the product of digits at tens place
- 1 (first) ← Square of digit at tens place

$31^2 =$ Square of digit at unit place in $31 \times 31 = 1 \times 1 = 1$
Double of the product of digits at tens place $(1 \times 3) \times 2 = 6$
Square of digit at tens place $= 3 \times 3 = 9$
$= 961$.

12×12 = Square of digit at unit place = $2 \times 2 = 4$

Double of the product of digits at ones and tens place $(1 \times 2)\ 2 = 4$

Square of digit at tens place $= 1 \times 1 = 1$

So square of 12 = 144.

To find the square of three digit numbers we divide it into two parts and use Anurupyen method. Anurupyen means "Similarity and proportions".

For example: If we want to find the square of 152 then divide 152 in two parts 15 and 2.

$152 \times 152 =$

- Square of digit at unit place $= 2^2 = 4$
- Double of the product of digits at ones & tens place $= 2 \times 15 \times 2 = 60$
- Square of digit at tens place $= 15^2 = 225$

225/60/4

225+6/04

=23104

Here we observe the following for the number whose square is to be evaluated:

1. To find the square of the digit at tens place in the first part from the right.
2. Multiply the digits at middle part of main number and double it.
3. Square the second digit in the main number in third part.
4. Arrange the number.

Example 28 Find the square of 43.

Solution

	III	II	I
$43^2 =$	4^2	4×3	3^2
		4×3	
	16	12	9
		+12	
	16	24	9
	16+2		49
	1849		

(Carry over number of middle part (II) is added with 16)

Example 29 Find $(132)^2$

Solution

13 2

Divide 132 in to two parts 13 and 2.

III	II	I
$(13)^2$	13×2	2^2
	$+13 \times 2$	
169	26	4
	26	
169	52	4
169+5		24
17424		

Exercise 6.6

1. Find the square using above method:

   (i) 18 (ii) 42 (iii) 83 (iv) 127 (v) 136

6.10 Square Root

When a number x is multiplied by itself then the value so obtained is x^2, called square number of the number x. Let us understand this way that x^2, is the pair $x \times x$. So, the square root of x^2 is x.
16 is a square number which is a pair of 4×4. So, the square root of 16 is 4.
Symbolic notation of square root is $\sqrt{\ }$

Digits of square root

Number of digits in a square of a number is double of the digits in it or one less than the double of the digits. Similarly, if the number of digits in the square root of a square number is even then it is half of it and one more than the half if odd. Let us look at the table given below:

No. of digits in a square number is odd				No. of digits in a square number is even			
Square Number	No. of Digits	Square Root	No. of Digits	Square Root	No. of Digits	Square Root	No. of Digits
1	1	1	$\frac{1+1}{2}=1$	16	2	4	$\frac{2}{2}=1$
100	3	10	$\frac{3+1}{2}=2$	81	2	9	$\frac{2}{2}=1$
961	3	31	$\frac{3+1}{2}=2$	1024	4	32	$\frac{4}{2}=2$
16641	5	129	$\frac{5+1}{2}=3$	108900	6	330	$\frac{6}{2}=3$

The number of digits in the square root of a perfect square number is equal to the number of pairs formed in it from the right hand side (i.e., from unit place) no matter the last pair contains only one digit.

Identification of perfect square number

1. Unit place of the perfect square number is either of 0, 1, 4, 5, 6 and 9, i.e., a number can't be a perfect square number if its unit place digit is either of 2, 3, 7 and 8.
2. Number of zeros in the last places of a perfect square number is even and the numbers on the left hand side of zeros should be a square number.
3. If sum of the digits in a number is 2, 3, 5, 6 and 8, then it can't be a perfect square.

Vedic method of finding the square root

1. First of all find if the number is a perfect square.
2. If it is a perfect square then find the number of digits.
3. Identify the unit place

Unit digit in the number	Unit digit in the square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

Now, find by using the Vilokanam method; what is the digit at tens place in the perfect square number?

Number Group	Tens Digit in Square root
1 – 3	1
4 – 8	2
9 – 15	3
16 – 24	4
25 – 35	5
36 – 48	6
49 – 63	7
64 – 80	8
81 – 99	9

Group 1 – 3 means it contains the numbers 1, 2, 3 and the possible square root of them could be considered as 1.
Vilokanam method of finding the square root is explained with following example:

Example 30 Find the square root of the number 361.

Solution Following can be concluded by observing the number.

(i) Unit place of 361 is 1, so it could be a perfect square.

(ii) Sum of digits = 3+6+1 = 10, so the sum of digits is 1+0=1, it could be a perfect square.

(iii) There should be two digits in the square root of the number.

(iv) Making the pair in the number 361 we find that the second pair contains only 3. So tens place digit in the square root is 1.

(v) Ultimate digit in the number is 1, so, the ultimate digit in the square root would be either 1 or 9 and tens place digit would be 1 because 3 lies in the number group 1 – 3.

(vi) The square root of 361 could be 11 or 19.

(vii) Multiply the tens place digit by its successor.
Product = 1 × 2 = 2, 3 in second pair > product 2.
So, larger of the two is the square root, i.e., 19.

Example 31 Find the square root of 5184.

Solution

(i) First pair = 84 and second pair = 51

(ii) Ultimate digit in first pair = 4 so, the ultimate digit in possible square root could be 2 or 8

(iii) The greatest square root included in 51 is 7, so the possible square root is 72 or 78. Product 7 × 8 = 56.

(iv) 51 < 56, so smaller of the two would be the square root.
Square Root = 72.

Note: This method is applicable only up to four digit numbers

Exercise 6.7

Find the square root by using Vilokanam method:.

(1)	169	(2)	324	(3)	576	(4)	2025
(5)	3025	(6)	9025	(7)	1024	(8)	441

6.11 Division Operation

When a number is subtracted from a number repeatedly, then successive subtraction is called division operation. The number from which subtraction is carried out is called **dividend** and the number which is subtracted is called **divisor**. The number of times we subtract a given number is called **quotient**. The number left out by subtracting a number up to maximum time is called **remainder**. Remainder is always less than the divisor.

Example 32 Subtracting 2 successively from 10.

Solution 10–2 = 8, 8–2 = 6, 6–2 = 4, 4–2 = 2, 2–2 = 0

Here 10 is dividend and 2 is divisor. Subtraction is made 5 times and the remainder is less than the divisor. So, quotient = 5 and remainder = 0.

6.11.1 ParavartyaYojyet Method (Transpose and Apply Method)

This method is used when the divisor is near to the base. In this method, the dividend is divided by the base of divisor and estimated quotient and remainder are calculated. This method has two categories:

(a) When divisor is greater than base.

(b) When divisor is less than base.

(a) When divisor is greater than base

(i) Calculate the deviation of divisor from the base.

(ii) Calculate the correction factor by transposing the deviation.(change the sign)

(iii) Divide by correction factor leaving first digit in the dividend

(iv) Divide the division operation in three parts. Learn by following example:

Example 33 Solve 4656 ÷ 11.

Solution

Dividend	11	4	6	5	6
Base	10		$\bar{4}$	-	-
Deviation	1			$\bar{2}$	-
Correction factor	$\bar{1}$				$\bar{3}$
Quotient		**4**	**2**	**3**	Reminder 3

Procedure

1. Make three parts for completing the division operation
2. Put divisor in first part, dividend in second part and in third part the digits equal to the number of zeros in the base.
3. Find base, deviation and correction factor.
4. Write below the first digit on the right of divisor.
5. Multiply this digit by correction factor write below the number after dividend
6. Subtract and write below and then multiply by correction factor. Repeat this process till we get digit in the third part.

Example 34 Solve 35984 ÷ 112

Solution

Dividend	1 1 2	3 5 9	8 4
Base	1 0 0	$\bar{3}$ $\bar{6}$	- -
Deviation	1 2	$\bar{2}$	$\bar{4}$
Correction factor	$\bar{1}\bar{2}$		$\bar{1}$ $\bar{2}$
Quotient		**3 2 1**	Reminder 32

(b) When divisor is less than base:

Solve according to the procedure used earlier. It is explained with following example:

Example 35 Solve 30103 ÷ 9

Solution

Dividend	9	3 0 1 0	3
Base	1 0	3 - -	-
Deviation	$\bar{1}$	3 -	-
Correction factor	1	4	4
Quotient		**3 3 4 4**	Reminder 7

Remember, this time the correction factor is positive. So it will be added to next number.

In the given example we need to divide by 9 which is one less than the nearest base 10.

We will write the first digit 3 as it is in the quotient and then multiply by the correction factor (+1) and add to next number 0. Write the quotient 3 below the horizontal number. Again, multiply this by correction factor and add to the next number and write as quotient. Repeat these steps till the end.

Example 36 Solve 11022 ÷ 89.

Solution

Dividend	8 9	1 1 0	2 2
Base	1 0 0	1 1	-
Deviation	$\bar{1}$ $\bar{1}$	2	2
Correction factor	1 1		3 3
Quotient		**1 2 3**	Reminder 75

Exercise 6.8

Solve following questions:

(1)	23244 ÷ 11	(2)	12064 ÷ 12	(3)	1234 ÷ 112
(4)	324842 ÷ 101	(5)	2012 ÷ 9	(6)	10321 ÷ 98

We Learnt

1. The addition and subtraction by calculating the deviation of numbers from 10 or multiples of 10 based upon Sankalan Vyavkalnabhyam formula
2. Learnt addition and subtraction by making two numbers near complete by using Poornapoornabhyam formula.
3. The subtraction by using NikhilamNavtahCharamdashatah formula.
4. Some interesting multiplication method of Vedic Mathematics, in which we learnt to multiply the numbers by 10, 100, 1000, 5, 50, 500 and 11 orally. Also learnt multiplication of numbers by 9, 99, 999 using EkNeunen method.
5. Learnt easy Vedic Mathematical methods for operations on fractions, square root using sub-formula Anurupen, Vilokanam and Nikhilam method respectively.