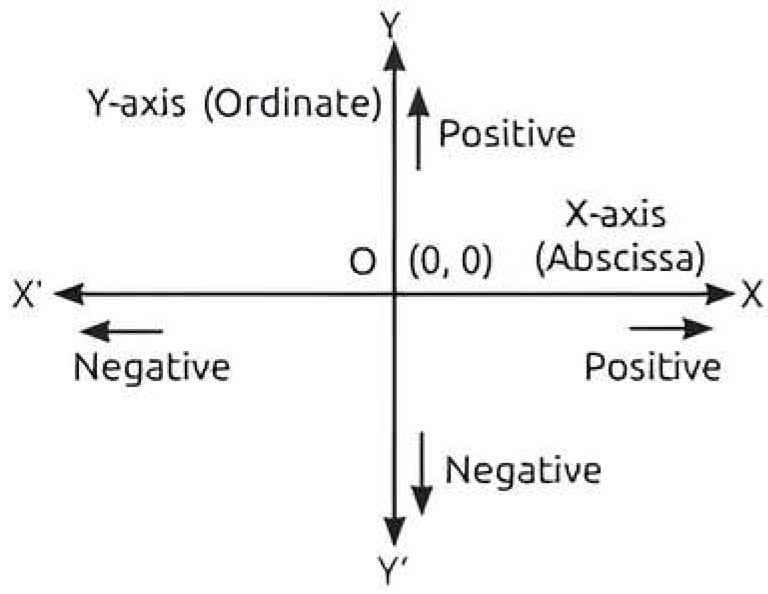
7 Coordinate Geometry

Fastrack« Revision

- **Cartesian System:** The system used for describing the position of a point in a plane with reference to two fixed mutually perpendicular lines is termed as the **cartesian system.**

  In a cartesian system,

  (i) Horizontal line XX' is called the X-axis, while vertical line YY' is called Y-axis.



  (ii) The axes XX′ and YY′ divide the plane into four parts called **quadrants**.

  (iii) The perpendicular distance from the Y-axis measured along the X-axis is called ***x*-coordinate** or **abscissa**.

  (iv) The perpendicular distance from the X-axis measured along the Y-axis is called ***y*-coordinate** or **ordinate**.

  (v) (0, 0) is the origin.

- **Distance Formula:** The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the cartesian plane is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

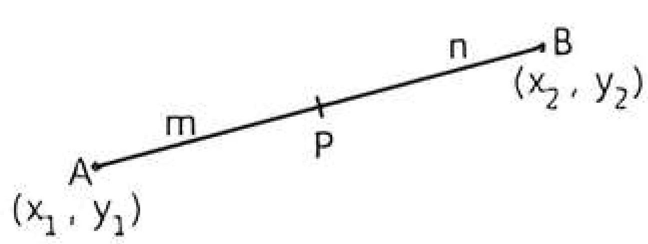
or $$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

  (i) The distance of a point $P(x, y)$ from the origin is $\sqrt{x^2 + y^2}$.

  (ii) Three points are collinear, if sum of two sides is equal to third side.

- **Section Formula for Internal Division:** If $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$, then

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$



- **Mid-point Formula:** If $P(x, y)$ is the mid-point of $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}$$

Knowledge BOOSTER

1. *In a triangle, sum of lengths of any two sides is greater than the length of third side.*
2. *A triangle is right-angled triangle iff sides of triangle satisfy Pythagoras theorem.*
   Or A triangle is right-angled, if sum of squares of any two sides is equal to square of third largest side.
3. *A triangle is equilateral iff its all sides are equal in length.*
4. *A triangle is isosceles iff its any two sides are equal in length.*
5. *A triangle is right-angled isosceles iff its two sides are equal in length and all its sides satisfy Pythagoras theorem.*
6. *A quadrilateral is a parallelogram iff its opposite sides are equal in length but the diagonals are not equal.*
7. *A quadrilateral is a rectangle iff its opposite sides are equal and diagonals are also equal.*
8. *A quadrilateral is a rhombus iff its all four sides are equal but the diagonals are not equal.*
9. *A quadrilateral is a square iff its all sides are equal and diagonals are also equal.*
10. *In parallelogram, rectangle, square and rhombus, diagonals bisect each other.*
11. *If the vertices of a triangle are* $A(x_1, y_1)$, $B(x_2, y_2)$ *and* $C(x_3, y_3)$, *then centroid of a* ΔABC *is*

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Practice Exercise

Multiple Choice Questions

Q 1. The distance of the point (3, 5) from X-axis (in units) is: [CBSE SQP 2023-24]

a. 3 b. −3 c. 5 d. −5

Q 2. The distance of the point (−6, 8) from origin is: [CBSE 2023]

a. 6 b. −6 c. 8 d. 10

Q 3. The distance between the points (3, 0) and (0, −3) is: [CBSE 2023]

a. $2\sqrt{3}$ units b. 6 units c. 3 units d. $3\sqrt{2}$ units

Q 4. The distance between the points $\left(\frac{-5}{2}, 7\right)$ and $\left(\frac{-1}{2}, 7\right)$ is: [CBSE 2023]

a. 3 b. 2 c. 4 d. 9

Q 5. If the distance between A(k, 3) and B(2, 3) is 5, then the value of k is:

a. 5 b. 6 c. 7 d. 8

Q 6. If the point (x, y) is equidistant from the points (2, 1) and (1, −2), then:

a. $x + 3y = 0$ b. $3x + y = 0$ c. $x + 2y = 0$ d. $3x + 2y = 0$

Q 7. The points (−4, 0), (4, 0) and (0, 3) are the vertices of a/an: [CBSE 2023]

a. right triangle b. isosceles triangle c. equilateral triangle d. scalene triangle

Q 8. The points (2, 4), (2, 6) and $(2+\sqrt{3}, 5)$ are the vertices of:

a. an equilateral triangle
b. an isosceles triangle
c. a right triangle
d. a right angled isosceles triangle

Q 9. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$ is:

a. $a^2 + b^2$ b. $a + b$ c. $a^2 - b^2$ d. $\sqrt{a^2 + b^2}$

Q 10. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is: [CBSE 2023]

a. ab b. $\frac{1}{2}ab$ c. $\frac{1}{4}ab$ d. $2ab$

Q 11. The centre of a circle is $(2a, a - 7)$. If the circle passes through the point (1, −9) and has diameter $10\sqrt{2}$ units, then the value of a is:

a. 9 b. $-\sqrt{3}$ c. $\sqrt{3}$ d. ± 3

Q 12. If $A(3, \sqrt{3})$, B(0, 0) and C(3, k) are the three vertices of an equilateral triangle ABC, then the value of k is: [CBSE 2021 Term-I]

a. 2 b. − 3 c. $\pm\sqrt{3}$ d. $\pm\sqrt{2}$

Q 13. ABCD is a rectangle whose three vertices are (4, 3), (4, 1) and (0, 1). The length of its diagonal is:

a. $2\sqrt{5}$ units b. $\sqrt{5}$ units c. $\frac{1}{\sqrt{5}}$ units d. $\frac{2}{\sqrt{5}}$ units

Q 14. If the vertices of a parallelogram PQRS taken in order are P(3, 4), Q(−2, 3) and R(−3, −2), then the coordinates of its fourth vertex S are: [CBSE SQP 2022-23]

a. (− 2, − 1) b. (−2, −3) c. (2, − 1) d. (1, 2)

Q 15. Points A(−1, y) and B(5, 7) lie on a circle with centre O(2, −3y). The values of y are: [CBSE 2021 Term-I]

a. 1, −7 b. −1, 7 c. 2, 7 d. −2, −7

Q 16. If P(−1, 1) is the mid-point of the line segment joining A(−3, b) and B(1, b + 4), then b =

a. 1 b. − 1 c. 2 d. 0

Q 17. In what ratio does the point P(3, 4) divides the line segment joining the points A(1, 2) and B(6, 7)?

a. 1 : 2 b. 2 : 3 c. 3 : 4 d. 1 : 1

Q 18. If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, y) in the ratio 3 : 1, then the value of y is: [CBSE 2020]

a. 4 b. 3 c. 2 d. 1

Q 19. If A(4, −2), B(7, −2) and C(7, 9) are the vertices of a △ABC, then △ABC is: [CBSE 2021 Term-I]

a. equilateral triangle
b. isosceles triangle
c. right angled triangle
d. isosceles right angled triangle

Q 20. In what ratio does X- axis divide the line segment joining the points A (2, −3) and B (5, 6)? [CBSE 2023]

a. 2 : 3 b. 2 : 1 c. 3 : 4 d. 1 : 2

Q 21. Y-axis divides the line segment joining the points $(-6, 2)$ and $(2, -6)$ in the ratio: [CBSE 2023]

a. 1 : 3 b. 3 : 2 c. 3 : 1 d. 2 : 3

Q 22. Point P divides the line segment joining R(–1, 3) and S(9, 8) in ratio $k : 1$. If P lies on the line $x - y + 2 = 0$, then value of k is: [CBSE SQP 2021 Term-I]

a. 2/3 b. 1/2 c. 1/3 d. 1/4

Assertion & Reason Type Questions

Directions (Q. Nos. 23-27): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true and Reason (R) is false
d. Assertion (A) is false and Reason (R) is true

Q 23. Assertion (A): The distance between the points $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$ is 2 units.

Reason (R): The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Q 24. Assertion (A): The point P(–4, 6) divides the join of A(–6, 10) and B(3, –8) in the ratio 2 : 7.

Reason (R): If the point $C(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}.$$

Q 25. Assertion (A): The point (0, 4) lies on Y-axis.

Reason (R): The x-coordinate of the point on Y-axis is zero. [CBSE SQP 2023-24]

Q 26. Assertion (A): The coordinates of the points which divide the line segment joining A(2, –8) and B(–3, –7) into three equal parts are $\left(\frac{1}{3}, -\frac{23}{3}\right)$ and $\left(-\frac{4}{3}, -\frac{22}{3}\right)$.

Reason (R): The points which divide AB in the ratio 1 : 3 and 3 : 1 are called points of trisection of AB.

Q 27. Assertion (A): The coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0), are $\left(\frac{17}{3}, 5\right)$.

Reason (R): Coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Fill in the Blanks Type Questions

Q 28. The angle subtended by joining points A(3, 0) and B(0, –2) to the origin point is ....

Q 29. If the point $P(k-1, 2)$ is equidistant from the points $A(3, k)$ and $B(k, 5)$, the value of k is/are ....

Q 30. The coordinates of a point on X-axis which is equidistant from the points (–3, 4) and (7, 6), are ....

Q 31. Suppose AB is a line segment and points P and Q are nearer to A and B on a line segment AB such that AP = PQ = QA, then P divides the line segment in the ratio ....

Q 32. The point which lies on the perpendicular bisector of the line segment joining the points A(–2, –5) and B(2, 5) is ....

True/False Type Questions

Q 33. The distance between points $P(a \sin\phi, 0)$ and $Q(0, -a\cos\phi)$ is a.

Q 34. The ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6) is 2/7. [NCERT EXERCISE]

Q 35. If the vertices of a triangle are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then the centroid of a $\triangle ABC$ is:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

Q 36. If three points are collinear, then area of triangle is not zero.

Q 37. If point P divides the line joining A and B in the ratio 1 : 1, then point P is the mid-point of AB.

Solutions

1. (c) We know that, the perpendicular distance from X-axis measured along the Y-axis is called y-coordinate.

The distance of the point (3, 5) from X-axis

= Numerical value of y-coordinate

= 5 units

2. (d) Given, $(-6, 8) \equiv (x_1, y_1)$ and origin $= (0, 0) = (x_2, y_2)$

∴ Distance of the point (–6, 8) from the origin

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0+6)^2 + (0-8)^2}$$

$$= \sqrt{36 + 64} = \sqrt{100} = 10$$

3. (d) Given, $(3, 0) = (x_1, y_1)$ and $(0, -3) = (x_2, y_2)$

$\therefore$ Distance between the points $(3, 0)$ and $(0, -3)$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-3)^2 + (-3-0)^2} = \sqrt{9+9} = 3\sqrt{2} \text{ units.}$$

4. (b) Given, $\left(\frac{-5}{2}, 7\right) = (x_1, y_1)$ and

$$\left(-\frac{1}{2}, 7\right) = (x_2, y_2)$$

$\therefore$ Distance between the points $\left(-\frac{5}{2}, 7\right)$ and $\left(-\frac{1}{2}, 7\right)$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(-\frac{1}{2} + \frac{5}{2}\right)^2 + (7-7)^2} = \sqrt{\left(\frac{4}{2}\right)^2 + (0)^2}$$

$$\Rightarrow \quad \sqrt{(4+0)} = 2.$$

5. (c) The distance between A and B is

$$AB = \sqrt{(k-2)^2 + (3-3)^2}$$

$$\Rightarrow \quad \sqrt{(k-2)^2} = 5 \quad \text{(given)}$$

Squaring on both sides, we have $(k-2)^2 = 25$

$$k - 2 = \pm 5$$

$$\Rightarrow \quad k = 2 \pm 5$$

$$\Rightarrow \quad k = 7 \text{ or } -3$$

6. (a) Let the points be $P(x, y)$, $A(2, 1)$ and $B(1, -2)$.

Since, P is equidistant from A and B.

$$\therefore \quad AP = BP$$

$$\Rightarrow \quad AP^2 = BP^2$$

$$\Rightarrow \quad (x-2)^2 + (y-1)^2 = (x-1)^2 + (y+2)^2$$

$$\Rightarrow \quad x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 + 4y + 4$$

$$\Rightarrow \quad -4x - 2y = -2x + 4y$$

$$\Rightarrow \quad 2x + 6y = 0$$

$$\Rightarrow \quad x + 3y = 0$$

7. (b) Let ABC be the triangle whose vertices are $A(-4, 0)$, $B(4, 0)$ and $C(0, 3)$.

TiP

In an isosceles triangle, any two sides are equal.

Here $\quad AB = \sqrt{(4+4)^2 + (0-0)^2} = \sqrt{(8)^2 + 0} = 8$

$$BC = \sqrt{(0-4)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

and $\quad AC = \sqrt{(0+4)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$

We have, $BC = AC = 5$

Hence, ΔABC is an isosceles triangle.

8. (a) Let ABC be the triangle whose vertices are $A(2, 4)$, $B(2, 6)$ and $C(2+\sqrt{3}, 5)$.

TiP

In an equilateral triangle, all sides are equal.

Here, $\quad AB = \sqrt{(2-2)^2 + (6-4)^2} = \sqrt{0^2 + 2^2} = 2$

$$BC = \sqrt{(2+\sqrt{3}-2)^2 + (5-6)^2}$$

$$= \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$$

and $\quad AC = \sqrt{(2+\sqrt{3}-2)^2 + (5-4)^2}$

$$= \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$$

We have, $AB = BC = AC = 2$

Hence, ΔABC is an equilateral triangle.

9. (d) The distance between points $P(a\cos\theta + b\sin\theta, 0)$ and $Q(0, a\sin\theta - b\cos\theta)$ is given by

$$PQ = \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$$

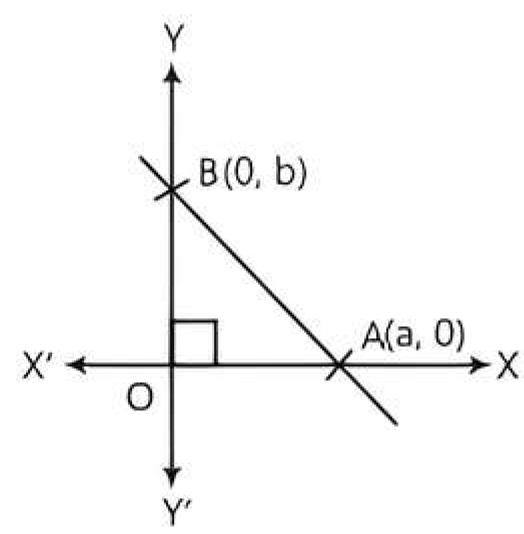
$$= \sqrt{\begin{aligned}&a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta \\ &+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta\end{aligned}}$$

$$= \sqrt{a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$

$$= \sqrt{a^2 + b^2} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

10. (b) Given equation of line is $\frac{x}{a} + \frac{y}{b} = 1$. ...(1)

The above eq. (1) is the intercept form of a line, which cut X-axis at point $(a, 0)$ and Y-axis at point $(0, b)$.



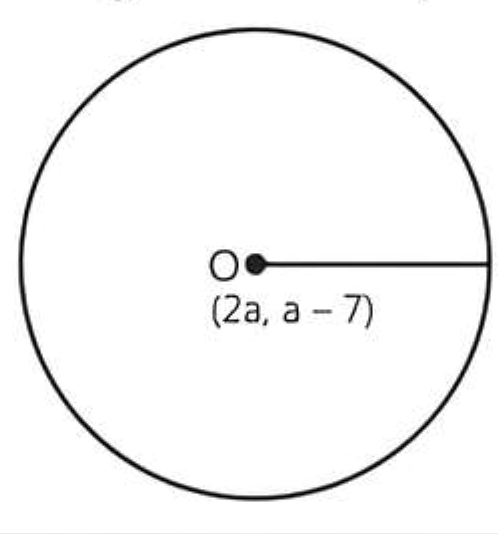
From figure, $OA = a$

and $\quad OB = b$

$\therefore$ The area of the triangle $AOB = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times OA \times OB = \frac{1}{2}ab$$

11. (d) Let $O(2a, a-7)$ be the centre and $A(1, -9)$ be any point through which circle passes.



$$\therefore \quad 2(OA) = 10\sqrt{2} \quad (\because \text{Diameter} = 2 \times \text{radius})$$

$$\Rightarrow \quad OA = 5\sqrt{2}$$

$$\Rightarrow \quad OA^2 = (5\sqrt{2})^2 = 50$$

$$\Rightarrow \quad (2a-1)^2 + (a-7+9)^2 = 50$$

$$\Rightarrow \quad 4a^2 + 1 - 4a + a^2 + 4 + 4a = 50$$

$$\Rightarrow \quad 5a^2 = 50 - 5 = 45$$

$$\Rightarrow \quad a^2 = 9$$

$$\Rightarrow \quad a = \pm 3$$

12. (c) Given vertices of an equilateral triangle are $A(3, \sqrt{3}), B(0, 0)$ and $C(3, k)$.

TiP

All sides of an equilateral triangle are equal.

$$\therefore \quad AB = BC$$

$$\Rightarrow \quad \sqrt{(0-3)^2 + (0-\sqrt{3})^2} = \sqrt{(3-0)^2 + (k-0)^2}$$

$$\Rightarrow \quad \sqrt{9+3} = \sqrt{9+k^2}$$

$$\Rightarrow \quad \sqrt{12} = \sqrt{9+k^2}$$

Squaring on both sides, we have

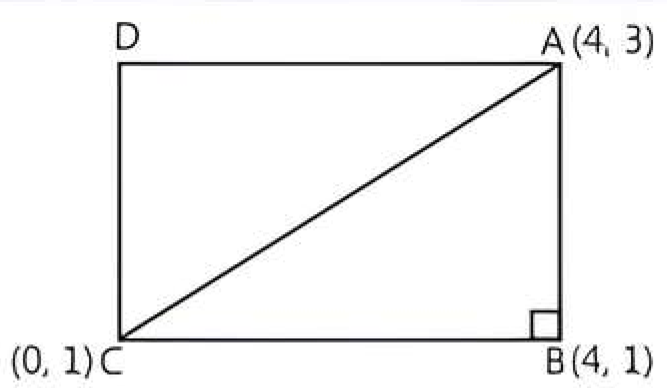
$$12 = 9 + k^2 \Rightarrow k^2 = 3$$

$$\Rightarrow \quad k = \pm\sqrt{3}$$

13. (a) Let the three vertices of rectangle ABCD be A(4, 3), B(4, 1) and C(0, 1).

TiP

Each adjacent sides of a rectangle intersect at right angle.



TR!CK

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

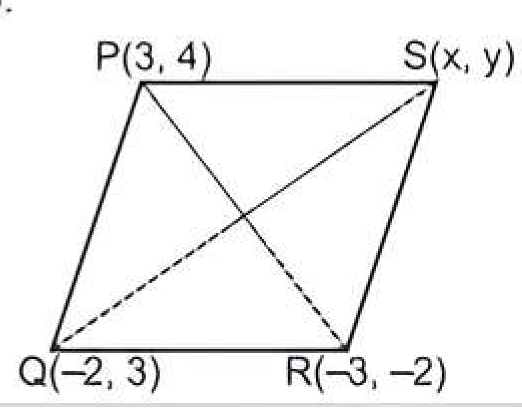
In right ΔABC,

Length of diagonal $AC = \sqrt{(0-4)^2 + (1-3)^2}$

(by Pythagoras theorem)

$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

14. (c) Let coordinates of fourth vertex of a parallelogram be $S(x, y)$.



TiP

In a parallelogram, diagonals intersect at mid-point.

$\therefore$ Mid-point of PR = Mid-point of QS

$$\Rightarrow \quad \left(\frac{3-3}{2}, \frac{4-2}{2}\right) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$$

$$\Rightarrow \quad (0, 1) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$$

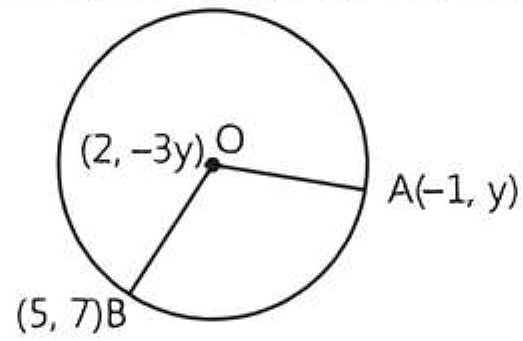
Equating the coordinates both sides,

$$0 = \frac{-2+x}{2}, 1 = \frac{3+y}{2}$$

$$\Rightarrow \quad x = 2, y = 2 - 3 = -1$$

Hence, coordinates of fourth vertex is S(2, −1).

15. (b) Since, OA and OB are the radii of a circle.



$$OA^2 = OB^2$$

$$\Rightarrow \quad (-1-2)^2 + (y+3y)^2 = (5-2)^2 + (7+3y)^2$$

$$\Rightarrow \quad 9 + 16y^2 = 9 + 49 + 9y^2 + 42y$$

$$\Rightarrow \quad 16y^2 - 9y^2 - 42y - 49 = 0$$

$$\Rightarrow \quad 7y^2 - 42y - 49 = 0$$

$$\Rightarrow \quad y^2 - 6y - 7 = 0$$

TR!CK

$$\because \quad 7 = 7 \times 1$$

Here we taken 7 and 1 as a factor of 7.

So, middle term −6 = 1 − 7.

$$\Rightarrow \quad y^2 - 7y + y - 7 = 0$$

$$\Rightarrow \quad y(y-7) + 1(y-7) = 0$$

$$\Rightarrow \quad (y+1)(y-7) = 0$$

$$\Rightarrow \quad y + 1 = 0 \quad \text{or} \quad y - 7 = 0$$

$$\Rightarrow \quad y = -1 \quad \text{or} \quad y = 7$$

16. (b) Since, P (−1, 1) is the mid-point of line segment joining $A(-3, b)$ and $B(1, b+4)$.

$$\therefore \quad (-1, 1) = \left(\frac{-3+1}{2}, \frac{b+b+4}{2}\right)$$

Equating y-coordinate, we get

$$1 = \frac{2b+4}{2} \Rightarrow 1 = b + 2$$

$$\Rightarrow \quad b = -1$$

17. (b) Let P(3, 4) divides the join of A(1, 2) and B(6, 7) in the ratio $k : 1$.

$\therefore$ Coordinates of P are $\dfrac{6k+1}{k+1} = 3$

$$\Rightarrow \quad 6k + 1 = 3k + 3 \Rightarrow k = \frac{2}{3}$$

and $\frac{7k+2}{k+1}=4$

$\Rightarrow \quad 7k+2=4k+4 \Rightarrow k=\frac{2}{3}$

Hence, the required ratio is $\frac{2}{3}:1$ *i.e.*, $2:3$.

18. (d)

TR!CK

If $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ then

$$x=\left(\frac{mx_2+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}\right).$$

$$P(6, 2)=\left(\frac{3\times 4+1\times 6}{3+1}, \frac{3\times y+1\times 5}{3+1}\right)$$

$$=\left(\frac{12+6}{4}, \frac{3y+5}{4}\right)=\left(\frac{9}{2}, \frac{3y+5}{4}\right)$$

On comparing y-coordinate,

$$2=\frac{3y+5}{4} \Rightarrow 3y+5=8 \Rightarrow y=1$$

19. (c) Given vertices of a triangle are $A(4, -2)$, $B(7, -2)$ and $C(7, 9)$.

Now, $AB=\sqrt{(7-4)^2+(-2+2)^2}=\sqrt{3^2+0}=3$

$BC=\sqrt{(7-7)^2+(9+2)^2}=\sqrt{0+11^2}=11$

and $CA=\sqrt{(4-7)^2+(-2-9)^2}=\sqrt{(-3)^2+(-11)^2}$

$=\sqrt{9+121}=\sqrt{130}$

Here we see that $AB \neq BC \neq CA$

Now $(AB)^2+(BC)^2=(3)^2+(11)^2$

$=9+121=130$

$=(CA)^2$

Hence, triangle ABC is a right angled triangle.

20. (d) Let the X-axis divides the join $(2, -3)$ and $(5, 6)$ in the ratio $k:1$.

Then Y-coordinate of X-axis is zero.

$\therefore \quad \frac{6k-3}{k+1}=0 \qquad [\because k \neq -1]$

$\Rightarrow \quad 6k-3=0 \Rightarrow k=\frac{3}{6}$

$\Rightarrow \quad k=\frac{1}{2}$ *i.e.*, $1:2$.

21. (c) Let the Y-axis divides the join $(-6, 2)$ and $(2, -6)$ in the ratio $k:1$. Then x-coordinate of Y-axis is zero.

$\therefore \quad \frac{2k-6}{k+1}=0 \qquad (\because k \neq -1)$

$\Rightarrow \quad 2k-6=0$

$\Rightarrow \quad k=\frac{6}{2}=3$ *i.e.*, $3:1$.

22. (a)

TiP

Coordinates of point P divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ is $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

Since, P divides the line segment joining $R(-1, 3)$ and $S(9, 8)$ in ratio $k:1$.

$\therefore$ Coordinates of P are $\left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right)$

Since, P lies on the line $x-y+2=0$.

Then, $\frac{9k-1}{k+1}-\frac{8k+3}{k+1}+2=0$

$\Rightarrow \quad 9k-1-8k-3+2k+2=0$

$\Rightarrow \quad 3k=2 \Rightarrow k=\frac{2}{3}$

23. (d) **Assertion (A):** Distance between points $A(\cos\theta, \sin\theta)$ and $B(\sin\theta, -\cos\theta)$ is given by

$$AB=\sqrt{(\sin\theta-\cos\theta)^2+(-\cos\theta-\sin\theta)^2}$$

$$=\sqrt{\begin{array}{l}\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta \\ \quad +\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta\end{array}}$$

$$=\sqrt{2(\sin^2\theta+\cos^2\theta)}=\sqrt{2}\text{ units}$$

So, Assertion (A) is false.

Reason (R): It is a true statement.

Hence, Assertion (A) is false but Reason (R) is true.

24. (a) **Assertion (A):** Let $P(-4, 6)$ divides $A(-6, 10)$ and $B(3, -8)$ in the ratio $k:1$.

Then, $\frac{k\times 3+1\times(-6)}{k+1}=-4$ and $\frac{k\times(-8)+1\times 10}{k+1}=6$

$\Rightarrow \quad 3k-6=-4k-4$ and $-8k+10=6k+6$

$\Rightarrow \quad 7k=2 \Rightarrow k=\frac{2}{7}$ and $14k=4 \Rightarrow k=\frac{2}{7}$

$\therefore$ Required ratio is $2/7$ *i.e.*, $2:7$.

So, Assertion (A) is true.

Reason (R) : It is also true.

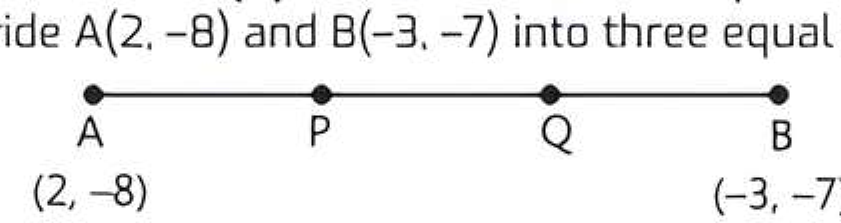
Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

25. Assertion (A) : X-coordinate of each point lies on Y-axis is zero. So, the point $(0, 4)$ always lies on Y-axis.

Thus, Assertion (A) is true.

Reason (R): It is a true statement. Hence, both Assertion (A) and Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

26. (c) **Assertion (A):** Let P and Q be the points which divide $A(2, -8)$ and $B(-3, -7)$ into three equal parts.



$\therefore \quad AP:PB=1:2$

So, coordinates of P

$$=\left(\frac{1\times(-3)+2\times 2}{1+2},\frac{1\times(-7)+2\times(-8)}{1+2}\right)$$

$$=\left(\frac{-3+4}{3},\frac{-7-16}{3}\right)=\left(\frac{1}{3},-\frac{23}{3}\right)$$

Also, $AQ:QB = 2:1$

$\therefore$ Coordinates of Q

$$=\left(\frac{2\times(-3)+1\times 2}{2+1},\frac{2\times(-7)+1\times(-8)}{2+1}\right)$$

$$=\left(\frac{-6+2}{3},\frac{-14-8}{3}\right)=\left(-\frac{4}{3},-\frac{22}{3}\right)$$

So, Assertion (A) is true.

Reason (R): It is false to say that in trisection point. point divide AB in the ratios 3 : 1 and 1 : 3.

Hence, Assertion (A) is true but Reason (R) is false.

27. (d) **Assertion (A):** The, coordinate of the centroid of a triangle with vertices (0, 6), (8, 12) and (8, 0) are

$$\left(\frac{0+8+8}{3},\frac{6+12+0}{3}\right)=\left(\frac{16}{3},6\right)$$

So, Assertion (A) is false.

Reason (R): It is a true statement.

Hence, Assertion (A) is false but Reason (R) is true.

28. Let $O \equiv (0, 0)$ be the origin.

$$\therefore \quad OA=\sqrt{(3-0)^2+(0-0)^2}=3$$

$$OB=\sqrt{(0-0)^2+(0+2)^2}=2$$

and $$AB=\sqrt{(3-0)^2+(0+2)^2}=\sqrt{9+4}=\sqrt{13}$$

$$\because \quad AB^2=OA^2+OB^2$$

$\therefore$ AB subtend right angle at origin.

29. $\because \quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$ (by distance formula)

$$\therefore (k-1-3)^2+(2-k)^2=(k-1-k)^2+(2-5)^2$$

$$\Rightarrow \quad (k-4)^2+(2-k)^2=(-1)^2+(-3)^2$$

$$\Rightarrow \quad k^2+16-8k+4+k^2-4k=1+9$$

$$\Rightarrow \quad 2k^2-12k+10=0$$

$$\Rightarrow \quad k^2-6k+5=0$$

TR!CK

$\because \quad 5 = 5\times 1$

Here we will take 5 and 1 as a factor of 5.

So, middle term $-6=-5-1$.

$$\Rightarrow \quad k^2-5k-k+5=0$$

$$\Rightarrow \quad k(k-5)-1(k-5)=0$$

$$\Rightarrow \quad (k-5)(k-1)=0$$

$$\Rightarrow \quad k=1, 5$$

30.

TiP

Ordinate of each point on X-axis is always zero.

Let required point be P(x, 0).

Also, let $A=(-3,4)$ and $B=(7,6)$

$$\therefore \quad AP = BP$$

$$\Rightarrow \quad AP^2=BP^2$$

$$\Rightarrow \quad (x+3)^2+(0-4)^2=(x-7)^2+(0-6)^2$$

(by distance formula)

$$\Rightarrow \quad x^2+9+6x+16=x^2+49-14x+36$$

$$\Rightarrow \quad 20x=60$$

$$\Rightarrow \quad x=3$$

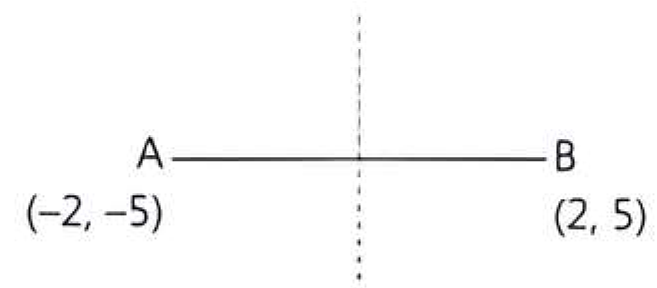
$\therefore$ Point is P(3, 0).

31. 1 : 2

32.

TiP

A point lies on the perpendicular bisector of AB *is equal to the mid-point of* AB.



Perpendicular bisector of AB = Mid-point of AB

$$=\left(\frac{-2+2}{2},\frac{-5+5}{2}\right)=(0,0)$$

33. The distance between points $P(a\sin\phi, 0)$ and $Q(0, -a\cos\phi)$ is

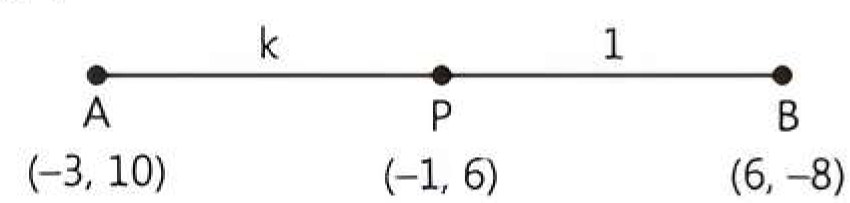
$$PQ=\sqrt{(0-a\sin\phi)^2+(-a\cos\phi-0)^2}$$

$$=\sqrt{a^2\sin^2\phi+a^2\cos^2\phi}=\sqrt{a^2(\sin^2\phi+\cos^2\phi)}$$

$$=\sqrt{a^2\times 1}=a \qquad [\because \sin^2\theta+\cos^2\theta=1]$$

Hence, given statement is true.

34. Let point P divides the line segment AB in the ratio $k:1$.



$\therefore$ The coordinates of $P=\left(\frac{6k-3}{k+1},\frac{-8k+10}{k+1}\right)$

$$\Rightarrow \quad (-1,6)=\left(\frac{6k-3}{k+1},\frac{-8k+10}{k+1}\right)$$

Equating the x and y-coordinates on both sides, we get

$$-1=\frac{6k-3}{k+1} \quad \text{and} \quad 6=\frac{-8k+10}{k+1}$$

$$\Rightarrow \quad -k-1=6k-3 \quad \text{and} \quad 6k+6=-8k+10$$

$$\Rightarrow \quad 7k=2 \quad \text{and} \quad 14k=4$$

$$\Rightarrow \quad k=\frac{2}{7} \quad \text{and} \quad k=\frac{2}{7}$$

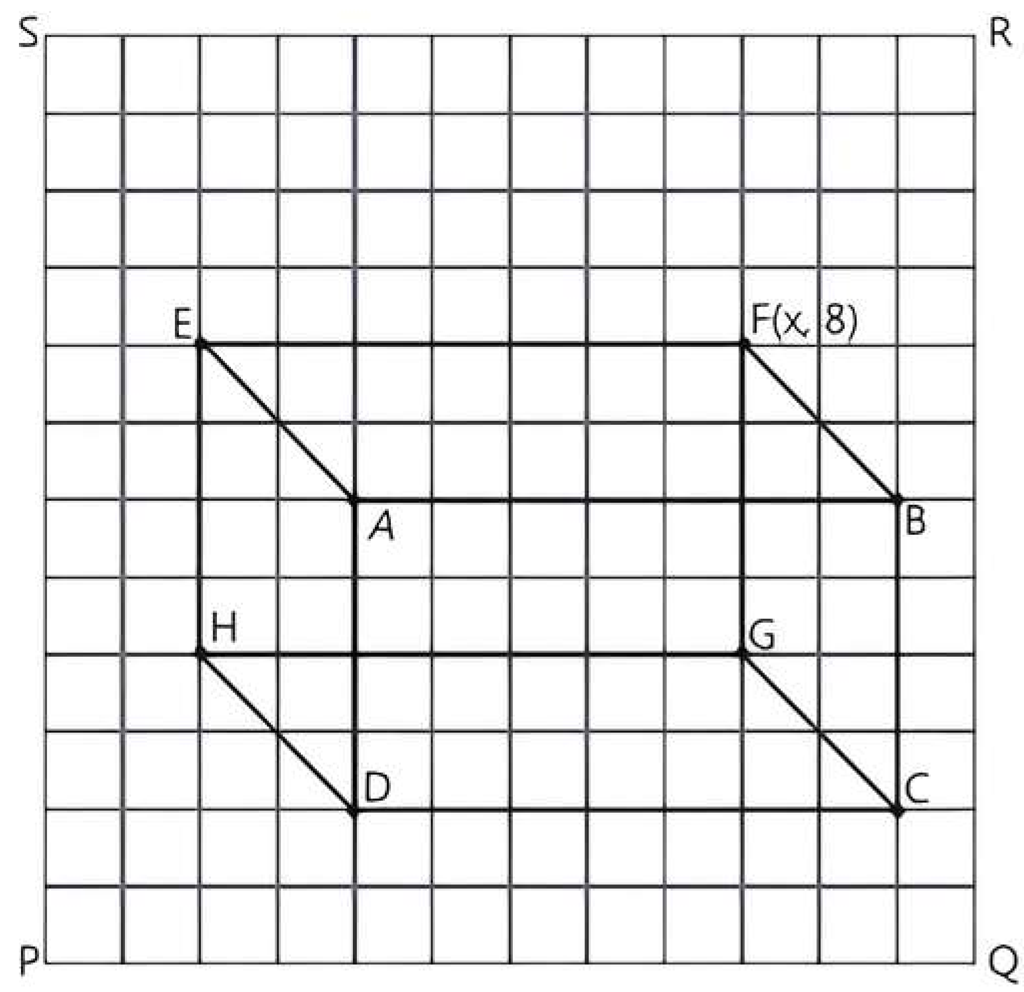
Hence, given statement is true.

35. True **36.** False **37.** True

Case Study Based Questions

Case Study 1

According to medical science and research, keeping an aquarium in the house helps in treating stress, anxiety and health problems associated with blood. It also provides visual stimulation that boost your focus and creativity. A sketch of an aquarium is drawn, which is given in the following figure.



Considering P as origin, solve the following questions.

Q 1. The coordinates of H are:

a. (4, 2) b. (4, 3) c. (2,4) d. (4, 8)

Q 2. Distance of the point G from the *Y*-axis is:

a. 3 units b. 4 units
c. 5 units d. 9 units

Q 3. Length of side HG =

a. 6 units b. 7 units c. $8\frac{3}{2}$ units d. 9 units

Q 4. The length of diagonal FD and the value of *x*, respectively are:

a. 8 units, 4 b. $\sqrt{8}$ units, 5
c. $\sqrt{15}$ units, 9 d. $\sqrt{61}$ units, 9

Q 5. If Q is considered as origin, then the coordinates of mid-point of BC are:

a. (–1, 4) b. (1, 6) c. (6, 1) d. (6, –1)

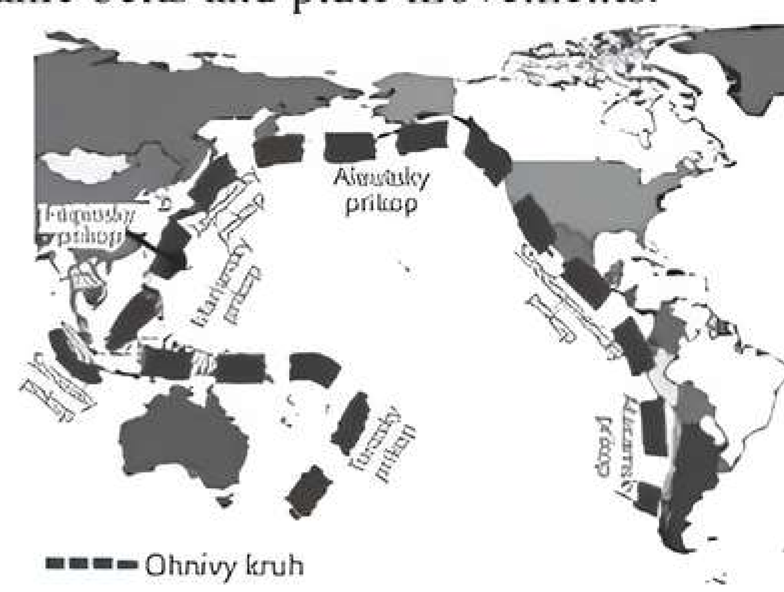
Solutions

1. We are given that P is origin.
   ∴ Coordinates of H are (2, 4).
   So, option (c) is correct.
2. Coordinates of G are (9, 4), therefore distance of *G* from *Y*-axis = 9 units.
   So, option (d) is correct.
3. Coordinates of H are (2, 4) and coordinates of G are (9, 4).
   Thus, $GH = \sqrt{(9-2)^2+(4-4)^2} = \sqrt{7^2+0} = 7$units
   So, option (b) is correct.
4. Coordinates of D are (4, 2) and coordinates of F are (9, 8).
   $\Rightarrow x = 9$
   Also, length of diagonal $FD = \sqrt{(4-9)^2+(2-8)^2}$
   $= \sqrt{25+36} = \sqrt{61}$ units
   So, option (d) is correct
5. If Q is origin, then
   Coordinates of B are (1, 6) and of C are (– 1, 2).
   Now, mid-point $BC = \left(\frac{(-1)+(-1)}{2}, \frac{6+2}{2}\right)$ i.e., (– 1, 4)
   So, option (a) is correct

Case Study 2

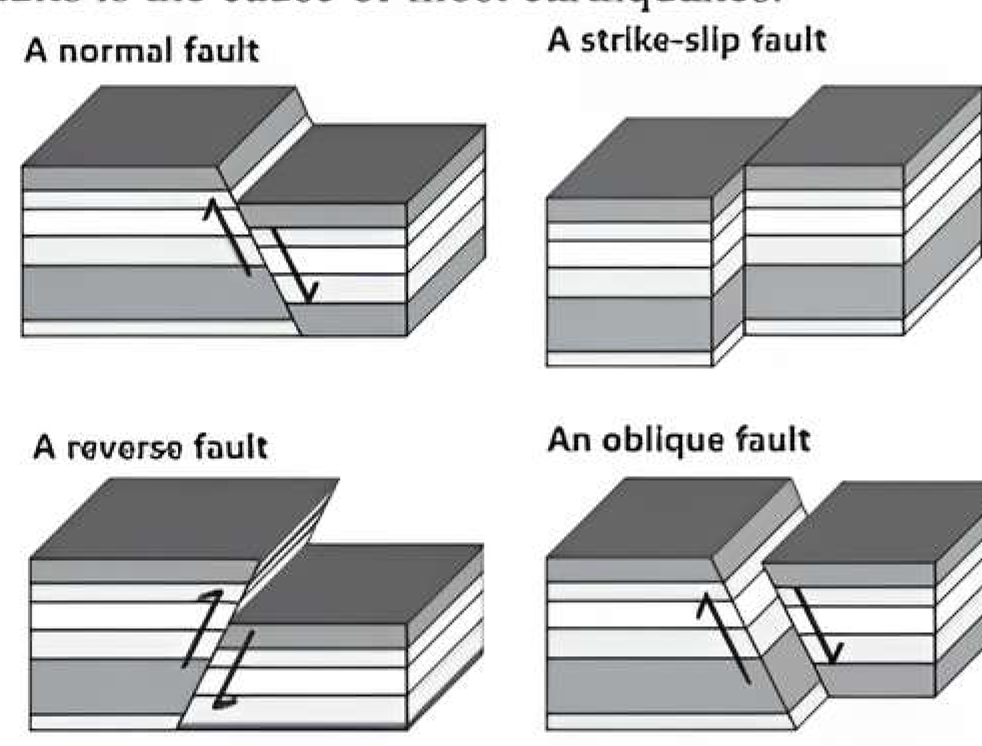
Pacific Ring of Fire

The Pacific Ring of Fire is a major area in the basin of the Pacific Ocean where many earthquakes and volcanic eruptions occur. In a large horseshoe shape, it is associated with a nearly continuous series of oceanic trenches, volcanic arcs, and volcanic belts and plate movements.

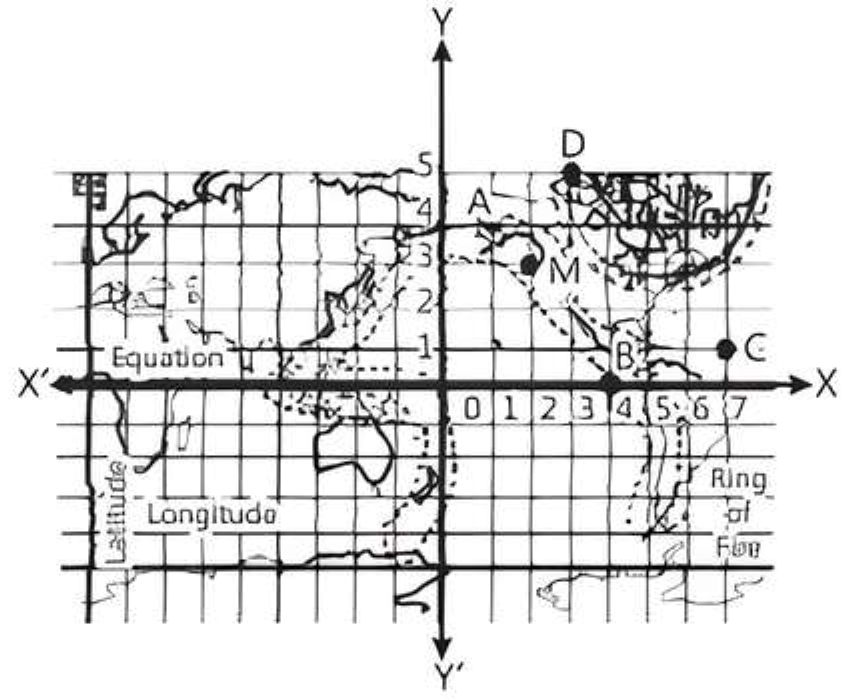


Fault Lines

Large faults within the Earth's crust result from the action of plate tectonic forces, with the largest forming the boundaries between the plates. Energy release associated with rapid movement on active faults is the cause of most earthquakes.



Positions of some countries in the pacific ring of fire is shown in the square grid below.



Based on the above information, solve the following questions: **[CBSE SQP 2021 Term-1]**

Q 1. The distance between the point Country A and Country B is:

a. 4 units b. 5 units c. 6 units d. 7 units

Q 2. Find a relation between x and y such that the point (x, y) is equidistant from the Country C and Country D:

a. $x - y = 2$ b. $x + y = 2$
c. $2x - y = 2$ d. $2x + y = 2$

Q 3. The fault line $3x + y - 9 = 0$ divides the line joining the Country P(1, 3) and Country Q(2, 7) internally in the ratio:

a. 3 : 4 b. 3 : 2
c. 2 : 3 d. 4 : 3

Q 4. The distance of the Country M from the X-axis is.

a. 1 units b. 2 units
c. 3 units d. 5 units

Q 5. What are the co-ordinates of the Country lying on the mid-point of Country A and Country D?

a. (1, 3) b. (2, 9/2)
c. (4, 5/2) d. (9/2, 2)

Solutions

1. From the figure,

coordinates of point A = (1, 4)

and coordinate of point B = (4, 0)

TiP

Distance between two points (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So, distance between country A and country B

$$= \sqrt{(4-1)^2 + (0+4)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

So, option (b) is correct.

2. From the figure,

coordinate of point C = (7, 1)

and coordinate of point D = (3, 5)

Given, the point P(x, y) is equidistant from Country C and Country D.

$$CP = DP$$

$$CP^2 = DP^2 \quad \text{(By distance formula)}$$

$$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$

$$x^2 + 49 - 14x + y^2 + 1 - 2y$$

$$= x^2 + 9 - 6x + y^2 + 25 - 10y$$

$$8x + 8y = 16$$

$$\Rightarrow x + y = 2$$

So, option (b) is correct.

3.

TiP

The coordinates of a point $P(x, y)$ which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are

$$\left\{\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right\}$$

The point which divides the line joining the Country P(1, 3) and Country Q(2, 7) in the ratio $k : 1$ is.

$$\left[\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right]$$

This point lie on the line $3x + y - 9 = 0$

$$\therefore \quad 3 \cdot \frac{2k+1}{k+1} + \frac{7k+3}{k+1} - 9 = 0$$

$$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0 \quad [\because k \neq -1]$$

$$\Rightarrow \quad 4k - 3 = 0$$

$$\Rightarrow \quad k = \frac{3}{4}$$

∴ Required ratio is 3 : 4.

So, option (a) is correct.

4. From the figure,

coordinates of the point M ≡ (2, 3)

∴ The distance of the Country M from the X-axis

= Perpendicular distance of the point M from the X-axis = 3 units

So, option (c) is correct.

5. From the figure,

TiP

Mid-point of the segment joining the points (x_1, y_1) and (x_2, y_2) is $\left\{\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right\}$

Coordinates of the point A ≡ (1, 4)

and coordinates of the point D ≡ (3, 5)

$$\text{Now, mid-point of AD} = \left\{\frac{1+3}{2}, \frac{4+5}{2}\right\} = \left\{\frac{4}{2}, \frac{9}{2}\right\}$$

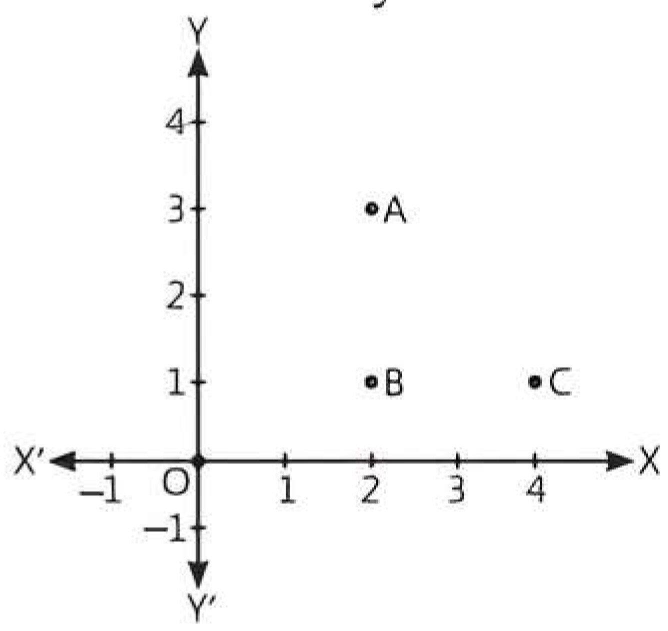
$$= \left(2, \frac{9}{2}\right)$$

∴ Required coordinates of the country lying on the mid–point of Country A and Country D is $\left(2, \frac{9}{2}\right)$

So, option (b) is correct.

Case Study 3

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A. Shagun's house is at B and library is at C.



Based on the above information, solve the following questions: **[CBSE SQP 2023-24]**

Q 1. How far is Alia's house from Shagun's house?

Q 2. How far is the library from Shagun's house?

Q 3. Show that for Shagun, school is farther compared to Alia's house and library.

Or

Show that Alia's house, Shagun's house and library for an isosceles right triangle.

Solutions

1. The coordinates of Alia's house and Shagun's house are A (2, 3) and B (2, 1) respectively.

TR!CK

The distance between $(x_1\ y_1)$ and (x_2, y_2) is .

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

∴ Distance of Alia's house from Shagun's house is,

$$BA = \sqrt{(2-2)^2+(3-1)^2} = \sqrt{(0)^2+(2)^2}$$
$$= \sqrt{0+4} = 2 \text{ units.}$$

2. The coordinates of Shagun's house and library are B (2, 1) and (4, 1) respectively.
Distance of library from Shagun's house is,

$$BC = \sqrt{(4-2)^2+(1-1)^2}$$
$$= \sqrt{(2)^2+(0)^2} = \sqrt{4+0} = 2 \text{ units.}$$

3. The coordinates of school, Alia's house, Shagun's house and library are O(0, 0), A (2, 3), B (2, 1) and C (4, 1)

Now, $BA = \sqrt{(2-2)^2+(3-1)^2} = \sqrt{(0)^2+(2)^2}$
$$= \sqrt{0+4} = 2 \text{ units.}$$
$$BC = \sqrt{(4-2)^2+(1-1)^2} = \sqrt{(2)^2+(0)^2}$$
$$= \sqrt{4+0} = 2 \text{ units.}$$
and $BO = \sqrt{(0-2)^2+(0-1)^2} = \sqrt{(-2)^2+(-1)^2}$
$$= \sqrt{4+1} = \sqrt{5} \text{ units.}$$

Here, BO is greater than BA and BC.
For Shagun, School (O) is father than Alia's house (A) and library (C). **Hence proved.**

Or

The coordinates of Alia's house, Shagun's house and library are A (2, 3), B (2, 1) and C (4, 1) respectively using distance formula,

$$AB = \sqrt{(2-2)^2+(1-3)^2} = \sqrt{(0)^2+(-2)^2}$$
$$= \sqrt{0+4} = 2 \text{ units.}$$
$$BC = \sqrt{(4-2)^2+(1-1)^2} = \sqrt{(2)^2+(0)^2}$$
$$= \sqrt{4+0} = 2 \text{ units.}$$
and $CA = \sqrt{(2-4)^2+(3-1)^2} = \sqrt{(-2)^2+(2)^2}$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

TR!CK

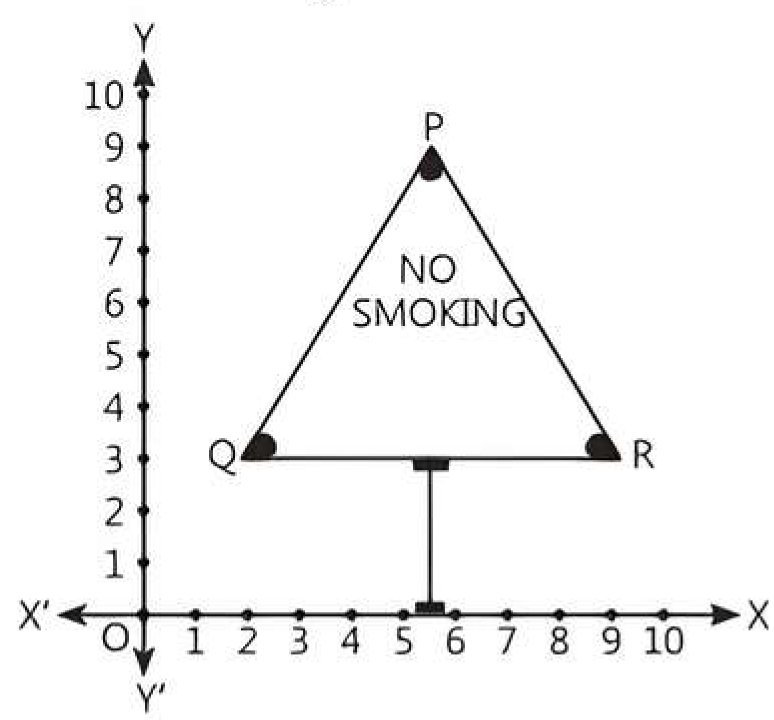
In, Isosceles right triangle ABC, right angle at B.

$$AB^2 + BC^2 = AC^2 \text{ and } AB = BC.$$

Here, $AB^2 + BC^2 = (2)^2 + (2)^2 = 4 + 4 = 8 = CA^2$ and $AB = BC$.
Therefore, A, B and C form an isosceles right triangle.
Hence proved.

Case Study 4

All of the persons know that smoking is injurious to health. So, some college students decided to start a campaign. To raise social awareness about hazards of smoking, they started "NO SMOKING" campaign. Some students were asked to prepare campaign banners in the shape of triangle which is as shown in the figure:



Based on the above information, solve the following questions:

Q 1. Find the coordinates of the mid-point of Q and R.

Q 2. Find the area of the triangle PQR.

Q 3. Find the point on *X*-axis, which is equidistant from points Q and R.

OR

Find the centroid of the triangle PQR.

Solutions

1. The coordinates of the vertices of Q and R are (2, 3) and (9, 3) respectively.

TR!CK

The mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$\therefore$ Mid-point of Q and R $=\left(\frac{2+9}{2}, \frac{3+3}{2}\right)=\left(\frac{11}{2}, 3\right)$

2. The coordinates of the vertices of P, Q and R are (6, 9), (2, 3) and (9, 3) respectively.

Now, base of the $\Delta PQR = QR = \sqrt{(9-2)^2+(3-3)^2}$

$= \sqrt{49+0} = 7$ units.

and height of the ΔPQR = Perpendicular distance from the vertex × P to the base $QR = (9-3) = 6$ units.

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 7 \times 6 = 21$ sq. units.

3. Let point on X-axis be P(x, 0)

Then $(QP)^2 = (PR)^2$

$\therefore$ $(x-2)^2+(0-3)^2=(9-x)^2+(3-0)^2$

$\Rightarrow$ $x^2+4-4x+9=81+x^2-18x+9$

$14x = 77$

$\Rightarrow$ $x = \frac{11}{2}$

$\therefore$ Point on X-axis is $\left(\frac{11}{2}, 0\right)$

Or

TR!CK

The centroid of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

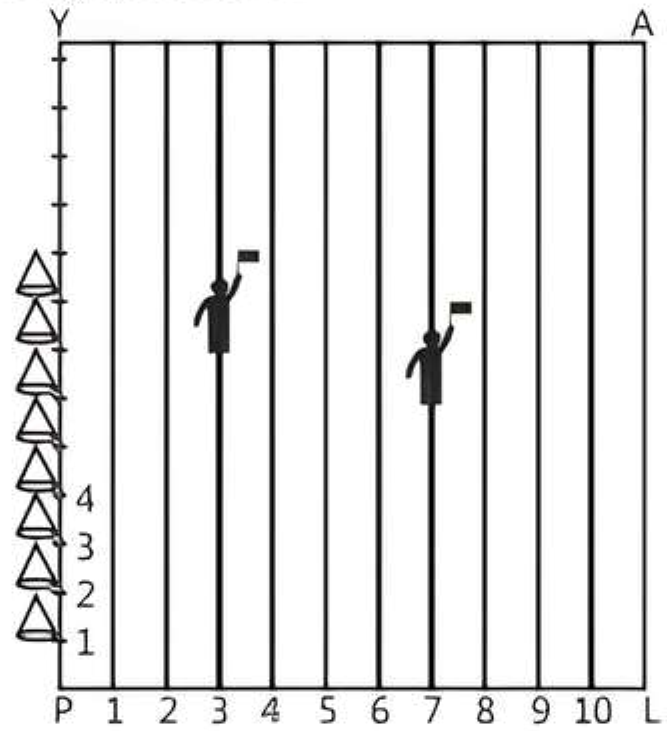
$\therefore$ Centroid of the $\Delta PQR = \left(\frac{6+2+9}{3}, \frac{9+3+3}{3}\right)$

$= \left(\frac{17}{3}, \frac{15}{3}\right) = \left(\frac{17}{3}, 5\right)$

Case Study 5

On Annual Sports Day of a school, parallel lines have been drawn with lime powder at a distance of 1 m from each other in a rectangular shaped school playground. 80 plastic cones have been placed at a distance of 1 m from each other along PY as shown in figure. Pushpendra runs $\frac{1}{4}$th the distance PY on the 3rd line and post a yellow flag. Pankaj runs $\frac{1}{5}$th the distance PY of the 7th line and posts a blue flag.

Based on the above information, solve the following questions:



Q 1. Find the coordinates of the yellow flag.

Q 2. What is the distance between both the flags?

Q 3. If Raman has to post a green flag exactly halfway between the line segment joining the two flags, where should he post his flag?

OR

If Raman change his position and post a green flag at a point between the line segment joining the two flags, then find the coordinate of the green flag which divides the line segment internally in the ratio 1 : 2.

Solutions

1. $\frac{1}{4}$th the distance $PY = \frac{80}{4} m = 20 m$

$\therefore$ Coordinates of yellow flag = (3, 20)

2. $\frac{1}{5}$th the distance $PY = \frac{80}{5} m = 16 m$

$\therefore$ Coordinates of yellow flag = (7, 16) Distance between both flags

$= \sqrt{(7-3)^2+(16-20)^2} = \sqrt{(4)^2+(-4)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ m

3. Position of green flag = Mid-point of yellow and blue flag

$= \left(\frac{3+7}{2}, \frac{20+16}{2}\right) = \left(\frac{10}{2}, \frac{36}{2}\right) = (5, 18)$

Hence, Raman should post his green flag at 18 m on 5th line.

Or

Given points : (3, 20) and (7, 16)

Here, $x_1 = 3$, $y_1 = 20$, $x_2 = 7$, $y_2 = 16$

and ratio $m_1 : m_2 = 1 : 2$

Let the point of division be P(x, y):

Then from division formula:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{1\times 7 + 2\times 3}{1+2} = \frac{13}{3} \text{ and } y = \frac{1\times 16 + 2\times 20}{1+2} = \frac{56}{3}$$

Therefore, the coordinate of the green flag is $\left(\frac{13}{3}, \frac{56}{3}\right)$

Very Short Answer Type Questions

Q 1. Find the distance of the point P(–3, 4) from the X-axis.

Q 2. Find the distance of a point P(x, y) from the origin. [CBSE 2018]

Q 3. Find the distance between the points (a, b) and (–a, –b). [CBSE 2019]

Q 4. Find the value of a, so that the point (3, a) lies on the line represented by $2x - 3y = 5$. [CBSE 2019, 17]

Q 5. Find the value of x for which the distance between the points A (x, 2) and B (9, 8) is 10 units. [CBSE 2019]

Q 6. Find the value (s) of y for which the distance between the points A (3, –1) and B (11, y) is 10 units. [CBSE 2023]

Q 7. Show that the points (–2, 3), (8, 3) and (6, 7) are the vertices of a right-angled triangle. [CBSE 2023]

Q 8. The vertices of a triangle are (–2, 0), (2, 3) and (1, –3). Is the triangle equilateral, isosceles or scalene? [CBSE 2023]

Q 9. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4). [CBSE 2019]

Q 10. Find the value(s) of 'x' so that PQ = QR, where the coordinates of P, Q and R are (6, –1), (1, 3) and (x, 8) respectively. [CBSE 2023]

Short Answer Type-I Questions

Q 1. Write the coordinates of a point on X-axis which is equidistant from the points (–3, 4) and (7, 6).

Q 2. Prove that the points (3, 0), (6, 4) and (–1, 3) are the vertices of a right-angled isosceles triangle. [CBSE 2016]

Q 3. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q (2, –5) and R (–3, 6), find the coordinate of P. [CBSE 2016]

Q 4. Find the coordinates of the point which divides the line segment joining the points (7, –1) and (–3, 4) internally in the ratio 2 : 3. [CBSE 2023]

Q 5. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, –3). Hence find m. [CBSE 2018]

Q 6. A line intersects Y-axis and X-axis at points P and Q, respectively. If R (2, 5) is the mid-point of line segment PQ, then find the coordinates of P and Q. [CBSE 2023]

Q 7. The mid-point of the line segment joining A (2a, 4) and B (–2, 3b) is (1, 2a +1). Find the values of a and b. [CBSE 2019]

Q 8. Find the ratio in which the line segment joining the points A (6, 3) and B (–2, –5) is divided by X-axis. Also find the coordinates of this point of X-axis. [CBSE 2023]

Short Answer Type-II Questions

Q 1. If the point P(x, y) is equidistant from the point A(a + b, b – a) and B(a – b, a + b), prove that bx = ay. [CBSE 2016]

Q 2. If the point Q (0, 1) is equidistant from the points P (5, –3) and R (x, 6); find the value of x. Also, find the distance PR. [CBSE 2023]

Q 3. If P (9a – 2, – b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b. [NCERT EXEMPLAR; CBSE 2016]

Q 4. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, –2) and Q(3, 7)? Also find the value of y. [CBSE 2017]

Q 5. If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and $x + y - 10 = 0$, find the value of k. [NCERT EXEMPLAR; CBSE 2020]

Q 6. Two vertices of a △ABC are given by A(6, 4) and B(–2, 2) and its centroid is G(3, 4). Find the coordinates of the third vertex C of △ABC. [V. Imp.]

Long Answer Type Questions

Q 1. Show that the points A (1, 7), B (4, 2), C (–1, –1) and D (–4, 4) are vertices of the square ABCD. [CBSE 2023]

Q 2. Name the type of quadrilateral formed, if any of the following points (–1, –2), (1, 0), (–1, 2) and (–3, 0). Also give reason for your answer. [NCERT EXERCISE, V. Imp.]

Q 3. Show that the points A (–3, 2), B (–5, –5), C(2, –3) and D (4, 4) are vertices of a rhombus ABCD. Is it also a square? [CBSE 2023]

Q 4. Prove that the points A (–1, 0), B (3, 1), C (2, 2) and D (–2, 1) are the vertices of a parallelogram ABCD. Is it also a rectangle? [CBSE 2023]

Q 5. If A (2, –1), B (3, 4), C (–2, 3) and D (–3, –2) are four points in a plane, show that ABCD is a rhombus but not a square. Find the area of the rhombus.

Q 6. If A (–2, 1), B (a, 0), C(4, b) and D (1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Also, find the length of its sides. *[CBSE 2018]*

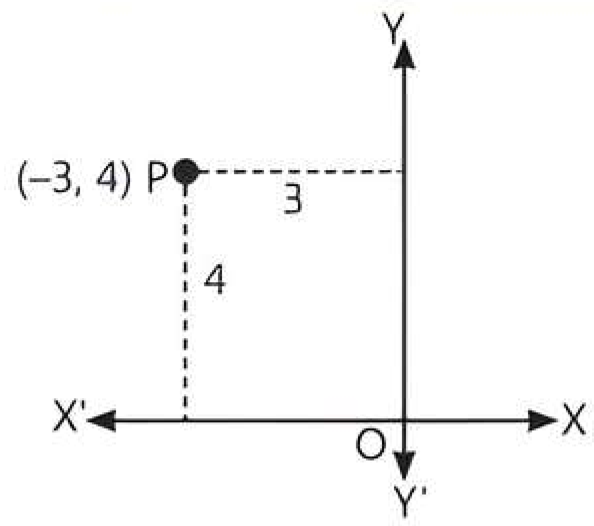
Solutions

Very Short Answer Type Questions

1.

TiP

The perpendicular distance from the X-axis measured along the Y-axis is equivalent to ordinate.



Distance of the point P(–3, 4) from the X-axis = Ordinate of point P (–3, 4) = 4 units

This is also clear from the adjoining figure that the distance of P from X-axis is 4 units.

2.

TR!CK

The distance between two points is

$$d = \sqrt{(\text{difference of abscissa})^2 + (\text{difference of ordinat })^2}$$

Distance between P(x, y) and origin *i.e.*, O(0, 0) is given by, $PO = \sqrt{(0-x)^2 + (0-y)^2} = \sqrt{x^2 + y^2}$.

3. Distance between (a, b) and (–a, –b) is given by

$$d = \sqrt{(-a-a)^2 + (-b-b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \text{ units.}$$

4.

TiP

Every solution of the linear equation is a point lying on the graph of the linear equation.

Since (3, a) lies on $2x - 3y = 5$

$\therefore \quad 2 \times 3 - 3 \times a = 5$

$\Rightarrow \quad 6 - 3a = 5$

$\Rightarrow \quad -3a = 5 - 6 = -1$

Hence, $\quad a = \dfrac{1}{3}$

5. By using the given condition,

$$AB = 10$$

$\Rightarrow \quad (AB)^2 = (10)^2$

TiP

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$\Rightarrow \quad (9 - x)^2 + (8 - 2)^2 = 100$

$\Rightarrow \quad 81 + x^2 - 18x + 36 = 100$

$\Rightarrow \quad x^2 - 18x + 17 = 0$

$\Rightarrow \quad x^2 - (17 + 1)x + 17 = 0$

$\Rightarrow \quad x(x - 17) - 1(x - 17) = 0$

$\Rightarrow \quad (x - 17)(x - 1) = 0 \Rightarrow x = 1, 17$

Hence, the values of x are 1 and 17.

6. By using the given condition,

$$AB = 10$$

$\Rightarrow \quad (AB)^2 = (10)^2$

$\Rightarrow \quad (11 - 3)^2 + (y + 1)^2 = 100$

$\Rightarrow \quad (8)^2 + (y^2 + 1 + 2y) = 100$

$\Rightarrow \quad 64 + y^2 + 2y + 1 = 100$

$\Rightarrow \quad y^2 + 2y - 35 = 0$

$\Rightarrow \quad y^2 + 7y - 5y - 35 = 0$

(By splitting the middle term)

$\Rightarrow \quad y(y + 7) - 5(y + 7) = 0$

$\Rightarrow \quad (y + 7)(y - 5) = 0$

$\Rightarrow \quad y = -7, 5.$

Hence, the values of y are –7 and 5.

7. Let the points are A (-2, 3), B (8, 3) and C (6, 7). Which are the vertices of ΔABC.

Now $AB = \sqrt{(8+2)^2 + (3-3)^2} = \sqrt{(10)^2 + (0)^2} = 10$

$BC = \sqrt{(6-8)^2 + (7-3)^2}$

$= \sqrt{(-2)^2 + (4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

and $CA = \sqrt{(-2-6)^2 + (3-7)^2}$

$= \sqrt{(-8)^2 + (-4)^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

$\because \quad AB \neq BC \neq CA$

But $BC^2 + CA^2 = (2\sqrt{5})^2 + (4\sqrt{5})^2$

$= 20 + 80 = 100 = (10)^2 = AB^2$

So, by Pythagoras theorem, Δ ABC is a right-angled triangle *i.e.*, the given points are the vertices of a right-angled triangle. **Hence proved.**

8. Let the points are A (–2, 0), B(2, 3) and C (1, –3) which are the vertices of ΔABC.

Now $AB = \sqrt{(2+2)^2 + (3-0)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5.$

$$BC = \sqrt{(1-2)^2 + (-3-3)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

and $CA = \sqrt{(-2-1)^2 + (0+3)^2} = \sqrt{(-3)^2 + (3)^2}$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$$

$\because$ AB $\neq$ BC $\neq$ CA

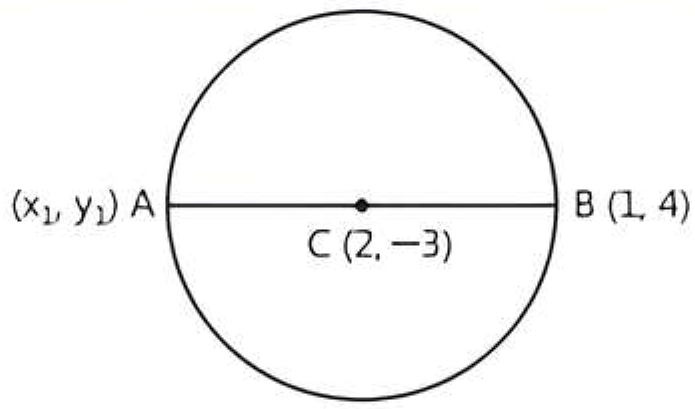
$\therefore$ ΔABC is a scalene triangle *i.e.*, the given points are the vertices of a scalene triangle.

9. Let the coordinate of point A be (x_1, y_1).

$\because$ Centre C of a circle is the mid-point of AB.

TiP

Centre of a circle is the mid-point of its diameter.



$\therefore$ Mid-point of $AB \equiv \left(\frac{x_1+1}{2}, \frac{y_1+4}{2}\right)$

$$\Rightarrow \quad (2, -3) = \left(\frac{x_1+1}{2}, \frac{y_1+4}{2}\right)$$

On equating x and y-coordinates, we get

$$2 = \frac{x_1+1}{2} \quad \text{and} \quad -3 = \frac{y_1+4}{2}$$

$\Rightarrow \quad x_1 = 4 - 1$ and $y_1 = -6 - 4$

$\Rightarrow \quad x_1 = 3$ and $y_1 = -10$

Hence, coordinate of point A is $(3, -10)$.

Short Answer Type-I Questions

1. Let the required point on X-axis be $P(x_1, 0)$.

Since, required point is equidistant from the points A(−3, 4) and B(7, 6).

TR!CK

The distance between (x_1, y_1) *and* (x_2, y_2) *is*

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$\therefore \quad PA = PB$

$\Rightarrow \sqrt{(-3-x_1)^2 + (4-0)^2} = \sqrt{(7-x_1)^2 + (6-0)^2}$

$\Rightarrow \quad \sqrt{9 + x_1^2 + 6x_1 + 16} = \sqrt{49 + x_1^2 - 14x_1 + 36}$

$\Rightarrow \quad x_1^2 + 6x_1 + 25 = x_1^2 - 14x_1 + 85$

(squaring on both the sides)

$\Rightarrow \quad 6x_1 + 14x_1 = 85 - 25$

$\Rightarrow \quad 20x_1 = 60 \Rightarrow x_1 = \frac{60}{20} = 3$

Hence, the required coordinates are P(3, 0).

2. Let A(3, 0), B(6, 4) and C(-1, 3) be the given points.

Then, $AB = \sqrt{(6-3)^2 + (4-0)^2}$

$$= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units.}$$

TiP

In a right-angled isosceles triangle ABC,

$AB = AC$, $BC^2 = AB^2 + AC^2$

or $BC = AC$, $AB^2 = BC^2 + AC^2$

or $BC = AB$, $AC^2 = AB^2 + BC^2$

$$BC = \sqrt{(-1-6)^2 + (3-4)^2}$$

TR!CK

The distance between (x_1, y_1) *and* (x_2, y_2) *is*

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$$= \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1}$$

$$= \sqrt{50} = 5\sqrt{2} \text{ units}$$

and $AC = \sqrt{(-1-3)^2 + (3-0)^2}$

$$= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

Here, $\quad AB = AC$

and $\quad BC^2 = AB^2 + CA^2$

(by Pythagoras theorem)

i.e., $\quad (5\sqrt{2})^2 = 5^2 + 5^2 = 25 + 25 = 50$

$\Rightarrow \quad 50 = 50$

Hence, given triangle is a right-angled isosceles.

Hence proved.

3. Let the y-coordinate of P be k, then its x-coordinate will be $2k$ by given condition.

So, the coordinates of P, Q and R are $(2k, k)$, $(2, -5)$ and $(-3, 6)$ respectively.

According to the given condition, PQ = PR

$\Rightarrow \quad PQ^2 = PR^2$

TR!CK

The distance between (x_1, y_1) *and* (x_2, y_2) *is*

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$\Rightarrow (2-2k)^2 + (-5-k)^2 = (-3-2k)^2 + (6-k)^2$

$\Rightarrow 4 + 4k^2 - 8k + 25 + k^2 + 10k$

$$= 9 + 4k^2 + 12k + 36 + k^2 - 12k$$

$\Rightarrow \quad 5k^2 + 2k + 29 = 5k^2 + 45$

$\Rightarrow \quad 2k = 16$

$\Rightarrow \quad k = \frac{16}{2} \Rightarrow k = 8$

Hence, coordinates of P are $(2 \times 8, 8)$ *i.e.*, (16, 8).

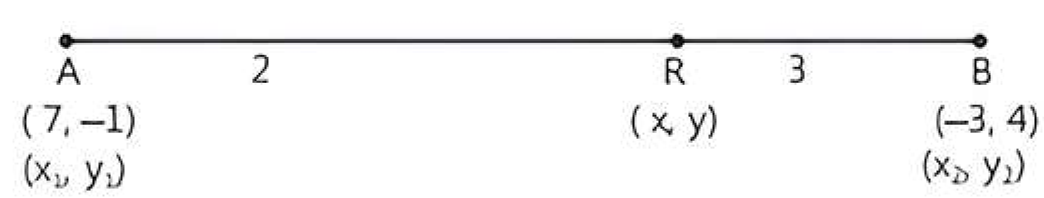
4.

Here, $\quad x_1 = -4$, $y_1 = 0$

$x_2 = 0$, $y_2 = 6$ and $m_1 : m_2 = 2 : 3$.

TR!CK

The coordinates of a point which divides the line segment joining the points (x_1, y_1) *and* (x_2, y_2) *internally in the ratio* $m_1 : m_2$ *are* $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.



By section formula,

$$R\ (x,\ y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{2(-3)+3\times 7}{2+3}, \frac{2\times 4+3\times(-1)}{2+3}\right)$$

$$(\because m_1 = 2,\ m_2 = 3)$$

$$= \left(\frac{-6+21}{5}, \frac{8-3}{5}\right) = \left(\frac{15}{5}, \frac{5}{5}\right) = (3, 1)$$

$\therefore \quad R\ (x,\ y) = (3, 1)$

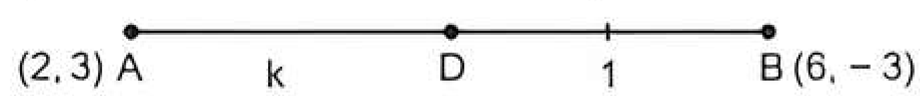
5. Let the ratio in which the line segment joining A(2, 3) and B(6, −3) is divided by point P(4, m) be $k : 1$.

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

Therefore,

$$P(4, m) \equiv P\left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$$



On comparing both sides, we get

$$\frac{6k+2}{k+1} = 4 \Rightarrow 6k+2 = 4k+4 \Rightarrow 2k = 2 \Rightarrow k = 1$$

and $\frac{-3k+3}{k+1} = m \Rightarrow m = \frac{-3(1)+3}{1+1} \qquad (\because k = 1)$

$$= \frac{-3+3}{2} = \frac{0}{2} = 0$$

Hence, required ratio is 1 : 1 and, $m = 0$

6. We know that the coordinates of the points at X and Y-axes are $(x, 0)$ and $(0, y)$ respectively.

TiPs

- *The ordinate of each point lying on X-axis is zero.*
- *The abscissa of each point lying on Y-axis is zero.*

Let the coordinates of the points P and Q be $(0, b)$ and $(a, 0)$ respectively.

Mid-point of PQ *i.e.*, R = (2, 5) (given)

TiP

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\Rightarrow \quad \left(\frac{0+a}{2}, \frac{b+0}{2}\right) = (2, 5)$$

$$\Rightarrow \quad \left(\frac{a}{2}, \frac{b}{2}\right) = (2, 5) \quad \text{(using mid-point formula)}$$

$$\Rightarrow \quad \frac{a}{2} = 2 \text{ and } \frac{b}{2} = -5$$

$$\therefore \quad a = 4 \text{ and } b = -10$$

Hence, the coordinates of P and Q are (0, 10) and (4, 0) respectively.

7. By using given conditions, coordinate of mid-point of A $(2a, 4)$ and B $(-2, 3b)$ = $(1, 2a+1)$

$$\Rightarrow \quad \left(\frac{2a-2}{2}, \frac{4+3b}{2}\right) = (1,\ 2a+1)$$

TR!CK

The mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

On equating the coordinates of x and y both sides, we get

$$\frac{2a-2}{2} = 1 \quad \text{and} \quad \frac{4+3b}{2} = 2a+1$$

$$\Rightarrow 2a-2 = 2 \quad \text{and} \quad 4+3b = 4a+2$$

$$\Rightarrow 2a = 4 \quad \text{and} \quad 2+3b = 4a$$

$$\Rightarrow a = 2 \quad \text{and} \quad 2+3b = 4\times 2$$

$$\Rightarrow a = 2 \quad \text{and} \quad 3b = 6$$

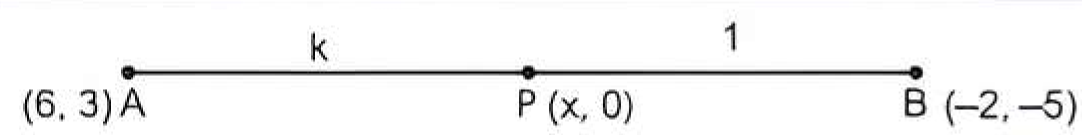
$$\Rightarrow a = 2 \quad \text{and} \quad b = 2$$

Hence, values of a and b are 2 and 2.

8. Let any point on X-axis be P(x, 0). Let point P (x, 0) divides the line joining points A (6, 3) and B (−2, −5) in the ratio $k : 1$.

TiP

Ordinate of each point on X-axis is always zero.



By using internal division formula,

$$\text{Coordinate of P} = \left(\frac{-2k+6}{k+1}, \frac{-5k+3}{k+1}\right)$$

TR!CK

The coordinates of the point P(x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$.

$$\Rightarrow (x, 0) = \left(\frac{-2k+6}{k+1}, \frac{-5k+3}{k+1}\right) \qquad ...(1)$$

$$\Rightarrow \quad 0 = \frac{-5k+3}{k+1} \Rightarrow -5k+3 = 0 \Rightarrow k = \frac{3}{5}$$

Hence, required ratio is 3 : 5.

$\therefore$ From eq. (1),

$$x = \frac{-2k+6}{k+1} \qquad ...(2)$$

Put $k = \frac{3}{5}$ in eq. (2), we get

$$x = \frac{-2\times\frac{3}{5}+6}{\frac{3}{5}+1} = \frac{-6+30}{3+5} = \frac{24}{8} = 3$$

Hence, coordinate of point P on X-axis is (3, 0).

Short Answer Type-II Questions

1. Given, $PA = PB$

$\Rightarrow \quad (PA)^2 = (PB)^2$

TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$$\Rightarrow \{(a+b)-x\}^2 + \{(b-a)-y\}^2 = \{(a-b)-x\}^2 + \{(a+b)-y\}^2$$

$$\Rightarrow (a+b)^2 + x^2 - 2(a+b)x + (b-a)^2 + y^2 - 2(b-a)y = (a-b)^2 + x^2 - 2(a-b)x + (a+b)^2 + y^2 - 2(a+b)y$$

$$\Rightarrow -2(a+b)x - 2(b-a)y = -2(a-b)x - 2(a+b)y \quad [\because (a-b)^2 = (b-a)^2]$$

$$\Rightarrow (a+b)x + (b-a)y = (a-b)x + (a+b)y$$

$$\Rightarrow (a+b-a+b)x = (a+b-b+a)y$$

$$\Rightarrow 2bx = 2ay \Rightarrow bx = ay$$

Hence proved.

2. Given, Q (0, 1) is equidistant from P (5,–3) and R(x, 6).

$\therefore \quad PQ = RQ$

$\Rightarrow \quad PQ^2 = RQ^2$

$\Rightarrow \quad (0-5)^2 + (1+3)^2 = (0-x)^2 + (1-6)^2$ (By distance formula)

$\Rightarrow \quad 25 + 16 = x^2 + 25$

$\Rightarrow \quad x^2 = 16 \Rightarrow x = \pm 4$

$\therefore$ Coordinates of point R are $(\pm 4, 6)$.

Now, distance $PR = \sqrt{(4-5)^2 + (6+3)^2}$ (when $x = 4$)

$= \sqrt{(-1)^2 + (9)^2} = \sqrt{1+81}$

$= \sqrt{82}$ units

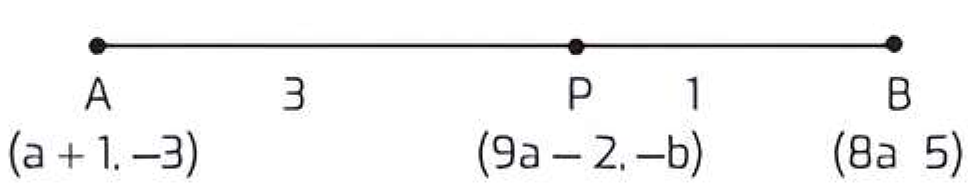
or distance $PR = \sqrt{(-4-5)^2 + (6+3)^2}$ (when $x = -4$)

$= \sqrt{(-9)^2 + (9)^2} = \sqrt{81+81}$

$= \sqrt{2 \times 81} = 9\sqrt{2}$ units.

3. Given that point $P(9a-2, -b)$ divides AB in the ratio 3 : 1.

$\therefore \quad AP : PB = m : n = 3 : 1$



TR!CK

The coordinates of the point $P(x, y)$ which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

Using section formula for internal division,

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$\Rightarrow \quad 9a - 2 = \frac{3 \times (8a) + 1 \times (3a+1)}{3+1}$$

$$\Rightarrow \quad (9a-2) \times 4 = 24a + 3a + 1$$

$$\Rightarrow \quad 36a - 8 = 27a + 1$$

$$\Rightarrow \quad 36a - 27a = 1 + 8 \Rightarrow 9a = 9$$

$$\Rightarrow \quad a = \frac{9}{9} = 1$$

and $\quad y = \dfrac{my_2 + ny_1}{m+n}$

$$\Rightarrow \quad -b = \frac{3 \times 5 + 1 \times (-3)}{3+1}$$

$$\Rightarrow -b = \frac{15-3}{4} = \frac{12}{4} = 3 \quad \Rightarrow \quad b = -3$$

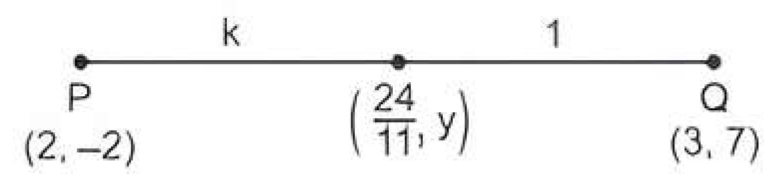
Hence, the values of a and b are 1 and –3 respectively.

4. Let the ratio in which the line segment joining P(2, –2) and Q(3, 7) is divided by point $\left(\frac{24}{11}, y\right)$ be $k : 1$.

TR!CK

The coordinates of the point $P(x, y)$ which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

$$\therefore \left(\frac{24}{11}, y\right) = \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$



On comparing both sides, we get

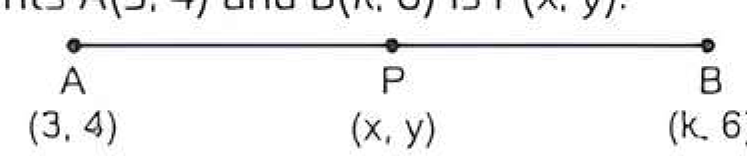
$$\frac{3k+2}{k+1} = \frac{24}{11} \Rightarrow 33k + 22 = 24k + 24$$

$$\Rightarrow \quad 9k = 2 \quad \Rightarrow \quad k = \frac{2}{9}$$

and $y = \dfrac{7k-2}{k+1} = \dfrac{7\left(\frac{2}{9}\right) - 2}{\frac{2}{9} + 1} = \dfrac{14-18}{2+9} = \dfrac{-4}{11}$

Hence, the required ratio is 2 : 9 and the value of y is $\frac{-4}{11}$.

5. Given, mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y).



TiP

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$\therefore$ Using mid-point formula,

$$x = \frac{3+k}{2} \text{ and } y = \frac{4+6}{2} = 5$$

Now, put the above values of x and y in the equation

$$x + y - 10 = 0$$

$$\therefore \quad \frac{3+k}{2} + 5 - 10 = 0$$

$$\Rightarrow \quad k + 3 = 5 \times 2 = 10$$

$$\Rightarrow \quad k = 10 - 3 = 7$$

$$\therefore \quad k = 7$$

6. Let the coordinates of third vertex C be (x_3, y_3).

Given, A(6, 4), B(−2, 2) and G(3, 4) are the two vertices and a centroid of ΔABC respectively.

TR!CK

If the vertices of a triangle are $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3), then centroid of the triangle is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

The coordinates of centroid of a triangle are given as,

$$x = \frac{x_1+x_2+x_3}{3} \Rightarrow 3 = \frac{6+(-2)+x_3}{3}$$

$$\Rightarrow 9 = 4 + x_3 \quad \Rightarrow x_3 = 9 - 4 = 5$$

and $y = \frac{y_1+y_2+y_3}{3}$

$$\Rightarrow 4 = \frac{4+2+y_3}{3} \Rightarrow 12 = 6 + y_3$$

$$\Rightarrow y_3 = 12 - 6 = 6$$

Hence, the coordinates of the third vertex C of ΔABC are (5, 6).

Long Answer Type Questions

1. Given, points are A (1, 7), B (4, 2), C (−1, −1) and D (−4, 4).

To identify that ABCD is a square, we have to find its edges or sides.

Now, $AB = \sqrt{(4-1)^2+(2-7)^2}$ (By distance formula)

$$= \sqrt{(3)^2+(-5)^2} = \sqrt{9+25} = \sqrt{34}.$$

TR!CK

Distance between (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

$$BC = \sqrt{(-1-4)^2+(-1-2)^2} = \sqrt{(-5)^2+(-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34}.$$

$$CD = \sqrt{(-4+1)^2+(4+1)^2} = \sqrt{(-3)^2+(5)^2}$$

$$= \sqrt{9+25} = \sqrt{34}$$

and $DA = \sqrt{(1+4)^2+(7-4)^2} = \sqrt{(5)^2+(3)^2}$

$$= \sqrt{25+9} = \sqrt{34}$$

Here, we see that AB = BC = CD = DA.

It means given ABCD may be formed either square or rhombus.

Now we further determine the diagonals, so that exact shape of figure will be clear.

Now diagonal of a quadrilateral are

$$AC = \sqrt{(-1-1)^2+(-1-7)^2} = \sqrt{(-2)^2+(-8)^2}$$

$$= \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

and $BD = \sqrt{(-4-4)^2+(4-2)^2}$

$$= \sqrt{(-8)^2+(2)^2} = \sqrt{64+4} = \sqrt{68}$$

$$= 2\sqrt{17}$$

Here, AC = BD

TiP

In a square ABCD, AB = BC = CD = DA and diagonal AC = diagonal BD.

It means, given points formed a square having all sides are equal and diagonals are also equal.

Hence proved.

2. Given vertices of a quadrilateral are A(-1, -2), B(1, 0), C(-1, 2) and D(-3 , 0).

To identify the type of quadrilateral, we have to find its edges or sides.

Now sides of a quadrilateral are

$AB = \sqrt{(1+1)^2+(0+2)^2}$ (Using distance formula)

TR!CK

Distance between (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

$$= \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2+(2-0)^2}$$

$$= \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$CD = \sqrt{(-3+1)^2+(0-2)^2}$$

$$= \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

and $DA = \sqrt{(-1+3)^2+(-2-0)^2} = \sqrt{(2)^2+(-2)^2}$

$$= \sqrt{4+4} = 2\sqrt{2}$$

Here we see that AB = BC = CD = DA

It means given quadrilateral may be either square or rhombus.

Now we further determine the diagonals, so that exact shape of figure will be clear.

Now diagonals of a quadrilateral are

$$AC = \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{0+4^2} = 4$$

and $BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{(-4)^2+0} = 4$

Here, AC = BD

TiP

In a square ABCD, AB = BC = CD = DA and diagonal AC = diagonal BD.

It means, given vertices of a quadrilateral is a square having all sides are equal and diagonals are also equal.

3. Given points are A (–3, 2), B (–5, –5), (2, –3) and D (4, 4).

To identify the type of quadrilateral, we have to find its edges or sides.

Now, $AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{(-2)^2 + (-7)^2}$

$= \sqrt{4+49} = \sqrt{53}$ [By distance formula]

$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{(7)^2 + (2)^2}$

$= \sqrt{49+4} = \sqrt{53}$

$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{(2)^2 + (7)^2} = \sqrt{4+49}$

$= \sqrt{53}$

and $DA = \sqrt{(-3-4)^2 + (2-4)^2} = \sqrt{(-7)^2 + (-2)^2}$

$= \sqrt{49+4} = \sqrt{53}$.

Here, we see that AB = BC = CD = DA.

It means given quadrilateral may be either square or rhombus.

Now we furthar determine the digonals, so that exact shape of figure will be clear.

Now, diagonals of a quadrilateral are

$AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{(5)^2 + (-5)^2}$

$= \sqrt{25+25} = 5\sqrt{2}$

and $BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{(9)^2 + (9)^2}$

$= \sqrt{81+81} = 9\sqrt{2}$

Here, AC ≠ BD.

It means, given points formed a rhombus having all sides are equal but diagonals are not equal.

Hence proved.

Rhombus ABCD does not form a square because its diagonals are not equal in length while square have equal diagonals inlength as well as all four sides are equal in length also.

4. Given points are A(–1, 0), B (3, 1), C(2, 2) and D (–2, 1).

To identify the type of quadrilateral, we have to find its edges or sides.

Now, $AB = \sqrt{(3+1)^2 + (1-0)^2} = \sqrt{(4)^2 + (1)^2}$

[By distance formula]

$= \sqrt{16+1} = \sqrt{17}$

$BC = \sqrt{(2-3)^2 + (2-1)^2} = \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

$CD = \sqrt{(-2-2)^2 + (1-2)^2} = \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{16+1} = \sqrt{17}$

and $DA = \sqrt{(-1+2)^2 + (0-1)^2} = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{1+1} = 2.$

Here we see that AB = CD and BC = DA.

It means given quadrilateral may be either parallelogram or rectangle.

Now, we further determine the diagonals, so that exact shape of figure will be clear.

Now, diagonals of a quadrilateral are

$AC = \sqrt{(2+1)^2 + (2-0)^2} = \sqrt{(3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

and $BD = \sqrt{(-2-3)^2 + (1-1)^2} = \sqrt{(-5)^2 + (0)^2}$

$= \sqrt{25+0} = 5.$

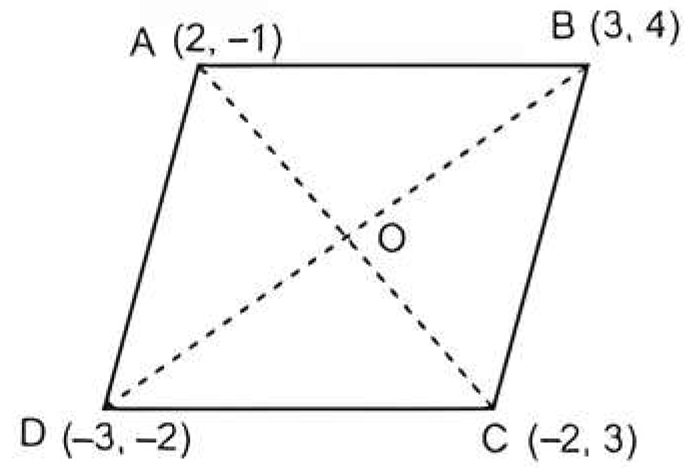
Here, AC ≠ BD

It means , given points formed a parallelogram having opposite sides are equal but diagonals are not equal. **Hence proved.**

Parallelogram ABCD does not form a rectangle because its diagonals are not equal in length while rectangle have equal diagonals in length as well as opposite sides are also equal.

5. $AB = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{1^2 + 5^2}$

$= \sqrt{1+25} = \sqrt{26}$ units



TR!CK

The distance between (x_1, y_1) and (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$BC = \sqrt{(-2-3)^2 + (3-4)^2}$

$= \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$ units

$CD = \sqrt{(-3+2)^2 + (-2-3)^2}$

$= \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$ units

and $DA = \sqrt{(2+3)^2 + (-1+2)^2}$

$= \sqrt{5^2 + 1^2} = \sqrt{25+1} = \sqrt{26}$ units

Here, $AB = BC = CD = DA = \sqrt{26}$ units

Now, $AC = \sqrt{(-2-2)^2 + (3+1)^2} = \sqrt{(-4)^2 + 4^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

and $BD = \sqrt{(-3-3)^2 + (-2-4)^2}$

$= \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ units

So, AC ≠ BD

In a quadrilateral, if the diagonals are not equal but sides are equal, then quadrilateral is a rhombus, while in case of a square, diagonals are equal.

Hence, given quadrilateral is a rhombus but not a square. **Hence proved.**

TR!CK

Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

where d_1 and d_2 are diagonals.

$\therefore$ Area of rhombus ABCD

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24 \text{ sq. units}$$

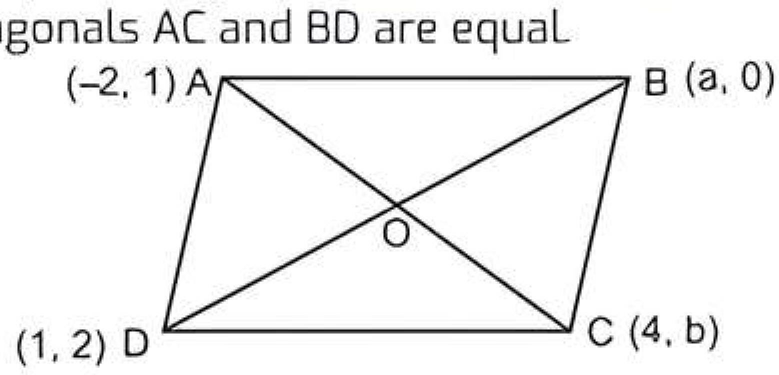
Hence, the area of rhombus ABCD is 24 sq. units.

6.

TiPs

- *Diagonals of a parallelogram bisect each other at a point.*
- *In a parallelogram, the length of opposite sides are equal.*

In parallelogram, coordinates of mid-point of diagonals AC and BD are equal.



$$\therefore \left(\frac{-2+4}{2}, \frac{1+b}{2}\right) \equiv \left(\frac{a+1}{2}, \frac{2+0}{2}\right)$$

(using mid-point formula)

$$\Rightarrow \left(1, \frac{1+b}{2}\right) \equiv \left(\frac{a+1}{2}, 1\right)$$

$$\Rightarrow 1 = \frac{a+1}{2} \quad \text{and} \quad \frac{1+b}{2} = 1$$

$$\Rightarrow 2 = a+1 \quad \text{and} \quad 1+b = 2$$

$$\Rightarrow a = 2-1 = 1 \quad \text{and} \quad b = 2-1 = 1$$

So, the coordinates of B and C are (1, 0) and (4, 1) respectively.

TR!CK

The distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(\textit{difference of abscissa})^2 + (\textit{difference of ordinate})^2}$$

Now,

$$AB = \sqrt{(1+2)^2 + (0-1)^2} = \sqrt{3^2 + 1^2} = \sqrt{10} \text{ units}$$

$$DC = AB = \sqrt{10} \text{ units}$$

(Opposite sides of a parallelogram are equal)

and $AD = \sqrt{(1+2)^2 + (2-1)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$ units

$$BC = AD = \sqrt{10} \text{ units}$$

Hence, $DC = AB = \sqrt{10}$ units and $BC = AD = \sqrt{10}$ units.

Chapter Test

Multiple Choice Questions

Q 1. If the points A (4,5) and B (x, 4) are on the circle with centre C(2,2), then the value of x is:

a. 5 b. 6 c. 2 d. 3

Q 2. The distance between the points A(0, 6) and B(0, 2) is:

a. 8 b. 6 c. 4 d. 2

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The value of y is 6, for which the distance between the points P(2,−3) and Q(10, y) is 10.

Reason (R): The distance between two points A(x_1, y_1) and B(x_2, y_2) is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Q 4. Assertion (A): The distance of the point (2, 11) from the Y-axis is 2 units.

Reason (R): The distance of the point (x, y) from Y-axis is x unit.

Fill in the Blanks

Q 5. The distance of the point (5, −4) from X-axis is ....

Q 6. The image of a point (−4, 5) with respect to Y-axis is ....

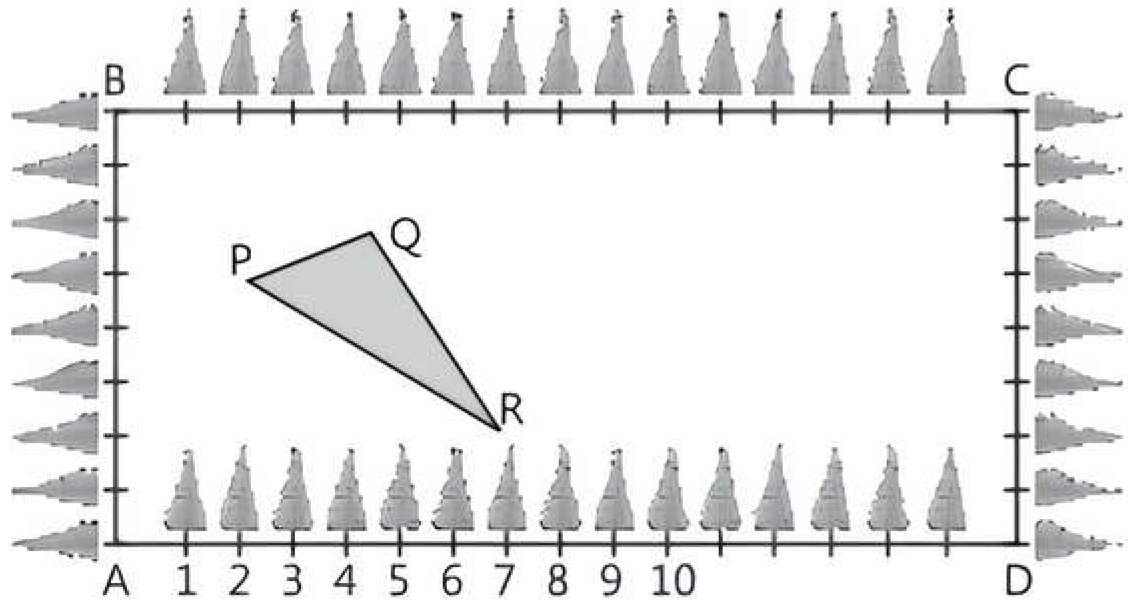
True/False

Q 7. If the vertices of a parallelogram taken in order are A(−1, 6), B(2, −5) and C(7, 2), then the fourth vertex is (4, 1).

Q 8. The ratio in which the point divides the line joining the points A(1, 2) and B(−1, 1) internally in the ratio 1 : 2 is $\left(\frac{1}{3}, \frac{5}{3}\right)$.

Case Study Based Question

Q 9. The class X students of a school in Rajinder Nagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Mango are planted on the boundary at the distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in figure.



Based on the above figure, solve the following questions:

(i) What will be the coordinates of the vertices of ΔPQR if C is the origin?

(ii) Taking A as origin, then find the coordinates of vertices of the triangle PQR.

(iii) If A is the origin, then find the perimeter of ΔPQR.

OR

If A is the origin, then find the coordinates of the point M on PR such that PM : MR = 2 : 1.

Very Short Answer Type Questions

Q 10. Find the distance between the points (1, 0) and (2, cot θ).

Q 11. If the coordinates of the mid-point of the line segment joining (–8, 13) and (x, 7) is (4, 10), then find the value of x.

Short Answer Type-I Questions

Q 12. Show that the points P(9, 0), Q(9, 6), R(–9, 6) and S (–9, 0) are the vertices of a parallelogram.

Q 13. If the distance between the points (2, –2) and (–1, x) is 5, then find the possible values of x.

Short Answer Type-II Questions

Q 14. If vertices of a ΔABC are A(7, 3), B(5, 3) and (3, –1), then find the length of the median through vertex A.

Q 15. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, –2) and Q(3, 7)? Also find the value of y.

Long Answer Type Question

Q 16. Points A (–1, y) and B(5, 7) lie on a circle with centre C(2, –3y). Find the value of y. Also, find the radius of the circle.