CHAPTER -09

DIFFERENTIAL EQUATIONS

TWO MARKS QUESTION

1. Find order and degree of the differential equation $\frac{dy}{dx} - \cos x = 0$ (U)

2. Find order and degree of the differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$ (U)

3. Find order and degree of the differential equation $y''' + y^2 + e^{y'} = o$ (U)

4. Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \sin\left(\frac{d^3y}{dx^3}\right) = 0$ (U)

5. Find order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ (U)

6. Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ (U)

7. Find order and degree of the differential equation $y' + 5y = 0$ (U)

8. Find order and degree of the differential equation $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$ (U)

9. Find order and degree of the differential equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$ (U)

10. Find order and degree of the differential equation $y' + y = e^x$ (U)

11. Find order and degree of the differential equation $y''' + (y')^2 + 2y = 0$ (U)

12. Find order and degree of the differential equation $y'' + 2y' + \sin y = 0$ (U)

13. Find order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ (U)

14. Find order and degree of the differential equation $2x^2\left(\frac{d^2y}{dx^2}\right) - 3\left(\frac{dy}{dx}\right) + y = 0$ (U)

15. Find order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$ (U)

16. Find order and degree of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$ (U)

17. Find order and degree of the differential equation $\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$ (U)

18. Find the number of arbitrary constants in the general solution of differential equation of fourth order also find the number of arbitrary constants in the particular solution of differential equation of third order.
(U)

19. Find the general solution of a differential equation: $\frac{dx}{dy} + P_1x = Q_1$. (U)

THREE MARKS QUESTIONS

1. Verify that the function y= e^{-3x} is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ (A)

2. Verify that the function y= a cosx +b sinx, where a,b $\in$ R is a solution of the differential equations $\frac{d^2y}{dx^2} + y = 0$. (A)

3. Verify that the function y= $e^x + 1$ is a solution of the differential equation $y'' - y' = 0$. (A)

4. Verify that the function $y = x^2 + 2x + c$ is a solution of the differential equation y'-2x-2=0. (A)

5. Verify that the function y=cosx+C is a solution of the differential equation y'+sinx=0. (A)

6. Verify that the function $y = \sqrt{1+x^2}$ is a solution of the differential equation $y' = \frac{xy}{1+x^2}$. (A)

7. Verify that the function y=Ax is a solution of the differential equation xy'=y($x \neq 0$). (A)

8. Verify that the function $y = x\sin x$ is a solution of the differential equation
$xy' = y + x\sqrt{x^2 - y^2}$ ($x \neq 0$ and x>y or x<-y) (A)

9. Verify that the function xy=logy+C is a solution of the differential equation
$y' = \frac{y^2}{1-xy}$ ($xy \neq 1$) (A)

10. Verify that the function y- cosy =x is a solution of the differential equation (ysiny+cosy+x)y'=y. (A)

11. Verify that the function $x+y=\tan^{-1}y$ is a solution of the differential equation $y^2y' + y^2 + 1 = 0$. (A)

12. Verify that the function $y=\sqrt{a^2 - x^2}$ $x \in (-a,a)$ is a solution of the differential equation $x+y\frac{dy}{dx} = 0 (y \neq 0)$. (A)

13. Verify that the function $y=ae^x+be^{-x}+x^2$ is a solution of the differential equation $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$ (A)

14. Verify that the function $y=e^x(a\cos x+b\sin x)$ is a solution of the differential equation . $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$. (A)

15. Verify that the function y=xsin3x is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$. (A)

16. Verify that the function $x^2 = 2y^2 \log y$ is a solution of the differential equation $(x^2 + y^2)\frac{dy}{dx} - xy = 0$. (A)

17. Form the differential equation representing the family of curves y=asin(x+b), where a,b are arbitrary constants. (A)

18. Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin. (A)

19. Form the differential equation of the family of circles touching the x-axis at origin. (A)

20. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis. (A)

21. Form the differential equation representing the family of circles touching the y-axis at origin. (A)

22. Form the differential equation representing the family of parabolas having vertex at origin and x-axis along positive y-axis. (A)

23. Form the differential equation representing the family of ellipses having foci on y-axis and centre at origin. (A)

24. Form the differential equation representing the family of hyperbolas having foci on x-axis and centre at origin. (A)

25. Form the differential equation representing the family of circles having centre on y-axis and radius 3 units. (A)

26. Form the differential equation representing the family of curves given by $(x-a)^2 + 2y^2 = a^2$, where a is an arbitrary constant. (A)

27. Form the differential equation of the family of circles in the first quadrant which touch the coordinate axes. (A)

FIVE MARKS QUESTIONS

1. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}, (y \neq 2)$ (A)

2. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. (A)

3. Find the particular solution of the differential equation

   $\frac{dy}{dx} = -4xy^2$ given that y=1, when x=0. (A)

4. Find the equation of the curve passing through the point (1, 1) whose differential equation is $xdy = (2x^2+1)dx(x \neq 0)$. (A)

5. Find the equation of a curve passing through the point (-2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$. (A)

6. In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs.1000 double itself? (A)

7. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$. (A)

8. Find the general solution of the differential equation $\frac{dy}{dx} = \sqrt{4-y^2}(-2 < y < 2)$. (A)

9. Find the general solution of the differential equation $\frac{dy}{dx} + y = 1(y \neq 1)$. (A)

10. Find the general solution of the differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. (A)

11. Find the general solution of the differential equation $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$ (A)

12. Find the general solution of the differential equation $\frac{dy}{dx} = (1+x^2)(1+y^2)$. (A)

13. Find the general solution of the differential equation $y \log y dx - x dy = 0$. (A)

14. Find the general solution of the differential equation $x^5 \frac{dy}{dx} = -y^5$. (A)

15. Find the general solution of the differential equation $\frac{dy}{dx} = \sin^{-1} x$. (A)

16. Find the general solution of the differential equation

$e^x \tan y dx + (1-e^x)\sec^2 y dy = 0$. (A)

17. Find the general solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$. (A)

18. Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$. (A)

19. Find the particular solution of the differential equation

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x; y = 1$ when x=0. (A)

20. Find the particular solution of the differential equation $x(x^2 - 1)\frac{dy}{dx} = 1; y = 0$ when x=2. (A)

21. Find the particular solution of the differential equation

$\cos\left(\frac{dy}{dx}\right) = a(a \in R); y = 2$ when x=0. (A)

22. Find the particular solution of the differential equation $\frac{dy}{dx} = y \tan x; y = 1$ when x=0. (A)

23. Find the particular solution of the differential equation

$(1+e^{2x})dy + (1+y^2)e^x dx,$ given that y=1 when x=0 (A)

24. Find the particular solution of the differential equation $(x-y)(dx+dy) = dx - dy$ given that y=-1, when x=0. (A)

25. Find the equation of a curve passing through the point(0,0) and whose differential equation is $y' = e^x \sin x$. (A)

26. For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$, find the solution curve passing through the point (1,-1). (A)

27. Find the equation of curve passing through the point (0,-2) given that at any point (x,y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point. (A)

28. At any point (x,y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contanct to the point (-4,-3). Find the equation of the curve given that it passes through (-2,1). (A)

29. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds. (A)

30. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs.100 if Rs.100 double itself in 10 years($\log_e 2=0.6931$). (A)

31. In a bank principal increases continuously at the rate of 5% per year. An amount of Rs.1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5}=1.648$). (A)

32. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present? (A)

33. Show that the differential equation $(x-y)\frac{dy}{dx}=x+2y$ is homogeneous and solve it. (S)

34. Show that the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$ is homogeneous and solve it.

(S)

35. Show that the differential equation $2ye^{\frac{x}{y}}dx+(y-2xe^{\frac{x}{y}})dy=0$ is homogeneous and find its particular solution, given that, x=0 when y=1. (S)

36. Show that the family of curves for which the slope of the tangent at any point (x,y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2-y^2=cx$

37. Show that the differential equation $(x^2+xy)dy=(x^2+y^2)dx$ is homogeneous and solve it. (S)

38. Show that the differential equation $y'=\frac{x+y}{x}$ is homogeneous and solve it. (S)

39. Show that the differential equation $(x-y)dy-(x+y)dx=0$ is homogeneous and solve it. (S)

40. Show that the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is homogeneous and solve it. (S)

41. Show that the differential equation $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is homogeneous and solve it. (S)

42. Show that the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$ is homogeneous and solve it. (S)

43. Show that the differential equation
$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}ydx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}xdy$ is homogeneous and solve it.
(S)

44. Show that the differential equation $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$ is homogeneous and solve it. (S)

45. Show that the differential equation $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$
is homogeneous and solve it. (S)

46. Show that the differential equation $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$
is homogeneous and solve it. (S)

47. For, (x+y)dy+(x-y)dx=0, find the particular solution satisfying the given condition,
y=1 when x=1. (S)

48. For $x^2dy + (xy + y^2)dx$, find the particular solution satisfying the given condition,
y=1 when x=1. (S)

49. For, $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0;$ find the particular solution satisfying the given condition,
$y = \pi/4$ when x=1. (S)

50. For, $\frac{dy}{dx} - \frac{y}{x} + \cos ec\left(\frac{y}{x}\right) = 0;$ find the particular solution satisfying the given condition, y=0
when x=1. (S)

51. For, $2xy + y^2 - 2x^2\frac{dy}{dx} = 0;$ find the particular solution satisfying the given condition,
y=2 when x=1. (S)

52. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$. (A)

53. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\neq 0)$. (A)

54. Find the general solution of the differential equation $ydx-(x+2y^2)dy=0$. (A)

55. Find the general solution of the differential equation $\frac{dy}{dx}+2y=\sin x$. (A)

56. Find the general solution of the differential equation $\frac{dy}{dx}+3y=e^{-2x}$. (A)

57. Find the general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$. (A)

58. Find the general solution of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x(0\leq x\leq \pi/2)$. (A)

59. Find the general solution of the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x(0\leq x\leq \pi/2)$. (A)

60. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$. (A)

61. Find the general solution of the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$. (A)

62. Find the general solution of the differential equation $(1+x^2)dy+2xydx=\cot xdx(x\neq 0)$. (A)

63. Find the general solution of the differential equation $x\frac{dy}{dx}+y-x+xy\cot x=0(x\neq 0)$. (A)

64. Find the general solution of the differential equation $(x+y)\frac{dy}{dx}=1$. (A)

65. Find the general solution of the differential equation $ydx+(x-y^2)dy=0$. (A)

66. Find the general solution of the differential equation $(x+3y^2)\frac{dy}{dx}=y(y>0)$. (A)

67. For, $\frac{dy}{dx}+2y\tan x=\sin x$, find the particular solution satisfying the given condition y=0 when x=$\frac{\pi}{3}$. (A)

68. For, $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, find the particular solution satisfying the given condition y=0 when x=1. (A)

69. For, $\frac{dy}{dx} - 3y\cot x = \sin 2x$ find the particular solution satisfying the given condition y=2 when x= $\frac{\pi}{2}$. (A)

70. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinates of the point. (S)

71. Find the equation of a curve passing through the point (0,2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. (S)

72. Find the general solution of the differential equation $x\frac{dy}{dx} - y = 2x^2$. (A)

73. Find the general solution of the differential equation $(1-y^2)\frac{dx}{dy} + yx = ay(-1 < y < 1)$. (A)

74. Verify that the function $y = c_1 e^{ax}\cos bx + c_2 e^{ax}\sin bx,$ where c_1, c_2 are arbitrary constants is a solution of the differential equation: $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$. (S)

75. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes. (S)

76. Find the particular solution of the differential equation: $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given that y=0 when x=0. (S)

77. Solve the differential equation: $(xdy - ydx)y\sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\left(\frac{y}{x}\right)$. (S)

78. Solve the differential equation: $(\tan^{-1} y - x)dy = (1+y^2)dx$. (S)

79. Prove that $x^2 - y^2 = c(x^2+y^2)^2$ is the general solution of differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$, where c is parameter. (S)

80. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$ is given by (x+y+1)=A(1-x-y-2xy), where A is parameter. (S)

81. Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is sinxcosydx+cosxsinydy=0. (S)

82. Find the particular solution of the differential equaton: $(1+e^{2x})dy+(1+y^2)e^x dx=0$ given that y=1 when x=0. (S)

83. Solve the differential equation: $ye^{\frac{x}{y}}dx=(xe^{\frac{x}{y}}+y^2)dy(y\neq 0)$. (S)

84. Find a particular solution of the differential equation (x-y)(dx+dy)=dx-dy, given that y=-1, when x=0. (S)

85. Solve the differential equation : $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1(x\neq 0)$. (S)

86. Find a particular solution of the differential equation :

$\frac{dy}{dx}+y\cot x=4x\operatorname{cosec}x(x\neq 0)$, given that y=0 when x=$\frac{\pi}{2}$. (S)

87. Find a particular solution of the differential equation $(x+1)\frac{dy}{dx}=2e^{-y}-1$, given that y=0 when x=0. (S)

88. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009? (S)

89. Find the general solution of the differential equation: $\frac{ydx-xdy}{y}=0$. (A)

90. Find the general solution of the differential equation: $e^x dy+(ye^x+2x)dx=0$. (A)