Quadratic Equations

MATHEMATICS REASONING

1. The roots of $ax^2+bx+c=0,\ a\neq 0$ are real and unequal, if b^2-4ac is .________.

(a) =0 (B) > 0
(c) <0 (d) ≥ 0

2. If $x=\sqrt{2+\sqrt{2+\sqrt{2+.........}}}$, then________.

(a) $x=1$ (b) $0<x<1$
(c) x is infinite (d) $x=2$

3. If 2 is a root of the equation $x^2+bx+12=0$ and the equation $x^2+bx+q=0$ has equal : roots, then q is equal to

(a) 8 (b) –8
(c) 16 (d) –16

4. If one root of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ is 1, then the other root is __.

(a) $\dfrac{b(c-a)}{a(b-c)}$ (b) $\dfrac{a(b-c)}{c(a-b)}$
(c) $\dfrac{a(b-c)}{b(c-a)}$ (d) $\dfrac{c(a-b)}{a(b-c)}$

5. If the roots of the equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal. Then _______.

(a) $2b=a+c$ (b) $2a=b+c$
(c) $2c=a+b$ (d) $\dfrac{1}{b}=\dfrac{1}{a}+\dfrac{1}{c}$

6. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as –6 and 1 respectively. The correct roots are ——.

(a) $3,-2$ (b) $-3,2$
(c) $-6,-1$ (d) $6,-1$

7. If $\sqrt{x-1}-\sqrt{x+1}+1=0$, then 4x is equal to_____.

(a) $4\sqrt{-1}$ (b) 0
(c) 5 (d) $1\frac{1}{4}$

8. If one of roots of $2x^2+ax+32=0$ is twice the other root, then the value of a is _____.

(a) $-3\sqrt{2}$ (b) $8\sqrt{2}$
(c) $12\sqrt{2}$ (d) $-2\sqrt{2}$

9. For what value of a, the roots of the equation $2x^2+6x+a=0$, satisfy the condition $\left(\dfrac{\alpha}{\beta}\right)+\left(\dfrac{\beta}{\alpha}\right)<2$ (where α and β are the roots of equation).

(a) $a<0$ (b) $-1<a<0$
(c) $-1<a<1$ (d) None of these

10. Roots of the quadratic equation $x^2+x-(a+1)(a+2)=0$ are _____.

(a) $-(a+1),(a+2)$
(b) $(a+1),-(a+2)$
(c) $(a+1),(a+2)$
(d) $-(a+1),-(a+2)$

11. The roots of the equation $3\sqrt{x}+5(x)^{\frac{1}{2}}=\sqrt{2}$ can be found by solving

(a) $9x^2+28x+25=0$
(b) $9x^2+30x+25=0$
(c) $9x^2+28x-25=0$
(d) $16x^2+22x-30=0$

12. If the roots of the equation $(a^2+b^2)x^2-2b(a+c)x+(b^2+c^2)=0$ are equal, then _____.

(a) $2b=a+c$ (b) $b^2=ac$
(c) $b=\dfrac{2ac}{a+c}$ (d) $b=ac$

13. Two numbers whose sum is 12 and the absolute value of whose difference is 4 are the roots of the equation _____.

(a) $x^2 - 12x + 30 = 0$

(b) $x^2 - 12x + 32 = 0$

(c) $2x^2 - 6x + 7 = 0$

(d) $2x^2 - 24x + 43 = 0$

14. The roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are _____.

(a) $1, -8$ (b) $1, -2$

(c) $\frac{2}{3}, \frac{1}{3}$ (d) $-2, -8$

15. In the equation $\frac{x(x-1)-(m+1)}{(x-1)(m-1)} = \frac{x}{m}$, the roots are equal when m = _____.

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 0

(d) 1

EVERYDAY MATHEMATICS

16. In a bangle shop, if the shopkeeper displays; the bangles in the form of a square then he is left with 38 bangles. If he wanted to increase the size of square by one unit each side of the square he found that 25 bangles fall short of in completing the square.
The actual number of bangles which he had with him in the shop was _____.

(a) 1690

(b) 999

(c) 538

(d) Can't be determined

17. A man walks a distance of 48 km in a given time. If he walks 2 km/hr. faster, he will perform the journey 4 hrs. before. His; normal rate of walking, is _____.

(a) 3 km/hr.

(b) 4 km/hr.

(c) - 6 km/hr. or 4 km/hr.

(d) 5 km/hr.

18. In the Maths Olympiad of 2020 at Animal Planet, two representatives from the donkey's side, while solving a quadratic equation, committed the following mistakes.
(i) One of them made a mistake in the constant term and got the roots as 5 I and 9.
(ii) Another one committed an error in the coefficient of x and he got the roots as 12 and 4.
But in the meantime, they realised that they are wrong and they managed to get it right jointly. Find the quadratic equation.

(a) $x^2 + 4x + 14 = 0$

(b) $2x^2 + 7x - 24 = 0$

(c) $x^2 - 14x + 48 = 0$

(d) $3x^2 - 17x + 52 = 0$

19. Rs. 6500 were divided equally among a certain number of persons. If there had been 15 more persons, each would have got Rs. 30 less. Find the original number of persons.

(a) 50

(b) 60

(c) 45

(d) 55

20. Swati can row her boat at a speed of 5 km/hr in still water. If it takes her 1 hour more to row the Boat 5.25 km upstream than to return downstream, find the speed of the stream.

(a) 5 km/hr.

(b) 2 km/hr.

(c) 3 km/hr.

(d) 4 km/hr.

ACHIEVERS SECTION (HOTS)

21. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$

(d) $5x^2 - 3x + 1 = 0$

22. Read the statements carefully. .

Statement - I: The quadratic equation $ax^2+bx+c=0$ has two distinct real roots, if $b^2-4ac>0$.

Statement - II: The quadratic equation $2(a^2+b^2)x^2+2(a+b)x+1=0$ has no real roots, when $a\neq b$.

(a) Both Statement - I and Statement - II are true.

(b) Statement -1 is true but Statement - II is false.

(c) Statement -1 is false but Statement - II is true. ;

(d) Both Statement - I and Statement - II are false.

23. If the roots of the equation $x^2+2cx+ab=0$ are real and unequal, then the equation $x^2-2(a+b)x+a^2+b^2+2c^2=0$ has _____ roots.

(a) Real

(b) Equal

(c) No real

(d) Can't be determined

24. Read the statement carefully and state 'T' for true and 'F' for false.

(i) The value of $2+\cfrac{1}{2+\cfrac{1}{2+......\infty}}$ is $\sqrt{2}$.

(ii) A line segment AB of length 2 m is divided at C into two parts such that $AC^2=AB-CB$ The length of the part CB is $3+\sqrt{5}$.

(iii) Every quadratic equation can have at most two real roots.

(iv) A real number a is said to be root of the quadratic equation $ax^2+bx+c=0$, if $a\alpha^2+b\alpha+c=0$.

	(i)	(ii)	(iii)	(iv)
(a)	F	T	T	T
(b)	F	T	T	F
(c)	T	F	F	T
(d)	F	F	T	T

25. The denominator of a fraction is one more than twice the numerator. If the sum of the A r. fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$

(C) $\frac{4}{3}$ (d) $\frac{3}{4}$

ANSWER KEY

1.	B	**2.**	D	**3.**	C	**4.**	D	**5.**	B
6.	D	**7.**	C	**8.**	C	**9.**	D	**10.**	B
11.	A	**12.**	B	**13.**	B	**14.**	A	**15.**	B
16.	B	**17.**	B	**18.**	C	**19.**	A	**20.**	B
21.	B	**22.**	C	**23.**	C	**24.**	D	**25.**	A

HINTS AND SOLUTION

1. (b): Given equation is $ax^2+bx+c=0$

Roots are real and unequal, if $b^2-4ac>0$

2. (d): $x=\sqrt{2+\sqrt{2+\sqrt{2+....}}}$ ...(i)

Squaring both sides of (1), we get

$x^2=2+\sqrt{2+\sqrt{2+\sqrt{2+....}}}$

$\Rightarrow\ x^2=2+x \ \Rightarrow\ x^2-x-2=0$

$\Rightarrow\ x=\frac{1\pm\sqrt{1+18}}{2}=\frac{1\pm\sqrt{9}}{2}$

Since x cannot be negative, therefore, neglect $\frac{1-\sqrt{9}}{2}$

Thus, $x=2$

3. (c): Since $x=2$ is a root of the equation

$x^2+bx+12=0 \ \Rightarrow\ (2)^2+b(2)+12=0$

$\Rightarrow\ 2b=-16 \ \Rightarrow\ b=-8$

Then, the equation x^2+bx+q becomes

$x^2-8x+q=0$ ...(ii)

Since (1) has equal roots $\Rightarrow\ b^2-4ac=0$

$\Rightarrow\ (-8)^2-4(1)q=0 \ \Rightarrow\ q=16$

4. (d): Given equation is

$a(b-c)x^2+b(c-a)x+c(a-b)=0$

Let α be the other root. then

Product of roots $=\alpha\times 1=\dfrac{c(a-b)}{a(b-c)}$

$\Rightarrow \quad \alpha=\dfrac{c}{a}\left(\dfrac{a-b}{b-c}\right)$

5. (b): Since the given equation has equal roots,

$\therefore \quad D=0$

$\Rightarrow \quad (b-c)^2-4(c-a)(a-b)=0$

$\Rightarrow \; 4a^2+b^2+c^2-4ab+2bc-4ac=0$

$\Rightarrow \; (-2a)^2+(b^2)+(c)^2+2(-2a)(b)+2bc$
$+2(-2a)(c)=0$

$\Rightarrow \quad (-2a+b+c)^2=0$

$\Rightarrow \; -2a+b+c=0 \;\Rightarrow 2a=b+c$

6. (d): Let the equation be $x^2+ax+b=0$

Its roots are 3 and 2

$\therefore$ Sum of roots, $5=-a$

and product of roots, $6=b$

Equation is $x^2-5x+6-0$

Now constant term is wrong and it is given that correct constant term is -6.

$\therefore \quad x^2-5x-6=0$ is the correct equation.

Its roots are -1 and 6.

7. (c): Given equation is

$\sqrt{x-1}-\sqrt{x+1}=-1$ ...(1)

Rationalising (1), we get

$\dfrac{(x-1)-(x+1)}{\sqrt{x-1}+\sqrt{x+1}}=-1 \;\Rightarrow\; \sqrt{x-1}+\sqrt{x+1}=2$ ....(2)

Adding (1) and (2), we get

$2\sqrt{x-1}=1 \;\Rightarrow\; x-1=\dfrac{1}{4} \;\Rightarrow\; 4x=5$

8. (c): Equation is $2x^2+ax+32=0$

Let one root be α, then other would be 2α.

Now, $\alpha\times 2\alpha=16 \;\Rightarrow\; \alpha=\pm 2\sqrt{2}$

and $\alpha+2\alpha=-a/2 \;\Rightarrow\; 3\alpha=-a/2 \;\Rightarrow$ $6\alpha=-a$

$\Rightarrow \quad \pm 12\sqrt{2}=-a$ or $a=\pm 12\sqrt{2}$

9. (d): Given equation is $2x^2+6x+a=0$

Now, $\left(\dfrac{\alpha}{\beta}\right)+\left(\dfrac{\beta}{\alpha}\right)<2$

$\Rightarrow \; \dfrac{\alpha^2+\beta^2}{\alpha\beta}<2 \;\Rightarrow\; \dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}<2$

$\Rightarrow \; \dfrac{9-a}{a/2}<2 \Rightarrow 9-a<a \; \left[\begin{array}{l}\because\; \alpha+\beta=-3 \\ \text{and } \alpha\beta=\dfrac{a}{2}\end{array}\right]$

$\Rightarrow \; \dfrac{9}{2}<a$

10. (b): Given equation is

$x^2+x-(a+1)(a+2)=0$

$\Rightarrow x^2+(a+2)x-(a+1)x-(a+1)(a+2)=0$

$\Rightarrow \quad x(x+(a+2))-(a+1)(x+(a+2))=0$

$\Rightarrow \; (x-(a+1))(x+(a+2))=0$

$\Rightarrow \quad x=(a+1)$ or $x=-(a+2)$

11. (a): We have. $3x^{1/2}+5x^{-1/2}=\sqrt{2}$

$\Rightarrow \quad 3x^{1/2}+\dfrac{5}{x^{1/2}}=\sqrt{2}$

$\Rightarrow \quad 3x+5=(2x)^{1/2}$ ....(1)

Squaring both sides of (1), we have

$9x^2+25+30x=2x$

$\Rightarrow \; 9x^2+28x+25=0$

12. (b): Since roots of the given equation are equal

$\therefore D=0$

$\Rightarrow \; (-2b(a+c))^2-4(a^2+b^2)(b^2+c^2)=0$

$\Rightarrow \; 4b^2(a^2+c^2+2ac)-4(a^2b^2+a^2c^2+b^4$
$+b^2c^2)=0$

$\Rightarrow a^2b^2+b^2c^2+2ab^2c-a^2b^2-a^2c^2-b^4-b^2c^2=0$

$\Rightarrow 2ab^2c-a^2c^2-b^4=0 \Rightarrow$

$b^4+a^2c^2-2ab^2c=0$

$\Rightarrow (b^2-ac)^2=0 \;\Rightarrow\; b^2=ac$

13. (b): Let the two roots be a and b, then

$a+b=12$ ...(1) and $a-b=4$ ...(2)

$\Rightarrow \quad a=8$ and $b=4$ (from (1) and (2))

Required equation is $x^2-12x+32=0$

14. (a): Given equation is $x^{2/3}+x^{1/3}-2=0$

Let $y = x^{1/3} \Rightarrow y^2 + y - 2 = 0$

$\Rightarrow y^2 + 2y - y - 2 = 0 \Rightarrow (y-1)(y+2) = 0$

$\Rightarrow y = 1$ or $y = -2 \Rightarrow x^{1/3} = 1$ or $x^{1/3} = -2$

$\Rightarrow x = 1$ or $x = -8$

15. (b) We have, $\dfrac{x(x-1)-(m+1)}{(x-1)(m-1)} = \dfrac{x}{m}$

$\Rightarrow (x^2 - x)m - (m^2 + m) = x(x-1)(m-1)$

$\Rightarrow x^2m - mx - m^2 - m = x^2m - x^2 - xm + x$

$\Rightarrow (x^2 - m^2) - (m + x) = 0$

$\Rightarrow (x+m)(x - m - 1) = 0$

Now, since roots are equal

$\Rightarrow -m = m + 1 \Rightarrow m = \dfrac{-1}{2}$

16. (b): Let the number of bangles in a side of square = x

According to the question,

$x^2 + 38$ = Total no. of bangles ...(1)

Also, $(x+1)^2 - 25 =$ Total no. of bangles ....(2)

From (1) and (2), we have

$x^2 + 38 = (x+1)^2 - 25$

$\Rightarrow 38 + 24 = 2x \Rightarrow x = 31$

$\therefore$ Total no. of bangles $= (31)^2 + 38 = 999$

17. (b): Let the speed of man be x km/hr. and the time taken by him to cover 48 km with speed x be t.

According to the question, $48 = x \times t$ ...(1)

Also, $48 = (x+2)(t-4)$ ....(2)

$\Rightarrow 48 = xt - 4x + 2t - 8$

$\Rightarrow 56 = 48 - 4x + 2t$ [Using (1)]

$\Rightarrow 8 = -4x + 2t \Rightarrow 4 = -2x + t$

$\Rightarrow 4 = -2x + \dfrac{48}{x}$ [Using (1)]

$\Rightarrow 2x = -x^2 + 24 \Rightarrow x^2 + 2x - 24 = 0$

$\Rightarrow x = \dfrac{-2 \pm \sqrt{4+96}}{2} = 4$ or -6

Since, speed cannot be negative.

Required speed is $4\ km/hr$

18. (c): For 1st one,

Let the equation be $x^2 + ax + b = 0$

Since roots are 5 and 9

$\therefore a = -14$ and $b = 45$

For 2nd one,

Let the equation be $x^2 + px + q = 0$

Since roots are 12 and 4.

$p = -16$ and $q = 48$

Now, according to the question, b and p both are wrong.

Therefore, the correct equation would be

$x^2 - 14x + 48 = 0$

19. (a): Let x be the number of persons and y be the amount received by each person.

According to the question, $xy = 6500$ ...(1)

Also, $(x+15)(y-30) = 6500$ ...(2)

$\Rightarrow xy - 30x + 15y - 450 = 6500$

$\Rightarrow 6500 - 30x + 15y - 450 = 6500$ [From (1)]

$\Rightarrow 2x - y + 30 = 0$

$\Rightarrow 2x - \dfrac{6500}{x} + 30 = 0$ [From (1)]

$\Rightarrow 2x^2 - 6500 + 30x = 0$

$\Rightarrow x^2 + 15x - 3250 = 0$

$\Rightarrow (x-50)(x+65) = 0$

$\Rightarrow x = 50$ or $x = -65$

Since, number of persons cannot be negative.

Hence, number of persons = 50

20. (b)

21. (b): (a) $2x^2 - 3\sqrt{2}x + \dfrac{9}{4} = 0$

$D = b^2 - 4ac = 18 - 4(2)\left(\dfrac{2}{9}\right) = 18 - 18 = 0$

$\therefore$ Given have equal roots.

(b) $x^2 + x - 5 = 0$

$D = 1 - 4(1)(-5) = 1 + 20 = 21 > 0$

$\therefore$ Given equation has real and distinct roots.

(C) $x^2 + 3x + 2\sqrt{2} = 0$

$D = (3)^2 - 4(1)(2\sqrt{2}) = 9 - 8\sqrt{2} < 0$

$\therefore$ Given equation does not have real roots.

(D) $5x^2 - 3x + 1 = 0$

$D = (-3)^2 - 4(5)(1) = 9 - 20 = -11 < 0$

$\therefore$ Given equation does not have real roots.

22. (c): Statement - I is false, since the quadratic equation $ax^2+bx+c=0$ has two distinct real roots, if $b^2-4ac>0$.
Also, given equation is
$2(a^2+b^2)x^2+2(a+b)x+1=0$
$D=b^2-4ac=(2(a+b))^2-4(2a^2+2b^2)(1)$
$=4a^2+4b^2+8ab-8a^2-8b^2$
$=-4a^2-4b^2+8ab=-4(a-b)^2<0$
$\therefore$ Given equation has no real roots. Hence, statement - II is true.

23. (c): The given equation $x^2+2cx+ab=0$ has real and unequal roots.
$\Rightarrow \quad D=(2c)^2-4ab>0$
$\Rightarrow \quad 4c^2-4ab>0 \Rightarrow c^2-ab>0$
Now, the equation
$x^2-2(a+b)x+a^2+b^2+2c^2=0$
$\therefore \; D=(-2(a+b))^2-4(a^2+b^2+2c^2)$
$=4a^2+4b^2+8ab-4a^2-4b^2-8c^2$
$=8ab-8c^2=8(ab-c^2)<0$
$(\because c^2-ab>0)$
Hence, the equation has no real roots.

24. (d): (i) Let $x=2+\cfrac{1}{2+\cfrac{1}{2+....\infty}}$
$\Rightarrow \; x=2+\frac{1}{x} \Rightarrow x^2-2x-1=0$
$\Rightarrow \; x=1+\sqrt{2}$ or $1-\sqrt{2}$
(ii) Let $AC=x$, then $CB=2-x$
Since, $AC^2=AB\times CB$
$\Rightarrow \; x^2=2(2-x) \Rightarrow x^2+2x-4=0$
$\Rightarrow \; x=-1+\sqrt{5}$ or $-1-\sqrt{5}$ (Not possible)
$\therefore \; CB=2-x=2-(\sqrt{5}-1)=(3-\sqrt{5})m$
(iii) True, every quadratic equation has maximum of two real roots.
(iv) True.

25. (a): Let the fraction be $\frac{x}{y}$.
According to the question,
$y=2x+1$ ....(1)

Also, $\frac{x}{y}+\frac{y}{x}=2\frac{16}{21}=\frac{58}{21}$
$\Rightarrow \; \frac{x}{2x+1}+\frac{2x+1}{x}=\frac{58}{21}$ (From(1))
$\Rightarrow \; \frac{x^2+4x^2+1+4x}{x(2x+1)}=\frac{58}{21}$
$\Rightarrow \; 105x^2+84x+21=116x^2+58x$
$\Rightarrow \; (x-3)(11x+7)=0$
$\Rightarrow \; x=3,$ or $x=-\frac{7}{11}$ (Not possible)
$\therefore \; y=7 \; \therefore$ Required fraction $=\frac{3}{7}$