GATE 2023
Electronics & Communications Engineering
Exam Held on : 05-02-2023
Afternoon Session

SECTION - A GENERAL APTITUDE

Q.1 "I cannot support this proposal. My ________ will not permit it."

(a) conscious (b) consensus
(c) conscience (d) consent

[1 Mark : MCQ]

Ans. (c)

End of Solution

Q.2 Courts : _______ : : Parliament : Legislature

(a) Judiciary (b) Executive
(c) Governmental (d) Legal

[1 Mark : MCQ]

Ans. (a)

End of Solution

Q.3 What is the smallest number with distinct digits whose digits add up to 45?

(a) 123555789 (b) 123457869
(c) 123456789 (d) 99999

[1 Mark : MCQ]

Ans. (c)

The digits should be distinct and smallest number is 123456789.

End of Solution

Q.4 In a class of 100 students,

(i) there are 30 students who neither like romantic movies nor comedy movies,
(ii) the number of students who like romantic movies is twice the number of students who like comedy movies, and
(iii) the number of students who like both romantic movies and comedy movies is 20.

How many students in the class like romantic movies?

(a) 40 (b) 20
(c) 60 (d) 30

[1 Mark : MCQ]

Ans. (c)

Let students who like Romantic Movies = R.
Students who like Comedy Movies = C.
Given $R = 2C$
Also, 30 students do not like Romantic and Comedy Movies both.

$\therefore$ $100 - 30 = 70 = n(R \cap C)$

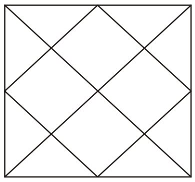
and $n(C \cap R) = 20$

$$n(R \cap C) = n(R) + n(C) - n(C \cap R)$$
$$70 = 2C + C - 20$$

$3C = 90$
$C = 30$
$R = 2C = 60$

End of Solution

Q.5 How many rectangles are present in the given figure?



(a) 8 (b) 9
(c) 10 (d) 12

[1 Mark : MCQ]

Ans. **(c)**

Number of rectangles = 10 as square is also called as rectangle.

End of Solution

Q.6 Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.

Based only on the information provided above, which one of the following options can be logically inferred with certainty?

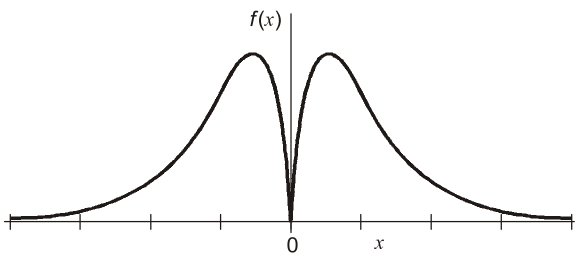
(a) All creatures with claws are animals.
(b) Some creatures with claws are non-ferocious.
(c) Some non-ferocious creatures have claws.
(d) Some ferocious creatures are creatures with claws.

[2 Marks : MCQ]

Ans. **(d)**

End of Solution

Q.7 Which one of the following options represents the given graph?



(a) $f(x) = x^2 2^{-|x|}$ (b) $f(x) = x2^{-|x|}$
(c) $f(x) = |x| 2^{-x}$ (d) $f(x) = x2^{-x}$

[2 Marks : MCQ]

Ans. (a)

Since, the given function is an even function.

Option (d) is only represents the even function.

End of Solution

Q.8 Which one of the following options can be inferred from the given passage alone?

When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

[Excerpt from The Truth about Stories by T. King]

(a) It is a child's description of what he or she likes.
(b) It is an adult's memory of what he or she liked as a child.
(c) The child in the passage read stories about interplanetary travel only in parts.
(d) It teaches us that stories are good for children.

[2 Marks : MCQ]

Ans. (b)

End of Solution

Q.9 Out of 1000 individuals in a town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid-positive individuals. The strategy is to:

(i) Collect saliva samples from all 1000 individuals and randomly group them into sets of 5.
(ii) Mix the samples within each set and test the mixed sample for covid.
(iii) If the test done in (ii) gives a negative result, then declare all the 5 individuals to be covid negative.
(iv) If the test done in (ii) gives a positive result, then all the 5 individuals are separately tested for covid.

Given this strategy, no more than _______ testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

(a) 700
(b) 600
(c) 800
(d) 1000

[2 Marks : MCQ]

Ans. (a)

End of Solution

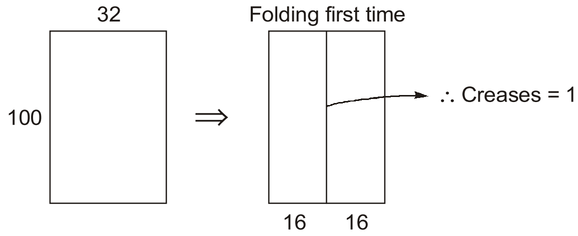
Q.10 100 cm × 32 cm rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions 100 cm × 1 cm.

The total number of creases visible when the sheet is unfolded is _______.

(a) 32
(b) 5
(c) 31
(d) 63

[2 Marks : MCQ]

Ans. (c)



$\therefore$ After folding 5 times $= 1 + 2^1 + 2^2 + 2^3 + 2^4$

$= 1 + 2 + 4 + 8 + 16 = 31$

End of Solution

■■■■

SECTION - B TECHNICAL

Q.11 Let $v_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$ and $v_2 = \begin{bmatrix}2\\1\\3\end{bmatrix}$ be two vectors. The value of the coefficient α in the expression $v_1 = \alpha v_2 + e$, which minimizes the length of the error vector e, is

(a) $\frac{7}{2}$

(b) $-\frac{2}{7}$

(c) $\frac{2}{7}$

(d) $-\frac{7}{2}$

[1 Mark : MCQ]

Ans. **(c)**

$$e = V_1 - \alpha V_2$$

$$e = (i + 2k + 0k) - \alpha(2i + j + 3k)$$

$$\hat{e} = (1-2\alpha)\hat{i} + (2-\alpha)\hat{j} + (0-3\alpha)\hat{k}$$

$$|\hat{e}| = \sqrt{(1-2\alpha)^2 + (2-\alpha)^2 + (-3\alpha)^2}$$

$$|\hat{e}|^2 = 5 + 14\alpha^2 - 8\alpha \text{ to be minimum at } \frac{\partial e^2}{\partial \alpha} = 28\alpha - 8 = 0$$

$\therefore$ $$\alpha = \frac{2}{7} \text{ stationary point}$$

End of Solution

Q.12 The rate of increase, of a scalar field $f(x, y, z) = xyz$ in the direction $v = (2,1,2)$ at a point (0,2,1) is

(a) $\frac{2}{3}$

(b) $\frac{4}{3}$

(c) 2

(d) 4

[1 Mark : MCQ]

Ans. **(b)**

$$f(x, y, z) = xyz$$

$$\overline{\nabla f} = \hat{i} f_x + \hat{j} f_y + \hat{k} f_z$$

$$= \hat{i}(yz) + \hat{j}(xz) + \hat{k}(xy)$$

$$\overline{\nabla f}_{(0,2,1)} = \hat{i}(2) + 0\hat{j} + 0\hat{k}$$

Directional derivative,

$$D.D = \overline{\nabla f} \cdot \frac{\bar{a}}{|\bar{a}|}$$

$$= \left(2\hat{i} + 0\hat{j} + 0\hat{k}\right) \cdot \frac{\left(2\hat{i} + \hat{j} + 2\hat{k}\right)}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{4}{\sqrt{9}} = \frac{4}{3}$$

End of Solution

Q.13 Let $w^4 = 16j$. Which of the following cannot be a value of w?

(a) $2e^{\frac{j2\pi}{8}}$ (b) $2e^{\frac{j\pi}{8}}$

(c) $2e^{\frac{j5\pi}{8}}$ (d) $2e^{\frac{j9\pi}{8}}$

[1 Mark : MCQ]

Ans. **(a)**

$$w = (2)j^{1/4}$$

$$w = 2(0 + j)^{1/4}$$

$$w = 2\left[e^{j(2n+1)\pi/2}\right]^{1/4}$$

$$= 2\left[e^{j(2n+1)\pi/8}\right]$$

For $n = 0$, $w = e^{j\pi/8}$

For $n = 2$, $w = 2e^{5\pi j/8}$

For $n = 4$, $w = 2e^{9\pi j/8}$

End of Solution

Q.14 The value of the contour integral, $\oint_C \left(\dfrac{z+2}{z^2+2z+2}\right)dz$, where the contour C is $\left\{z : \left|z+1-\dfrac{3}{2}j\right| = 1\right\}$, taken in the counter clockwise direction, is

(a) $-\pi(1 + j)$ (b) $\pi(1 + j)$

(c) $\pi(1 - j)$ (d) $-\pi(1 - j)$

[1 Mark : MCQ]

Ans. **(b)**

$$I = \oint_c \frac{z+2}{z^2+2z+2}dz; \; c = \left|z+1-\frac{3}{2}i\right| = 1$$

Poles are given $(z + 1)^2 + 1 = 0$

$$z + 1 = \pm\sqrt{-1}$$

$$z = -1 + j, -1 - j$$

where $-1 - i$ lies outside 'c'

$$z = (-1, 1) \text{ lies inside '}c\text{'}.$$

by CRT

$$\oint_c f(z)dz = 2\pi i \text{ Res } (f(z), z = -1 + j)$$

$$= 2\pi i\left(\frac{z+2}{2(z+1)}\right)_{z=-1+i}$$

$$= 2\pi i\left(\frac{-1+j+2}{2(-1+j+1)}\right)$$
$$= \pi(1 + j)$$

End of Solution

Q.15 Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1 \le k \le n\}$ and $\{v_k \mid 1 \le k \le n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B = P^{-1} AB$, respectively, are

(a) $\{\lambda_k \det(A) \mid 1 \le k \le n\}$ and $\{Pv_k \mid 1 \le k \le n\}$

(b) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{v_k \mid 1 \le k \le n\}$

(c) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{Pv_k \mid 1 \le k \le n\}$

(d) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P^{-1}v_k \mid 1 \le k \le n\}$

[1 Mark : MCQ]

Ans. (c)

$$B = P^{-1} AP$$
$$\Rightarrow \quad A = PBP^{-1}$$

$\Rightarrow$ A, B are called matrices similar.

$\Rightarrow$ Both A, B have same set 7 eigen values

But eigen vectors of A, B are different.

Let $BX = \lambda X$

$$\Rightarrow \quad (P^{-1}AP)X = \lambda X$$
$$\Rightarrow \quad A(PX) = \lambda(PX)$$

$\therefore$ Eigen vectors of A are PX.

End of Solution

Q.16 In a semiconductor, if the Fermi energy level lies in the conduction band, then the semiconductor is known as

(a) degenerate n-type.
(b) degenerate p-type.
(c) non-degenerate n-type.
(d) non-degenerate p-type.

[1 Mark : MCQ]

Ans. (a)

As the Fermi lies inside the conduction band hence it is degenerate n-type semiconductor.

End of Solution

Q.17 For an intrinsic semiconductor at temperature $T = 0\ K$, which of the following statement is true?

(a) All energy states in the valence band are filled with electrons and all energy states in the conduction band are empty of electrons.

(b) All energy states in the valence band are empty of electrons and all energy states in the conduction band are filled with electrons.

(c) All energy states in the valence and conduction band are filled with holes.

(d) All energy states in the valence and conduction band are filled with electrons.

[1 Mark : MCQ]

Ans. (a)

Intrinsic semiconductor at T = 0 K behaves as an insulator.

Hence, valence band is completely filled with electron and conduction band is completely empty.

End of Solution

Q.18 A series *RLC* circuit has a quality factor Q of 1000 at a center frequency of 10^6 rad/s. The possible values of R, L and C are

(a) $R = 1\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

(b) $R = 0.1\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

(c) $R = 0.01\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

(d) $R = 0.001\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

[1 Mark : MCQ]

Ans. (d)

Given: $Q = 1000$ and $\omega_o = 10^6$ rad/sec

We know, for series *RLC* circuit,

$$Q = \frac{\omega_o L}{R}$$

Also,
$$\omega_o = \sqrt{\frac{1}{LC}}$$

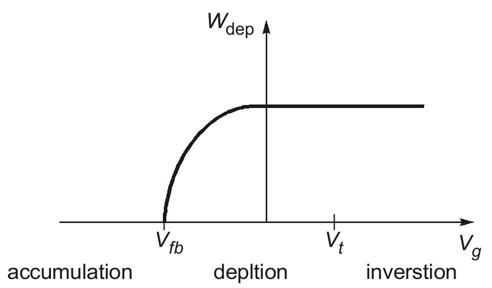
$$Q = \frac{1}{\sqrt{LC}} \times \frac{1}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

So, $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 0.001$

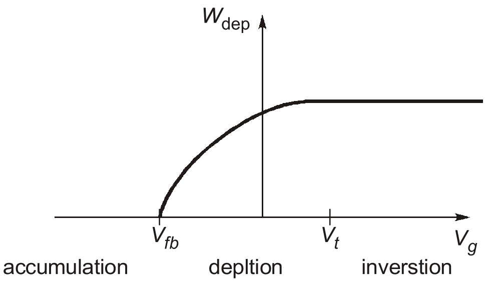
End of Solution

Q.19 For a MOS capacitor, V_{fb} and V_t are the flat-band voltage and the threshold voltage, respectively. The variation of the depletion width (W_{dep}) for varying gate voltage (V_g) is best represented by

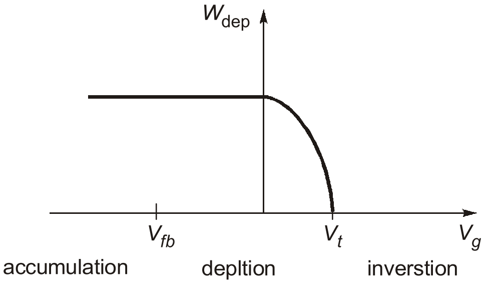
(a)



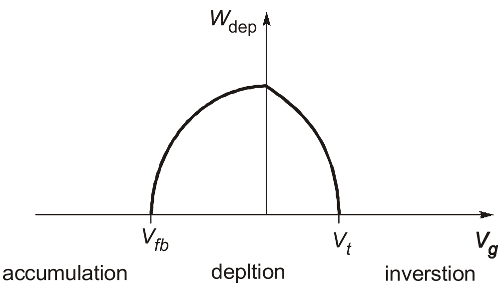
(b)



(c)



(d)



[1 Mark : MCQ]

Ans. **(b)**

$\because$ We know $V_G < V_{FB}$ then accumulation mode.

$\therefore$ In accumulation mode $\boxed{W_d = 0}$ because there is no depletion charge.

Now, $V_{FB} < V_G < V_T \Rightarrow$ then depletion and inversion mode.

$\therefore$ Depletion width is available.

$\therefore$ $V_G > V_T \Rightarrow$ Strong inversion.

$\therefore$ Depletion width $W_d \Rightarrow$ Constant.

And $W_d = \sqrt{\dfrac{2\epsilon\,|\phi_S|}{qN_s}}$ and $|\phi_S| \propto V_G$

But after strong inversion, W_d remains constant.

$\therefore$ Correction option is (b).

End of Solution

Q.20 Consider a narrow band signal, propagating in a lossless dielectric medium ($\epsilon_r = 4$, $\mu_r = 1$), with phase velocity v_p and group velocity v_g. Which of the following statement is true? (c is the velocity of light in vacuum.)

(a) $v_p > c$, $v_g > c$
(b) $v_p < c$, $v_g > c$
(c) $v_p > c$, $v_g < c$
(d) $v_p < c$, $v_g < c$

[1 Mark : MCQ]

Ans. (d)

- Phase velocity, $V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \in_0}} , \frac{1}{\sqrt{\mu_r \in_r}} = \frac{C}{\sqrt{\mu_r \in_r}}$

$\therefore \quad V_p < C$

- Group velocity, $V_g = \frac{d\omega}{d\beta} = \frac{V_p}{1 - \frac{\omega}{V_p}\frac{dV_p}{d\omega}}$

Here, $V_p \neq f(\omega)$

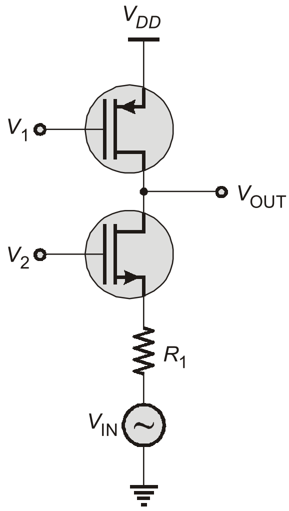
$\therefore \quad V_g = V_p < C$

Hence, $V_p < C$

$V_g < C$

End of Solution

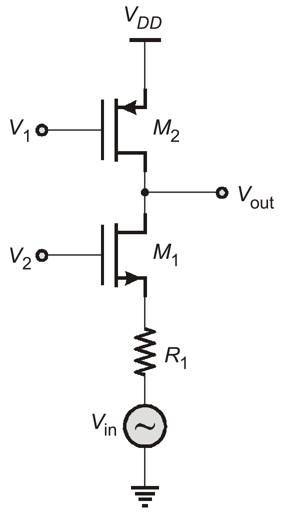
Q.21 In the circuit shown below, V_1 and V_2 are bias voltages. Based on input and output impedances, the circuit behaves as a



(a) voltage controlled voltage source.
(b) voltage controlled current source.
(c) current controlled voltage source.
(d) current controlled current source.

[1 Mark : MCQ]

Ans. **(d)**



Here from circuit,

M_1 is common-gate amplifier and M_2 behaves as an active load.

By using properties of common gate (CG) amplifier,

Input impedance (R_i) is low

Output impedance (R_0) is high

So, it is a current amplifier.

Current amplifier is a current controlled current source.

End of Solution

Q.22 A cascade of common-source amplifiers in a unity gain feedback configuration oscillates when

(a) the closed loop gain is less than 1 and the phase shift is less than 180°.

(b) the closed loop gain is greater than 1 and the phase shift is less than 180°.

(c) the closed loop gain is less than 1 and the phase shift is greater than 180°.

(d) the closed loop gain is greater than 1 and the phase shift is greater than 180°.

[1 Mark : MCQ]

Ans. **(∗)**

For oscillation,

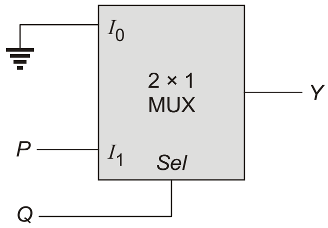
1. Loop gain magnitude ≥ 1.
2. Phase of loop gain = 360°

So, correct answer is option (d).

∗ In options, closes loop gain is mentioned technically it should be loop gain.

End of Solution

Q.23 In the circuit shown below, P and Q are the inputs. The logical function realized by the circuit shown below is



(a) $Y = PQ$ (b) $Y = P + Q$

(c) $Y = \overline{PQ}$ (d) $Y = \overline{P + Q}$

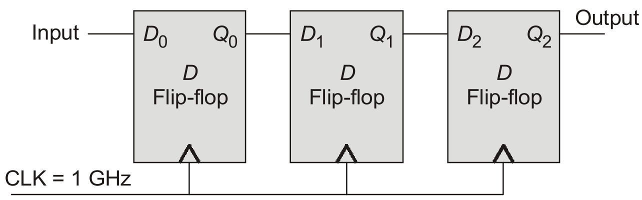
[1 Mark : MCQ]

Ans. **(a)**

$$\text{Output} = \bar{Q} \cdot I_0 + Q \cdot I_1$$
$$= \bar{Q} \cdot 0 + Q \cdot P$$
$$= PQ$$

End of Solution

Q.24 The synchronous sequential circuit shown below works at a clock frequency of 1 GHz. The throughput, in M bits/s, and the latency, in ns, respectively, are



(a) 1000, 3 (b) 333.33, 1

(c) 2000, 3 (d) 333.33, 3

[1 Mark : MCQ]

Ans. **(a)**

The given circuit is a type of SISO.

$\therefore$ Latency $= n \times T_{clk}$ ..... n = number of flip flops

$= 3 \times 1$ ...... $T_{clk} = \dfrac{1}{f_{clk}} = 1\text{ns}$

$= 3$ ns

Now, Throughput = Number of bits/sec

$\because$ 1 bit = 1 nsec

$\therefore$ Throughput $= 10^9$ bits/sec

$= 1000$ Mbps

End of Solution

Q.25 The open loop transfer function of a unity negative feedback system is $G(s) = \dfrac{k}{s(1+sT_1)(1+sT_2)}$, where k, T_1 and T_2 are positive constants. The phase cross-over frequency, in rad/s, is

(a) $\dfrac{1}{\sqrt{T_1 T_2}}$ (b) $\dfrac{1}{T_1 T_2}$

(c) $\dfrac{1}{T_1\sqrt{T_2}}$ (d) $\dfrac{1}{T_2\sqrt{T_1}}$

[1 Mark : MCQ]

Ans. **(a)**

We know phase crossover frequency is that frequency at which phase of the open loop transfer function is –180°.

$$\therefore \quad G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

$$G(j\omega) = \frac{K}{(j\omega)(1+j\omega T_1)(1+j\omega T_2)}$$

$$\text{Phase of } G(j\omega) = \phi = -90 - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

$\therefore$ At $\omega = \omega_{pc}$, $\phi = -180$

$$\therefore \quad -180 = -90 - \tan^{-1}(\omega_{pc}T_1) - \tan^{-1}(\omega_{pc}T_2)$$

$$90 = \tan^{-1}(\omega_{pc}T_1) + \tan^{-1}(\omega_{pc}T_2)$$

$$\tan^{-1}\left(\frac{\omega_{pc}T_1 + \omega_{pc}T_2}{1-\omega_{pc}^2 T_1 T_2}\right) = 90$$

$$1-\omega_{pc}^2 T_1 T_2 = 0$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1 T_2}}$$

End of Solution

Q.26 Consider a system with input $x(t)$ and output $y(t) = x(e^t)$. The system is

(a) Causal and time invariant (b) Non-causal and time varying

(c) Causal and time varying (d) Non-causal and time invariant

[1 Mark : MCQ]

Ans. **(b)**

We have, $y(t) = x(e^t)$

At $t = 0$

$$y(0) = x(1)$$

i.e. present value of output depends on future value of input, hence it is non-causal.

For Time Variant:

Delay the input,

$$y(t) = x(e^t - t_o) \quad \text{...(i)}$$

Delay the output,

$$y(t - t_o) = x(e^{t - t_o}) \quad \text{...(ii)}$$

i.e. equations (i) ≠ (ii)

Hence, it is time variant system.

End of Solution

Q.27 Let $m(t)$ be a strictly band-limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10B$, the energy in the signal $m(t)\cos\omega_0 t$ is

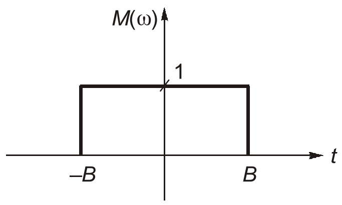
(a) $\frac{E}{4}$ (b) $\frac{E}{2}$

(c) E (d) $2E$

[1 Mark : MCQ]

Ans. (b)

$m(t) \rightleftharpoons$



$$\text{Energy } (E) = \frac{1}{2\pi}\int_{-B}^{B} (1)^2 \cdot d\omega$$

$$E = \frac{B}{\pi}$$

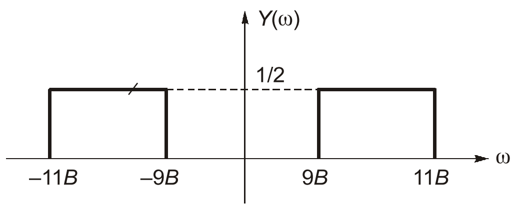
Now, let

$$y(t) = m(t)\cos\omega_0 t$$

$$Y(\omega) = \frac{1}{2}[M(\omega - \omega_0) + M(\omega + \omega_0)]$$

Here;

$$\omega_0 = 10B$$



Now,

$$\text{Energy } (E') = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 \cdot d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-11B}^{-9B}\left(\frac{1}{2}\right)^2 \cdot d\omega + \int_{9B}^{11B}\left(\frac{1}{2}\right)^2 \cdot d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{4}\times 2B + \frac{1}{4}\times 2B\right]$$

$$E' = \frac{B}{2\pi} = \frac{1}{2}\left(\frac{B}{\pi}\right) = \frac{E}{2}$$

End of Solution

Q.28 The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

(a) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$

(b) $\dfrac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

(c) $\sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$

(d) $\sqrt{\pi}\, e^{-\frac{\omega^2}{2}}$

[1 Mark : MCQ]

Ans. **(c)**

We know; $e^{-at^2};\, a > 0 \xleftrightarrow{FT} \sqrt{\frac{\pi}{a}} \cdot e^{-\omega^2/4a}$

Here; $a = 1$

$\therefore$ $X(\omega) = \sqrt{\pi} \cdot e^{-\omega^2/4}$

End of Solution

Q.29 In the table shown below, match the signal type with its spectral characteristics.

	Signal type		Spectral Characteristics
(i)	Continuous, aperiodic	**(a)**	Continuous, aperiodic
(ii)	Continuous, periodic	**(b)**	Continuous, periodic
(iii)	Discrete, aperiodic	**(c)**	Discrete, aperiodic
(iv)	Discrete, periodic	**(d)**	Discrete, periodic

(a) (i) → (a), (ii) → (b), (iii) → (c), (iv) → (d)

(b) (i) → (a), (ii) → (c), (iii) → (b), (iv) → (d)

(c) (i) → (d), (ii) → (b), (iii) → (c), (iv) → (a)

(d) (i) → (a), (ii) → (c), (iii) → (d), (iv) → (b)

[1 Mark : MCQ]

Ans. (b)

Signal Types	Spectral Characteristics
• Continuous and aperiodic	• Aperiodic and continuous
• Continuous and periodic	• Aperiodic and discrete
• Discrete and aperiodic	• Periodic and continuous
• Discrete and periodic	• Periodic and discrete

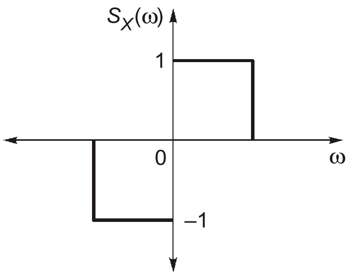
End of Solution

Q.30 For a real signal, which of the following is/are valid power spectral density/densities?

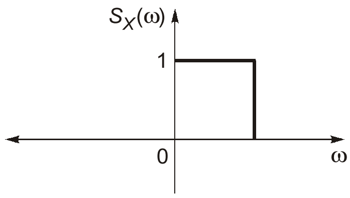
(a) $S_X(\omega) = \dfrac{2}{9+\omega^2}$

(b) $S_X(\omega) = e^{-\omega^2}\cos^2\omega$

(c)



(d)



[1 Mark : MSQ]

Ans. (a, b)

(i) $S_x(\omega) \geq 0$

(ii) $S_x(\omega)$ is even function

Hence, options (a) and (b) are valid power spectral densities.

End of Solution

Q.31 The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB. The resolution of the ADC is _____ bits. (rounded off to the nearest integer).

[1 Mark : NAT]

Ans. (10)

We know that for sinusoidal input, the signal to noise ratio (SNR) is given as,

$$\text{SNR} = 1.76 + 6.02\,n \text{ dB}$$

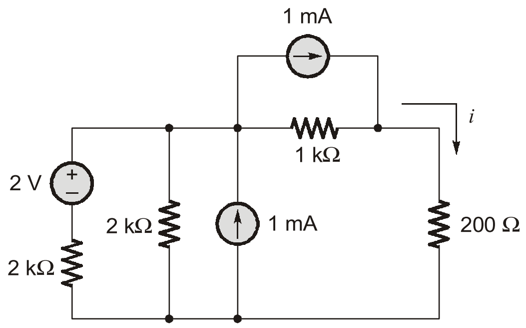
$$61.96 \text{ dB} = 1.76 + 6.02\,n \text{ dB}$$

$$6.02\,n = 61.96 - 1.76$$

$$\therefore \quad n = 10 \text{ bits}$$

End of Solution

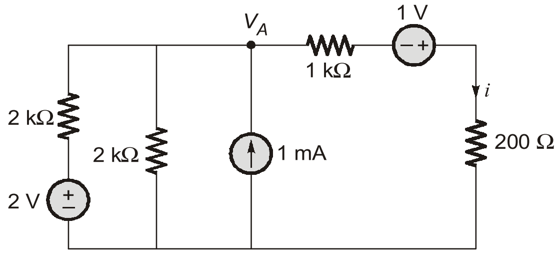
Q.32 In the circuit shown below, the current i flowing through 200 Ω resistor is ______ mA. (rounded off to two decimal places).



[1 Mark : NAT]

Ans. **(1.36)**

By applying source transformation,



Apply nodal at node V_A,

$$\frac{V_A - 2}{2\,\text{k}\Omega} + \frac{V_A}{2\,\text{k}\Omega} + \frac{V_A + 1}{1.2\,\text{k}\Omega} = 1\text{ mA}$$

$$V_A\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{1.2}\right] = 1 + \frac{2}{2} - \left(\frac{1}{1.2}\right)$$

$$V_A = 0.636\text{ V}$$

The current through 200 Ω resistor,

$$i = \frac{V_A + 1}{1.2\,\text{k}\Omega} = \frac{0.636 + 1}{1.2}$$

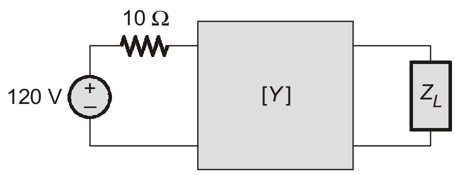
$$i = 1.36\text{ mA}$$

End of Solution

Q.33 For the two port network shown below, the [Y]-parameters is given as

$$[Y] = \frac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & 4/3 \end{bmatrix} S$$

The value of load impedance Z_L, in Ω, for maximum power transfer will be ______. (rounded off to the nearest integer).

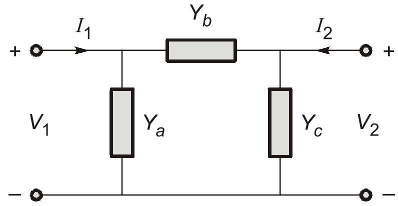


[1 Mark : NAT]

Ans. (80)

$$[Y] = \begin{bmatrix} \frac{2}{100} & -\frac{1}{100} \\ -\frac{1}{100} & \frac{4}{300} \end{bmatrix}$$

For the given Y-parameter the two-port network is



$$Y_{11} = Y_a + Y_b = \frac{2}{100}$$

$$Y_{12} = Y_{21} = -Y_b = -\frac{1}{100}$$

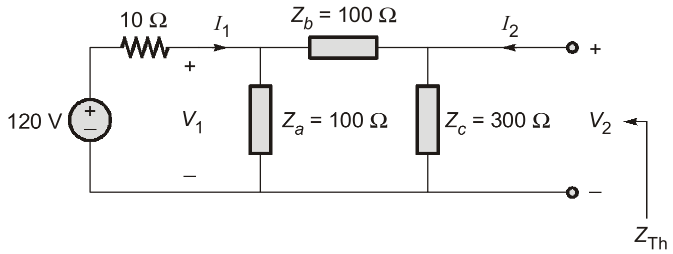
$$Y_{22} = Y_b + Y_c = \frac{4}{300}$$

On solving,
$$Y_b = \frac{1}{100}\ S$$

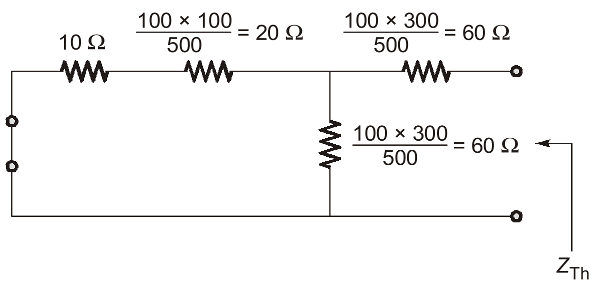
$$Y_a = \frac{1}{100}\ S$$

$$Y_c = \frac{1}{300}\ S$$

The network becomes,



Converting Δ – to ⅄,



$$Z_{Th} = 60 + [(20 + 10) \parallel 60]$$

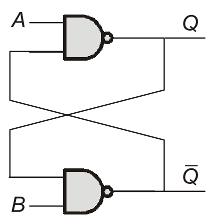
$$= 60 + \frac{30 \times 60}{30 + 60} = 80\ \Omega$$

For maximum power transfer,

$$Z_L = Z_{Th} = 80\ \Omega$$

End of Solution

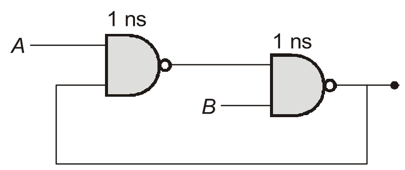
Q.34 For the circuit shown below, the propagation delay of each NAND gate is 1 ns. The critical path delay, in ns, is ________ (rounded off to the nearest integer).



[1 Mark : NAT]

Ans. (2)

The given circuit can be drawn as;

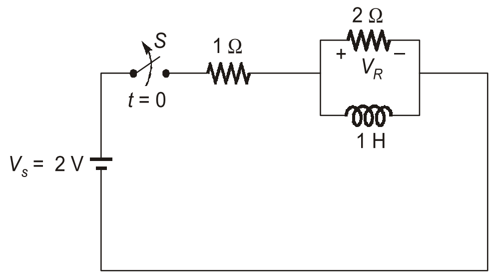


$\therefore$ The critical path delay = 1 ns + 1 ns

= 2 ns

End of Solution

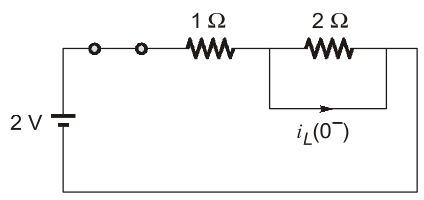
Q.35 In the circuit shown below, switch S was closed for a long time. If the switch is opened at $t = 0$, the maximum magnitude of the voltage V_R, in volts, is ________ (rounded off to the nearest integer).



[1 Mark : NAT]

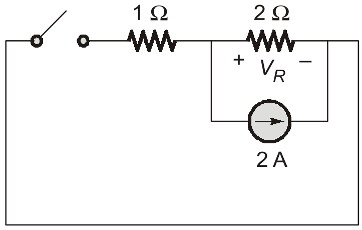
Ans. (4)

At $t = 0^-$



$$i_L(0^-) = \frac{2}{1} = 2\text{ A}$$

At $t = 0^+$



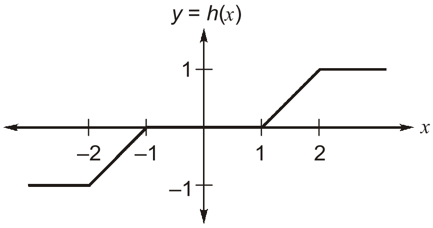
$$V_R = -2 \times 2 = -4$$

Magnitude of voltage V_R,

$$|V_R| = 4$$

End of Solution

Q.36 A random variable X, distributed normally as $N(0,1)$, undergoes the transformation $Y = h(X)$, given in the figure. The form of the probability density function of Y is (In the options given below, a, b, c are non-zero constants and $g(y)$ is piece-wise continuous function)



(a) $a\delta(y - 1) + b\delta(y + 1) + g(y)$

(b) $a\delta(y + 1) + b\delta(y) + c\delta(y - 1) + g(y)$

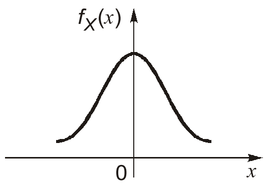
(c) $a\delta(y + 2) + b\delta(y) + c\delta(y - 2) + g(y)$

(d) $a\delta(y + 1) + b\delta(y - 2) + g(y)$

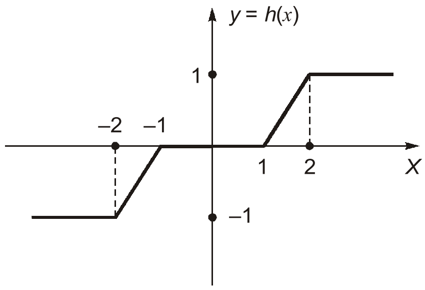
[2 Marks : MCQ]

Ans. **(b)**

$$X = N(0, 1)$$



$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



$Y = -1 \;;\; x \le -2$

$\quad 0 \;;\; -1 \le x \le 1$

$\quad 1 \;;\; x \ge 2$

$\quad x + 1 \;;\; -2 \le x \le -1$

$\quad x - 1 \;;\; 1 \le x \le 2$

Y is taking discrete set of values and a continuous range of values, so it is mixed random variable.

From the given options, density function of 'Y' will be.

$$f_Y(y) = a\delta(y+1) + b\delta(y) + c\delta(y-1) + g(y)$$

End of Solution

Q.37 The value of the line integral $\int_P^Q (z^2 dx + 3y^2 dy + 2xz\,dz)$ along the straight line joining the points $P(1, 1, 2)$ and $Q(2, 3, 1)$ is

(a) 20
(b) 24
(c) 29
(d) –5

[2 Marks : MCQ]

Ans. (b)

$\int_P^Q z^2 dx + 3y^2 dy + 2xzdz$ along the line joining the points $P(1, 1, 2)$ and $Q(2, 3, 1)$ is

$$= \int_{P(1,2)}^{P(2,1)} z^2 dx + 2xydz + \int_{y=1}^{3} 3y^2 dy$$

$$= \left(xz^2\right)_{(1,2)}^{(2,1)} + \left(y^3\right)_1^3$$

$$= (2 \times 1^2 - 1 \times 2^2) + (3^3 - 1^3)$$

$$= -2 + 26 = 24$$

End of Solution

Q.38 Let x be an $n \times 1$ real column vector with length $l = \sqrt{x^T x}$. The trace of the matrix $P = xx^T$ is

(a) l^2 (b) $\frac{l^2}{4}$

(c) l (d) $\frac{l^2}{2}$

[2 Marks : MCQ]

Ans. **(a)**

Given, $$l = \sqrt{x^T x},\ P = \left(xx^T\right)_{n \times n}$$

Let $$(x)_{n \times 1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ x_n \end{bmatrix}$$

$$l = \sqrt{x^T x} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots x_n^2}$$

$$P = xx^T$$

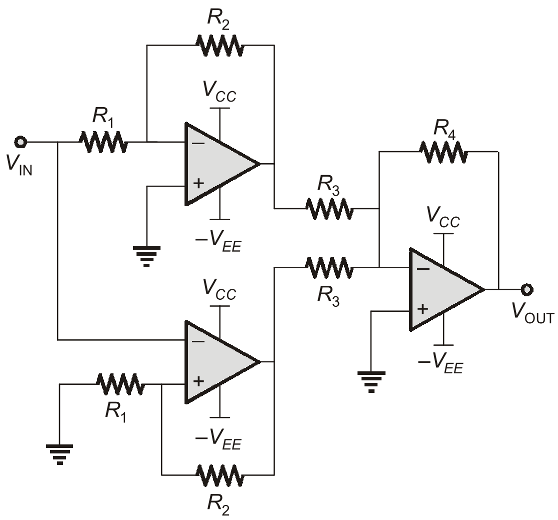
$$= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ x_n \end{bmatrix} \left[x_1\, x_2\, x_3 \ldots x_n\right]$$

$$P = \begin{bmatrix} x_1^2 & & & & \\ & x_1^2 & & & \\ & & - & & \\ & & & - & \\ & & & & x_n^2 \end{bmatrix}$$

$$\text{Trace of } P = x_1^2 + x_2^2 + \ldots.. + x_n^2 = l^2$$

End of Solution

Q.39 The $\frac{V_{OUT}}{V_{IN}}$ of the circuit shown below is



(a) $-\frac{R_4}{R_3}$

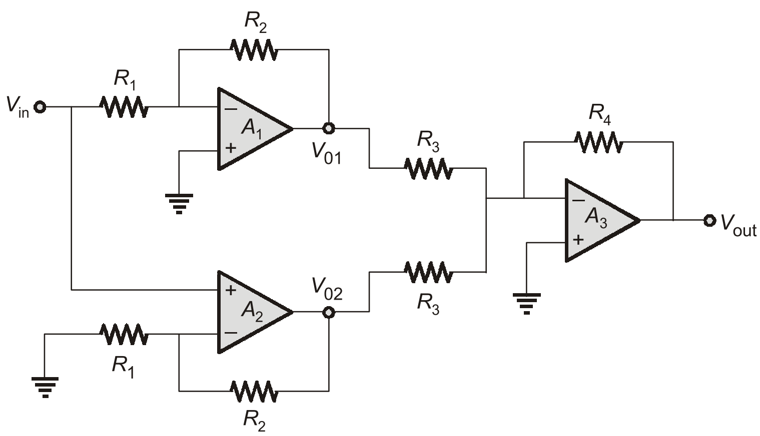
(b) $\frac{R_4}{R_3}$

(c) $1+\frac{R_4}{R_3}$

(d) $1-\frac{R_4}{R_3}$

[2 Marks : MCQ]

Ans. **(a)**



Here, A_1 is an inverting amplifier and A_2 is a non-inverting amplifier.

$$V_{01} = \frac{-R_2}{R_1} V_{in}$$

$$V_{02} = \left(1+\frac{R_2}{R_1}\right) V_{in}$$

Also, A_3 is an inverting summing amplifier,

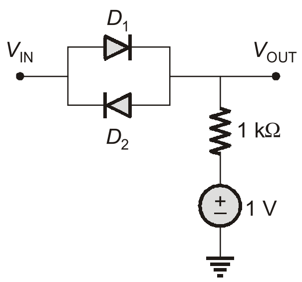
$$V_{out} = \frac{-R_4}{R_3}V_{01} - \frac{R_4}{R_3}V_{02} = \frac{-R_4}{R_3}\left[\frac{R_2}{R_1}V_{in} + \left(1+\frac{R_2}{R_1}\right)V_{in}\right]$$

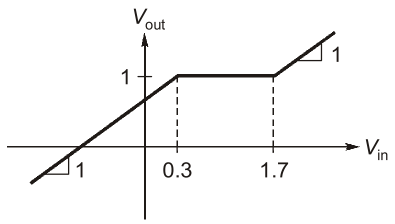
$$V_{out} = \frac{-R_4}{R_3}V_{in}$$

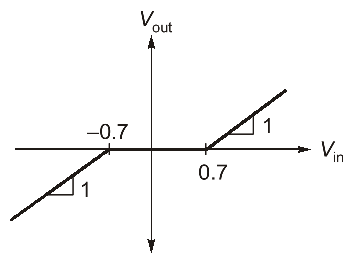
$$\text{Gain, } \frac{V_{out}}{V_{in}} = \frac{-R_4}{R_3}$$

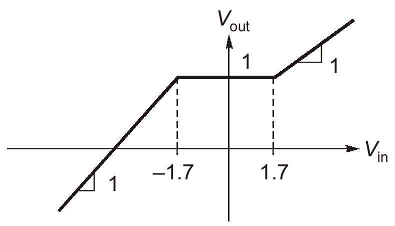
End of Solution

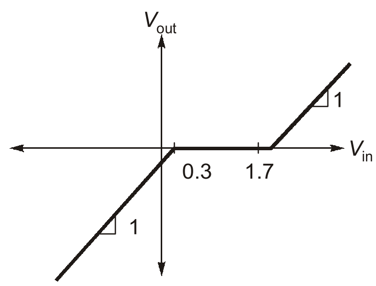
Q.40 In the circuit shown below, D_1 and D_2 are silicon diodes with cut-in voltage of 0.7 V. V_{IN} and V_{OUT} are input and output voltages in volts. The transfer characteristic is



(a)


(b)


(c)


(d)


[2 Marks : MCQ]

Ans. **(a)**

Case I:

$$V_\gamma = 0.7\,V$$

For the +ve half cycle if input V_{in},

$$D_1 \to \text{ON and } D_2 \to \text{OFF}$$

For diode D_1: $\quad V_{in} - 1V > 0.7$

$$V_{in} > 1.7V$$

$$V_0 = V_{in} - 0.7$$

Case II:

For the +ve half cycle if input V_{in},

$$D_1 \rightarrow \text{OFF and } D_2 \rightarrow \text{ON}$$

For diode D_2: $\quad 1 - V_{in} > 0.7$

$$V_{in} < 0.3\text{V}$$

$$V_0 = V_{in} + 0.7$$

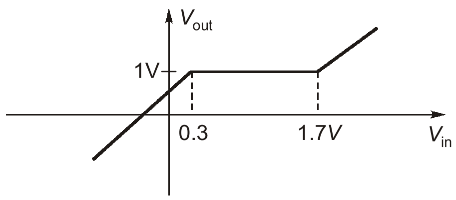
Case III:

$$0.3V < V_{in} < 1.7V$$

$$D_1 \rightarrow \text{OFF and } D_2 \rightarrow \text{OFF}$$

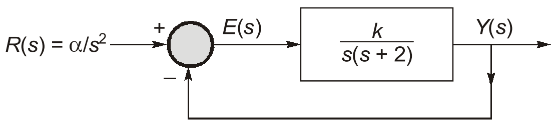
$$V_0 = 1\text{V}$$

Transfer characteristics,



End of Solution

Q.41 A closed loop system is shown in the figure where $k > 0$ and $\alpha > 0$. The steady state error due to a ramp input ($R(s) = \alpha/s^2$) is given by



(a) $\frac{2\alpha}{k}$ (b) $\frac{\alpha}{k}$

(c) $\frac{\alpha}{2k}$ (d) $\frac{\alpha}{4k}$

[2 Marks : MCQ]

Ans. **(a)**

Given, input is $r(t) = \alpha t u(t)$

$$R(s) = \frac{\alpha}{s^2}$$

From the figure,

$$G(s)H(s) = \frac{K}{s(s+2)}$$

Now steady state error for Ramp input is

$$e_{ss} = \frac{\alpha}{K_v}, \text{ where } \alpha \text{ is the magnitude of Ramp input}$$

$$K_v = \lim_{s\to 0}[sG(s)H(s)]$$

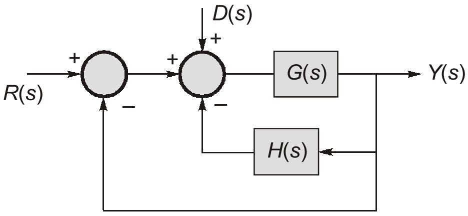
$$K_v = \lim_{s\to 0}\left[\frac{s \times K}{s(s+2)}\right] = \frac{K}{2}$$

$$\therefore \qquad e_{ss} = \frac{\alpha \times 2}{K}$$

$$e_{ss} = \frac{2\alpha}{K}$$

End of Solution

Q.42 In the following block diagram, $R(s)$ and $D(s)$ are two inputs. The output $Y(s)$ is expressed as $Y(s) = G_1(s)R(s) + G_2(s)D(s)$.
$G_1(s)$ and $G_2(s)$ are given by



(a) $G_1(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

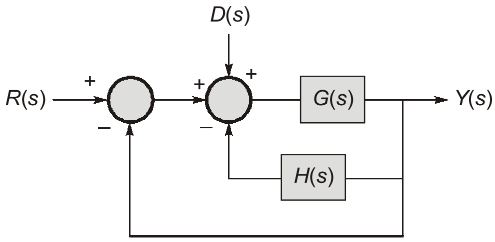
(b) $G_1(s) = \dfrac{G(s)}{1+G(s)+H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+H(s)}$

(c) $G_1(s) = \dfrac{G(s)}{1+G(s)+H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

(d) $G_1(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s) = \dfrac{G(s)}{1+G(s)+H(s)}$

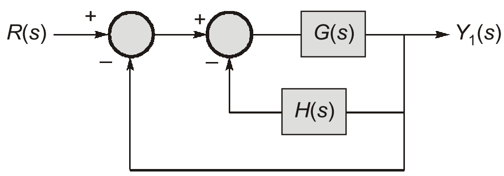
[2 Marks : MCQ]

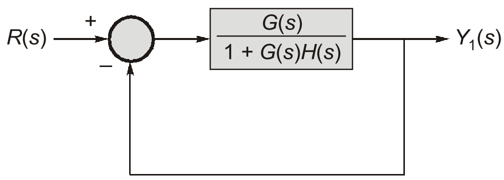
Ans. (a)



$$Y(s) = \underbrace{G_1(s)R(s)}_{Y_1(s)} + \underbrace{G_2(s)D(s)}_{Y_2(s)}$$

Considering first $R(s)$ only, then $Y(s)$ is $Y_1(s)$





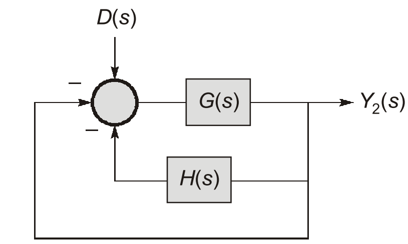
$$\frac{Y_1(s)}{R(s)} = \frac{\dfrac{G(s)}{1+G(s)H(s)}}{1+\dfrac{G(s)}{1+G(s)H(s)}}$$

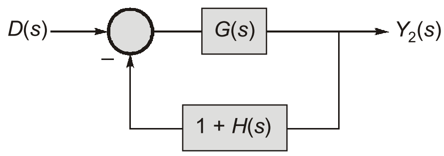
$$\frac{Y_1(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)+G(s)}$$

$$Y_1(s) = \left[\frac{G(s)}{1+G(s)+G(s)H(s)}\right]R(s)$$

$$\therefore \qquad G_1(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$$

Now considering $D(s)$ only, then $Y(s)$ is $Y_2(s)$





$$\frac{Y_2(s)}{D(s)} = \frac{G(s)}{1+G(s)[1+H(s)]}$$

$$Y_2(s) = \left[\frac{G(s)}{1+G(s)+G(s)H(s)}\right]D(s)$$

$$\therefore \qquad G_2(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$$

Hence, $G_1(s)$ and $G_2(s)$ both are equal.

End of Solution

Q.43 The state equation of a second order system is

$\dot{x}(t) = Ax(t)$, $x(0)$ is the initial condition.

Suppose λ_1 and λ_2 are two distinct eigenvalues of A and v_1 and v_2 are the corresponding eigenvectors. For constants α_1 and α_2, the solution, $x(t)$, of the state equation is

(a) $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_i$

(b) $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} v_i$

(c) $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} v_i$

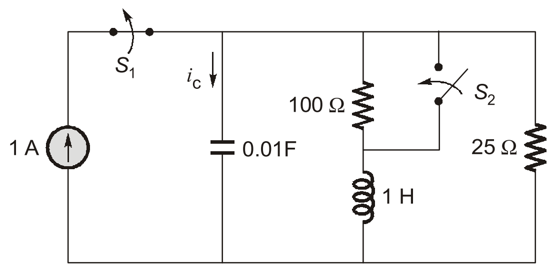
(d) $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} v_i$

[2 Marks : MCQ]

Ans. (a)

End of Solution

Q.44 The switch S_1 was closed and S_2 was open for a long time. At $t = 0$, switch S_1 is opened and S_2 is closed, simultaneously. The value of $i_c(0^+)$, in amperes, is



(a) 1

(b) –1

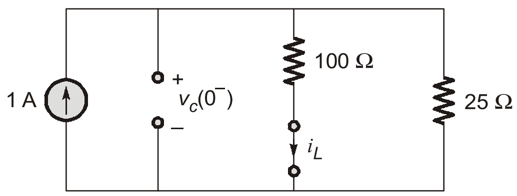
(c) 0.2

(d) 0.8

[2 Marks : MCQ]

Ans. (b)

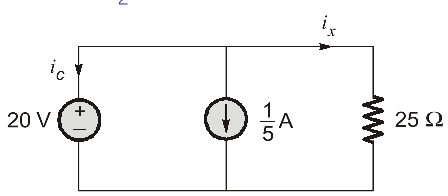
At $t = 0^-$; $S_1 \rightarrow$ closed, $S_2 \rightarrow$ opened



$$i_L(0^-) = \frac{1 \times 25}{100 + 25} = 0.2 \text{ A}$$

$$v_c(0^-) = \frac{1}{5} \times 100 = 20 \text{ V}$$

At $t = 0^+$; $S_1 \rightarrow$ opened, $S_2 \rightarrow$ closed



$$i_x = \frac{20}{25} = \frac{4}{5}\text{ A} = 0.8\text{ A}$$

By KCL: $\quad -i_c = i_x + 0.2 = 0.8 + 0.2$

$\Rightarrow \quad i_c = -1$ A

End of Solution

Q.45 Let a frequency modulated (FM) signal

$x(t) = A\cos(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda)$, where $m(t)$ is a message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5(x(t))^2$. Let B_T denote the FM bandwidth. The minimum value of ω_c required to recover $x(t)$ from $y(t)$ is

(a) $B_T + W$

(b) $\frac{3}{2}B_T$

(c) $2B_T + W$

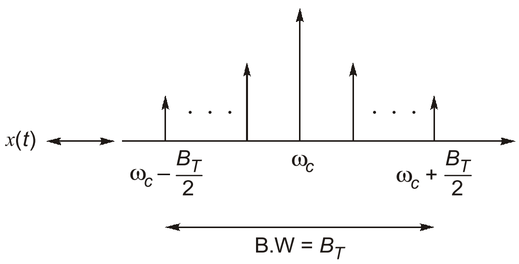
(d) $\frac{5}{2}B_T$

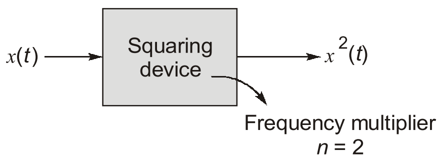
[2 Marks : MCQ]

Ans. **(b)**

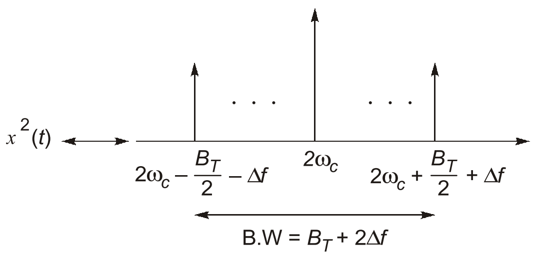
$$x(t) = A\cos\left[\omega_c t + K_f \int_{-\infty}^{t} m(\lambda)\,d\lambda\right]$$

B.W. $[x(t)] \rightarrow BT = 2[\Delta f + \omega]$



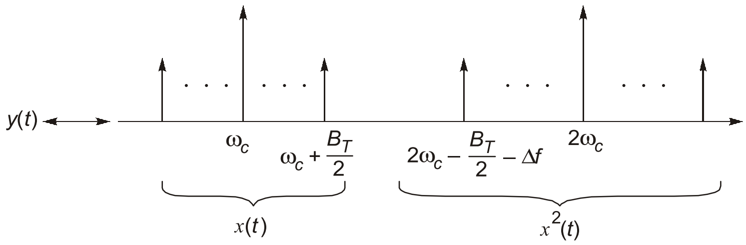


$$x^2(t) \rightarrow \left.\begin{matrix}\Delta f' = 2\Delta f \\ \omega_c' = 2\omega_c\end{matrix}\right\}$$



$$\text{BW}[x^2(t)] = 2[\Delta f' + \omega]$$
$$= 2[2\Delta f + \omega] = BT + 2\Delta f$$

$$y(t) = 2x(t) + 5x^2(t)$$



To recover $x(t) \rightarrow$ $\quad 2\omega_C - \frac{B_T}{2} - \Delta f > \omega_c + \frac{B_T}{2}$

$$\omega_c > \Delta f + BT$$

$$\omega_c > \Delta f + 2\Delta f + 2\omega$$

$$\omega_c > 3\Delta f + 2\omega$$

$$\omega_c > \frac{3}{2}\{2[\Delta f + \omega]\} - \omega$$

$$\omega_c > \frac{3}{2}B_T - \omega$$

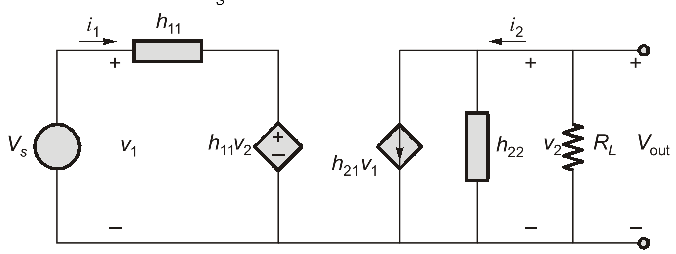
Compared to FM BW, message BW is very small. So, that it can be ignored.

$$\omega_C > \frac{3}{2}B_T$$

$$[\omega_c]_{min} = \frac{3}{2}B_T$$

End of Solution

Q.46 The h-parameters of a two port network are shown below. The condition for the maximum small signal voltage gain $\frac{V_{out}}{V_s}$ is



(a) $h_{11} = 0$, $h_{12} = 0$, h_{21} = very high and $h_{22} = 0$
(b) h_{11} = very high, $h_{12} = 0$, h_{21} = very high and $h_{22} = 0$
(c) $h_{11} = 0$, h_{12} = very high, h_{21} = very high and $h_{22} = 0$
(d) $h_{11} = 0$, $h_{12} = 0$, h_{21} = very high and h_{22} = very high

[2 Marks : MCQ]

Ans. **(*)**

Dependent current source should have $h_{21}\, I_1$ instead of $h_{21}\, V_1$ according to h-parameter.

$$A_v = \frac{V_{out}}{V_s} = \frac{-h_{21} I_1 \times \left(\frac{1}{h_{22}} \| R_L\right)}{h_{11} I_1 + h_{12} V_2}$$

To achieve maximum $\frac{V_{out}}{V_s}$,

$$h_{11} = 0, \quad h_{12} = 0$$
$$h_{21} = \text{Very high}, \quad h_{22} = 0$$

Hence, answer should be (a) according to $h_{21}\, I_1$.

End of Solution

Q.47 Consider a discrete-time periodic signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$, where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$ and $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\frac{4\pi n}{5}$ is

(a) –10
(b) 10
(c) –2
(d) 2

[2 Marks : MCQ]

Ans. **(a)**

Let, $I = \sum_{n=0}^{4} x(n) \sin\frac{4\pi n}{5}$

$$= \frac{1}{2j} \sum_{n=0}^{4} x(n) \cdot \left[e^{j\frac{4\pi n}{5}} - e^{-j\frac{4\pi n}{5}}\right]$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{4} x(n)\, e^{j\frac{4\pi n}{5}} - \sum_{n=0}^{4} x(n) \cdot e^{-j\frac{4\pi n}{5}}\right] \quad ...(i)$$

As we know, $$a_k = \frac{1}{N} \sum_{n=0}^{4} x(n) \cdot e^{-jK \cdot \frac{2\pi}{N} n}$$

$$= \frac{1}{5} \sum_{n=0}^{4} x(n) \cdot e^{-j\frac{2\pi}{N} kn}$$

Put $K = 2$; $$a_2 = \frac{1}{5} \sum_{n=0}^{4} x(n) \cdot e^{-j\frac{4\pi n}{5}}$$

Put $K = -2$; $$a_{-2} = \frac{1}{5} \sum_{n=0}^{4} x(n) \cdot e^{j\frac{4\pi n}{5}}$$

From equation (i), $$I = \frac{1}{2j}[5a_{-2} - 5a_2]$$

$$= \frac{5}{2j}[a_3 - a_2] \qquad \left[\because \begin{array}{l} a_{-2} = a_{-2+N} \\ \quad\;\; = a_{-2+5} \\ \quad\;\; = a_3 \end{array}\right]$$

$$\Rightarrow \qquad I = \frac{5}{2j}[-2j - 2j] = -10$$

End of Solution

Q.48 Let an input $x[n]$ having discrete time Fourier transform.

$X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-3j\Omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-j2\Omega}$. The output $y[n]$ of the system is

(a) $\delta[n] + \delta[n-1] - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(b) $\delta[n] - \delta[n-1] - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(c) $\delta[n] - \delta[n-1] - \frac{1}{2}\delta[n-2] + \frac{5}{2}\delta[n-3] - \delta[n-5]$

(d) $\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2] + \frac{5}{2}\delta[n-3] + \delta[n-5]$

[2 Marks : MCQ]

Ans. (c)

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$$

$$= \left[1 - e^{-j\Omega} + 2e^{-3j\Omega}\right]\left[1 - \frac{1}{2}e^{-2j\Omega}\right]$$

$$= 1 - e^{-j\Omega} + 2.5e^{-3j\Omega} - 0.5e^{-j2\Omega} - e^{-j5\Omega}$$

Taking IDTFT;

$$y[n] = \delta[n] - \delta[n-1] - 0.5\delta[n-2] + 2.5\delta[n-3] - \delta[n-5]$$

End of Solution

Q.49 Let $x(t) = 10\cos(10.5Wt)$ be passed through an LTI system having impulse response $h(t) = \pi\left(\frac{\sin Wt}{\pi t}\right)^2 \cos 10Wt$. The output of the system is

(a) $\left(\frac{15W}{4}\right)\cos(10.5Wt)$

(b) $\left(\frac{15W}{2}\right)\cos(10.5Wt)$

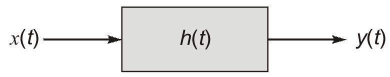
(c) $\left(\frac{15W}{8}\right)\cos(10.5Wt)$

(d) $(15W)\cos(10.5Wt)$

[2 Marks : MCQ]

Ans. **(a)**

Given $h(t)$ is Real and Even. When sinusoidal input applied to LTI system having even impulse response, then output will also be sinusoidal.

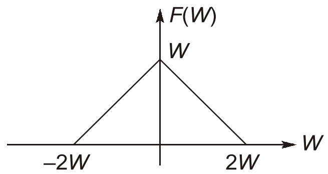


here, $$y(t) = H(W)\big|_{W=10.5W} \cdot 10\cos(10.5Wt)$$

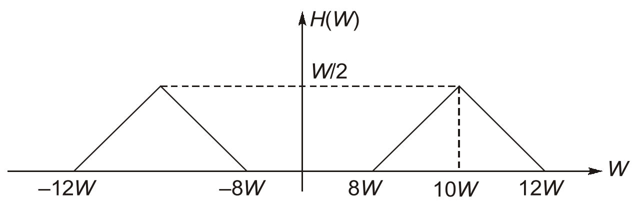
let, $$h(t) = f(t)\cos 10Wt$$

where, $$f(t) = \pi\left(\frac{\sin Wt}{\pi t}\right)^2$$

$$f(t) \xrightleftharpoons{FT}$$



Now; $$H(W) = \frac{1}{2}[F(W + 10W) + F(W - 10W)]$$



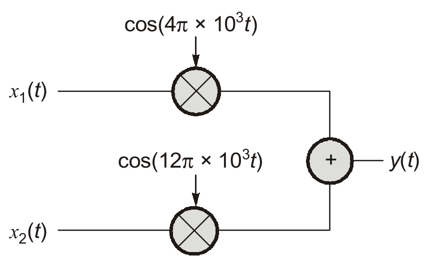
$\therefore$ $$H(W)\big|_{W\,=\,10.5W} = \frac{3}{8}W$$

Hence, $$y(t) = \left(\frac{3}{8}W\right)(10\cos 10.5Wt)$$

$$= \frac{15}{4}W\cos 10.5Wt$$

End of Solution

Q.50 Let $x_1(t)$ and $x_2(t)$ be two band-limited signals having bandwidth $B = 4\pi \times 10^3$ rad/s each. In the figure below, the Nyquist sampling frequency, in rad/s, required to sample $y(t)$, is

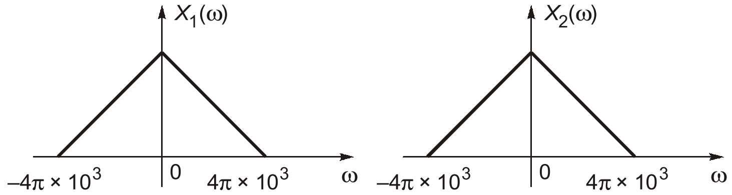


(a) $20\pi \times 10^3$
(b) $40\pi \times 10^3$
(c) $8\pi \times 10^3$
(d) $32\pi \times 10^3$

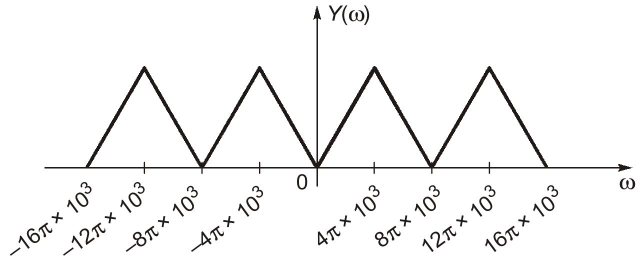
[2 Marks : MCQ]

Ans. (d)

Given that, $x_1(t)$ and $x_2(t)$ are two bandlimited signals having bandwidth $B = 4\pi \times 10^3$ rad/sec.



and $$y(t) = x_2(t)\cos(12\pi \times 10^3 t) + x_1(t)\cos(4\pi \times 10^3 t)$$



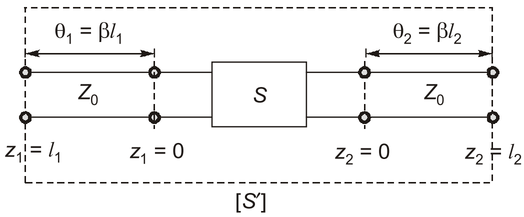
So,
$$\text{Nyquist rate} = 2\omega_{max} = 2[16\pi \times 10^3] = 32\pi \times 10^3 \text{ rad/sec}$$

End of Solution

Q.51 The S-parameters of a two port network is given as

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

with reference to Z_0. Two lossless transmission line sections of electrical lengths $\theta_1 = \beta l_1$ and $\theta_2 = \beta l2$ are added to the input and output ports for measurement purposes, respectively. The S-parameters $[S']$ of the resultant two port network is



(a) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

(b) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

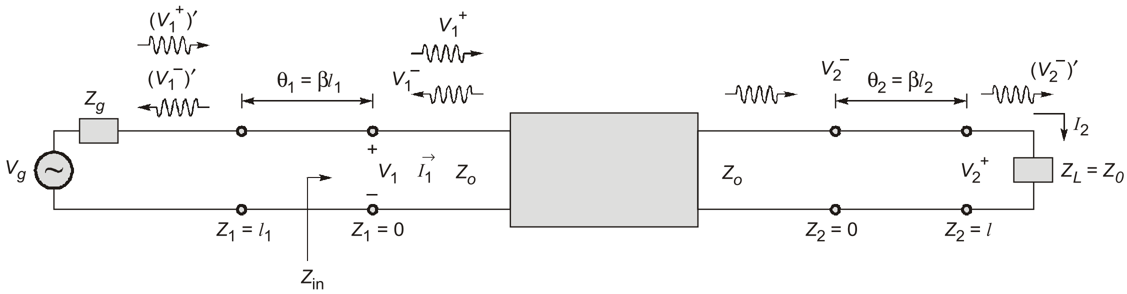
(c) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{ej(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(d) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{ej(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

[2 Marks : MCQ]

Ans. (a)

Let us evaluate S_{11} and S_{21} first at $V_2^+ = 0$



(a) S11 :

$$V_1 = V_1^+ + V_1^- = (V_1^+)' e^{-j\beta l_1} + (V_1^-) e^{+j\beta l_1}$$

$$\therefore \quad V_1^+ = (V_1^+)' e^{-j\beta l_1}$$

$$V_1^- = (V_1^-) e^{+j\beta l_1}$$

$$S_{11} = \frac{V_1^-}{V_1^+} = \frac{(V_1^-) e^{+j\beta l_1}}{(V_1^+)' e^{-j\beta l_1}} = S'_{11} e^{+j2\beta l_1}$$

$$\Rightarrow \quad S'_{11} = S_{11} e^{-j2\beta l_1} = S_{11} e^{-j2\theta_1}$$

$$\therefore \quad S'_{11} = S_{11} e^{-j2\theta_1}$$

(b) S_{21} :

$$V_2 = V_2^+ e^{+j\beta l_2} + V_2^- e^{-j\beta l_2} = (V_2^+)' + (V_2^-)'$$

Here, $V_2^+ = 0$

Hence, $\quad V_2 = (V_2^-)' = V_2^- e^{-j\beta l_2}$

$$\Rightarrow \quad V_2^- = (V_2^-)' e^{+j\beta l_2}$$

From previous discussion in S_{11},

$$V_1^+ = (V_1^+) e^{-j\beta l_1}$$

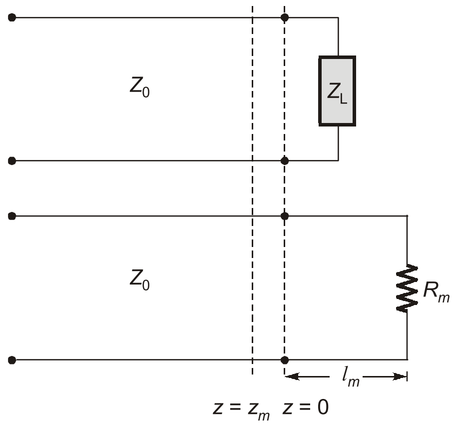
$$\therefore \quad S_{21} = \frac{V_2^-}{V_1^+} = \frac{(V_2^-)' e^{+j\beta l_2}}{(V_1^+)' e^{-j\beta l_1}} = S'_{21} e^{+j(\beta l_2 + \beta l_1)}$$

$$\Rightarrow \quad S'_{21} = S_{21} e^{-j(\beta l_2 + \beta l_1)}$$

$$\Rightarrow \quad S'_{21} = S_{21} e^{-j(\theta_1 + \theta_2)}$$ (looking at options (a) is correct)

End of Solution

Q.52 The standing wave ratio on a 50 Ω lossless transmission line terminated in an unknown load impedance is found to be 2.0. The distance between successive voltage minima is 30 cm and the first minimum is located at 10 cm from the load. Z_L can be replaced by an equivalent length l_m and terminating resistance R_m of the same line. The value of R_m and l_m, respectively, are



(a) R_m = 100 Ω, l_m = 20 cm
(b) R_m = 25 Ω, l_m = 20 cm
(c) R_m = 100 Ω, l_m = 5 cm
(d) R_m = 25 Ω, l_m = 5 cm

[2 Marks : MSQ]

Ans. **(b, c)**

Given $S = 2$, $Z_{min} = 10$ cm, $Z_0 = 50\ \Omega$

As we know that, $|\Gamma| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}$

Now, distance between successive voltage minima = 30 cm

$$\Rightarrow \quad \frac{\lambda}{2} = 30 \text{ cm}$$

$$\Rightarrow \quad \lambda = 60 \text{ cm}$$

Also, for minima,

$$2\beta Z_{min} = (2n + 1)\pi + \theta_\Gamma$$

At $n = 0$, 1st minima, $Z_{min} = 10$ cm

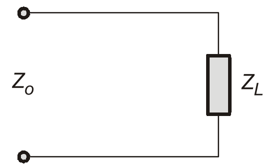
$$\frac{4\pi}{\lambda} Z_{min} = \pi + \theta_\Gamma$$

$$\Rightarrow \quad \frac{4\pi}{60} * 10 = \pi + \theta_\Gamma$$

$$\Rightarrow \quad \frac{2\pi}{3} - \pi = \theta_\Gamma$$

$$\Rightarrow \quad \theta_\Gamma = \frac{-\pi}{3} \qquad \therefore \Gamma = \frac{1}{3}\angle -60^\circ$$

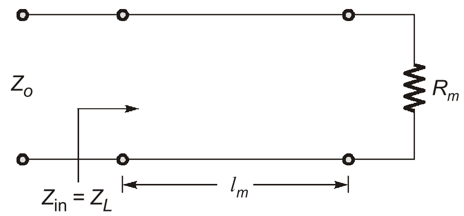
Now, $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$



$$\Rightarrow \qquad Z_L = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right]$$

$$\Rightarrow \qquad Z_L = 50\left[\frac{1+0.33e^{-j\frac{\pi}{3}}}{1-0.33e^{-j\frac{\pi}{3}}}\right]$$

$$\Rightarrow \qquad Z_L = 67.97\ \angle{-32.67^\circ}$$



Now, $$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$$

$$\Rightarrow \qquad Z_{in} = 50\left[\frac{R_m + j50\tan\beta l_m}{50 + jR_m\tan\beta l_m}\right]$$

Here, $Z_{in} = Z_L = 67.97\ \angle{-32.67^\circ}$

Going through options,

$$\left.\begin{aligned} R_m &= 100\ \Omega \text{ and } L_m = 5\text{ cm} \\ \text{and } R_m &= 25\ \Omega \text{ and } L_m = 20\text{ cm}\end{aligned}\right\}$$ satisfy this identity, hence option (b) and (c) are correct.

End of Solution

Q.53 The electric field of a plane electromagnetic wave is

$E = a_x C_{1x}\cos(\omega t - \beta z) + a_y C_{1y}\cos(\omega t - \beta z + \theta)$ V/m

Which of the following combination(s) will give rise to a left handed elliptically polarized (LHEP) wave?

(a) $C_{1x} = 1,\ C_{1y} = 1,\ \theta = \pi/4$ (b) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = \pi/2$

(c) $C_{1x} = 1,\ C_{1y} = 2,\ \theta = 3\pi/2$ (d) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = 3\pi/4$

[2 Marks : MSQ]

Ans. **(a, b, d)**

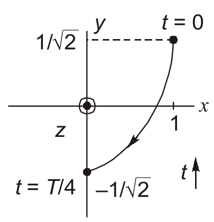
Given, $\vec{E} = \hat{a}_x C_{1x}\cos(\omega t - \beta z) + \hat{a}_y C_{1y}\cos(\omega t - \beta z + \theta)$

at $z = 0$

$$\vec{E} = C_{1x}\cos\omega t\,\hat{a}_x + C_{1y}\cos(\omega t + \theta)\hat{a}_y$$

Going by options,

Option (a) $$\vec{E} = \cos\omega t\,\hat{a}_x + \cos(\omega t + \pi/4)\hat{a}_y$$



at $t = 0$, $\omega t = 0$, $\vec{E} = \hat{a}_x + \dfrac{1}{\sqrt{2}}\hat{a}_y$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = 0 - \dfrac{1}{\sqrt{2}}\hat{a}_y$

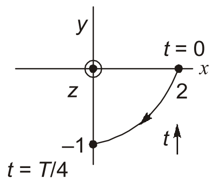
$\Rightarrow$ Hence, it is LHEP.

Option (b) $\vec{E} = 2\cos\omega t\,\hat{a}_x + \cos(\omega t + \pi/2)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = 2\hat{a}_x$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = -1\hat{a}_y$

$\Rightarrow$ Hence, it is LHEP.

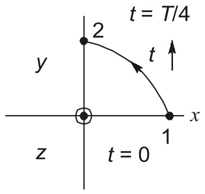


Option (c) $\vec{E} = \cos\omega t\,\hat{a}_x + 2\cos(\omega t + 3\pi/2)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = \hat{a}_x$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = 2\hat{a}_y$

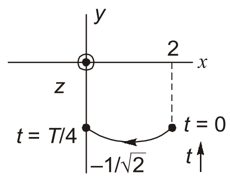
$\Rightarrow$ Hence, it is RHEP.



Option (d) $\vec{E} = 2\cos\omega t\,\hat{a}_x + \cos(\omega t + 3\pi/4)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = 2\hat{a}_x - \dfrac{1}{\sqrt{2}}\hat{a}_y$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = 0 - \dfrac{1}{\sqrt{2}}\hat{a}_y = \dfrac{-1}{\sqrt{2}}\hat{a}_y$



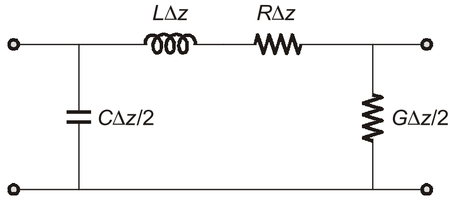
$\Rightarrow$ Hence, it is LHEP.

$\therefore$ Option (a), (b) and (d) are correct.

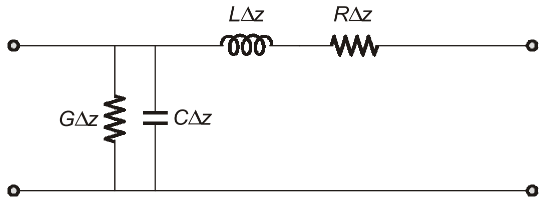
End of Solution

Q.54 The following circuit(s) representing a lumped element equivalent of an infinitesimal section of a transmission line is/are

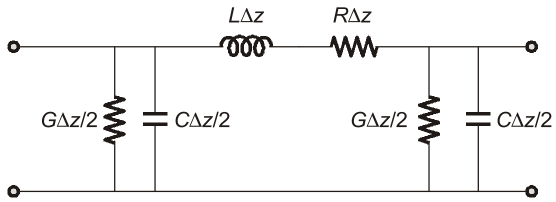
(a)

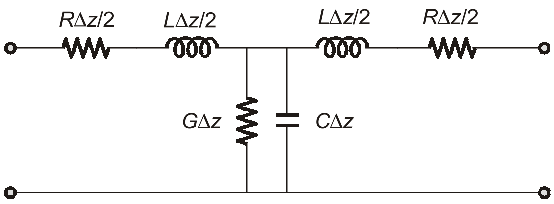


(b)



(c)



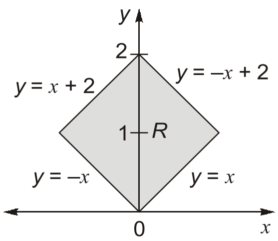
(d) 

[2 Marks : MSQ]

Ans. (b, c, d)

End of Solution

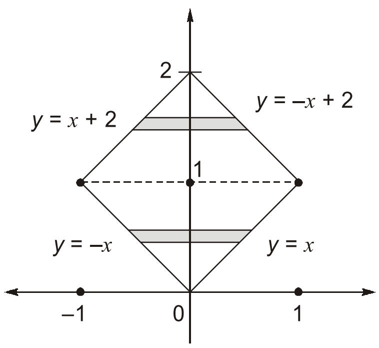
Q.55 The value of the integral $\iint_R xy\, dx\, dy$ over the region R, given in the figure, is _______. (rounded off to the nearest integer).



[2 Marks : NAT]

Ans. (0)

$$I = \iint_R xy\, dx\, dy$$



$$= \int_{y=0}^{1} \int_{x=-y}^{y} xy\,dx\,dy + \int_{y=1}^{2} \int_{x=y-2}^{2-y} xy\,dx\,dy$$

$$= \int_0^1 y\left(\frac{x^2}{2}\right)_{-y}^{y} dy + \int_1^2 y\left(\frac{x^2}{2}\right)_{y-2}^{2-y} dy$$

$$= 0 + 0 = 0$$

End of Solution

Q.56 In an extrinsic semiconductor, the hole concentration is given to be $1.5n_i$ where n_i is the intrinsic carrier concentration of 1×10^{10} cm^{-3}. The ratio of electron to hole mobility for equal hole and electron drift current is given as __________.
(rounded off to two decimal places).

[2 Marks : NAT]

Ans. **(2.25)**

Given, intrinsic carrier concentration $n_i = 1 \times 10^{10}$ cm^{-3}

Hole concentration, $p = 1.5 \times n_i$

$p = 1.5 \times 10^{10}$ cm^{-3}

Given, electron and hole current are equal

$$I_{p\text{ drift}} = I_{n\text{ drift}}$$

$$pq\,\mu_p\,EA = nq\,\mu_n\,EA$$

$$1.5 \times 10^{10}\,\mu_p = n\,\mu_n \quad \text{...(i)}$$

But according to mass action law,

$$np = n_i^2$$

$$\therefore \quad n = \frac{n_i}{1.5} = \frac{10^{10}}{1.5}\text{cm}^{-3}$$

Put in equation (i)

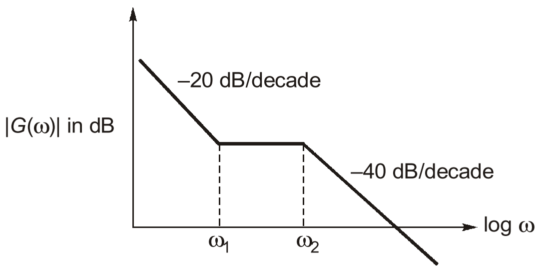
$$\therefore \quad 1.5 \times 10^{10}\,\mu_p = \frac{10^{10}}{1.5} \times \mu_n$$

$$\frac{\mu_n}{\mu_p} = 2.25$$

End of Solution

Q.57 The asymptotic magnitude Bode plot of a minimum phase system is shown in the figure.

The transfer function of the system is $(s) = \dfrac{k(s+z)^a}{s^b(s+p)^c}$, where k, z, p, a, b and c are positive constants. The value of $(a + b + c)$ is __________.
(rounded off to the nearest integer).



[2 Marks : NAT]

Ans. **(4)**

From the Bode magnitude plot, it is clear that there is one pole at origin,

$$\therefore \quad b = 1$$

and at frequency ω_1, system has a zero

$\therefore \quad a = 1$

and at frequency ω_2, system have two poles

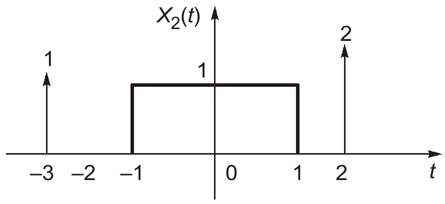
$\therefore \quad c = 2$

$\therefore \quad a + b + c = 1 + 1 + 2$

$a + b + c = 4$

End of Solution

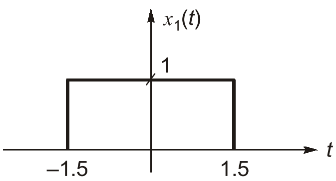
Q.58 Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is ________. (rounded off to the nearest integer).



[2 Marks : NAT]

Ans. (15)

$$x_1(t) = u(t + 1.5) - u(t - 1.5)$$



$$\Rightarrow \quad x_1(t) = \text{rect}\left(\frac{t}{3}\right)$$

$$x_1(t) = \text{rect}\left(\frac{t}{3}\right) \xleftrightarrow{FT} 3\,Sa(1.5\omega)$$

Now, $$x_2(t) = \delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$$

Taking Fourier transform

$$X_2(\omega) = e^{3j\omega} + 2\,Sa(\omega) + 2e^{-2j\omega}$$

$\because$ $$y(t) = x_1(t) * x_2(t)$$

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$$

We know, $$Y(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\omega t} \cdot dt$$

$\therefore$ $$\int_{-\infty}^{\infty} y(t) = Y(0)$$

$\therefore$ $$Y(0) = X_1(0) \cdot X_2(0)$$

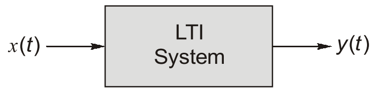
$$= 3[1 + 2 + 2] = 15$$

End of Solution

Q.59 Let $X(t)$ be a white Gaussian noise with power spectral density $\frac{1}{2}$ W/Hz. If $X(t)$ is input to an LTI system with impulse response $e^{-t}u(t)$. The average power of the system output is _______ W. (Rounded off to two decimal place).

[2 Marks : NAT]

Ans. (0.25)



$$h(t) = e^{-t}\, u(t)$$

Given: Input PSD

$$\Rightarrow \qquad S_X(f) = \frac{1}{2}\text{ W/Hz}$$

We know output PSD,

$$S_Y(f) = S_X(f)\,|H(f)|^2$$

$$S_Y(f) = \frac{1}{2}|H(f)|^2$$

$$\text{Power}\,[y(t)] = \int_{-\infty}^{\infty} S_Y(f)df = \int_{-\infty}^{\infty}\frac{1}{2}|H(f)|^2\,df$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} h^2(t)dt = \frac{1}{2}\int_{0}^{\infty} e^{-2t}dt$$

$$= \frac{1}{2}\times\frac{1}{2} = \frac{1}{4} = 0.25\text{ W}$$

End of Solution

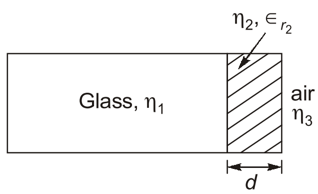
Q.60 A transparent dielectric coating is applied to glass ($\epsilon_r = 4$, $\mu_r = 1$) to eliminate the reflection of red light ($\lambda_0 = 0.75$ μm). The minimum thickness of the dielectric coating, in μm, that can be used is _________ (rounded off to two decimal places).

[2 Marks : NAT]

Ans. (0.133)

For no reflection, impedance must be matched.

Hence, η_2 acts like a quarter wave impedance transformer.



So,

(i) $$\eta_2 = \sqrt{\eta_1 \cdot \eta_3} \quad \Rightarrow \quad \epsilon_{r_2} = \sqrt{\epsilon_{r_1} \cdot \epsilon_{r_3}} \quad \Rightarrow \quad \epsilon_{r_2} = 2$$

(ii) For impedance matching,

$$d = (2n+1)\frac{\lambda}{4};\ n = 0, 1, 2...$$

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{\lambda_0}{\sqrt{\epsilon_{r_2}}}$$

Here, $$\lambda = \frac{0.75 \times 10^{-6}}{\sqrt{2}} = 0.53 \times 10^{-6}$$

Hence, for minimum distance, $n = 0$

So, $$d = \frac{\lambda}{4} = \frac{0.53 \times 10^{-6}}{4} = 0.133\ \mu\text{m}$$

End of Solution

Q.61 In a semiconductor device, the Fermi-energy level is 0.35 eV above the valence band energy. The effective density of states in the valence band at $T = 300$ K is $1 \times 10^{19}\ \text{cm}^{-3}$. The thermal equilibrium hole concentration in silicon at 400 K is ______ $\times 10^{13}\ \text{cm}^{-3}$. (rounded off to two decimal places).

Given kT at 300 K is 0.026 eV.

[2 Marks : NAT]

Ans. **(63.36)**

Given, $E_F - E_V = 0.35$ eV [Considering it is given at 400 K]

Also, $V_{T_1} = KT_1 = 0.026$ eV at $T_1 = 300$ K

$$\therefore \quad \frac{V_{T_1}}{V_{T_2}} = \frac{T_1}{T_2} \Rightarrow V_{T_2} = \frac{T_2}{T_1} \times V_{T_1}$$

$$\therefore \quad V_{T_2} = \frac{400}{300} \times 0.026$$

$$V_{T_2} = 0.03466 \text{ eV at } T_2 = 400 \text{ K}$$

Now, $N_v = 1 \times 10^{19}/\text{cm}^3$ at $T_1 = 300$ K

$$N_v \propto T^{3/2}$$

$$\frac{N_{V_2}}{N_{V_1}} = \left(\frac{T_2}{T_1}\right)^{3/2}$$

$$N_{V_2} = \left(\frac{T_2}{T_1}\right)^{3/2} NV_1 \qquad (\because T_2 = 400 \text{ K})$$

$$= \left(\frac{400}{300}\right)^{3/2} N_{V_1}$$

$$N_{V_2} = 1.5396 \times 10^{19}/\text{cm}^3$$

Now, hole concentration at 400 K is given as

$$p = N_v\, e^{-(E_F - E_V)/kT_2} = 1.5396 \times 10^{19} \times e^{-0.35\text{ eV}/0.03466\text{ eV}}$$

$$p = 63.36 \times 10^{13}\ \text{cm}^{-3}$$

End of Solution

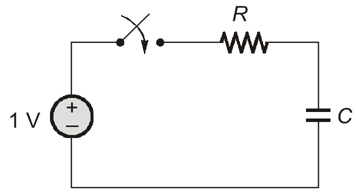
Q.62 A sample and hold circuit is implemented using a resistive switch and a capacitor with a time constant of 1 μs. The time for sampling switch to stay closed to charge a capacitor adequately to a full scale voltage of 1 V with 12-bit accuracy is ______ μs. (rounded off to two decimal places)

[2 Marks : NAT]

Ans. **(8.3177)**

Given: Time constant (τ) of 1 μsec.

Full scale voltage = 1 V



The voltage across capacitor is given as

$$V_c(t) = V_{in}(1 - e^{-t/\tau})$$

$$\therefore \quad V_c(t) = (1 - e^{-t/1\ \mu sec}) \quad \text{...(i)}$$

To calculate the voltage to stay closed to charge capacitor adequately to a full scale voltage with 12-bit accuracy is given by

$$v_c(t) = V_{ref}\left[1 - \frac{1}{2^n}\right]$$

$$n = \text{Number of bit} = 12$$

$$\therefore \quad V_c(t) = \left[1 - \frac{1}{4096}\right] \quad \text{...(ii)}$$

Comparing equations (i) and (ii), we get,

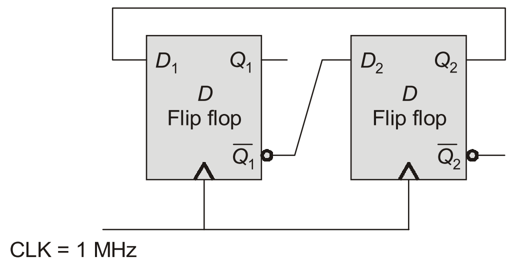
$$e^{-t/1\mu sec} = \frac{1}{4096}$$

$$-\frac{t}{1\mu sec} = \ln\left\{\frac{1}{4096}\right\}$$

$$t = 8.3177\ \mu sec$$

End of Solution

Q.63 In a given sequential circuit, initial states are $Q_1 = 1$ and $Q_2 = 0$. For a clock frequency of 1 MHz, the frequency of signal Q_2 in kHz, is ___________. (rounded off to the nearest integer).



[2 Marks : NAT]

Ans. (250)

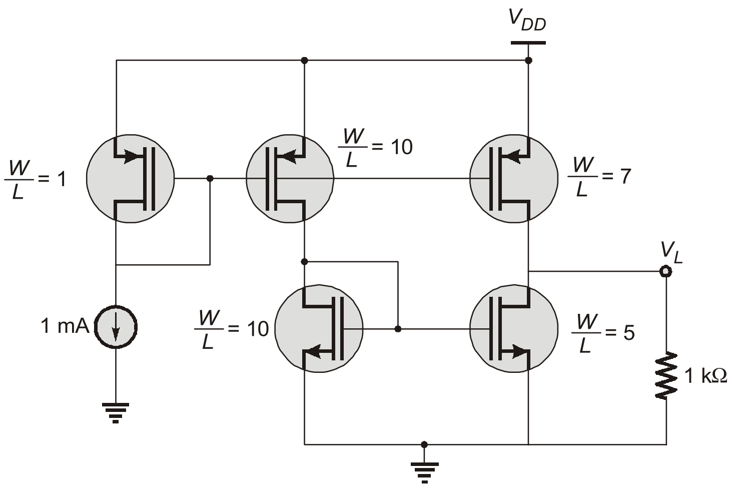
Clk	$D_1 = Q_2$	$D_2 = \overline{Q_1}$	Q_1	Q_2
Initial			1	0
1	0	0	0	0
2	0	1	0	1
3	1	1	1	1
4	1	0	1	0

Therefore, the given counter is having MOD-4

$\therefore$ The frequency of signal $Q_2 = \frac{f_i}{4} = \frac{1000}{4}$ kHz = 250 kHz

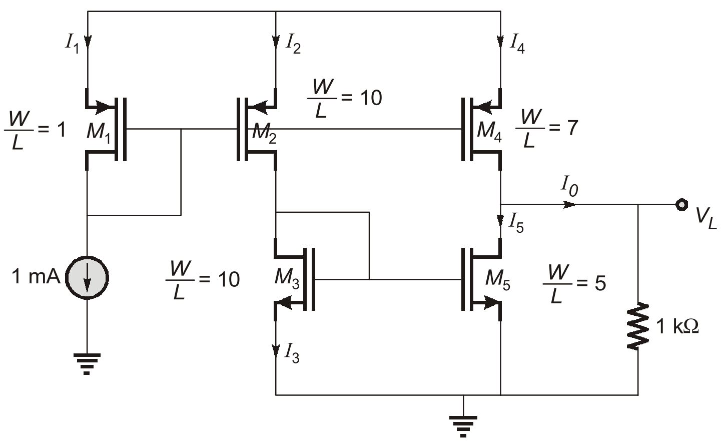
End of Solution

Q.64 In the circuit below, the voltage V_L is ______ V. (rounded off to two decimal places)



[2 Marks : NAT]

Ans. (2)



We know,

$$I_D \propto \left(\frac{W}{L}\right)$$
$$I_1 = 1 \text{ mA}$$
$$I_2 = \frac{10}{1} \times 1 = 10 \text{ mA}$$
$$I_3 = 10 \text{ mA}$$
$$I_4 = 7 \text{ mA}$$
$$I_5 = 5 \text{ mA}$$
$$I_0 = I_4 - I_5 = 7 - 5 = 2 \text{ mA}$$
$$V_L = 2 \times 1 = 2\text{V}$$

End of Solution

Q.65 The frequency of occurrence of 8 symbols (a-h) is shown in the table below. A symbol is chosen and it is determined by asking a series of "yes/no" questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is _______.

(rounded off to two decimal places).

Symbols	a	b	c	d	e	f	g	h
Frequency of occurrence	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

[2 Marks : NAT]

Ans. **(1.984)**

The average number of questions when asked in the most efficient sequence, to determine the chosen symbol = min possible number of questions per symbol (H)

$$H = \sum_i P_x(x_i) \log_2 \frac{1}{P_x(x_i)}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{32}\log_2 32 + \frac{1}{64}\log_2 64 + 2 \times \frac{1}{128}\log_2 128$$

$$= 1.984 \frac{\text{Questions}}{\text{Symbol}}$$

End of Solution

■■■■