Mathematics

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Question	Very Short Answer (VSA) 1 Mark	Short Answer– I (SA I) 2 Marks	Short Answer– II (SA II) 2 Marks	Long Answer (LA) 5 Marks	Total Marks	% Weightage
1.	**Remembering**	2	2	2	2	20	25%
2.	**Understanding**	2	1	1	4	23	29%
3.	**Application**	2	2	3	1	19	24%
4.	**High Order Thinking Skills**	-	1	4	-	14	17%
5.	**Inferential and Evaluative**	-	-	-	1	4	5%
	Total	6 × 1 = 6	6 × 2 = 12	10 × 3 = 30	8 × 4 = 32	80	100%

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General Instructions:

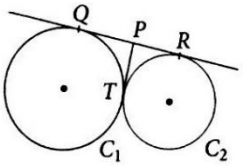
(i) All question are compulsory
(ii) The question paper consists of 30 question divided into four section A, B, C and D.
(iii) Section A contains 6 questions of I mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

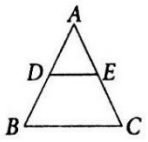
1. Find the discriminant of the equation $\sqrt{7}y^2 - 6y - 13\sqrt{7} = 0$.

2. The decimal expansion of the rational number $\frac{37}{2^3.5^4}$, will terminate after how many places of decimals?

3. In the adjoining figure, QR is common tangent to the circles C_1 and C_2 touching externally at T. Find the length of QR if $QP = 3.2$ cm



4. In ΔABC shown below, $DE \parallel BC$. If $BC = 8$ cm, $DE = 6$ cm and area of $\Delta ADE = 45$ cm^2, what is the area of ΔABC?



5. If $7 \tan \theta = 4$, then find the value of $\frac{7\sin\theta - 3\cos\theta}{7\sin\theta + 3\cos\theta}$.

6. The mid-point of the line joining $(3a, 4)$ and $(-2, 2b)$ is $(2, 2a + 2)$. Find the values of a and b.

SECTION - B

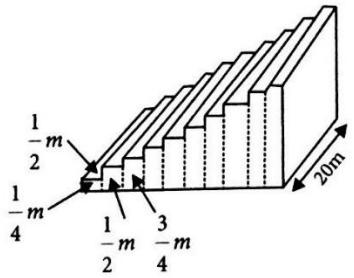
7. Using Euclid's division algorithm, find HCF of 15 and 575.

8. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, then find the value of $\frac{m}{n}+\frac{n}{m}$.

9. Find the roots of $6x^2 - \sqrt{2}x - 2 = 0$ by factorisation of the corresponding quadratic equation.

10. Find the distance between the points $P\left(\frac{\sin\theta}{2}, 0\right)$ and $Q\left(0, \frac{\cos\theta}{2}\right)$.

11. A small terrace at a hockey ground comprises of 10 steps each of which is 20 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. Calculate the total volume of concrete required to build the terrace.



12. Show that the pair of linear equations $x = 2y$ and $y = 2x$ have unique solution at (0, 0). Justify your answer.

SECTION - C

13. Find a quadratic polynomial, whose sum and product of the zeroes are $-\frac{8}{3}$ and $\frac{4}{3}$, respectively. Also, find the zeroes of this polynomial by factorisation.

14. If the centroid of the triangle formed by the points $A(a, b)$, $B(b, c)$ and $C(c, a)$ is at the origin. What is the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$?

15. A bag contains 18 balls out of which x balls are red.
    (i) If one ball is drawn at random from the bag, what is the probability that it is red?
    (ii) If 2 more red balls are put in the bag, the probability of drawing a red ball will be $\frac{9}{8}$ times than that in part (i). Find x.

OR

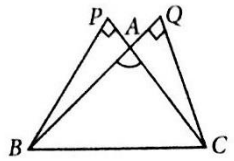
Two customers Neha and Nancy are visiting a particular shop in the same week (Monday to Friday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on :

(i) The same day

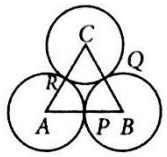
(ii) Consecutive days

(iii) Different days

16. In ΔABC, $\angle A$ is obtuse, $PB \perp PC$ and $QC \perp QB$. Prove that $AB \times AQ = AC \times AP$.



17. In the adjoining figure three circles with centres, A, B and C, respectively touch each other externally. If $AB = 5$ cm, $BC = 7$ cm and $CA = 6$ cm, then find the radius of the circle with centre A.



18. Two chairs and three tables cost ₹ 5650 whereas three chairs and two tables cost ₹ 7100. Find the cost of a chair and a table separately.

OR

Solve the following system of equations : $\frac{27}{x+y} - \frac{15}{x-y} = -2$ and $\frac{30}{x+y} - \frac{1}{x-y} = 3$

19. If the p^{th} term of an A.P. is a and the q^{th} term is b, then show that the sum of its first $(p + q)^{th}$ terms is $\frac{P+q}{2}\left\{a+b+\frac{a-b}{p-q}\right\}$.

20. A conical vessel with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel.

$\left[\text{Take } \pi = \frac{22}{7}\right]$

OR

A container is in the form of the frustum of a cone. It its height is 16 cm and the radii of its lower and upper ends are 8 cm and 20 cm respectively, then find the slant height of the container and also the cost of milk that the container can hold, if the cost of milk is ₹ 30 per litre. [Take $\pi = 3.14$]

21. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is a positive integer.

22. Find mode from the following frequency distribution :

Wages (in ₹)	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50
Number of Workers	22	45	67	73	85	190	64	55

OR

The mean of the following frequency distribution is 57.6 and the sum of the observation is 50. Find the missing frequencies f_1 and f_2.

Class Interval	0–20	20–40	40–60	60–80	10–100	100–120
Frequency	7	8	f_1	10	f_2	5

SECTION - D

23. Draw two circles with P and Q as their centres and of radii 2 cm and 4 cm such that $PQ = 7$ cm. Construct tangents to each circle from the centre of the other circle.

24. A highway leads to the foot of 300 m high tower. An observatory is set at the top of the tower. It sees a car moving towards it with an angle of depression of 30°. After 15 seconds angle of depression becomes 60.

(i) Find the distance travelled by the car during this time.

(ii) How this observatory is helpful to regulate the traffic on the highway?

OR

A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 60° with it. The distance between the foot of the tree to the point where the top touches the ground is 10 m. The teacher asked the students to find the height of the tree. All the students failed but Neeraj took initiative and calculated it correctly using trigonometry.

(i) What height Neeraj calculated?

(ii) What quality of Neeraj is depicted here?

25. Find the solution of the equation $\sqrt{x^2-16}-(x-4)=\sqrt{x^2-5x+4}$.

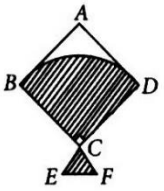
26. The frequency distribution of marks obtained by 230 candidates in a medical entrance test is as follows :

Marks	400 – 450	450–500	500–550	550–600	600–650	650–700	770–750	750–800	Total
Frequency	20	35	40	32	24	27	18	34	230

27. If $a_1, a_2, \ldots, a_n$ are in AP where, $a_i > 0$ for all i, then show that

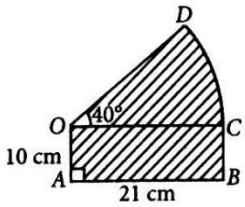
$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots..+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}.$$

28. The figure given alongside shows a kite, in which BCD is in the shape of a quadrant of a circle of radius 42 cm. $ABCD$ is a square and ΔCEF is an isosceles right-angled triangle whose equal sides are 6 cm long. Find the area of the shaded region.



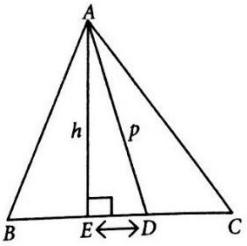
OR

Find the area of the shaded region.



29. Prove that $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$.

30. In the given figure, D is the mid-point of side BC and $AE \perp BC$. If $BC = a$, $AC = b$, $AB = c$, $AE = h$, $DE = x$ and $AD = p$. Prove that



(i) $b^2 = P^2 + ax + \frac{a^2}{4}$

(ii) $c^2 = P^2 - ax + \frac{a^2}{4}$

(iii) $b^2 + c^2 = 2P^2 + \frac{a^2}{2}$

OR

In the given figure, $ABCD$ is a rhombus, DPR and CBR are straight lines. Show that $DP \times CR = DC \times PR$