# **JEE Advanced 2024**

# Sample Paper - 3

Time Allowed: 3 hours Maximum Marks: 180

#### **General Instructions:**

This question paper has THREE main sections and four sub-sections as below.

#### **MRQ**

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

#### **MCQ**

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

#### NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

#### **MATCH**

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +3 marks for the correct response and -1 for the incorrect response.

## **Mathematics (MRQ)**

- 1. The tangent PT and the normal PN to the parabola  $y^2 = 4ax$  at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose
  - a) directrix is x = 0

b) focus is (a, 0)

c) vertex is  $\left(\frac{2a}{3},0\right)$ 

- d) latus rectum is  $\frac{2a}{3}$
- 2. Let  $h(x) = min \{x, x^2\}$ , for every real number of x, Then

[4]

[4]

- a) h is continuous for all x
- b) h is differentiable for all x
- c) h is not differentiable at two values of x
  - d) h'(x) = 1, for all x > 1
- 3. The option(s) with the values of a and L that satisfy the following equation is(are)

$$rac{\int_{0}^{4\pi} e^{t} (\sin^{6} at + \cos^{4} at) dt}{\int_{0}^{\pi} e^{t} (\sin^{6} at + \cos^{4} at) dt} = L$$
 ?

a) 
$$a=2, L=rac{e^{4\pi}+1}{e^{\pi}+1}$$

b) 
$$a=4, L=rac{e^{4\pi}-1}{e^{\pi}-1}$$

C) 
$$a=4, L=rac{e^{4\pi}+1}{e^{\pi}+1}$$

d) 
$$a=2,~\mathrm{L}=rac{e^{4\pi}-1}{e^{\pi}-1}$$

### **Mathematics (MCQ)**

4. Coefficient of 
$$t^{24}$$
 in  $(1 + t^2)^{12} (1 + t^{12})(1 + t^{24})$  is

[3]

[3]

[4]

a) 
$${}^{12}C_6$$

b) 
$$^{12}C_6 + 1$$

c) 
$$^{12}C_6 + 2$$

d) 
$$^{12}C_6 + 3$$

5. Let  $f(x) = (1 + b^2)x^2 + 2bx + 1$  and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

c) 
$$[0, \frac{1}{2}]$$

d) 
$$[\frac{1}{2}, 1]$$

6. A straight line L through the point (3, -2) is inclined at an angle 60° to the line  $\sqrt{3}x + y =$  [3] 1. If L also intersects the X-axis, then the equation of L is

a) 
$$\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$$

b) 
$$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

c) 
$$y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$$

d) 
$$\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$$

7. Let g (x) = 1 + x - [x] and f(x) =  $\begin{cases} -1, & x < 0 \\ 0, & x = 0, \text{ then for all x, f[f(x)) is equal to} \\ 1, & x > 0 \end{cases}$ 

c) 
$$f(x)$$

## **Mathematics (NUM)**

8. Let  $\overbrace{75\cdots57}^r$  denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + ... +  $\overbrace{75\cdots57}^{98}$ . If S =  $\overbrace{75\cdots57+m}^{99}$ , where m and n are natural numbers less than 3000, then the value of m + n is

- 9. If the variable line  $3x + 4y = \alpha$  lies between the two circles  $(x 1)^2 + (y 1)^2 = 1$  and  $(x 9)^2 + (y 1)^2 = 4$ , without intercepting a chord on either circle, then the sum of all the integral values of  $\alpha$  is \_\_\_\_\_.
- 10. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a=6, b=10 and the area of the triangle is  $15\sqrt{3}$ , if  $\angle A\dot{C}B$  is obtuse and if r denotes the radius of the incircle of the triangle, then  $r^2$  is equal to

- 11. If the vectors  $a\hat{i} + \hat{j} + \hat{k}$ ,  $\hat{i} + b\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} + c\hat{k}$  ( $a \neq b \neq c \neq 1$ ) are coplanar, then the value of  $\frac{1}{(1-a)} + \frac{1}{(1-b)} + \frac{1}{(1-c)} = \underline{\qquad}$ .
- 12. The total number of distinct  $x \in [0, 1]$  for which  $\int_{0}^{x} \frac{t^2}{1+t^4} dt = 2x 1$  is:
- 13. The number of values of  $\theta$  in the interval,  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  such that  $\theta \neq \frac{n\pi}{5}$  for  $n = 0, \pm 1, \pm 2$  and **[4]** tan  $\theta = \cot 5\theta$  as well as  $\sin 2\theta = \cos 4\theta$  is

### **Mathematics (MATCH)**

14. Let 
$$z_k = \cos(\frac{2k\pi}{10}) + i\sin(\frac{2k\pi}{10})$$
; k = 1, 2, ..., 9.

List-I	List-II
(P) For each $z_k$ there exists as $z_j$ such that $z_k.z_j = 1$	(1) True
(Q) There exists a $k \in \{1, 2,, 9\}$ such that $z_1.z = z_k$ has no solution z in the set of complex numbers	(2) False
(R) $\frac{ 1-z_1  1-z_2 \dots  1-z_9 }{10}$ equal	(3) 1
(S) $1-\sum\limits_{k=1}^{9}\cos\left(rac{2k\pi}{10} ight)$ equal	(4) 2

[3]

[3]

15. Let H:  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , here a > b > 0, be a hyperbola in the xy-plane whose conjugate axis

LM subtends an angle of  $60^{\rm O}$  at one of its vertices N. Let the area of the triangle LMN be  $4\sqrt{3}$ .

- <b>v</b> - ·	
List I	List II
P. The length of the conjugate axis of H is	1. 8
Q. The eccentricity of H is	2. $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	3. $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	4. 4

a) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  3; R  $\rightarrow$  1; S  $\rightarrow$  2

b) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  2; R  $\rightarrow$  1; S  $\rightarrow$  3

c) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  1; R  $\rightarrow$  3; S  $\rightarrow$  2

d) P 
$$\rightarrow$$
 3; Q  $\rightarrow$  4; R  $\rightarrow$  2; S  $\rightarrow$  1

16. Let p, q, r be nonzero real numbers that are, respectively, the 10<sup>th</sup>, 100<sup>th</sup> and 1000<sup>th</sup> terms of a harmonic progression. Consider the system of linear equations

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qr x + pr y + pq z = 0$$
.

List-I	List-II

(I) If $\frac{q}{r}$ = 10, then the system of linear equations has	(P) $x = 0$ , $y = \frac{10}{9}$ , $z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq$ 100, then the system of linear equations has	(Q) $x = \frac{10}{9}$ , $y = -\frac{1}{9}$ , $z = 0$ solution
(III) If $\frac{p}{q} \neq$ 10, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q}$ = 10, then the system of linear equations has	(S) no solution
	(T) at least one solution

a) (I) 
$$\rightarrow$$
 (Q); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (R)

b) (I) 
$$\rightarrow$$
 (Q); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (R)

c) (I) 
$$\rightarrow$$
 (T); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (T)

d) (I) 
$$\rightarrow$$
 (T); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (T)

17. Let  $\ell_1$  and  $\ell_2$  be the lines  $\vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$  and  $\vec{r}_2 = (\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{k})$ , respectively. Let X **[3]** be the set of all the planes H that contain the line  $\ell_1$ . For a plane H, let d(H) denote the smallest possible distance between the points of  $\ell_2$  and H. Let H<sub>0</sub> be a plane in X for which d(H<sub>0</sub>) is the maximum value of d(H) as H varies over all planes in X. Match each entry in List-I to the correct entries in List-II.

List - I	List -
(P) The value of d(H <sub>0</sub> ) is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from $H_0$ is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H <sub>0</sub> is	(3) 0
(S) The distance of origin from the point of intersection of planes $y = z$ , $x = 1$ and $H_0$ is	$(4) \sqrt{2}$
	$(5) \frac{1}{\sqrt{2}}$

a) (P) 
$$\rightarrow$$
 (5), (Q)  $\rightarrow$  (1), (R)  $\rightarrow$  (4), (S)  $\rightarrow$  (2)

b) (P) 
$$\rightarrow$$
 (5), (Q)  $\rightarrow$  (4), (R)  $\rightarrow$  (3), (S)  $\rightarrow$  (1)

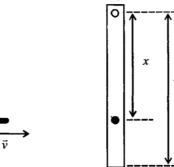
c) (P) 
$$\rightarrow$$
 (2), (Q)  $\rightarrow$  (1), (R)  $\rightarrow$  (3), (S)  $\rightarrow$  (2)

d) (P) 
$$\rightarrow$$
 (2), (Q)  $\rightarrow$  (4), (R)  $\rightarrow$  (5), (S)  $\rightarrow$  (1)

[4]

### Physics (MRQ)

18. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular



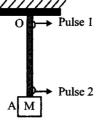
a) 
$$\omega=rac{3vx}{L^2+3x^2}$$

b) 
$$\omega_M=rac{v}{2L}\sqrt{3}$$

c) 
$$\omega = \frac{12vx}{L^2 + 12x^2}$$

d) 
$$x_M=rac{L}{\sqrt{3}}$$

19. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength  $\lambda_0$  is produced at point O on the rope. The pulse takes time  $T_{OA}$  to reach point A. If the wave pulse of wavelength  $\lambda_0$  is produced at point A (Pulse 2) without disturbing the position of M it takes time  $T_{AO}$  to reach O. Which of the following options is/are correct?



- a) The w avelength of Pulse 1 becomes longer when it reaches point A
- b) The time  $T_{AO} = T_{AO}$
- c) The velocity of any pulse along the rope is independent of its frequency and wavelength
- d) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
- 20. Two metallic rings A and B, identical in shape and size but having different resistivities  $\rho_A$  [4] and  $\rho_B$ , are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights  $h_A$  and  $h_B$ , respectively, with  $h_A > h_B$ . The possible relation(s) between their resistivities and their masses  $m_A$  and  $m_B$  is(are)



a)  $\rho_A$  <  $\rho_B$  and m<sub>A</sub> < m<sub>B</sub>

b)  $\rho_A$  >  $\rho_B$  and m<sub>A</sub> = m<sub>B</sub>

c)  $\rho_A < \rho_B$  and m<sub>A</sub> = m<sub>B</sub>

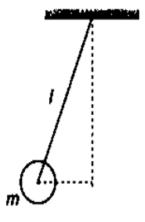
d)  $\rho_A > \rho_B$  and m<sub>A</sub> > m<sub>B</sub>

- 21. The unit of surface tension may be expressed as:
  - a) joule metre<sup>-2</sup>

b) newton metre<sup>-2</sup>

c) joule metre

- d) newton metre
- 22. A ball of mass (m) 0.5 kg is attached to the end of a string having length (l) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is



a) 27

b) 36

c) 9

- d) 18
- 23. The radius of a planet is  $\frac{1}{4}$  of the earth's radius and its acceleration due to gravity is double that of the earth's acceleration due to gravity. How many times will the escape velocity at the planet's surface be as compared to its value on the earth's surface?
- [3]

[3]

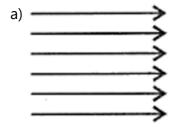
a) 2

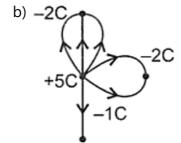
b)  $\sqrt{2}$ 

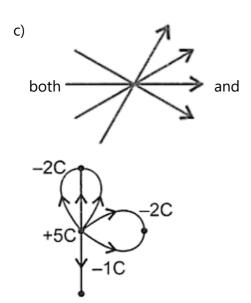
c)  $\frac{1}{\sqrt{2}}$ 

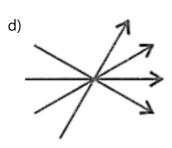
- d)  $2\sqrt{2}$
- 24. Which of the following configurations of electric lines of force is not possible?







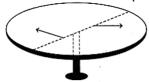




### **Physics (NUM)**

- 25. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance [4] of 25 cm in front of it to 50 cm, the magnification of its image changes from  $m_{25}$  to  $m_{50}$ . The ratio is  $\frac{m_{25}}{m_{50}}$  is
- 26. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm<sup>-2</sup> and it is absorbed by the water over an effective area of 0.05 m<sup>2</sup>. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be \_\_\_\_\_\_.

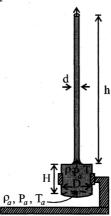
  (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s<sup>-1</sup>, Heat capacity of water = 4200 Jkg<sup>-1</sup> K<sup>-1</sup>)
- 27. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms<sup>-1</sup> with respect to the ground. The rotational speed of the platform in rad s<sup>-1</sup> after the balls leave the platform is



28. A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure  $P_a$  and its temperature becomes T = 360 K. The hot air with density  $\rho$  rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density  $\rho_a = 1.2$  kg m<sup>-3</sup>, pressure  $P_a$  and temperature  $T_a = 300$  K enters the furnace. Assume air as an ideal gas, neglect the variations in  $\rho$  and T inside the chimney and the furnace. Also ignore the viscous effects.

[4]

[Given: The acceleration due to gravity  $g = 10 \text{ ms}^{-2}$  and  $\pi = 3.14$ ]

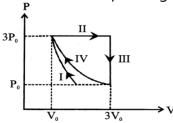


Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is  $\_\_\_\_$  gm s<sup>-1</sup>.

- 29. The Bohr radius of the fifth electron of phosphorous atom (atomic number = 15) acting as **[4]** a dopant in silicon (relative dielectric constant = 12) is  $\underline{\qquad}$   $\overset{\circ}{A}$ .
- 30. A particle, of mass  $10^{-3}$  kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field  $\vec{E}$  (t) = E<sub>0</sub> sin  $\omega t \hat{i}$ , where E<sub>0</sub> = 1.0 NC<sup>-1</sup> and  $\omega$  =  $10^3$  rad s<sup>-1</sup>. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms<sup>-1</sup>, attained by the particle at subsequent times is \_\_\_\_\_\_.

### **Physics (MATCH)**

31. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II.



LIST - I	LIST - II	
P. In process I	1. Work done by the gas is zero	
Q. In process II	2. Temperature of the gas remains unchanged	
R. In process III	3. No heat is exchanged between the gas and its surroundings	
S. In process IV	4. Work done by the gas is 6P <sub>0</sub> V <sub>0</sub>	

a) P 
$$ightarrow$$
 3; Q  $ightarrow$  4; R  $ightarrow$  1; S  $ightarrow$  2

b) P 
$$\rightarrow$$
 3; Q  $\rightarrow$  4; R  $\rightarrow$  2; S  $\rightarrow$  1

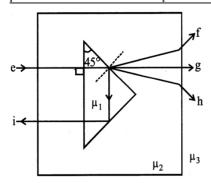
c) P 
$$\rightarrow$$
 1; Q  $\rightarrow$  3; R  $\rightarrow$  2; S  $\rightarrow$  4

d) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  3; R  $\rightarrow$  1; S  $\rightarrow$  2

32. A right angled prism of refractive index  $\mu_1$  is placed in a rectangular block of refractive index  $\mu_2$ , which is surrounded by a medium of refractive index  $\mu_3$ , as shown in the figure. A ray of light **e** enters the rectangular block at normal incidence. Depending upon the

Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

List-I	List-II
$(P)\;e\to f$	(1) $\mu_1>\sqrt{2}\mu_2$
(Q) $e \rightarrow g$	(2) $\mu_2>\mu_1$ and $\mu_2>\mu_3$
(R) $e \rightarrow h$	(3) $\mu_1 = \mu_2$
(S) $e \rightarrow i$	(4) $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$



[3]

33. List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

List - I	List - II
(P) $^{238}_{92}\mathrm{U}  ightarrow ^{234}_{91}\mathrm{Pa}$	(1) one a particle and one $eta^+$ particle
(Q) $^{214}_{82} \ \mathrm{Pb}  o ^{210}_{82} \ \mathrm{Pb}$	(2) three $\beta^-$ particles and one a particle
(R) $^{210}_{81}\mathrm{Tl}  ightarrow ^{206}_{.82}\mathrm{Pb}$	(3) two $\beta^-$ particles and one a particle
(S) $^{228}_{91}$ Pa $ ightarrow$ $^{224}_{88}$ Ra	(4) one a particle and one $\beta^-$ particle
	(5) one a particle and two $\beta^+$ particles

a) 
$$ext{P} 
ightarrow 5, ext{Q} 
ightarrow 1, ext{R} 
ightarrow 3, ext{ S} 
ightarrow 2$$

b) 
$$ext{P} 
ightarrow 5, ext{Q} 
ightarrow 3, ext{R} 
ightarrow 1, \; ext{S} 
ightarrow 4$$

c) 
$$P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$$

d) 
$$ext{P} 
ightarrow 4, ext{Q} 
ightarrow 1, ext{R} 
ightarrow 2, \; ext{S} 
ightarrow 5$$

34. A musical instrument is made using four different metal strings 1,2,3 and 4 with mass per unit length  $\mu$ ,  $2\mu$ ,3 $\mu$  and  $4\mu$  respectively. The instrument is played by vibrating the strings by varying the free length in between the range L<sub>0</sub> and 2L<sub>0</sub>. It is found that in string-1 ( $\mu$ ) at free length L<sub>0</sub> and tension T<sub>0</sub> the fundamental mode frequency is f<sub>0</sub>.

List - I gives the above four strings while list - II lists the magnitude of some quantity.

List-I	List-II
(I) String - 1 (μ)	(P) 1
	·

List-I	List-II
(II) String - 2 (2µ)	(Q) $\frac{1}{2}$
(III) String - 3 (3μ)	(R) $\frac{1}{\sqrt{2}}$
(IV) String - 4 (4μ)	$(S)\frac{1}{\sqrt{3}}$
	(T) $\frac{3}{16}$
	(U) 1/16

The length of the strings 1,2,3 and 4 are kept fixed at  $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ , and  $\frac{7L_0}{4}$ , respectively.

Strings 1, 2, 3, and 4 are vibrated at their 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, and 14<sup>th</sup> harmonics, respectively such that all the strings have same frequency The correct match for the tension in the four strings in the units of T<sub>0</sub> will be

a) (I) 
$$\rightarrow$$
 (T), (II) $\rightarrow$  (Q), (III) $\rightarrow$  (R), (IV) $\rightarrow$  (U)

b) (I) 
$$\rightarrow$$
 (P), (II) $\rightarrow$  (R), (III) $\rightarrow$  (T), (IV) $\rightarrow$  (U)

c) (I) 
$$\rightarrow$$
 (P), (II) $\rightarrow$  (Q), (III) $\rightarrow$  (T), (IV) $\rightarrow$  (U)

d) (I) 
$$\rightarrow$$
 (P), (II) $\rightarrow$  (Q), (III) $\rightarrow$  (R), (IV) $\rightarrow$  (T)

### **Chemistry (MRQ)**

35. Which of the following will undergo aldol condensation?

[4]

- a) trideuteroacetaldehyde
- b) benzaldehyde

c) propanaldehyde

d) acetaldehyde

36. In the decay sequence,

[4]

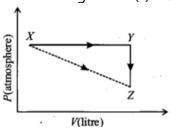
 $^{238}_{92}\mathrm{U} \xrightarrow{-x_1} ^{234}_{90}\mathrm{Th} \xrightarrow{-x_2} ^{234}_{91}\mathrm{Pa} \xrightarrow{-x_2} ^{234}\mathrm{Z} \xrightarrow{-x_4} ^{230}_{90}\mathrm{Th}$  x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub> and x<sub>4</sub> are particles/radiation emitted by the respective isotopes. The correct option(s) is (are)

a)  $x_3$  is  $\gamma$ - ray

b) Z is an isotope of uranium

c) x<sub>2</sub> is  $\beta$  -ray

- d) x<sub>1</sub> will deflect towards negatively charged plate
- 37. For an ideal gas, consider only P- V work in going from an initial state X to the final state [4] Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice(s) is (are) correct? [Take  $\Delta S$  as change in entropy and was work done].



a) 
$$w_{X 
ightarrow Y 
ightarrow Z} = w_{X 
ightarrow Y}$$

b) 
$$w_{X o Z}=w_{X o Y^+}+w_{Y o Z}$$

## **Chemistry (MCQ)**

38. The correct order of increasing C—O bond length of CO,  $CO_3^{2-}$ , CO<sub>2</sub> is

[3]

a)  $CO < CO_3^{2-} < CO_2$ 

b)  $CO_2 < CO_3^{2-} < CO$ 

c) CO < CO $_2$  <  $\mathrm{CO}_3^{2-}$ 

- d)  $CO_3^{2-}$  <  $CO_2$  < CO
- 39. For the reversible reaction,  $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$  at 500° C, the value of  $K_p$  is 1.44  $\times$  [3]  $10^{-5}$  when partial pressure is measured in atmosphere. The corresponding value of  $K_C$  with concentration in mol/L is
  - a)  $\frac{1.44\times10^{-5}}{(0.082\times773)^{-3}}$

b)  $\frac{1.44 \times 10^{-5}}{(0.082 \times 773)^{-2}}$ 

c)  $\frac{1.44 \times 10^{-5}}{(0.082 \times 500)^{-2}}$ 

- d)  $\frac{1.44 \times 10^{-5}}{\left(8.314 \times 773\right)^{-2}}$
- 40. The compounds which gives the most stable carbonium ion on dehydration is

[3]

CH<sub>3</sub> CH<sub>3</sub> CH<sub>3</sub> CH<sub>3</sub> CH<sub>3</sub>

- b) OH  $CH_3 \stackrel{|}{C}H CH_2CH_3$
- c) CH<sub>3</sub>CH<sub>2</sub>— CH<sub>2</sub>— CH<sub>2</sub>OH
- d)  $\mathrm{CH_3} \mathrm{C}_{\mathrm{H}_2}\mathrm{H} \mathrm{CH_2OH}_{\mathrm{CH_2}}$
- 41. Hydrogen bonding is maximum in

[3]

a) triethylamine

b) ethanol

c) diethyl ether

d) ethyl chloride

### **Chemistry (NUM)**

- 42. 20 mL of calcium hydroxide was consumed when it was reacted with 10 mL of unknown solution of H<sub>2</sub>SO<sub>4</sub>. Also 20 mL standard solution of 0.5 M HCl containing 2 drops of phenolphthalein was titrated with calcium hydroxide the mixture showed pink colour when burette displaced the value of 35.5 mL whereas the burette showed 25.5 mL initially. The concentration of H<sub>2</sub>SO<sub>4</sub> is \_\_\_\_\_ M (Nearest Integer)
- 43. The vapour pressure of pure benzene is 639.7 mm of mercury and the vapour of a solution **[4]** of a solute in benzene at the same temperature is 631.9 mm of mercury. Calculate the molality of the solution.
- 44. From the following data, calculate the enthalpy change for the combustion of cyclopropane at 298 K. The enthalpy of formation of CO<sub>2</sub>(g), H<sub>2</sub>O(l) and propene(g) are

- -393.5, -285.8 and 20.42 kJ mol<sup>-1</sup> respectively. The enthalpy of isomerisation of cyclopropane to propene is -33.0 kJ mol<sup>-1</sup>.
- 45. What is the maximum number of electrons that may be present in all the atomic orbitals with principal quantum number 3 and azimuthal quantum number 2?
- 46. The rate of a first-order reaction is 0.04 mol litre<sup>-1</sup> s<sup>-1</sup> at 10 minutes and 0.03 mol litre<sup>-1</sup>  $s^{-1}$  at 20 minutes after initiation. Find the half-life of the reaction.
- 47. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound, is CH<sub>3</sub>CH<sub>2</sub>CH(CH<sub>3</sub>)CH<sub>2</sub>CH<sub>2</sub>

### **Chemistry (MATCH)**

48. Match List-I with List-II:

List-I (Reagents used)	List-II (Compound with functional group detected)
(A) Alkaline solution of copper sulphate and sodium citrate	(I) HO O
(B) Neutral FeCl <sub>3</sub> solution	(II) NH <sub>2</sub>
(C) Alkaline chloroform solution	(III) O CHO
(D) Potassium iodide and sodium hypochlorite	(IV) OH

[3]

[3]

49. Match each coordination compound in List-I with an appropriate pair of characteristics from List- II and select the correct answer using the code given below the lists. {en = H<sub>2</sub>NCH<sub>2</sub>CH<sub>2</sub>NH<sub>2</sub>; atomic numbers : Ti = 22 ; Cr = 24; Co = 27 ;Pt = 78}

List-I	List-II
(A) [Cr(NH <sub>3</sub> ) <sub>4</sub> Cl <sub>2</sub> ]Cl	(p) Paramagnetic and exhibits ionisation isomerism
(B) [Ti(H <sub>2</sub> O) <sub>5</sub> Cl](NO <sub>3</sub> ) <sub>2</sub>	(q) Diamagnetic and exhibits cis-trans isomerism
(C) [Pt(en)(NH <sub>3</sub> )Cl]NO <sub>3</sub>	(r) Paramagnetic and exhibits cis-trans isomerism
(D) [Co(NH <sub>3</sub> ) <sub>4</sub> (NO <sub>3</sub> ) <sub>2</sub> ]NO <sub>3</sub>	(s) Diamagnetic and exhibits ionisation isomerism

50. An aqueous solution of X is added slowly to an aqueous solution of Y as shown in List I. The variation in conductivity of these reactions is given in List II. Match list I with List II and select the correct answer using the code given below the lists:

List-I	List-II	
$(P)_{X}^{(C_2H_5)_3N+CH_3COOH}$	(1) Conductivity decreases and then increases	
(Q) $\frac{\text{KI}(0.1\text{M})}{\text{X}}$ + AgNO <sub>3</sub> (0.01M)	(2) Conductivity decreases and then does not change much	
(R) $CH_3 \underset{X}{COOH} + \underset{Y}{KOH}$	(3) Conductivity increases and then does not change much	
(S) $NaOH + H_Y$	(4) Conductivity does not change much and then increases	

51. Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in List-II and choose the correct option.

List- I	List- II
$(P) P2O3 + 3H2O \rightarrow$	(1) P(O)(OCH <sub>3</sub> )Cl <sub>2</sub>
(Q) $P_4$ + 3NaOH + 3H <sub>2</sub> O $\rightarrow$	(2) H <sub>3</sub> PO <sub>3</sub>
(R) $PCl_5 + CH_3COOH \rightarrow$	(3) PH <sub>3</sub>
(S) $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow$	(4) POCI <sub>3</sub>
	(5) H <sub>3</sub> PO <sub>4</sub>

a) P 
$$ightarrow$$
 2; Q  $ightarrow$  3; R  $ightarrow$  1; S  $ightarrow$  5

b) P 
$$ightarrow$$
 2; Q  $ightarrow$  3; R  $ightarrow$  4; S  $ightarrow$  5

c) P 
$$ightarrow$$
 3; Q  $ightarrow$  5; R  $ightarrow$  4; S  $ightarrow$  2

d) P 
$$\rightarrow$$
 5; Q  $\rightarrow$  2; R  $\rightarrow$  1; S  $\rightarrow$  3

[3]

## **JEE Advanced 2024**

# Sample Paper - 3

## **Solution**

### **Mathematics (MRQ)**

- 1. **(b)** focus is (a, 0)
  - (c) vertex is  $\left(\frac{2a}{3},0\right)$

**Explanation:** Let  $P(at^2, 2at)$  be any point on the parabola  $y^2 - 4ax$ .

 $\therefore$  Tangent to the parabola at P is  $y = \frac{x}{t} + at$ ,

which meets the axis of parabola i.e x-axis at T ( $-at^2$ , 0).

Also normal to parabola at P is  $t \times t + y = 2at + at^3$ 

which meets the axis of parabola at  $N(2a + at^2, 0)$ 

Let G (x, y) be the centriod of  $\triangle$ PTN, then

$$x=rac{at^2-at^2+2a+at^2}{3}$$
 and y =  $rac{2at}{3}$ 

$$\Rightarrow x = rac{2a+at^2}{3}$$
 ...(i) and y =  $rac{2at}{3}$  ...(ii)

Eliminating t from (i) and (ii), we get the locus of centriod G

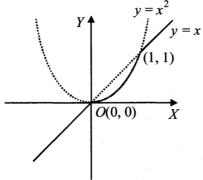
as 3x = 2a + a 
$$\left(\frac{3y}{2a}\right)^2 \Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2}{3}a\right)$$
,

which is a parabola with vertex  $(\frac{2a}{3},0)$ , directrix as  $x-\frac{2a}{3}=-\frac{a}{3}$ , latus rectum as  $\frac{4a}{3}$  and focus as (a, 0).

- 2. (a) h is continuous for all x
  - (c) h is not differentiable at two values of x
  - **(d)** h'(x) = 1, for all x > 1

**Explanation:** From the figure it is clear that

$$h(x) = \left\{ egin{aligned} x, & ext{if } x \leq 0 \ x^2, & ext{if } 0 < x < 1 \ x, & ext{if } x \geq 1 \end{aligned} 
ight.$$



From the graph it is clear that h is continuous for all  $x \in R$ , h' (x) = 1 for all x > 1 and h is not differentiable at x = 0 and 1 as there are sharp turns at x = 0 and 1.

3. **(b)** 
$$a=4, L=rac{e^{4\pi}-1}{e^{\pi}-1}$$

(d) 
$$a=2, \; \mathrm{L}=rac{e^{4\pi}-1}{e^{\pi}-1}$$

**Explanation:** Let  $f(t) = e^t (\sin^6 at + \cos^4 at)$ 

$$\therefore f(k\pi+t) = e^{k\pi+t} \left(\sin^6 a(k\pi+t) + \cos^4 a(k\pi+t)\right) = e^{k\pi}f(t)$$

$$\therefore \frac{\int_0^{4\pi} e^t (\sin^6 at + \cos^4 at) dt}{\int_0^{\pi} e^t (\sin^6 at + \cos^4 at) dt}$$

$$=\frac{\frac{(1+e^{\pi}+e^{2x}+e^{3\pi})\int_0^{\pi}e^t(\sin^6at+\cos^4at)dt}{\int_0^{\pi}e^t(\sin^6at+\cos^4at)dt}}=1+e^{\pi}+e^{2\pi}+e^{3\pi}=\frac{e^{4\pi}-1}{e^{\pi}-1}$$

### Mathematics (MCQ)

(c) 
$$^{12}C_6+2$$

**Explanation:** Here, Coefficient of  $t^{24}$  in  $\{(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})\}$ 

= Coefficient of 
$$t^{24}$$
 in  $\{(1 + t^2)^{12} \cdot (1 + t^{12} + t^{24} + t^{36})\}$ 

= Coefficient of 
$$t^{24}$$
 in  $\{(1+t^2)^{12}+t^{12}(1+t^2)^{12}+t^{24}(1+t^2)^{12}\}$ ; [neglecting  $t^{36}(1+t^2)^{12}$ ]

= Coefficient of 
$$t^{24} = (^{12}C_{12} + ^{12}C_{6} + ^{12}C_{0}) = 2 + ^{12}C_{6}$$

5.

**Explanation:** 
$$f(x) = (1 + b^2)x^2 + 2bx + 1$$

It is a quadratic expression with coefficient of  $x^2 = 1 + b^2 > 0$ .

 $\therefore$  f(x) represents an upward parabola whose minimum value is  $\frac{-D}{4a}$ . Here D is being the discriminant.

$$\therefore m(b) = -rac{4b^2 - 4(1 + b^2)}{4(1 + b^2)} = rac{1}{1 + b^2}$$

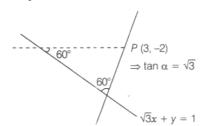
Now, 
$$\frac{1}{1+b^2} > 0$$
 and  $b^2 \ge 0 \Rightarrow 1 + b^2 \ge 1$ 

$$\Rightarrow \frac{1}{1+b^2} \le 1$$
. Hence m(b) = (0, 1]

6.

**(b)** 
$$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

**Explanation:** A straight line passing through P and making an angle of  $\alpha = 60^{\circ}$ , is given by  $\frac{y-y_1}{x-x_1} = \tan(\theta \pm \alpha)$ 



$$\Rightarrow \sqrt{3}x + y = 1$$

$$\Rightarrow y = -\sqrt{3}x + 1, \text{ then tan } \theta = -\sqrt{3}$$

$$\Rightarrow \frac{y+2}{x-3} = \frac{\tan \theta \pm \tan \alpha}{1 \mp \tan \theta \tan \alpha}$$

$$\Rightarrow \frac{y+2}{x-3} = \frac{\tan\theta \pm \tan\alpha}{1 \mp \tan\theta \tan\alpha}$$

$$\frac{y+2}{x-3} = \frac{-\sqrt{3}+\sqrt{3}}{1-(-\sqrt{3})(\sqrt{3})}$$

and 
$$\frac{y+2}{x-3} = \frac{-\sqrt{3}-\sqrt{3}}{1+(-\sqrt{3})(\sqrt{3})}$$

$$\Rightarrow$$
y + 2 = 0

and 
$$\frac{y+2}{x-3} = \frac{-2\sqrt{3}}{1-3} = \sqrt{3}$$

$$y + 2 = \sqrt{3}x - 3\sqrt{3}$$

Neglecting, y + 2 = 0, as it does not intersect Y-axis.

7. **(a)** 1

**Explanation:** g(x) = 1 + x - [x] is greater than 1

since 
$$x - [x] > 0$$

$$f[g(x)] = 1$$
, since  $f(x) = 1$  for all  $x > 0$ 

Mathematics (NUM)

8. 1219.0

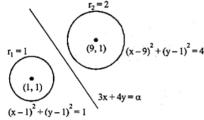
Given S = 77 + 757 + 7557 + ... + 
$$\overbrace{75...57}^{98}$$

$$\underbrace{\frac{10 \text{ S} = 770 + 7570 + ... + 75...570 + 75...570}_{-9 \text{ S} = 77 - 13 - 13... - 13} - \underbrace{75...570}_{98}}_{98 \text{ times}}$$
9 S = -77 + 13 × 98 +  $\underbrace{75....57}_{99}$  + 13

$$S = \frac{\overbrace{75...57}^{99} + 1210}{9}$$
  
 $\Rightarrow$  m = 1210 and n = 9  $\Rightarrow$  m + n = 1219

#### 9. 165.0

Explanation:



We can say line lies between the two circles or both centres should lie on either side of the line as well as line can be tangent to circle.

$$(3 + 4 - \alpha) \cdot (27 + 4 - \alpha) < 0$$

$$(7 - \alpha) \cdot (31 - \alpha) < 0 \Rightarrow a \in (7, 31) \dots (i)$$

 $d_1$  = distance of (1, 1) from line

 $d_2$  = distance of (9, 1) from line

$$d_1 \geq r_1 \Rightarrow rac{|7-lpha|}{5} \geq 1 \Rightarrow lpha(-\infty,2] \cup [12,\infty)$$
 ...(ii)

$$\mathrm{d}_2 \geq \mathrm{r}_2 \Rightarrow rac{|31-lpha|}{5} \geq 2 \Rightarrow lpha \in (-\infty,21] \cup [41,\infty)$$
 ...(iii)

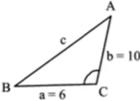
(i) 
$$\cap$$
 (ii)  $\cap$  (iii)  $\Rightarrow \alpha \in [12, 21]$ 

Sum of integers = 12 + 13 + ... + 21 = 165

#### 10.3.0

### **Explanation:**

We know that area of triangle  $=\frac{1}{2}ab\sin C$ 



$$\Rightarrow rac{1}{2} imes 6 imes 10 imes \sin C = 15\sqrt{3} \Rightarrow \sin C = rac{\sqrt{3}}{2}$$

$$\Rightarrow C = 120^{\circ}$$
 as C is obtuse angle.

Now 
$$\cos C=rac{a^2+b^2-c^2}{2ab}$$
, [cosine rule]  $\therefore \cos 120^\circ=rac{36+100-c^2}{120}$ 

$$\therefore \cos 120^{\circ} = \frac{36+100-c^2}{120}$$

$$\Rightarrow c^2=196 \Rightarrow c=14, \therefore s=rac{a+b+c}{2}=15$$

Hence radius of incircle,  $r=rac{\triangle}{s}=rac{15\sqrt{3}}{15}=\sqrt{3}$ 

$$\therefore r^2 = 3$$

11. 1.0

Given that the vectors  $\vec{a}=a\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+b\hat{j}+\hat{k}$  and  $\vec{c}=\hat{i}+\hat{j}+c\hat{k}$  where  $\mathbf{a}\neq\mathbf{b}\neq\mathbf{c}\neq\mathbf{1}$  are coplanar

$$\therefore [ec{a}ec{b}ec{c}] = 0 \Rightarrow egin{vmatrix} a & 1 & 1 \ 1 & b & 1 \ 1 & 1 & c \end{bmatrix} = 0$$

Applying  $\mathrm{C}_1=C_1-C_2, C_2=C_2-C_3$ 

$$\begin{vmatrix} a-1 & 0 & 1 \\ 1-b & b-1 & 1 \\ 0 & 1-c & c \end{vmatrix} = 0$$

Taking (1 - a), (1 - b), (1 - c) common from  $R_1, R_2$  and  $R_3$  respectively, we get

$$\Rightarrow (1-a)(1-b)(1-c) \begin{vmatrix} -1 & 0 & \frac{1}{1-a} \\ 1 & -1 & \frac{1}{1-b} \\ 0 & 1 & \frac{c}{1-c} \end{vmatrix} = 0$$

Applying  $R_2 o R_2 + R_1$ 

$$\Rightarrow (1-a)(1-b)(1-c) egin{bmatrix} -1 & 0 & rac{1}{1-a} \ 0 & -1 & rac{1}{1-b} + rac{1}{1-a} \ 0 & 1 & rac{c}{1-c} \end{bmatrix} = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)(-1)\Big[-rac{c}{1-c}-rac{1}{1-b}-rac{1}{1-a}\Big]=0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[\frac{1}{1-a} + \frac{1}{1-b} + \frac{c}{1-c}\right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[rac{1}{1-a}+rac{1}{1-b}-rac{(1-c)-1}{1-c}
ight]=0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[rac{1}{1-a} + rac{1}{1-b} + rac{1}{1-c} - 1
ight] = 0$$

But 
$$a \neq b \neq c \neq 1$$

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1 = 0 \Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

12. 1

**Explanation:** 

Let 
$$f(x) = \int_{0}^{x} \frac{t^2}{1+t^4} dt - 2x + 1$$

$$\Rightarrow f'(x) = rac{x^2}{1+x^4}$$
 - 2 < 0  $orall x \in [0, 1]$ 

∴ f is decreasing on [0, 1]

Also 
$$f(0) = 1$$

and f(1) = 
$$\int_{0}^{1} \frac{t^2}{1+t^4} dt - 1$$

For 
$$0 \le t \le 1 \Rightarrow 0 \le \frac{t^2}{1+t^4} < \frac{1}{2}$$

$$\therefore \int\limits_0^1 rac{t^2}{1+t^4} dt < rac{1}{2}$$

$$\Rightarrow$$
 f(1) < 0

∴ f(x) crosses x-axis exactly once in [0, 1]

$$\therefore$$
 f(x) = 0 has exactly one root in [0, 1]

13. 3

$$an heta = \cot 5 heta, heta 
eq rac{n\pi}{5}$$

$$\Rightarrow \cos \theta \cos 5\theta - \sin 5\theta \sin \theta = 0 \Rightarrow \cos 6\theta = 0$$

$$\Rightarrow 6\theta = \frac{-5\pi}{2}, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$\Rightarrow \theta = \frac{-5\pi}{12}, \frac{-\pi}{4}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$$

Again  $\sin 2\theta = \cos 4\theta = 1 - 2\sin^2 2\theta$ 

$$ightarrow 2\sin^2 2 heta + \sin 2 heta - 1 = 0 \Rightarrow \sin 2 heta = -1, rac{1}{2}$$

$$\Rightarrow 2 heta=rac{-\pi}{2},rac{\pi}{6},rac{5\pi}{6}\Rightarrow heta=rac{-\pi}{4},rac{\pi}{12},rac{5\pi}{12}$$

So, common solutions are  $\theta = \frac{-\pi}{4} \cdot \frac{\pi}{12}$  and  $\frac{5\pi}{12}$ 

... Number of solutions = 3

### **Mathematics (MATCH)**

**Explanation:** (P) 
$$ightarrow$$
 (1) :  $z_k = \cos \frac{2k\pi}{10} + i \sin \frac{2k\pi}{10}$ , k = 1 to 9

$$\therefore z_k = e^{irac{2k\pi}{10}}$$

Now 
$$z_k z_j = 1 \Rightarrow z_j = rac{1}{z_k} = e^{-irac{2k\pi}{10}} = \overline{z_k}$$

We know if  $z_k$  is  $10^{th}$  root of unity so will be  $\bar{z}_k$ .

$$\therefore$$
 For every  $z_k$ , there exist  $z_i = \bar{z}_k$ 

Such that 
$$z_k \cdot z_j = z_k \cdot \bar{z}_k = 1$$

Hence the statement is true.

(Q) 
$$ightarrow$$
 (2) z<sub>1</sub>= z<sub>k</sub>  $\Rightarrow$   $z=rac{z_k}{z_1}$  for  $z_1 
eq 0$ 

$$\therefore$$
 We can always find a solution of  $z_1.z = z_k$ 

Hence the statement is false.

(R) 
$$\rightarrow$$
 (3): We know  $z^{10}$  -1 = (z - 1)(z - z<sub>1</sub>)...(z - z<sub>9</sub>)

$$\Rightarrow$$
 (z - z<sub>1</sub>) (z - z<sub>2</sub>)...(z - z<sub>9</sub>) =  $\frac{z^{10}-1}{z-1}$ 

$$= 1 + z + z^2 + ...z^9$$

For 
$$z = 1$$
, we get  $(1 - z_1)(1 - z_2)...(1 - z_9) = 10$ 

$$\therefore \frac{|1-z_1||1-z_2|...|1-z_9|}{10} = 1$$

(S) 
$$\rightarrow$$
 (4): 1, Z<sub>1</sub>, Z<sub>2</sub>, ..., Z<sub>9</sub> are 10th roots of unity.

$$\therefore Z^{10} - 1 = 0$$

From equation 
$$1 + Z_1 + Z_2 + ... + Z_9 = 0$$

$$Re(1) + Re(Z_1) + Re(Z_2) + ... + Re(Z_9) = 0$$

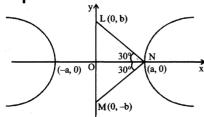
$$\Rightarrow$$
 Re(Z<sub>1</sub>) + Re(Z<sub>2</sub>) + ... Re(Z<sub>9</sub>) = -1

$$\Rightarrow\sum_{K=1}^9\cosrac{2k\pi}{10}=-1\Rightarrow 1-\sum_{K=1}^9\cosrac{2k\pi}{10}$$
 = 2

Hence ((P) - (1), (Q) - (2), (R) - (3), (S) - (4)) is the correct option.

15. (a) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  3; R  $\rightarrow$  1; S  $\rightarrow$  2

**Explanation:** Area of  $\triangle$ LMN =  $4\sqrt{3}$  (given)



$$\Rightarrow \frac{1}{2} \times LM \times ON = 4\sqrt{3} \Rightarrow \frac{1}{2}(2b)(\sqrt{3}b) = 4\sqrt{3}$$

$$\therefore b^2 = 4 \Rightarrow b = 2$$

So, length of the conjugate axis of hyperbola = 2b = 4

Now tan 30° = 
$$\frac{OL}{ON} = \frac{a}{b} \Rightarrow$$
 a =  $\sqrt{3}b \Rightarrow$  a =  $2\sqrt{3}$ 

$$\therefore$$
 b<sup>2</sup> = a<sup>2</sup> (e<sup>2</sup> - 1)  $\Rightarrow$  4 = 12(e<sup>2</sup> - 1)  $\Rightarrow$  e<sup>2</sup> = 1 +  $\frac{1}{3}$  =  $\frac{4}{3}$ 

$$\therefore$$
 The eccentricity of hyperbola = e =  $\frac{2}{\sqrt{3}}$  and

The distance between the foci of hyperbola = 2ae

$$=2\times2\sqrt{3}\times=\frac{2}{\sqrt{3}}=8$$

And length of latus ractum of hyperbola

$$=\frac{2b^2}{a}=\frac{2\times 4}{2\sqrt{3}}=\frac{4}{\sqrt{3}}$$

16. (a) (l) 
$$\rightarrow$$
 (Q); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (R)

**Explanation:** We have system of linear equations

$$x + y + z = 1 ...(i)$$

$$10x + 100y + 1000z = 0$$

$$x + 10y + 100z = 0 ...(ii)$$

$$qrx + pry + pqz = 0 ...(iii)$$

$$\Rightarrow \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0 \ (\because p, q, r \neq 0)$$

Let 
$$p = \frac{1}{a+9d}$$
,  $q = \frac{1}{a+99d}$ ,  $r = \frac{1}{a+999d}$ 

Now, equation (iii) is

$$(a + 9d)x + (a + 99d)y + (a + 999d)z = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 100 \\ a + 9d & a + 99d & a + 999d \end{vmatrix} = 0$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 10 & 100 \\ 0 & a + 99d & a + 999d \end{vmatrix} = 900(d - a)$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 100 \\ a + 9d & 0 & a + 999d \end{vmatrix} = 990(a - d)$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 0 \\ a+9d & a+99d & 0 \end{vmatrix} = 90(d-a)$$

Let option I: If  $\frac{q}{r} = 10 \Rightarrow a = d$ 

$$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

Since eq. (i) and eq. (ii) represents non-parallel planes and eq. (ii) and eq. (iii) represents same plane

 $\Rightarrow$  Infinitely many solutions

So, option I  $\rightarrow$  P, Q, R, T

Option II:  $\frac{p}{r} \neq 100 \Rightarrow a \neq d$ 

$$\Delta$$
 = 0,  $\Delta_x$ ,  $\Delta_y$ ,  $\Delta_z \neq 0$ 

No solution

So, option II 
$$\rightarrow$$
 S

Option III: 
$$\frac{p}{a} \neq 10 \Rightarrow a \neq d$$

No solution

So, option III 
$$\rightarrow$$
 S

Option IV: If 
$$\frac{p}{q} = 10 \Rightarrow a = d$$

Infinitely many solution

Hence, IV 
$$\rightarrow$$
 P, Q , R , T

17.

**(b)** (P) 
$$\rightarrow$$
 (5), (Q)  $\rightarrow$  (4), (R)  $\rightarrow$  (3), (S)  $\rightarrow$  (1)

**Explanation:** For largest possible distance between plane  $H_0$  and  $I_2$ , the line  $I_2$  must be parallel to plane  $H_0$ .

 $\therefore$  H<sub>0</sub> will be the plane containing the line I<sub>1</sub> and parallel to I<sub>2</sub>

Normal vector 
$$\overrightarrow{\mathbf{n}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{k}$$

∴ H<sub>0</sub> : x - z = 
$$\frac{c}{(0,0,0)}$$
 ⇒ c = 0

∴ 
$$H_0$$
:  $x - z = 0$ 

(P) Distance of point (0,1, -1) from H<sub>0</sub>.

d (H<sub>0</sub>) = 
$$\left| \frac{0 - (1)}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

(Q) The distance of the point (0, 1, 2) from 
$$H_0 = \left| \frac{0-2}{\sqrt{2}} \right| = \sqrt{2}$$

(R) The distance of origin from 
$$H_0 = \left| \frac{0}{\sqrt{2}} \right| = 0$$

(S) Point of intersection of planes 
$$y = z$$
,  $x = 1$  and  $H_0$  is  $(1, 1, 1)$ .

Distance = 
$$\sqrt{1+1+1} = \sqrt{3}$$
.

## Physics (MRQ)

18. **(a)** 
$$\omega = \frac{3vx}{L^2 + 3x^2}$$

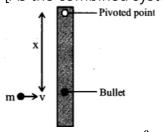
(b) 
$$\omega_M = rac{v}{2L} \sqrt{3}$$

(d) 
$$x_M=rac{L}{\sqrt{3}}$$

Explanation: From angular momentum conservation about the pivoted point.

$$\mathrm{mvx} = \left(rac{\mathrm{mL}^2}{3} + \mathrm{mx}^2
ight) \omega$$

[As the combined system rotates with angular speed  $\omega$  about the pivot]



$$\therefore \omega = \frac{mvx}{\frac{mL^2}{3} + mx^2} = \frac{3vx}{L^2 + 3x^2}$$

$$\omega=rac{3vx}{L^2+3x^2}$$

Hence option ( $\omega=rac{3vx}{L^2+3x^2}$ ) is correct.

For maximum angular velocity,  $\frac{d\omega}{dx}$  = 0

$$rac{\mathrm{d}}{\mathrm{dx}} \Big( rac{\mathrm{L}^2}{\mathrm{x}} + 3 \mathrm{x} \Big) = 0$$

$$\Rightarrow rac{ ext{L}^2}{x^2} + 3 = 0 \Rightarrow x = rac{ ext{L}}{\sqrt{3}}$$

$$\therefore X_m = \frac{L}{\sqrt{3}}$$

So option ( $x_M = \frac{L}{\sqrt{3}}$ ) is correct.

$$\omega_m=rac{3vx}{L^2+3x^2}=rac{3vrac{L}{\sqrt{3}}}{L^2+3\left(rac{L}{\sqrt{3}}
ight)^2}=rac{\sqrt{3}}{2L}V$$

Hence option ( $\omega_M = \frac{v}{2L}\sqrt{3}$ ) is correct.

19. **(b)** The time  $T_{AO} = T_{AO}$ 

(c) The velocity of any pulse along the rope is independent of its frequency and wavelength **Explanation:** Wavelength of pulse,  $\lambda=\frac{v}{f}=\frac{1}{f}\sqrt{\frac{T}{\mu}}$  or,  $T\propto\sqrt{T}$ 

Where T = tension of string.

Here 
$$T_1 > T_2 :: \lambda_1 > \lambda_2$$

The velocities of the two pulses cannot be same at midpoint as velocity being vector quantity has direction.

 $V=\sqrt{\frac{T}{\mu}}$ , so speed at any position will be same for both pulses, therefore time taken by both pulses will be same ie.,  $T_{AO}=T_{OA}$ 

20. (a)  $\rho_A < \rho_B$  and m<sub>A</sub> < m<sub>B</sub>

(c) 
$$\rho_A < \rho_B$$
 and m<sub>A</sub> = m<sub>B</sub>

**Explanation:** Induced emf  $e = -\frac{d\phi}{dt}$ . For identical rings induced emf will be same. But current will be different. Given  $h_A > h_Q$ .

Hence, 
$$V_A > V_B$$
 as  $\left(h = rac{v^2}{2g}
ight)$ 

If  $\rho_A > \rho_B$ , then, I<sub>A</sub> < I<sub>B</sub>. In this case given condition can be fulfilled if m<sub>A</sub> < m<sub>B</sub>.

If  $\rho_A < \rho_B$ , then, I<sub>A</sub> > I<sub>B</sub>. In this case given condition can be fulfilled if m<sub>A</sub>  $\leq$  m<sub>B</sub>.

## Physics (MCQ)

21. (a) joule metre $^{-2}$ 

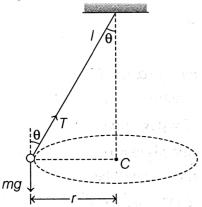
**Explanation:** Unit of surface tension is Nm<sup>-1</sup>

Also J 
$$m^{-2} = Nmm^{-2} = Nm^{-1}$$

22.

**(b)** 36

# **Explanation:**



Tcos $\theta$  component will cancel mg.

 $Tsin\theta$  component will provide necessary centripetal force to the ball towards centre C.

∴ Tsin
$$\theta$$
 = mr $\omega^2$  = m (Isin $\theta$ ) $\omega^2$  or T -  $\frac{m}{\omega^2}$ 

$$\therefore \omega = \sqrt{rac{T}{ml}}$$
 or  $\omega_{
m max} = \sqrt{rac{T_{
m max}}{ml}} = \sqrt{rac{324}{0.5 imes 0.5}} = 36 {
m rad/s}$ 

23.

(c) 
$$\frac{1}{\sqrt{2}}$$

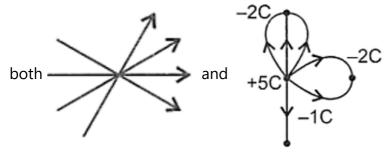
**Explanation:**  $V_S = \sqrt{2gR}$ 

$$rac{V_{S_1}}{V_{S_2}} = \sqrt{rac{g_1 R_1}{g_2 R_2}} = \sqrt{rac{(2g)(R/4)}{(g)R}} = rac{1}{\sqrt{2}}$$

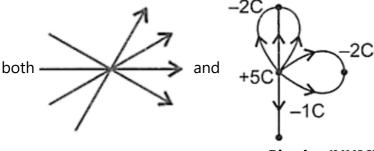
$$V_{S_1}=rac{V_{S_2}}{\sqrt{2}}$$

24.

(c)



## **Explanation:**



**Physics (NUM)** 

25. 6.0

**Explanation:** 

When 
$$u = -25$$
 cm

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} + \frac{1}{-25} = \frac{1}{100} \Rightarrow v = 100 \text{ cm}$$

$$m_{25} = \frac{-v}{u} = \frac{-100}{-25} = 4$$

When 
$$u = -50$$
 cm

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} + \frac{1}{-50} = \frac{3}{100} \Rightarrow v = \frac{100}{3} \text{ cm}$$

$$m_{50} = \frac{-v}{u} = \frac{-1000}{3} \times \frac{-1}{50} = \frac{2}{3}$$
  
So,  $\frac{m_{25}}{m_{50}} = \frac{4}{\frac{2}{3}} = 6$ 

So, 
$$\frac{m_{25}}{m_{50}} = \frac{4}{\frac{2}{3}} = 6$$

26. 8.33

**Explanation:** 

Rate of loss of heat

$$rac{dQ}{dt} = \sigma \mathrm{eA}(\mathrm{T}^4 - T_0^4) ...(\mathrm{i})$$

$$\Rightarrow rac{\mathrm{dQ}}{\mathrm{Adt}}$$
 =  $\mathrm{e}\left(\mathrm{T}_0 + \Delta\mathrm{T}
ight)^4 - \mathrm{T}_0^4
ight)$  =  $\sigma\mathrm{T}_0^4\left[\left(1 + rac{\Delta\mathrm{T}}{\mathrm{T}_0}
ight)^4 - 1
ight]$ 

= 
$$e\sigma T_0^4 \left[ \left( 1 + 4 rac{\Delta T}{T_0} 
ight) - 1 
ight]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T ...(ii)$$

Now from eq. (i)

$$ms\frac{dQ}{dt} = \sigma eA(T^4 - T_0^4) [\because Q = ms\Delta T]$$

$$\Rightarrow \frac{dT}{dt} = \frac{\sigma e A}{ms} \left[ (T_0 - \Delta T)^4 - T_0^4 \right]$$

= 
$$\frac{\sigma eA}{ms}T_0^4 imes \left[ \left(1 + \frac{\Delta T}{T_0}\right)^4 - 1 \right]$$

$$\frac{dT}{dt} = \frac{\sigma e A}{ms} T_0^4 \cdot 4\Delta T$$

$$rac{dT}{dt} = K\Delta T$$
; (K =  $rac{4\sigma eAT_0^3}{ms}$  Constant for Newton's law of cooling)

$$\Rightarrow 4\sigma eAT_0^3 = rac{K}{A} ext{(ms)}$$

From eq. (i)

$$rac{dQ}{Adt} = e\sigma T_0^3 \cdot 4\Delta T$$

Since, rate of loss of heat = heat received per second

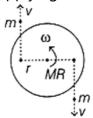
700 = (K/A) (ms) 
$$\Delta$$
T [K × ms = 4200 × 10<sup>-3</sup>]  
 $\Rightarrow \Delta T = \frac{700 \times A}{K \times ms} = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200} = \frac{50}{6} = \frac{25}{3}$ 

$$\Delta T = 8.33$$

### 27.4

**Explanation:** 

Applying conservation of angular momentum



$$2mvr - \frac{MR^2}{2} = 0$$

$$\omega=rac{4mvr}{MR^2}$$

Substituting the values, we get

$$\omega = \frac{(4)(5 \times 10^{-2})(9)(\frac{1}{4})}{45 \times 10^{-2} \times \frac{1}{4}} \Rightarrow \omega = 4 \text{ rad/s}$$

#### 28.47.10

**Explanation:** 

Since, pressure P = constant  $\rho_a T_a = \rho T$ 

$$\Rightarrow$$
 1.2  $\times$  300 =  $\rho$ (360)  $\therefore \rho$  = 1 kg/m<sup>3</sup>

Applying Bernoullis theorem between upper and bottom point

Assuming velocity of hot air inside the furnace  $\simeq 0\,$ 

$$P_a + 0 + 0 = P_a - \rho_a g(h) + \rho g(h) + \frac{1}{2} \rho V^2$$

$$\therefore V = \sqrt{\frac{2(\rho_a - \rho)g \times 9}{\rho}} = \sqrt{2(0.2)90} = 6$$

Therefore the steady mass flow rate of air existing the chimney

$$Q = \rho \pi \left(\frac{d^2}{4}\right) V = 1 \times 3.14 \frac{\times (0.1)^2}{4} \times 6$$

$$= 0.0471 \text{ kg/s} = 47.10 \text{ gms}^{-1}$$

29. 3.81

The fifth valence electron of phosphorous is in its third shell, i.e., n = 3. For phosphorous, Z = 15.

∴ Bohr's radius for n<sup>th</sup> orbit

$$r_{\text{n}} = \left(\frac{n^2}{Z}\varepsilon_r\right) r_{\text{0}} = \frac{3^2}{15} \times 12 \times 0.529 \stackrel{o}{A} = 3.81 \stackrel{o}{A}$$

30.2

**Explanation:** 

$$\therefore$$
 F = ma  $\therefore$  qE =  $m \frac{dv}{dt}$   
 $\Rightarrow$  dv =  $\frac{qEdt}{m}$  =  $\frac{q\sin 1000t\hat{i}}{m}dt$  ( $\cdot$  E =  $\sin 10^3$  t  $\hat{i}$  given)

$$\therefore \int\limits_0^v dv = rac{q}{m} \int\limits_0^{rac{\pi}{a}} \sin 1000 t \ ext{dt [max. speed is at } rac{T}{2} = rac{2\pi}{\omega imes 2}]$$

$$V = -\frac{q}{m} \left[ \frac{\cos 1000t}{1000} \right]_0^{\frac{\pi}{\omega}} = -\frac{1}{10^{-3}} \times \frac{\left[ \cos 1000t \right]_0^{\frac{\pi}{\omega}}}{1000} \text{ (:: m 10}^{-3} \text{ kg, q = 1C, E}_0 = 1 \text{ NC}^{-1} \text{ and w = 10}^3 \text{ rads}^{-1} \text{ given)}$$

$$\therefore$$
 V = -[cos 1000  $\times \frac{\pi}{1000}$  - cos 0] = -[-1-1] = 2 ms<sup>-1</sup>

Hence maximum speed attained by the particle.

### **Physics (MATCH)**

31. **(a)** 
$$P \rightarrow 3$$
;  $Q \rightarrow 4$ ;  $R \rightarrow 1$ ;  $S \rightarrow 2$ 

**Explanation:** Process I is adiabatic therefore  $\Delta Q = 0$ 

Process II is isobaric P = constant therefore  $W = P(V_2 - V_1)$ 

$$= 3P_0(3V_0 - V_0) = 6P_0V_0$$

Process III is isochoric V = constant therefore

$$W - P(V_2 - V_1) = 0$$

Process IV is isothermal, temperature T = constant,  $\therefore \Delta u = 0$ 

32.

**Explanation:**  $e \to f$ . When the ray enters from the rectangular block to prism then angle of incidence > angle of refraction, so  $\mu_2 > \mu_1$  The ray then moves away from the normal when it emerges out of the rectangular block. Therefore  $\mu_2 > \mu_3$ .

 ${\sf e} o {\sf g}$ . As there is no deviation of the ray as it emerges out of the prism,  $\therefore \mu_2 = \mu_1.$ 

e  $\to$  h. As the ray emerges out of prism, it moves away from the normal.  $\therefore \mu_2 < \mu_1$ . And the ray moves away from the normal as it emerges out of the rectangular block,  $\therefore \mu_2 > \mu_3$ . e  $\to$  i. At the prism surface, total internal reflection has taken place.

$$\therefore$$
 Critical angle 45° > C  $\Rightarrow$  sin 45° > sin C  $\Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \therefore \mu_1 > \sqrt{2}\mu_2$ 

33.

(c) 
$$P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$$

**Explanation:** In  $\alpha$ -decay mass number (A) decreases by 4 units and atomic number (Z) decreases by 2 units. In  $\beta^-$ decay A does not change but Z increases by 1 unit.

In  $\beta^+$  decay A does not change but Z decreases by 1 unit.

(P) 
$$_{92}\mathrm{U}^{238} \rightarrow _{91}\mathrm{Pa}^{234}$$

$$\mathrm{N}_1=rac{238-234}{4}=1
ightarrow 1lpha$$

$${
m N}_2-{
m N}_3=(92-91)-\left(rac{4}{2}
ight)=-1 o 1eta^-$$

 $\therefore 1\alpha$  and  $1\beta$  emission.

(Q) 
$$_{82} \text{ Pb}^{214} \rightarrow_{82} \text{ Pb}^{210}$$

$$N_1=rac{214-210}{4}=1
ightarrow 1lpha$$

$$m N_2 - N_3 = (82 - 82) - \left(rac{4}{2}
ight) = -2 
ightarrow 2eta^{-1}$$

 $\therefore 1\alpha$  and  $2\beta$  emission.

(R) 
$$_{81} \text{ T} \ell^{210} \rightarrow_{82} \text{ Pb}^{206}$$

$$\mathrm{N}_1=rac{210-206}{4}=1
ightarrow 1lpha$$

$$m N_2 - N_3 = (81 - 83) - rac{4}{2} = -3 
ightarrow 3eta^-$$

 $\therefore 1\alpha$  and  $3\beta^-$ emission.

(S) 
$$_{91} {\rm ~Pa^{228}} \rightarrow _{88} {\rm Ra^{224}}$$

$$N_1 = \frac{228 - 224}{4} = 1\alpha$$

$$N_2 - N_3 = (91 - 88) - \frac{4}{2} = 1\beta^+$$

 $\therefore 1\alpha$  and  $1\beta^+$ emission.

34.

(c) (I) 
$$\rightarrow$$
 (P), (II) $\rightarrow$  (Q), (III) $\rightarrow$  (T), (IV) $\rightarrow$  (U)

Explanation: As 
$$v = \frac{p}{2\ell} \sqrt{\frac{T}{m}} \mathrel{\therefore} T = \frac{v^2\ell^2 \; m}{p^2}$$

String - 1 
$$\mathrm{T}_0 = rac{\mathrm{f}_0^2 4\,\mathrm{L}_0^2 \mu}{\ell^2}$$

String - 2 
$$T_2=rac{f_0^24\left(rac{3}{2}
ight)^2L_0^2(2\mu)}{\left(3
ight)^2}=rac{T_0}{2}$$

String - 3 
$$T_3=rac{f_0^24\left(rac{5}{2}
ight)^2L_0^2(3\mu)}{5^2}=rac{3}{16}\;T_0$$

String - 4 
$$\mathrm{T}_4=rac{\mathrm{f}_0^2 4 \left(rac{7}{4}
ight)^2 \mathrm{L}_0^2 (4 \mu)}{\left(14
ight)^2}=rac{\mathrm{T}_0}{16}$$

## Chemistry (MRQ)

- 35. (a) trideuteroacetaldehyde
  - (c) propanaldehyde
  - (d) acetaldehyde

**Explanation:** Carbonyl compounds having  $\alpha$  - H or  $\alpha$  - D undergo aldol condensation.

a. 
$$\overset{lpha}{C}H_3CHO$$

b. 
$$CH_3\stackrel{lpha}{C}H_2CHO$$

C. 
$$(N_0 \alpha - H)$$

d. 
$$\overset{\alpha}{C}D_3CHO \ (\alpha-D \text{ present })$$

- 36. **(b)** Z is an isotope of uranium
  - (c)  $x_2$  is  $\beta$  -ray
  - (d) x<sub>1</sub> will deflect towards negatively charged plate

Explanation: 
$$^{238}_{92}\mathrm{U}\longrightarrow ^{234}_{90}\mathrm{Th}+^{4}_{2}\mathrm{He};~x_{1}=lpha$$

$$^{234}_{90}\mathrm{Th}\longrightarrow ^{234}_{91}\mathrm{Pa}+^{0}_{-1}\mathrm{e};\,x_{2}=eta$$

$$^{33}_{91}$$
 Pa  $\longrightarrow$   $^{33}_{92}$ Z  $+$   $^{0}_{-1}$ e;  $x_3=eta$ 

$$^{91}_{92} \longrightarrow ^{92}_{90} ext{Th} + ^{4}_{2} ext{He}; x_{4} = lpha$$

37. (a) 
$$w_{X 
ightarrow Y 
ightarrow Z} = w_{X 
ightarrow Y}$$

(d) 
$$\Delta S_{X o Z} = \Delta S_{X o Y} + \Delta S_{Y o Z}$$

**Explanation:**  $\Delta S_{X \to Z} = \Delta S_{X \to Y} + \Delta S_{Y \to Z}$  [Entropy is a state function, hence additive]  $w_{X \to Y \to Z} = w_{X \to Y}$  [Work done in  $Y \to Z$  is zero because it is an isochoric process].

### **Chemistry (MCQ)**

38.

(d) 
$$CO_3^{2-}$$
 <  $CO_2$  <  $CO$ 

**Explanation:** Bond length  $\propto \frac{1}{\rm Bond\, order}$ 

Bond order : CO, = 2, CO = 3,  $CO_3^{2-} = 1 + \frac{1}{3} = \frac{4}{3}$ 

Therefore, order of bond length is  $CO_3^{2-} < CO_2 < CO$ 

39.

**(b)** 
$$\frac{1.44\times10^{-5}}{(0.082\times773)^{-2}}$$

**Explanation:**  $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) \ \Delta n = -2$ 

$$K_p = K_c (RT)^{\Delta n}$$

$$K_c = \frac{K_p}{(RT)^{\Delta n}} = \frac{1.44 \times 10^{-5}}{(0.082 \times 773)^{-2}}$$

**Explanation:** 
$$CH_3 - CH_3 -$$

41.

## (b) ethanol

# **Explanation:**

Ethanol is capable in forming intermolecular H-bonds:

## Chemistry (NUM)

42. 1.0

**Explanation:** 

Reaction with HCI

$$Ca(OH)_2 + 2HCI \rightarrow CaCl_2 + 2H_2O$$

Volume of  $Ca(OH)_2 = 10$  ml, Volume of HCI = 20 ml

Concentration of HCI = 0.5 M

No. of millimoles of HCl = 10, No. of milli moles of  $Ca(OH)_2 = 5$ 

i.e. 
$$\mathrm{M_{Ca(OH)_2}}$$
 =  $\frac{\mathrm{no.~of~milli~moles}}{\mathrm{V(ml)}} = \frac{5}{10}$  = 0.5 M

Reaction with H<sub>2</sub>SO<sub>4</sub>

$$\text{Ca(OH)}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{CaSO}_4 + 2\text{H}_2\text{O}$$

No. of milli moles of Ca(OH)<sub>2</sub> =  $20 \times 0.5 = 10$ 

i.e. no. of milli moles of  $H_2SO_4 = 10$ 

$$\Rightarrow M_{\rm H_2SO_4}$$
 =  $\frac{\rm no.~of~milli~moles}{\rm V(ml)}$  =  $\frac{10}{10}$  = 1 M

43. 0.156

$$\frac{P^{O}-P}{P^{o}}=\frac{n}{N}$$
 [Raoult's Equation]

Let the molality of the solution = m

Now, the solution contains 'm' moles of solute per 1000 g of benzene.

Vapour pressure of benzene, P° = 639.7 mm

Vapour pressure of solution, P = 631.9 mm

Moles of benzene (Mol. wt. 78), N =  $\frac{1000}{78}$ 

Moles of solute, n = ?

Substitute these values in the Raoult's equation

$$\frac{P^{O}-P}{P^{o}} = \frac{n}{N} \text{ or } \frac{639.7-631.9}{639.7} = \frac{n \times 78}{1000}$$
or  $\frac{7.8}{639.7} = \frac{78n}{1000}$ 

$$\therefore n = \frac{1000 \times 7.8}{78 \times 639.7} = 0.156$$

Hence, molality of solution = 0.156 m

### 44. -2091.32

**Explanation:** 

$$\begin{array}{c} \text{CH}_2 \\ \text{H}_2\text{C} & \text{CH}_2 \\ \Delta \text{H} = \text{-}33.0 \text{ kJ} \dots \text{(i)} \\ \text{C(s)} + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g}); \ \Delta \text{H} = \text{-}393.5 \text{ kJ} \dots \text{(ii)} \\ \text{H}_2(\text{g}) + \frac{1}{2}\text{O}_2(\text{g}) \rightarrow \text{H}_2\text{O(l)}; \ \Delta \text{H} = \text{-}285.8 \text{ kJ} \dots \text{(iii)} \\ \text{3C(s)} + \text{3H}_2(\text{g}) \rightarrow \text{CH}_3\text{-CH} = \text{CH}_2(\text{g}); \ \Delta \text{H} = 20.42 \text{ kJ} \dots \text{(iv)} \end{array}$$

The required reaction is

$$H_2C$$
  $CH_2 + \frac{9}{2}O_2 \rightarrow 3CO_2 + 3H_2O; \Delta H = ?$ 

To calculate the value of  $\Delta H$  follow the following steps, (iv)-(i) yields

#### 45. 10

**Explanation:** 

For n = 3 and 1 = 2 (i.e., 3d orbital), the values of m varies from -2 to +2, i.e. -2, -1, 0, +1, +2 and for each m there are 2 values of s, i.e.  $+\frac{1}{2}$  and  $-\frac{1}{2}$ .

... The maximum no. of electrons in all the five d-orbitals is 10.

#### 46. 24.14

**Explanation:** 

$$r_1 = kc_1$$
 and  $r_2 = kc_2$ 

Since, rate of first-order reaction is directly proportional to the concentration of its reactant,  $\therefore \frac{r_1}{r_2} = \frac{c_1}{c_2} = \frac{0.04}{0.03}$ 

According to first-order reaction

$$k = \frac{2.303}{t_{20} - t_{10}} \log \frac{c_1}{c_2}$$

On substituting the various values k = 0.0287 min<sup>-1</sup>  $t_{\frac{1}{2}} = \frac{0.693}{k} = \frac{0.693}{0.0287} = 24.14$  min

47.8

Explanation:

$$\begin{array}{c} & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & \\ & & \\ & & \\ & & \\ & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ &$$

I has one chiral carbon = two isomers

II has two chiral carbons and no symmetry = four isomers.

III and IV have no chiral carbon, no stereoisomers.

### **Chemistry (MATCH)**

48.

### **Explanation:**

- A. Alkaline solution of copper sulphate and sodium citrate is known as Benedict's solution and it is used to test aliphatic aldehydes. Hence it can be used to test compound (III)
- B. Neutral FeCl<sub>3</sub> solution is used to test phenolic compound (IV)
- C. Alkaline chloroform solution is used to test primary amines (II)
- D.  $2KI + NaOCI + H_2O \rightarrow NaCI + I_2 + 2KOH$

Potassium iodide and sodium hypochlorite gives (I2 + KOH) which is used to test those

compounds which have  $\rm CH_3-\stackrel{O}{C}-~or~CH_3-\stackrel{OH}{CH}-group (iodoform~test).$  Hence the compound is (I)

49.

#### **Explanation:**

Complex	Magnetic character	Isomerism
A. [Cr(NH <sub>3</sub> ) <sub>4</sub> Cl <sub>2</sub> ]Cl	Cr <sup>3+</sup> is d <sup>3</sup> hence paramagnetic	cis-trans
B. [Ti(H <sub>2</sub> O) <sub>5</sub> Cl](NO <sub>3</sub> ) <sub>2</sub>	Ti <sup>3+</sup> i s d <sup>1</sup> , hence paramagnetic	ionization
C. [Pt(en)(NH <sub>3</sub> )Cl]NO <sub>3</sub>	Pt <sup>2+</sup> is d <sup>8</sup> , complex is square planar, all electrons are paired, hence diamagnetic	ionization
D. [Co(NH <sub>3</sub> ) <sub>4</sub> (NO <sub>3</sub> ) <sub>2</sub> ]NO <sub>3</sub>	Co <sup>3+</sup> is d <sup>6</sup> , all electrons are paired due to strong ligands, hence diamagnetic	cis-trans

50.

**Explanation:** (P) 
$$(C_2H_5)_3N + CH_3COOH \longrightarrow (C_2H_5)_3NH^+CH_3COO^-$$

Initially conductivity increases because on neutralisation ions are created. After that it becomes practically constant because X alone cannot form ions.

(Q) 
$$KI_X(0.1\mathrm{M}) + \mathrm{AgNO}_3(0.01\mathrm{M}) o Agl \downarrow + KNO_3$$

Number of ions in the solution remains constant as only AgNO<sub>3</sub> precipitated as Agl. Thereafter, conductance increases due to increase in number of ions.

- (R) Initially conductance decreases due to the decrease in the number of  $\bar{O}H$  ions as  $OH^-$  is getting replaced by  $CH_3COO^-$  which has poorer conductivity. Thereafter, it slowly increases due to the increase in number of  $H^+$  ions.
- (S) Initially it decreases due to decrease in  $H^+$  ions and then increases due to the increase in  $OH^-$  ions.

51.

**(b)** P 
$$\to$$
 2; Q  $\to$  3; R  $\to$  4; S  $\to$  5

**Explanation:** (P)  $P_2O_3 + 3H_2O \rightarrow 2H_3PO_3$ 

(Q) 
$$P_4$$
 +  $3NaOH$  +  $3H_2O \rightarrow 3NaH_2PO_2$  +  $PH_3$ 

(R) 
$$PCI_5 + CH_3COOH \rightarrow CH_3COCI + POCI_3 + HCI$$

(S) 
$$H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow 4Ag + 4HNO_3 + H_3PO_4$$