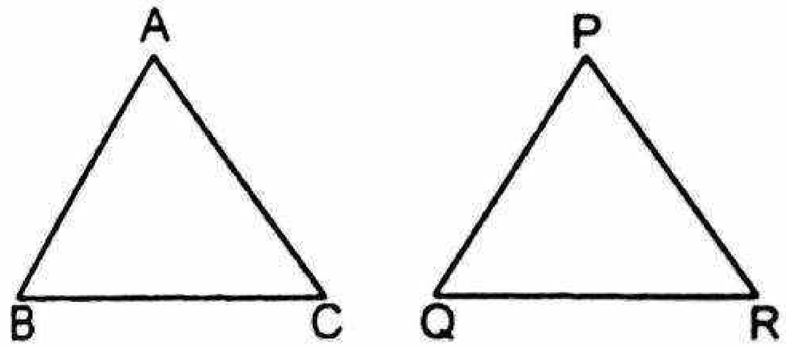
CHAPTER 13

TRIANGLES
(Congruences and Similarity)

Congruency of triangles - Two triangles are are said to be congruent if they are equal in all respects. i.e.

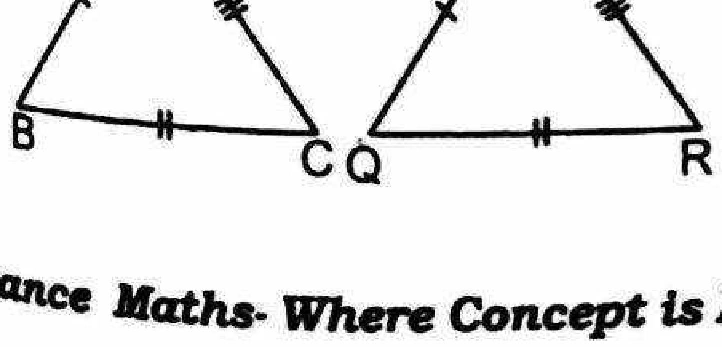
1. Each of the three sides of one triangle must be equal the three respective sides of the other.
2. Each of the three angles of the one triangle must be equal to the three respective angles of the other.



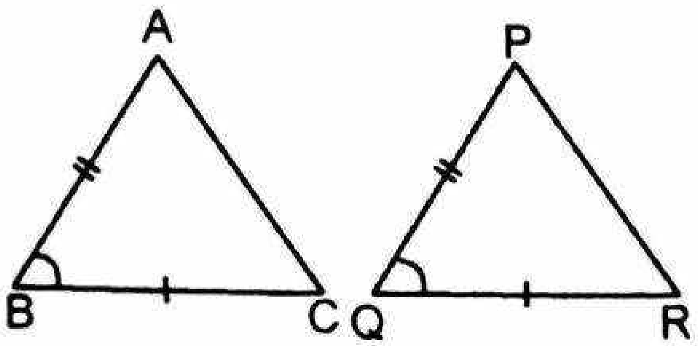
i.e. $\left.\begin{array}{l} AB = PQ \\ AC = PR \\ CB = QR \end{array}\right\} and \left.\begin{array}{l} \angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R \end{array}\right\}$

Tests of congruency - With the help of the following given tests, we can deduce without having detailed information about triangles that whether the given two triangles are congruent or not.

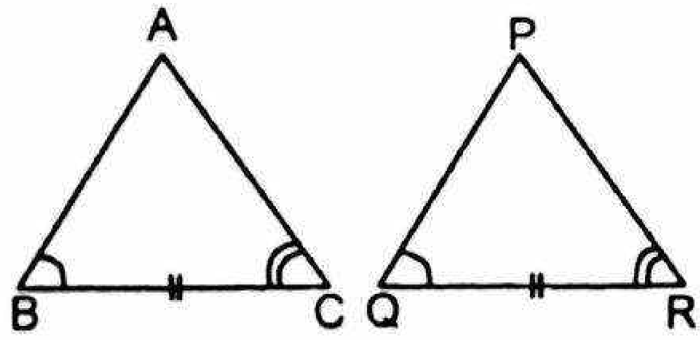
1. S-S-S (Side-Side-Side)
if AB = PQ, AC = PR, BC = QR
then :- $\Delta ABC \cong \Delta PQR$



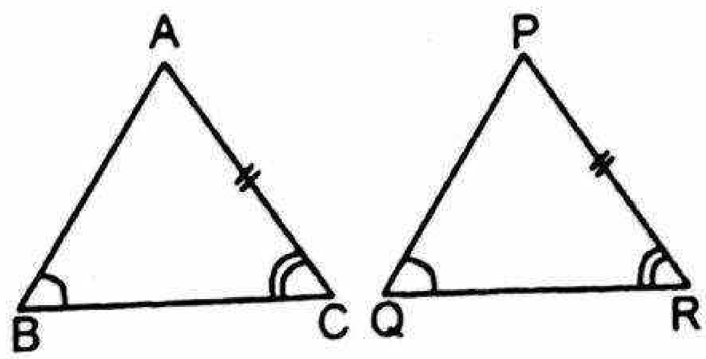
2. S-A-S (Side-Angle-Side) if AB = PQ, $\angle ABC = \angle PQR$, BC = QR then - $\Delta ABC \cong \Delta PQR$



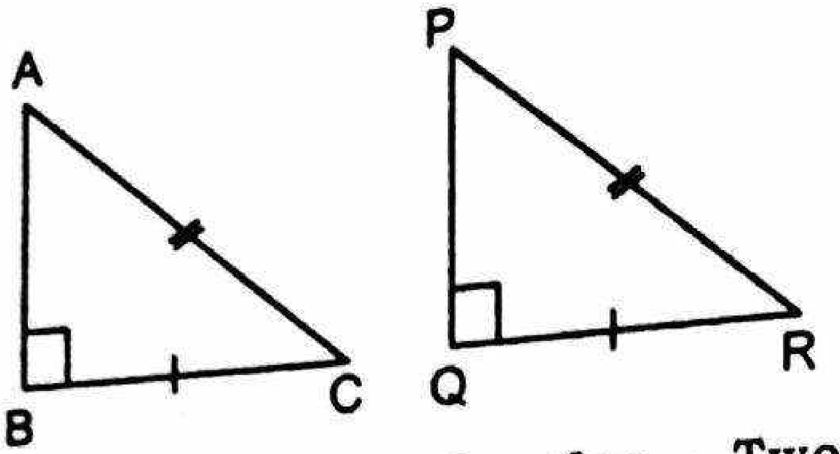
3. A-S-A (Angle-Side-Angle)
if $\angle ABC = \angle PQR$, BC = QR, $\angle ACB = \angle PRQ$ then- $\Delta ABC \cong \Delta PQR$



4. A-A-S (Angle-Angle-Side) if $\angle ABC = \angle PQR$, $\angle ACB = \angle PRQ$ and AC = PR (or AB = PQ) then - $\Delta ABC \cong \Delta PQR$



5. R-H-S (Right angle-Hypotenuse-Side)
if AC = PR, $\angle B = \angle Q$ and BC = QR
$\therefore \Delta ABC \cong \Delta PQR$

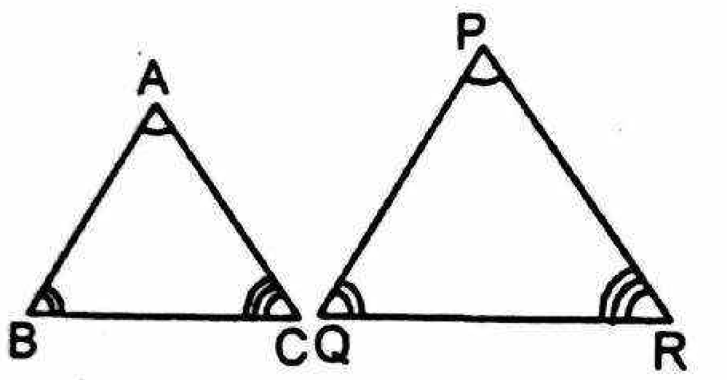


Similarity of triangles - Two triangles are said to be similar if the corresponding angles are congruent and their corresponding sides are in proportion. The symbol for similarity is '~'

If $\Delta ABC \sim \Delta PQR$ then

$\angle ABC \cong \angle PQR, \angle BCA \cong QRP,$

$\angle BAC \cong \angle QPR$



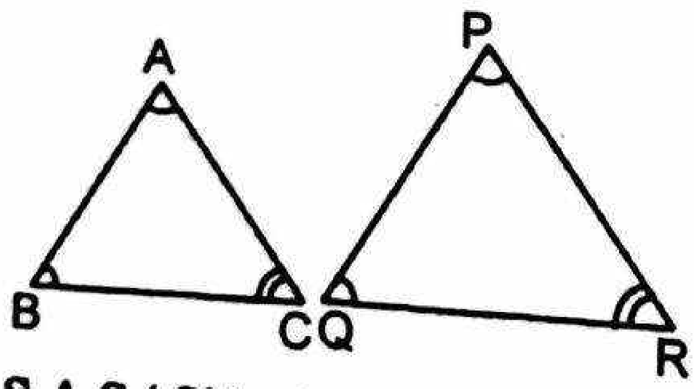
Tests for Similarity -

1. A-A (Angle-Angle)

   if $\angle ABC \cong \angle PQR$

   and $\angle ACB \cong \angle PRQ$

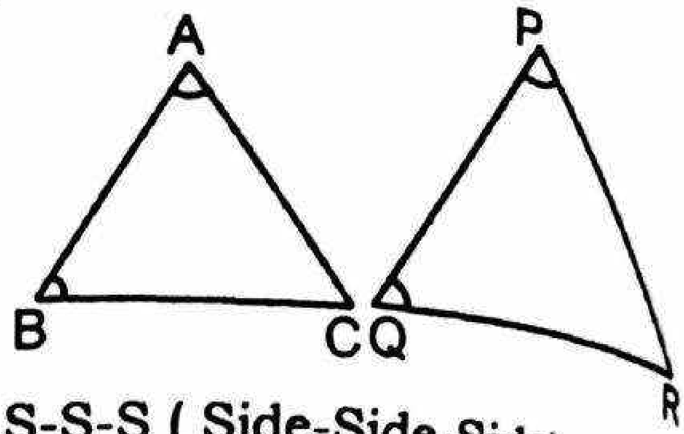
   then, $\Delta ABC \sim \Delta PQR$



2. S-A-S (Side-Angle-Side)

   if $\frac{AB}{PQ} = \frac{BC}{QR} and \angle ABC = \angle PQR$

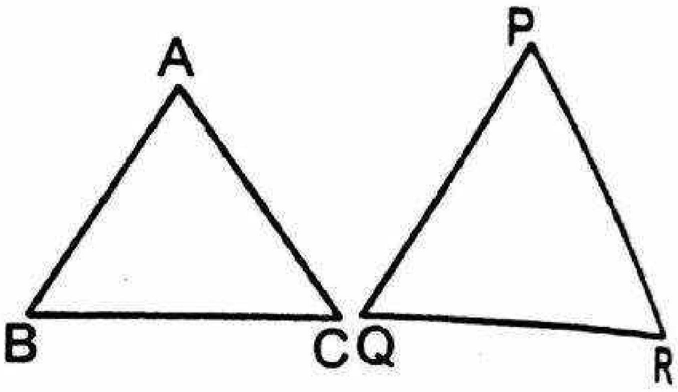
   then, $\Delta ABC \sim \Delta PQR.$



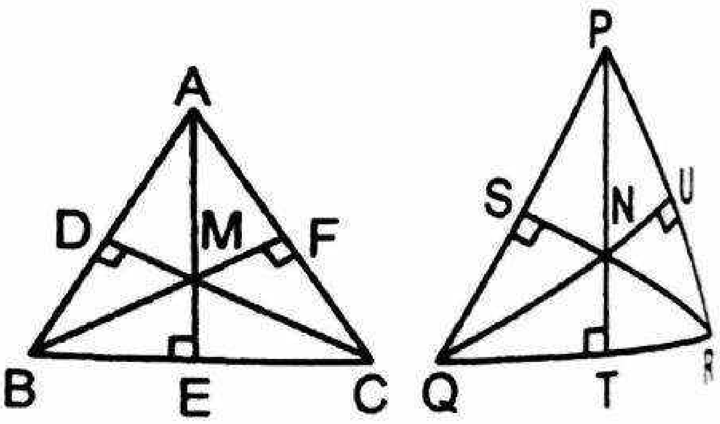
3. S-S-S (Side-Side-Side)

   if $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

   then, $\Delta ABC \sim \Delta PQR.$



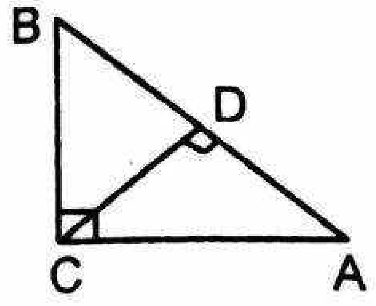
Properties of Similar triangles - If the two triangles are similar, then for the proportional / corresponding sides we have the following results.



1. Ratio of sides = Ratio of heights (altitudes)
   = Ratio of medians
   = ratio of angle bisectors
   = Ratio of inradii
   = Ratio of circumradii
2. Ratio of areas = Ratio of squares of corresponding sides. i.e. if $\Delta ABC \sim \Delta PQR$, then

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

3. In a right angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too.

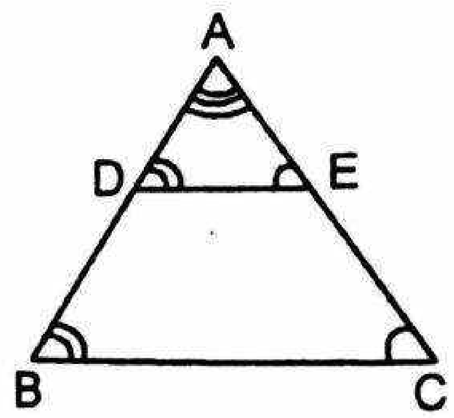


i.e., $\Delta BCA \sim \Delta BDC \sim \Delta CDA$.

4. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

i.e. $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ & $\dfrac{AD}{AB} = \dfrac{AE}{AC} = \dfrac{DE}{BC}$

and $\Delta ADE \sim \Delta ABC$

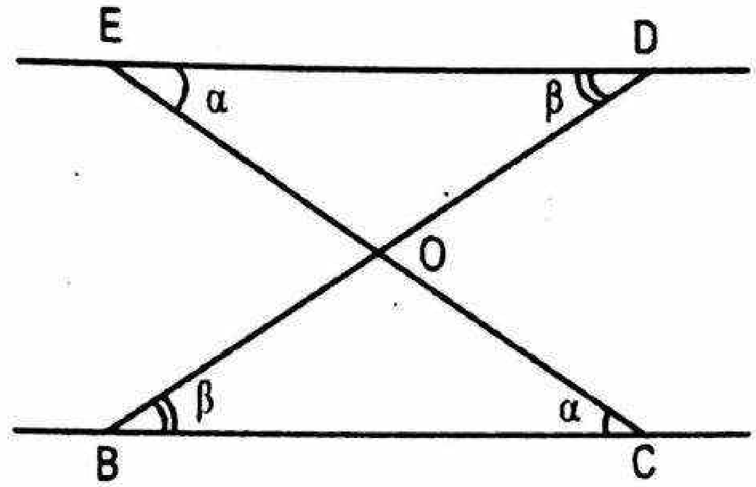


- If D and E are the mid-points of AB and AC & DE | | BC then -

$$DE = \frac{1}{2} BC$$

- Vice versa is also true i.e.
If a line divides any two sides in the same ratio $\left(i.e. \dfrac{AD}{DB} = \dfrac{AE}{EC} \right)$ then the line is parallel to third line
i.e. DE || BC

5. Two triangles b/w the two parallel lines will always be similar.



If ED | | BC then $\Delta EOD \sim \Delta COB$

6. If the area of two similar triangles are equal then the triangles are congruent.
If Area of ABC = Area of PQR

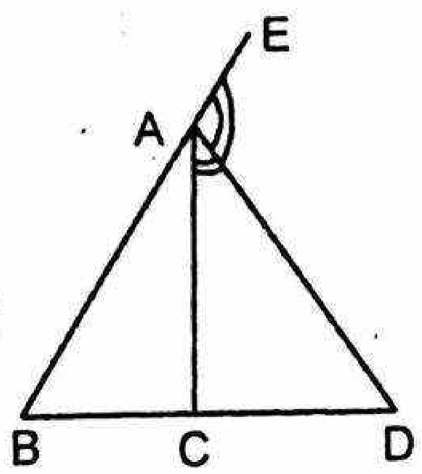
$\Delta ABC \sim \Delta PQR$

⇓

$\Delta ABC \cong \Delta PQR$

⇒ If in ΔABC, bisector of external angle A, intersects BC produced to D, then

$\dfrac{AB}{AC} = \dfrac{BD}{CD}$



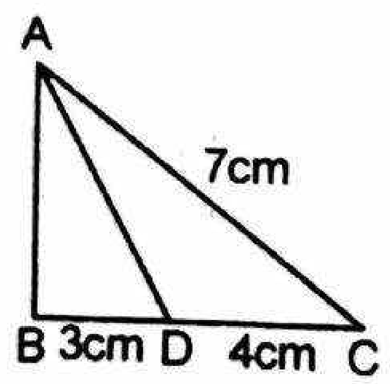
Exercise
LEVEL - 1

1. The areas of two similar triangles are in the ratio 9 : 16. Their corresponding sides will be in the ratio :
(a) 3 : 5 (b) 3 : 4
(b) 4 : 5 (d) 4 : 3

2. In Δ ABC and DEF, if $\angle A = 50°$, $\angle B = 70°$, $\angle C = 60°$, $\angle E = 70°$, $\angle F = 50°$, $\angle D = 60°$ then :
(a) $\Delta ABC \sim \Delta DEF$
(b) $\Delta ABC \sim \Delta EFD$
(c) $\Delta ABC \sim \Delta DFE$
(d) $\Delta ABC \sim \Delta FED$

3. The corresponding sides of two similar triangles are in the ratio 1 : 3. Their altitude will be in the ratio:
(a) 1 : 3 (b) 3 : 1
(c) 1 : 9 (d) 9 : 1

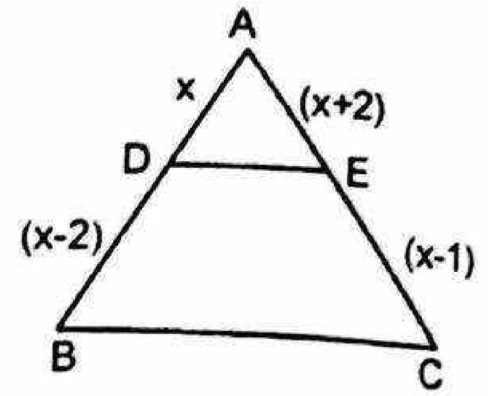
4. In the given figure, if AD is bisector of $\angle$BAC then AB is :
(a) 6cm (b) 5 cm
(c) 5.25cm (d) 5.75 cm



5. ABC is a triangle and P is any point on AB such that $\angle ACP = \angle ABC$, if AC = 9cm, CP = 12cm and BC = 15cm, then AP is equal to :
(a) 11.2 cm (b) 10.2 cm
(c) 8.0 cm (d) 7.2 cm

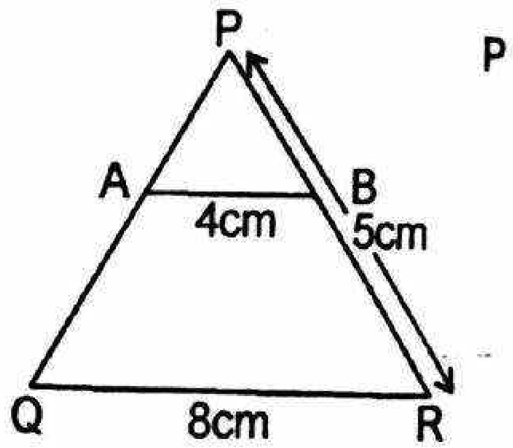
6. If the three side of one triangle are equal to the corresponding sides of the other triangle then the triangle are :
(a) congruent (b) similar
(c) congruent and similar.
(d) None of these

7. In the given figure, DE || BC, then the value of x is :



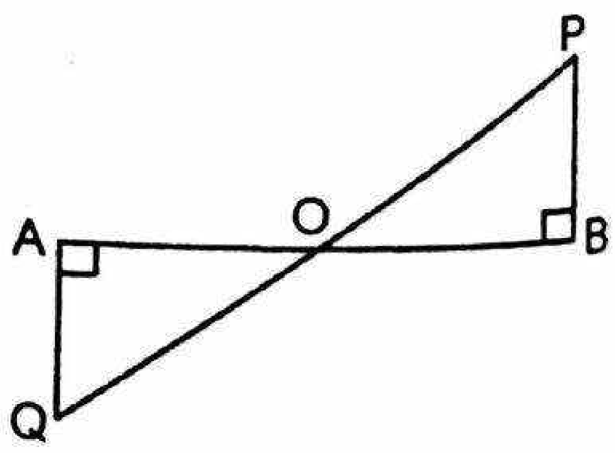
(a) 3 (b) 4.5
(c) 6 (d) 4

8. In the given figure, AB || QR. Find the length of PB :



(a) 2.5 cm (b) 2 cm
(c) 3 cm (d) 3.5 cm

9. In the given figure, QA and PB are perpendiculars to AB. If AO = 9cm, BO = 6cm and BP = 8cm. Find AQ :



(a) 11cm (b) 9 cm
(c) 14cm (d) 12 cm

10. Two triangles ABC and DEF are similar to each other in which AB = 10cm, DE = 8cm. Then the ratio of the areas of triangles ABC and DEF is :

(a) 4 : 5 (b) 25 : 16
(c) 64 : 125 (d) 4 : 7

11. In ΔABC, PQ || BC. If AP : PB = 1 : 2 and AQ = 3cm, AC is :
(a) 6 cm (b) 9 cm
(c) 12 cm (d) 8 cm

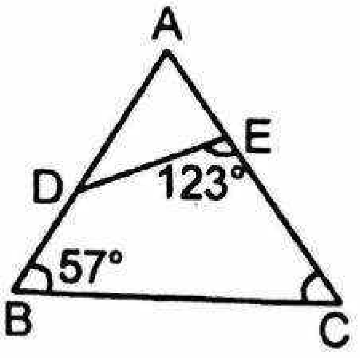
12. If ΔABC is similar to ΔDEF, such that $\angle$A = 47° and $\angle$E = 63°, then $\angle$C is equal to :
(a) 40° (b) 70°
(c) 65° (d) 37°

13. If G be the centroid of ΔABC and the area of ΔGBD is 6 sq.cm, where D is the mid-point of side BC, then the area of ΔABC is :
a. 18 sq.cm b. 12 sq.cm
c. 24 sq.cm d. 36 sq.cm

14. The perimeters of two similar

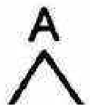
LEVEL - 2

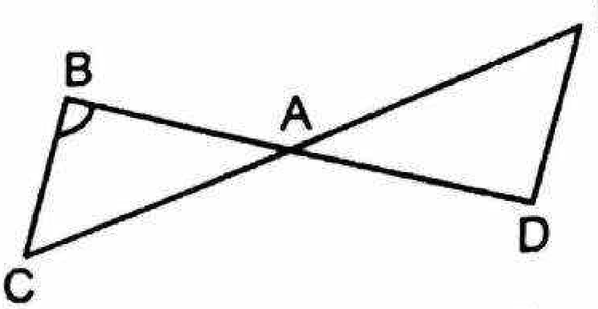
1. In the given figure, AD = 11cm, AB = 18 cm and AE = 9 cm. Find EC :



(a) 13 cm (b) 14cm
(b) 8 cm (d) 11cm

2. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if AE = 12. Find AB :

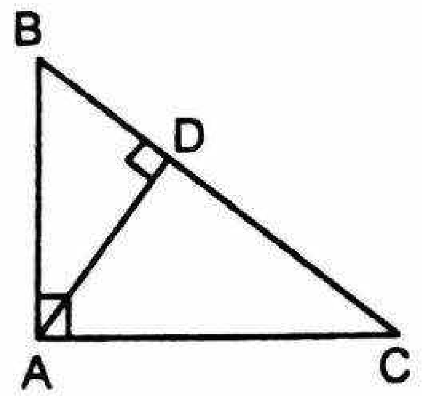




:

(a) 6 cm (b) 7 cm
(c) 5.5 cm (d) 7.5cm

5. Which of the following is true in the given figure, where AD is the altitude to the hypotenuse of a right angle triangled Δ ABC ?



(i) $\Delta ABD \sim \Delta CAD$
(ii) $\Delta ABD \cong \Delta CDA$
(iii) $\Delta ADB \sim \Delta CAB$

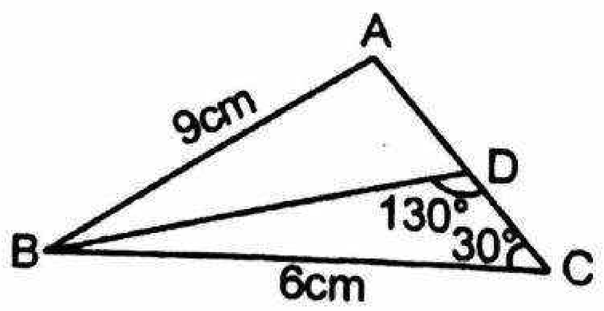
Of these statements , the correct ones are :

(a) (i) and (ii) (b) (ii) and (iii)
(c) (ii) and (iii) (d) (i),(ii) and (iii)

6. In Δ ABC, D is a point on BC such that $\frac{AB}{AC} = \frac{BD}{DC}$ .If $\angle B = 70°, \angle C = 50°$, then the value of $\angle$ BAD :

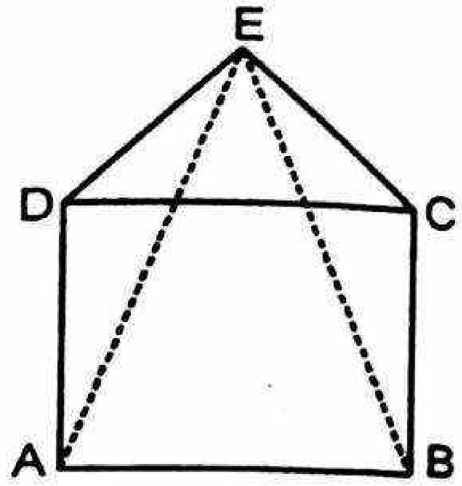
(a) 30° (b) 60°
(c) 40° (d) 50°

7. In the given figure, AD : DC = 3 : 2, then $\angle$ ABC :



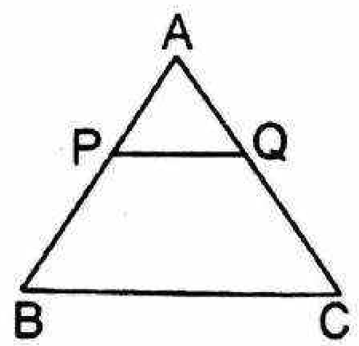
(a) 30° (b) 40°
(c) 45° (d) 50°

8. In the given figure, ABCD is a square and DCE is an equilateral triangle, then $\angle$ DAE will be :



(a) 45° (b) 30°
(c) 15° (d) $22\frac{1°}{2}$

9. In the given triangle ABC, BP = 3 AP, QC = 3AQ and BC = 36cm. Find the value of PQ ?



(a) 9cm (b) 8cm
(c) 6cm (d) 7cm

10. In a triangle ABC, $\angle BAC = 90°$ and AD is perpendicular to BC. If AD = 6cm and BD = 4cm, then the length of BC is :

(a) 8cm (b) 10cm
(c) 9cm (d) 13cm

11. The points D and E are taken on the sides AB and AC of Δ ABC such that $AD = \frac{1}{3}AB$, $AE = \frac{1}{3}AC$. If the length of BC is 15cm, then the length the length of DE is :

(a) 10cm (b) 8cm
(c) 6cm (d) 5cm

12. If G is centroid and AD, BE, CF are three medians of ΔABC with area $72cm^2$, then the area of ΔBDG is :
(a) $12\ cm^2$ (b) $16\ cm^2$
(c) $24\ cm^2$ (d) $8\ cm^2$

13. The three medians AD, BE and CF of ΔABC intersect at G. If the area of ΔABC is 60sq.cm then the area of the quadrilateral BDGF is :
(a) 10 sq.cm (b) 15 sq.cm
(c) 20 sq.cm (d) 30 sq.cm

14. D is any point on side AC of ΔABC. If P,Q,X,Y are the mid-points of AB, BC, AD and DC respectively, then the ratio of PX and QY is :
(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 2 : 3

15. ABC is an equilateral triangle. P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \| \overline{BC}$. If $\overline{PQ}$ = 5cm the area of ΔAPQ is :
(a) $\frac{25}{4}$ sq.cm (b) $\frac{25}{\sqrt{3}}$ sq.cm
(c) $\frac{25\sqrt{3}}{4}$ sq.cm (d) $25\sqrt{3}$ sq.cm

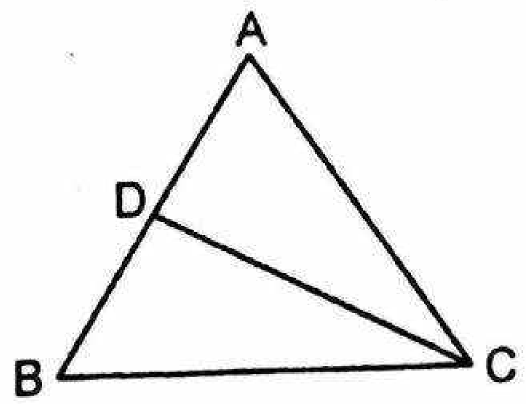
16. In a right angled ΔABC, $\angle ABC = 90°$; $BN \perp AC$, AB = 6cm, AC = 10cm. Then AN : NC is :
(a) 3 : 4 (b) 3 : 16
(c) 1 : 4 (d) 9 : 16

17. A straight line parallel to base BC of the triangle ABC intersects AB and AC at the points D and E respectively. If the area of the ΔACD is 36sq cm, then the area of ΔABE is :
(a) 36 sq.cm (b) 18 sq.cm
(c) 12 sq.cm
(d) None of these

18. In ΔABC, P and Q are the middle points of the sides AB and AC respectively. R is a point on the segment PQ such that PR : RQ = 1 : 2, If PR = 2cm, then BC =
(a) 4 cm (b) 2 cm
(c) 12 cm (d) 6 cm

19. ABC is a right-angled triangle. AD is perpendicular to the hypotenuse BC. If AC = 2AB, then the alue of BD is :
(a) $\frac{BC}{2}$ (b) $\frac{BC}{3}$
(c) $\frac{BC}{4}$ (d) $\frac{BC}{5}$

20. In the given figure, $\angle BAC = \angle BCD$, AB = 32cm and BD = 18cm, then the ratio of perimeter of ΔBCD and ΔABC is :



(a) 4 : 3 (b) 8 : 5
(c) 5 : 8 (d) 3 : 4

LEVEL - 3

1. Find the maximum area that can be enclosed in a triangle of perimeter 24cm :

(a) 32 cm^2 (b) $16\sqrt{3}$ cm^2

(c) $16\sqrt{2}$ cm^2 (d) 27cm^2

2. The lengths of perpendiculars drawn from any point in the interior of an equilateral triangle to the respective side of the triangle are P_1, P_2 and P_3, then the side of triangle is :

(a) $\frac{5}{\sqrt{3}}(P_1+P_2+P_3)$

(b) $\frac{1}{\sqrt{3}}(P_1+P_2+P_3)$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (d) $\frac{\sqrt{2}-1}{\sqrt{2}}$

5. D and E are the mid-points of AB and AC of ΔABC, BC is produced to any point P; DE, DP and EP are joined. then, area of :

(a) $\Delta PED = \frac{1}{4}\Delta ABC$

(b) $\Delta PED = \Delta BEC$

(c) $\Delta ADE = \Delta BEC$

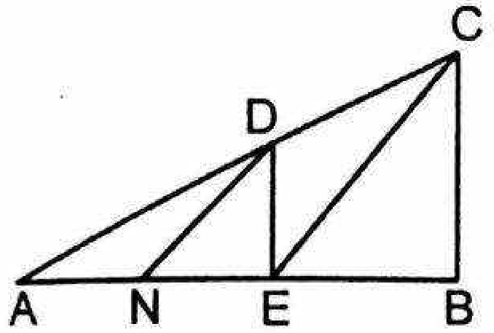
(d) $\Delta BDE = \Delta BEC$

6. ABC is a right angled triangle, right angled at C and P is the length of

(a) 12.5cm (b) 15cm
(c) 17.5cm (d) 20cm

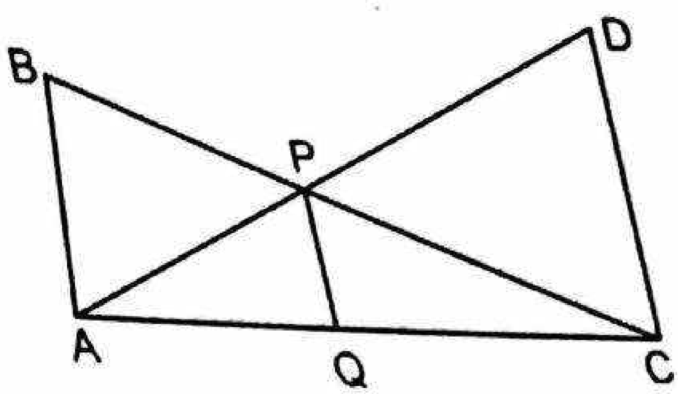
8. In a ΔABC, D is the mid-point of BC and E is the mid-point of AD. The line BE is extended and it intersects AC at T. If AB = 18cm, BC = 17cm and AC = 15cm. Find TC ?
(a) 8cm (b) 9cm
(d) 10cm (d) 7cm

9. In the given figure, DE || BC and EC || ND, AE : EB = 4 : 5, then EN : EB is :



a. 5 : 9 b. 9 : 4
c. 4 : 5 d. 4 : 9

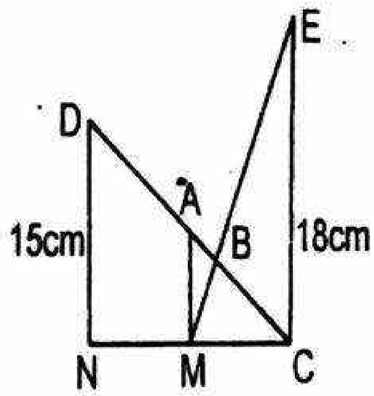
10. In the given figure, AB || CD || PQ, AB = 12cm, CD = 18cm and AC = 6cm. Then PQ is :



(a) $\frac{36}{5}cm$ (b) $\frac{18}{5}cm$

(c) 9cm (d) $\frac{14}{5}cm$

11. In the given figure, EC || AM || DN and AB = 5cm,B C = 10cm. Find DC:
(a) 19 cm (b) 20cm
(c) 25 cm (d) 17.5cm



Hints and Solutions:

LEVEL-1

1.(b) Required ratio = $\sqrt{\frac{9}{16}}=\frac{3}{4}=3:4$

2.(d) ∵ in △ABC,

$\angle A = 50°$, $\angle B = 70°$, $\angle C = 60°$

and in △DEF,

$\angle E = 70°$, $\angle F = 50°$ and $\angle D = 60°$

∴ in △FED,

$\angle F = 50°$, $\angle E = 70°$ and $\angle D = 60°$

∴ △ABC ~ △FED.

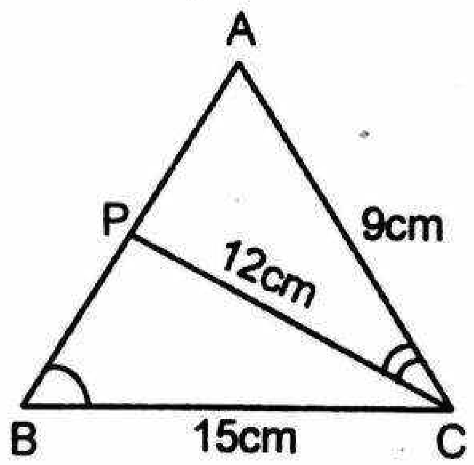
3.(a) Ratio of altitude = Ratio of corresponding sides

= 1 : 3

4.(c) $\frac{AB}{AC}=\frac{BD}{DC}\Rightarrow AB=\frac{3}{4}\times 7$

$=\frac{21}{4}=5.25cm$

5.(d)



In △APC and △ABC,

$\angle ACP = \angle ABC$ and $\angle A = \angle A$ (common)

∴ △APC ~ △ABC

∴ $\frac{AP}{AC}=\frac{PC}{BC}\Rightarrow\frac{AP}{9}=\frac{12}{15}$

⇒ AP = 7.2cm

6.(c) Here, triangles are congruent. But congruent triangles are always similar.

7.(d) $\frac{AD}{DB}=\frac{AE}{EC}$ (by basic proportionality theorem ∵ DE || Bc)

$\Rightarrow \frac{x}{x-2}=\frac{x+2}{x-1}$

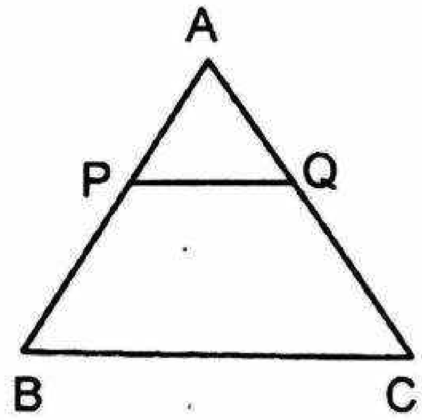
$\Rightarrow x^2-x=x^2-4 \Rightarrow x=4$

8.(a) $\frac{AB}{QR}=\frac{PB}{PR}\Rightarrow\frac{4}{8}=\frac{PB}{5}\Rightarrow PB=2.5cm$

9.(d) $\frac{AO}{BO}=\frac{AQ}{BP}\Rightarrow\frac{9}{6}=\frac{AQ}{8}\Rightarrow AQ=12cm$

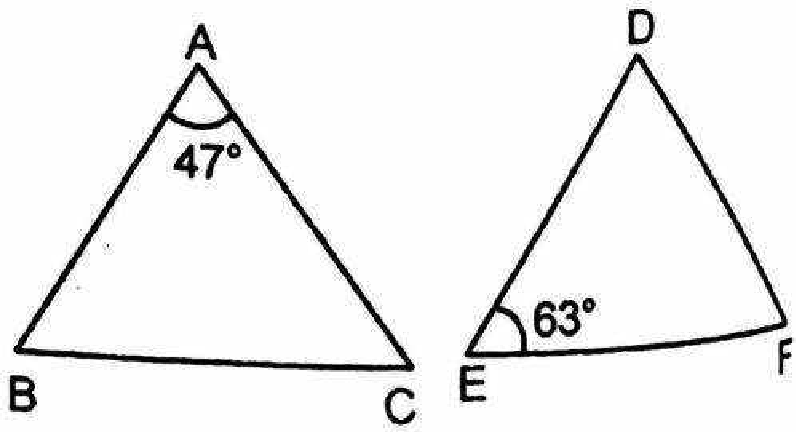
10.(b) $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}=\frac{AB^2}{DE^2}=\frac{100}{64}=\frac{25}{16}$

11.(b)



$\frac{AP}{BP}=\frac{AQ}{QC}=\frac{1}{2}\Rightarrow QC=2AQ=6cm$

12.(b)



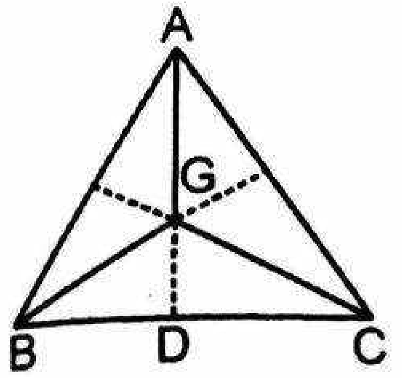
△ABC ~ △DEF

∴ $\angle A = 47° = \angle D$

$\angle B = \angle E = 63°$

∴ $\angle C = 180° - 47° - 63° = 70°$

13.(d)



Area of Δ ABC = $6 \times ar(\Delta BGD)$

= 6×6 = 36sq.cm

14.(c) $\frac{AB}{PQ} = \frac{perimeter of \Delta ABC}{perimeter of \Delta PQR} = \frac{36}{24}$

$\Rightarrow AB = \frac{36}{24} \times 10 = 15cm$

LEVEL-2

1.(a) Δ ADE ~ Δ ACB (A-A-A properly)

$\because$ $\angle A = \angle A$, $\angle AED = \angle ABC$ & $\angle ADE = \angle ACB$)

$\therefore$ $\frac{AE}{AB} = \frac{AD}{AC} \Rightarrow \frac{9}{18} = \frac{11}{AC}$

$\Rightarrow$ AC = 22cm

$\therefore$ EC = AC - AE

= 22 - 9

= 13cm

2.(d) in Δ ABC and Δ ADE,

$\angle BAC = \angle DAE$

= 180°- (75° + 65°)

= 40°

$\angle AED = 75° = \angle ABC$

$\therefore$ Δ AED ~ Δ ABC

$\therefore$ $\frac{DE}{BC} = \frac{AE}{AB} = \frac{AD}{AC} \Rightarrow \frac{2}{3} = \frac{12}{AB}$

$\Rightarrow$ AB = 18cm

3.(c) $\frac{AB}{AC} = \frac{BD}{CD} \Rightarrow \frac{12}{8} = \frac{BC + CD}{CD}$

$\Rightarrow \frac{3}{2} = \frac{4 + CD}{CD} \Rightarrow CD = 8cm$

4.(c) $\because$ BC || DE

$\therefore$ $\angle ABC = \angle ADE$

$\angle ACB = \angle AED$ and $\angle BAC = \angle EAD$

$\therefore$ Δ ABC ~ Δ AED

$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{18}{9} = \frac{11}{AE}$

$\Rightarrow$ AE = 5.5cm

5.(d) In Δ ABD and Δ CAD

$\angle ADB = \angle ADC$ = 90° each

$\angle BAD = \angle ACD$

= 90° - $\angle B$ and AD = AD (common)

$\therefore$ $\Delta ADB \sim \Delta CAD$ and

$\Delta ABD \cong \Delta CAD$

In ΔADB and ΔCAB

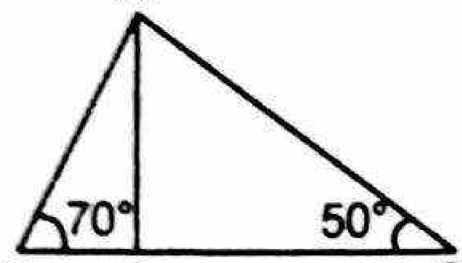
$\angle ADB = \angle BAC = 90°$ each

and $\angle ABC = \angle ABD$

$\therefore$ $\Delta ADB \sim \Delta CAB$

Here, (i),(ii) and (iii) are correct statements.

6.(a)



$\angle A + 70° + 50° = 180°$

$\Rightarrow \angle A = 60°$

$\because \dfrac{AB}{AC} = \dfrac{BD}{DC}$

$\therefore$ AD is the bisector of $\angle BAC$.

$\therefore \angle BAD = \dfrac{1}{2}\angle BAC = \dfrac{1}{2} \times 60° = 30°$

7.(b) $\dfrac{AD}{DC} = \dfrac{3}{2}$ and $\dfrac{AB}{BC} = \dfrac{3}{2}, \therefore \dfrac{AD}{DC} = \dfrac{AB}{AC}$

$\therefore$ BD is the bisector of $\angle B$

Now, $\angle CBD = 180° - (130° + 30°)$

$= 20°$

$\therefore \angle B = 2(\angle CBD) = 40°$

8.(c) $\angle ADE = (90° + 60°) = 150°$

$\because$ DE = DC = EC_(i)(equilateral Δ DEC)

and AD = DC = AB = BC ______(ii) (square)

$\therefore$ from (i) and (ii) AD = DE

$\therefore \angle DEA = \angle DAE = x°$(Let) (in Δ ADE)

$\therefore x + x + 150° = 180° \Rightarrow x = 15°$

9.(a) AB = AP + BP = AP + 3AP

$= 4AP \Rightarrow \dfrac{AB}{AP} = \dfrac{4}{1}$ &

AC = AQ + QC = AQ + 3AQ

$= 4AQ \Rightarrow \dfrac{AC}{AQ} = \dfrac{4}{1}$

in ΔABC and ΔAPQ

$\angle BAC = \angle PAQ$ (common) and

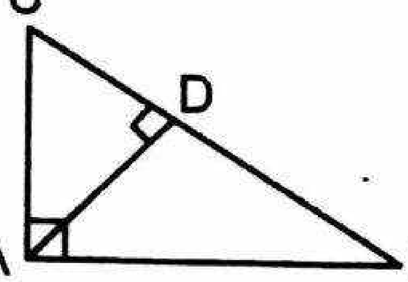
$\dfrac{AB}{AP} = \dfrac{AC}{AQ}$

$\therefore \Delta ABC \sim \Delta APQ$

$\therefore \dfrac{AB}{AP} = \dfrac{AC}{AQ} = \dfrac{BC}{PQ} = 4$

$\Rightarrow \dfrac{BC}{PQ} = \dfrac{4}{1} \Rightarrow \dfrac{36}{PQ} = 4 \Rightarrow PQ = 9cm$

10.(d)



AB =

$\sqrt{AD^2 + BD^2} = \sqrt{36+16} = \sqrt{52}cm$

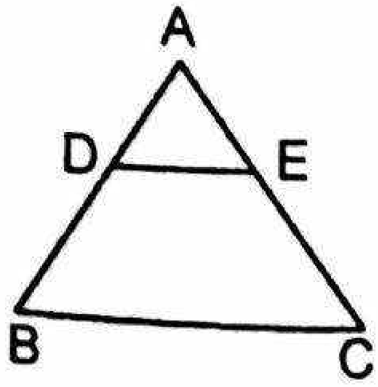
$\Delta ABC \sim \Delta ABD$

$\therefore \dfrac{AB}{BC} = \dfrac{BD}{AB}$

$\Rightarrow AB^2 = BC \times BD$

$\Rightarrow 52 = BC \times 4 \Rightarrow BC = 13cm$

11.(d)

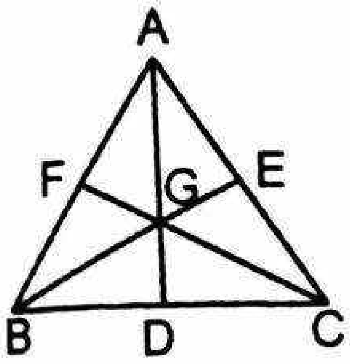


$$\frac{AD}{DB} = \frac{AE}{AC} = \frac{1}{3}$$

$\therefore \Delta ABC \sim \Delta ADE$

$$\therefore \frac{DE}{BC} = \frac{1}{3} \Rightarrow DE = \frac{15}{3} = 5cm$$

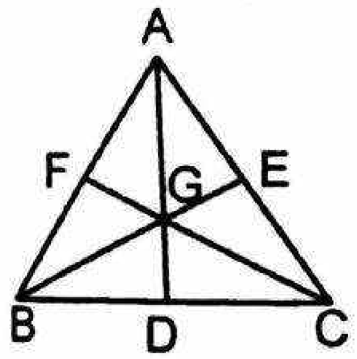
12.(a)



$$\text{Area of } \Delta BDG = \frac{1}{6} \times \text{ Area of } \Delta ABC$$

$$= \frac{1}{6} \times 72 = 12cm^2$$

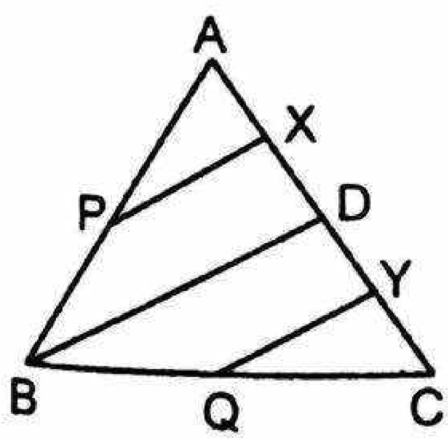
13.(c)



$$\text{Required area } = \frac{1}{3} \times 60$$

$= 20$ sq.cm.

14.(b) (By mid-point theorem)

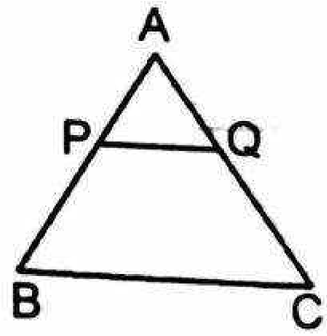


PX || BD and $PX = \frac{1}{2}BD$

QY || BD and $QY = \frac{1}{2}BD$

$\therefore$ PX : QY = 1 : 1

15.(c)



PQ || BC

$\therefore \angle APQ = \angle ABC = 60°$

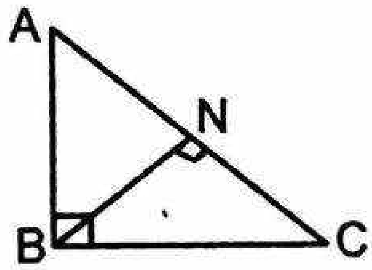
& $\angle AQP = \angle ACB = 60°$

$$\therefore \text{Area of } \Delta APQ = \frac{\sqrt{3}}{4} \times (PQ)^2$$

$$= \frac{\sqrt{3}}{4} \times 25$$

$$= \frac{25\sqrt{3}}{4} sq.cm$$

16.(d)



in ΔABC & ΔBNC,

$\angle ABC = \angle BNC = 90°$

and $\angle C = \angle C$ (common)

$\therefore \Delta ABC \sim \Delta BNC$

and $BC = \sqrt{10^2 - 6^2} = 8cm$

$$\therefore \frac{AC}{BC} = \frac{BC}{NC}$$

$$\Rightarrow \frac{10}{8} = \frac{8}{NC}$$

$\Rightarrow NC = 6.4$

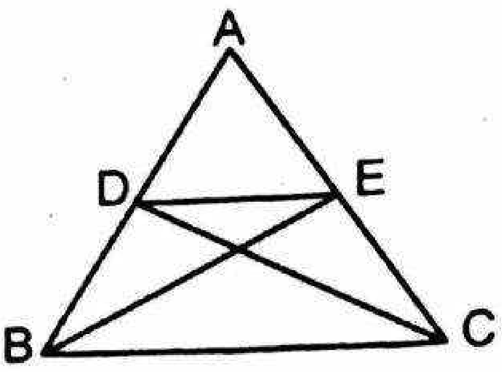
$\therefore$ AN = 10 - 6.4

= 3.6

$\therefore$ AN : NC = 3.6 : 6.4

= 9:16

17.(a)



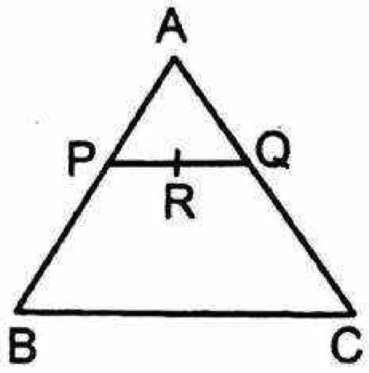
ΔDBC & ΔEBC lie on the same base BC and be tween same parallel lines .

$\therefore$ ar (ΔDBC) = ar (ΔEBC)

$\Rightarrow$ ar (ΔABC) - ar (ΔACD) - ar (ΔABC) - ar (ΔABE)

$\Rightarrow$ ar (ΔACD) = ar (ΔABE) = 36sq.cm

18.(c)



$\frac{PR}{RQ} = \frac{1}{2} \Rightarrow \frac{2}{RQ} = \frac{1}{2} \Rightarrow RQ = 4$

$\therefore$ PQ = PR + RQ

= 2 + 4 = 6cm

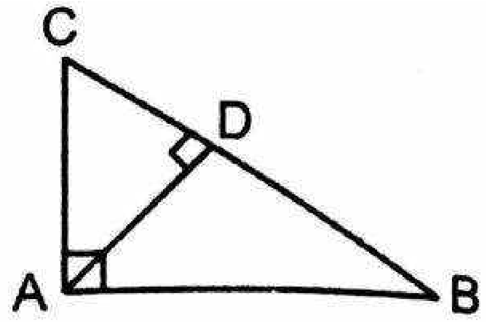
The line joining the mid-points of two sides of a triangle is parallel to and half of the third side.

$\therefore$ BC = 2PQ

= 2 × 6

= 12cm.

19.(b)



In ΔABD and ΔACD

$\angle ADC = \angle ADB = 90°$

$\angle CAD = \angle ABD = 90° - \angle ACB$

$\Rightarrow \Delta CAD \sim \Delta ABD$

$\therefore \frac{AC}{AB} = \frac{CD}{BD}$

$\Rightarrow \frac{AC + AB}{AB} = \frac{CD + BD}{BD} \quad \frac{3AB}{AB} = \frac{BC}{BD}$

$\Rightarrow BD = \frac{BC}{3}$

20.(d) In ΔABC and ΔBDC

$\angle BAC = \angle BCD$ (given)

and $\angle B = \angle B$ (common)

$\therefore \Delta ABC \sim \Delta CBD$

$\frac{AB}{BC} = \frac{BC}{BD} \Rightarrow \frac{32}{BC} = \frac{BC}{18}$

$\Rightarrow BC^2 = 18 \times 32$

$\Rightarrow$ BC = 24cm

$\therefore$ perimeter of ΔBCD : perimeter of ΔABC = BC : AB

= 24 : 32

= 3 : 4

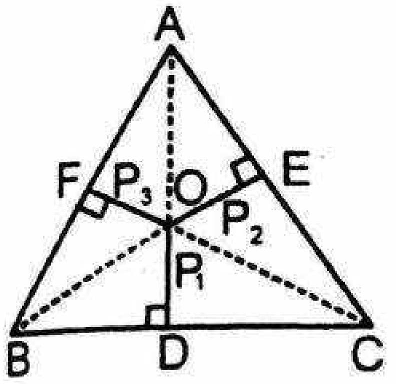
LEVEL-3

1.(b) For the given perimeter of a triangle the maximum area is enclosed by an equilateral triangle.

$\therefore$ $3a = 24$ cm $\Rightarrow a = 8$cm

$\therefore$ Area = $\frac{\sqrt{3}}{4}a^2 \; \frac{\sqrt{3}}{4} \times (8)^2 = 16\sqrt{3}cm^2$

2.(d)



Let the side of Δ ABC be a . O is the point in the interior of Δ ABC.

OD,OE,OF are perpendiculars

ar (Δ OAB) + ar (Δ OBC) + ar (Δ OAC) = ar (Δ ABC)

$$\Rightarrow \frac{1}{2}a \times P_3 + \frac{1}{2}a \times P_1 + \frac{1}{2}a \times P_2 = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow \frac{1}{2}a(P_1 + P_2 + P_3) = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$$

3.(c) $a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3) = \frac{2}{\sqrt{3}}(6 + 8 + 10)$

$\frac{48}{\sqrt{3}} = 16\sqrt{3}$cm

4.(d) XY || AC (given)

$\therefore$ $\angle BXY = \angle A$ and $\angle BYX = \angle C$

$\therefore$ $\Delta ABC \sim \Delta XBY$

$\therefore$ $\frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \left(\frac{AB}{XB}\right)^2$ ....(i)

Also, ar(Δ ABC) = 2 ar(Δ XYB) (given)

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = 2$ ....(ii)

therefore from (i) & (ii)

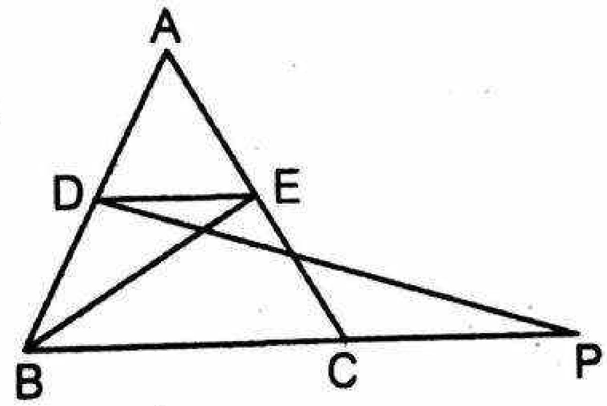
$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1} = \frac{AB}{XB} = \frac{\sqrt{2}}{1}$$

or $\frac{XB}{AB} = \frac{1}{\sqrt{2}}$ or $1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\Rightarrow \frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

5.(a) (By mid-point theorem)



DE || BC

DE = $\frac{1}{2}BC$

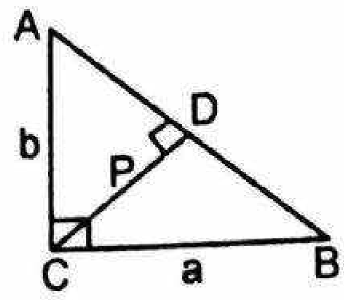
$\Rightarrow$ ar (Δ BDE) = $\frac{1}{4} \times ar(\Delta ABC)$

and Δ BDE = Δ PED

[∵ both triangles lie on the same base DE and be tween two parallel lines DE and BP.]

$\therefore$ ar (Δ PED) = $\frac{1}{4} \times ar(\Delta ABC)$

6.(b)



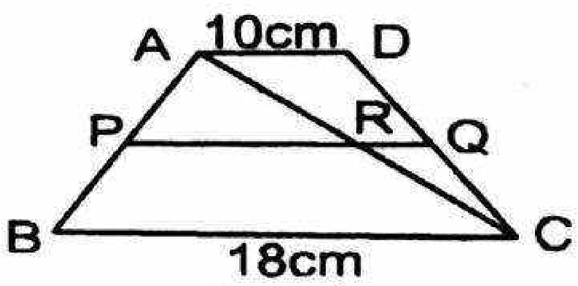
$\therefore$ $\Delta ABC \sim \Delta BDC$

$\therefore \frac{CD}{AC} = \frac{BC}{AB} \Rightarrow \frac{p}{b} = \frac{a}{c} \Rightarrow pc = ab$

$\Rightarrow P\sqrt{a^2+b^2} = ab \Rightarrow P2\ (a^2+b^2) = a^2b^2$

$\Rightarrow \frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} \Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

7.(b)



Now in Δ APR and Δ ABC

$\angle APR = \angle ABC$ (∵ PQ | | AD | | BC)

and ∵ ARP = $\angle ACB$ (∵ PQ | | BC)

$\therefore$ $\Delta APR \sim \Delta ABC$

$\therefore \frac{AP}{AB} = \frac{PR}{BC} \Rightarrow PR = \frac{AP}{AB} \times BC$

$= \frac{AP}{AP+PB} \times BC$

$\Rightarrow PR = \frac{5}{8} \times 18 = \frac{45}{4} cm$

and $\frac{AP}{PB} = \frac{AR}{RC} = \frac{5}{3}$

similarly, $\Delta RCQ \sim \Delta CAD$

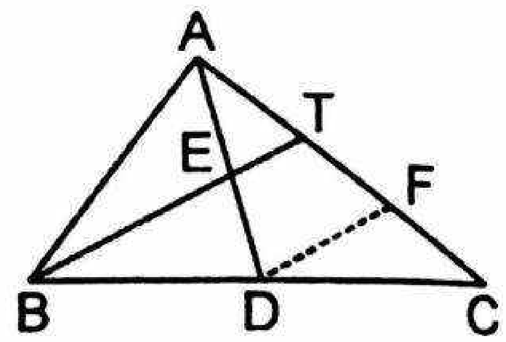
$\therefore \frac{RQ}{AD} = \frac{RC}{AC}$

$\Rightarrow RO = \frac{RC}{AR+RC} \times AD = \frac{3}{8} \times 10$

$= \frac{15}{4}$ cm

$\therefore$ PQ = PR + RO = $\frac{45}{4} + \frac{15}{4} = 15$cm

8.(c)



Draw DF | | ET

in Δ ADF, E id the mid-point of AD and DF | | ET

$\therefore$ T will be the mid-point of AF.

i.e. AT = TF __________(i)

Now, in Δ BTC,

D is the mid-point of BC & DF | | ET | | BT

$\therefore$ F will be the mid-point of TC

i.e TF = FC __________(ii)

$\therefore$ from (i) and (ii)

AT = TF = FC

$\therefore$ AC = 15cm, AT = TF = FC = $\frac{AC}{3}$

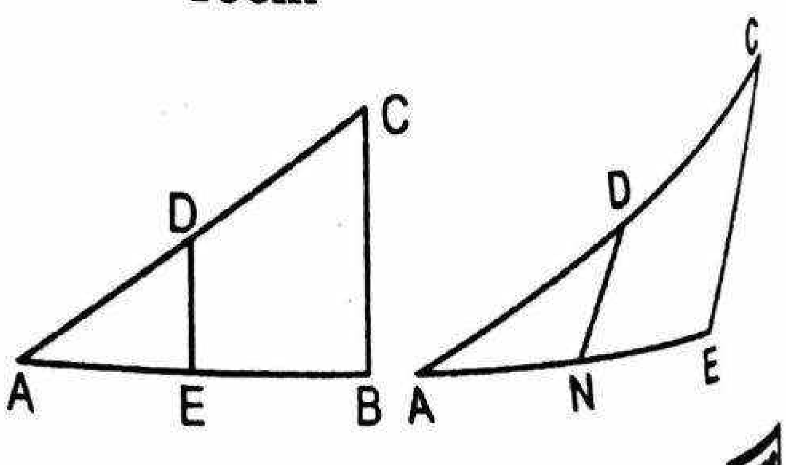
= 5cm

$\therefore$ TC = TF + FC

= 5 + 5

= 10cm

9.(d)



in ΔABC,

$\because DE \parallel BC$

$\therefore \frac{AD}{DC}=\frac{AE}{EB}=\frac{4}{5}=4:5$ ______(i)

in ΔAEC

$EC \parallel ND$

$\therefore \frac{AN}{NE}=\frac{AD}{DC}=\frac{4}{5}=4:5$ ______(ii)

Let AE = 40 $\therefore$ EB = 50 and

$\therefore EN = 40\times\frac{5}{9}=\frac{200}{9}$

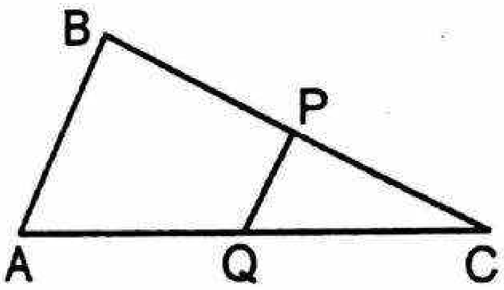
$EN : EB = \frac{200}{9}:50=4:9$

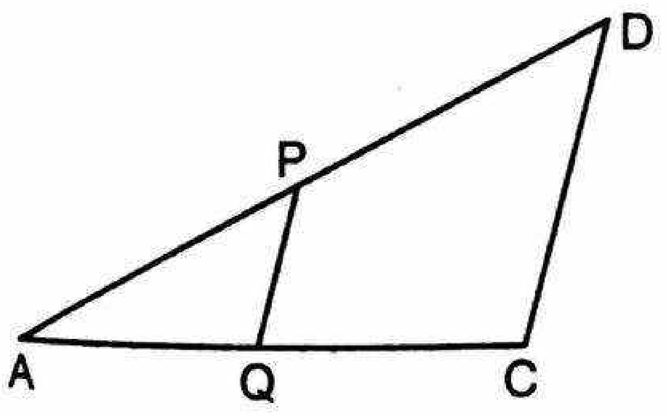
Note (short-cut) - AE : EB = 4 : 5

$\therefore$ EN : EB = 4 : (4 + 5)

= 4 : 9

10.(a)





$PQ \parallel AB$

$\therefore \Delta ABC \sim \Delta PQC$

$\frac{AB}{PQ}=\frac{AC}{QC}$

$\Rightarrow PQ=\frac{AB}{AC}\times QC$ ______(i)

$\because PQ \parallel CD$

$\therefore \Delta ACD \sim \Delta APQ$

$\therefore \frac{CD}{PQ}=\frac{AC}{AQ}$

$\therefore \Delta ACD \sim \Delta APQ$

$\Rightarrow \frac{CD}{PQ}=\frac{AC}{AQ}$

$\Rightarrow PQ = \frac{AQ}{AC}\times CD$ ______(ii)

from (i) & (ii) - $\frac{AQ}{AC}\times CD=\frac{AB}{AC}\times QC$

$\Rightarrow AQ \times CD = AB \times QC$

$\Rightarrow (AC - QC)\times 18 = 12 \times QC$

$\Rightarrow (6 - QC) \times 18 = 12 \times QC$

$\Rightarrow (6 - QC) \times 3 = 2QC$

$\Rightarrow 5QC = 18 \Rightarrow QC = \frac{18}{5}$

$\therefore$ from (i) $PQ=\frac{12}{6}\times\frac{18}{5}=\frac{36}{5}cm$

Alternatively :

Let $QC = x \Rightarrow AQ = 6 - x$

In ΔABC,

$\frac{PQ}{AB}=\frac{QC}{AC}$

$\frac{PQ}{12}=\frac{x}{6} \Rightarrow PQ = 2x$ ______(i)

In ΔACD,

$\frac{PQ}{CD}=\frac{AQ}{AC}$

$\frac{PQ}{18}=\frac{6-x}{6}$

$\Rightarrow$ PQ = 18 – 3x ______(ii)

From (i) and (ii), 2x = 18 – 3x

$$x = \frac{18}{5}$$

$\therefore$ From (i), $PQ = 2x = 2 \times \frac{18}{5} = \frac{36}{5} cm$

11.(c) in ΔABM and ΔBEC

$\angle BAM = \angle BCE$

$\angle BMA = \angle BEC$ ($\because$ AM || EC)

$\therefore$ $\Delta ABM \sim \Delta BEC$

$\therefore \frac{AB}{BC} = \frac{AM}{EC} \Rightarrow \frac{5}{10} = \frac{AM}{18} \Rightarrow AM = 9$ cm

$\because$ AM || DN

$\therefore$ $\Delta AMC \sim \Delta DNC$

$\therefore \frac{DN}{AM} = \frac{DC}{AC} \Rightarrow \frac{15}{9} \frac{DC}{15}$

$\Rightarrow DC = \frac{15 \times 15}{9} = 25cm$

Answer-Key

LEVEL - 1

1. (b)	2. (d)	3. (a)
4. (c)	5. (d)	6. (c)
7. (d)	8. (a)	9. (d)
10. (b)	11. (b)	12. (b)
13. (d)	14. (c)	

LEVEL - 2

1. (a)	2. (d)	3. (c)
4. (c)	5. (d)	6. (a)
7. (b)	8. (c)	9. (a)
10. (d)	11. (d)	12. (a)
13. (c)	14. (b)	15. (c)
16. (d)	17. (a)	18. (c)
19. (b)	20. (d)	

LEVEL - 3

1. (b)	2. (d)	3. (c)
4. (d)	5. (a)	6. (b)
7. (b)	8. (c)	9. (d)
10. (a)	11. (c)	