Chapter 4

Dynamics

CONTENTS

I. Newton

Dynamics is, in general, the science of motion. If we consider the motion of a particle, the subject is called dynamics of a particle.

The study of dynamics is essential for those interested in defence the motion of projectiles and rockets can be discussed with its help. Dynamics is indispensable for those interested in space flights for calculating the paths of satellites and of all other space vehicles which may be sent to the moon or to outer space. The study of dynamics has been useful to mathematics in another way – the discovery of calculus was, to a large extent, facilitated by the attempts of Fermat, Newton and others to understand continuous motion.

Dynamics

4.1 Introduction

Dynamics is that branch of mechanics which deals with the study of laws governing motions of material system under the action of given forces.

(1) **Displacement :** The displacement of a moving point is its change of position. To know the displacement of a moving point, we must know both the length and the direction of the line joining the two positions of the moving point. Hence the displacement of a point involves both magnitude and direction *i.e.,* it is a vector quantity.

(2) **Speed :** The speed of a moving point is the rate at which it describes its path. The speed of a moving particle or a body does not give us any idea of its direction of motion; so it is a scalar quantity. In M.K.S. or S.I. system, the unit of speed is *metre per second* (m/s).

Average speed : The average speed of moving particle in a time-interval is defined as the distance travelled by the particle divided by the time interval.

If a particle travels a distance s in time-interval t, then Average speed = $\frac{s}{t}$

Note : ❑ The average speed of a particle gives the overall "rapidity" with which the particle moves in the given interval of time.

Instantaneous speed : The instantaneous speed or simply speed of a moving particle is defined as the rate of change of distance along its path, straight or curved.

If s is the distance travelled by a particle along its path, straight or curved, in time t, then

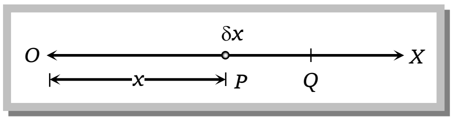
Instantaneous speed = $\frac{ds}{dt}$ or speed at time $t = \frac{ds}{dt}$

Note : ❑ The average speed is defined for a time interval and the instantaneous speed is defined at a particular instant.

4.2 Velocity and Acceleration

(1) **Velocity :** The velocity of a moving point is the rate of its displacement. It is a vector quantity.

Let a particle starting from the fixed point O and moving along the straight line OX describes the distance $OP = x$ in time t. If the particle describes a further distance $PQ = \delta x$ in time δt, then $\frac{\delta x}{\delta t}$ is called the average velocity of the particle in time δt.



And $\lim_{\delta t \to 0} \frac{\delta x}{\delta t} = \frac{dx}{dt} = v$ is called the instantaneous velocity (or simply velocity) of the particle at time t.

Note : ❑ Average velocity in time t is the mean of the initial and final velocity.

(2) **Acceleration :** The rate of change of velocity of a moving particle is called its acceleration. It is a vector quantity.

The acceleration of a moving point at time t at distance x is given by, $f = \frac{v_2 - v_1}{t} \Rightarrow f = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v.\frac{dv}{dx}$

In M.K.S. or S.I. system, the unit of acceleration is m/sec^2.

It should be noted that $a > 0$ if v increases with time and $a < 0$ if v decreases with time. A negative acceleration is called retardation.

Clearly, retardation means decrease in the velocity.

Example: 1 The average speed of a bicycle over a journey of 20 *km*; if it travels the first 10 *km*. at 15 *km/hr* and the second 10 *km*. at 10 *km/hr*, is

(a) 12 *km/hr* (b) 10 *km/hr* (c) 15 *km/hr* (d) None of these

Solution: (a) Time taken by the bicycle to travel the first 10 *km* = $\frac{10}{15}$ *hour* = $\frac{2}{3}$ *hour*

Time taken by the bicycle to travel the second 10 *km* = $\frac{10}{10}$ *hour* = 1 *hour*

Total distance travelled = 10 + 10 = 20 *km*

Total time taken = $\frac{2}{3} + 1 = \frac{5}{3}$ *hour*.

$\therefore$ Average speed of bicycle = $\frac{\text{Distance}}{\text{Time}} = \frac{20}{5/3} = \frac{60}{5} = 12$ *km/hr*.

Example: 2 If a particle moves in a straight line according to the formula, $x = t^3 - 6t^2 - 15t$, then the time interval during which the velocity is negative and acceleration is positive, is

(a) [0, 5] (b) $(2, \infty)$ (c) (2, 5) (d) None of these

Solution: (c) We have $x = t^3 - 6t^2 - 15t$

$\therefore\ v = \frac{dx}{dt} = 3t^2 - 12t - 15$ and $a = \frac{d^2x}{dt^2} = 6t - 12$

Now $v < 0$ and $a > 0 \Rightarrow 3(t^2 - 4t - 5) < 0$ and $6(t-2) > 0 \Rightarrow -1 < t < 5$ and $t > 2 \Rightarrow t \in (2, 5)$.

Example: 3 A particle moves in a fixed straight path so that $S = \sqrt{1+t}$. If v is the velocity at any time t, then its acceleration is

(a) $-2v^3$ (b) $-v^3$ (c) $-v^2$ (d) $2v^3$

Solution: (a) We have $s = \sqrt{1+t}$

$$\Rightarrow \frac{ds}{dt} = \frac{1}{2\sqrt{1+t}} \Rightarrow \frac{d^2s}{dt^2} = \frac{-1}{4(1+t)^{3/2}} = \frac{-1}{4}\left(\frac{1}{\sqrt{1+t}}\right)^3 \Rightarrow \frac{d^2s}{dt^2} = \frac{-1}{4}\left(2\frac{ds}{dt}\right)^3 \Rightarrow \frac{d^2s}{dt^2} = -2v^3.$$

Example: 4 Two particles are moving with uniform velocities u and v respectively along X and Y axes, each directed towards the origin. If the particles are at distances a and b from the origin, the time at which they will be nearest to each other will be equal to

(a) $\frac{au}{u^2 + v^2}$ (b) $\frac{bu}{u^2 + v^2}$ (c) $\frac{au + bv}{u^2 + v^2}$ (d) $\frac{au}{bv}$

Solution: (c) Let the two particles be at A and B respectively at $t = 0$ and at time t their positions be P and Q respectively.

Then, $AP = ut$ and $BQ = vt$

$\therefore \; OP = a - ut$ and $OQ = b - vt$

Now, $PQ^2 = OP^2 + OQ^2 \Rightarrow \quad PQ^2 = (a - ut)^2 + (b - vt)^2$

$\Rightarrow \dfrac{d}{dt}(PQ^2) = -2u(a - ut) - 2v(b - vt)$ and $\dfrac{d^2}{dt^2}(PQ^2) = 2(u^2 + v^2)$

For maximum or minimum value of PQ^2, we must have

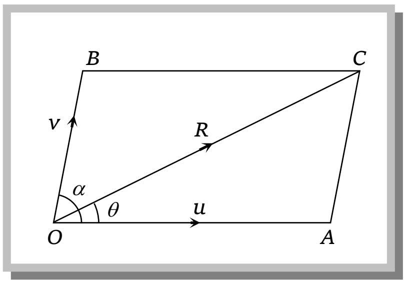
$\dfrac{d}{dt}(PQ^2) = 0 \Rightarrow -2u(a - ut) - 2v(b - vt) = 0 \Rightarrow t = \dfrac{au + bv}{u^2 + v^2}$

Clearly, $\dfrac{d^2}{dt^2}(PQ^2) > 0$ for all t. Hence, PQ is least at $t = \dfrac{au + bv}{u^2 + v^2}$.

4.3 Resultant and Components of Velocities

(1) **Resultant of velocities :** The resultant of two velocities possessed by a particle is given by the law of parallelogram of velocities.

Parallelogram law of velocities : If a moving particle has two simultaneous velocities represented in magnitude and direction by the two sides of a parallelogram drawn from an angular point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.



Let a particle at *O* has simultaneously two velocities *u* and *v* represented in magnitude and direction by *OA* and *OB* respectively of the parallelogram *OACB*. The diagonal *OC* represents the resultant velocity.

Let *R* be the resultant velocity.

If $\angle AOB = \alpha$ and $\angle AOC = \theta$, then the magnitude of their resultant velocity is $\boxed{R = \sqrt{u^2 + v^2 + 2uv\cos\alpha}}$ and, the direction of this resultant velocity makes an angle θ with the direction of u such that $\boxed{\tan\theta = \dfrac{v\sin\alpha}{u + v\cos\alpha}}$

Case I : When $\alpha = 0$, then $R_{max} = u + v$

Case II : When $\alpha = \pi$, then $R_{min} = |u - v|$

Case III : When $\alpha = \pi/2$, *i.e.*, u and v are at right angle to each other, then

$R = \sqrt{u^2 + v^2}$ and $\theta = \tan^{-1}\left(\dfrac{v}{u}\right)$.

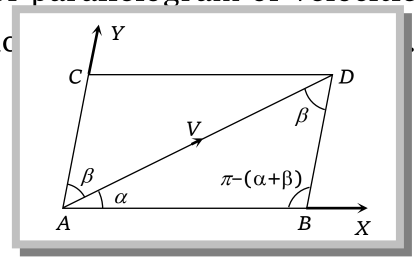
Case IV : When $u = v$, then $R = 2u\cos\dfrac{\alpha}{2}$ and $\theta = \dfrac{\alpha}{2}$.

Note : ❑ The angle made by the direction of the resultant velocity with the direction of v is given by $\tan^{-1}\left(\dfrac{u\sin\alpha}{v + u\cos\alpha}\right)$.

❑ If the direction of resultant velocity R makes an angle θ with the direction of $\vec{u}$, then

$$\sin\theta = \frac{v\sin\alpha}{R} \text{ and } \cos\theta = \frac{u + v\cos\alpha}{R}$$

(2) **Components of a velocity in two given directions :** If components of a velocity V in two given directions making angles α and β with it, then by the law of parallelogram of velocities the required components of V are represented by AB and $AC = BD$ alc .



$$\frac{AB}{\sin ADB} = \frac{BD}{\sin BAD} = \frac{AD}{\sin ABD} \quad i.e., \quad \frac{AB}{\sin\beta} = \frac{BD}{\sin\alpha} = \frac{AD}{\sin(\alpha+\beta)}$$

$$\therefore \quad AB = AD.\frac{\sin\beta}{\sin(\alpha+\beta)} \text{ and } BD = AD.\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

Hence, the components of velocity V in the directions making angle α and β are $\frac{V\sin\beta}{\sin(\alpha+\beta)}$ and $\frac{V\sin\alpha}{\sin(\alpha+\beta)}$ respectively.

Important Tips

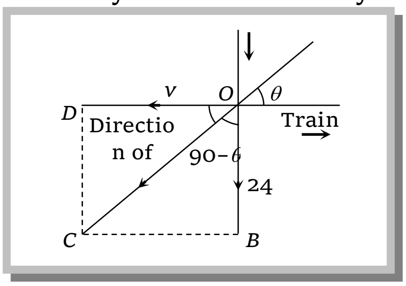
☞ *If components of velocity V are mutually perpendicular, then* $\beta = 90^o - \alpha$. *Thus components of V are* $V\cos\alpha$ *along AX and* $V\sin\alpha$ *along AY.*

Example: 5 Drops of water falling from the roof of the tunnel seems from the window of the train to be falling from an angle making $\tan^{-1}\frac{1}{2}$ from the horizontal. It is known that their velocities are 24 *decimeter/sec*. Assuming the resistance of air negligible, what will be the velocity of the train

(a) 42 *decimeter/sec.* (b) 48 *decimeter/sec.* (c) 45 *decimeter/sec.* (d) 44 *decimeter/sec.*

Solution: (b) Let the velocity of train is *v decimeter/sec.*

Let the real velocity of drops in magnitude and direction may be represented by *OB* and the velocity of train in opposite direction is represented by *OD*, then $OB = 24$, $OD = BC = v$



Completing parallelogram *OBCD*, *OC* will represent the relative velocity of drops.

If the drops are seen in a direction making an angle θ with the horizontal, *i.e.*, $\angle COD = \theta$, then $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ or $\tan\theta = \frac{1}{2}$ or $\cot\theta = 2$

In ΔOBC, from sine formula, $\frac{OB}{\sin\theta} = \frac{BC}{\sin(90-\theta)}$ or $\frac{24}{\sin\theta} = \frac{v}{\cos\theta}$ or

$v = 24\cot\theta = 24 \times 2 = 48$ *decimeter/second.*

Example: 6 If u and v be the components of the resultant velocity w of a particle such that $u = v = w$, then the angle between the velocities is

(a) 60^o (b) 150^o (c) 120^o (d) 30^o

Solution: (a) Let α be the angle between the components of the resultant velocity *w*.

Then $w^2 = u^2 + v^2 + 2uv\cos\alpha \Rightarrow u^2 = 2u^2(1+\cos\alpha) \Rightarrow 4\cos^2\frac{\alpha}{2} = 1 \Rightarrow \cos\frac{\alpha}{2} = \frac{1}{2} \Rightarrow \alpha = 60^o$.

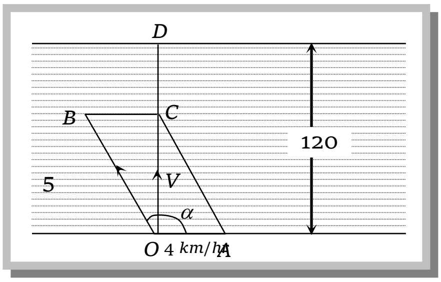
Example: 7 A man swims at a speed of 5 *km/hr*. He wants to cross a canal 120 *metres* wide, in a direction perpendicular to the direction of flow. If the canal flows at 4 *km/hr*, the direction and the time taken by the man to cross the canal are

(a) $\tan^{-1}\left(\frac{3}{4}\right)$, 2.4 min. (b) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$, 144 sec. (c) $\tan^{-1}\left(\frac{1}{2}\right)$, 100 sec. (d) None of these

Solution: (b) Suppose the man swims in a direction making an angle α with the direction of current.

Since the man wants to cross the canal in a direction perpendicular to the direction of flow.

Therefore, $\tan\frac{\pi}{2} = \frac{5\sin\alpha}{4+5\cos\alpha} \Rightarrow 4 + 5\cos\alpha = 0 \Rightarrow \cos\alpha = \frac{-4}{5}$



$\Rightarrow \alpha = \cos^{-1}\left(\frac{-4}{5}\right) \Rightarrow \alpha = \pi - \cos^{-1}\frac{4}{5} \Rightarrow \alpha = \pi - \tan^{-1}\frac{3}{4}$

Let *V* be the resultant velocity, then

$V^2 = 4^2 + 5^2 + 2\times4\times5\cos\alpha$

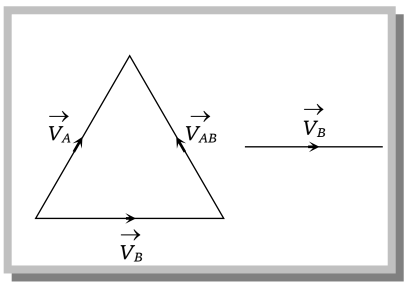
$V^2 = 16 + 25 + 40\times\frac{-4}{5} = 9 \Rightarrow V = 3km/hr.$

Time taken by the man to cross the canal = $\frac{120}{3\times1000}hr = \frac{1}{25}hr = \frac{60}{25}\text{min.} = 2.4\text{ min.} = 144\text{ sec.}$

4.4 Relative Velocity

Let *A* and *B* are two bodies in motion. The velocity of *A* relative to *B* is the velocity with which *A* appears to move as viewed by *B i.e.,* velocity of *A* relative to *B* = Resultant of velocity of *A* and reversed velocity of *B*.

Apparent velocity of *A* as seen from *B* means velocity of *A* relative to *B*.



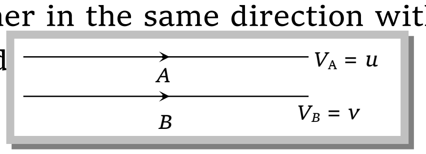
The relative velocity of *A* with respect to *B* is also called the velocity of *A* relative to *B* and is denoted by $\vec{V}_{AB}$ and $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$.

If two bodies *A* and *B* are moving with velocities of magnitudes V_A and V_B respectively inclined at an angle α with each other, then

$$\vec{V}_{AB} = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\alpha}$$

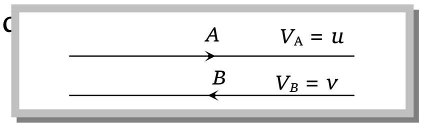
If the direction of the relative velocity of *A* with respect to *B* makes an angle θ with the direction of velocity of *B*, then $\tan\theta = \frac{V_A\sin\alpha}{V_B - V_A\cos\alpha}$

Case I : When the particles *A* and *B* move parallel to each other in the same direction with velocities *u* and *v* respectively. Here $\alpha = 0$, then $V_{AB} = (u-v)$ in the d



$V_{BA} = (v-u)$ in the direction of *B*

Case II : When the particles A and B move parallel to each other in opposite directions with velocities u and v respectively. Here $\alpha = \pi$, then $V_{AB} = (u+v)$ in the c



$V_{BA} = (v+u)$ in the direction of B.

Note : ❑ True velocity of A = Resultant of the relative velocity of A with respect to B and the true velocity of B

i.e., $\vec{V}_A = \vec{V}_{AB} + \vec{V}_B$ [Since $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$]

Example: 8 A train of length 200 m travelling at 30 *m/sec*. overtakes another of length 300 m travelling at 20 *m*/sec. The time taken by the first train to pass the second is

(a) 30 *sec.* (b) 50 *sec.* (c) 10 *sec.* (d) 40 *sec.*

Solution: (b) Distance covered by the first train to pass the second = (200 + 300)*m* = 500 *metres*.

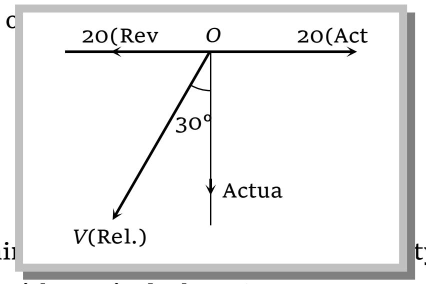
Since both the trains are travelling in the same direction. ∴ Velocity of first train relative to second = (30 – 20) = 10 *m/sec*.

∴ Time taken by the first train to pass the second = $\frac{500}{10} = 50$ *second*.

Example: 9 To a man running at a speed of 20 *km/hr*, the rain drops appear to be falling at an angle of 30^o from the vertical. If the rain drops are actually falling vertically downwards, their velocity in *km/hr*. is

(a) $10\sqrt{3}$ (b) 10 (c) $20\sqrt{3}$ (d) 40

Solution: (c) Velocity of rain relative to man = Actual velocity of rain – Velocity c



Resolving horizontally and vertically,

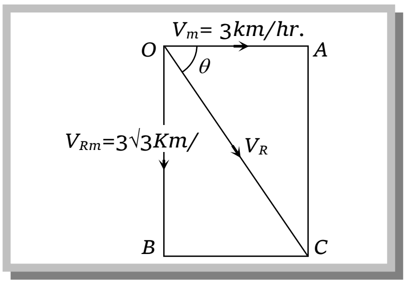
$V\cos 30^o = u$ and $V\sin 30^o$ = 20 *i.e.,* $\frac{V}{2} = 20$, $\therefore V = 40$

$\therefore 40\frac{\sqrt{3}}{2} = u \Rightarrow u = 20\sqrt{3}\, km/hr$.

Example: 10 A man is walking at the rate of 3 *km/hr* and the rain appears to hi ty of $3\sqrt{3}\, km/hr$. If the actual direction of the rain makes an angle θ with vertical, then θ =

(a) 15^o (b) 30^o (c) 45^o (d) 60^o

Solution: (d) Let $\vec{V}_M$ denotes the velocity of man. Then, $V_M = |\vec{V}_M| = 3km/hr$. It appears to the man that rain is falling vertically at the rate of $3\sqrt{3}km/hr$. This means that the relative velocity of rain with respect to man is $3\sqrt{3}km/hr$, in a direction perpendicular to the velocity of the man. Let $\vec{V}_{RM}$ denote the relative velocity of rain with respect to man and $\vec{V}_R$ be the velocity of rain.



Let $\overrightarrow{OA} = \vec{V}_R$, $\overrightarrow{OB} = \vec{V}_{RM}$. Complete the parallelogram *OACB*.

Then, the diagonal $\overrightarrow{OC}$ represents true velocity V_R of the rain.

$\therefore\ V_R^2 = V_M^2 + V_{RM}^2 + 2V_M.V_{RM}\cos 90^o$ [Using $w^2 = v^2 + u^2 + 2uv\cos\theta$]

$\Rightarrow V_R = \sqrt{3^2 + (3\sqrt{3})^2 + 2\times 3\times 3\sqrt{3}\times 0} = 6km/hr.$

Let θ be the angle between the direction of velocity of man and velocity of rain. Then,

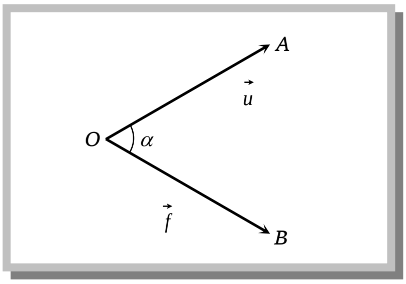
$\tan\theta = \frac{V_{RM}\sin 90^o}{V_M + V_{RM}\cos 90^o} \Rightarrow \tan\theta = \frac{3\sqrt{3}}{3} = \sqrt{3}$

Thus, the actual velocity of the rain is 6 *km/hr* in a direction making an angle of 60^o with the direction of motion of the man.

Example: 11 The particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle *w.r.t.* the first is least after a time **[AIEEE 2003]**

(a) $\frac{u \sin\alpha}{f}$ (b) $\frac{f \cos\alpha}{u}$ (c) $u \sin\alpha$ (d) $\frac{u \cos\alpha}{f}$

Solution: (d) After *t*, velocity = $f \times t$



$V_{BA} = \vec{f}t + (-\vec{u})$

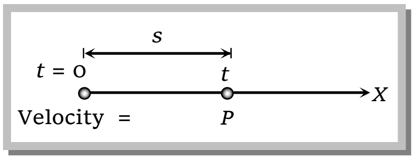
$V_{BA} = \sqrt{f^2t^2 + u^2 - 2fut\cos\alpha}$

For max. and min., $\frac{d}{dt}(V_{BA}^2) = 2f^2t - 2fu\cos\alpha = 0$

$t = \frac{u\cos\alpha}{f}$

4.5 Rectilinear motion with uniform Acceleration

A point or a particle moves in a straight line, starting with initial velocity *u*, and moving with constant acceleration *f* in its direction of motion. If *v* be its final velocity at the end of time *t* and *s* be its distance at the instant, from its starting point, then the equations describing the motion of the particle are,



(i) $v = u + ft$ (ii) $s = ut + \frac{1}{2}ft^2$ (iii) $v^2 = u^2 + 2fs$

(iv) If s_n is the distance travelled by the particle in n^{th} second, then $s_n = u + \frac{1}{2}f(2n-1)$

Important Tips

☞ *If a particle moves in a straight line with initial velocity u m/sec. and constant acceleration f m/sec², then distance travelled in t seconds is given by,* $s = ut + \frac{1}{2}ft^2 = \frac{1}{2}[2ut + ft^2] = \frac{1}{2}[u + (u + ft)]t = \left(\frac{u+v}{2}\right).t$, *where* $v = u + ft$

= *(Average velocity) × t* {*Average velocity* = $\frac{u+v}{2}$}

Example: 12 A body is moving in a straight line with uniform acceleration. It covers distances of 10 *m* and 12 *m* in third and fourth seconds respectively. Then the initial velocity in *m/sec.* is

(a) 2 (b) 3 (c) 4 (d) 5

Solution: (d) Let initial velocity is *u m/sec.* and acceleration is $f\,m/\sec^2$.

So, $u + \frac{1}{2}f(2\times3-1) = 10$ or $u + \frac{5}{2}f = 10$ .....(i) and $u + \frac{1}{2}f(2\times4-1) = 12$ or $u + \frac{7}{2}f = 12$ ......(ii)

Subtracting (i) from (ii), we get $0 + \frac{2}{2}f = 2$ or $f = 2m/\sec^2$

Substituting value of *f* in equation (i) , $u + \frac{5}{2}\times2 = 10$ or $u + 5 = 10$ or $u = 5m/\sec$.

Example: 13 Two trains *A* and *B*, 100 *kms* apart, are travelling to each other with starting speed of 50 *km/hr* for both. The train *A* is accelerating at 18 km/hr^2 and *B* is decelerating at 18 m/h^2. The distance where the engines cross each other from the initial position of *A* is

(a) 50 *kms* (b) 68 *kms* (c) 32 *kms* (d) 59 *kms*

Solution: (d) Let engine of train *A* travel *x km* and cross engine of train *B* in *t* hours.

$u = 50 km/hr$,

Acceleration $f = 18 km/hr^2$, so $s = 50t + \frac{1}{2} \times 18t^2$, $s = 50t + 9t^2$ .....(i)

So distance travelled by engine of train *B* will be $(100 - s)$ in *t* hour.

$u = 50 km/hr.$, Acceleration = –18 *km/hr*

$\therefore\ 100 - s = 50t - \frac{1}{2} \times 18t^2 \Rightarrow 100 - s = 50t - 9t^2$ .....(ii)

Adding (i) and (ii), we get $100 = 100t, \therefore t = 1$ *hour*. From (i), $s = 50 \times 1 + 9 \times 1^2 = 50 + 9 = 59 kms$.

Example: 14 A particle is moving with a uniform acceleration. If during its motion, it moves *x*, *y* and *z* distance in p^{th}, q^{th} and r^{th} seconds respectively, then

(a) $(q-r)x + (r-p)y + (p-q)z = 1$ (b) $(q-r)x + (r-p)y + (p-q)z = -1$

(c) $(q-r)x + (r-p)y + (p-q)z = 0$ (d) $(q+r)x + (r+p)y + (p+q)z = 0$

Solution: (c) Let *u* be the initial velocity and *f* be the acceleration of the particle.

Then $s_{pth} = u + \frac{1}{2} f(2p-1) = x$, $s_{qth} = u + \frac{1}{2} f(2q-1) = y$, $s_{rth} = u + \frac{1}{2} f(2r-1) = z$

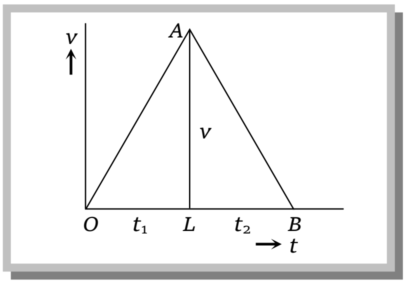
Now, multiplying (i) by $(q-r)$, (ii) by $(r-p)$, (iii) by $(p-q)$ and adding, we get $x(q-r) + y(r-p) + z(p-q) = 0$.

Example: 15 A body travels a distance *s* in *t* seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration *f* and in the second part with constant retardation *r*. The value of *t* is given by **[AIEEE 2003]**

(a) $2s\left(\frac{1}{f} + \frac{1}{r}\right)$ (b) $\frac{2s}{\frac{1}{f} + \frac{1}{r}}$ (c) $\sqrt{2s(f+r)}$ (d) $\sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$

Solution: (d) Portion *OA*, *OB* corresponds to motion with acceleration '*f*' and retardation '*r*' respectively.

Area of $\Delta OAB = s$ and $OB = t$. Let $OL = t_1, LB = t_2$ and $AL = v$



$s = \frac{1}{2} OB.AL = \frac{1}{2} t.v$ and $v = \frac{2s}{t}$

Also, $f = \frac{v}{t_1}, t_1 = \frac{v}{f} = \frac{2s}{tf}$ and $r = \frac{v}{t_2}, t_2 = \frac{v}{r} = \frac{2s}{tr}$

$t = t_1 + t_2 = \frac{2s}{tf} + \frac{2s}{tr}$; $t = \left(\frac{1}{f} + \frac{1}{r}\right)\frac{2s}{t} \Rightarrow t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$.

4.6 Motion Under Gravity

When a body is let fall in vacuum towards the earth, it will move vertically downward with an acceleration which is always the same at the same place on the earth but which varies slightly from place to place. This acceleration is called acceleration due to gravity. Its value in M.K.S. system is $9.8\, m/\sec^2$, in C.G.S. system $981\, cm/\sec^2$ and in F.P.S. system $32\, ft/\sec^2$. It is always denoted by *g*.

(1) **Downward motion:** If a body is projected vertically downward from a point at a height *h* above the earth's surface with velocity *u*, and after *t* second its velocity becomes *v*, the equation of its motion are $v = u + gt$,

$h = ut + \frac{1}{2}gt^2$, $v^2 = u^2 + 2gh$, $s_t = u + \frac{1}{2}g(2t-1)$. In particular, if the body starts from rest or is simply let fall or dropped, then, $v = gt$, $h = \frac{1}{2}gt^2$, $v^2 = 2gh$. $(\because u = 0)$

(2) **Upward motion :** When a body be projected vertically upward from a point on the earth's surface with an initial velocity u and if the direction of the upward motion is regarded as *+ve,* the direction of acceleration is *–ve* and it is, therefore, denoted by – *g*. The body thus moves with a retardation and its velocity gradually becomes lesser and lesser till it is zero. Thus, for upward motion, the equations of motion are,

$$v = u - gt,\ \ h = ut - \frac{1}{2}gt^2,\ \ v^2 = u^2 - 2gh,\ s_t = u - \frac{1}{2}g(2t-1).$$

(3) **Important deductions**

(i) **Greatest height attained :** Let H be the greatest height. From the result, $v^2 = u^2 - 2gh$.

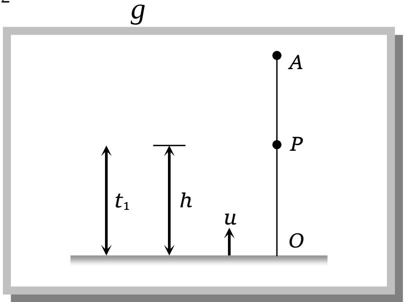
We have, $0 = u^2 - 2gH$ $\therefore\ H = \frac{u^2}{2g}$. Hence greatest height $(H) = \frac{u^2}{2g}$.

(ii) **Time to reach the greatest height :** Let T be the time taken by the particle to reach the greatest height.

From the result, $v = u - gt$

We have, $0 = u - gT$ *i.e.,* $T = \frac{u}{g}$. Therefore time to reach the greatest height $(T) = \frac{u}{g}$

(iii) **Time for a given height :** Let t be the time taken by the body to reach at a given height *h*. Then, $h = ut - \frac{1}{2}gt^2 \Rightarrow gt^2 - 2ut + 2h = 0$. Hence, $t_1 = \frac{u - \sqrt{u^2 - 2gh}}{g}$ and $t_2 = \frac{u + \sqrt{u^2 - 2gh}}{g}$



Clearly t_1 and t_2 are real; if $u^2 \geq 2gh$. If $u^2 = 2gh$, then $t_1 = t_2 = \frac{u}{g}$, which is the time taken to reach the highest point.

So, let $u^2 > 2gh$.

The lesser value of t_1 gives the time when the body is going up *i.e.,* it is time from *O* to *P* and the larger value t_2 gives the total time taken by the body to reach the highest point and then coming back to the given point, *i.e.,* it is the time from *O* to *A* and then *A* to *P*.

(iv) **Time of flight :** It is the total time taken by the particle to reach the greatest height and then return to the starting point again. When the particle returns to the starting point, $h = 0$. Therefore, from the result, $h = ut - \frac{1}{2}gt^2$.

We have, $0 = ut - \frac{1}{2}gt^2$ or $t = 0$ or $\frac{2u}{g}$

$t = 0$ corresponds to the instant with the body starts, and $t = 2\,u/g$ corresponds to the time when the particle after attaining the greatest height reaches the starting point. $\therefore$ Time of flight $= \frac{2u}{g}$.

Example: 16 A stone is dropped from a certain height which can reach the ground in 5 *sec*. If the stone is stopped after 3 *seconds* of its fall and then time taken by the stone to reach the ground for the remaining distance is **[MNR 1985; UPSEAT 2000]**

(a) 2 *seconds* (b) 3 *seconds* (c) 4 *seconds* (d) None of these

Solution: (c) Let distance travelled in 5 *sec*. is h_1 *metre*. $\therefore$ $h_1 = 0 + \frac{1}{2}g \times 5 \times 5$ or $h_1 = \frac{25}{2}g$

Let distance travelled in 3 *seconds* is h_2 *metre*. So, $h_2 = 0 + \frac{1}{2}g \times 3 \times 3$ or $h_2 = \frac{9}{2}g$

So remaining distance $h = h_1 - h_2$ or $h = \frac{25}{2}g - \frac{9}{2}g$ or $h = \frac{16}{2}g = 8g$.

Let time taken by the stone to reach the ground for the remaining distance $8g$ is t second.

$\therefore \quad 8g = 0 + \frac{1}{2}gt^2$ or $t^2 = 16, \therefore t = 4$ *sec*.

Example: 17 A man in a balloon, rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 *seconds* after the balloon is let go from the ground. The greatest height above the ground reached by the ball is

(a) 14.7 *m* (b) 19.6 *m* (c) 9.8 *m* (d) 24.5 *m*

Solution: (a) Velocity after 2 *seconds* = (4.9)2 = 9.8 *m*/*sec*.

Distance covered in 2 *seconds* = $\frac{1}{2}ft^2 = \frac{1}{2}(4.9)(2)^2 = 9.8m$

Again after the release of ball, velocity of the ball = 9.8 *m*/*sec*.

$v = 0$; Using $v^2 = u^2 - 2gh$, we get $0 = (9.8)^2 - 2(9.8)h \Rightarrow h = \frac{9.8}{2} = 4.9$ *metre*.

Hence greatest height attained = 9.8 + 4.9 = 14.7 *m*.

Example: 18 A particle is dropped under gravity from rest from a height $h(g = 9.8m/\sec^2)$ and then it travels a distance $\frac{9h}{25}$ in the last second. The height h is

(a) 100 *metre* (b) 122.5 *metre* (c) 145 *metre* (d) 167.5 *metre*

Solution: (b) $S_n = u + \frac{1}{2}f(2n-1) = \frac{1}{2}g(2n-1)$ $\quad (\because u = 0, f = g)$

$\frac{9h}{25} = \frac{1}{2}g(2n-1)$ and $h = \frac{1}{2}gn^2$

$\therefore \quad \frac{9}{25}.\frac{1}{2}gn^2 = \frac{1}{2}g(2n-1) \Rightarrow \frac{9}{25}n^2 = (2n-1) \Rightarrow 9n^2 = 50n - 25$

$\Rightarrow \quad 9n^2 - 50n + 25 = 0 \Rightarrow n = \frac{50 \pm \sqrt{2500 - 900}}{18} = \frac{50 \pm 40}{18}$; $n = 5$ or $\frac{5}{9}$

Since $\frac{5}{9} < 1$, $\therefore$ rejecting this value. We have $n = 5$; $\therefore$ $h = \frac{1}{2}(9.8) \times 25 = 122.5\,metre$.

4.7 Laws of Motion

(1) Newton's laws of motion

(i) **First law :** Every body continues in its state of rest or of uniform motion in a straight line except when it is compelled by external impressed forces to change that state.

(ii) **Second law :** The rate of change of momentum is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts.

$$\frac{d}{dt}(mv) \propto F \Rightarrow m\frac{dv}{dt} \propto F \quad \Rightarrow \quad m\frac{dv}{dt} = KF \Rightarrow F = mf \qquad \text{(for } K = 1)$$

Force = mass × acceleration. The S.I. unit of force is Newton and C.G.S. unit of force is dyne.

1 Newton = 1 $kg\text{-}m/sec^2$, 1 Dyne = 1 $gm\text{-}cm/sec^2$

(iii) **Third law :** To every action there is an equal and opposite reaction, or the actions and reaction are always equal and opposite.

Note : ❑ 1 Newton = 10^5 Dynes.

❑ The action and reaction do not act together on the same body or the same part of the body.

(2) **Weight :** The weight of a body is the force with which it is attracted by the earth towards its centre.

For a body of mass *m*, the weight *W* is given by $W = mg$ (By Newton's second law of motion)

Note : ❑ Let W_1 and W_2 be the weights of two bodies of masses m_1 and m_2 respectively at a place on the earth then, $W_1 = m_1 g$ and $W_2 = m_2 g \Rightarrow \frac{W_1}{W_2} = \frac{m_1}{m_2}$

❑ 1 *gm.wt.* = *g* dynes = 981 dynes

1 *kg. wt.* = *g* Newtons = 9.81 *N*.

(3) **Momentum of a body :** It is the quantity of motion in a body and is equal to the product of its mass (*m*) and velocity (*v*) with which it moves. Thus, momentum of the body is *mv*. The units of momentum are *gm-cm/sec* or *kg-m/sec*.

Momentum = Mass × Velocity = *m.v*.

(4) **Impulse of a body :** The impulse of a force in a given time is equal to the product of the force and the time during which it acts. The impulse of a force *F* acting for a time *t* is therefore *F.t*.

Example: 19 A train weighing *W* tons is moving with an acceleration *f* ft/sec^2. When a carriage of weight *w* tons is suddenly detached from it, then the change in the acceleration of the train is

(a) $\frac{Wf}{W-w}\ ft/\sec^2$ (b) $\frac{W}{W-w}\ ft/\sec^2$ (c) $\frac{wf}{W-w}\ ft/\sec^2$ (d) $\frac{w}{W-w}\ ft/\sec^2$

Solution: (c) Mass of train = *W* × 2240 *lbs*.

∴ Pull of the engine = *W* × 2240 × *f Poundals*

When a carriage of mass *W* tons is detached, mass of the train = (*W* – *w*) tons = (*W* – *w*) . 2240 *lbs*

Pull is the same as before *i.e.,* *W* × 2240 × *f Poundals*.

∴ New acceleration = $f_1 = \frac{W \times 2240 \times f}{(W-w)2240}$

$$f_1 = \frac{Wf}{W-w}\ ft/\sec^2$$

Change in acceleration = $f_1 - f = \frac{Wf}{W-w} - f = \left(\frac{W-W+w}{W-w}\right)f = \frac{wf}{W-w}$.

Example: 20 A hockey stick pushes a ball at rest for 0.01 second with an average force of 50 *N*. If the ball weighs 0.2 *kg*, then the velocity of the ball just after being pushed is

(a) 3.5 *m/sec.* (b) 2.5 *m/sec.* (c) 1.5 *m/sec.* (d) 4.5 *m/sec.*

Solution: (b) If *v m/sec.* is the velocity of the ball just after being pushed, then 50 × 0.01 = 0.2 × *v*[∵ Impulse = change of mo

v = 2.5 *m/sec.*

Example: 21 A mass *m* is acted upon by a constant force *P lb.wt.*, under which in *t* seconds it moves a distance of *x* feet and acquires a velocity *v ft/sec*. Then *x* is equal to

(a) $\frac{gP}{2mt^2}$ (b) $\frac{mg}{2v^2P}$ (c) $\frac{gt^2}{2Pm}$ (d) $\frac{mv^2}{2gP}$

Solution: (d) Force = *P lb.wt.*= *Pg* poundals and mass = *m lbs*

$\therefore\ f = \frac{F}{m} = \frac{Pg}{m}\ ft/\sec^2$

Now initial velocity = 0, final velocity = *v ft/sec*.

Distance = *x* and time = *t*

Hence $x = \frac{v^2 - u^2}{2f} = \frac{v^2 - 0}{2\frac{Pg}{m}} = \frac{mv^2}{2Pg}$.

Example: 22 An engine and train weight 420 *tons* and the engine exerts a force of 7 *tons*. If the resistance to motion be 14 *lbs. wt. per ton*, then the time, the train will take to acquire a velocity of 30 *m/hr*. from rest is **[BIT Ranchi 1994]**

(a) 2.2 *m* (b) 2.6 *m* (c) 2.8 *m* (d) 3 *m*

Solution: (a) Effective force = Pull of the engine – Resistance

= 7 tons – 420 × 14 *lbs* = 7 × 2240 *lbs* – 5880 *lbs* = 9800 *lbs* = 9800 × 32 Poundals

Also acceleration = $\frac{P}{m} = \frac{9800 \times 32}{420 \times 2240} = \frac{1}{3}\ ft/\sec^2$

Again initial velocity = 0 and final velocity = 30 *m.p.h.* = 44 *ft/sec*.

Use $v = u + ft$, we get $44 = 0 + \frac{1}{3}t$

$\therefore\ t = 132$ *second* = $2\frac{12}{60}$ *minute*; $t = 2\frac{1}{5}$ *minute* = 2.2 *minute*.

4.8 Motion of a Body released from a Balloon or a Lift

(1) When a lift is ascending with uniform acceleration of *f m/sec*² and after *t second* a body is dropped from it, then at the time when the body is dropped:

(i) Initial velocity of the body is same as that of the lift and is in the same direction. So, the velocity of the body is *ft m/sec*.

(ii) Initial velocity of the body relative to the lift = Velocity of the body – Velocity of the lift = *ft* – *ft* = 0

(iii) Acceleration of the body = *g m/sec*² in downward direction.

(iv) Acceleration of the lift = *f m/sec*² in upward direction.

(v) Acceleration of the body relative to the lift

= Acceleration of the body – Acceleration of the lift = *g* – (– *f*) = *f* + *g* in downward direction.

(2) When a lift is ascending with uniform acceleration of $f\ m/\sec^2$ and after *t second* a body is thrown vertically upward with velocity $v\ m/\sec$, then at that time, we have the following :

(i) Initial velocity of the body = *v* + velocity of lift = *v* + *ft*, in upward direction

(ii) Initial velocity of the body relative to the lift = Velocity of the body – Velocity of lift = (*v* + *ft*) – *ft* = *v m/sec*.

(iii) Acceleration of the body relative to the lift in vertically downward direction is $(f + g)$ m/sec^2.

(3) When a lift is descending with uniform acceleration $f\, m/\sec^2$ and after time t a body is dropped from it. Then at that time, we have the following

(i) Velocity of the body = Velocity of the lift = ft m/sec in downward direction

(ii) Acceleration of the body relative to the lift in downward direction

= Acceleration of the body – Acceleration of lift = $g - f\ m/\sec^2$

4.9 Apparent weight of a Body resting on a moving Horizontal plane or a Lift

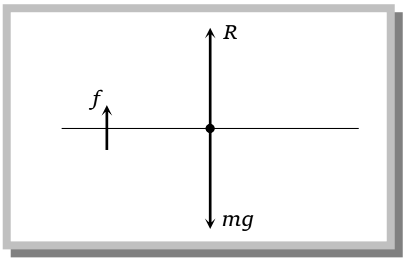
Let a body of mass m be placed in a lift moving with an acceleration f and R is the normal reaction, then

(1) **When the lift is rising vertically upwards :** Effective force in upward direction = Sum of the external forces in the same direction.

$\Rightarrow\ mf = R - Mg \Rightarrow R = m(f+g)$

Clearly, the pressure R exerted by the body on the plane is greater than the actual weight mg of the body. This pressure is also known as apparent weight.

If a man of mass m is standing on a lift which is moving vertically upwards with an acceleration, then $R = m(g+f)\ \Rightarrow\ R = mg\left(1+\frac{f}{g}\right)$



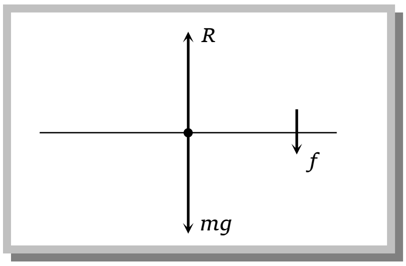
$\Rightarrow R$ = (weight of the man) $\left(1+\frac{f}{g}\right)$

$\Rightarrow R$ = Weight of the man + $\frac{f}{g}$ (weight of the man)

Thus, the apparent weight of the man is $\frac{f}{g}$ times more than the actual weight.

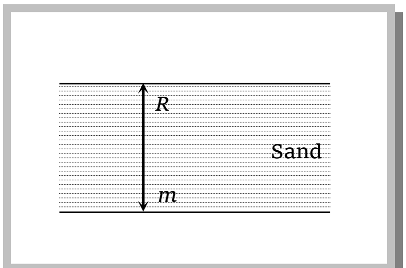
(2) **When the lift is descending vertically downwards :** Effective force in downward direction = Sum of the forces in the same direction.

$\Rightarrow\quad mf = mg - R\quad \Rightarrow\quad R = m(g-f)$ ......(ii)



Clearly, the pressure exerted by the body on the plane is less than its actual weight when the plane moves vertically downwards.

If a man of mass m in is standing on a lift, which is moving vertically downward with an acceleration f, then the pressure $R = m(g-f)$ = $mg\left(1-\frac{f}{g}\right)$ $\Rightarrow R$ = weight of the man – $\frac{f}{g}$ (weight of the man)



Thus, the apparent weight of the man is $\frac{f}{g}$ times less than his actual weight.

Note : ❑ Effective force stopping a falling body = $F - mg$

❑ If R be the resistance of sand on a body of mass m falling in sand, then effective force = $R - mg$.

Important Tips

☞ *If the plane moves vertically upward with retardation equal to g i.e.,* $f = -g$ *, then from* $R = m(f + g)$ *, we get* $R = 0$ *. Thus there is no pressure of the body on the plane when the plane rises vertically with retardation equal to g.*

☞ *If the plane moves down freely under gravity i.e., with acceleration equal to g, then from* $R = m(g - f)$ *, we get* $R = 0$ *. Thus there is no pressure of the body on the plane, when it moves vertically downwards with an acceleration equal to g.*

Example: 23 A paracute weighing 112 *lbs. wt.* falling with a uniform acceleration from rest, describes 16 *ft.* in the first 4 *sec.* Then the resultant pressure of air on the parachute is

(a) 85 *lbs. wt.* (b) 95 *lbs. wt.* (c) 105 *lbs. wt.* (d) 115 *lbs. wt.*

Solution: (c) Let *f* be the uniform acceleration. Here $u = 0$, $s = 16$, $t = 4\,\text{sec}$.

Since $s = ut + \frac{1}{2}ft^2$, $\therefore$ $16 = 0 + \frac{1}{2}f(4)^2$. $\therefore$ $f = 2\,ft/\sec^2$

Equation of motion is $mf = mg - R$ (Downward)

$\therefore$ $R = m(g - f)$ = 112 (32 – 2) = 112 × 30 Poundals $= \frac{112 \times 30}{32}$ *lbs. wt.* = 105 *lbs. wt.*

Example: 24 A man falls vertically under gravity with a box of mass '*m*' on his head. Then the reaction force between his head and the box is

(a) *mg* (b) 2 *mg* (c) 0 (d) 1.5 *mg*

Solution: (c) Let reaction be *R*. Since the man falls vertically under gravity $\therefore \ f = g$

From $mg - R = mf$, we have $mg - R = mg$. $\therefore R = 0$.

Example: 25 A man of mass 80 *kg*. is travelling in a lift. The reaction between the floor of the lift and the man when the lift is ascending upwards at 4 m/sec^2 is

(a) 1464.8 *N* (b) 1784.8 *N* (c) 1959.8 *N* (d) 1104.8 *N*

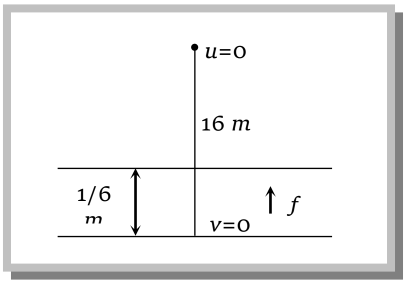
Solution: (d) When the lift is ascending, we have $R = mg + ma = m(g+a)$ = $80(9.81+4) = 80(13.81)$ = 1104.8 *N*.

Example: 26 A particle of mass *m* falls from rest at a height of 16 *m* and is brought to rest by penetrating $\frac{1}{6}m$ into the ground. If the average vertical thrust exerted by the ground be 388 *kg. wt*, then the mass of the particle is

(a) 2 *kg* (b) $3kg$ (c) $4kg$ (d) $8kg$

Solution: (c) Let $f\ m/\sec^2$ be the retardation produced by the thrust exerted by the ground and let $v\,m/\sec$. be the velocity with which the mass *m* penetrates into the ground. Then, $v^2 = 0^2 + 2g \times 16$ $[\because v^2 = u^2 + 2gh]$

$\Rightarrow v = \sqrt{2 \times 9.8 \times 16} = 4\sqrt{19.6} = 17.70\ m/\sec$.



Since the mass *m* penetrates through $\frac{1}{6}m$ in the ground.

Therefore, its final velocity is zero.

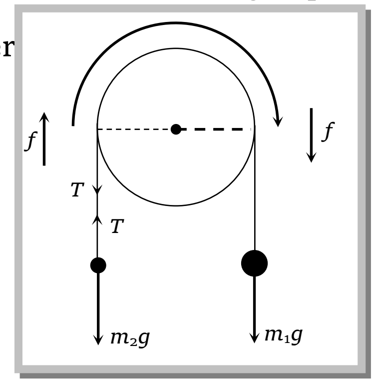
So, $0^2 = v^2 - 2fs \Rightarrow 0 = (4\sqrt{19.6})^2 - 2f \times \frac{1}{6}$

$\Rightarrow f = \frac{16 \times 19.6 \times 6}{2} = 940.8\, m/\sec^2 \Rightarrow (388 - m) = \frac{940.8\, m}{9.8}$ $[\because \ g = 9.8\, m/\sec^2]$

$\Rightarrow 388 - m = 96m \Rightarrow m = \frac{388}{97} = 4$. Hence, the mass of the particle is 4 *kg*.

4.10 Motion of two particles connected by a String

(1) **Two particles hanging vertically :** If two particles of masses m_1 and m_2 $(m_1 > m_2)$ are suspended freely by a light inextensible string which passes over a smooth fixed light pulley, then the particle of mass $m_1 (> m_2)$ will move downwards, with Acceler

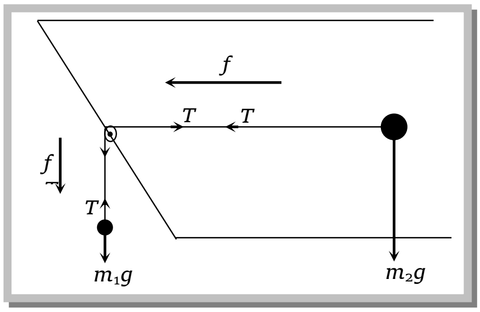


Tension in the string $T = \frac{2m_1 m_2 g}{m_1 + m_2}$

Pressure on the pulley $= 2T = \frac{4m_1 m_2 g}{m_1 + m_2}$

(2) **One particle on smooth horizontal table :** A particle of mass m_1 attached at one end of light inextensible string is hanging vertically, the string passes over a pulley at the end of a smooth

horizontal table, and a particle of mass m_2 placed on the table is attached at the other end of the string. Then for the system,

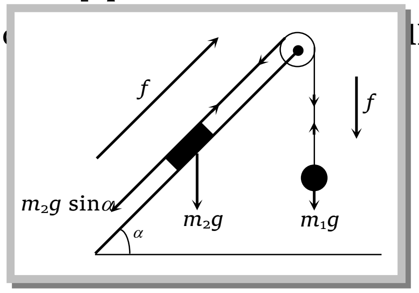


Acceleration $f = \dfrac{m_1 g}{m_1 + m_2}$

Tension in the string $T = \dfrac{m_1 m_2 g}{m_1 + m_2}$

Pressure on the pulley$= T\sqrt{2} = \dfrac{\sqrt{2} m_1 m_2 g}{m_1 + m_2}$.

(3) **One particle on an inclined plane :** A particle of mass m_1 attached at one end of a light inextensible string is hanging vertically, the string passes over a pulley at the end of a smooth plane inclined at an angle α to the horizontal, and a particle of mass m_2 placed on this inclined plane is attached at the other end of the string. If the particle (… lly downwards, then for the system,



Acceleration $f = \dfrac{(m_1 - m_2 \sin\alpha)}{m_1 + m_2} g$

Tension in the string $T = \dfrac{m_1 m_2 (1 + \sin\alpha)}{m_1 + m_2} g$

Pressure on the pulley $= \sqrt{\{2(1 + \sin\alpha)\}} T = \dfrac{\sqrt{2} m_1 m_2 g (1 + \sin\alpha)^{3/2}}{m_1 + m_2}$

Example: 27 The sum of two weights of an Atwood's machine is 16 *lbs*. The heavier weight descends 64 *ft* in 4 *seconds*. The value of weights in *lbs* are

(a) 10, 6 (b) 9, 7 (c) 8, 8 (d) 12, 4

Solution: (a) Let the two weights be m_1 and m_2 (*lbs.*)

$(m_1 > m_2)$, $\therefore$ $m_1 + m_2 = 16$ ......(i)

Now, the distance moved by the heavier weight in 4 *seconds* from rest is 64 *ft*.

If acceleration = *f*, then $64 = 0 + \frac{1}{2} f (4)^2 \Rightarrow f = 8 ft/\sec^2$

But $f = \dfrac{m_1 - m_2}{m_1 + m_2} g \Rightarrow \dfrac{m_1 - m_2}{16} \times 32 = 8 \Rightarrow m_1 - m_2 = 4$ ......(ii)

From (i) and (ii), $m_1 = 10$ *lbs*, $m_2 = 6 lbs$.

Example: 28 Two weights W and W' are connected by a light string passing over a light pulley. If the pulley moves with an upward acceleration equal to that of gravity, the tension of the string is

(a) $\dfrac{WW'}{W + W'}$ (b) $\dfrac{2WW'}{W + W'}$ (c) $\dfrac{3WW'}{W + W'}$ (d) $\dfrac{4WW'}{W + W'}$

Solution: (d) Suppose W descends and W' ascends. If f is the acceleration of the weights relative to the pulley, then since the pulley ascends with an acceleration f, the actual acceleration of W and W' are $(f - g)$ (downwards) and $(f + g)$ (upwards) respectively. Let T be tension in the string.

$\therefore\ Wg - T = W(f-g)$ and $T - W'g = W'(f+g)$ or $g - \frac{T}{W} = f - g$ and $\frac{T}{W'} - g = f + g$

Subtracting, We get $\frac{T}{W} + \frac{T}{W'} - 2g = 2g$ or $T = \frac{4WW'}{W+W'}$ $kg.\ wt.$

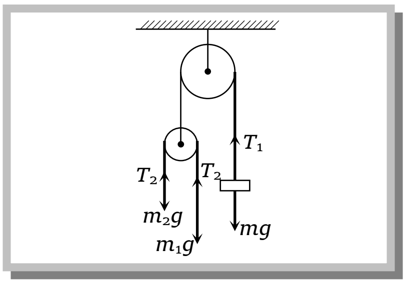
Example: 29 To one end of a light string passing over a smooth fixed pulley is attached a particle of mass M, and the other end carries a light pulley over which passes a light string to whose ends are attached particles of masses m_1 and m_2. The mass M will remain at rest or will move with a uniform velocity, if **[MNR 1993]**

(a) $\frac{1}{M} = \frac{1}{m_1} + \frac{1}{m_2}$ (b) $\frac{2}{M} = \frac{1}{m_1} + \frac{1}{m_2}$ (c) $\frac{4}{M} = \frac{1}{m_1} + \frac{1}{m_2}$ (d) $\frac{8}{M} = \frac{1}{m_1} + \frac{1}{m_2}$

Solution: (c) If the mass M has no acceleration, then $Mg = T_1$

But $T_1 = 2T_2 = 2\left(\frac{2m_1m_2}{m_1+m_2}g\right) = \frac{4m_1m_2}{m_1+m_2}g$

$\therefore\ Mg = \frac{4m_1m_2}{m_1+m_2}g$ or $\frac{4}{M} = \frac{m_1+m_2}{m_1m_2} = \frac{1}{m_1} + \frac{1}{m_2}$



4.11 Impact of Elastic Bodies

(1) **Elasticity :** It is that property of bodies by virtue of which they can be compressed and after compression they recover or tend to recover their original shape. Bodies possessing this property are called elastic bodies.

(2) **Law of conservation of momentum :** It states "the total momentum of two bodies remains constant after their collision of any other mutual action." Mathematically $m_1.u_1 + m_2.u_2 = m_1.v_1 + m_2.v_2$

where m_1 = mass of the first body, u_1 = initial velocity of the first body, v_1 = final velocity of the first body.

m_2, u_2, v_2 = Corresponding values for the second body.

(3) **Coefficient of restitution :** This constant ratio is denoted by e and is called the coefficient of restitution or coefficient of elasticity. The values of e varies between the limits 0 and 1. The value of e depends upon the substances of the bodies and is independent of the masses of the bodies. If the bodies are perfectly elastic, then $e = 1$ and for inelastic bodies $e = 0$.

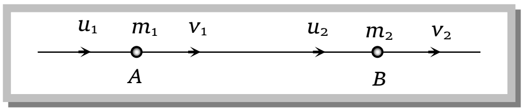
(4) **Impact :** When two bodies strike against each other, they are said to have an impact. It is of two kinds : Direct and Oblique.

(5) **Newton's experimental law of impact :** It states that when two elastic bodies collide, their relative velocity along the common normal after impact bears a constant ratio of their relative velocity before impact and is in opposite direction.

If u_1 and u_2 be the velocities of the two bodies before impact along the common normal at their point of contact and v_1 and v_2 be their velocities after impact in the same direction, $v_1 - v_2 = -e(u_1 - u_2)$

Case I : If the two bodies move in direction shown in diagram given below, then

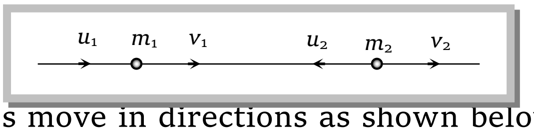
(a) $v_1 - v_2 = -e(u_1 - u_2)$ and (b) $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$



Case II : If the direction of motion of two bodies before and after the impact are as shown below, then by the laws of direct impact, we have

(a) $v_1 - v_2 = -e[u_1 - (-u_2)]$ or $v_1 - v_2 = -e(u_1 + u_2)$

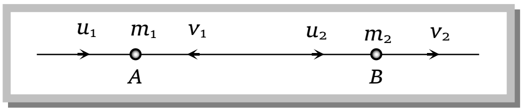
and (b) $m_1v_1 + m_2v_2 = m_1u_1 + m_2(-u_2)$ or $m_1v_1 + m_2v_2 = m_1u_1 - m_2u_2$



Case III : If the two bodies move in directions as shown below, then by the laws of direct impact, we have

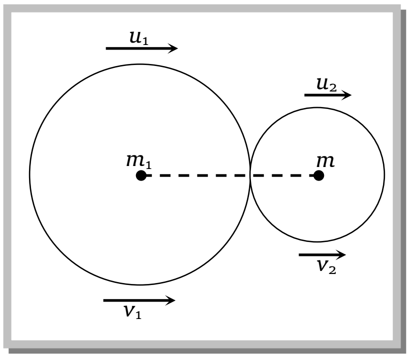
(a) $v_1 - (-v_2) = -e[-u_1 - (-u_2)]$ or $v_1 + v_2 = -e(u_2 - u_1)$

and (b) $m_1v_1 + m_2(-v_2) = m_1(-u_1) + m_2(-u_2)$ or $m_1v_1 - m_2v_2 = -(m_1u_1 + m_2u_2)$



(6) **Direct impact of two smooth spheres :** Two smooth spheres of masses m_1 and m_2 moving along their line of centres with velocities u_1 and u_2 (measured in the same sense) impinge directly. To find their velocities immediately after impact, e being the coefficient of restitution between them.

Let v_1 and v_2 be the velocities of the two spheres immediately after impact, measured along their line of centres in the same direction in which u_1 and u_2 are measured. As the spheres are smooth, the impulsive action and reaction between them will be along the common normal at the point of contact. From the principle of conservation of momentum,



$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \quad .....(i)$$

Also from Newton's experimental law of impact of two bodies,

$$v_2 - v_1 = e[u_1 - u_2] \quad .....(ii)$$

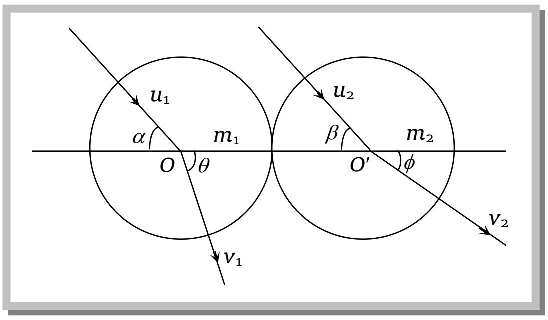
Multiplying (ii) by m_2 and subtracting from (i), we get

$$\Rightarrow (m_1 + m_2)v_1 = (m_1 - em_2)u_1 + m_2u_2(1+e)$$

$$\therefore v_1 = \frac{(m_1 - em_2)u_1 + m_2u_2(1+e)}{m_1 + m_2} \quad .....(iii)$$

(7) **Oblique impact of two spheres :** A smooth spheres of mass m_1, impinges with a velocity u_1 obliquely on a smooth sphere of mass m_2 moving with a velocity u_2. If the direction of motion before impact make angles α and β respectively with the line joining the centres of the spheres, and if the coefficient of restitution be e, to find the velocity and directions of motion after impact,

Let the velocities of the sphere after impact be v_1 and v_2 in directions inclined at angles θ and ϕ respectively to the line of centres. Since the spheres are smooth, there is no force perpendicular to the line joining the centres of the two balls and therefore, velocities in that direction are unaltered.



$v_1 \sin\theta = u_1 \sin\alpha$ .....(i)

$v_2 \sin\phi = u_2 \sin\beta$ .....(ii)

And by Newton's law, along the line of centres,

$v_2 \cos\phi - v_1 \cos\theta = -e(u_2 \cos\beta - u_1 \cos\alpha)$ .....(iii)

Again, the only force acting on the spheres during the impact is along the line of centres. Hence the total momentum in that direction is unaltered.

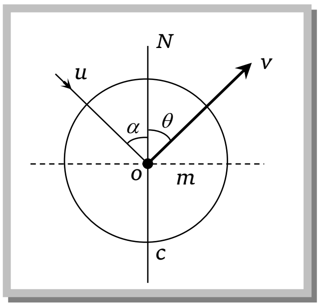
$\therefore m_1 v_1 \cos\theta + m_2 v_2 \cos\phi = m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta$ .....(iv)

The equations (i), (ii), (iii) and (iv) determine the unknown quantities.

(8) **Impact of a smooth sphere on a fixed smooth plane :** Let a smooth sphere of mass m moving with velocity u in a direction making an angle α with the vertical strike a fixed smooth horizontal plane and let v be the velocity of the sphere at an angle θ to the vertical after impact.

Since, both the sphere and the plane are smooth, so there is no change in velocity parallel to the horizontal plane.

$\therefore v \sin\theta = u \sin\alpha$ .....(i)



And by Newton's law, along the normal CN, velocity of separation = e.(velocity of approach)

$\therefore v\cos\theta - 0 = eu\cos\alpha$

$v\cos\theta = eu\cos\alpha$ ....(ii)

Dividing (i) by (ii), we get $\cot\theta = e\cot\alpha$

***Particular case* :** If $\alpha = 0$ then from (i), $v\sin\theta = 0 \Rightarrow \sin\theta = 0$

$\therefore \theta = 0$; $\because v \neq 0$ and from (ii) $v = eu$

Thus if a smooth sphere strikes a smooth horizontal plane normally, then it will rebound along the normal with velocity, e times the velocity of impact *i.e.* velocity of rebound = e.(velocity before impact).

Rebounds of a particle on a smooth plane : If a smooth ball falls from a height h upon a fixed smooth horizontal plane, and if e is the coefficient of restitution, then whole time before the rebounding ends $= \sqrt{\left(\frac{2h}{g}\right)} \cdot \frac{1+e}{1-e}$

And the total distance described before finishing rebounding $= \frac{1+e^2}{1-e^2} h$.

Example: 30 A ball is dropped from a height h on a horizontal plane and the coefficient of restitution for the impact is e, the velocity with which the ball rebounds from the floor is

(a) eh (b) egh (c) $e\sqrt{gh}$ (d) $e\sqrt{2gh}$

Solution: (d) Let u be the velocity with which the ball strikes the ground. Then, $u^2 = 2gh \Rightarrow u = \sqrt{2gh}$ .....(i)

Suppose the ball rebounds with velocity v, then $V = -e(-u-0) \Rightarrow v = ue \Rightarrow v = e\sqrt{2gh}$, [from (i)].

Example: 31 A sphere impinges directly on a similar sphere at rest. If the coefficient of restitution is $\frac{1}{2}$, the velocities after impact are in the ratio

(a) 1 : 2 (b) 2 : 3 (c) 1 : 3 (d) 3 : 4

Solution: (c) From principle of conservation of momentum ,

$mv_1 + m'v_1' = mu_1 + m'u_1'$ ......(i)

From Newton's rule for relative velocities before and after impact.

$v_1 - v_1' = -e(u_1 - u_1')$ ......(ii)

Here $m = m'$, $e = \frac{1}{2}$ and let $u_1' = 0$

Then from (i) $mv_1 + mv_1' = mu_1$ or $v_1 + v_1' = u_1$ ......(iii)

From (ii), $v_1 - v_1' = \frac{-1}{2}u_1$ ....(iv)

Adding (iii) and (iv) , $2v_1 = \frac{1}{2}u_1$ or $v_1 = \frac{1}{4}u_1$ ......(v)

From (iii) and (v), $v_1' = u_1 - \frac{1}{4}u_1 = \frac{3}{4}u_1$. Hence $v_1 : v_1' = 1 : 3$.

Example: 32 A ball impinges directly upon another ball at rest and is itself reduced to rest by the impact. If half of the K.E. is destroyed in the collision, the coefficient of restitution, is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

Solution: (d) Let masses of the two balls be m_1 and m_2 .

By the given question, we have

m_1	m_2
u_1	$u_2 = 0$
$v_1 = 0$	v_2

$\therefore$ $m_1u_1 + m_2.0 = m_1.0 + m_2v_2$ or $m_1u_1 = m_2v_2$ .....(i) and $0 - v_2 = e(0 - u_1)$ *i.e.,* $v_2 = eu_1$ .....(ii)

Again for the given condition, K.E. (before impact) = 2 × K.E. after impact

$\therefore$ $\frac{1}{2}m_1u_1^2 + 0 = 2\left(0 + \frac{1}{2}m_2v_2^2\right)$ .....(iii) or $\frac{1}{2}m_1u_1^2 = 2.\frac{1}{2}m_2v_2^2$

But from (i), $m_1u_1 = m_2v_2$, $\therefore$ $\frac{1}{2}u_1 = v_2$ *i.e.,* $u_1 = 2v_2$. Putting in (ii), we get $v_2 = e.2v_2$, $\therefore$ $e = \frac{1}{2}$.

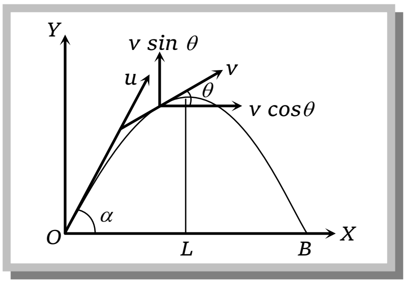
4.12 Projectile Motion

When a body is thrown in the air, not vertically upwards but making an acute angle α with the horizontal, then it describes a curved path and this path is a Parabola.

The body so projected is called a projectile. The curved path described by the body is called its trajectory.

The path of a projectile is a parabola whose equation is

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$



Its vertex is $A\left(\frac{u^2 \sin\alpha\cos\alpha}{g}, \frac{u^2 \sin^2\alpha}{2g}\right)$.

Focus is $S\left(\frac{u^2 \sin 2\alpha}{2g}, \frac{-u^2 \cos 2\alpha}{2g}\right)$

(1) **Some important deductions about projectile motion:** Let a particle is projected with velocity u in a direction making angle α with OX. Let the particle be at a point $P(x,y)$ after time t and v be its velocity making an angle θ with OX.

(i) Time of flight $= \frac{2u\sin\alpha}{g}$

(ii) Range of flight *i.e.*, Horizontal Range $R = \frac{u^2 \sin 2\alpha}{g}$

Maximum horizontal Range $= \frac{u^2}{g}$ and this happens, when $\alpha = \frac{\pi}{4}$

(iii) Greatest height $= \frac{u^2 \sin^2\alpha}{2g}$

Time taken to reach the greatest height $= \frac{u\sin\alpha}{g}$

(iv) Let $P(x,y)$ be the position of the particle at time t. Then $x = (u\cos\alpha)t$, $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

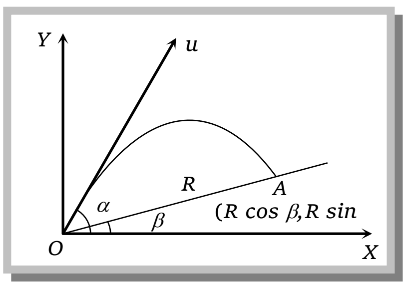
(v) Horizontal velocity at time $t = \frac{dx}{dt} = u\cos\alpha$

Vertical velocity at time $t = \frac{dy}{dt} = u\sin\alpha - gt$

(vi) Resultant velocity at time $t = \sqrt{u^2 - 2ugt\sin\alpha + g^2t^2}$. And its direction is $\theta = \tan^{-1}\left(\frac{u\sin\alpha - gt}{u\cos\alpha}\right)$

(vii) Velocity of the projectile at the height h $= \sqrt{u^2 - 2gh}$ and its direction is $\theta = \tan^{-1}\left[\frac{\sqrt{u^2\sin^2\alpha - 2gh}}{u\cos\alpha}\right]$

(2) **Range and time of flight on an inclined plane**



Let OX and OY be the coordinate axes and OA be the inclined plane at an angle β to the horizon OX. Let a particle be projected from O with initial velocity u inclined at an angle α with the horizontal OX. The equation of the trajectory is $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$.

(i) Range and time of flight on an inclined plane with angle of inclination β are given by

$R = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$ and maximum range up the plane $= \frac{u^2}{g(1+\sin\beta)}$, where $2\alpha - \beta = \frac{\pi}{2}$

(ii) Time of flight $T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$

(iii) Range down the plane $= \frac{2u^2\cos\alpha\sin(\alpha+\beta)}{g\cos^2\beta}$

(iv) Maximum range down the plane $= \frac{u^2}{g(1-\sin\beta)}$, where $2\alpha + \beta = \frac{\pi}{2}$

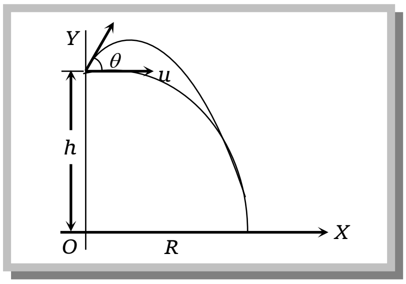
(v) Time of flight $= \frac{2u\sin(\alpha+\beta)}{g\cos\beta}$, down the plane

(vi) Condition that the particle may strike the plane at right angles is $\cot\beta = 2\tan(\alpha-\beta)$.

Example: 33 Two stones are projected from the top of a cliff h *metres* high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals

(a) $\sqrt{\frac{2u}{gh}}$ (b) $2g\sqrt{\frac{u}{h}}$ (c) $2h\sqrt{\frac{u}{g}}$ (d) $u\sqrt{\frac{2}{gh}}$

Solution: (d) $R = u\sqrt{\frac{2h}{g}} = (u\cos\theta)\times t$; $t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$ .....(i)



Now $h = (-u\sin\theta)t + \frac{1}{2}gt^2$, '$t$' from (i)

$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$, $h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$, $\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0$; $\therefore \tan\theta = u\sqrt{\frac{2}{hg}}$.

Example: 34 A gun projects a ball at an angle of 45^o with the horizontal. If the horizontal range is 39.2 m, then the ball will rise to

[AMU 1999]

(a) 9.8 m (b) 4.9 m (c) 2.45 m (d) 19.6 m

Solution: (a) Horizontal range = $\frac{u^2\sin 2\alpha}{g} = 39.2 \Rightarrow \frac{u^2\sin 90^o}{g} = 39.2 \Rightarrow \frac{u^2}{g} = 39.2$

Greatest height attained = $\frac{u^2\sin^2\alpha}{2g}$ = $\frac{39.2}{2}\sin^2 45^o$ = $(19.6)\frac{1}{2} = 9.8m$.

Example: 35 If the range of any projectile is the distance equal to the height from which a particle attains the velocity equal to the velocity of projection, then the angle of projection will be

(a) 60^o (b) 75^o (c) 36^o (d) 30^o

Solution: (b) Range = $\frac{u^2\sin 2\alpha}{g}$, where α is the angle of projection.

Also, $u^2 = 2gh \Rightarrow \frac{u^2}{2g} = h$

By the given condition, $\frac{u^2}{2g} = \frac{u^2 \sin 2\alpha}{g} \Rightarrow \frac{1}{2} = \sin 2\alpha$, $\therefore$ $2\alpha = 30^o$ or 150^o

$\therefore$ $\alpha = 15^o$ or 75^o. Hence, required angle of projection = 75^o.

Example: 36 A particle is thrown with the velocity v such that its range on horizontal plane is twice the maximum height obtained. Its range will be

(a) $\frac{2v^2}{3g}$ (b) $\frac{4v^2}{3g}$ (c) $\frac{4v^2}{5g}$ (d) $\frac{v^2}{7g}$

Solution: (c) Range = 2 × (maximum height)

$\therefore$ $\frac{v^2 \sin 2\alpha}{g} = 2\frac{v^2 \sin^2 \alpha}{2g} \Rightarrow \sin 2\alpha = \sin^2 \alpha$

$\Rightarrow 2\sin\alpha\cos\alpha - \sin^2\alpha = 0 \Rightarrow \sin\alpha(2\cos\alpha - \sin\alpha) = 0 \Rightarrow \sin\alpha = 0$ or $\tan\alpha = 2$

But $\alpha \neq 0$, $\therefore \sin\alpha \neq 0$, $\therefore \tan\alpha = 2$. Now $R = \frac{v^2 \sin 2\alpha}{g} = \frac{v^2}{g}.\frac{2\tan\alpha}{1+\tan^2\alpha} \Rightarrow \frac{v^2}{g}.\frac{2.2}{1+4} = \frac{4v^2}{5g}$.

Example: 37 Let u be the velocity of projection and v_1 be the velocity of striking the plane when projected so that the range up the plane is maximum, and v_2 be the velocity of striking the plane when projected so that the range down the plane is maximum, then u is the

(a) A.M. of v_1, v_2 (b) G.M. of v_1, v_2 (c) H.M. of v_1, v_2 (d) None of these

Solution: (b) Let u be the velocity of projection and β be the inclination of the plane with the horizon. Let R be the maximum range up the plane. Then, $R = \frac{u^2}{g(1+\sin\beta)}$

The coordinates of the point where the projectile strikes the plane are $(R\cos\beta, R\sin\beta)$. It is given that v_1 is the velocity of striking the plane when projected so that the range up the plane is maximum.

$\therefore$ $v_1^2 = u^2 - 2gR\sin\beta$

$\Rightarrow$ $v_1^2 = u^2 - 2g\sin\beta.\frac{u^2}{g(1+\sin\beta)} \Rightarrow v_1^2 = u^2\left(\frac{1-\sin\beta}{1+\sin\beta}\right)$ .....(i)

Replacing β by $-\beta$ in (i), we get $v_2^2 = u^2\left(\frac{1+\sin\beta}{1-\sin\beta}\right)$ .....(ii)

From (i) and (ii), we get $v_1^2.v_2^2 = u^4 \Rightarrow v_1 v_2 = u^2$ *i.e.*, u is the G.M. of v_1, v_2.

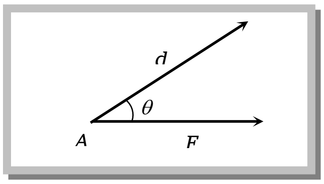
4.13 Work, Power and Energy

(1) **Work :** Work is said to be done by a force when its point of application undergoes a displacement. In other words, when a body is displaced due to the action of a force, then the force is said to do work. Work is a scalar quantity.

Work done = force × displacement of body in the direction of force

$W = \vec{F}.\vec{d}$

$W = Fd\cos\theta$, where θ is the angle between F and d.



(2) **Power :** The rate of doing work is called power. It is the amount of work that an agent is capable of doing in a unit time. 1 *watt* = 10^7 *ergs per sec* = 1 *joule* per *sec*., 1 H.P. = 746 watt.

(3) **Energy :** Energy of a body is its capacity for doing work and is of two kinds :

(i) **Kinetic energy :** Kinetic energy is the capacity for doing work which a moving body possesses by virtue of its motion and is measured by the work which the body can do against any force applied to stop it, before the velocity is destroyed. $K.E.=\frac{1}{2}mv^2$

(ii) **Potential energy :** The potential energy of a body is the capacity for doing work which it possesses by virtue of its position of configuration. $P.E.=mgh$.

Example: 38 Two bodies of masses m and $4m$ are moving with equal momentum. The ratio of their K.E. is **[MNR 1990; UPS**

(a) 1 : 4 (b) 4 : 1 (c) 1 : 1 (d) 1 : 2

Solution: (b) Momentum of first body = mu, where u is the velocity of the first body.

$\therefore$ Momentum of second body = $4mv$, where v is the velocity of the second body.

By the given condition, $mu=4mv \Rightarrow u=4v$

$\therefore$ Ratio of K.E. = $\frac{\frac{1}{2}mu^2}{\frac{1}{2}.4mv^2}=\frac{(4v)^2}{4v^2}=4$. Hence, the ratio of K.E. is 4 : 1.

Example: 39 A bowler throws a bumper with a speed of $24m/\sec$. The moment the ball touches the ground, it losses its energy by 1.5 *kgm*. If the weight of the ball is 225 *gm*, then speed of the ball at which it hits the bat is **[UPSEAT 2000]**

(a) 2.22 *m/sec.* (b) 22.2 *m/sec.* (c) 4.44 *m/sec.* (d) 44.4 *m/sec.*

Solution: (b) Weight of the ball = $w=225$ *gm* = 0.225 *kg*

Velocity of the ball = *v* = 25 *m/sec.*

Loss of energy = 1.5 *kg.m*

$\therefore$ K.E. of the ball before it touches the ground = $\frac{1}{2}\frac{wv^2}{g}=\frac{1}{2}.\frac{0.225}{9.81}\times(25)^2=7.17kgm$.

K.E. of the ball after bumping = K.E. before touching the ground – loss of K.E. = 7.17 – 1.5 = 5.67 *kg-m.*

Let v_1 be the velocity of the ball at which it hits the bat.

Now K.E. of the ball at the time of hitting the bat $=\frac{1}{2}\frac{w}{g}v_1^2 \Rightarrow$ $5.67=\frac{1}{2}\times\frac{0.225}{9.81}v_1^2$ $\Rightarrow$

$v_1^2=494 \Rightarrow v_1=22.2\ m/\sec$.

Example: 40 A particular starts from rest and move with constant acceleration. Then the ratio of the increase in the K.E. in m^{th} and $(m+1)^{th}$ second is

(a) $m:m$ (b) $m+1:m+1$ (c) $2m-1:2m+1$ (d) None of these

Solution: (c) Required ratio = $\frac{\text{Increase in K.E. in } m^{th} \text{second}}{\text{Increase in K.E. in } (m+1)^{th} \text{second}}$ = $\frac{\text{Work done against the force in } m^{\text{th}} \text{second}}{\text{Work done against the force in } (m+1)^{th} \text{second}}$

$$=\frac{P\times\left[0+\frac{1}{2}f(2m-1)\right]}{P\times\left[0+\frac{1}{2}f(2(m+1)-1)\right]}=\frac{2m-1}{2m+2-1}=\frac{2m-1}{2m+1}.$$

Assignment

Velocity and Acceleration, Resultant and Component of Velocities

Basic Level

1. The initial velocity of a particle is $u(\text{at } t=0)$, and the acceleration f is given by at. Which of the following relation is valid

(a) $v=u+at^2$ (b) $v=u+\frac{at^2}{2}$ (c) $v=u+at$ (d) $v=u$

2. A particle is moving in a straight line such that the distance described s and time taken t are given by $t=as^2+bs+c$, $a>0$. If v is the velocity of the particle at any time t, then its acceleration is

(a) $-2av$ (b) $-2av^2$ (c) $-2av^3$ (d) None of these

3. If a particle, moving in a straight line, covers a distance s in time t, given by the relations $s^2=at^2+2bt+c$, then its acceleration is

(a) $\frac{b^2-ac}{s^3}$ (b) $\frac{ac-b^2}{s^3}$ (c) $\frac{ac-b^2}{s^2}$ (d) $\frac{ac-b^2}{s}$

4. The speed v of a body moving on a straight track varies according to $v=\begin{cases}2t+13, & 0\le t\le 5\\ 3t+8, & 5<t\le 7\\ 4t+1, & t>7\end{cases}$.

The distances are measured in *metres* and time t in *seconds*. The distance in *metres* moved by the particle at the end of 10 *seconds* is

(a) 127 (b) 247 (c) 186 (d) 313

5. If the velocity of a particle moving in a straight line is given by $v^2=se^s$, then its acceleration is

(a) $\frac{v^2}{2s}$ (b) $\frac{v^2}{2s}(s+1)$ (c) $\frac{v^2}{2}(s-1)$ (d) $\frac{v}{2}(s+1)$

6. The position at any time t, of a particle moving along x-axis is given by the relation $s=t^3-9t^2+24t+6$, where s denotes the distance in *metre* from the origin. The velocity v of the particle at the instant when the acceleration becomes zero, is given by

(a) $v=3$ (b) $v=-3$ (c) $v=0$ (d) $v=-6$

7. For a particle moving in a straight line, if time t be regarded as a function of velocity v, then the rate of change of the acceleration a is given by

(a) $a^2\frac{d^2t}{dv^2}$ (b) $a^3\frac{d^2t}{dv^2}$ (c) $-a^3\frac{d^2t}{dv^2}$ (d) None of these

8. If the law of motion of a particle moving in a straight line is given by $ks=\log\left(\frac{1}{v}\right)$, then its acceleration a is given by

(a) $a=-kv$ (b) $a=-kv^3$ (c) $a=-kv^2$ (d) None of these

9. A point moves rectilinearly with deceleration whose modulus depends on the velocity of the particle as $a\sqrt{v}$, where a is a positive constant. At the initial moment its velocity is equal to v_0. The time it takes before it comes to rest is

(a) $2\frac{\sqrt{v_0}}{a}$ (b) $\frac{\sqrt{v_0}}{a}$ (c) $\frac{v_0}{a}$ (d) $\frac{a}{\sqrt{v_0}}$

10. The law of motion of a particle moving in a straight line is given by $s=\frac{1}{2}vt$, where v is the velocity at time t and s is the distance covered. Then acceleration is

(a) A function of t (b) a function of s (c) a function of v (d) constant

11. If the displacement of a particle varies with time as $\sqrt{x}=t+7$, then

(a) The velocity of the particle is inversely proportional to t (b) The velocity of the particle is proportional to t

(c) The velocity of the particle is proportional to $\sqrt{t}$ (d) The particle moves with a constant acceleration

12. The x and y displacement of a particle in the xy-plane at any instant are given by $x=at^2$ and $y=2at$, where a is a constant. The velocity of the particle at any instant is given by

(a) $4a\sqrt{t^2+4}$ (b) $2a\sqrt{t^2+1}$ (c) $4a\sqrt{t^2+1}$ (d) $\frac{a}{2}\sqrt{t^2+4}$

13. The acceleration of a particle, starting from rest, varies with time according to the relation $a=-s\omega^2\sin\omega t$. The displacement of this particle at time t will be

(a) $s\sin\omega t$ (b) $s\omega\cos\omega t$ (c) $s\omega\sin\omega t$ (d) $-\frac{1}{2}(s\omega^2\sin\omega t)t^2$

14. A particle moves along a straight line in such a way that its distance from a fixed point on the line, at any time t from the start, is given by the equation $s=6-2t+3t^3$. Its acceleration after 1 *second* of motion is

(a) 12 (b) 16 (c) 18 (d) None of these

15. A particle moves in a straight line with a velocity given by $\frac{dx}{dt}=x+1$. The time taken by the particle to traverse a distance of 99 *metres* is

(a) $\log_{10} e$ (b) $2\log_e 10$ (c) $2\log_{10} e$ (d) $\frac{1}{2}\log_{10} e$

16. A passenger travels along the straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2. The average velocity is given by

(a) v_1+v_2 (b) $\frac{v_1+v_2}{2}$ (c) $\frac{2v_1v_2}{v_1+v_2}$ (d) $\sqrt{v_1v_2}$

17. If a particle moves along a straight line according to the law $s^2=at^2+2bt+c$, then its acceleration is given by

(a) $\frac{a-v}{s}$ (b) $\frac{a-v^2}{s}$ (c) $\frac{a-v^2}{s^2}$ (d) $\frac{a-v}{s^2}$

18. If a particle has two velocities each equal to u in magnitude and their resultant is also of magnitude u, then the angle between the two velocities is

(a) 60^o (b) 30^o (c) 90^o (d) 120^o

19. If two velocities u and v are inclined at such an angle that the resultant of $2u$ and v inclined at the same angle is at right angle to v, then the resultant of u and v is of magnitude

(a) $2u$ (b) v (c) $2v$ (d) u

20. If a particle having simultaneous velocities 3 *m/sec.*, 5 *m/sec.* and 7 *m/sec.* at rest, then the angle between the first two velocities is

(a) 120^o (b) 150^o (c) 60^o (d) 90^o

21. The greatest and least magnitudes of the resultants of two velocities of constant magnitudes are u and v respectively. If a particle has these velocities inclined at an angle 2α, then the resultant velocity is of magnitude

(a) $\sqrt{u^2\cos^2\alpha+v^2\sin^2\alpha}$ (b) $\sqrt{u^2\sin^2\alpha+v^2\cos^2\alpha}$ (c) $\sqrt{u^2\cos\alpha+v^2\sin\alpha}$ (d) None of these

22. A particle possesses simultaneously two velocities 10 *m/sec*. and 15 *m/sec*. in directions inclined at an angle of 60^o, then its resultant velocity is

(a) 15 *m/sec*. (b) $5\sqrt{19}\ m/\sec$ (c) 25 *m/sec* (d) None of these

23. A particle is moving with a velocity of 30 *m/sec*. The components of the velocity in *m/sec* at angle 30^o and 45^o in opposite sides to its direction are

(a) $\sqrt{3}-1, \sqrt{3}+1$ (b) $30(\sqrt{3}-1), 15(\sqrt{6}-\sqrt{3})$ (c) $30(\sqrt{3}+1), 30(\sqrt{3}-1)$ (d) None of these

24. If *OP* makes 4 revolutions in one *second*, the angular velocity in radians per second is

(a) π (b) 2π (c) 4π (d) 8π

25. A velocity $\frac{1}{4} m/s$ is resolved into two components along *OA* and *OB* making angles 30^o and 45^o respectively with the given velocity, then the component along *OB* is **[AIEEE 2004]**

(a) $\frac{1}{8}(\sqrt{6}-\sqrt{2}) m/s$ (b) $\frac{1}{4}(\sqrt{3}-1) m/s$ (c) $\frac{1}{4} m/s$ (d) $\frac{1}{8} m/s$

Advance Level

26. Two straight railways converge to a level crossing at an angle α and two trains are moving towards the crossing with velocities *u* and *v*. If *a* and *b* are the initial distances of the trains from the crossing, the least distance between them will be after time *t* given by

(a) $\frac{(au+bv)+(av+bu)\cos\alpha}{u^2+v^2+2uv\cos\alpha}$ (b) $\frac{(au+bv)-(av+bu)\cos\alpha}{u^2+v^2-2uv\cos\alpha}$ (c) $\frac{(au+bv)-(av+bu)\cos\alpha}{u^2+v^2+2uv\cos\alpha}$ (d) None of these

27. A particles moves from rest, away from a fixed point *O*, with an acceleration $\frac{\mu}{x^2}$, where *x* is the distance of the particle from *O*. If it is at rest, then its distance from *O* is *b*. The velocity when it is at a distance 2*b* from *O* is

(a) $\frac{\mu}{b}$ (b) $\frac{\mu}{b^2}$ (c) $\sqrt{\frac{\mu}{b^2}}$ (d) $\sqrt{\frac{\mu}{b}}$

28. The velocity *v* of a particle is at any time related to the distance travelled by the particle by the relation $v=as+b$, where $a>0$ and $b\le a/2$. Which of the following statements will be true for this motion (Given $s=0$ when $t=0$)

(a) The displacement of the particle at time *t* is $s=\frac{b}{a}(e^{at}-1)$ (b) The particle will experience a retardation if $b>0$

(c) The particle will be at rest at $t=0$ (d) The motion of the particle is under constant acceleration

29. A particle moving in a straight line is subject to a resistance which produces a retardation kv^3, where *v* is the velocity and *k* is a constant. If *u* is the initial velocity of the particle, then

(a) $v=\frac{u}{1+kxu}$ (b) $v=\frac{u}{1+xu}$ (c) $v=\frac{ku}{1+kxu}$ (d) $v=\frac{u}{1-kxu}$

30. A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If *u* be the speed of the man in still water and *v* be that of the stream, then $t_1:t_2=$

(a) $u+v:u-v$ (b) $u-v:u+v$ (c) $\sqrt{u+v}:\sqrt{u-v}$ (d) $\sqrt{u-v}:\sqrt{u+v}$

Relative velocity

Basic Level

31. Two trains, each 250 *m* long, are moving towards each other on parallel lines with velocities of 20 *km/hr* and 30 *km/hr* respectively. The time that elapses from the instant when they first meet until they have cleared each other is

(a) 20 *sec.* (b) 36 *sec.* (c) 30 *sec.* (d) None of these

32. A train *A* is moving towards east with a velocity of 30 *km/hr* and another train *B* is moving on parallel lines towards west with a velocity of 40 *km/hr.* The relative velocity of train *A* with respect to train *B* is

(a) 10 *km/hr* (b) 70 *km/hr* towards east (c) 70 *km/hr* towards west (d) None of these

33. Two scooterists *P* and *Q* are moving due north at 48 *km/hr* and 36 *km/hr* respectively. The velocity of *P* relative to *Q* is

(a) 12 *km/hr* due south (b) 12 *km/hr* due north (c) 84 *km/hr* due south (d) 84 *km/hr* due north

34. If two particles, *A* and *B*, moves with speed *u* and 2*u* respectively in two straight lines inclined at an angle α, then the relative velocity of *B* with respect to *A* is

(a) $u\sqrt{5+4\cos\alpha}$ (b) $u\sqrt{5-4\cos\alpha}$ (c) $u\sqrt{4-5\cos\alpha}$ (d) $u\sqrt{4+5\cos\alpha}$

35. A railway train, moving at the rate of 44 *m/sec*, is struck by a stone, moving horizontally and at right angles to the train with velocity of 33 *m/sec*. The magnitude and direction of the velocity with which the stone appears to meet the train is

(a) $50, \tan^{-1}\frac{3}{4}$ (b) $55, \tan^{-1}\left(\frac{-3}{4}\right)$ (c) $40, \cos^{-1}\frac{3}{4}$ (d) None of these

36. To a boy cycling at the rate of 4 *km/hr* eastward, the wind seems to blow directly, from the north. But when he cycles at the rate of 7 *km/hr*, it seems to blow from north-east. The magnitude of the actual velocity of the wind is

(a) $5/\sqrt{2} km/hr$ (b) $5\sqrt{2}\ km/hr$ (c) $5 km/hr$ (d) $5\frac{1}{2} km/hr$

37. If a particle *A* is moving along a straight line with velocity 3 *m/sec* and another particle *B* has a velocity 5 *m/sec.* at an angle of 60° to the path of *A*, then the velocity *B* relative to *A*

(a) $\sqrt{39}\ m/\sec$ (b) $\sqrt{19}\ m/\sec$ (c) $19\ m/\sec$ (d) None of these

38. A train *A* is moving towards east with a velocity of 30 *km/h* and another train *B* is moving on parallel lines towards west with a speed of 40 *km/h*. The velocity of train *A* relative to train *B* is

(a) 10 *km/h* (b) 70 *km/h* towards east (c) 70 *km/h* towards west (d) None of these

39. A car is travelling at a velocity of 10 *km/h* on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}\ km/hr$ when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of the car

(a) 135^o (b) 45^o (c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}(1/\sqrt{2})$

40. A man wishes to cross a river to an exactly opposite point on the other bank, if he can swim with twice the velocity of the current, then the inclination to the current of the direction in which he should swim is

(a) 90^o (b) 120^o (c) 150^o (d) None of these

41. A ship is moving with velocity 12 *km/hr* in east direction and another ship is moving with velocity 16 *km/hr* in north direction. The relative velocity of second ship with respect to first ship will be

(a) 20 *km/hr* (b) 22 *km/hr* (c) 18 *km/h* (d) $20\sqrt{2}$ *km/h*

42. A particle moves towards east from a point *A* to a point *B* at the rate of 4 *km/h* and then towards north from *B* to *C* at the rate of 5 *km/h*. If *AB* = 12 *km* and *BC* = 5 *km*, then its average speed for its journey from *A* to *C* and resultant average velocity direct from *A* to *C* are respectively

(a) $\frac{13}{9}$ *km/h* and $\frac{17}{9}$ *km/h* (b) $\frac{13}{4}$ *km/h* and $\frac{17}{4}$ *km/h* (c) $\frac{17}{9} km/h$ and $\frac{17}{9}$ *km/h*(d)

Advance Level

43. A person travelling towards the north-east, finds that the wind appears to blow from north, but when he doubles his speed it seems to come from a direction inclined at an angle $\tan^{-1}\frac{1}{2}$ on the east of north. The true direction of the wind is towards

(a) North-east (b) North (c) East (d) None of these

44. A man is walking towards north with speed 4.5 *km/hr* . Another man is running towards west with speed 6 *km/hr*. The magnitude and direction of the relative velocity of the second with respect to first is

(a) 7.5 *km/hr* at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ south of west (b) 7.5 *km/hr* at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ west of south

(c) 7.5 *km/hr* south-west (d) None of these

45. A man is swimming in a lake in a direction 30^o east of north with a speed of 5 *km/hr* and a cyclist is going on the road along the lake shore towards east at a speed of 10 *km/hr*. The direction of the swimmer relative to the cyclist is

(a) 30^o west of north (b) West-north (c) 60^o west of north (d) None of these

46. Two cars *A* and *B* are moving uniformly on two straight roads at right angles to one another at 40 and 20 *km/hr* respectively. A passes the intersection of the road when *B* has still to move 50 *km* to reach it. The shortest distance between the two cars and the time when they are closest are

(a) $20\sqrt{5}\ km, 30$ minutes (b) 20 *km*, 10 *minutes* (c) 20 *km*, 20 *minutes* (d) None of these

47. A man is tavelling in a train moving at the rate of $60\sqrt{3}$ *km/hr* and the rain is falling vertically at the rate of 60 *km/hr*. The magnitude and direction of the apparent velocity of the rain to the man sitting in the train

(a) 120 *km/hr*, making an angle of 60^o with the motion of the train

(b) 120 *km/hr* making an angle of 30^o with the motion of the train

(c) 120 *km/hr* making an angle of 45^o with the motion of the train

(d) None of these

48. A person travelling towards eastwards at the rate of 4 *km/hr*. finds that the wind seems to blow directly from the north. On doubling his speed it appears to come from north-east. The velocity and direction of the wind are

(a) $4\sqrt{2}\ km/hr, 90^o$ (b) $5\sqrt{2}\ km/hr, 60^o$ (c) $4\sqrt{2}\ km/hr, 135^o$ (d) None of these

49. A boat takes 10 *minutes* to cross a river in a straight line from a point *A* on the bank to a point *B* on the other bank and 20 *minutes* to do the return journey. The current flows at the rate of 3 *km/hr* and the speed of the boat relative to the water is 6 *km/hr*. The width of the river and the down stream distance from *A* to *B* are

(a) $\frac{\sqrt{15}}{4}, \frac{3}{4}$ (b) $\frac{\sqrt{10}}{4}, \frac{1}{3}$ (c) $\sqrt{6}, \frac{1}{2}$ (d) None of these

50. If a moving particle has two equal velocities inclined at an angle 2α such that their resultant velocity is twice as great as when they are inclined at an angle 2β, then

(a) $\cos\alpha = 2\cos\beta$ (b) $\cos\beta = 2\cos\alpha$ (c) $\cos\alpha = 3\cos\beta$ (d) $\cos\beta = 3\cos\alpha$

51. The speed of a boat in a river is *u m/sec* and that of the current is v $m/\sec$. The boat traverse *a* distance of *d metres* down the stream and then comes back to its original position. The average speed of the boat for to and fro journey is

(a) $\frac{u^2-v^2}{u^2}$ (b) $\frac{u^2-v^2}{v^2}$ (c) $\frac{u^2-v^2}{u}$ (d) $\frac{u^2-v^2}{v}$

52. A thief, when detected, jumps out of a running train at right angles to its direction with a velocity of 5 *m/min*. If the velocity of the train is 36 *km/hr*, then the angle θ between the direction in which the thief falls and the direction of motion of the train is given by

(a) $\tan^{-1}\left(\frac{5}{36}\right)$ (b) $\tan^{-1}\left(\frac{1}{20}\right)$ (c) $\tan^{-1}\left(\frac{5}{120}\right)$ (d) None of these

53. A 30 *m* wide canal is flowing at the rate of 20 *m/min*. A man can swim at the rate of 25 *m/min*. in still water. The time taken by him to cross the canal perpendicular to the flow is

(a) 1.0 *min* (b) 1.5 *min.* (c) 2.0 *min.* (d) 2.5 *min.*

54. A man crosses a 320 *m* wide river perpendicular to the current in 4 *minutes*. If in still water he can swim with a speed 5/3 times that of the current, then the speed of the current in *m/min* is

(a) 30 (b) 40 (c) 50 (d) 6

Rectilinear motion with acceleration

Basic Level

55. A body starts from rest with a uniform acceleration of $8m/\sec^2$. Then the time it will take in traversing the second *metre* of its journey is

(a) $\sqrt{2}$ *sec* (b) $\frac{1}{2}$ *sec* (c) $\left(\frac{\sqrt{2}-1}{2}\right)$ *sec* (d) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$ *sec*

56. A body starts from rest and moves with a uniform acceleration. The ratio of the distance covered in n^{th} *sec* to the distance covered in *n seconds* is

(a) $\frac{2}{n}-\frac{1}{n^2}$ (b) $\frac{1}{n^2}-\frac{1}{n}$ (c) $\frac{2}{n^2}-\frac{1}{n}$ (d) $\frac{2}{n}+\frac{1}{n^2}$

57. If a particle moves in a straight line with uniform acceleration, the distance traversed by it in consecutive seconds are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

58. If a point moves with constant acceleration from *A* to *B* in the straight line *AB* has velocities *u* and *v* at *A* and *B* respectively, then the velocity at *C*, the mid-point of *AB* is

(a) $\frac{u+v}{2}$ (b) $\sqrt{u^2+v^2}$ (c) $\sqrt{\frac{u^2+v^2}{2}}$ (d) None of these

59. A point is moving with uniform acceleration ; in the eleventh and fifteenth seconds from the commencement it moves through 720 and 960 *cm* respectively. Its initial velocity, and the acceleration with which it moves are

(a) 60, 40 (b) 70, 30 (c) 90, 60 (d) None of these

60. A particle is moving in a straight line with initial velocity *u* and uniform acceleration *f*. If the sum of the distances travelled in t^{th} and $(t+1)^{th}$ seconds is 100 *cm*, then its velocity after *t* seconds, in *cm/sec*. is

(a) 20 (b) 30 (c) 50 (d) 80

61. If the coordinates of a point moving with the constant acceleration be x_1, x_2, x_3 at the instants t_1, t_2, t_3 respectively, then

$x_1(t_2-t_3)+x_2(t_3-t_1)+x_3(t_1-t_2)=$

(a) $f(t_1-t_2)(t_2-t_3)(t_3-t_1)$ (b) $2f(t_1-t_2)(t_2-t_3)(t_3-t_1)$ (c) $\frac{f}{2}(t_1-t_2)(t_2-t_3)(t_3-t_1)$ (d) None of these

62. A body is in motion along a straight line. As it crosses a fixed point, a stop watch is started. The body travels a distance of 180 *cm* in the first three seconds and 220 *cm* in the next five *seconds*. The velocity of the body after 9 *seconds* is

(a) 66 *cm/sec* (b) 30 *cm/sec* (c) 36 *cm/sec* (d) 45 *cm/sec*

63. A body starts from rest and moves in a straight line with uniform acceleration F, the distances covered by it in second, fourth and eighth seconds are

(a) In arithmetic progression (b) In geometrical progression (c) In the ratio 1 : 3 : 7

64. A bullet of mass 0.006 *kg* travelling at 120 *metres/sec* penetrates deeply into a fixed target and is brought to rest in 0.01 *sec*. The distance through which it penetrates the target is

(a) 3 *cm* (b) 6 *cm* (c) 30 *cm* (d) 60 *cm*

65. A person travelling on a straight line moves with uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity 'v' is given by

(a) $v = \frac{v_1 + v_2}{2}$ (b) $v = \sqrt{v_1 v_2}$ (c) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (d) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

66. A particle starts with a velocity of 200 *cm/sec* and moves in a straight line with a retardation of 10 cm/sec^2. Then the time it takes to describe 1500 *cm* is

(a) 10 *sec*, 30 *sec* (b) 5 *sec*, 15 *sec*. (c) 10 *sec* (d) 30 *sec*

Advance Level

67. For $\frac{1}{m}$ of the distance between two stations a train is uniformly accelerated and $\frac{1}{n}$ of the distance it is uniformly retarded, it starts from rest at one station and comes to rest at the other. Then the ratio of its greatest velocity to its average velocity is

(a) $m+n+1:1$ (b) $\left(\frac{1}{m}+\frac{1}{n}\right):1$ (c) $\frac{1}{m}+\frac{1}{n}+1:1$ (d) $m+n+1:mn$

68. A train starts from station A with uniform acceleration f_1 for some distance and then goes with uniform retardation f_2 for some more distance to come to rest at B. If the distance between stations A and B is 4 *km* and the train takes 4 *minutes* to complete this journey, then $\frac{1}{f_1} + \frac{1}{f_2} =$

(a) 1 (b) 2 (c) 4 (d) None of these

69. A bullet moving at 100 *m/sec* is fired into a wood-block in which it penetrates 50 *cm*. If the same bullet moving with the same velocity were fired into a similar piece of wood but only 12.5 *cm* thick, then the velocity with which it emerges is

(a) 500 *m/sec* (b) $\frac{500}{\sqrt{3}} m/\sec$ (c) $500\sqrt{3} m/\sec$ (d) None of these

70. A body traversed half the distance with velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. The mean velocity of the body averaged over the whole time of motion is

(a) $\frac{v_0 + v_1 + v_2}{3}$ (b) $\frac{2v_0 + v_1 + v_2}{4}$ (c) $\frac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}$ (d) $\frac{v_0(v_1 + v_2)}{v_0 + v_1 + v_2}$

71. Two points move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity u and the second starts from rest with constant acceleration f. The distance between the two points will be maximum at time

(a) $t = \frac{2u}{f}$ (b) $t = \frac{u}{f}$ (c) $t = \frac{u}{2f}$ (d) $t = \frac{u^2}{f}$

72. A train starts from rest from a station with constant acceleration for 2 *minutes* and attains a constant speed. It then runs for 11 *minutes* at this speed and retards uniformly during the next 3 *minutes* and stops at the next station which is 9 *km* off. The maximum speed (in *km/hr*) attained by the train is

(a) 30 (b) 35 (c) 40 (d) 45

73. A point moves from rest with constant acceleration. If it covered $\frac{9}{25}$ part of its total distance in its last second of motion, then upto what time it travelled

(a) 5 *second* (b) $\frac{5}{9}$ *second* (c) (a) and (b) both are true (d) $6\frac{1}{3}$ *second*

Motion under gravity

Basic Level

74. If a particle is thrown vertically upwards with a velocity of u *cm/sec* under gravity, then the time for the particle to come to earth again is

[MNR 1995]

(a) $\frac{u}{g}$ sec (b) $\frac{2u}{g}$ sec (c) $\frac{u}{2g}$ sec (d) None of these

75. Two balls are projected at the same instant, from the same point with the same velocity, one vertically upwards and other vertically downwards. If first takes t_1 *sec* and second takes t_2 *sec* to reach the ground, then $t_1 t_2 =$

(a) $\frac{h}{g}$ (b) $2gh$ (c) $\frac{2h}{g}$ (d) gh

76. If a particle is projected vertically upwards and is at a height h after t_1 seconds and again after t_2 seconds, then $h =$ **[UPSEAT 1993, 1999]**

(a) gt_1t_2 (b) $\sqrt{gt_1t_2}$ (c) $2gt_1t_2$ (d) $\frac{1}{2}gt_1t_2$

77. From the top of a tower, 98 *m* high, a body is projected vertically upwards with a velocity of 39.2 *m/sec*. The velocity with which it strikes the ground is

(a) 58 *m/sec* (b) 60 *m/sec* (c) 58.8 *m/sec* (d) 55 *m/sec*

78. If the acceleration of falling bodies on the moon is 1.6 m/sec^2 and t_1 and t_2 seconds are timings of free fall from equal altitude above the moon's and earth's surface, then $t_1 : t_2 =$

(a) $7 : 2\sqrt{2}$ (b) $2\sqrt{2} : 7$ (c) $\sqrt{2} : 7$ (d) $2 : 7$

79. A house has multi-storeys. The lowest storey is 20 *ft* high. A stone which is dropped from the top of the house passes the lowest storey in $\frac{1}{4}$ sec. The height of the house is

(a) 100 *ft* (b) 110 *ft* (c) 110.25 *ft* (d) None of these

80. Two bodies of different masses m_1 and m_2 are dropped from different heights h_1 and h_2. The ratio of the times taken by the two bodies to fall through these distances is

(a) $h_1 : h_2$ (b) $\sqrt{h_1} : \sqrt{h_2}$ (c) $h_1^2 : h_2^2$ (d) $h_2 : h_1$

81. The time to slide down the chord through the highest point of a vertical circle is

(a) Variable (b) Constant

(c) Dependent on the position of the chord (d) None of these

82. Two particles A and B are dropped from the height of 5 *m* and 20 *m* respectively. Then the ratio of time taken by A to that taken by B, to reach the ground is

(a) 1 : 4 (b) 2 : 1 (c) 1 : 2 (d) 1 : 1

83. A body is projected upwards with a certain velocity , and it is found that when in its ascent, it is 29430 *cm* from the ground it takes 4 *seconds* to return to the same point, again. The velocity of projection of the body is

(a) 7000 *cm/sec* (b) 7848 *cm/sec* (c) 8000 *cm/sec* (d) None of these

84. A particle is projected from the top of tower 5 *m* high and at the same moment another particle is projected upward from the bottom of the tower with a speed of 10 *m/s*, meet at distance '*h*' from the top of tower, then *h*

(a) 1.25 *m* (b) 2.5 *m* (c) 3 *m* (d) None of these

Advance Level

85. If a body is projected vertically upwards with velocity u and t seconds after words another body is similarly projected with the same velocity, then the two bodies will meet after T *seconds* of the projection of the second body, where T =

(a) $\frac{u-gt}{2g}$ (b) $\frac{u-2gt}{2g}$ (c) $\frac{2u-gt}{g}$ (d) $\frac{2u-gt}{2g}$

86. A stone falling from the top of a vertical tower described m *metres*, when another is let fall from a point n *metres* below the top. If the two stones fall from rest and reach the ground together, then the time taken by them to reach the ground is

(a) $\frac{n+m}{\sqrt{2gm}}$ (b) $\frac{n+m}{\sqrt{2gn}}$ (c) $\frac{n-m}{\sqrt{2gm}}$ (d) $\frac{m-n}{\sqrt{2gn}}$

87. Let $g_1\,m/\sec^2$, $g_2\,m/\sec^2$ be the accelerations due to gravity at two places P and Q. If a particle occupies n *seconds* less and acquires a velocity of m *metre/sec* more at place P than place Q in falling through the same distance, then m/n equals

(a) $g_1 g_2$ (b) $\sqrt{\frac{g_1}{g_2}}$ (c) $\sqrt{\frac{g_2}{g_1}}$ (d) $\sqrt{g_1 g_2}$

88. After a ball has been falling under gravity for 5 *seconds* it passes through a pane of glass and loses half of its velocity and now reaches the ground in one second. The height of the glass above the ground is

(a) 2000 *m* (b) 2500 *m* (c) 2943 *m* (d) None of these

89. A tower is 61.25 *m* high. A rigid body is dropped from its top and at the same instant another body is thrown upwards from the bottom of the tower with such a velocity that they meet in the middle of the tower. The velocity of the projection of the second body is

(a) 20 *m/sec* (b) 25 *m/sec* (c) 24.5 *m/sec* (d) None of these

90. A particle is dropped from the top of a tower h *metres* high and at the same moment another particle is projected upwards from the bottom of the tower. If the two particles meet when the upper one has described $\left(\frac{1}{n}\right)^{th}$ of the distance, then the velocities when they meet are in the ratio

(a) $2:n-2$ (b) $(n-2):2$ (c) $(n+2):2$ (d) $2:n+2$

91. A particle was dropped from the top of the tower of height h and at the same time another particle is thrown vertically upwards form the bottom of the tower with such a velocity that it can just reach the top of the tower. The two particles meet at a height

[UPSEAT 1998]

(a) $\frac{h}{2}$ (b) $\frac{3}{5}h$ (c) $\frac{3h}{4}$ (d) $\frac{h}{4}$

92. A stone is dropped from an aeroplane which is rising with acceleration f and t *seconds* after this another stone is dropped. The distance between the two stones at time T after the *second* stone is dropped is

(a) $\frac{1}{2}(g+f)(t+T)$ (b) $\frac{1}{2}(g+f)(t+2T)$ (c) $\frac{1}{2}(g+f)(2t+T)$ (d) $\frac{1}{2}(g-f)(t+2T)$

93. A stone is dropped slowly from the top of the wall and it reaches the surface of the water with the velocity 3924 *cm/sec,* if sound of splash is heard after $4\frac{109}{475}$ *seconds*, then the velocity of sound will be

(a) 312 *metre/sec* (b) 302 *metre/sec.* (c) 321 *metre/sec* (d) 342 *metre/sec*

Laws of motion, Apparent weight of a body on lift

Basic Level

94. A man on a lift ascending with an acceleration $f\, m/\sec^2$ throws a ball vertically upwards with a velocity of $v\, m/\sec$ relative to the lift and catches it again in *t seconds*. The value of *t* is

(a) $\frac{2v}{f-g}$ (b) $\frac{v}{f-g}$ (c) $\frac{v}{f+g}$ (d) $\frac{2v}{f+g}$

95. A body weighs most **[Roorkee 1994]**

(a) At the earth's surface (b) Above the earth's surface (c) Inside the earth (d) At the centre of the earth

96. A dyne is the force which produces an acceleration of $1 cm/\sec^2$ when acted on a mass of

(a) $1 mg$ (b) $10 gm$ (c) $1 gm$ (d) $1 kg$

97. A balloon of mass *M* ascends with a uniform acceleration *f*. If a certain part of the balloon is detached in such a way that the acceleration is doubled, then the mass of the detached portion is

(a) $\frac{fM}{f+g}$ (b) $\frac{fM}{f+2g}$ (c) $\frac{fM}{2f+g}$ (d) $\frac{gM}{2f+g}$

98. In a rectilinear motion a particle of mass *m* changes its velocity from *u* to *v* in describing a distance *x*. If *F* is the constant force which produces the changes, then $F=$

(a) $\frac{1}{2}m(v^2-u^2)$ (b) $\frac{1}{2x}m(v^2-u^2)$ (c) $\frac{1}{2x}m(v^2+u^2)$ (d) None of these

99. A cricket ball of mass 200 *gm* moving with a velocity of 20 *m/sec* is brought to rest by a player in 0.1 *sec*. The average force applied by the player is

(a) 4×10^3 *dynes* (b) 4×10^4 *dynes* (c) 4×10^5 *dynes* (d) 4×10^6 *dynes*

100. A train whose mass is 16 metric tons, moves at the rate of 72 *km/hr*. After applying breaks it stops in 500 *metre*. What is the force exerted by breaks obtaining it to be uniform

(a) 800 *N* (b) 1600 *N* (c) 3200 *N* (d) 6400 *N*

101. A mass of 8 *kg* is rolled a grass with a velocity of 28 *m/sec*. If the resistance be $\left(\frac{1}{10}\right)^{th}$ of the weight, then the body comes to rest after travelling

(a) 200 *m* (b) 400 *m* (c) 600 *m* (d) 800 *m*

102. If a force F_1 acts on a mass of 10 *kg* and in one-fifth of a second produces in it a velocity of 2 *m/sec* and the other force F_2 acting on a mass of 625 *kg* in a minute produces in it a velocity of 18 *km/hr*, then $F_1 : F_2$

(a) 24 : 25 (b) 48 : 25 (c) 24 : 5 (d) 48 : 125

103. In a diving competition, the boards fixed at a height of 10 *m* above the water level. A competitor jumps from the board and dives to a depth of 5 *m*. If the mass of the competitor is 60 *kg*, then the resistance offered by the water is

(a) 588 *N* (b) 1176 *N* (c) 1764 *N* (d) None of these

104. A man weighing 60 *kg* jumps off a railway train running on horizontal rails at 20 *km/h* with a packet weighing 10 *kg* in his hand. The thrust of the packet on his hand is

(a) 0 (b) 10 *kg wt.* (c) 50 *kg wt.* (d) 70 *kg wt.*

105. A hockey stick pushes a ball at rest for 0.01 *sec* with an average force of 50 *N*. If the ball weighs 0.2 *kg*, then the velocity of the ball just after being pushed is

(a) 3.5 *m/sec* (b) 2.5 *m/sec* (c) 1.5 *m/sec* (d) 4.5 *m/sec*

106. A bullet of mass 10 *gram* fired into a wall with a velocity of 10 *m/sec* loses its velocity in penetrating through 5 *cm* into the wall. The average force exerted by the wall is

(a) 10^4 *gm wt* (b) 10^6 *dynes* (c) 10^5 *dynes* (d) None of these

107. If body of mass *M kg* and at rest is acted upon by a constant force of *W kg* weight, then in seconds it moves through a distance of

(a) $\frac{gTW}{2M}$ *metre* (b) $\frac{gTW^2}{2M}$ *metre* (c) $\frac{g^2TW}{2M}$ *metre* (d) $\frac{gT^2W}{2M}$ *metre*

108. A train is moving with constant velocity. If the resistance of its motion is 10 *lbs per ton* (of mass) and the force exerted by the engine is 200 *lbs wt*, then the mass of engine is

(a) 20 *tons* (b) 200 *tons* (c) 2000 *tons* (d) 2 *tons*

109. If the barrel of the gun is cut down 50 *cm*, then a bullet of 49 *kilogram* fire out with velocity 361 *m/sec* instead of 441 *m/sec*. The approximate thrust of gas on the bullet will be

(a) 317.6 metric ton weight (b) 318.4 metric ton weight (c) 319.3 metric ton weight (d) 320.8 metric ton weight

110. A cart of 100 *kg* is free to move on smooth rails and a block of 20 *kg* is resting on it. Surface of contact between the cart and the block is smooth. A force of 60 *Newton* is applied to the cart. Acceleration of 20 *kg*, block in *metres per second*^{2} is **[UPSEAT 1993]**

(a) 3 (b) 0.6 (c) 0.5 (d) 0

111. A man having mass 70 *kilogram* is standing in a lift which is moving with uniform acceleration of 25 *cm/sec*^{2}. What will be the reaction of floor when lift coming down

(a) $\frac{70\times 956}{981} kg\text{-}wt$ (b) $\frac{70\times 1006}{981} kg\text{-}wt$ (c) $\frac{70\times 25}{981} kg\text{-}wt$ (d) $\frac{70\times 981}{25} kg\text{-}wt$

Advance Level

112. From the gun cartage of mass *M*, a fire arm of mass *m* with velocity *u* relative to gun cartage is fired. The real velocities of fire arms and gun cartage will be respectively

(a) $\frac{Mu}{M+m} = \frac{Mu}{M-u}$ (b) $\frac{Mu}{M+m} = \frac{mu}{M+m}$ (c) $\frac{M+m}{Mu} = \frac{M+m}{mu}$ (d) $\frac{M+m}{M-m} = \frac{M+m}{Mm}$

113. The shortest time from rest to rest in which a steady load of *P* tons can lift a weight of *W* tons through a vertical distance *h* feet is

(a) $\sqrt{\left(\frac{2h}{g}.\frac{P}{P-W}\right)}$ (b) $\sqrt{\left(\frac{2h}{g}.\frac{P}{P+W}\right)}$ (c) $\sqrt{\left(\frac{2h}{g}.\frac{P+W}{P-W}\right)}$ (d) None of these

114. A shot, whose mass is 400 *kg*, is discharged from a 80 metric ton gun with a velocity of 490 *m/sec*. The necessary force required to stop the gun after a recoil of 1.6 *m* is

(a) 245/16 metric ton (b) 15 metric ton (c) 20 metric ton (d) None of these

115. A rough plane is 100 *ft* long and is inclined to the horizon at an angle $\sin^{-1}(3/5)$, the coefficient of friction being 1/2, and a body slides down it from rest at the highest point, the velocity on reaching the bottom would be

(a) $16/\sqrt{5}$ *ft/sec* (b) $16\ ft/\sec$ (c) $16\sqrt{5}\ ft/\sec$ (d) $16/\sqrt{7}\ ft/\sec$.

116. A particle slide down a rough inclined plane whose inclination to the horizontal is 45^o and whose coefficient of friction is 3/4. The time of descending the distance $4\sqrt{8/5}$ *m* down the plane is

(a) 0.8 *sec* (b) 1.2 *sec* (c) 1.4 *sec* (d) 1.62 *sec*

117. The times of ascent and descent of a particle projected along an inclined plane of inclination α are t_1 and t_2 respectively, the coefficient of friction is

(a) $\frac{t_2-t_1}{t_2+t_1}\tan\alpha$ (b) $\frac{t_2+t_1}{t_2-t_1}\tan\alpha$ (c) $\frac{t_2^2-t_1^2}{t_2^2+t_1^2}\tan\alpha$ (d) $\frac{t_2^2+t_1^2}{t_2^2-t_1^2}\tan\alpha$

Motion of two particles connected by a string

Basic Level

118. A pulley carrying a total load *W* hangs in a loop of a chord which passes over two fixed pulleys and has unequal weights *P* and *Q* freely suspended from the ends, each segment of the chord vertical. If *W* remains at rest, then *W* =

(a) $\frac{PQ}{P+Q}$ (b) $\frac{2PQ}{P+Q}$ (c) $\frac{3PQ}{P+Q}$ (d) $\frac{4PQ}{P+Q}$

119. Two particles of masses m_1 and m_2 are connected by a light inextensible string m_2 is placed on a smooth horizontal table and the string passes over a light pulley at the edge of the table and m_1 is hanging freely. If m_1 is replaced by m_2 and m_2 is replaced by m_3, then the acceleration of the system remains unaltered if m_1, m_2, m_3 are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

120. A light string passing over a light smooth pulley carries masses of 3 *kg* and 5 *kg* at its ends. If the string breaks after the masses have moved 9 *m*, then the 3 *kg* mass will farther rise $(g = 10 m/\sec^2)$

(a) 1.75 *m* (b) 1.95 *m* (c) 2.05 *m* (d) 2.25 *m*

121. A body of mass 90 *gm* is placed on a smooth table from the distance 2.45 *metre* from end and is attached to a rope which is hanging from the end of table, then time taken by body to reach to end of table will be

(a) $\sqrt{2}$ sec (b) $\sqrt{3}$ sec (c) 2 sec (d) $\sqrt{5}$ sec

122. Two bodies of mass 8 and 10 *gm* is attached to a light rope which is passing over a smooth pulley. If this system is given to a velocity $\frac{3}{16} g\, cm/\sec$. then small body will move downwards and heavy body will move upwards, then after what time they will move in opposite directions

(a) $\frac{25}{16}$ sec (b) $\frac{23}{14}$ sec (c) $\frac{27}{16}$ sec (d) $\frac{81}{512}$ sec

123. Two masses m_1 and m_2 are connected by a light inextensible string and suspended over a smooth fixed pulley. Then **[Roorkee 1994]**

(a) Pressure on the pulley = $m_1 g$ (b) Pressure on the pulley = $m_2 g$

(c) Pressure $< (m_1 + m_2)g$ (d) Pressure > $(m_1 + m_2)g$

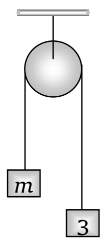
124. Two strings pass over a smooth pulley, on one side they are attached to masses of 3 and 4 *kg* respectively, and on the other to a mass of 5 *kg*. Then the tensions of the strings are

(a) 2, 3 *kg wt*. (b) 5/2, 10/3 *kg wt*. (c) 3, 4 *kg wt*. (d) None of these

125. A body of mass 5 *gram* is placed on a smooth table and is connected by a string passing over a light smooth pulley at the edge with a body of mass 10 *gram*. The common acceleration is

(a) 2*g*/3 (b) 3 *g*/2 (c) 2.5 *g* (d) 0.5 *g*

126. Two masses are attached to the pulley as shown in fig., find acceleration of centre of mass



(a) $\frac{g}{4}$ (b) $\frac{-g}{4}$ (c) $\frac{g}{2}$ (d) $\frac{-g}{2}$

Advance Level

127. A light string passing over a light smooth pulley carries masses of 3 *kg* and 5 *kg* at its ends. If the string breaks after the masses have moved 9 *m*, how much further the 3 *kg* mass will rise (Take *g* = 10 m/sec^2)

(a) 1.75 *m* (b) 1.95 *m* (c) 2.05 *m* (d) 2.25 *m*

128. A mass 2*Q* on a horizontal table, whose coefficient of friction is $\sqrt{3}$ is connected by a string with a mass $6Q$ which hangs over the edge of the table. Eight seconds after the commencement of the motion, the string breaks. The distance of the new position of equilibrium of 2*Q* from its initial position is

(a) 117.6 *m* (b) 120.4 *m* (c) 130.4 *m* (d) None of these

129. A mass of 6 *kg* slides down a smooth inclined plane whose height is half its length, and draws another mass from rest over a distance 3 *m* in 5 *sec* along a smooth horizontal table which is level with the top of the plane over which the string passes, the mass on the table is

(a) 86.5 *kg* (b) 96.5 *kg* (c) 106.5 *kg* (d) 116.5 *kg*

130. Masses of 5 *kg* and 3 *kg* rest on two inclined planes each inclined at 30^o to the horizontal and are connected by a string passing over the common vertex. After 2 *second* the mass of 5 *kg*. is removed. How far up the plane will the 3 *kg*. mass continue to move

(a) 1.9/8 *m* (b) 2.9/8 *m* (c) 3.9/8 *m* (d) 4.9/8 *m*

Impact of elastic ladies

Basic Level

131. Two equal perfectly elastic balls impinges directly, then after impact they

(a) Are at rest (b) Interchange their velocities

(c) Move with the same velocities (d) Move with twice velocities

132. A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution is *e*, their velocities after the impact are as

[UPSEAT 1999]

(a) 1 : *e* (b) $e:1$ (c) $1+e:1-e$ (d) $1-e:1+e$

133. A ball is dropped from a height of 22.5 *metre* on a fixed horizontal plane. If $e=2/5$, then it will stop rebounding after

(a) 5 *sec.* (b) 6 *sec.* (c) 7 *sec.* (d) 8 *sec.*

134. An elastic ball with coefficient of elasticity 1/2 is dropped from rest at a height *h* on a smooth floor. The total distance covered by the ball is

(a) More than 2*h* (b) Less than 2*h* but more than (3/2)*h*

(c) Less than (3/2)*h* but more that (4/3)*h* (d) Less then (4/3)*h*

135. Hailstorm are observed to strike the surface of a frozen lake in a direction making an angle of 30^o to the vertical and to rebound at an angle of 60^o to the vertical. Assuming the contact to be smooth, the coefficient of restitution is **[MNR 1986]**

(a) 1/3 (b) 2/3 (c) $1/\sqrt{3}$ (d) None of these

136. Any heavy elastic ball falls from the ceiling of any room and after rebounding two times reaches the half of the height of ceiling. The coefficient of restitution is

(a) $(0.50)^{1/2}$ (b) $(0.50)^{1/3}$ (c) $(0.50)^{1/4}$ (d) $(0.25)^{1/2}$

137. A ball of 1 *kg* moving with velocity 7 *m/sec* overtakes and collides with a ball of mass 2 *kg* moving with velocity 1 *m/sec*. in the same direction. If $e=3/4$, the velocity of the lighter ball after impact is

(a) 120 *m/sec* (b) $\frac{1}{2}$ *m/sec* (c) 1 *m/sec* (d) 0 *m/sec*

138. A ball is dropped from a height of 25 *dm* above a fixed horizontal plane. If it rebounds to a height of 16 *dm*, then the coefficient of restitution is

(a) 16/25 (b) 0.8 (c) 16 *g*/25 (d) 0.8 *g*

Advance Level

139. A ball falls from a height *h* upon a fixed horizontal plane, *e* is the coefficient of restitution, the whole distance described by the ball before it comes to rest is

(a) $\frac{1+e^2}{1-e^2}h$ (b) $\frac{1-e^2}{1+e^2}h$ (c) $\frac{1+e^2}{(1-e^2)h}$ (d) $\frac{1-e^2}{(1+e^2)h}$

140. A ball is thrown from a point at a distance c from a smooth vertical wall and against the wall and returns to the point of projection. If e as the coefficient of restitution, α the angle of projection, the time of flight of the ball is

(a) $\left[\frac{2(1-e)c}{eg}\tan\alpha\right]^{1/2}$ (b) $\left[\frac{2(1+e)c}{eg}\tan\alpha\right]^{1/2}$ (c) $2(1+e)c\tan\alpha$ (d) None of these

141. A ball of mass 8 *kg* and moving with velocity 4 *m/sec* collides with another ball of mass 10 *kg* moving with velocity 2 *m/sec* in the same direction. If the coefficient of restitution is 1/2, the velocities (in *m/sec*) of the balls after impact are **[MNR 1983]**

(a) 0, 0 (b) 7/3, 10/3 (c) 2/3, 5/3 (d) 2, 2

142. Three balls of masses m_1, m_2, m_3 are lying in straight line. The first ball is moved with a certain velocity so that it strikes the second ball directly, then the second ball collides with the third. If the coefficient of elasticity for each ball is e and after impact first ball comes to rest, while after second impact the second ball comes to rest. Then m_1, m_2, m_3 are in

(a) A.P., (b) G.P. (c) H.P. (d) None of these

143. A sphere impings directly on an equal sphere which is at rest. Then the original kinetic energy lost is equal

(a) $\frac{1+e^2}{2}$ times the initial K.E.(b) $\frac{1-e^2}{2}$ (c) $\frac{1-e^2}{2}$ times the initial K.E. (d) None of these

Projectile motion

Basic Level

144. A particle is projected with velocity $2\sqrt{(2g)}$ so that it just clears two walls of equal height 2 *metre*, which are at a distance of 4 *metre* from each other. What is the time of passing from one wall to another

(a) $\sqrt{(2/g)}$ (b) $\sqrt{(2g)}$ (c) $2\sqrt{(2/g)}$ (d) $\sqrt{(g/2)}$

145. A particle is thrown over a triangle from one end of horizontal base. If α, β are the base angles and θ the angle of projection, then

(a) $\tan\theta = \tan\alpha - \tan\beta$ (b) $\tan\theta = \tan\beta - \tan\alpha$ (c) $\tan\theta = \tan\alpha + \tan\beta$ (d) None of these

146. A particle is projected down an inclined plane with a velocity of 21 *m/sec* at an angle of 60^o with the horizontal. Its range on the inclined plane, inclined at an angle of 30^o with the horizontal is

(a) 21 *dm* (b) 2.1 *dm* (c) 30 *dm* (d) 6 *dm*

147. If you want to kick a football to the maximum distance the angle at which it should be kicked is (assuming no resistance) **[MNR 1981, 95]**

(a) 45^o (b) 90^o (c) 30^o (d) 60^o

148. The path of projectile in vacuum is a **[MNR 1971; UPSEAT 1998]**

(a) Straight line (b) Circle (c) Ellipse (d) Parabola

149. A particle is projected under gravity $(g = 9.81 m/\sec^2)$ with a velocity of $29.43\, m/\sec$ at an elevation of 30°. The time of flight in seconds to a height of 9.81 *m* are

(a) 5, 1, 5 (b) 1, 2 (c) 1, 5, 2 (d) 2, 3

150. From the top of a tower of height 100 *m*, a ball is projected with a velocity of 10 *m/sec*. It takes 5 *seconds* to reach the ground. If $g = 10 m/\sec^2$, then the angle of projection is

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

151. A particle is projected with initial velocity u making an angle α with the horizontal, its time of flight will be given by

[MNR 1979; UPSEAT 1998]

(a) $\frac{2u\sin\alpha}{g}$ (b) $\frac{2u^2\sin\alpha}{g}$ (c) $\frac{u\sin\alpha}{g}$ (d) $\frac{u^2\sin\alpha}{g}$

152. The escape velocity for a body projected vertically upwards is 11.2 *km/sec*. If the body is projected in a direction making an angle of 60^o with the vertical, then the escape velocity will be

(a) 11.2 *km/sec* (b) $5.6\sqrt{3} km/\sec$ (c) 5.6 *km/sec* (d) None of these

153. A particle is projected with the speed of $10\sqrt{5} m/\sec$ at an angle of 60^o from the horizontal. The velocity of the projectile when it reaches the height of 10 *m* is ($g = 9.8\, m/\sec^2$)

(a) $4\sqrt{(19)}\, m/\sec$ (b) $\sqrt{(179)}\, m/\sec$ (c) $15\, m/\sec$ (d) $5\sqrt{(15)}\, m/\sec$

154. From the top of a hill of height 150 *m*, a ball is projected with a velocity of 10 *m/sec*. It takes 6 *second* to reach the ground. The angle of projection of the ball is

(a) 15^o (b) 30^o (c) 45^o (d) 60^o

155. A cricket ball is thrown from the top of a cliff 200 *m* high with a velocity of 80 *m/sec*. at an elevation of 30^o above the horizon, the horizontal distance from the foot of the cliff to the point where it hits the ground is (take $g = 10 m/\sec^2$)

(a) 595.3 *m* (b) 695.3 *m* (c) 795.3 *m* (d) 895.3 *m*

156. A particle is projected with a velocity of 39.2 *m/sec* at an angle of 30^o to the horizontal. It will move at right angle to the direction of projection after the time

(a) 8 *sec* (b) 5 *sec* (c) 6 *sec* (d) 10 *sec*

157. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and *R* be the maximum range on the horizontal plane. Then R_1, R, R_2 are in

(a) Arithmetic-Geometric progression (A.G.P.) (b) A.P.
(c) G.P. (d) H.P.

158. If t_1 and t_2 are the times of flight of two particles having the same initial velocity *u* and range *R* on the horizontal, then $t_1^2 + t_2^2$ is equal to

(a) 1 (b) $4u^2/g^2$ (c) $u^2/2g$ (d) u^2/g

159. A particle is projected at an angle of 45^o with a velocity of 9.8 *metre per second*. The horizontal range will be

(a) 9.8 *metre* (b) 4.9 *metre* (c) $9.8/\sqrt{2}$ *metre* (d) $9.8\sqrt{2}$ *metre*

160. Two balls are projected respectively from the same point in directions inclined at 60^o and 30^o to the horizontal. If they attain the same height, the ratio of their velocities of projection is

(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$ (c) $1:1$ (d) $1:2$

161. If a projectile having horizontal range of 24 acquires a maximum height of 8, then its initial velocity and the angle of projection are

[Roorkee Screeninig 1990]

(a) $24\sqrt{g}, \sin^{-1}(0.6)$ (b) $8\sqrt{g}, \sin^{-1}(0.8)$ (c) $5\sqrt{g}, \sin^{-1}(0.8)$ (d) $5\sqrt{g}, \sin^{-1}(0.6)$

162. The range of a projectile fixed at an angle of 15^o is 50 *m*, if it is fixed with the same speed at an angle of 45^o, then the range will be

[UPSEAT 2002]

(a) 50 *m* (b) 100 *m* (c) 150 *m* (d) None of these

163. A particle is thrown with velocity *u* at an angle of 30^o from horizontal line when it becomes perpendicular to its original position

[UPSEAT 2002]

(a) $\frac{2u}{g}$ (b) $2ug$ (c) $\frac{u\sqrt{3}}{g}$ (d) None of these

164. *AB* is the vertical diameter of a circle in a vertical plane. Another diameter *CD* makes an angle of 60^o with *AB*, then the ratio of the time taken by a particle to slide along *AB* to the time taken by it to slide along *CD* is

(a) 1 : 1 (b) $\sqrt{2}:1$ (c) $1:\sqrt{2}$ (d) $3^{1/4}:2^{1/2}$

165. A particle is projected up a smooth inclined plane of inclination 60^o along the line of greatest slope. If it comes to instantaneous rest after 2 *second* then the velocity of projection is $(g = 9.8 m/\sec^2)$

(a) 9.8 *m/s* (b) 10 *m/s* (c) 16.97 *m/s* (d) 19.6 *m/s*

166. A body is projected through an angle α from vertical so that its range is half of maximum range, α is

(a) 60^o (b) 75^o (c) 30^o (d) 22.5^o

Advance Level

167. The angular elevation of an enemy's position on a hill *h* feet high is β. Show that is order to shell if the initial velocity of the projectile must not be less than

(a) $[gh(1+\sin\beta)]^{1/2}$ (b) $[gh(1-\sin\beta)]^{1/2}$ (c) $[gh(1+\text{cosec }\beta)]^{1/2}$ (d) $[gh(1-\text{cosec }\beta)]^{1/2}$

168. The ratio of the greatest range up an inclined plane through the point of projection and the distance through which a particle falls freely during the corresponding time of flight is

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 3

169. A stone is projected so that its horizontal range is maximum and equal to 80 *ft*. Its time of flight and the height it rises are

(a) $\sqrt{3}, 1$ (b) $\sqrt{4}, 15$ (c) $\sqrt{5}, 20$ (d) None of these

170. The velocity and direction of projection of a shot which passes in horizontal direction just over the top of a wall which is 50 yds. away and 75 feet high

(a) $40, 30^o$ (b) $40\sqrt{6}, 45^o$ (c) $50, 60^o$ (d) None of these

171. A shot fired from a gun on top of a tower, 272 feet high hits the ground at a distance of 4352 feet in 17 *seconds*. The velocity and direction of projection are

(a) 256, 30^o (b) $256\sqrt{2}, 45^o$ (c) $180, 60^o$ (d) None of these

172. If the time taken in slipping down on smooth inclined plane is twice to the time taken in falling from the vertical height of that plane, then the inclination of plane will be

(a) 45^o (b) 60^o (c) 75^o (d) 30^o

Work power and Energy

Basic Level

173. A labour throws 12 bricks per minute from the ground so as to just reach the roof 3.3 *metres* high. If each brick weights 3.75 *kg*, the horse power at which he is working, is

(a) 0.0325 (b) 0.325 (c) 3.25 (d) None of these

174. A weight of 10 metric tons is dragged in half an hour through a distance of 110 *metre* up a rough inclined plane inclined at an angle of 30^o to the horizon, the coefficient of friction being $1/\sqrt{3}$. The horse power (nearly) of the engine by which this work will be done is

(a) 6 (b) 8 (c) 10 (d) 20

175. A body is 3 *kg* is projected upwards with such a velocity that it can reach the height 196 *metres* only. The kinetic energy of the body at the time of projection is

(a) 5000 *Joule* (b) 5762.4 *Joule* (c) 6000 *Joule* (d) None of these

176. A bullet of 125 *grams* strikes a target with a velocity of 400 *metres per second* and is embedded in it. If the target weighs 10 *kg* and is free to move, then the velocity of the target after impact is

(a) 400/81 *m/sec* (b) 400 *m/sec* (c) 300 *m/sec* (d) None of these

177. A bullet is shot with a velocity of 600 *m/sec* into a target weighing 12 *kg* and is free to move with a velocity 1.5 *m/sec* after impact. Then the percentage loss of kinetic energy in the impact is

(a) 79.75 % (b) 89.75 % (c) 99.75 % (d) None of these

178. A 15 *kg* block is moving on ice with a speed of 5 *metre per second* when a 10 *kg* block is dropped onto it vertically. The two together move with a velocity which in *metre per second* is

(a) 3 (b) $\sqrt{(15)}$ (c) 5 (d) Indeterminate

179. A ball weighing 0.01 *kg* hits a hard surface vertically with a speed of 5 *m/sec* and rebounds with the same speed. The ball reamins in contact with the surface for 0.01 *sec*. The average force exerted by the surface on the ball (in Newton) is

(a) 0.1 (b) 1.0 (c) 5.0 (d) 10.0

Advance Level

180. A labour has to throw bricks near mistry 16 feet vertically above. He throws the bricks in such a manner that the brick reach the mistry with the velocity of 16 *ft/sec*. If the throws bricks such that bricks just reach the mistry, then the portion of the energy saved is

(a) 1/3 (b) 1/4 (c) 1/5 (d) 1/6

181. A hammer of mass 2 *kg* falls vertically through 1 *metre* on the top of a nail of mass 100 *gm* and drives it a distance of 10 *cm* in the ground. The resistance of the ground is

(a) 3441/210 *kg wt* (b) 4441/210 *kg wt* (c) 5441/210 *kg wt* (d) None of these

182. A bullet of mass *m* penetrates a thickness *a* of a plate of mass *M* at rest. If this plate is free to move, then the thickness to which the bullet will penetrate is

(a) $Ma/(m+M)$ (b) $ma/(m+M)$ (c) $(M-m)a/(m+M)$ (d) None of these

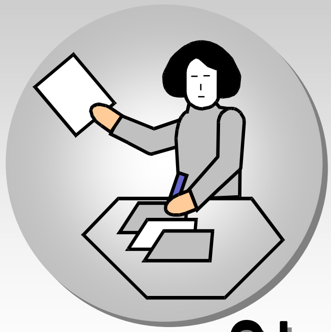
183. A glass marble, whose mass is $(1/10) kg$ falls from a height of 2.5 *m* and rebounds to a height of 1.6 *m*. Then the average force between the marble and the floor, if the time during which they are in contact be one-tenth of a second, is

(a) 10.58 *N* (b) 11.58 *N* (c) 12.58 *N* (d) 13.58 *N*

184. A fire engine lifts 50 *kg* water up to 2 *m* height per minutes and throws it out with the velocity of 19.62 *m/sec*. The horse power of engine will be

(a) 0.12 (b) 0.24 (c) 0.36 (d) 0.48

* * *



Answer Sheet

Dynamics *Assignment (Basic and Advance Level)*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	c	b	b	b	b	c	c	a	d	d	b	a	c	b	c	b	d	d	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	b	b	d	a	c	d	a	a	c	b	b	b	b	b	c	b	b	a	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
a	d	c	a	c	a	b	c	a	a	c	b	c	d	c	a	a	c	c	c
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
c	b	d	d	a	a	c	b	c	c	b	c	a	b	c	d	c	b	c	b
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
b	c	b	a	d	c	d	c	d	b	c	b	d	d	a	b	c	b	d	d
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
b	b	c	a	b	b	d	a	d	c	a	b	a	a	c	d	c	d	b	d
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
d	c	c	b	a	b	d	a	d	d	b	d	a	b	a	c	d	b	a	b
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160

b	b	c	c	c	d	a	d	b	a	a	c	a	b	c	a	d	b	a	b
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
c	b	d	c	c	b	c	c	c	b	b	d	a	b	b	a	c	a	d	c
181	182	183	184																
b	a	d	b																