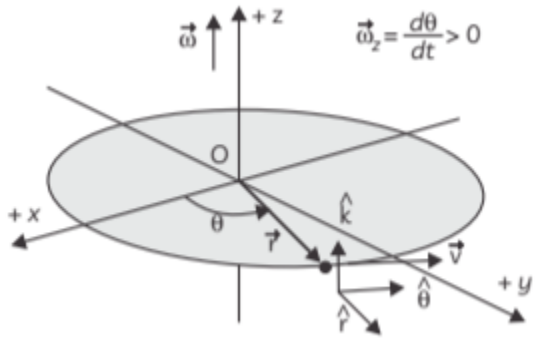
Systems of Particles and Rotational Motion

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Every particle of a rotating body moves in a circle. The linear velocity of the particle is related to the angular velocity. The relation between these two quantities involves a vector product for rotation about a fixed axis, the direction of the vector does not change with time. Its magnitude may, however, change from instant to instant. For the more general rotation, both the magnitude and the direction of may change from instant to instant.



(A) What is the angular velocity of a bike wheel if it takes 3 seconds to complete one revolution?

(B) A 95 kg person is riding a Ferris wheel with a radius of 10 m. The wheel rotates at a constant angular rate of one revolution per minute. Determine the rider's linear velocity.

(C) Two men stand facing each other on two separate boats drifting on quiet water. Both are holding the end of a rope. Why do the two boats always meet at the same location, whether each guy pulls alone or both full together? Will the time taken in the two scenarios be different? Ignore friction.

Ans. (A) The definition of angular velocity is

$$\omega = \frac{\Delta\theta}{\Delta t}$$

By identifying the given information to be $\Delta\theta = 2$ and $\Delta t = 3s$, we can plug this into the equation to calculate the angular velocity:

$$\omega = \frac{2\pi}{3s} = 2.09\ s^{-1}$$

(B)

$$\text{Convert } \frac{\text{rotations}}{\text{minute}} \text{ to } \frac{\text{meters}}{\text{second}}:$$

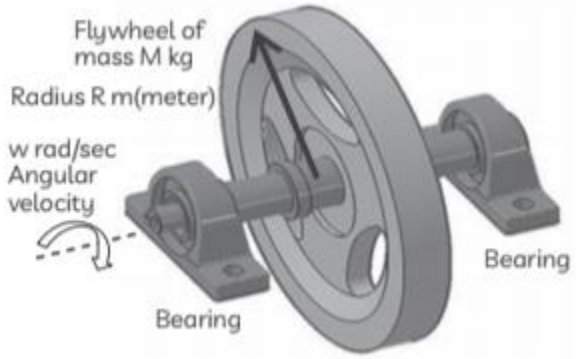
$$= 1\,\frac{\text{rotations}}{\text{minute}} \times \frac{1\text{minute}}{60\,\text{seconds}}$$

$$= \frac{2\times\pi\times 10\text{metres}}{1\,\text{rotation}}$$

$$= \frac{1.05\text{metres}}{\text{second}}$$

(C) The men on the two boats floating onwater constitute a single system. So, the forces applied by the two men are internal. Whether each man pulls separately or both pull together, the centre of mass of the system of boats remains fixed due to the absence of any external forces. Consequently, the two boats meet at a fixed point, which is the centre of mass of the system.

2. A heavy wheel called a flywheel is linked to the shaft of a steam engine, automobile vehicle's sudden increase or drop in speed. It enables a gradual shift in speed and prevents jarring motion, ensuring a comfortable ride for passengers.



(A) A flywheel is so constructed that almost the whole of its mass is concentrated at its rim, because:

(a) it increases the moment of inertia of the flywheel

(b) it decreases the moment of inertia of the flywheel

(c) it increases the speed of the flywheel

(d) it increases the power of the flywheel

(B) Assertion (A): A wheel moving down a perfectly frictionless inclined plane shall undergo slipping (not rolling).

Reason (R): For rolling torque is required, which is provided by tangential frictional

force.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

(C) A circular disc constructed of iron and aluminium must have the greatest moment of inertia about the geometrical axis. How this is feasible?

(a) Aluminium at interior surrounded to it.
(b) Iron at the interior and aluminium surrounding to it.
(c) Using iron and aluminium layers in an alternative order.
(d) Sheet of iron is used at both the external surface and aluminium sheet's internal layer.

(D) A ferries wheel has a trip duration of 3 minutes, which means it takes three minutes to complete one full revolution. What is the angular velocity of the ferries wheel in rad/s if it only takes people around once?

(a) 18.85
(b) 0.35
(c) 0.035
(d) 2.094

(E) A CD rotates at a rate of 5 rad/s in the positive counterclockwise direction. After pressing play, the disk is speeding up at a rate of 2 rad/s2. What is the angular velocity of the CD in rad/s after 4 seconds?

(a) 13
(b) 19.5
(c) 6
(d) 10

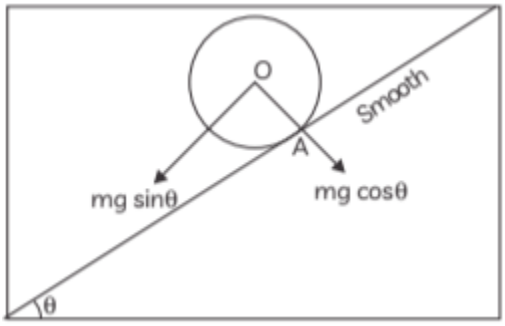
Ans. (A) (a) it increases the moment of inertia of the flywheel

Explanation: The concentration of the mass at the rim increases the moment of inertia (1) of the flywheel. Such a wheel gains or loses some kinetic energy of rotation $\frac{1}{2} I\omega^2$. It brings about a relatively smaller change in its angular speed . Hence, such a flywheel helps in maintaining uniform rotation.

(B) (b) Both A and R are true and R is not correct explanation of A.

Explanation: As the surface of the inclined plane is smooth, there is no friction between it and the wheel. The wheel rolls along the gradient as a result of mgsine.

A body's rotational motion is caused by torque. Yet, there is no torque occurring on the wheel around point O. As a result, there is no rotational motion on the wheel. As a result, the wheel moves downward on the inclined plane surface. The point of contact between the surface and the body is always at rest during a pure rolling action. As a result, there is no labor against the friction.



(C) (a) Aluminium at interior surrounded to it.

Explanation: Moment of inertia depends on the distribution of mass about the axis of rotation. Density of iron is more than that of aluminium, therefore for moment of inertia to be maximum, the iron should be far away from the axis. Thus, aluminium should be at the interior and iron surrounds it.

(D) (c) 0.035

Explanation: Angular velocity, in rad/s, is given by the length travelled divided by the time taken to travel the length:

$$\omega = \frac{\theta}{t}.$$

The amount of time taken to make one revolution is 3 min. One revolution is equal to 2π rad, and 3 minutes is equal to 180 seconds. Divide the radian value by the seconds, value to get the angular velocity.

$$\omega = \frac{2 \times \pi}{180}$$

$$\omega = 0.035 \text{ rad/s}$$

(E) (a) 13

Explanation: Given initial angular velocity, angular acceleration, and time we can easily solve for final angular velocity with:

$\omega = \omega_0 + \alpha t$

$\omega = 5 \text{ rad/s} + 2 \text{ rad/s} (4)$

$= 13 \text{ rad/s}$

3. A Point object is only a hypothetical concept. In actual practice, we have a bodies or object which have a definite size. An extended object or a real object is made up of large number of particles. Whereas a point object/ mass can have only translation motion, an extended object can have translational motion, rotational motion and a combination of translational and rotational motion as well. The motion of the system of particles or an extended object is quite complicated. This is because every particle of the system moves in a different manner than the other particles of the system. Therefore it describe the overall motion of a body or a system of particles in a simple manner. We define the concept 'Centre of Mass!

(A) The center of mass of a system of particles does not depend on:

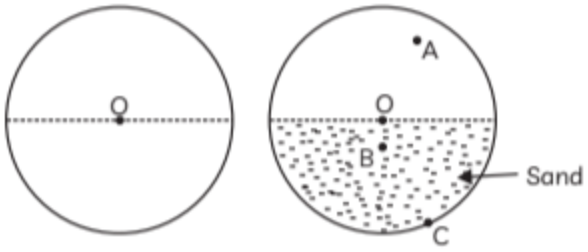
(a) position of particles

(b) relative particles distance between the

(c) forces acting on the particles

(d) mass of the particles

(B) The center of mass of a hollow sphere is at its center. Centre of mass of the hollow sphere when filled half with sand:



(a) shifts to A

(b) shifts to B

(c) shifts to C

(d) remains at O (center of the sphere)

(C) For which of the following does the center of mass lie outside the body:

(a) a pencil

(b) a dice

(c) a bangle

(d) a shot put

(D) Assertion (A): Moment of inertia of a body is same, whatever be the axis of

rotations.

Reason (R): Moment of inertia depe- nds only a distribution of mass.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

(E) A couple is acting on a two-particle system. The resultant motion will be:

(a) purely rotational motion

(b) purely translatory motion

(c) both (a) and (b)

(d) neither (a) nor (b)

Ans. (A) (c) forces acting on the particles

Explanation: The position of centre of mass is given by:

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\ldots\ldots}{m_1 + m_2 + m_3 + \ldots\ldots\ldots}$$

Where $r_1, r_2, r_3, \ldots\ldots\ldots$

represents position of m_1, m_2, m_3, ... respectively

From above formula it is clear that $\vec{r}_{COM}$ is

independent of force. Option (c) will be correct option.

(B) (b) shifts to B

Explanation: The hollow sphere's center will have gravity when no sand is thrown into it. Thus, when we begin to pour sand, the bottom begins to fill. Because there is more mass concentrated at the bottom in the initial condition, the center of gravity will begin to shift from its starting position downward. The center of gravity rises when the sphere is halfway full because mass accumulates on the upper half. Once the sphere is completely filled with sand, the center of gravity will once more be at the center.

(C) (c) a bangle

Explanation: A bangle has a ring-like form, and its center of mass, which is outside the ring or bangle, is where the ring's center of mass is located.

(D) (b) Both A and R are true and R is not the correct explanation of A.

Explanation: We know that

$$I = \frac{m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^3 + ... + m_n r_n^2}{m_1 + m_2 + m_3}$$

Where r_1, r_2, r_3 are distances of mass m_1, m_2, m_3 etc. from the axis. From the relation it is clear that it depends upon distribution of the masses and position of axis.
So, assertion is true.
We know that
Angular momentum = Ia
Torque = Ia
If we compare these equations with equations like linear momentum = mv, force = ma, we find that I represents mass in angular motion. As mass represents inertia in linear motion. I represents inertia in angular motion.
But assertion and reason are mutually exclusive. So, (b) is the answer.
(E) (a) purely rotational motion.
Explanation: A pair is made up of two equal and opposing forces with parallel action axes and some lateral separation. As a result, a couple's net force (or resultant) is a null vector; as a result, there won't be any translational acceleration and just rotational motion.