

# Class IX Chapter 7 – Triangles

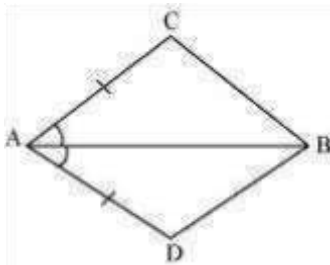
## Maths

### Exercise 7.1 Question

1:

In quadrilateral ACBD,  $AC = AD$  and  $AB$  bisects  $\angle A$  (See the given figure). Show that

$\triangle ABC \cong \triangle ABD$



Answer:  $\triangle ABC \cong \triangle ABD$ . What can you say about  $BC$  and  $BD$ ?

In  $\triangle ABC$  and  $\triangle ABD$ ,

$AC = AD$  (Given)

$\angle CAB = \angle DAB$  ( $AB$  bisects  $\angle A$ )

$AB = AB$  (Common)

$\therefore \triangle ABC \cong \triangle ABD$  (By SAS congruence rule)

$\therefore BC = BD$  (By CPCT)

Therefore,  $BC$  and  $BD$  are of equal lengths.

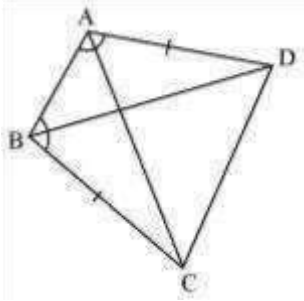
Question 2:

ABCD is a quadrilateral in which  $AD = BC$  and  $\angle DAB = \angle CBA$  (See the given figure).  
Prove that

(i)  $\triangle ABD \cong \triangle BAC$

(ii)  $BD = AC$

(iii)  $\angle ABD = \angle BAC$ .



Answer:

In  $\triangle ABD$  and  $\triangle BAC$ ,

$AD = BC$  (Given)

$\angle$

$\angle DAB = \angle CBA$  (Given)

$AB = BA$  (Common)

$\therefore \triangle ABD \cong \triangle BAC$  (By SAS congruence rule)

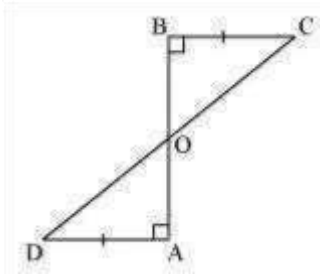
$\therefore BD = AC$  (By CPCT) And,  $\angle ABD$

$= \angle BAC$  (By CPCT)

Question 3:

AD and BC are equal perpendiculars to a line segment AB (See the given figure).

Show that CD bisects AB.



Answer:

In  $\Delta BOC$  and  $\Delta AOD$ ,

$\angle BOC = \angle AOD$  (Vertically opposite angles)

$\angle CBO = \angle DAO$  (Each  $90^\circ$ )

$BC = AD$  (Given)

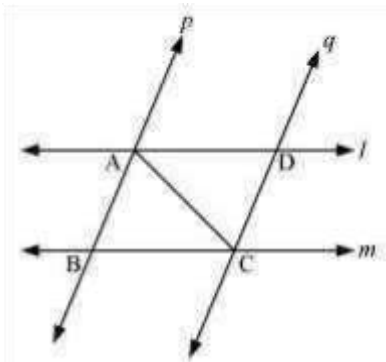
$\therefore \Delta BOC \cong \Delta AOD$  (AAS congruence rule)

$\therefore BO = AO$  (By CPCT)

$\Rightarrow$  CD bisects AB.

Question 4:  $l$  and  $m$  are two parallel lines intersected by another pair of parallel lines  $p$  and  $q$  (see

the given figure). Show that  $\Delta ABC \cong \Delta CDA$ .



Answer:

In  $\Delta ABC$  and  $\Delta CDA$ ,

$\angle BAC = \angle DCA$  (Alternate interior angles, as  $p \parallel q$ )

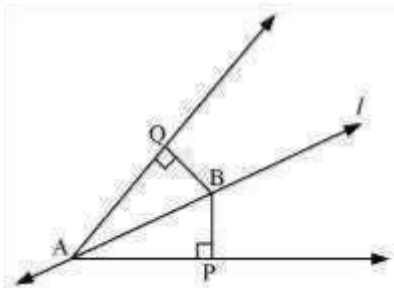
$AC = CA$  (Common)

$\angle BCA = \angle DAC$  (Alternate interior angles, as  $l \parallel m$ )

$\therefore \triangle ABC \cong \triangle CDA$  (By ASA congruence rule)

Question 5:

Line  $l$  is the bisector of an angle and  $B$  is any point on  $l$ .  $BP$  and  $BQ$  are perpendiculars from  $B$  to the arms of  $\angle A$  (see the given figure). Show that: i)  $\triangle APB \cong \triangle AQB$  (ii)  $BP = BQ$  or  $B$  is equidistant from the arms of  $\angle A$ .



Answer:

In  $\triangle APB$  and  $\triangle AQB$ ,

$\angle APB = \angle AQB$  (Each  $90^\circ$ )

$\angle PAB = \angle QAB$  ( $l$  is the angle bisector of  $\angle A$ )

$AB = AB$  (Common)

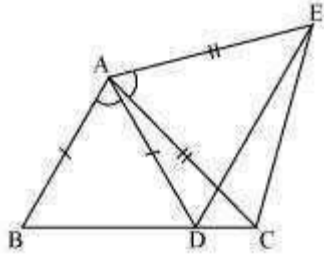
$\therefore \triangle APB \cong \triangle AQB$  (By AAS congruence rule)  $\therefore BP =$

$BQ$  (By CPCT)

$\therefore B$  is equidistant from the arms of  $\angle A$ . Or,  
it can be said that  $B$  is equidistant from the arms of  $\angle A$ .

Question 6:

In the given figure,  $AC = AE$ ,  $AB = AD$  and  $\angle BAD = \angle EAC$ . Show that  $BC = DE$ .



Answer:

It is given that  $\angle BAD = \angle EAC$

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC$$

$$\angle BAC = \angle DAE$$

In  $\triangle BAC$  and  $\triangle DAE$ ,  $AB = AD$

(Given)  $\angle BAC =$

$\angle DAE$  (Proved above)

$AC = AE$  (Given)

$\therefore \triangle BAC \cong \triangle DAE$  (By SAS congruence rule)

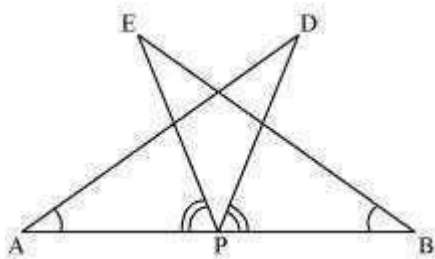
$\therefore BC = DE$  (By CPCT)

Question 7:

$AB$  is a line segment and  $P$  is its mid-point.  $D$  and  $E$  are points on the same side of  $AB$  such that  $\angle BAD = \angle ABE$  and  $\angle EPA = \angle DPB$  (See the given figure). Show that i)

$\therefore$   
 $\triangle DAP \cong \triangle EBP$  (

(ii)  $AD = BE$



Answer:

It is given that  $\angle EPA = \angle DPB$

$$\therefore \angle EPA + \angle DPE = \angle DPB + \angle DPE$$

$$\therefore \angle DPA = \angle EPB$$

In  $\triangle DAP$  and  $\triangle EBP$ ,

$$\angle DPA = \angle EPB \text{ (Given)}$$

$AP = BP$  (P is mid-point of AB)

$$\therefore \angle DPA = \angle EPB \text{ (From above)}$$

$$\therefore \triangle DAP \cong \triangle EBP \text{ (ASA congruence rule)}$$

$$\therefore AD = BE \text{ (By CPCT)}$$

Question 8:

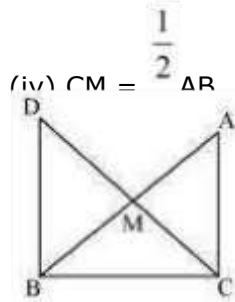
In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that  $DM = CM$ . Point D is joined to point

B (see the given figure). Show that: i)

$$\triangle AMC \cong \triangle BMD \text{ (}$$

ii)  $\angle DBC$  is a right angle. ( iii)

$$\triangle DBC \cong \triangle ACB \text{ (}$$



Answer:

(i) In  $\triangle AMC$  and  $\triangle BMD$ ,  
 $AM = BM$  (M is the mid-point of AB)

$\angle AMC = \angle BMD$  (Vertically opposite angles)

$CM = DM$  (Given)

$\therefore \triangle AMC \cong \triangle BMD$  (By SAS congruence rule)

$\therefore AC = BD$  (By CPCT) And,

$\angle ACM = \angle BDM$  (By CPCT) ii)

$\therefore \angle ACM = \angle BDM$  (

However,  $\angle ACM$  and  $\angle BDM$  are alternate interior angles.

Since alternate angles are equal,

It can be said that  $DB \parallel AC$

$\therefore \angle DBC + \angle ACB = 180^\circ$  (Co-interior angles)

$\therefore \angle DBC + 90^\circ = 180^\circ$

$\therefore \angle DBC = 90^\circ$

(iii) In  $\triangle DBC$  and  $\triangle ACB$ ,  
 $DB = AC$  (Already proved)

$\angle DBC = \angle ACB$  (Each  $90^\circ$ )

$BC = CB$  (Common)

$\therefore \triangle DBC \cong \triangle ACB$  (SAS congruence rule) iv)

$\triangle DBC \cong \triangle ACB$  (

$\therefore AB = DC$  (By CPCT)

$\therefore AB = 2\text{ CM}$

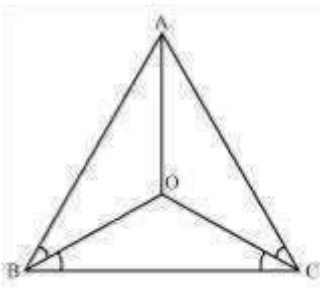
$\therefore CM = \frac{1}{2} AB$

#### Exercise 7.2 Question

1:

In an isosceles triangle  $ABC$ , with  $AB = AC$ , the bisectors of  $\angle B$  and  $\angle C$  intersect each other at  $O$ . Join  $A$  to  $O$ . Show that:

i)  $OB = OC$  (ii)  $AO$  bisects  $\angle A$  ( Answer:



(i) It is given that in triangle  $ABC$ ,  $AB = AC$



$\therefore \angle ACB = \angle ABC$  (Angles opposite to equal sides of a triangle are equal)

$$\therefore \frac{1}{2} \angle ACB = \frac{1}{2} \angle ABC$$

$$\therefore \angle OCB = \angle OBC$$

$\therefore$

$OB = OC$  (Sides opposite to equal angles of a triangle are also equal)

(ii) In  $\triangle OAB$  and  $\triangle OAC$ ,  $AO = AO$  (Common)

$AB = AC$  (Given)

$OB = OC$  (Proved above)

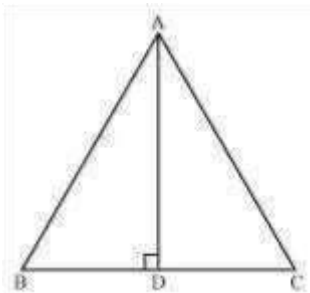
Therefore,  $\triangle OAB \cong \triangle OAC$  (By SSS congruence rule)

$$\therefore \angle BAO = \angle CAO \text{ (CPCT)}$$

$\therefore AO$  bisects  $\angle A$ .

Question 2:

In  $\triangle ABC$ ,  $AD$  is the perpendicular bisector of  $BC$  (see the given figure). Show that  $\triangle ABC$  is an isosceles triangle in which  $AB = AC$ .



Answer:

In  $\triangle ADC$  and  $\triangle ADB$ ,

$AD = AD$  (Common)

$\angle ADC = \angle ADB$  (Each  $90^\circ$ )

$CD = BD$  (AD is the perpendicular bisector of BC)

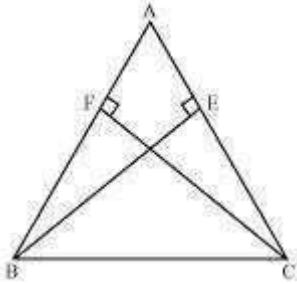
$\therefore \triangle ADC \cong \triangle ADB$  (By SAS congruence rule)

$\therefore AB = AC$  (By CPCT)

Therefore, ABC is an isosceles triangle in which  $AB = AC$ .

Question 3:

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.



Answer:

In  $\triangle AEB$  and  $\triangle AFC$ ,

$\angle AEB$  and  $\angle AFC$  (Each  $90^\circ$ )  $\angle A =$

$\angle A$  (Common angle)

$AB = AC$  (Given)

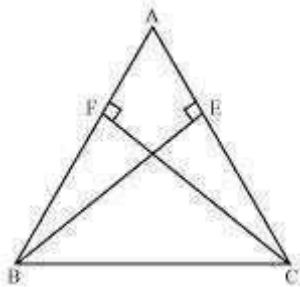
$\therefore \triangle AEB \cong \triangle AFC$  (By AAS congruence rule)  $\therefore BE = CF$  (By CPCT)

Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the

given figure). Show that

(i)  $\triangle ABE \cong \triangle ACF$



Answer:

(ii)  $AB = AC$ , i.e., ABC is an isosceles triangle.

(i) In  $\triangle ABE$  and  $\triangle ACF$ ,

$\angle ABE$  and  $\angle ACF$  (Each  $90^\circ$ )

$\angle A = \angle A$  (Common angle)

$BE = CF$  (Given)

$\therefore \triangle ABE \cong \triangle ACF$  (By AAS congruence rule)

(ii) It has already been proved that

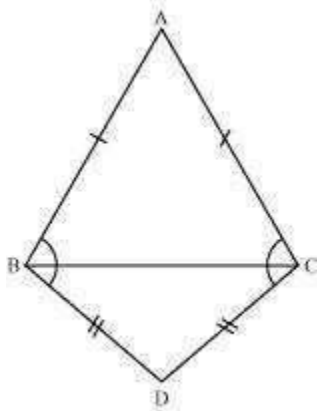
$\triangle ABE \cong \triangle ACF$

$\therefore AB = AC$  (By CPCT)

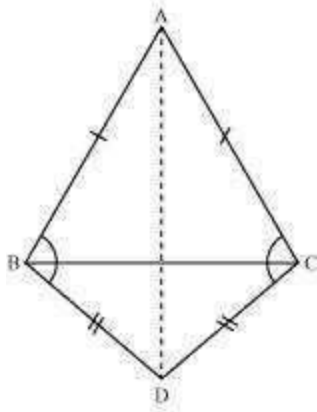
Question 5:

$\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles on the same base  $BC$  (see the given figure).

Show that  $\angle ABD = \angle ACD$ .



Answer:



Let us join AD.

In  $\triangle ABD$  and  $\triangle ACD$ ,

$AB = AC$  (Given)

$BD = CD$  (Given)

$AD = AD$  (Common side)

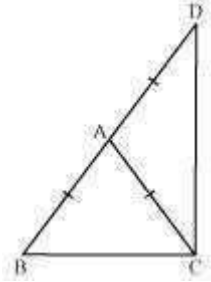
$\therefore \triangle ABD \cong \triangle ACD$  (By SSS congruence rule)

$\therefore \angle ABD = \angle ACD$  (By CPCT)

Question 6:

$\triangle ABC$  is an isosceles triangle in which  $AB = AC$ . Side BA is produced to D such that AD

$= AB$  (see the given figure). Show that  $\angle BCD$  is a right angle.



Answer:

In  $\triangle ABC$ ,

$AB = AC$  (Given)

$\therefore \angle ACB = \angle ABC$  (Angles opposite to equal sides of a triangle are also equal)

In  $\triangle ACD$ ,

$AC = AD$

$\therefore \angle ADC = \angle ACD$  (Angles opposite to equal sides of a triangle are also equal)

In  $\triangle BCD$ ,

$\angle ABC + \angle BCD + \angle ADC = 180^\circ$  (Angle sum property of a triangle)

$\therefore \angle ACB + \angle ACB + \angle ACD + \angle ACD = 180^\circ$

$\therefore 2(\angle ACB + \angle ACD) = 180^\circ$

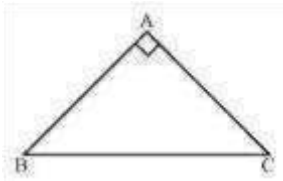
$\therefore 2(\angle BCD) = 180^\circ$

$\therefore \angle BCD = 90^\circ$

Question 7:

$\triangle ABC$  is a right angled triangle in which  $\angle A = 90^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$ .

Answer:



It

is given that

$$AB = AC$$

$\therefore \angle C = \angle B$  (Angles opposite to equal sides are also equal)

In  $\triangle ABC$ ,

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$90^\circ + \angle B + \angle C = 180^\circ$$

$$90^\circ + \angle B + \angle B = 180^\circ$$

$$2 \angle B = 90^\circ$$

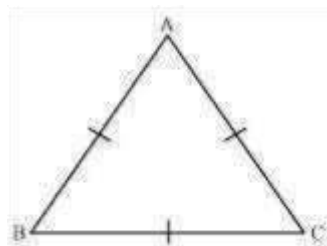
$$\angle B = 45^\circ$$

$$\angle B = \angle C = 45^\circ$$

Question 8:

Show that the angles of an equilateral triangle are  $60^\circ$  each.

Answer:



Let us consider that ABC is an equilateral triangle.

Therefore,  $AB = BC = AC$

$$AB = AC$$

$\therefore \angle C = \angle B$  (Angles opposite to equal sides of a triangle are equal)

Also,

$$AC = BC$$

$$\therefore \angle B = \angle A \text{ (Angles opposite to equal sides of a triangle are equal)}$$

Therefore, we obtain  $\angle A$

$$= \angle B = \angle C$$

In  $\triangle ABC$ ,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\therefore \angle A + \angle A + \angle A = 180^\circ$$

$$\therefore 3\angle A = 180^\circ$$

$$\therefore \angle A = 60^\circ$$

$\therefore \angle A = \angle B = \angle C = 60^\circ$  Hence, in an equilateral triangle, all interior angles are of measure  $60^\circ$ .

### Exercise 7.3

Question 1:

$\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles on the same base  $BC$  and vertices  $A$  and  $D$  are on the same side of  $BC$  (see the given figure). If  $AD$  is extended to intersect

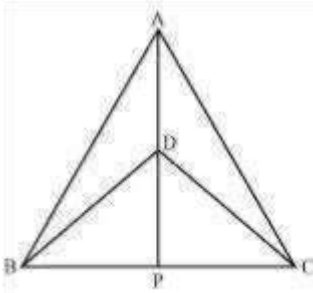
$BC$  at  $P$ , show that

i)  $\triangle ABD \cong \triangle ACD$  ( ii)  $\triangle ABP \cong \triangle ACP$

( iii)  $AP$  bisects  $\angle A$  as well as  $\angle D$ . (



(iv) AP is the perpendicular bisector of BC.



Answer:

(i) In  $\triangle ABD$  and  $\triangle ACD$ ,

$AB = AC$  (Given)

$BD = CD$  (Given)

$AD = AD$  (Common)

$\therefore \triangle ABD \cong \triangle ACD$  (By SSS congruence rule)

$\therefore \angle BAD = \angle CAD$  (By CPCT)

$\therefore \angle BAP = \angle CAP \dots (1)$

(ii) In  $\triangle ABP$  and  $\triangle ACP$ ,

$AB = AC$  (Given)

$\angle BAP = \angle CAP$  [From equation (1)]

$AP = AP$  (Common)

$\therefore \triangle ABP \cong \triangle ACP$  (By SAS congruence rule)

$\therefore BP = CP$  (By CPCT) ... (2)

(iii) From equation (1),

$\angle BAP = \angle CAP$

Hence, AP bisects  $\angle A$ .

In  $\triangle BDP$  and  $\triangle CDP$ ,

$BD = CD$  (Given)

$DP = DP$  (Common)

$BP = CP$  [From equation (2)]

$\therefore \triangle BDP \cong \triangle CDP$  (By S.S.S. Congruence rule)

$\therefore \angle BDP = \angle CDP$  (By CPCT) ... (3) Hence,

AP bisects  $\angle D$ . iv)  $\triangle BDP \cong \triangle CDP$

$\triangle CDP$  (

$\therefore \angle BPD = \angle CPD$  (By CPCT) .... (4)

$\therefore$

$\therefore \angle BPD + \angle CPD = 180$  (Linear pair angles)  
 $\angle BPD + \angle BPD = 180$

$\therefore$

$\angle BPD \times 2 = 180$  [From equation (4)]

$\therefore$

$\angle BPD = 90$  ... (5)

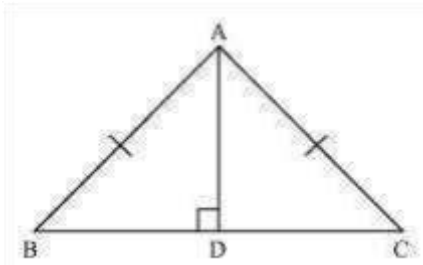
From equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.

Question 2:

AD is an altitude of an isosceles triangles ABC in which  $AB = AC$ . Show that

i) AD bisects BC (ii) AD bisects  $\angle A$ . (

Answer:



(i) In  $\triangle BAD$  and  $\triangle CAD$ ,

$\angle ADB = \angle ADC$  (Each  $90^\circ$  as AD is an altitude)

$AB = AC$  (Given)

$AD = AD$  (Common)

$\therefore \triangle BAD \cong \triangle CAD$  (By RHS Congruence rule)

$\therefore BD = CD$  (By CPCT)

Hence, AD bisects BC.

(ii) Also, by CPCT,

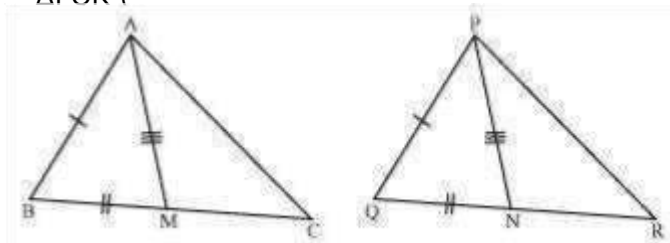
$\angle BAD = \angle CAD$  Hence, AD

bisects A.  $\therefore$

Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of  $\triangle PQR$  (see the given figure). Show that: i)  $\triangle ABM$

$\triangle PQN$  ( ii)  $\triangle ABC \cong \triangle PQR$  (



$\therefore$

Answer:

(i) In  $\triangle ABC$ , AM is the median to BC.

$$\therefore BM = \frac{1}{2} BC$$

$$\therefore QN = \frac{1}{2} QR$$

However,  $BC = QR$

$$\therefore \frac{1}{2} BC = \frac{1}{2} QR$$

$$\therefore BM = QN \dots (1)$$

In  $\triangle ABM$  and  $\triangle PQN$ , In  $\triangle PQR$ , PN is the median to QR.

$AB = PQ$  (Given)

$BM = QN$  [From equation (1)]

$AM = PN$  (Given)

$\therefore \triangle ABM \cong \triangle PQN$  (SSS congruence rule)

$\therefore \angle ABM = \angle PQN$  (By CPCT)

$\therefore \angle ABC = \angle PQR \dots (2)$

(ii) In  $\triangle ABC$  and  $\triangle PQR$ ,

$AB = PQ$  (Given)

$\angle ABC = \angle PQR$  [From equation (2)]

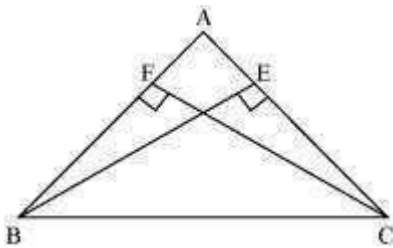
$BC = QR$  (Given)

$\therefore \triangle ABC \cong \triangle PQR$  (By SAS congruence rule)

Question 4:

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Answer:



In  $\triangle BEC$  and  $\triangle CFB$ ,

$$\angle BEC = \angle CFB \text{ (Each } 90^\circ)$$

$$BC = CB \text{ (Common)}$$

$$BE = CF \text{ (Given)}$$

$$\therefore \triangle BEC \cong \triangle CFB \text{ (By RHS congruency)}$$

$$\therefore \angle BCE = \angle CBF \text{ (By CPCT)}$$

$$\therefore AB = AC \text{ (Sides opposite to equal angles of a triangle are equal)}$$

Hence,  $\triangle ABC$  is isosceles.

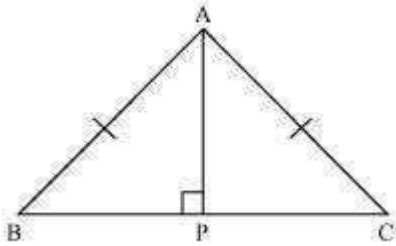
Question 5:

∴

∴

Answer:

ABC is an isosceles triangle with  $AB = AC$ . Drawn  $AP \perp BC$  to show that  $\angle B = \angle C$ .



In  $\triangle APB$  and  $\triangle APC$ ,

$\angle APB = \angle APC$  (Each  $90^\circ$ )

$AB = AC$  (Given)

$AP = AP$  (Common)

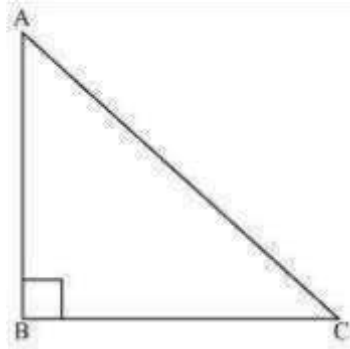
$\therefore \triangle APB \cong \triangle APC$  (Using RHS congruence rule)

$\therefore \angle B = \angle C$  (By using CPCT)

Exercise 7.4 Question 1:

Show that in a right angled triangle, the hypotenuse is the longest side.

Answer:



Let us consider a right-angled triangle ABC, right-angled at B.

In  $\triangle ABC$ ,

$$\therefore \angle A + \angle B + \angle C = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$\therefore \angle A + 90^\circ + \angle C = 180^\circ$$

$$\therefore \angle A + \angle C = 90^\circ$$

Hence, the other two angles have to be acute (i.e., less than  $90^\circ$ ).

$\therefore \angle B$  is the largest angle in  $\triangle ABC$ .

$\therefore \angle B > \angle A$  and  $\angle B > \angle C$

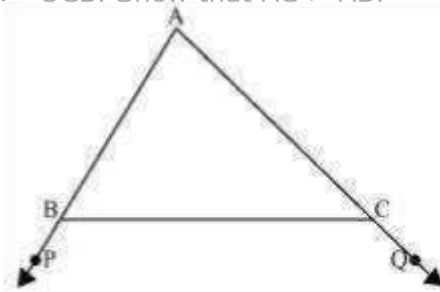
$\therefore AC > BC$  and  $AC > AB$

[In any triangle, the side opposite to the larger (greater) angle is longer.] Therefore, AC is the largest side in  $\triangle ABC$ .

However, AC is the hypotenuse of  $\triangle ABC$ . Therefore, hypotenuse is the longest side in a right-angled triangle.

Question 2:

In the given figure sides AB and AC of  $\triangle ABC$  are extended to points P and Q respectively. Also,  $\angle PBC < \angle QCB$ . Show that  $AC > AB$ .



Answer:

In the given figure,

$$\angle ABC + \angle PBC = 180^\circ \text{ (Linear pair)}$$

$$\therefore \angle ABC = 180^\circ - \angle PBC \dots (1)$$

Also,

$$\angle ACB + \angle QCB = 180^\circ$$

$\therefore$

$\therefore$



$$\angle ACB = 180^\circ - \angle QCB \dots (2)$$

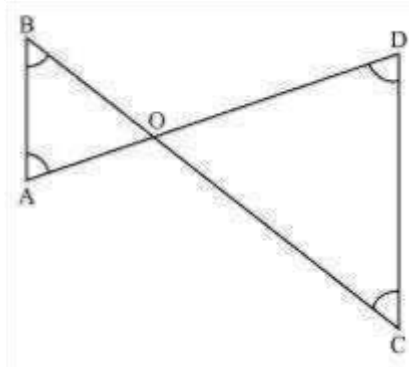
As  $\angle PBC < \angle QCB$ ,

$$\therefore 180^\circ - \angle PBC > 180^\circ - \angle QCB$$

$$\therefore \angle ABC > \angle ACB \text{ [From equations (1) and (2)] } \therefore AC >$$

AB (Side opposite to the larger angle is larger.) Question 3:

In the given figure,  $\angle B < \angle A$  and  $\angle C < \angle D$ . Show that  $AD < BC$ .



Answer:

In  $\triangle AOB$ ,

$$\because \angle B < \angle A \quad AO < BO \text{ (Side opposite to smaller angle is smaller)} \dots (1)$$

In  $\triangle COD$ ,

$$\because \angle C < \angle D$$

$$\therefore OD < OC \text{ (Side opposite to smaller angle is smaller)} \dots (2)$$

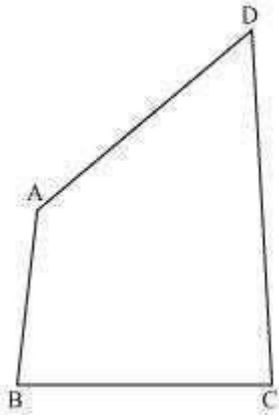
On adding equations (1) and (2), we obtain

$$AO + OD < BO + OC$$

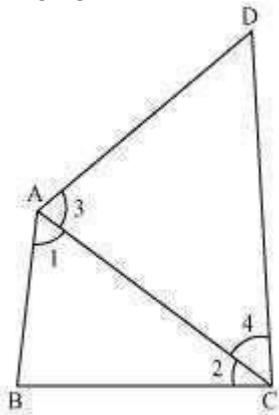
$$AD < BC$$

Question 4:

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see the given figure). Show that  $\angle A > \angle C$  and  $\angle B > \angle D$ .



Answer:



Let us join AC.  
In  $\triangle ABC$ ,

$AB < BC$  (AB is the smallest side of quadrilateral ABCD)

$\therefore \angle 2 < \angle 1$  (Angle opposite to the smaller side is smaller) ... (1)

In  $\triangle ADC$ ,

$AD < CD$  (CD is the largest side of quadrilateral ABCD)

$\therefore \angle 4 < \angle 3$  (Angle opposite to the smaller side is smaller) ... (2)

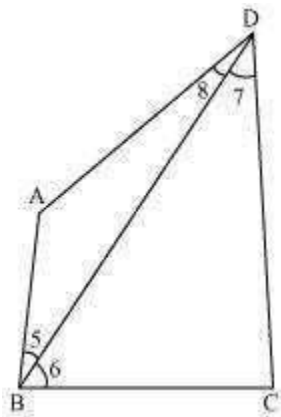
On adding equations (1) and (2), we obtain

$$\angle 2 + \angle 4 < \angle 1 + \angle 3$$

$$\angle C < \angle A$$

$$\therefore \angle A > \angle C$$

Let us join BD.



In  $\triangle ABD$ ,

$AB < AD$  (AB is the smallest side of quadrilateral ABCD)

$$\therefore \angle 8 < \angle 5 \text{ (Angle opposite to the smaller side is smaller) ... (3)}$$

In  $\triangle BDC$ ,

$BC < CD$  (CD is the largest side of quadrilateral ABCD)

$$\therefore \angle 7 < \angle 6 \text{ (Angle opposite to the smaller side is smaller) ... (4)}$$

On adding equations (3) and (4), we obtain

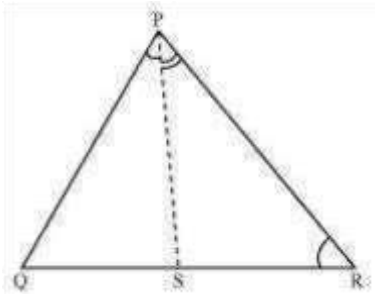
$$\angle 8 + \angle 7 < \angle 5 + \angle 6$$

$$\therefore \angle D < \angle B$$

$$\therefore \angle B > \angle D$$

5:

In the given figure,  $PR > PQ$  and PS bisects  $\angle QPR$ . Prove that  $\angle PSR > \angle PSQ$ .



Answer:

As  $PR > PQ$ ,

$\therefore \angle PQR > \angle PRQ$  (Angle opposite to larger side is larger) ... (1) PS is the bisector of  $\angle QPR$ .

$$\therefore \angle QPS = \angle RPS \dots (2)$$

$\therefore \angle PSR$  is the exterior angle of  $\triangle PQS$ .

$$\therefore \angle PSR = \angle PQR + \angle QPS \dots (3)$$

$\therefore$

$\therefore \angle PSQ$  is the exterior angle of  $\triangle PRS$ .

$$\angle PSQ = \angle PRQ + \angle RPS \dots (4)$$

Adding equations (1) and (2), we obtain

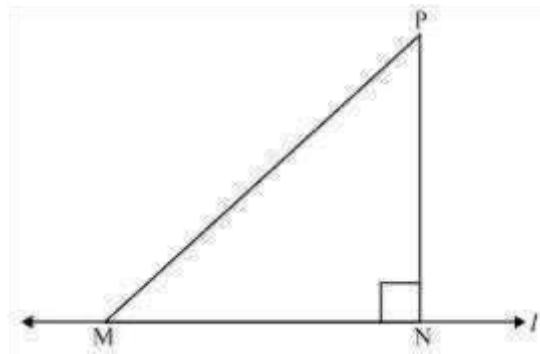
$$\angle PQR + \angle QPS > \angle PRQ + \angle RPS$$

$$\therefore \angle PSR > \angle PSQ \text{ [Using the values of equations (3) and (4)]}$$

Question 6:

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Answer:



Let us take a line  $l$  and from point  $P$  (i.e., not on line  $l$ ), draw two line segments  $PN$  and  $PM$ . Let  $PN$  be perpendicular to line  $l$  and  $PM$  is drawn at some other angle.

In  $\triangle PNM$ ,

$$\therefore \angle N = 90^\circ$$

$$\therefore \angle P + \angle N + \angle M = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$\therefore \angle P + \angle M = 90^\circ$$

Clearly,  $\angle M$  is an acute angle.

$$\therefore \angle M < \angle N$$

$$\therefore PN < PM \text{ (Side opposite to the smaller angle is smaller)}$$

Similarly, by drawing different line segments from  $P$  to  $l$ , it can be proved that  $PN$  is smaller in comparison to them.

Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

#### Exercise 7.5 Question

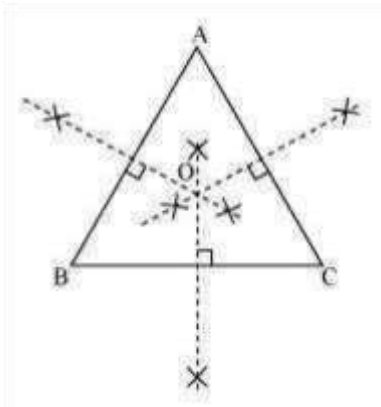
1:

ABC is a triangle. Locate a point in the interior of  $\triangle ABC$  which is equidistant from all the vertices of  $\triangle ABC$ .

Answer:

Circumcentre of a triangle is always equidistant from all the vertices of that triangle.

Circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.



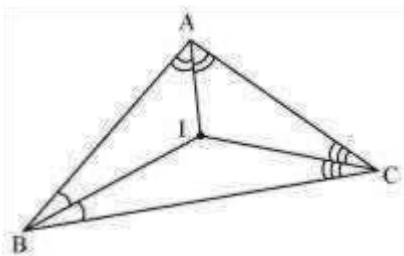
In  $\triangle ABC$ , we can find the circumcentre by drawing the perpendicular bisectors of sides AB, BC, and CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of  $\triangle ABC$ .

Question 2:

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Answer:

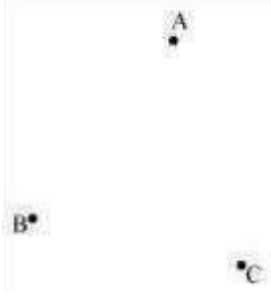
The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.



Here, in  $\triangle ABC$ , we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of  $\triangle ABC$ .

Question 3:

In a huge park people are concentrated at three points (see the given figure)



A: where there are different slides and swings for children,

B: near which a man-made lake is situated,

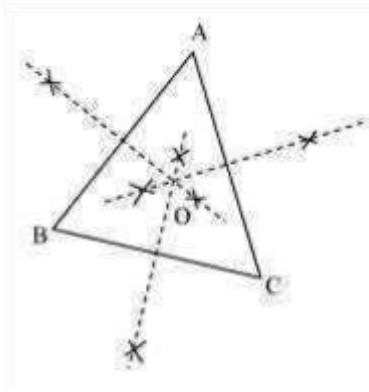
C: which is near to a large parking and exit.

Where should an ice-cream parlour be set up so that maximum number of persons can approach it?

(Hint: The parlor should be equidistant from A, B and C) Answer:

Maximum number of persons can approach the ice-cream parlour if it is equidistant from A, B and C. Now, A, B and C form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre O of  $\triangle ABC$ .

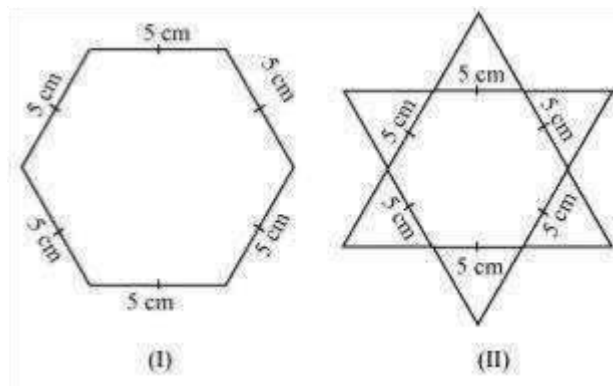




In this situation, maximum number of persons can approach it. We can find circumcentre O of this triangle by drawing perpendicular bisectors of the sides of this triangle.

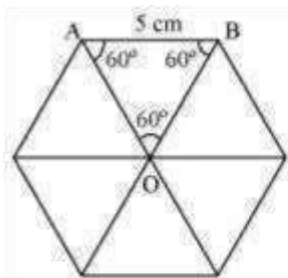
Question 4:

Complete the hexagonal and star shaped rangolies (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



Answer:

It can be observed that hexagonal-shaped rangoli has 6 equilateral triangles in it.



$$= \frac{\sqrt{3}}{4} (\text{side})^2 = \frac{\sqrt{3}}{4} (5)^2$$

Area of  $\triangle OAB$

$$= \frac{\sqrt{3}}{4} (25) = \frac{25\sqrt{3}}{4} \text{ cm}^2$$

$$= 6 \times \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2} \text{ cm}^2$$

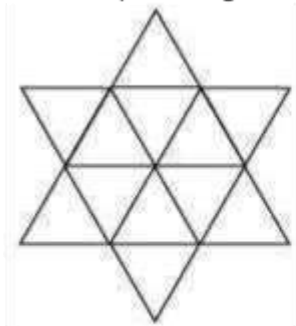
Area of hexagonal-shaped rangoli

$$\text{Area of equilateral triangle having its side as 1 cm} = \frac{\sqrt{3}}{4} (1)^2 = \frac{\sqrt{3}}{4} \text{ cm}^2$$

Number of equilateral triangles of 1 cm side that can be filled

$$\text{in this hexagonal-shaped rangoli} = \frac{\frac{75\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}} = 150$$

Star-shaped rangoli has 12 equilateral triangles of side 5 cm in it.



$$\text{Area of star-shaped rangoli} = 12 \times \frac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$$

Number of equilateral triangles of 1 cm side that can be filled

$$\text{in this star-shaped rangoli} = \frac{75\sqrt{3}}{\frac{\sqrt{3}}{4}} = 300$$

Therefore, star-shaped rangoli has more equilateral triangles in it.