

wrong statement.

correct statement.

14. **Assertion (A):** State variables are required to specify the equilibrium state of the system. [1]
Reason (R): Pressure is an intensive state variable.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.
15. **Assertion:** The plane of the orbit of an artificial satellite must contain the centre of the earth. [1]
Reason: For the orbital motion of satellite, the necessary centripetal force is provided by gravitational pull of earth on satellite.
- a) Assertion and reason both are correct statements and reason is correct explanation for assertion. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
c) Assertion is correct statement but reason is wrong statement. d) Assertion is wrong statement but reason is correct statement.
16. **Assertion (A):** $v = u + at$ formula is applied for projectile but cannot be applied for uniform circular motion. [1]
Reason (R): $v = u + at$ is applicable when an acceleration (a) is uniform.
- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

Section B

17. Find the ratio of the length of a closed pipe to that of an open pipe in order that the second overtone of the former is in unison with the fourth overtone of the latter. [2]
18. Distinguish between the dimensions and unit of a physical quantity. [2]
19. The rotational kinetic energy of a body is given by $E = \frac{1}{2}I\omega^2$, where ω is the angular velocity of the body. Use the equation to obtain a dimensional formula for moment of inertia I. Also write its SI unit. [2]
20. A rocket is set for vertical firing. If the exhaust speed is 1200 ms^{-1} , how much gas must be ejected per second to supply the thrust needed [2]
- i. to overcome the weight of the rocket,
ii. to give to the rocket an initial vertical upward acceleration of 19.6 ms^{-2} ? Given mass of rocket = 6000 kg.
21. Why is the weight of a body at the poles more than the weight at the equator? Explain. [2]

OR

The radius of the earth's orbit around the sun is $1.5 \times 10^{11} \text{ m}$. Calculate the angular and linear velocity of the earth. Through how much angle does the earth revolve in 2 days?

Section C

22. In giving a patient a blood transfusion, the bottle is set up so that the level of blood is 1.3 m above needle, which has an internal diameter of 0.36 mm and is 3 cm in length. If 4.5 cm^3 of blood passes through the needle in one minute, calculate the viscosity of blood. The density of blood is 1020 kgm^{-3} . [3]
23. Explain the following [3]
- i. Hot tea cools rapidly when poured into the saucer from the cup.

ii. Temperature of a hot liquid falls rapidly in the beginning but slowly afterwards.

iii. A hot liquid cools faster if outer surface of the container is blackened.

24. Define $v = u + at$ from velocity time graph. [3]

25. A stone of mass m tied to the end of a string revolves in a vertical circle of radius R . The net forces at the lowest and highest points of the circle directed vertically downwards are: [Choose the correct alternative] [3]

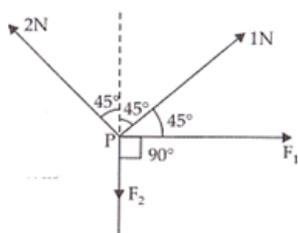
| | Lowest Point | Highest Point |
|-----|-------------------------|-------------------------|
| (a) | $T - mg$ | $T + mg$ |
| (b) | $T + mg$ | $T - mg$ |
| (c) | $mg + T - (mv_1^2) / R$ | $mg - T + (mv_1^2) / R$ |
| (d) | $mg - T - (mv_1^2) / R$ | $mg + T + (mv_1^2) / R$ |

26. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960 \text{ J}$, $Q_2 = -5585 \text{ J}$, $Q_3 = -2980 \text{ J}$ and $Q_4 = 3645 \text{ J}$ respectively. The corresponding works involved are $W_1 = 2200 \text{ J}$, $W_2 = -825 \text{ J}$, $W_3 = -1100 \text{ J}$ and W_4 respectively. Find [3]

i. W_4 and

ii. efficiency of the cycle.

27. There are four forces acting at a point P produced by strings as shown in the figure, which is at rest. Find the forces F_1 and F_2 . [3]



28. The flow of blood in a large artery of an anaesthetised dog is diverted through a Venturi meter. The wider part of the meter has a cross-sectional area equal to that of the artery. $A = 8 \text{ mm}^2$. The narrower part has an area $a = 4 \text{ mm}^2$. The pressure drop in the artery is 24 Pa . What is the speed of the blood in the artery? [3]

OR

During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa . At what height must the blood container be placed so that blood may just enter the vein?

[Use the density of whole blood $\rho = 1.06 \times 10^3 \text{ kgm}^{-3}$]

Section D

29. Read the text carefully and answer the questions: [4]

An **elastic collision** is a **collision** in which there is no net loss in kinetic energy in the system as a result of the **collision**. Both momentum and kinetic energy are conserved quantities in **elastic collisions**.

behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

OR

A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2 .

- a. What is his new angular speed? (Neglect friction.)
- b. Is kinetic energy conserved in the process? If not, from where does the change come about?

Solution

Section A

- (c) $\text{dyne} \times \text{cm}^4$
Explanation: Unit of $a = \text{Unit of } P \times \text{Unit of } V^2$
 $= \text{dyne cm}^{-2} \times (\text{cm}^3)^2$
 $= \text{dyne cm}^4$
- (a) 13.2 cm
Explanation: The fundamental frequency of an open organ pipe = Third harmonic of a closed pipe
 $\frac{v}{2l_1} = \frac{3v}{4l_2}$
 $\therefore l_1 = \frac{2l_2}{3} = \frac{2 \times 20}{3} \text{ cm} = 13.3 \text{ cm}$
- (a) 0.7 \AA
Explanation: $x_{CM} = \frac{12 \times 0 + 16 \times 1.2}{12 + 16} \simeq 0.7 \text{ \AA}$
- (c) Bernoulli's principle
Explanation: An aeroplane gets a dynamic upward lift in accordance with Bernoulli's principle.
- (d) acceleration
Explanation: Acceleration due to gravity is independent of the mass of the body.
- (c) 40 cm
Explanation: $y = 5 \sin 2\pi \left[\frac{t}{0.04} - \frac{x}{40} \right]$
 $y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda} \right]$
 $\therefore \lambda = 40 \text{ cm}$
- (a) frame of reference consisting of a clock and a Cartesian system having three mutually \perp axes, (X, Y, and Z)
Explanation: Motion is a change in position of an object with time. In order to specify the position, we need to use a reference point and a set of axes. It is convenient to choose a rectangular coordinate system consisting of three mutually perpendicular axes, labelled X-, Y-, and Z- axes.
The point of intersection of these three axes is called origin (O) and serves as the reference point. The coordinates (x, y, z) of an object describe the position of the object with respect to this coordinate system.
To measure time, we position a clock in this system. This coordinate system along with a clock constitutes a frame of reference.
- (b) a diverging lens
Explanation: The speed of sound in H_2 is greater than that in air, so a balloon filled with H_2 will behave like a diverging lens.
- (b) decreases when density increases
Explanation: $v_c = \frac{Re\eta}{\rho D}$
Critical velocity decreases when density ρ increases or diameter D increases.
- (d) $\sqrt{2gR}$
Explanation: $\frac{1}{2}mv^2 = U_i - U_f = 0 - \left(-\frac{GMm}{R} \right)$
 $\frac{1}{2}mv^2 = \frac{gR^2m}{R}$ [GM = gR^2]
 $v = \sqrt{2gR}$

11.

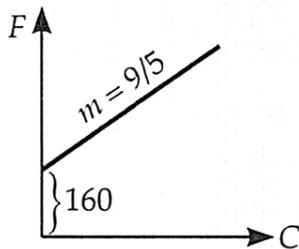
(b) $[ML^2T^{-1}]$

Explanation: [Angular momentum] = $[ML^2T^{-1}]$

12.

(c) intercepts the positive y-axis

Explanation:



As $\frac{C}{100} = \frac{F-32}{180}$

$\therefore F = \frac{9}{5}C + 160$

Thus the graph between C and F is a straight with positive intercept (=160) on Y-axis as shown in the figure.

13.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation: Assertion and reason both are correct statements but reason is not correct explanation for assertion.

14.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

15.

(a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: Assertion and reason both are correct statements and reason is correct explanation for assertion.

16.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: $v = u + at$ is applicable only when acceleration (a) is uniform.

So, this formula is applied for projectile since projectile motion is under uniform acceleration (g).

But in uniform circular motion acceleration is not uniform; hence this formula cannot be applied.

Hence both the assertion and reason are true but the reason does not explain the assertion.

Section B

17. Fundamental frequency of a closed organ pipe, $\nu = \frac{v}{4L}$

Fundamental frequency of an open organ pipe, $\nu' = \frac{v}{2L}$

Second overtone of the closed pipe = $5\nu = \frac{5v}{4L}$

Fourth overtone of the open pipe = $5\nu' = \frac{5v}{2L'}$

As the two overtones are in unison, therefore

$$\frac{5v}{4L} = \frac{5v}{2L'} \text{ or } \frac{L}{L'} = \frac{2}{4} = \frac{1}{2}$$

or $L : L' = 1 : 2$

18. Dimensions of a physical quantity are the powers to which the fundamental units must be raised to represent the unit of the given physical quantity. The unit is compact mathematical expression involving fundamental units. The unit may also be given some name, e.g., $[F] = [MLT^{-2}]$ and unit of force = $kg\ ms^{-2}$ or newton.

19. It is given that the rotational kinetic energy, $E = \frac{1}{2}I\omega^2 \Rightarrow I = \frac{[E]}{[\omega^2]}$

$$\text{Therefore, } I = \frac{[E]}{[\omega^2]} = \frac{[ML^2T^{-2}]}{[T^{-1}]^2} \left[\frac{ML^2T^{-2}}{T^{-2}} \right] = [ML^2]$$

Its SI unit is Joule.

20. i. Here $u = 1200\ ms^{-1}$, $m = 6000\ kg$,

Given: Thrust = Weight of rocket

$$\therefore u \frac{dm}{dt} = mg$$

$$\text{or } \frac{dm}{dt} = \frac{mg}{u} = \frac{6000 \times 9.8}{1200} = 49\ kg\ s^{-1}$$

ii. Here $u = 1200 \text{ ms}^{-1}$, $m = m_0 = 6000 \text{ kg}$, $t = 0$, $a = 29.6 \text{ ms}^{-2}$

$$\text{As } a = \left[\frac{u}{m_0 - t \frac{dm}{dt}} \right] \frac{dm}{dt} - g$$

$$\therefore 29.6 = \left[\frac{1200}{6000 - 0 \times \frac{dm}{dt}} \right] \frac{dm}{dt} - 9.8$$

$$\text{or } 29.6 + 9.8 = \frac{1200}{6000} \times \frac{dm}{dt}$$

$$\text{or } \frac{dm}{dt} = \frac{39.4 \times 6000}{1200} = 197 \text{ kg s}^{-1}$$

21. As $g = \frac{GM}{R^2}$ and the value of R at the poles is less than that at the equator, so g at poles is greater than g at the equator.

Now, as $g_p > g_e$, hence $mg_p > mg_e$

i.e., the weight of a body at the poles is more than the weight at the equator.

OR

Here $r = 1.5 \times 10^{11} \text{ m}$,

Period of revolution of the earth,

$$T = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s}$$

\therefore Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{365 \times 24 \times 60 \times 60}$$

$$= 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

Linear velocity,

$$v = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7} = 2.99 \times 10^4 \text{ ms}^{-1}$$

In 365 days, the earth revolves through an angle of 2π radians.

\therefore The angle through which the earth revolves in 2 days

$$= \frac{2\pi}{365} \times 2 = \frac{2 \times 3.14 \times 2}{365} = 0.0344 \text{ rad}$$

Section C

22. Length of the needle, $l = 3 \text{ cm}$

Radius of the needle, $r = \frac{0.36}{2} \text{ mm} = 0.018 \text{ cm}$

Volume of blood flowing out per second,

$$Q = \frac{\text{Total Volume}}{\text{Time}} = \frac{4.5}{60} = 0.075 \text{ cm}^3 \text{ s}^{-1}$$

Density of blood,

$$\rho = 1020 \text{ kg m}^{-3} = 1020 \times 10^{-3} \text{ g cm}^{-3} = 1.02 \text{ g cm}^{-3} \text{ (Given)}$$

The bottle is set up so that the level of blood is 1.3 m above needle, pressure difference,

$p = 1.3 \text{ m}$ column of blood

$$= 1.3 \times 100 \times 1.02 \times 980 \text{ dyne cm}^{-2}$$

$$\eta = \frac{\pi p r^4}{8 Q l} = \frac{3.142 \times 1.3 \times 100 \times 1.02 \times 980 \times (0.018)^4}{8 \times 0.075 \times 3}$$

$$= 0.238 \text{ poise}$$

23. i. The tea will cool faster in the saucer as surface area increases on pouring hot tea in saucer from the cup and the rate of loss of heat is directly proportional to surface area of the radiating surface

ii. A hot liquid cools rapidly in the beginning but slowly afterward because the temperature of a hot liquid falls exponentially in accordance with Newton's law of cooling. In other words, the rate of cooling is directly proportional to the temperature difference between hot liquid and the surroundings.

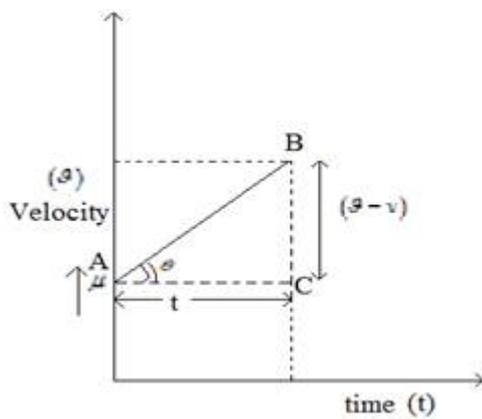
iii. When the outer surface of the container is blackened, the surface becomes good emitter of heat and so the hot liquid in it cools faster.

24. Consider a velocity-time graph for a uniformly accelerated body starting from rest is represented as follows.

Slope of $v - t$ graph

$$\tan \theta = \frac{v-u}{t}$$

But $\tan \theta = \text{acceleration (a)}$



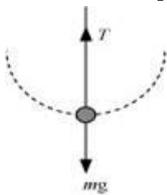
If acceleration is represented as a , then acceleration is defined as the rate of change in velocity

$$\Rightarrow a = \frac{v-u}{t}$$

$$v - u = at$$

$$v = u + at$$

25. The free body diagram of the stone at the lowest point is shown in the following figure. In this case tension, T in the string acts upwards and weight of the body, mg acts downwards. The net force $T - mg$, supplies required centripetal force to rotate the stone in the circular path.

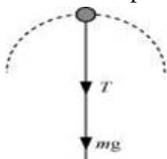


According to Newton's second law of motion, the net force acting on the stone at this point is equal to the centripetal force, i.e.,

$$F_{net} = T - mg = \frac{mv_1^2}{R} \dots(i)$$

Where, v_1 = Velocity at the lowest point

The free body diagram of the stone at the highest point is shown in the following figure. In this case both the tension T in the string and weight mg of the stone act downwards. The net force $T + mg$ provides required centripetal force to rotate the stone in the circular path.



Using Newton's second law of motion, we have:

$$T + mg = \frac{mv_2^2}{R} \dots(ii)$$

Where, v_2 = Velocity at the highest point

It is clear from equations (i) and (ii) that the net force acting at the lowest and the highest points are respectively $(T - mg)$ and $(T + mg)$.

26. i. By the first law of thermodynamics, $dQ = dU + dW$

But for a cyclic process, $dU = 0$

$$\therefore dQ = dW$$

$$\text{or } Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$$

$$\text{or } 5960 - 5585 - 2980 + 3645 = 2200 - 825 - 1100 + W_4$$

$$\text{or } W_4 = (5960 + 3645 + 825 + 1100) - (5585 + 2980 + 2200) = 11530 - 10765 = 765 \text{ J.}$$

ii. Efficiency,

$$\begin{aligned} \eta &= \frac{\text{Total work done}}{\text{Heat absorbed}} = \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4} \\ &= \frac{2200 - 825 - 1100 + 765}{5960 + 3645} \\ &= \frac{1040}{9605} = 0.1083 = 10.83 \% \end{aligned}$$

27. The given system is in mechanical equilibrium, hence sum of all the forces has to be equal to zero.

Net force along X and Y axes are also zero.

Resolving all forces along X-axis

$$F_x = 0$$

$$F_1 + 1 \sin 45^\circ - 2 \sin 45^\circ = 0$$

$$F_1 = 1 \sin 45^\circ$$

$$F_1 = \frac{1}{\sqrt{2}}$$

$$F_1 = 0.707 \text{ N}$$

Resolving all forces along Y-axis

$$F_y = 0$$

$$-F_2 + 1 \cos 45^\circ + 2 \cos 45^\circ = 0$$

$$-F_2 = -3 \cos 45^\circ$$

$$F_2 = 3 \cdot \frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = \frac{3 \times 1.414}{2}$$

$$= 3 \times 0.707 = 2.121 \text{ N.}$$

28. We take the density of blood from table to be $1.06 \times 10^3 \text{ g m}^{-3}$. The ratio of the area is $\left(\frac{A}{a}\right) = 2$.

| Fluid | $\rho(\text{kg m}^{-3})$ |
|--------------------|--------------------------|
| Water | 1.00×10^3 |
| Sea water | 1.03×10^3 |
| Mercury | 13.6×10^3 |
| Ethyl alcohol | 0.806×10^3 |
| Whole blood | 1.06×10^3 |
| Air | 1.29 |
| oxygen | 1.43 |
| Hydrogen | 9.0×10^{-2} |
| Interstellar space | $\approx 10^{-2}$ |

Using eq. speed of fluid through wide neck is given by $v_1 = \sqrt{\left(\frac{2\rho_m g h}{\rho}\right) \left(\left(\frac{A}{a}\right)^2 - 1\right)^{-1/2}}$ we obtain

$$v_1 = \sqrt{\frac{2 \times 24 \text{ Pa}}{1060 \text{ kg m}^{-3} \times (2^2 - 1)}} = 0.123 \text{ ms}^{-1}$$

OR

Gauge pressure is given by, $P = 2000 \text{ Pa}$

The density of whole blood is given by, $\rho = 1.06 \times 10^3 \text{ kg m}^{-3}$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Height of the blood container is $\Rightarrow h$

The pressure of the blood container is given by, $P = h\rho g$

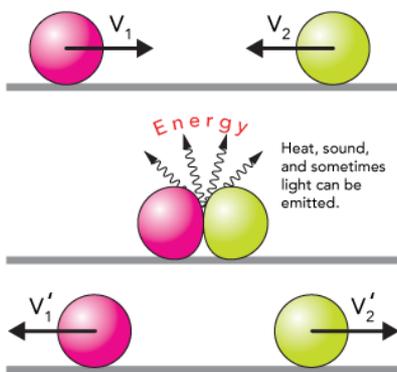
$$h = \frac{P}{\rho g} = \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925 \text{ m}$$

Thus, The blood may enter the vein if the blood container is kept at a height greater than 0.1925 m, i.e., about 0.2m.

Section D

29. Read the text carefully and answer the questions:

An **elastic collision** is a **collision** in which there is no net loss in kinetic energy in the system as a result of the **collision**. Both momentum and kinetic energy are conserved quantities in **elastic collisions**.



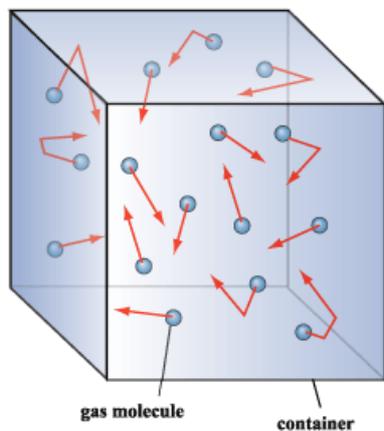
- (i) (c) circular motion
Explanation: circular motion
- (ii) (a) 1
Explanation: 1
- (iii) (d) momentum
Explanation: momentum

OR

- (c) equal
Explanation: equal
- (iv) (d) zero
Explanation: zero

30. Read the text carefully and answer the questions:

Gas molecules move in random motion inside the container. The **pressure exerted** by the gas is due to the continuous collision of the molecules against the walls of the container. Due to this continuous collision, the walls experience a continuous force which is equal to the total momentum imparted to the walls per second.



- (i) (d) 1:2
Explanation: 1:2
- (ii) (c) directly proportional to the density
Explanation: directly proportional to the density
- (iii) (a) pressure increase
Explanation: pressure increase
- (iv) (b) becomes double
Explanation: becomes double

OR

- (d) $M^1L^2T^{-2}K^{-1}$
Explanation: $M^1L^2T^{-2}K^{-1}$

Section E

31. Assume that $t = 0$ when $\theta = \theta_0$. Then,

$$\theta = \theta_0 \cos \omega t$$

Given a seconds pendulum $\omega = 2\pi$

$$\text{At time } t_1, \text{ let } \theta = \frac{\theta_0}{2}$$

$$\therefore \cos 2\pi t_1 = 1/2 \Rightarrow t_1 = \frac{1}{6}$$

$$\theta = -\theta_0 2\pi \sin 2\pi t \quad \left[\theta = \frac{d\theta}{dt} \right]$$

$$\text{At } t_1 = \frac{1}{6}$$

$$\theta = -\theta_0 2\pi \sin \frac{2\pi}{6} = -\sqrt{3}\pi\theta_0$$

Thus the linear velocity is

$$u = -\sqrt{3}\pi\theta_0 l \text{ perpendicular to the string.}$$

The vertical component is

$$u_y = -\sqrt{3}\pi\theta_0 t \sin \theta_0$$

and the horizontal component is

$$u_x = -\sqrt{3}\pi\theta_0 l \cos \theta_0$$

At the time it snaps, the vertical height is

$$H' = H + l \left(1 - \cos \left(\frac{\theta_0}{2} \right) \right)$$

Let the time required for fall be t , then

$$H' = u_y t + (1/2)gt^2$$

(notice g is also in the negative direction)

$$\text{or, } \frac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 l \sin \theta_0 t - H' = 0$$

$$\therefore t = \frac{-\sqrt{3}\pi\theta_0 l \sin \theta_0 \pm \sqrt{3\pi^2\theta_0^2 e^2 \sin^2 \theta_0 + 2gH'}}{-\sqrt{3}\pi\theta_0^2 \pm \sqrt{3\pi^2\theta_0^4 l^2 + 2gH'}} \cdot \frac{g}{g}$$

Neglecting terms of order θ_0^2 and higher.

$$\sqrt{\frac{2H'}{g}}$$

$$\text{Now } H' = H + l(1 - 1) = H \therefore t \sqrt{\frac{2H}{g}}$$

The distance travelled in the x -direction is $u_x t$ to the left of where it snapped.

$$X = \sqrt{3}\pi\theta_0 l \cos \theta_0 \sqrt{\frac{2H}{g}}$$

To order of θ_0

$$X = \sqrt{3}\pi\theta_0 l \sqrt{\frac{2H}{g}} = \sqrt{\frac{6H}{g}} \theta_0 l$$

At the time of snapping, the bob was

$$l \sin \theta_0 \quad l\theta_0 \text{ distance from A.}$$

Thus, the distance from A is

$$l\theta_0 - \sqrt{\frac{6H}{g}} l\theta_0 = l\theta_0 (1 - \sqrt{6H/g})$$

OR

Periodic function: Any function that repeats itself at regular intervals of its argument is called a periodic function. Consider the function $f(\theta)$ satisfying the property,

$$f(\theta + T) = f(\theta)$$

This indicates that the value of the function f remains same when the argument is increased or decreased by an integral multiple of T for all values of θ . A function f satisfying this property is said to be periodic having a period T . For example, trigonometric functions like $\sin \theta$ and $\cos \theta$ are periodic with a period of 2π radians, because

$$(\theta + 2\pi) = \sin \theta$$

$$\cos (\theta + 2\pi) = \cos \theta$$

If the independent variable θ stands for some dimensional quantity such as time t , then we can construct periodic functions with period T as follows:

$$f_1(t) = \sin \frac{2\pi t}{T} \text{ and } g_1(t) = \cos \frac{2\pi t}{T}$$

We can check the periodicity by replacing t by $t + T$. Thus

$$\begin{aligned} f_1(t + T) &= \sin \frac{2\pi}{T} (t + T) = \sin \left(\frac{2\pi t}{T} + 2\pi \right) \\ &= \sin \frac{2\pi t}{T} = f_1(t) \end{aligned}$$

Similarly, $g_1(t + T) = g_1(t)$

It can be easily seen that functions with period T/n , where $n = 1, 2, 3, \dots$ also repeat their values after a time T . Hence it is possible to construct two infinite sets of periodic functions such as

$$f_n(t) = \sin \frac{2\pi nt}{T} \quad n = 1, 2, 3, 4, \dots$$

$$g_n(t) = \cos \frac{2\pi nt}{T} \quad n = 0, 1, 2, 3, 4, \dots$$

In the set of cosine functions, we have included the constant function $g_0(t) = 1$.

The constant function 1 is periodic for any value of T and hence does not alter the periodicity of $g_n(t)$.

Fourier theorem: This theorem states that any arbitrary function $F(t)$ with period T can be expressed as the unique combination of sine and cosine functions $f_n(t)$ and $g_n(t)$ with suitable coefficients. Mathematically, it can be expressed as

$$F(t) = b_0 + b_1 \cos \frac{2\pi t}{T} + b_2 \cos \frac{4\pi t}{T} + b_3 \cos \frac{6\pi t}{T} + \dots + a_1 \sin \frac{2\pi t}{T} + a_2 \sin \frac{4\pi t}{T} + a_3 \sin \frac{6\pi t}{T} + \dots$$

$$= b_0 + b_1 \cos \omega t + b_2 \cos^2 \omega t + b_3 \cos^3 \omega t + \dots + a_1 \sin \omega t + a_2 \sin^2 \omega t + a_3 \sin^3 \omega t + \dots$$

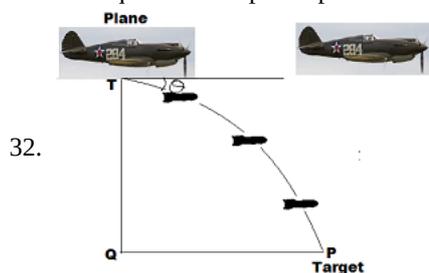
$$\text{or } F(t) = b_0 + \sum_n b_n \cos n\omega t + \sum_n a_n \sin n\omega t$$

$$\text{where } \omega = \frac{2\pi}{T}$$

The coefficients $b_0, b_1, b_2, \dots, a_1, a_2, a_3, \dots$ are called Fourier coefficients. These coefficients can be determined uniquely by a mathematical method called Fourier analysis. Suppose all the Fourier coefficients except a_1 and b_1 are zero, then

$$F(t) = a_1 \sin \frac{2\pi t}{T} + b_1 \cos \frac{2\pi t}{T}$$

This equation is a special periodic motion called simple harmonic motion (S.H.M.).



Let the pilot drops the bomb in t second before the point Q , vertically up the target T .

The horizontal velocity of the bomb will be equal to the velocity of the fighter plane, but the vertical component of it is zero. So, in time t bomb must cover the vertical distance TQ as free fall with the initial velocity zero.

$$\text{Given that : } u = 0, \quad H = 1.5 \text{ km} = 1500 \text{ m}, \quad g = + 10 \text{ m/s}^2$$

By Using the equation, $H = ut + \frac{1}{2}gt^2$, we get

$$1500 = 0 + \frac{1}{2}10t^2$$

$$t = \sqrt{\frac{1500}{5}} = \sqrt{300} = 10\sqrt{3} \text{ s}$$

\therefore Distance covered by plane or bomb in this time t , is given by $PQ = ut$

$$PQ = 200 \times 10\sqrt{3} = 2000\sqrt{3} \text{ m}$$

$$\tan \theta = \frac{TQ}{PQ} = \frac{1500}{2000\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{20 \times 3} = \frac{\sqrt{3}}{4}$$

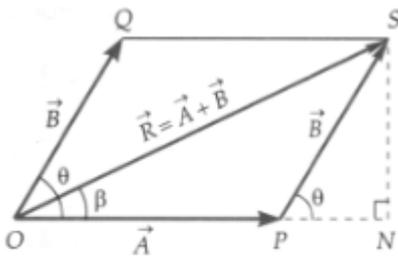
$$\tan \theta = \frac{1.732}{4} = 0.433 = \tan^{-1} 23^\circ 42'$$

$$\Rightarrow \theta = 23^\circ 42'$$

Thus the bomb should be thrown at an angle $23^\circ 42'$

OR

Let the two \vec{A} and \vec{B} inclined to each other at an angle θ be represented both in magnitude and direction by the adjacent sides \vec{OP} and \vec{OQ} of the parallelogram $OPSQ$. Then according to the parallelogram law of vector addition, the resultant of \vec{A} and \vec{B} is represented both in magnitude and direction by the diagonal \vec{OS} of the parallelogram.



Analytical treatment of parallelogram law.

Magnitude of resultant \vec{R} . Draw SN perpendicular to OP produced.

Then $\angle SPN = \angle QOP = \theta$, $OP = A$, $PS = OQ = B$, $OS = R$

From right angled $\triangle SNP$, we have

$$\frac{SN}{PS} = \sin \theta \text{ or } SN = PS \sin \theta = B \sin \theta$$

$$\text{and } \frac{PN}{PS} = \cos \theta \text{ or } PN = PS \cos \theta = B \cos \theta$$

Using Pythagoras theorem in right angled $\triangle ONQ$,

We get

$$OQ^2 = ON^2 + QN^2$$

$$= (OP + PN)^2 + QN^2$$

$$\text{or } R^2 = (A + B \cos \theta)^2 + (B \sin \theta)^2$$

$$= A^2 + B^2 \cos^2 \theta + 2 AB \cos \theta + B^2 \sin^2 \theta$$

$$= A^2 + B^2(\cos^2 \theta + \sin^2 \theta) + 2 AB \cos \theta$$

$$= A^2 + B^2 + 2 AB \cos \theta$$

$$\text{or } R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

33. Weight of car = 1800 Kg

Distance of COG from front axle = 1.05 m

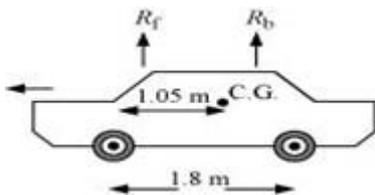
Distance of COG from back axle = 1.8 - 1.05 = 0.75 m

Vertical forces are balanced ,

So, At translational equilibrium:

$$R_1 + R_2 = mg$$

$$R_1 + R_2 = 1800 \times 9.8 = 17640$$



R_1 and R_2 are the forces exerted by the level ground on the front and back wheels respectively.

Angular momentum about centre of gravity is zero.

So,

$$R_1(1.05) = R_2(1.8 - 1.05)$$

$$R_1 \times 1.05 = R_2 \times 0.75$$

$$\frac{R_1}{R_2} = \frac{0.75}{1.05} = \frac{5}{7}$$

$$\frac{R_1}{R_2} = \frac{7}{5}$$

$$R_1 = 1.4 R_2 \dots (ii)$$

Solving equations (i) and (ii), we get:

$$1.4 R_2 + R_2 = 17640$$

$$R_2 = \frac{17640}{2.4} = 7350 \text{ N}$$

$$\therefore R_1 = 17640 - 7350 = 10290 \text{ N}$$

Therefore, the force exerted on each front wheel = $\frac{R_1}{2} = \frac{7350}{2} = 3675 \text{ N}$, and

The force exerted on each back wheel = $\frac{R_2}{2} = \frac{10290}{2} = 5145 \text{ N}$

OR

THE LAW OF CONSERVATION OF ANGULAR MOMENTUM STATES THAT: "When the net external torque acting on a system about a given axis is zero, the total **angular momentum** of the system about that axis remains constant." Mathematically, If then $I\omega = \text{constant}$.

In this problem, as all the forces are conservative in nature and external torque on the system is zero so angular momentum of the system will remain conserved although energy of the system may not remain constant if external forces are acting on the system.

a. Moment of inertia of the man-platform system $I = 7.6 \text{ kg m}^2$

Moment of inertia when the man stretches his hands to a distance of 90 cm:

$$\begin{aligned} & 2 \times mr^2 \\ & = 2 \times 5 \times (0.9)^2 \\ & = 8.1 \text{ kg m}^2 \end{aligned}$$

Initial moment of inertia of the system, $I_i = 7.6 + 8.1 = 15.7 \text{ kgm}^2$

Angular speed, $\omega_1 = 300 \text{ rev /min}$

Angular momentum, $L_i = I_i \omega_i = 15.7 \times 30 \dots\dots(i)$

Moment of inertia when the man folds his hands to a distance of 20 cm:

$$\begin{aligned} & 2 \times m^2 \\ & = 2 \times 5(0.2)^2 = 0.4 \text{ kgm}^2 \end{aligned}$$

Final moment of inertia, $I_f = 7.6 + 0.4 = 8 \text{ kgm}^2$

Final angular speed = ω_f

Final angular momentum, $L_f = I_f \omega_f = 0.79 \omega_f \dots (ii)$

From the conservation of angular momentum, we have:

$$\begin{aligned} I_i \omega_i & = I_f \omega_f \\ \therefore \omega_f & = \frac{15.7 \times 30}{8} = 58.88 \text{ rev/min} \end{aligned}$$

b. Kinetic energy is not conserved in the given process. In fact, with the decrease in the moment of inertia, kinetic energy increases. The additional kinetic energy comes from the work done by the man to fold his hands toward himself. (muscular work done by the man will be converted into kinetic energy)